

A **step** is any unit of work with bounded execution time (it doesn't keep getting slower with growing input size).

We classify algorithm complexity by classifying the **order of growth** of a function $f(N)$, where f gives the number of steps the algorithm must perform for a given input size.

Big O definition: if $f(N) \leq C * g(N)$ for large N values and some fixed constant C , then $f(N) \in O(g(N))$

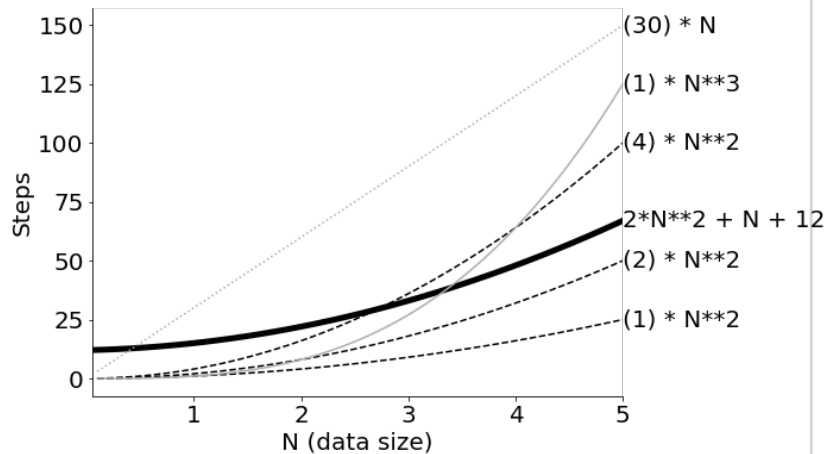
Let $f(N) = 2N^2 + N + 12$

If we want to show $f(N) \in O(N^3)$, what is a good lower bound on N ? Let's have $C=1$.

If we want to show $f(N) \in O(N^2)$, do you pick 1, 2, or 4 for the C ? After you pick C , what you choose for a lower bound on N ?

What is more informative to show?
 $f(N) \in O(N^3)$ or $f(N) \in O(N^2)$?

Somebody claims $f(N) \in O(N)$, offering $C=30$ and $N>0$. Suggest an N value to counter their claim.



Each of the following list operations are either $O(1)$ or $O(N)$, where N is list length. Circle those you think are $O(N)$.

`L.insert(0, x)` `L.pop(0)` `x = L[0]` `x = max(L)` `x = len(L)`
`L.append(x)` `L.append(x)` `L2.extend(L)` `x = sum(L)` `found = x in L`

```
def search(L, target):  
    for x in L:  
        if x == target: #line A  
            return True  
    return False
```

*assume this is asked
unless otherwise stated*

Let $f(N)$ be the number of times line A executes, with $N=\text{len}(L)$. What is $f(N)$ in each case?

Worst Case (target is at end of list): $f(N) = \underline{\hspace{2cm}}$

Best Case (target is at beginning of list): $f(N) = \underline{\hspace{2cm}}$

Average Case (target in middle of list): $f(N) = \underline{\hspace{2cm}}$

```
# assume L is already sorted, N=len(L)  
def search(L, target):  
    left_idx = 0 # inclusive  
    right_idx = len(L) # exclusive  
    while right_idx - left_idx > 1:  
        mid_idx = (right_idx + left_idx) // 2  
        mid = L[mid_idx]  
        if target >= mid:  
            left_idx = mid_idx  
        else:  
            right_idx = mid_idx  
  
    return right_idx > left_idx and L[left_idx] == target
```

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```
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        mid_idx = (right_idx + left_idx) // 2
        mid = L[mid_idx]
        if target >= mid:
            left_idx = mid_idx
        else:
            right_idx = mid_idx

    return right_idx > left_idx and L[left_idx] == target
```

how many times does this step run
when $N = 1$? $N = 2$? $N = 4$? $N = 8$?

If $f(N)$ is the number of times this
steps runs, then $f(N) = \underline{\hspace{2cm}}$

The complexity of binary search is
 $O(\underline{\hspace{2cm}})$