ME 531, Spring 2025
Assigned March 31, 2025

Problem Set #1
Due: by 11:59 pm, April 13

- Upload a scanned PDF to Gradescope
- Show all work and/or computer code used in your calculations

1. (10 points) Modern robotic manipulators that act directly upon their target environments must be controlled so that impact forces as well as steady-state forces do not damage the target. At the same time, the manipulator must provide sufficient force to perform the task. In order to develop a control system to regulate these forces, the robotic manipulator and target environment must be modeled. Assuming the model shown below in Fig. 1:

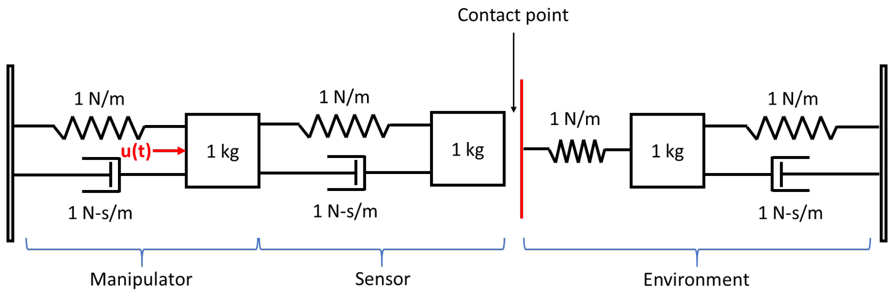


Figure 1. Sketch of a coupled manipulator, sensor, and environment.

a) **(5 points)** Represent in state space $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ the manipulator and its environment *under the condition that the manipulator is in constant contact with its target environment*.

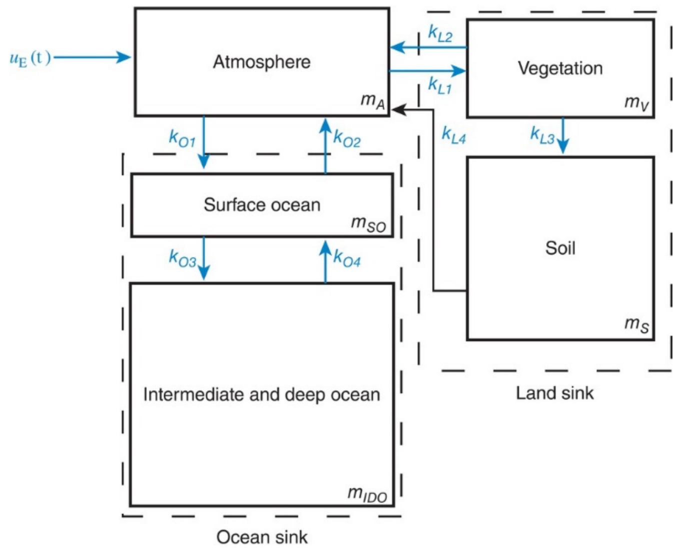
b) **(5 points)** Simulate the impulse response of the system (intuitively, we can think of this as a sudden 'kick' to the manipulator). An efficient method to simulate the impulse response is to use Matlab's `impulse` function with a state-space model `ss`. When creating a state-space model, you'll need to set the output matrix `C` and feedthrough matrix `D`. We haven't discussed output feedback yet – for now set `C` to a 6 x 6 identity matrix and `D` to a 6 x 1 column vector of zeros. Your final deliverable should be plots of the three position states over 20 seconds (you do not need to plot the velocity states).

2. (5 points) The figure below[1] shows a schematic description of the global carbon cycle (*Li et al.*). In the figure, $m_A(t)$ represents the amount of carbon in gigatons (GtC) present in the atmosphere of earth; $m_V(t)$ the amount in vegetation; $m_S(t)$ the amount in soil; $m_{SO}(t)$ the amount in surface ocean; and $m_{IDO}(t)$ the amount in intermediate and deep-ocean reservoirs. Let $u_E(t)$ stand for the human generated CO_2 emissions (GtC/yr). From the figure, the atmospheric mass balance in the atmosphere can be expressed as:

$$\frac{dm_A}{dt}(t) = u_E(t) - (k_{O1} + k_{L1})m_A(t) + k_{L2}m_V(t) + k_{O2}m_{SO}(t) + k_{L4}m_S(t)$$

where the k's are exchange coefficients (yr^{-1}).

Express the system in state-space form where each mass balance is a state variable.



[1]Li, S., Jarvis, A.J., and Leedal, D.T. Are response function representations of the global carbon cycle ever interpretable? *Tellus*, vol. 61B, 2009, pp. 361-371. (Fig. 1, p. 363).

3. (5 points) Tanks T_1 and T_2 initially contain 100 gal of water each. In T_1 the water is pure, whereas 150 lbs of fertilizer is dissolved in T_2. By circulating liquid at a rate of 2 gal/min and stirring (to keep the mixture uniform), the amounts of fertilizer $y_1(t)$ in T_1 and $y_2(t)$ in T_2 change with time t.

Plot the fertilizer content in each tank as a function of time. How long should we let the liquid circulate so that T_1 will contain at least half as much fertilizer as there will be left in T_2?