Lecture 5 Worksheet: Complexity Analysis

A **step** is any unit of work with bounded execution time (it doesn't keep getting slower with growing input size).

We classify algorithm complexity by classifying the **order of growth** of a function f(N), where f gives the number of steps the algorithm must perform for a given input size.

Big O definition: if **f(N) ≤ C * g(N)** for large **N** values and some fixed constant **C**, then **f(N) ∈ O(g(N))**

1

Let f(N) = $2N^2 + N + 12$

If we want to show f(N) ∈ O(N^3), what is a good lower bound on N? Let's have C=1.

N>3

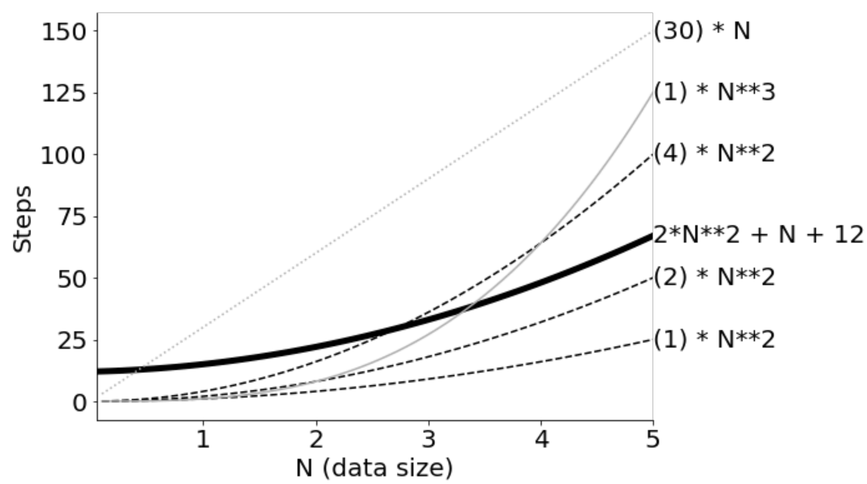
If we want to show f(N) ∈ O(N^2), do we pick 1, 2, or 4 for the C? After picking C, should we choose for N's lower bound?

C=4, N>3

What is more informative to show? f(N) ∈ O(N^3) or f(N) ∈ O(N^2)?

O(N^2)

Somebody claims f(N) ∈ O(N), offering C=30 and N>0. Suggest an N value to counter their claim. 1000



2

Each of the following list operations are either O(1) or O(N), where N is len(L). Circle those you think are O(N).

`L.insert(0, x)` `L.pop(0)` `x = L[0]` `x = max(L)` `x = len(L)`

`L.append(x)` `L.pop(-1)` `L2.extend(L)` `x = sum(L)` `found = X in L`

3

```
def search(L, target):
    for x in L:
        if x == target: #line A
            return True
    return False
```

assume this is asked unless otherwise stated

Let **f(N)** be the number of times line A executes, with `N=len(L)`. What is **f(N)** in each case?

Worst Case (target is at end of list): f(N) = N

Best Case (target is at beginning of list): f(N) = 1

Average Case (target in middle of list): f(N) = ~N/2

4

```
def selection_sort(L):
    for i in range(len(L)):
        idx_min = i
        for j in range(i, len(L)):
            if L[j] < L[idx_min]:
                idx_min = j
        L[idx_min], L[i] = L[i], L[idx_min] # swap values at i and idx_min

nums = [2, 4, 3, 1]
selection_sort(nums)
print(nums)
```

if this runs **f(N)** times, where **N=len(L)**,

then f(N) = 1+2+...+N = N*(N+1)/2

The complexity of selection sort is

O(N^2)

5

```python
# assume L is already sorted, N=len(L)
def binary_search(L, target):
    left_idx = 0 # inclusive
    right_idx = len(L) # exclusive
    while right_idx - left_idx > 1:
        mid_idx = (right_idx + left_idx) // 2
        mid = L[mid_idx]
        if target >= mid:
            left_idx = mid_idx
        else:
            right_idx = mid_idx

    return right_idx > left_idx and L[left_idx] == target
```

how many times does this step run when N = 1? N = 2? N = 4? N = 8?
0 1 2 3

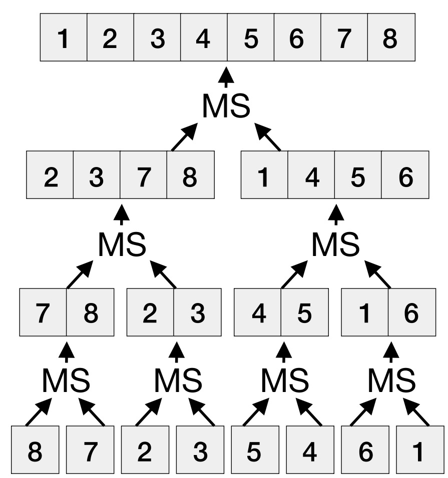
If **f(N)** is the number of times this step runs, then f(N) = $\log_2(N)$

The complexity of binary search is O(log N)

6

```python
def merge(L1, L2):
  rv = []
  idx1 = 0
  idx2 = 0

  while True:
    done1 = idx1 == len(L1)
    done2 = idx2 == len(L2)

    if done1 and done2:
      return rv

    choose1 = False
    if done2:
      choose1 = True
    elif not done1 and L1[idx1] < L2[idx2]:
      choose1 = True

    if choose1:
      rv.append(L1[idx1])
      idx1 += 1
    else:
      rv.append(L2[idx2])
      idx2 += 1

  return rv
```

```python
def merge_sort(L):
  if len(L) < 2:
    return L
  mid = len(L) // 2
  left = L[:mid]
  right = L[mid:]
  left = merge_sort(left)
  right = merge_sort(right)
  return merge(left, right)

merge_sort([4, 1, 2, 3])
```



`merge([1, 3], [2, 4])` will return [1, 2, 3, 4].

`merge(L1, L2)` implements an O(N) algorithm. But how can we measure the size of the input? N = len(L1) + len(L2)

If we double the list size, there will be ____ more level(s). Level count grows O(log N). Work per level is O(N). merge_sort complexity: O(N log N)

7

```python
nums = [...]

first100sum = 0

for x in nums[:100]:
    first100sum += x
print(x)
```

If we increase the size of nums from 20 items to 100 items, the code will probably take 5 times longer to run.

If we increase the size of nums from 100 to 1000, will the code take longer? Yes / No

The complexity of the code is O(1), with N=len(nums).