ME 531, Spring 2025 **Problem Set #4**
Assigned May 12, 2025 **Due: by 11:59 pm, May 25**

- Upload a scanned PDF to Gradescope
- Show all work and/or computer code used in your calculations

1. (10 points) Consider the system represented in state variable form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathrm{u}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x}$$

where $\mathbf{A} = \begin{bmatrix} 1 & 4 \\ -5 & 10 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & -4 \end{bmatrix}$

Verify that the system is observable. Then, design a full-state observer by placing the observer poles at $s_{1,2} = -1$. Plot the response of the estimation error $\mathbf{e} = \mathbf{x} - \hat{\mathbf{x}}$ with an initial estimation error of $\mathbf{e}(0) = \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathrm{T}}$.

2. (15 points) Design a compensator (i.e. full state feedback law + observer) for a spring, mass, damper system where m = 5 kg, k = 2 N/m, and b = 2 N-s/m. We have a laser range finder that provides feedback about the position of the mass. The system is deterministic (i.e. no white process or measurement noise). The control input is a force applied to the mass. The system has initial conditions [1; 1] (i.e. at t = 0 the mass is located at 1 m and moving at 1 m/sec), and we want to drive it back to the origin [0; 0]. The full-state feedback controller should have 4% overshoot and a 2 sec settling time. Turn in a well-documented program with the following three figures:

- A figure that compares the system response using state feedback and output feedback (i.e. an observer with a state estimate) where the system's initial state estimate error is [0.5; -0.5].
- A figure that shows convergence of the state estimate error to zero.
- A figure that shows the actuation expenditure using the state estimate.