ME 531, Spring 2025 **Problem Set #2**
Assigned April 14, 2025 **Due: by 11:59 pm, April 27**

- **Upload a scanned PDF to Gradescope**
- **Show all handwritten work and code used in your calculations**

1. (6 points) Consider the simple heat exchanger shown in Figure 1, in which f_C and f_H are the flows (assumed constant) of cold and hot water, T_H and T_C represent the temperatures in the hot and cold compartments, respectively, T_{Hi} and T_{Ci} denote the temperature of the hot and cold inflow, respectively, and V_H and V_C are the volumes of hot and cold water. The temperatures in both compartments evolve according to:

$$V_C \frac{dT_C}{dt} = f_C(T_{Ci} - T_C) + \beta(T_H - T_C)$$

$$V_H \frac{dT_H}{dt} = f_H(T_{Hi} - T_H) - \beta(T_H - T_C)$$

Let the inputs to this system be $u_1 = T_{Ci}$, $u_2 = T_{Hi}$, the outputs are $y_1 = T_C$ and $y_2 = T_H$, and assume that $f_C = f_H = 0.1\ (m^3/min), \beta = 0.2\ (m^3/min)$ and $V_H = V_C = 1\ (m^3)$.

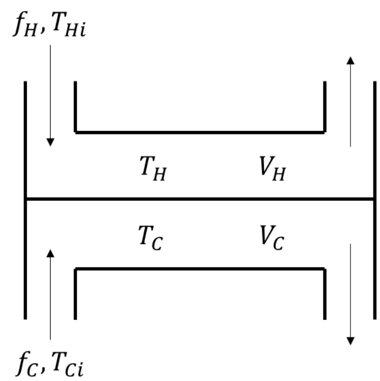

Figure 1. A simple heat exchanger.

(a) In the absence of any input, use the *eigendecomposition* to determine $y_1(t)$ and $y_2(t)$.

(b) Is the system stable? Why or why not?

2. (6 points) Consider the differential equation for the Van der Pol oscillator (use `ode45` or `scipy.integrate.solve_ivp`)

$$\ddot{y} - \mu(1 - y^2)\dot{y} + y = 0$$

which has a nonlinear damping term $-\mu(1 - y^2)\dot{y}$.

(a) Write this nonlinear ODE as a system of first order differential equations.

(b) For $\mu = 0.1$, numerically solve the equation for `t=0:100` sec for two sets of initial conditions: $y(0) = -3; \dot{y}(0) = 0$ and $y(0) = -1; \dot{y}(0) = 0$. Plot your solutions as trajectories over the vector field/phase portrait (y vs $\dot{y}$). Discretize the state space from `[-5:5; -5:5]`.

(c) Repeat part (b) for $\mu = 1$.

3. (4 points) Consider the nonlinear system

$$\dot{x} = 2x - 2x^2 - xy$$

$$\dot{y} = 2y - xy - 2y^2$$

(a) The system has a fixed point at (2/3, 2/3). Complete a Jacobian linearization at this fixed point.

(b) Is the linearized model stable or unstable at the fixed point? Why?

4. (4 points) Consider the second-order system

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}\mathbf{u}(t)$$

For what values of k_1 and k_2 is the system completely controllable?

5. (6 points) Consider the system

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\mathbf{u}(t)$$

(a) Design a state feedback law that places the eigenvalues at $\lambda = -5, -6$.

(b) Plot the response of the *uncontrolled* system for initial conditions $x_1(0) = 1$ and $x_2(0) = 1$.

(c) Plot the response of the *controlled* system for the same initial conditions as part (b).

(d) For the controlled system, plot the control input.