ME 531, Spring 2025 | Problem Set #3
Assigned April 28, 2025 | Due: by 11:59 pm, May 11

- Upload a scanned PDF to Gradescope
- Show all work and/or computer code used in your calculations

1. (10 points) Consider the cart-pole shown in Figure 1. The rod is massless and there is a small amount of linear rolling friction with damping coefficient b. Let M = 5 kg, m = 1 kg, L = 2 m, and b = 1 N-s/m. The cart starts with the initial conditions $[x, \dot{x}, \theta, \dot{\theta}] = [-1, 0,\ \pi + 0.1, 0]$. We want to drive the cart to the reference position $[x, \dot{x}, \theta, \dot{\theta}] = [0, 0,\ \pi, 0]$ with the pendulum stabilized in the upright configuration.

Assuming full-state feedback, design an LQR controller for the linearized system selecting the **Q** and **R** weights of your choice. Compare your LQR system response with 100 randomly generated sets of stable eigenvalues, chosen in the interval [-3.5, -0.5]. Your figure(s) should show the simulated response of the full *nonlinear system* for all controllers. Plot each set of state variables on a single plot (i.e. one plot for positions from all 101 controllers, one for velocities, etc.). Indicate the LQR controller response in each plot with a different color.

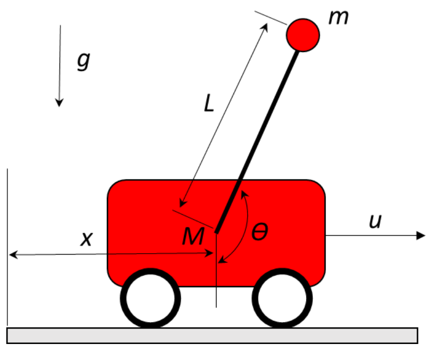

Figure 1. Cart-pole system.

2. (10 points) A high-performance helicopter has the model shown in Figure 2.

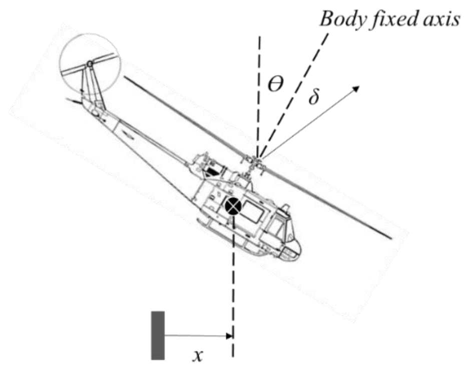

Figure 2. Pitch of a helicopter.

The goal is to control the pitch angle Θ of the aircraft by adjusting the rotor thrust angle δ. The equations of motion of the helicopter are

$$\frac{d^2\theta}{dt^2} = -\sigma_1 \frac{d\theta}{dt} - \alpha_1 \frac{dx}{dt} + n\delta$$

$$\frac{d^2x}{dt^2} = g\theta - \alpha_2\frac{d\theta}{dt} - \sigma_2\frac{dx}{dt} + g\delta$$

where *x* is translation in the horizontal direction. For a military high-performance helicopter, we find that

$\sigma_1 = 0.415$	$\alpha_2 = 1.43$
$\sigma_2 = 0.0198$	$n = 6.27$
$\alpha_1 = 0.0111$	$g = 9.8$

all in appropriate SI units.

(a) Find a state variable representation of this system where the state vector is $\mathbf{x} = [\theta\ \dot{\theta}\ \dot{x}]^T$.

(b) The aircraft is initially moving forward at 40 m/s with a pitch of 30 degrees ($\Theta = \pi/6$, $dx/dt = 40$) when we want to transition to a hover above the ground ($\Theta = 0$, $dx/dt = 0$). Design an LQR controller to achieve this behavior, selecting the weights of your choice. Provide a short written justification for your controller weights (~1-2 sentences) – remember, there are people on board the helicopter! Additionally, provide a well-documented Matlab/Python code that includes plots of the states and the control input.