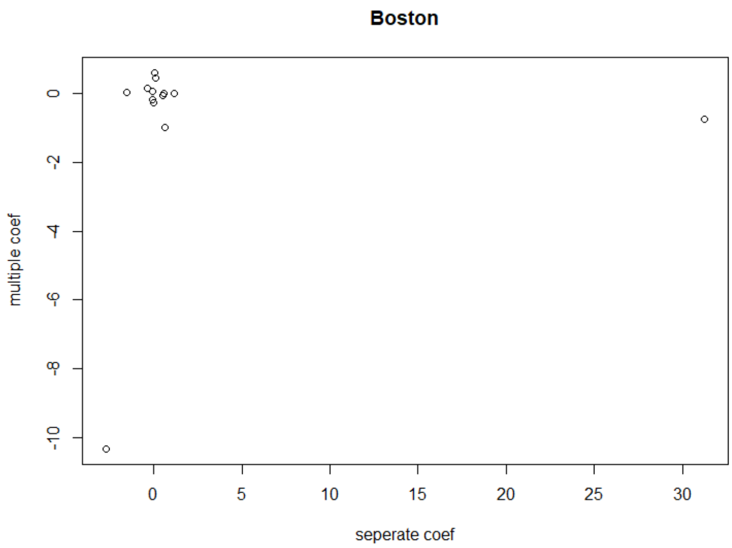
3.15 (a)

```
znm=lm(crim~zn)
indusm = lm(crim~indus)
noxm = lm(crim~nox)
rmm = lm(crim~rm)
agem = lm(crim~age)
dism = lm(crim~dis)
radm = lm(crim~rad)
taxm = lm(crim~tax)
ptratiom = lm(crim~ptratio)
blackm = lm(crim~black)
lstatm = lm(crim~lstat)
medvm = lm(crim~medv)
```

(c)



(d)

```
> bla=lm(crim~black+I(black^2)+I(black^3))
> summary(bla)

Call:
```

```
lm(formula = crim ~ black + I(black^2) + I(black^3))

Residuals:
    Min      1Q  Median      3Q     Max
-13.096  -2.343  -2.128  -1.439  86.790

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.826e+01  2.305e+00   7.924  1.5e-14 ***
black       -8.356e-02  5.633e-02  -1.483    0.139
I(black^2)   2.137e-04  2.984e-04   0.716    0.474
I(black^3)  -2.652e-07  4.364e-07  -0.608    0.544
---
Signif. codes:  0  ‘***’  0.001  ‘**’  0.01  ‘*’  0.05  ‘.
’  0.1  ‘ ’  1

Residual standard error: 7.955 on 502 degrees of freedom
Multiple R-squared:  0.1498,Adjusted R-squared:  0.1448
F-statistic: 29.49 on 3 and 502 DF,  p-value: < 2.2e-16
```

Through coding, we discover that there exists evidence of the non-linear association between all of the predictors and the response except for black.

4.1

$$P(X) = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}$$

$$\frac{P(X)}{1-P(X)} = \frac{\frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}}{\frac{1}{1+e^{\beta_0+\beta_1 X}}} = e^{\beta_0+\beta_1 X}$$

4.2

$$P_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\{-\frac{(x-\mu_k)^2}{2\sigma^2}\}}{\sum_{i=1}^{K} \pi_i \frac{1}{\sqrt{2\pi}\sigma} \exp\{-\frac{(x-\mu_i)^2}{2\sigma^2}\}}$$

$$\delta_k(x) = x\frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k)$$

Let $\delta_k(x) > \delta_i(x), \forall i \neq k$

Since exponential function is monotone increasing：

$$\exp(\delta_k(x)) > \exp(\delta_i(x)) \Rightarrow \pi_k \exp(\frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2}) > \pi_i \exp(\frac{\mu_i}{\sigma^2} - \frac{\mu_i^2}{2\sigma^2})$$

thus we prove that maximizing $\delta_k(x)$ is equivalent to maximizing $p_k(x)$.

4.3

$$P_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma_k} \exp\{-\frac{(x-\mu_k)^2}{2\sigma_k^2}\}}{\sum_{i=1}^K \pi_i \frac{1}{\sqrt{2\pi}\sigma_i} \exp\{-\frac{(x-\mu_i)^2}{2\sigma_i^2}\}}$$

Since we assume that the σ^2 of each class is not same, so we can't remove the σ^2.

$$\begin{aligned}\delta_k(x) &= \log(P_k(x)) + \log(\sum_{i=1}^K \pi_i \frac{1}{\sqrt{2\pi}\sigma_i} \exp\{-\frac{(x-\mu_i)^2}{2\sigma_i^2}\}) \\ &= \log(\pi_k) - \log(\sqrt{2\pi}\sigma_k) - \frac{(x-\mu_k)^2}{2\sigma_k^2}\end{aligned}$$

It is not linear, and we can see that it is quadratic.