[301] Randomness

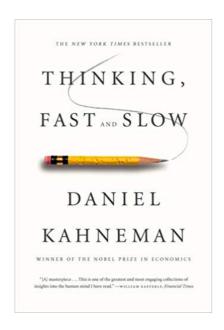
Tyler Caraza-Harter

Which series was randomly generated? Which did I pick by hand?

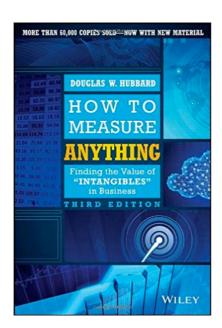




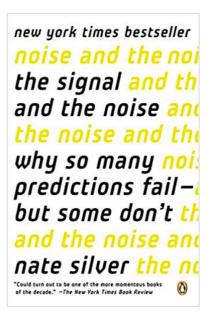
Announcement 1: Recommended popular stats books (for winter-break reading)



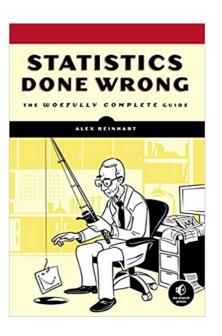
Thinking, Fast and Slow by Daniel Kahneman



How to Measure Anything by Douglas W. Hubbard



The Signal and the Noise by Nate Silver



Statistics Done Wrong by Alex Reinhart

Announcement 2: Course Evaluations

Section 1:

https://aefis.wisc.edu/index.cfm/page/AefisCourseSection.surveyResults?courseSectionid=580893

Section 2:

https://aefis.wisc.edu/index.cfm/page/AefisCourseSection.surveyResults?courseSectionid=580894

Evaluations are important generally, but especially this semester

- my first time teaching CS 301
- we made major changes to CS 301 this semester
- I promise to read every evaluation after the semester ends

Announcement 3: Final Exam Prep

Details: similar to midterms

- worth 20%
- 2 hours on Dec 19th at 7:45am (in the morning!)
- you can have a hand-written notesheet
- we'll use any extra time this Wed to review
- cumulative, across whole semester
- topics NOT included on the exam: beautifulsoup, regression, simulation

Recommended prep

- make sure you understand all the study sheet problems
- review the readings, especially anything I took the time to write myself
- review everything you got wrong on the midterms
- review the slides
- review the code you wrote for the projects

Comments on old finals

- we'll post them, because people ask for them
- content has evolved a lot in the last 3rd of CS 301, so they're not great review material

Games





Games





Security



Games

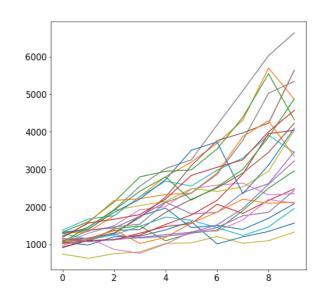




Security



Simulation



Games

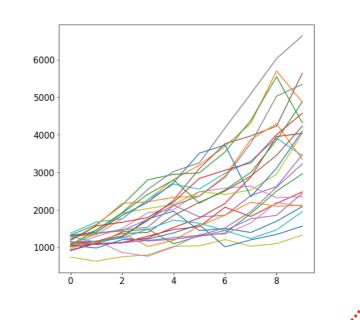




Security



Simulation



our focus

Outline

choice()

pseudorandom: debugging/seeding

visualization: bar plots vs. histograms

normal()

statistical significance: an intuitive approach

Previous (from random module that comes w/ Python):

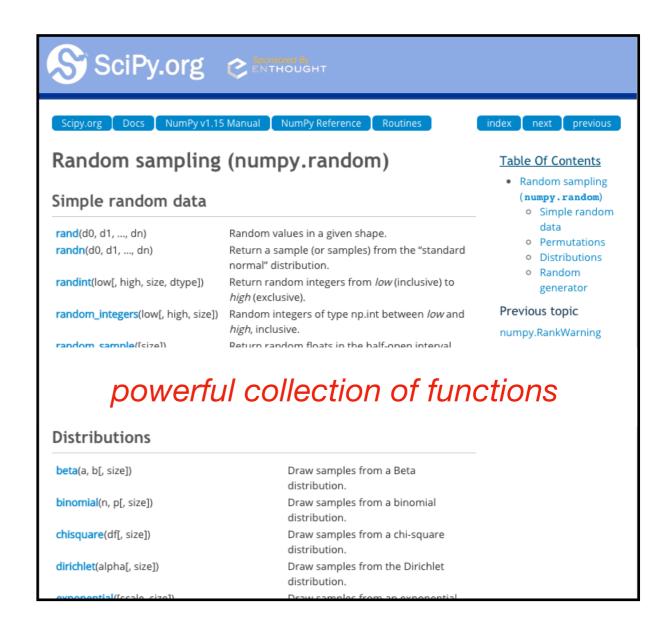
• choice, choices, randint

Previous (from random module that comes w/ Python):

choice, choices, randint

numpy.random:

- powerful collection of functions
- today: choice, normal



Previous (from random module that comes w/ Python):

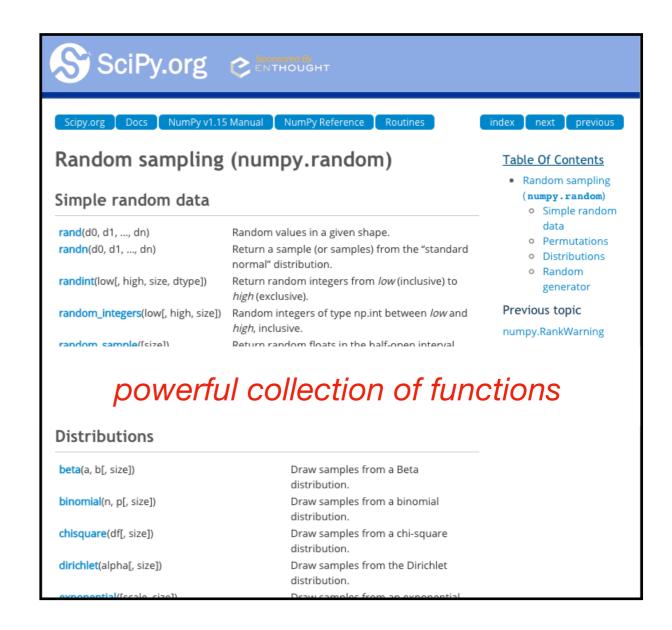
• choice, choices, randint

numpy.random:

- powerful collection of functions
- today: choice, normal

Series.line.hist:

- similar to bar plot
- visualize spread of random results



Previous (from random module that comes w/ Python):

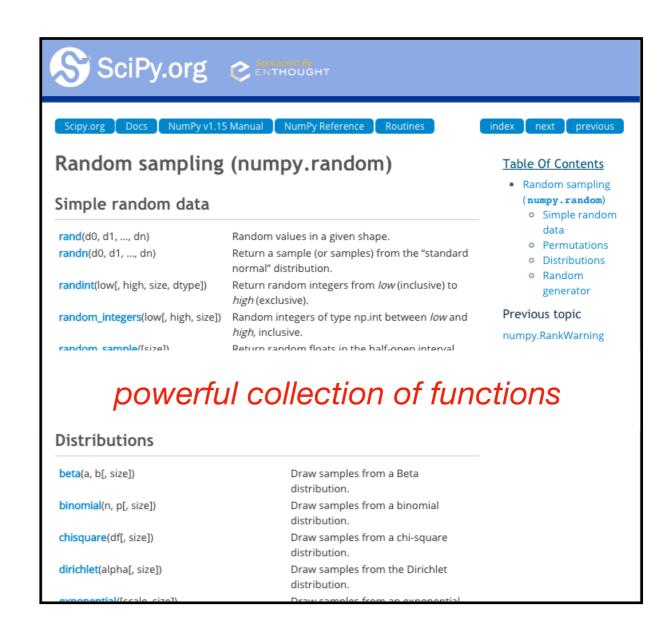
• choice, choices, randint

numpy.random:

- powerful collection of functions
- today: choice normal

Series.line.hist:

- similar to bar plot
- visualize spread of random results



from numpy.random import choice, normal

```
from numpy.random import choice, normal

result = choice(["rock", "paper", "scissors"])

list of things to randomly choose from
```

```
from numpy.random import choice, normal

result = choice(["rock", "paper", "scissors"])
print(result)
```

Output:

```
from numpy.random import choice, normal
result = choice(["rock", "paper", "scissors"])
print(result)
result = choice(["rock", "paper", "scissors"])
print(result)
                                 Output:
                                 scissors
                                 rock
```

```
from numpy.random import choice, normal
result = choice(["rock", "paper", "scissors"])
print(result)
result = choice(["rock", "paper", "scissors"])
print(result)
                                     Output:
                                     scissors
                                     rock
               each time choice is
            called, a value is randomly
           selected (will vary run to run)
```

```
from numpy.random import choice, normal
choice(["rock", "paper", "scissors"])
```

for simulation, we'll often want to compute many random results

```
from numpy.random import choice, normal
choice(["rock", "paper", "scissors"], size=5)
```

for simulation, we'll often want to compute many random results

```
from numpy.random import choice, normal
choice(["rock", "paper", "scissors"], size=5)
array(['rock', 'scissors', 'paper', 'rock', 'paper'], dtype='<U8')</pre>
```

```
from numpy.random import choice, normal
choice(["rock", "paper", "scissors"], size=5)
array(['rock', 'scissors', 'paper', 'rock', 'paper'], dtype='<U8')</pre>
```

1-dimensional ndarray with 5 items

```
from numpy.random import choice, normal

choice(["rock", "paper", "scissors"], size=5)

array(['rock', 'scissors', 'paper', 'rock', 'paper'], dtype='<U8')

1-dimensional ndarray with 5 items

choice(["rock", "paper", "scissors"], size=(3,2))</pre>
```

```
from numpy.random import choice, normal
choice(["rock", "paper", "scissors"], size=5)
array(['rock', 'scissors', 'paper', 'rock', 'paper'], dtype='<U8')
  1-dimensional ndarray with 5 items
choice(["rock", "paper", "scissors"], size=(3,2))
                              numpy shape tuple
```

```
from numpy.random import choice, normal
choice(["rock", "paper", "scissors"], size=5)
array(['rock', 'scissors', 'paper', 'rock', 'paper'], dtype='<U8')
   1-dimensional ndarray with 5 items
choice(["rock", "paper", "scissors"], size=(3,2))
array([['rock', 'scissors'],
      ['paper', 'rock'],
      ['scissors', 'paper']], dtype='<U8')
```

```
from numpy.random import choice, normal
choice(["rock", "paper", "scissors"], size=5)
array(['rock', 'scissors', 'paper', 'rock', 'paper'], dtype='<U8')
   1-dimensional ndarray with 5 items
choice(["rock", "paper", "scissors"], size=(3,2))
array([['rock', 'scissors'],
      ['paper', 'rock'],
      ['scissors', 'paper']], dtype='<U8')
 ???-dimensional ndarray with ??? items
```

```
from numpy.random import choice, normal
choice(["rock", "paper", "scissors"], size=5)
array(['rock', 'scissors', 'paper', 'rock', 'paper'], dtype='<U8')
   1-dimensional ndarray with 5 items
choice(["rock", "paper", "scissors"], size=(3,2))
array([['rock', 'scissors'],
      ['paper', 'rock'],
      ['scissors', 'paper']], dtype='<U8')
   2-dimensional ndarray with 6 items
```

```
from numpy.random import choice, normal
# random Series
choice(["rock", "paper", "scissors"], size=5)
```

```
from numpy.random import choice, normal
# random Series
Series(choice(["rock", "paper", "scissors"], size=5))
```

```
from numpy.random import choice, normal
# random Series
Series(choice(["rock", "paper", "scissors"], size=5))
          paper
       scissors
         paper
           rock
           rock
  dtype: object
# random DataFrame
DataFrame(choice(["rock", "paper", "scissors"], size=(5,3)))
 0 scissors scissors scissors
 1 scissors scissors
               rock
     rock scissors
               rock
 3 scissors scissors
               rock
          rock
    paper
               rock
```

Demo 1: exploring bias

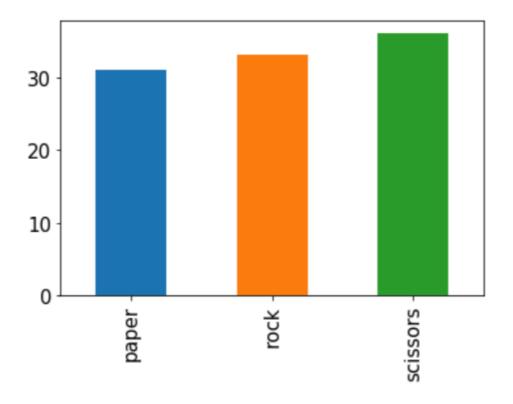
```
choice(["rock", "paper", "scissors"])
```

Question 1: how can we make sure the randomization isn't biased?

Demo 1: exploring bias

```
choice(["rock", "paper", "scissors"])
```

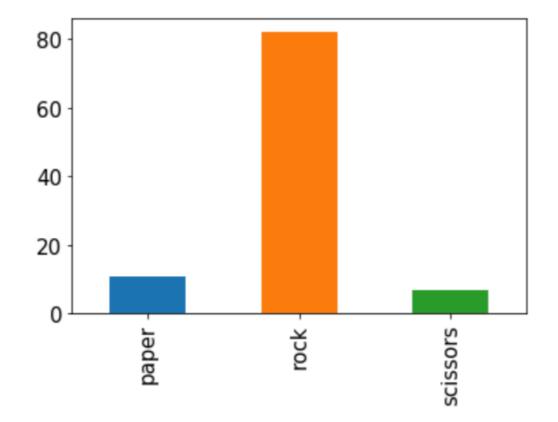
Question 1: how can we make sure the randomization isn't biased?



Demo 1: exploring bias

Question 1: how can we make sure the randomization isn't biased?

Question 2: how can we make it biased (if we want it to be)?



```
from numpy.random import choice, normal
# random string: rock, paper, or scissors
choice(["rock", "paper", "scissors"])
```

```
from numpy.random import choice, normal

# random string: rock, paper, or scissors
choice(["rock", "paper", "scissors"])

# random int: 0, 1, or 2
choice([0, 1, 2])
```

```
from numpy.random import choice, normal
# random string: rock, paper, or scissors
choice(["rock", "paper", "scissors"])
# random int: 0, 1, or 2
choice([0, 1, 2])
        same
# random int (approach 2): 0, 1, or 2
choice(3)
```

```
from numpy.random import choice, normal
# random string: rock, paper, or scissors
choice(["rock", "paper", "scissors"])
# random int: 0, 1, or 2
choice([0, 1, 2])
         same
# random int (approach 2): 0, 1, or 2
choice(3)
               random non-negative int
                 that is less than 3
```

Outline

choice()

pseudorandom: debugging/seeding

visualization: bar plots vs. histograms

normal()

statistical significance: an intuitive approach

```
s = Series(choice(10, size=5))

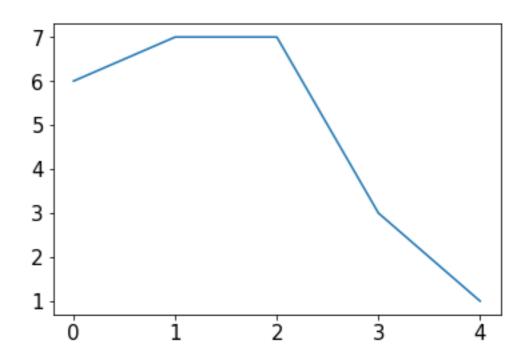
0    6
1    7
2    7
3    3
```

dtype: int64

```
s = Series(choice(10, size=5))
```

```
0 6
1 7
2 7
3 3
4 1
dtype: int64
```

```
s.plot.line()
```



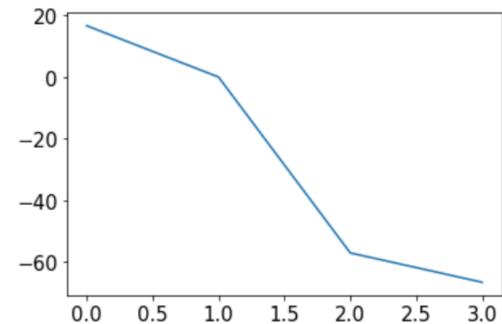
```
percents = []
for i in range(1, len(s)):
    diff = 100 * (s[i] / s[i-1] - 1)
    percents.append(diff)
```

what are we computing for diff?

```
s = Series(choice(10, size=5))
                                        6
                                        5
                                        4
                                        3
                                        2
dtype: int64
s.plot.line()
                                             20
percents = []
                                              0
for i in range(1, len(s)):
    diff = 100 * (s[i] / s[i-1] - 1)
                                             -20
    percents.append(diff)
Series(percents).plot.line()
                                             -40
                                            -60
                                                    0.5
                                                                    2.5
                                                        1.0
                                                           1.5
                                                                2.0
                                                0.0
                                                                        3.0
```

```
percents = []
for i in range(1, len(s)):
    diff = 100 * (s[i] / s[i-1] - 1)
    percents.append(diff)
Series(percents).plot.line()

    can you identify the bug in the code?
```



```
s = Series(choice(10, size=5))
                                           6
                                           5
                                           3
                                           2
dtype: int64
s.plot.line()
        /anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:14:
        RuntimeWarning: divide by zero encountered in long scalars
percents = []
                                                 0
for i in range(1, len(s)):
     diff = 100 * (s[i] / s[i-1] - 1)
                                                -20
     percents.append(diff)
Series(percents).plot.line()
                                                -40
                                                -60
    can you identify the bug in the code?
                                                       0.5
                                                                1.5
                                                                    2.0
                                                   0.0
                                                            1.0
                                                                             3.0
```

some bugs are easier to debug than others

- syntax or runtime errors easier than semantic bugs
- small inputs are easier than big inputs

a bug is reproducible if it shows up every time you run the program with the same inputs

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who had a non-reproducible bug for a project this semester?

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who had a non-reproducible bug for a project this semester?

non-reproducible bugs

- are hard to fix
- common with programs based on randomness

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who had a non-reproducible bug for a project this semester?

non-reproducible bugs

- are hard to fix
- common with programs based on randomness

fortunately, the random values we've been generating are not really, truly random. They're merely *pseudorandom*.

```
684, 559, 629, 192, 835, ...

37, 235, 908, 72, 767, ...

168, 527, 493, 584, 534, ...

874, 664, 249, 643, 952, ...

122, 174, 439, 709, 897, ...

867, 206, 701, 998, 118, ...

906, 713, 227, 980, 618, ...

... billions more ...
```

- can generate billions of different seemingly random sequences
- subsequent calls to choice progress along these sequences
- every program run starts with a different sequence
- we can choose our sequence

```
684, 559, 629, 192, 835, ...
37, 235, 908, 72, 767, ...
168, 527, 493, 584, 534, ...
874, 664, 249, 643, 952, ...
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168, 527, 493, 584, 534, ...
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122, 174, 439, 709, 897, ...
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```
684, 559, 629, 192, 835, ...

37, 235, 908, 72, 767, ...

168, 527, 493, 584, 534, ...

874, 664, 249, 643, 952, ... restart!

122, 174, 439, 709, 897, ...

867, 206, 701, 998, 118, ...

906, 713, 227, 980, 618, ...

billions more ...
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874, 664, 249, 643, 952, ...

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867, 206, 701, 998, 118, ...

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684, 559, 629, 192, 835, ...

37, 235, 908, 72, 767, ...

168, 527, 493, 584, 534, ...

874, 664, 249, 643, 952, ...

122, 174, 439, 709, 897, ...

867, 206, 701, 998, 118, ...

906, 713, 227, 980, 618, ...

billions more ...
```

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```
684, 559, 629, 192, 835, ...

37, 235, 908, 72, 767, ...

168, 527, 493, 584, 534, ...

874, 664, 249, 643, 952, ...

122, 174, 439, 709, 897, ...

867, 206, 701, 998, 118, ...

906, 713, 227, 980, 618, ...

billions more ...
```

- can generate **billions** of different seemingly random sequences
- subsequent calls to choice progress along these sequences
- every program run starts with a different sequence
- we can choose our sequence

```
0: 684, 559, 629, 192, 835, ...1: 37, 235, 908, 72, 767, ...
2: 168, 527, 493, 584, 534, ...

3: 874, 664, 249, 643, 952, ...

4: 122, 174, 439, 709, 897, ...

5: 867, 206, 701, 998, 118, ...
 6: 906, 713, 227, 980, 618, ...
                    ... billions more ...
```

seed

- can generate billions of different seemingly random sequences
- subsequent calls to choice progress along these sequences
- every program run starts with a different sequence
- we can choose our sequence

```
from numpy.random import choice, normal import numpy as np
```

```
np.random.seed(1)
choice(10, size=5)
array([5, 8, 9, 5, 0])
```

from numpy.random import choice, normal import numpy as np

```
np.random.seed(1)
choice(10, size=5)

np.random.seed(2)
choice(10, size=5)

array([8, 8, 6, 2, 8])
```

from numpy.random import choice, normal import numpy as np

```
np.random.seed(1)
choice(10, size=5)

np.random.seed(2)
choice(10, size=5)

np.random.seed(1)
choice(10, size=5)

array([5, 8, 9, 5, 0])

array([5, 8, 9, 5, 0])
```

from numpy.random import choice, normal import numpy as np

```
np.random.seed(1)
choice(10, size=5)

np.random.seed(2)
choice(10, size=5)

np.random.seed(1)
choice(10, size=5)

array([5, 8, 9, 5, 0])

array([5, 8, 9, 5, 0])

array([5, 8, 9, 5, 0])
```

from numpy.random import choice, normal import numpy as np

```
np.random.seed(1)
choice(10, size=5)

np.random.seed(2)
choice(10, size=5)

np.random.seed(1)
choice(10, size=5)

array([5, 8, 9, 5, 0])

array([5, 8, 9, 5, 0])

array([5, 8, 9, 5, 0])
```

Debug tip: if you have a bug related to randomness, find a seed that causes the bug to arise, then use that seed until you find the problem. (don't forget to remove it when you're done!)

Outline

choice()

pseudorandom: debugging/seeding

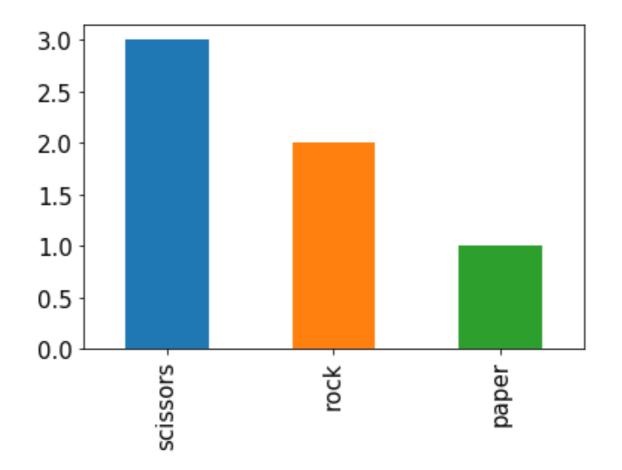
visualization: bar plots vs. histograms

normal()

statistical significance: an intuitive approach

Frequencies across categories

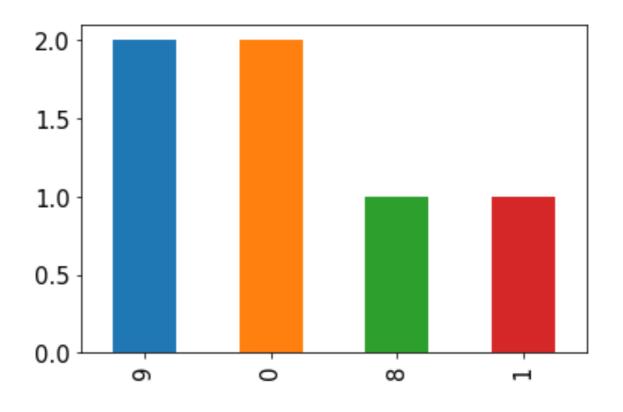
bars are a good way to view frequencies across categories



Frequencies across numbers

bars are a bad way to view frequencies across numbers

```
s = Series([0, 0, 1, 8, 9, 9])
s.value_counts().plot.bar()
```

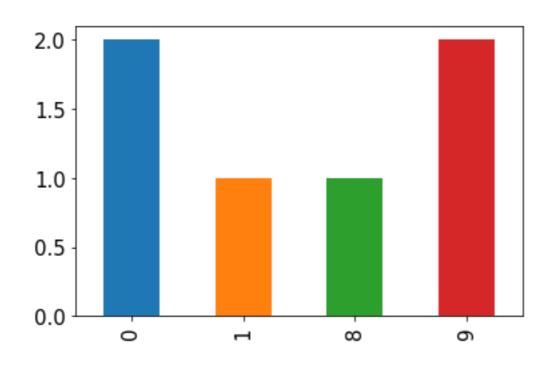


numbers not ordered

Frequencies across numbers

bars are a bad way to view frequencies across numbers

```
s = Series([0, 0, 1, 8, 9, 9])
s.value_counts().sort_index().plot.bar()
```

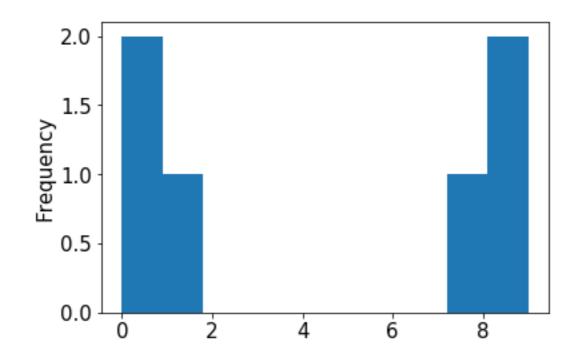


gap between 1 and 8 not obvious

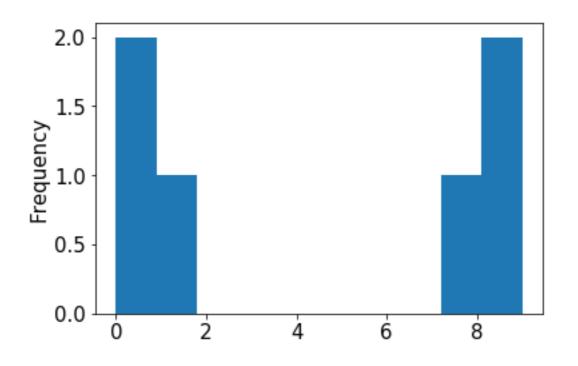
Frequencies across numbers

bars are a bad way to view frequencies across numbers

```
s = Series([0, 0, 1, 8, 9, 9])
s.value_counts().sort_index().plot.bar()
s.plot.hist()
```



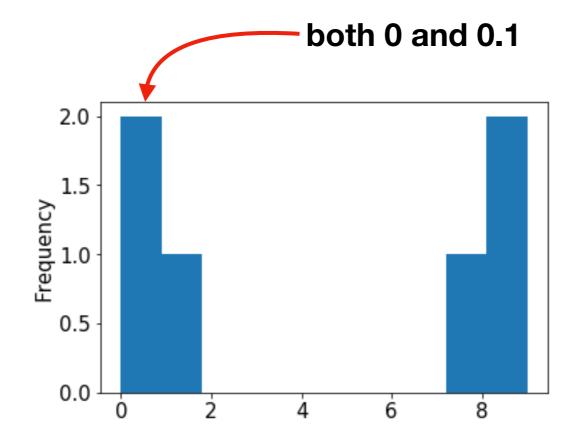
```
s = Series([0, 0, 1, 8, 9, 9])
s.value_counts().sort_index().plot.bar()
s.plot.hist()
```



this kind of plot is called a histogram

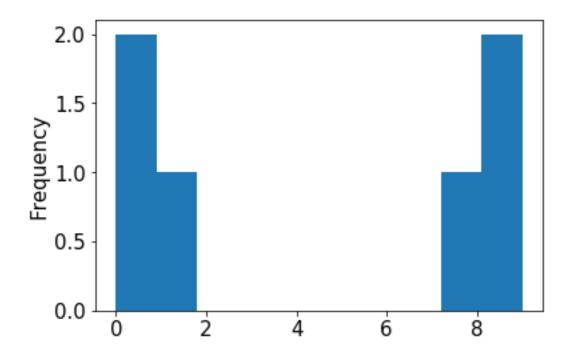
histograms are a good way to view frequencies across numbers

```
s = Series([0.1, 0, 1, 8, 9, 9.2])
s.value_counts().sort_index().plot.bar()
s.plot.hist()
```



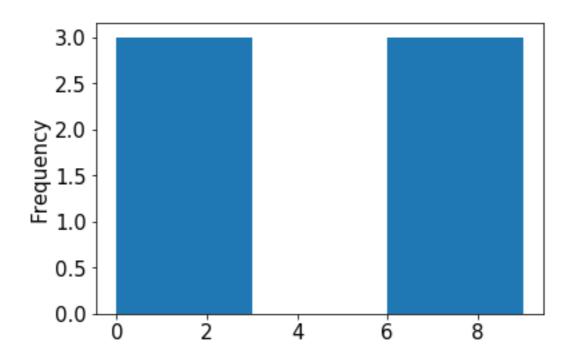
a histogram "bins" nearby numbers to create discrete bars

```
s = Series([0.1, 0, 1, 8, 9, 9.2])
s.value_counts().sort_index().plot.bar()
s.plot.hist(bins=10)
```



we can control the number of bins

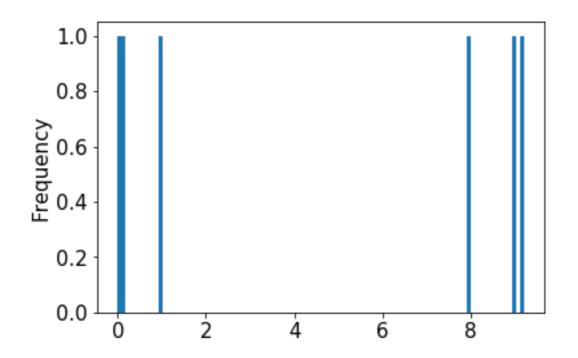
```
s = Series([0.1, 0, 1, 8, 9, 9.2])
s.value_counts().sort_index().plot.bar()
s.plot.hist(bins=3)
```



too few bins provides too little detail

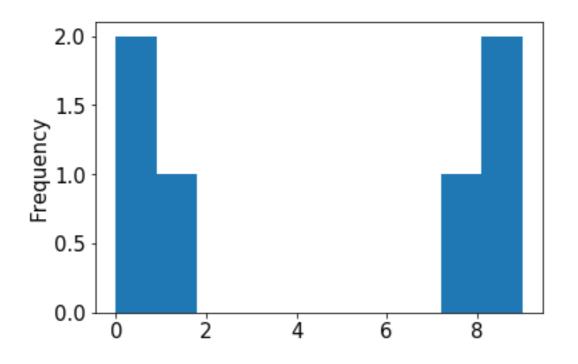
histograms are a good way to view frequencies across numbers

```
s = Series([0.1, 0, 1, 8, 9, 9.2])
s.value_counts().sort_index().plot.bar()
s.plot.hist(bins=100)
```



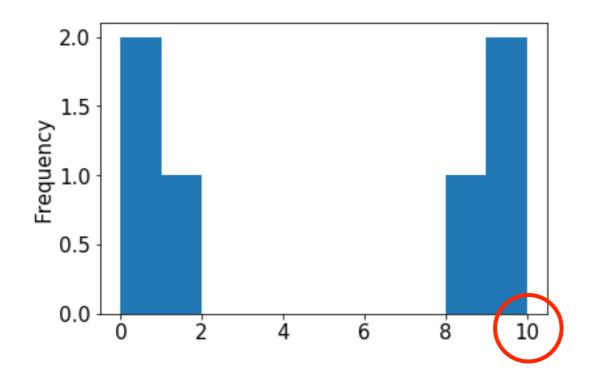
too many bins provides too much detail (equally bad)

```
s = Series([0.1, 0, 1, 8, 9, 9.2])
s.value_counts().sort_index().plot.bar()
s.plot.hist(bins=10)
```



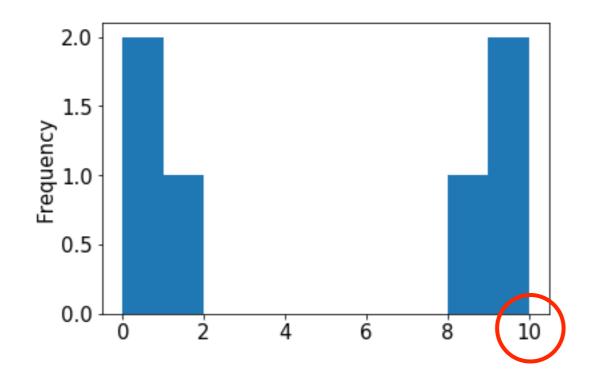
numpy chooses the default bin boundaries

```
s = Series([0.1, 0, 1, 8, 9, 9.2])
s.value_counts().sort_index().plot.bar()
s.plot.hist(bins=[0,1,2,3,4,5,6,7,8,9,10])
```



we can override the defaults

```
s = Series([0.1, 0, 1, 8, 9, 9.2])
s.value_counts().sort_index().plot.bar()
s.plot.hist(bins=range(11))
```



this is easily done with range

Demo 2: coin flips

If we flip 10 coins repeatedly, we'll get varying numbers of heads



Demo 2: coin flips

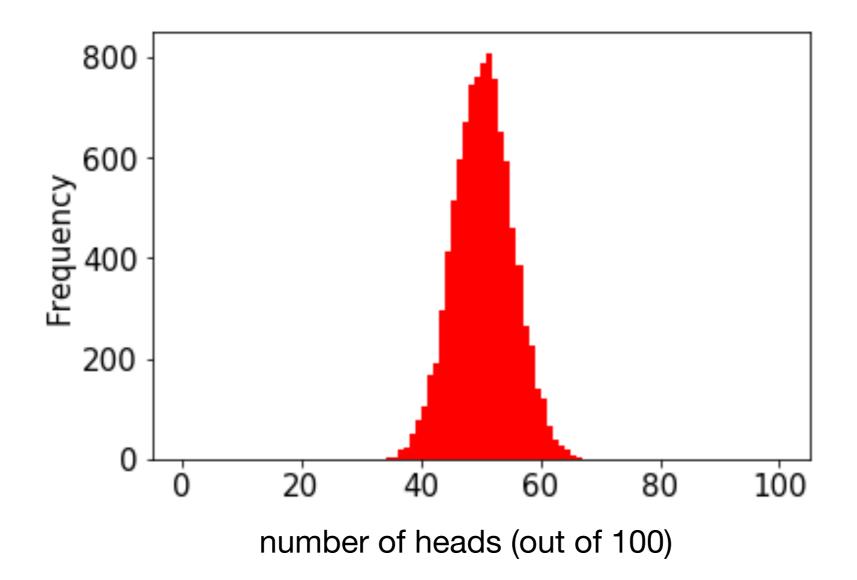
If we flip 10 coins repeatedly, we'll get varying numbers of heads

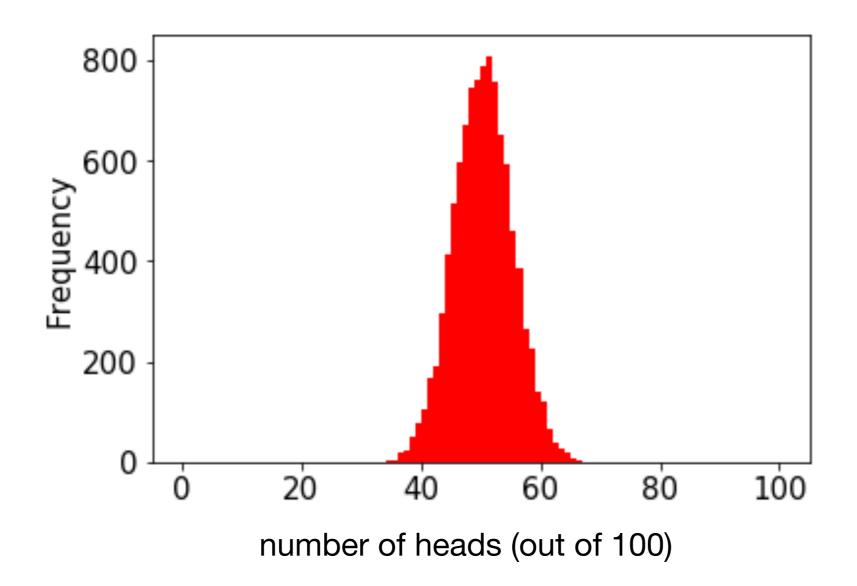


If we flip 100 coins, 10K times, how often do we get each head count?

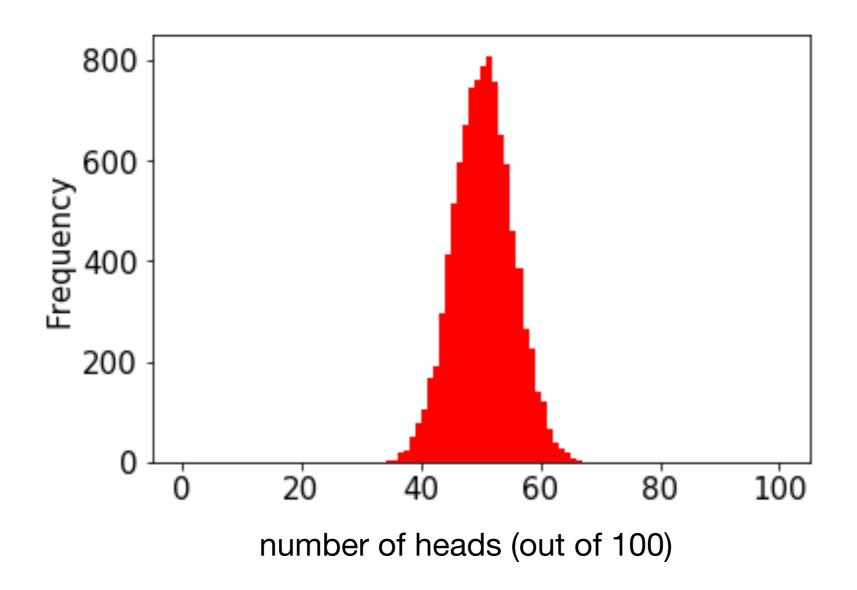
number of samples

sample size



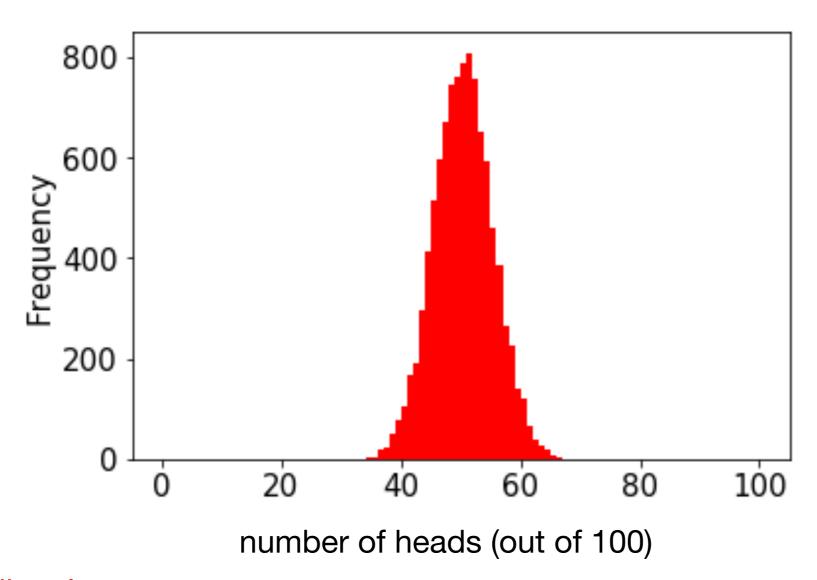


this shape resembles what we often call a normal distribution or a "bell curve"



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in general, if we take large samples enough times, the results will look like this (we won't discuss exceptions here)



numpy can directly generate random numbers fitting a normal distribution

this shape resembles what we often call a normal distribution or a "bell curve"

in general, if we take large samples enough times, the results will look like this (we won't discuss exceptions here)

Outline

choice()

pseudorandom: debugging/seeding

visualization: bar plots vs. histograms

normal()

statistical significance: an intuitive approach

```
from numpy.random import choice, normal
import numpy as np

for i in range(10):
    print(normal())
```

```
from numpy.random import choice, normal
import numpy as np
for i in range(10):
                                 Output:
    print(normal())
                                 -0.18638553993371157
                                 0.02888452916769247
                                 1.2474561113726423
                                 -0.5388224399358179
                                 -0.45143322136388525
                                 -1.4001861112018241
                                 0.28119371511868047
                                 0.2608861898556597
                                 -0.19246288728955144
                                 0.2979572961710292
```

```
from numpy.random import choice, normal
import numpy as np
for i in range(10):
                                   Output:
    print(normal())
                                    -0.18638553993371157
                                   0.02888452916769247
                                    1.2474561113726423
          average is 0 (over many calls)
                                    -0.5388224399358179
                                    -0.45143322136388525
         numbers closer to 0 more likely
                                    -1.4001861112018241
                                    0.28119371511868047
                   -x just as likely as x
                                    0.2608861898556597
                                    -0.19246288728955144
                                    0.2979572961710292
```

```
from numpy.random import choice, normal
import numpy as np

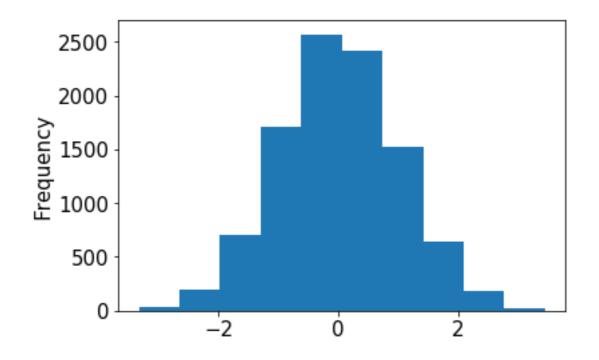
s = Series(normal(size=10000))
```

```
from numpy.random import choice, normal
import numpy as np

s = Series(normal(size=10000))
s.plot.hist()
```

```
from numpy.random import choice, normal
import numpy as np

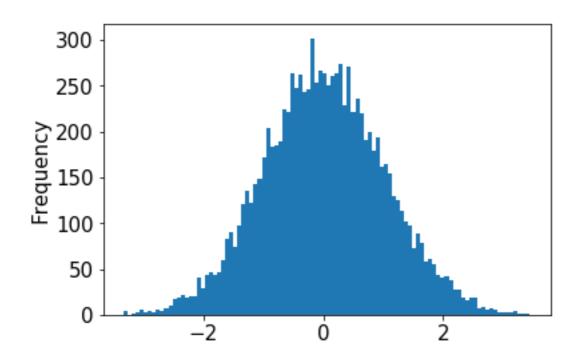
s = Series(normal(size=10000))
s.plot.hist()
```



```
from numpy.random import choice, normal
import numpy as np

s = Series(normal(size=10000))

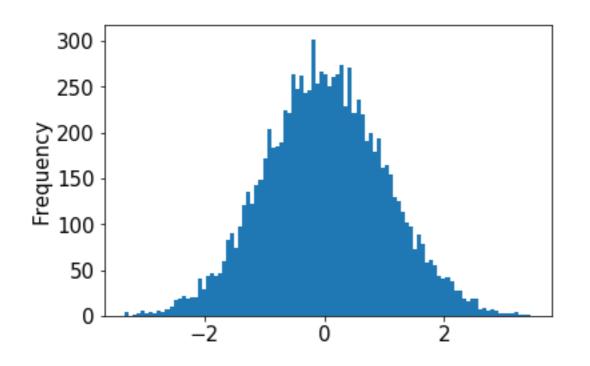
s.plot.hist(bins=100)
```



```
from numpy.random import choice, normal
import numpy as np

s = Series(normal(size=10000))

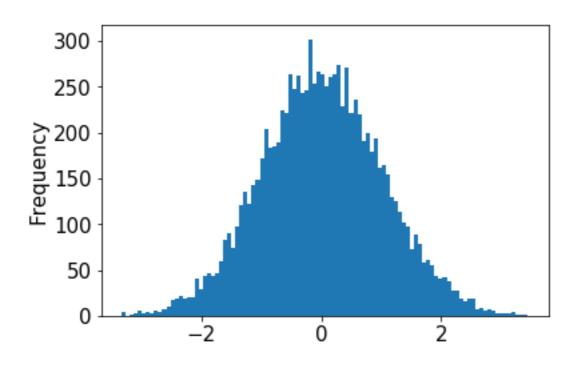
s.plot.hist(bins=100, loc=), scale=
```

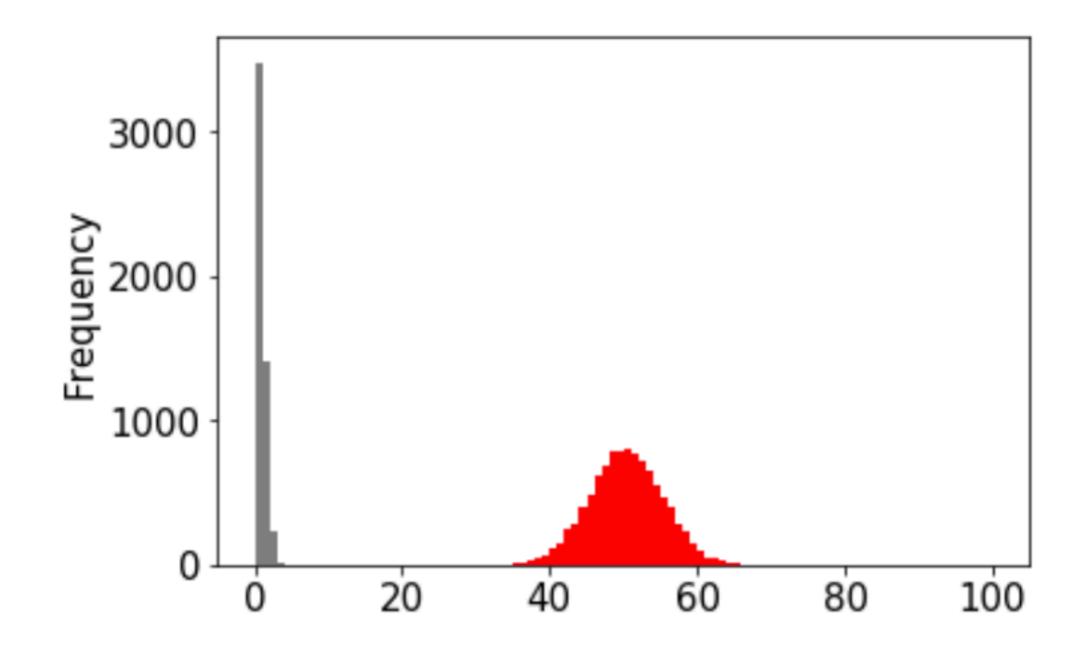


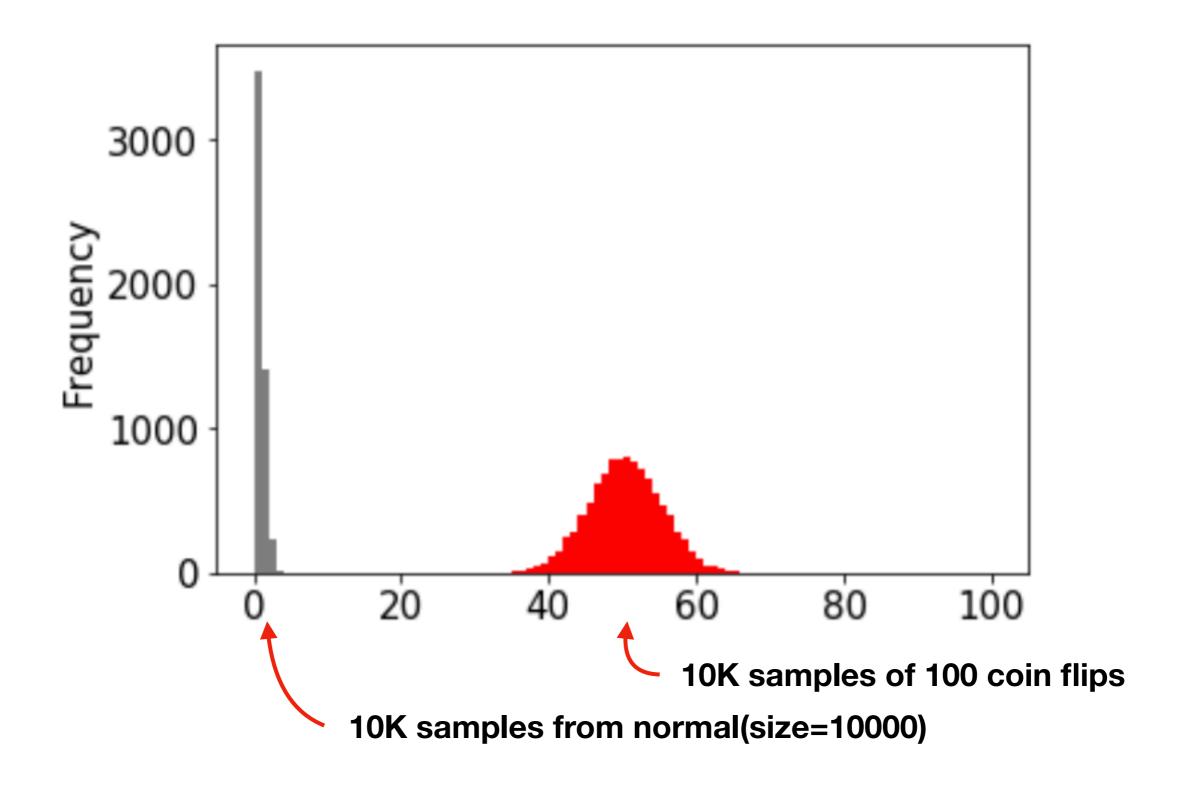
```
from numpy.random import choice, normal import numpy as np
```

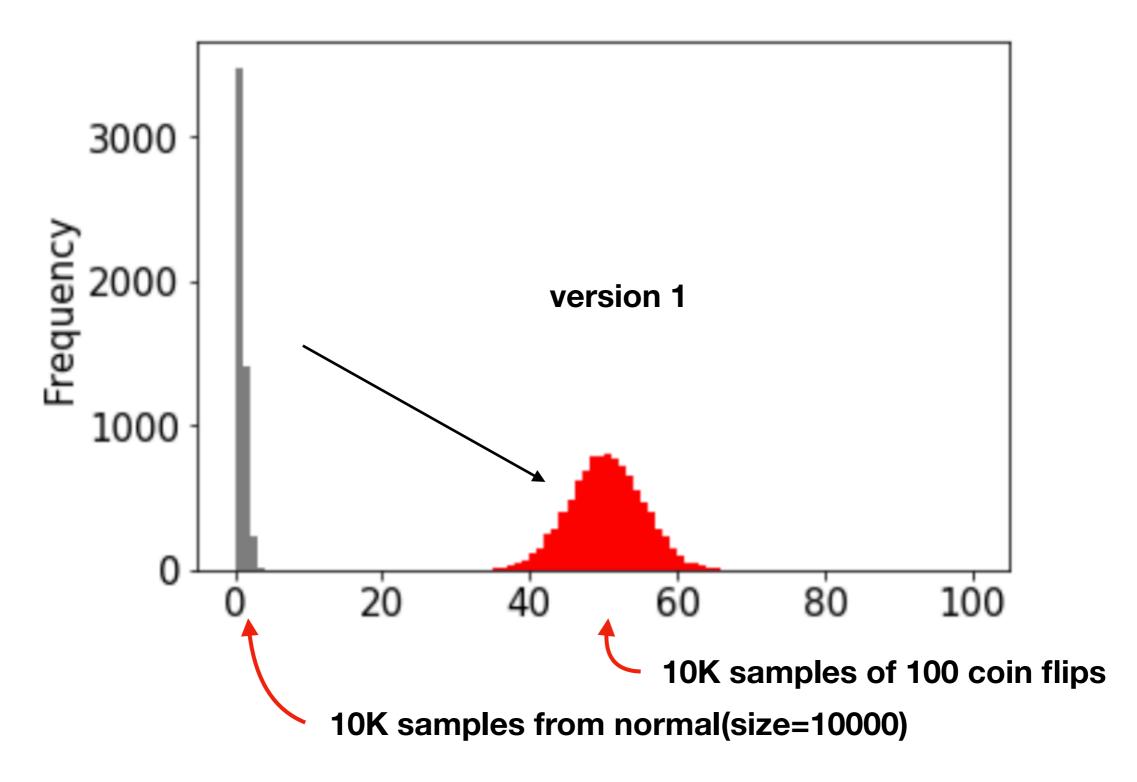
```
s = Series(normal(size=10000))
```

try plugging in different values (defaults are 0 and 1, respectively)

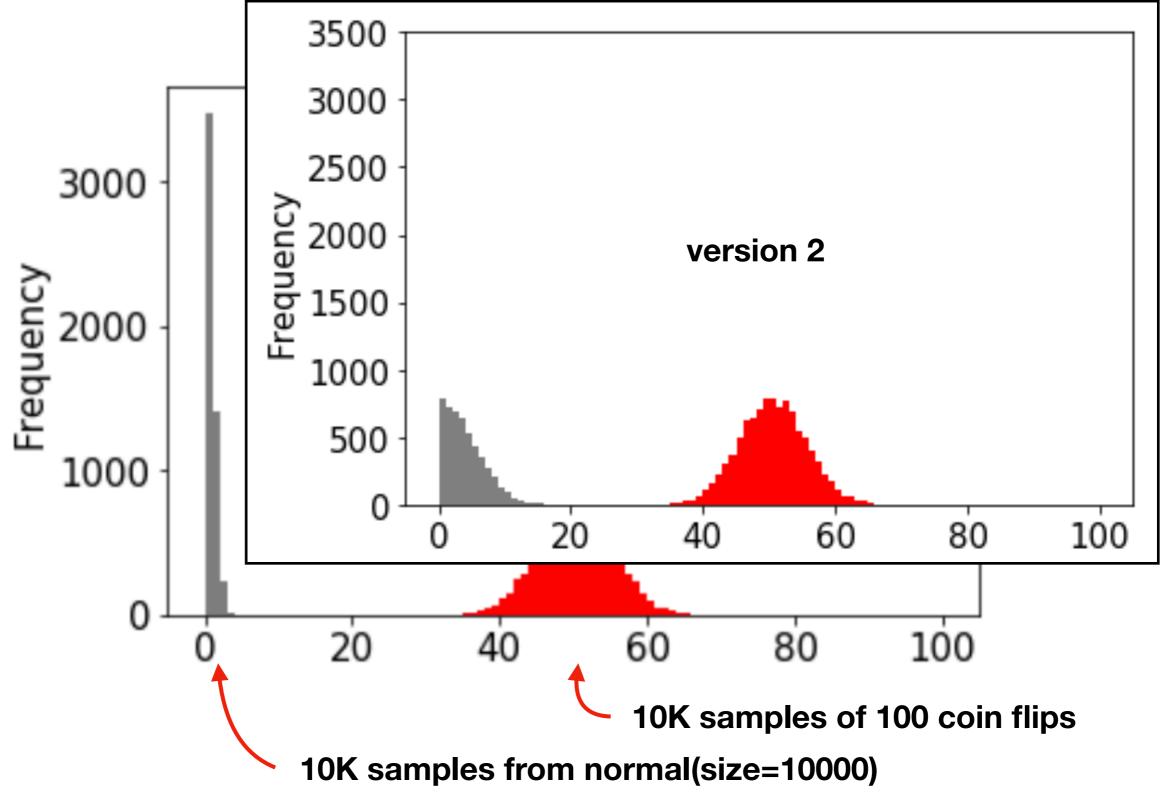




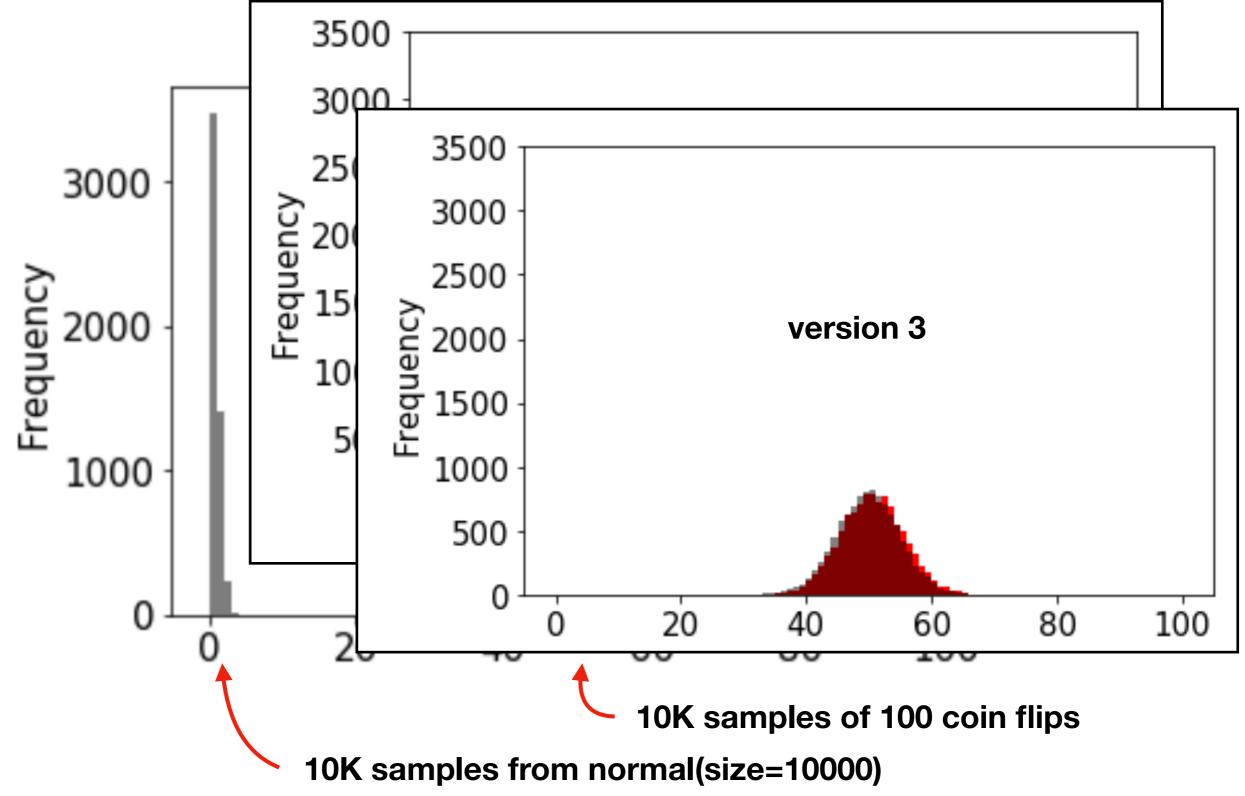




goal: play with loc and scale arguments to normal until gray overlaps red



goal: play with loc and scale arguments to normal until gray overlaps red



goal: play with loc and scale arguments to normal until gray overlaps red

Outline

choice()

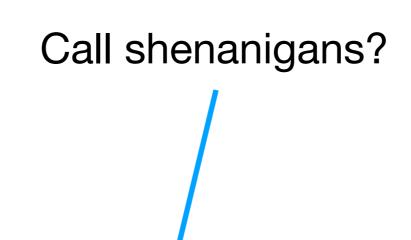
pseudorandom: debugging/seeding

visualization: bar plots vs. histograms

normal()

statistical significance: an intuitive approach









Call shenanigans?

a statistician might say we're trying to decide if the evidence that the coin isn't fair is statistically significant







Call shenanigans? No.

51 49



Call shenanigans? No.

Call shenanigans?



Call shenanigans? No.

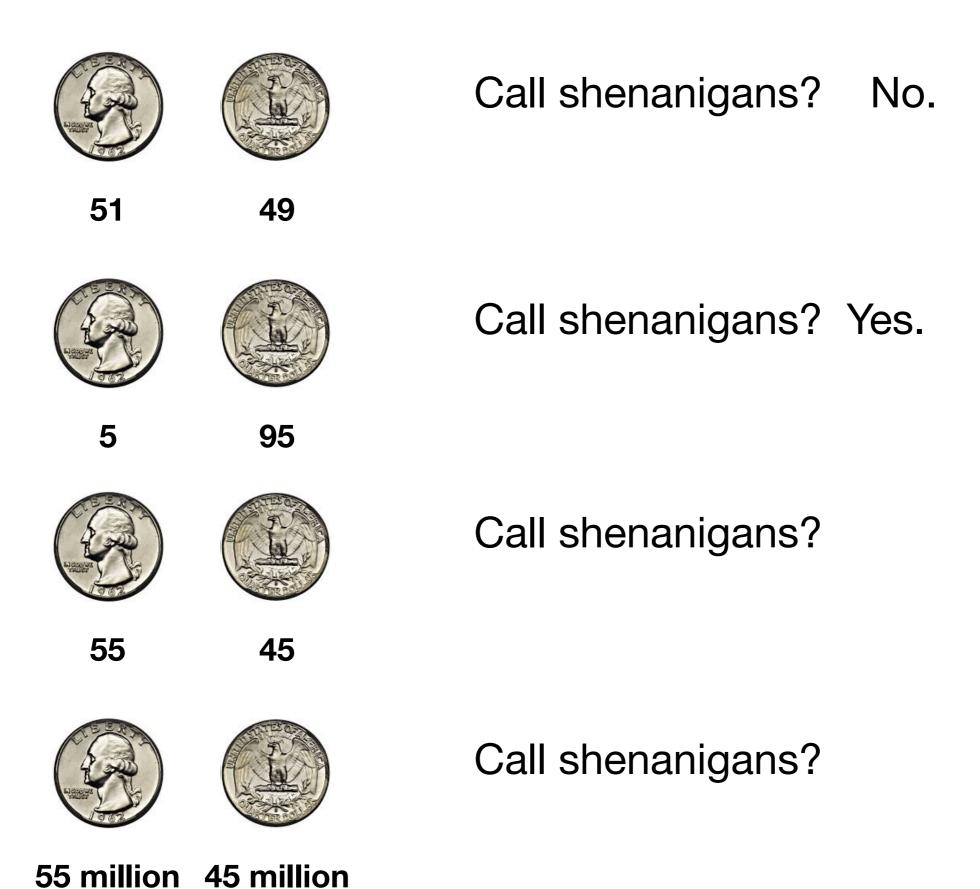
Call shenanigans? Yes.

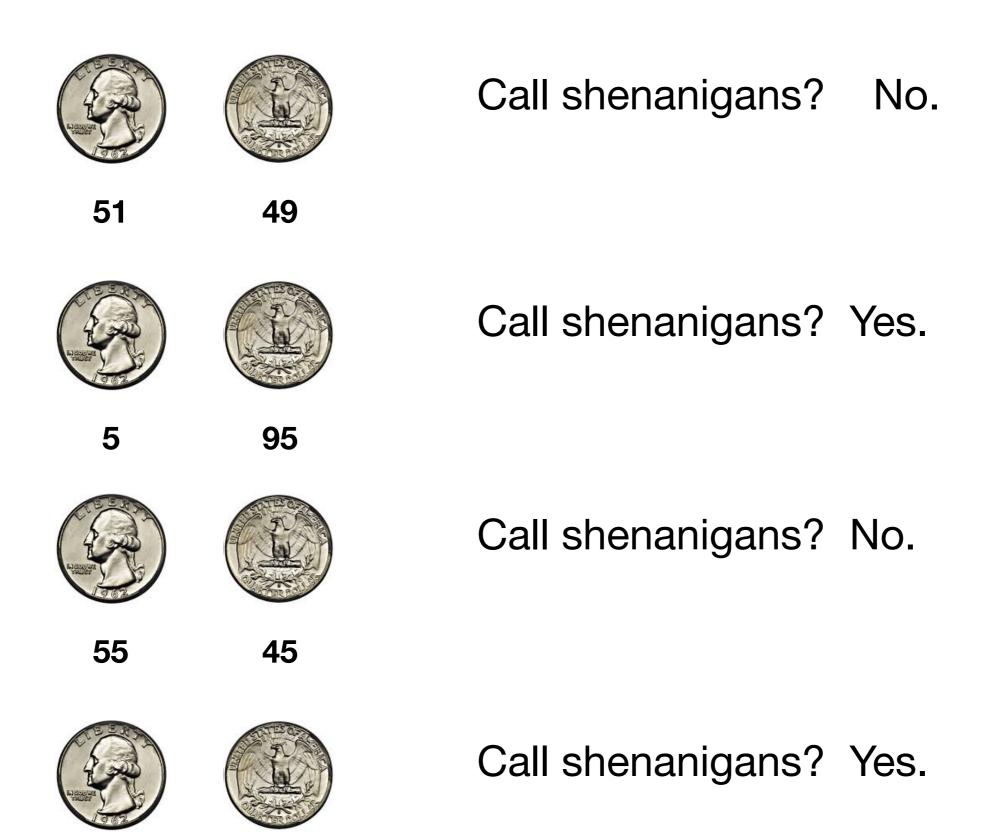


Call shenanigans? No.

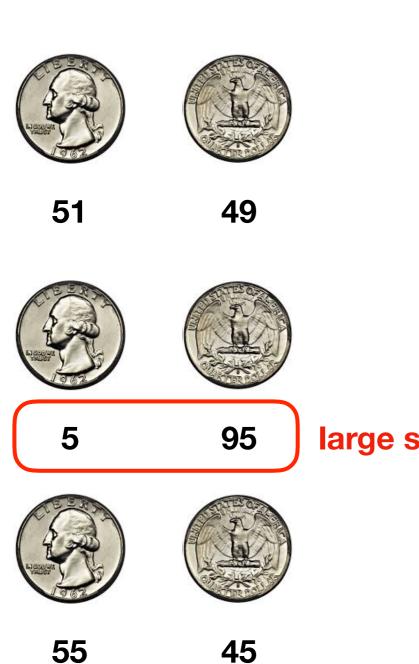
Call shenanigans? Yes.

Note: there is a non-zero probability that a fair coin will do this, but the odds are slim





55 million 45 million



Call shenanigans? No.

Call shenanigans? Yes.

large skew is good evidence of shenanigans

Call shenanigans? No.



Call shenanigans? Yes.

55 million 45 million

small skew over large samples is good evidence





40

Call shenanigans?



Call shenanigans?



Call shenanigans?

- 1. "flip" it 100 times using numpy.random.choice
- 2. count heads
- 3. repeat above 10K times



Call shenanigans?

- 1. "flip" it 100 times using numpy.random.choice
- 2. count heads
- 3. repeat above 10K times

```
[50, 61, 51, 44, 39, 43, 51, 49, 49, 38, ...]
```





Call shenanigans?

60 40

we got 10 more heads than we expect on average

Strategy: simulate a fair coin

- 1. "flip" it 100 times using numpy.random.choice
- 2. count heads
- 3. repeat above 10K times

[50, 61, 51, 44, 39, 43, 51, 49, 49, 38, ...]





Call shenanigans?

60

40

we got 10 more heads than we expect on average how common is this?

- 1. "flip" it 100 times using numpy.random.choice
- 2. count heads
- 3. repeat above 10K times

```
[50, 61, 51, 44, 39, 43, 51, 49, 49, 38, ...]
```





Call shenanigans?

60

40

we got 10 more heads than we expect on average how common is this?

- 1. "flip" it 100 times using numpy.random.choice
- 2. count heads
- 3. repeat above 10K times

Demo 5: Do front-row students score better?





	0	1	2	3	4	5	6	7	8	9
0	84	90	89	89	90	87	85	100	96	94
1	100	89	89	87	72	88	72	98	94	82
2	85	82	83	84	73	85	76	94	87	96
3	74	87	82	89	97	91	84	80	91	87
4	98	76	78	77	91	88	78	100	77	70
5	82	72	73	84	98	70	94	91	73	83
6	70	100	94	76	71	75	71	77	100	73
7	89	77	83	71	95	89	77	92	91	70
8	100	84	82	79	70	88	98	77	81	79
9	99	88	89	92	84	82	94	93	77	75

df.mean(axis=1)



0	90.4
1	87.1
2	84.5
3	86.2
4	83.3
5	82.0
6	80.7
7	83.4
8	83.8
9	87.3
dty	pe: float64

what are the odds that the front row would do so well by chance?