

Declarative Syntax Definition Grammars and Trees

Guido Wachsmuth

Assessment

last lecture

Identify three examples of polymorphism in the following Java expression: "1" + ((2 + 4) + 3.5)

- 1st and 2nd `+` symbolise different operations on different types
- 2nd and 3rd `+` symbolise same operation on different types
- `(2 + 4)` is of type `int`, but 3rd `+` requires a `float`
- `((2 + 4) + 3.5)` is of type `float`, but 1st `+` requires a `String`

What kinds of polymorphism do they represent?

- ad-hoc polymorphism, overloading
- ad-hoc polymorphism, type coercion, implicit conversion

What are the differences?

Assessment

last lecture

```
public class A {}

public class B extends A {}

public class C {
    public int m() { return 1 + 2; }
}

public class D {
    public String m() { return "1" + "2"; }
    public A m(A a1, A a2) { return a1; }
}

public class E extends D {
    public A m(B b1, B b2) { return b2; }
}
```

Type Systems

overriding

methods

- parameter types
- return type

covariance

- method in subclass
- return type: subtype of original return type

contravariance

- method in subclass
- parameter types: supertypes of original parameter types

Example: Java overloading vs. overriding

```
public class F {  
    public A m(B b) { System.out.println("F.m(B b)"); return b; }  
}  
  
public class G extends F {  
    public A m(A a) { System.out.println("G.m(A a)"); return a; }  
}  
  
public class H extends F {  
    public B m(B b) { System.out.println("H.m(B b)"); return b; }  
}  
  
A a = new A(); B b = new B(); F f = new F(); G g = new G(); H h = new H();  
A ab = b;  
  
f.m(b);  
g.m(a); g.m(b); g.m(ab);  
h.m(b);
```

Example: Java invariance

```
public class X {  
    public A a;  
    public A getA() { return a; }  
    public void setA(A a) { this.a = a; }  
}  
  
public class Y extends X {  
    public B a;  
    public B getA() { return a; }  
    public void setA(B a) { this.a = a; }  
}  
  
A a = new A(); B b = new B(); X y = new Y();  
  
y.getA(); y.setA(b); y.setA(a);  
  
String[] s = new String[3]; Object[] o = s; o[1] = new A();
```

P A R E N T A L

ADVISORY

THEORETICAL CONTENT

Overview today's lecture

theory

- formal languages
- formal grammars
- decidability & complexity
- syntax trees

practice

- syntax definition for MiniJava

first assignment

I

generative grammar



infinite productivity

EE A TOU
K-BRENTAN

LA DEUXIÈME ANNÉE

DÉ LATIN

finite models

Théorie 250 pages.

227 Exercices 100 pages.

Lexiques

24 pages blanches pour notes et 4 cartes

Study of Language

linguistics and friends

philosophy

linguistics

- lexicology
- grammar
- morphology
- syntax
- phonology
- semantics

interdisciplinary

- applied linguistics
- computational linguistics
- historical linguistics
- neurolinguistics
- psycholinguistics
- sociolinguistics

computer science

- grammar
- semantics

A Theory of Language

formal languages

vocabulary Σ

finite, nonempty set of elements (words, letters)
alphabet

string over Σ

finite sequence of elements chosen from Σ
word, sentence, utterance

formal language λ

set of strings over a vocabulary Σ
 $\lambda \subseteq \Sigma^*$



A Theory of Language

formal grammars

formal grammar G

derivation relation \Rightarrow_G

formal language $L(G) \subseteq \Sigma^*$

$$L(G) = \{w \in \Sigma^* \mid S \xrightarrow{G^*} w\}$$



Decimal Numbers

morphology

$$G = (N, \Sigma, P, S)$$

Num → Digit Num

Num → Digit

Digit → "0"

Digit → "1"

Digit → "2"

Digit → "3"

Digit → "4"

Digit → "5"

Digit → "6"

Digit → "7"

Digit → "8"

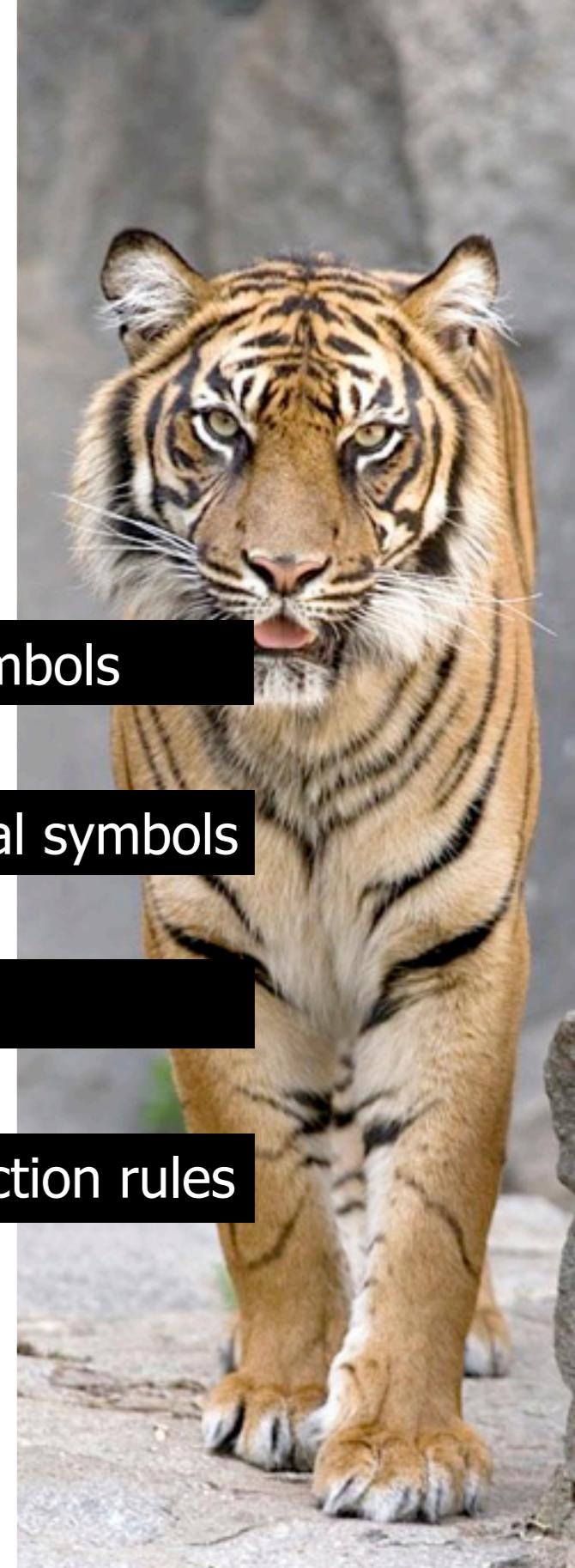
Digit → "9"

Σ : finite set of terminal symbols

N: finite set of non-terminal symbols

$S \in N$: start symbol

$P \subseteq (N \times N \cup \Sigma)^*$: set of production rules



Decimal Numbers

production

Num \Rightarrow

Digit Num \Rightarrow

4 Num \Rightarrow

4 Digit Num \Rightarrow

4 3 Num \Rightarrow

4 3 Digit Num \Rightarrow

4 3 0 Num \Rightarrow

4 3 0 Digit \Rightarrow

4 3 0 3

Num \rightarrow Digit Num

Digit \rightarrow "4"

Num \rightarrow Digit Num

Digit \rightarrow "3"

Num \rightarrow Digit Num

Digit \rightarrow "0"

Num \rightarrow Digit

Digit \rightarrow "3"

leftmost derivation

Decimal Numbers

production

Num \Rightarrow

Digit Num \Rightarrow

Digit Digit Num \Rightarrow

Digit Digit Digit Num \Rightarrow

Digit Digit Digit Digit \Rightarrow

Digit Digit Digit 3 \Rightarrow

Digit Digit 0 3 \Rightarrow

Digit 3 0 3 \Rightarrow

4 3 0 3

Num \rightarrow Digit Num

Num \rightarrow Digit Num

Num \rightarrow Digit Num

Num \rightarrow Digit

Digit \rightarrow "3"

Digit \rightarrow "0"

Digit \rightarrow "3"

Digit \rightarrow "4"

rightmost derivation

Binary Expressions

syntax

$G = (N, \Sigma, P, S)$

$Exp \rightarrow Num$

$Exp \rightarrow Exp "+" Exp$

$Exp \rightarrow Exp "-" Exp$

$Exp \rightarrow Exp "*" Exp$

$Exp \rightarrow Exp "/" Exp$

$Exp \rightarrow "(" Exp ")"$

Σ : finite set of terminal symbols

N : finite set of non-terminal symbols

$S \in N$: start symbol

$P \subseteq (N \times N \cup \Sigma)^*$: set of production rules



Binary Expressions

production

$\text{Exp} \Rightarrow$

$\text{Exp} + \text{Exp} \Rightarrow$

$\text{Exp} + \text{Exp} * \text{Exp} \Rightarrow$

$3 + \text{Exp} * \text{Exp} \Rightarrow$

$3 + 4 * \text{Exp} \Rightarrow$

$3 + 4 * 5$

$\text{Exp} \rightarrow \text{Exp} " + " \text{Exp}$

$\text{Exp} \rightarrow \text{Exp} " * " \text{Exp}$

$\text{Exp} \rightarrow \text{Num}$

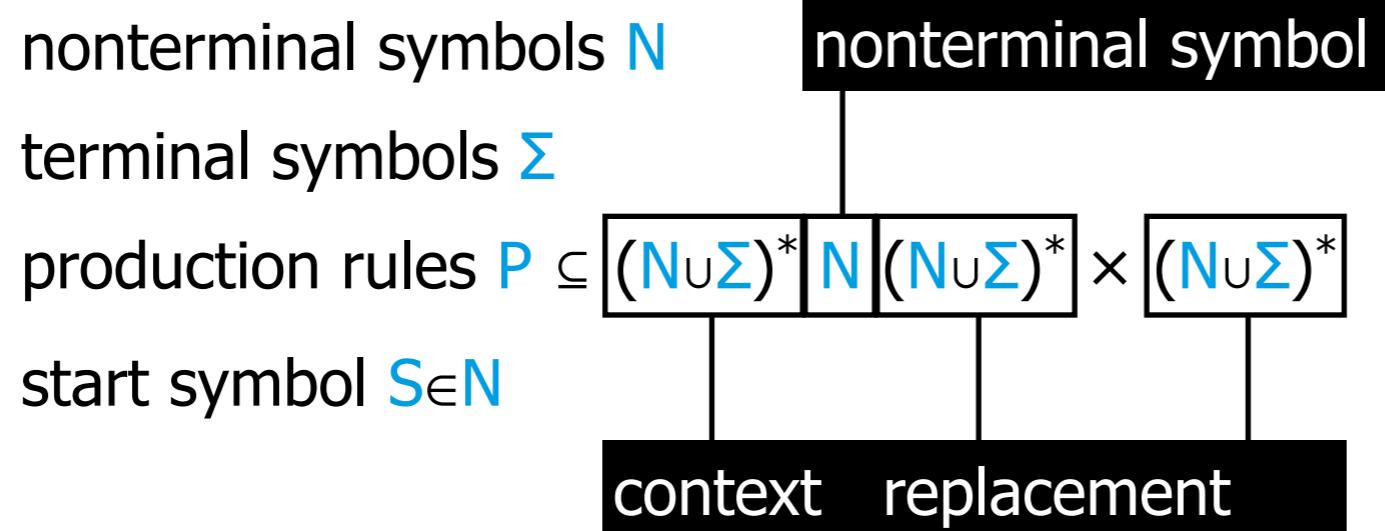
$\text{Exp} \rightarrow \text{Num}$

$\text{Exp} \rightarrow \text{Num}$

A Theory of Language

formal grammars

formal grammar $G = (N, \Sigma, P, S)$



A Theory of Language grammar classes

type-0, unrestricted: $P \subseteq (N \cup \Sigma)^* N (N \cup \Sigma)^* \times (N \cup \Sigma)^*$

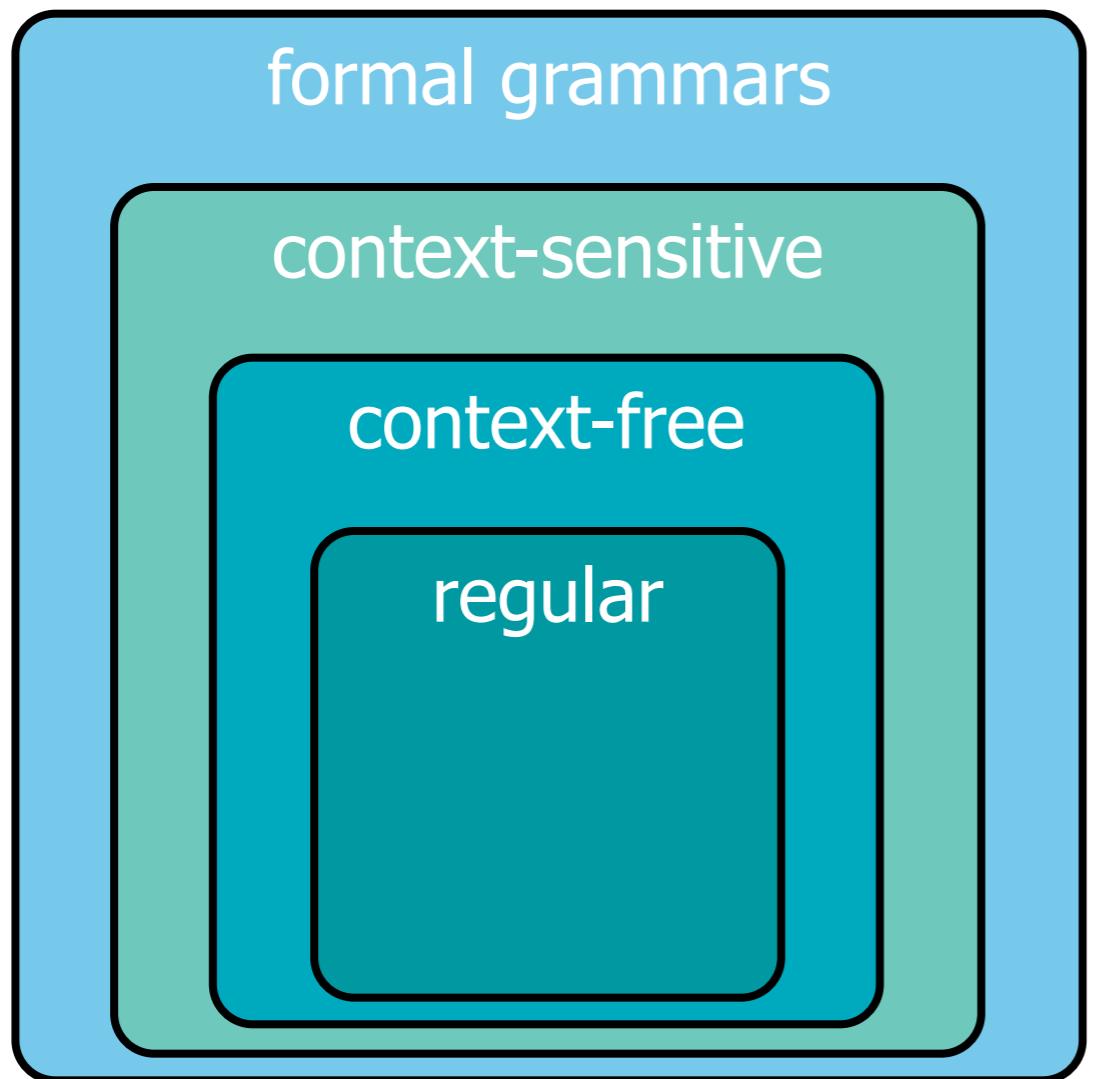
type-1, context-sensitive: $(a A c, a b c)$

type-2, context-free: $P \subseteq N \times (N \cup \Sigma)^*$

type-3, regular: (A, x) or (A, xB)



A Theory of Language grammar classes



A Theory of Language formal grammars

formal grammar G

derivation relation \Rightarrow_G

formal language $L(G) \subseteq \Sigma^*$

$$L(G) = \{w \in \Sigma^* \mid S \xrightarrow{G^*} w\}$$



A Theory of Language

formal languages

formal grammar $G = (N, \Sigma, P, S)$

derivation relation $\Rightarrow_G \subseteq (N \cup \Sigma)^* \times (N \cup \Sigma)^*$

$w \Rightarrow_G w' \Leftrightarrow$

$\exists (p, q) \in P: \exists u, v \in (N \cup \Sigma)^*:$

$w = u p v \wedge w' = u q v$

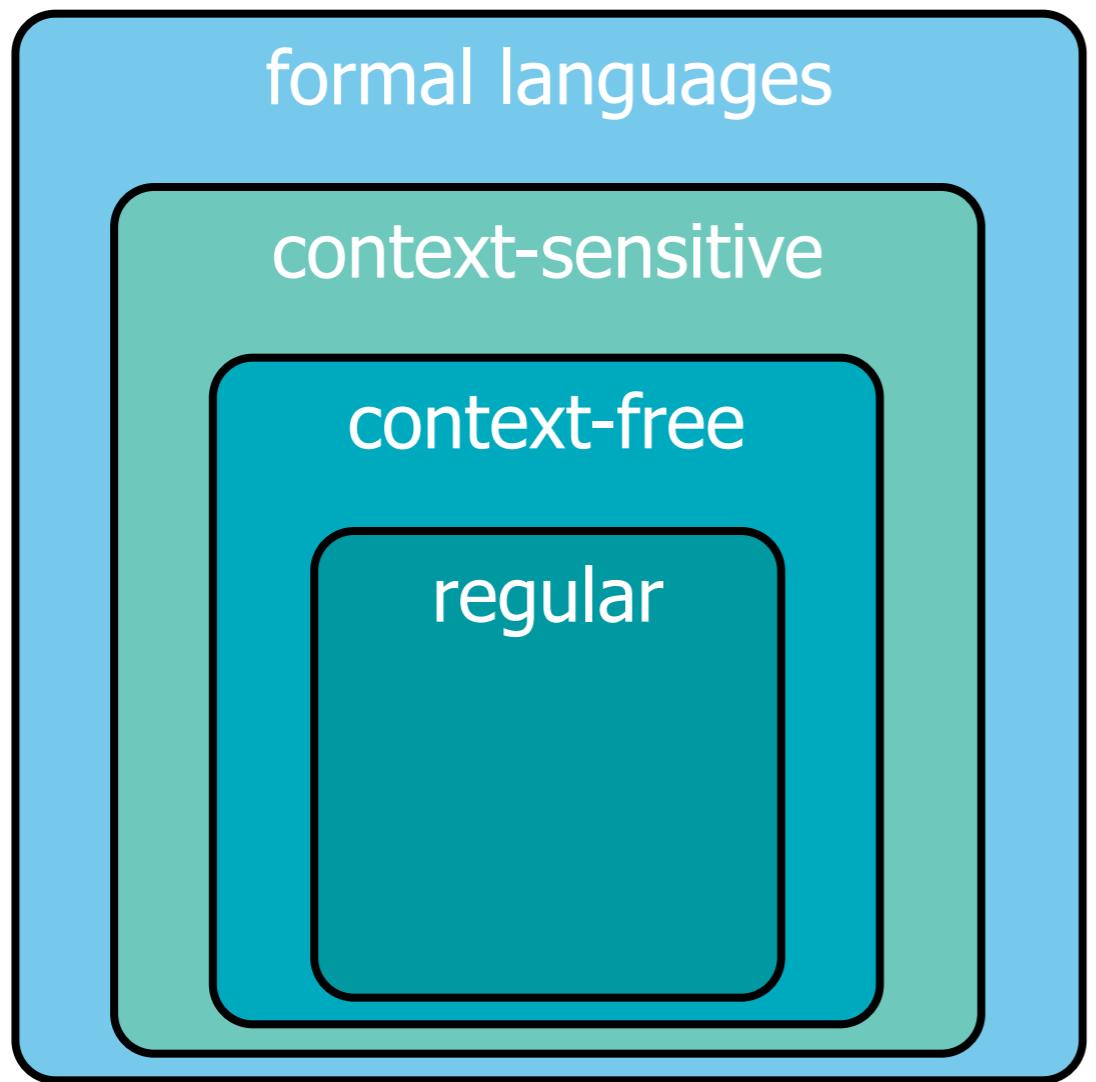
formal language $L(G) \subseteq \Sigma^*$

$L(G) = \{w \in \Sigma^* \mid S \xrightarrow{G}^* w\}$



A Theory of Language

language classes



II

word problem

Theoretical Computer Science

decidability & complexity

word problem $x_L: \Sigma^* \rightarrow \{0,1\}$

$w \rightarrow 1$, if $w \in L$

$w \rightarrow 0$, else

decidability

type-0: semi-decidable

type-1, type-2, type-3: decidable

complexity

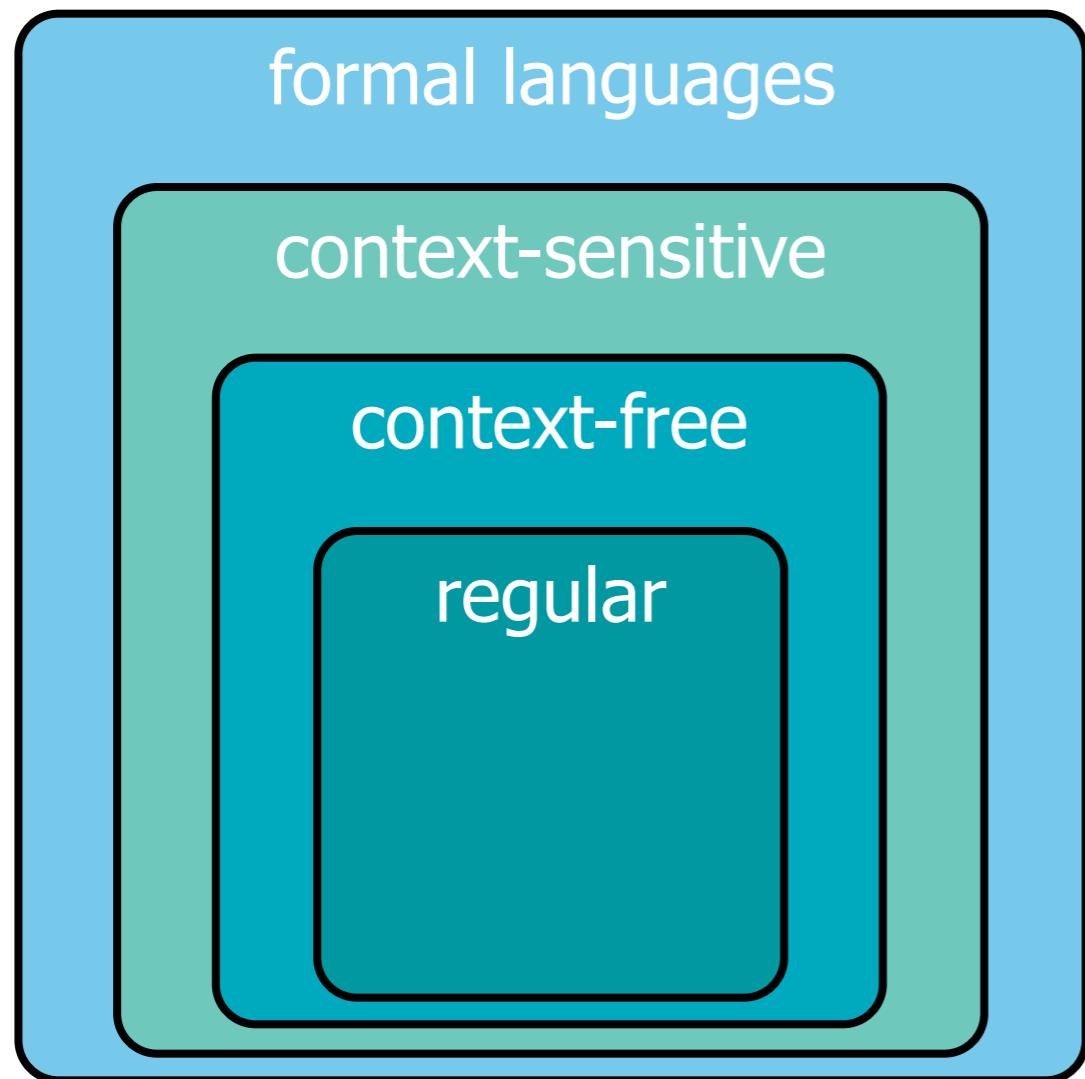
type-1: PSPACE-complete

PSPACE ⊇ NP ⊇ P

type-2, type-3: P

Theoretical Computer Science

decidability & complexity



Context-free Grammars

production vs. reduction rules

$\text{Exp} \rightarrow \text{Num}$

$\text{Exp} \rightarrow \text{Exp} "+" \text{Exp}$

$\text{Exp} \rightarrow \text{Exp} "-" \text{Exp}$

$\text{Exp} \rightarrow \text{Exp} "*" \text{Exp}$

$\text{Exp} \rightarrow \text{Exp} "/" \text{Exp}$

$\text{Exp} \rightarrow "(" \text{Exp} ")"$

$\text{Num} \rightarrow \text{Exp}$

$\text{Exp} "+" \text{Exp} \rightarrow \text{Exp}$

$\text{Exp} "-" \text{Exp} \rightarrow \text{Exp}$

$\text{Exp} "*" \text{Exp} \rightarrow \text{Exp}$

$\text{Exp} "/" \text{Exp} \rightarrow \text{Exp}$

$"(" \text{Exp} ")" \rightarrow \text{Exp}$

productive form

reductive form

Binary Expressions

reduction

$3 + 4 * 5 \Rightarrow$

Num → Exp

Exp + $4 * 5 \Rightarrow$

Num → Exp

Exp + Exp * $5 \Rightarrow$

Num → Exp

Exp + Exp * Exp \Rightarrow

Exp "*" Exp → Exp

Exp + Exp \Rightarrow

Exp "+" Exp → Exp

Exp

III

syntax trees

Context-free Grammars

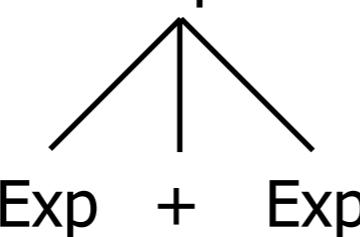
tree construction rules

Exp

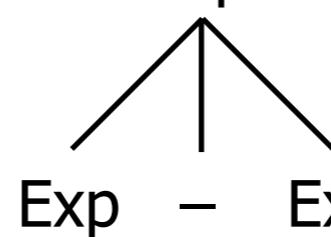


Num

Exp



Exp

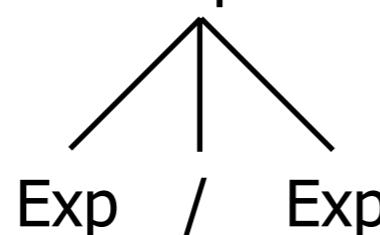


Exp

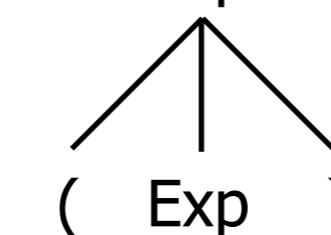


Exp * Exp

Exp

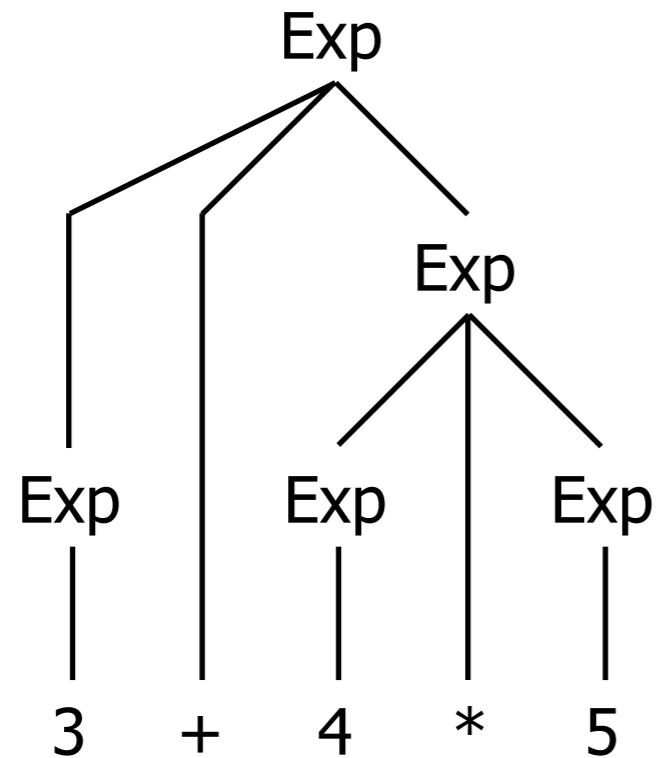


Exp



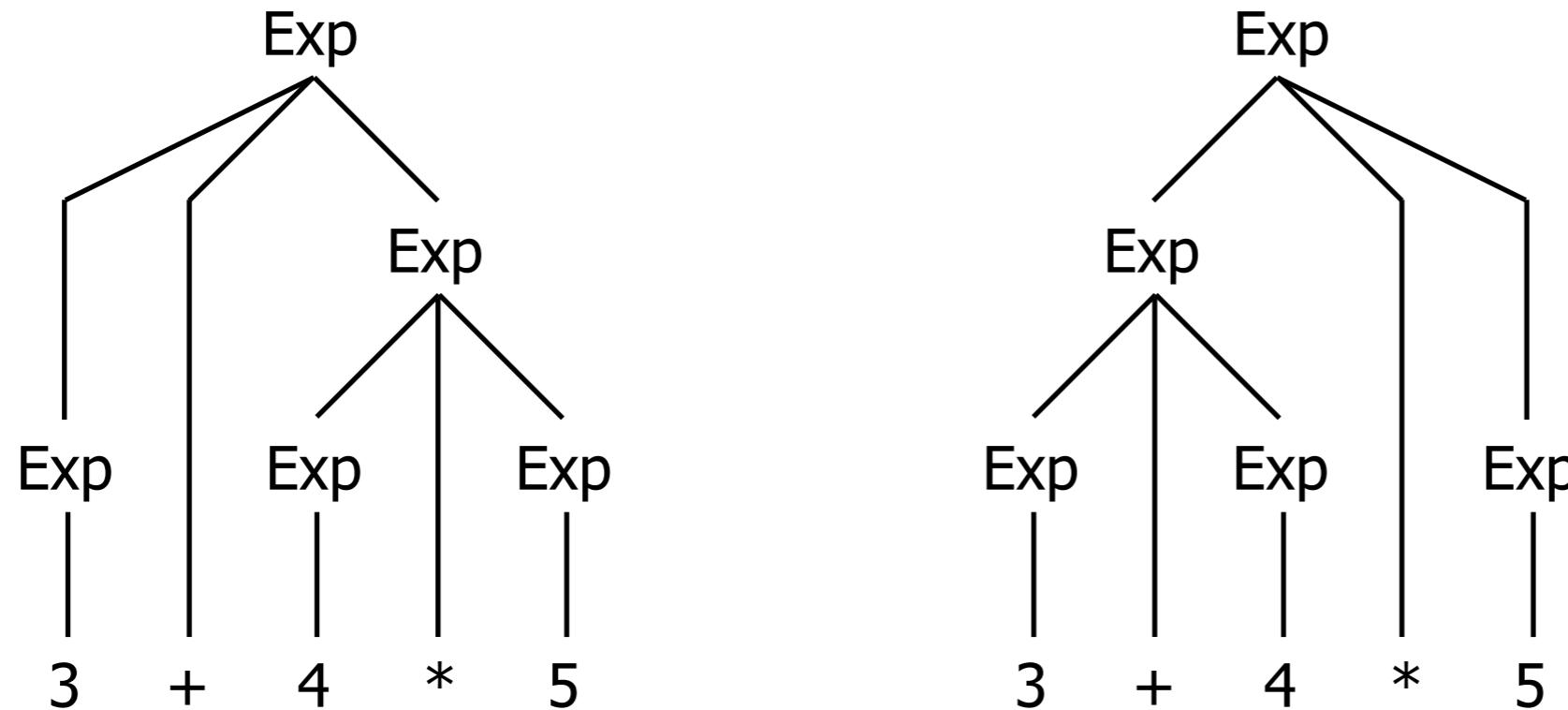
Binary Expressions

tree construction



Context-free Grammars

ambiguities



Abstract Syntax Trees

parse trees

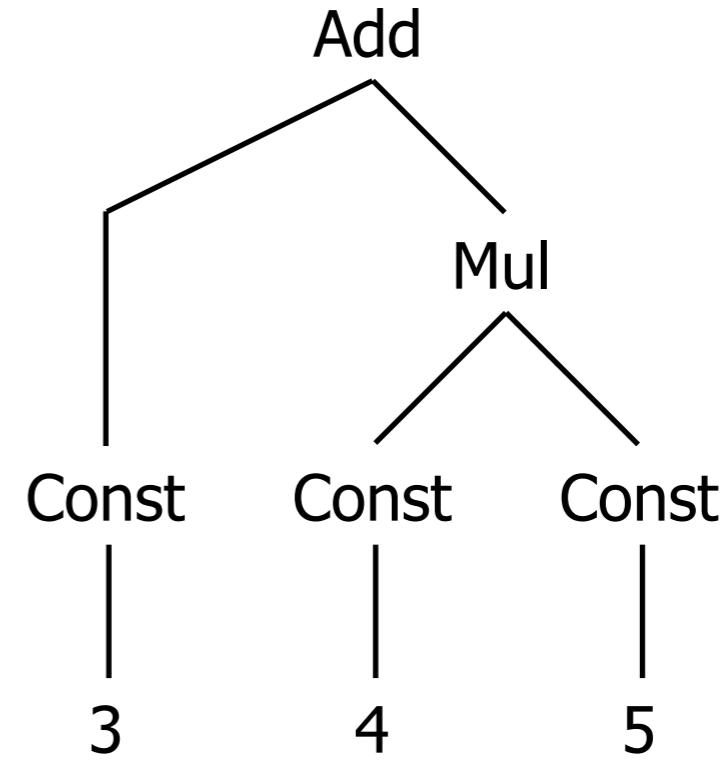
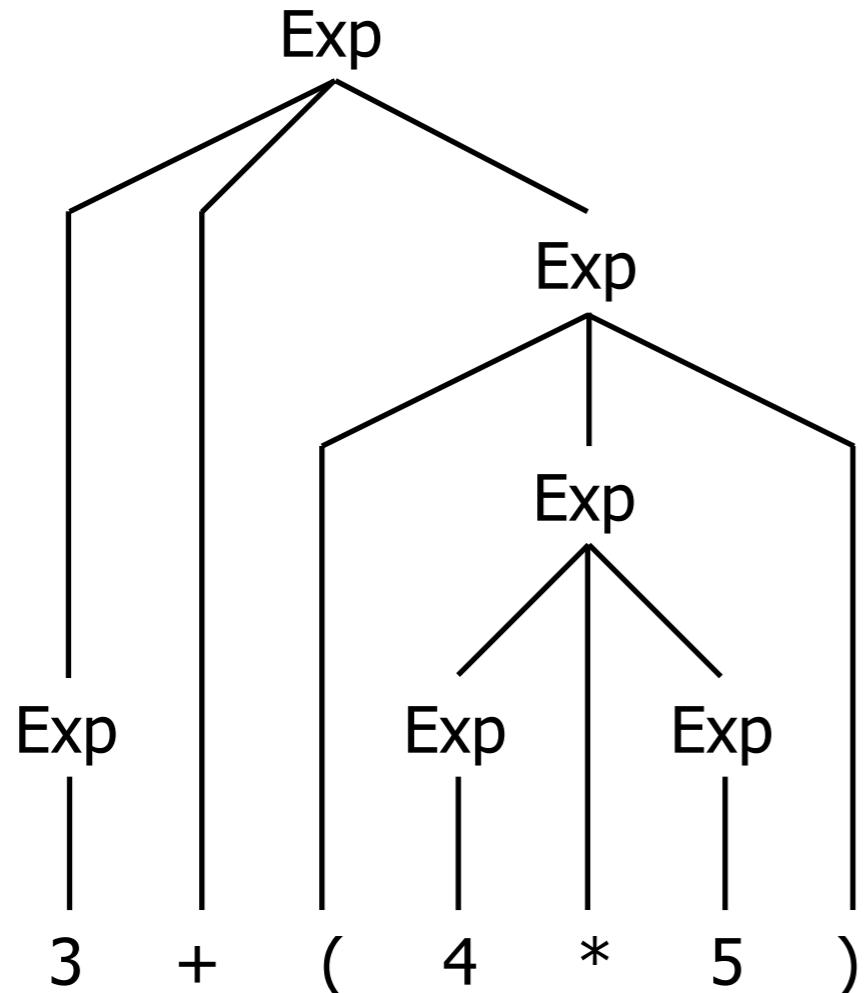
- parent node: nonterminal symbol
- children nodes: nonterminal and terminal symbols

abstract syntax trees

- abstract over terminal symbols
- convey information at parent nodes
- abstract over injective production rules

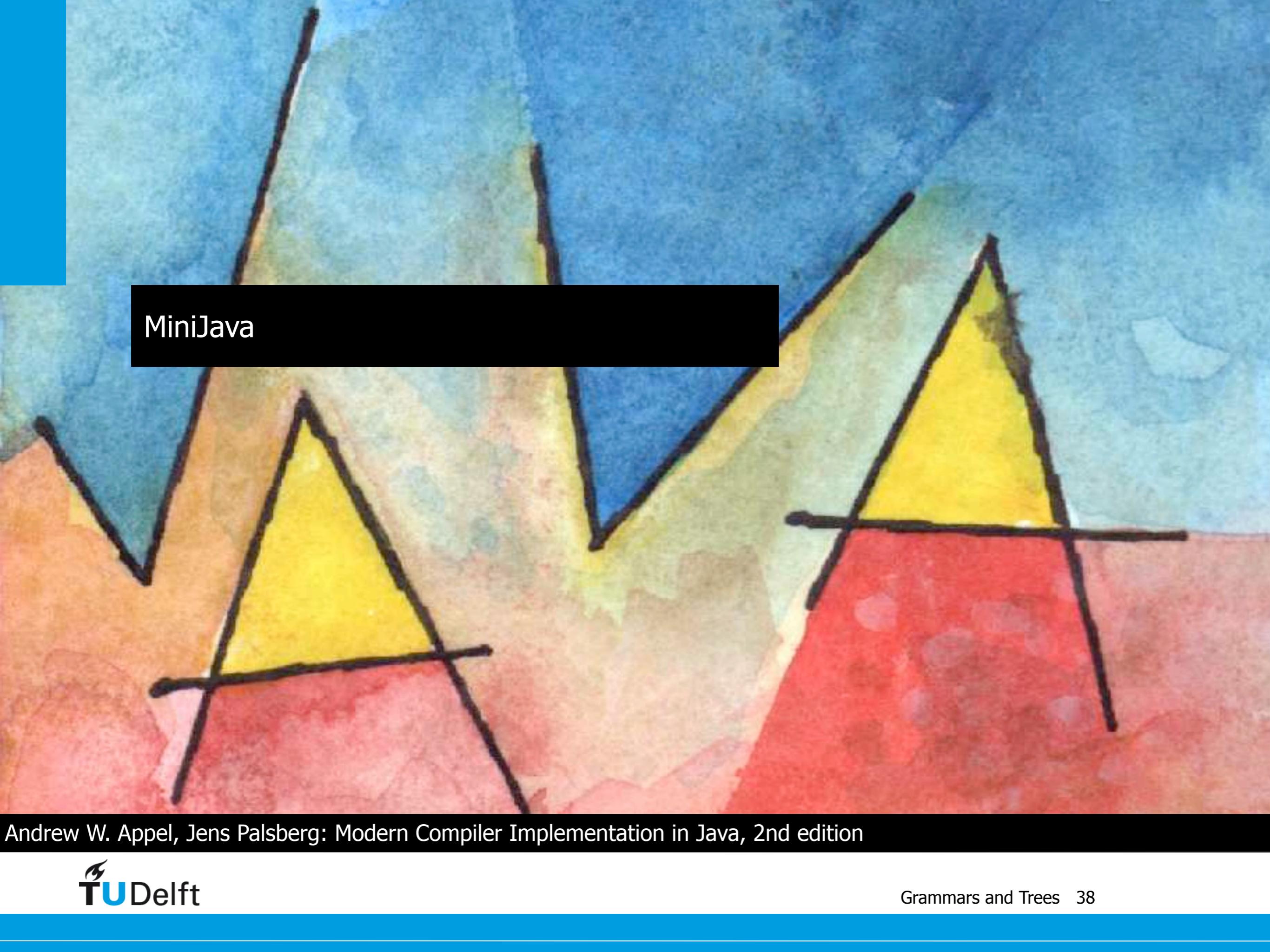
Binary Expressions

abstract syntax trees



coffee break



The background of the slide is a vibrant, abstract painting featuring large, overlapping triangles in shades of yellow, orange, red, green, blue, and purple against a light blue sky. A small white bird is visible in the upper right corner.

MiniJava

Andrew W. Appel, Jens Palsberg: Modern Compiler Implementation in Java, 2nd edition

MiniJava

lexical syntax

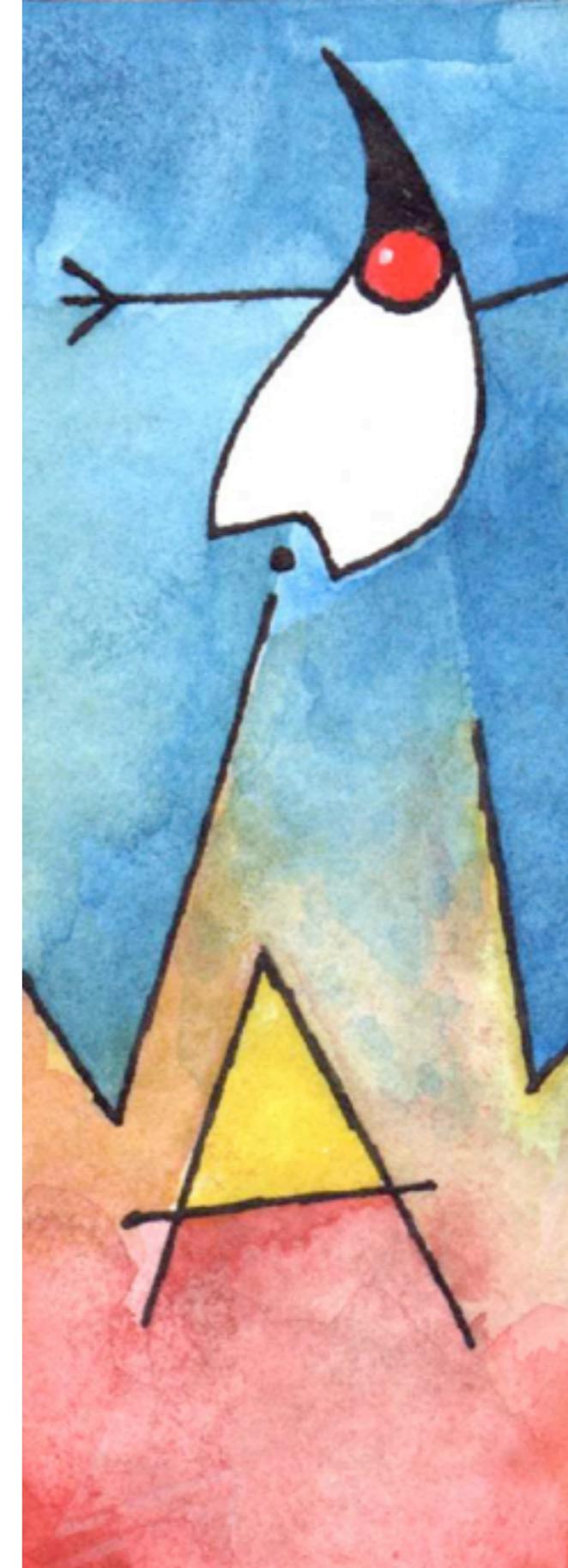
An **identifier** is a sequence of letters, digits, and underscores, starting with a letter. Uppercase letters are distinguished from lowercase.

A sequence of decimal digits is an **integer constant** that denotes the corresponding integer value.

A **binary operator** is one of

&& < + - *

A **comment** may appear between any two tokens. There are two forms of comments: One starts with `/*`, ends with `*/`, and may be nested; another begins with `//` and goes to the end of the line.



MiniJava Expressions

42

true

false

x

x*y

!x

(x)

Exp → Int

→ "true"

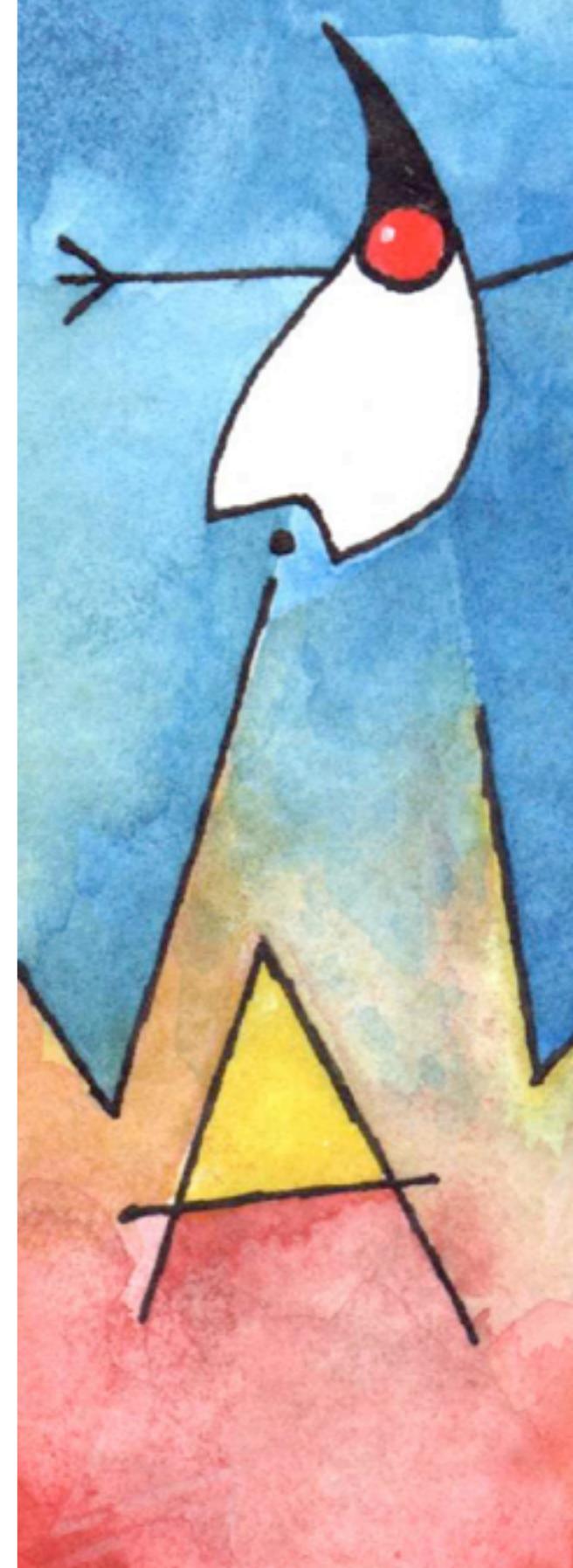
→ "false"

→ Id

→ Exp Op Exp

→ "!" Exp

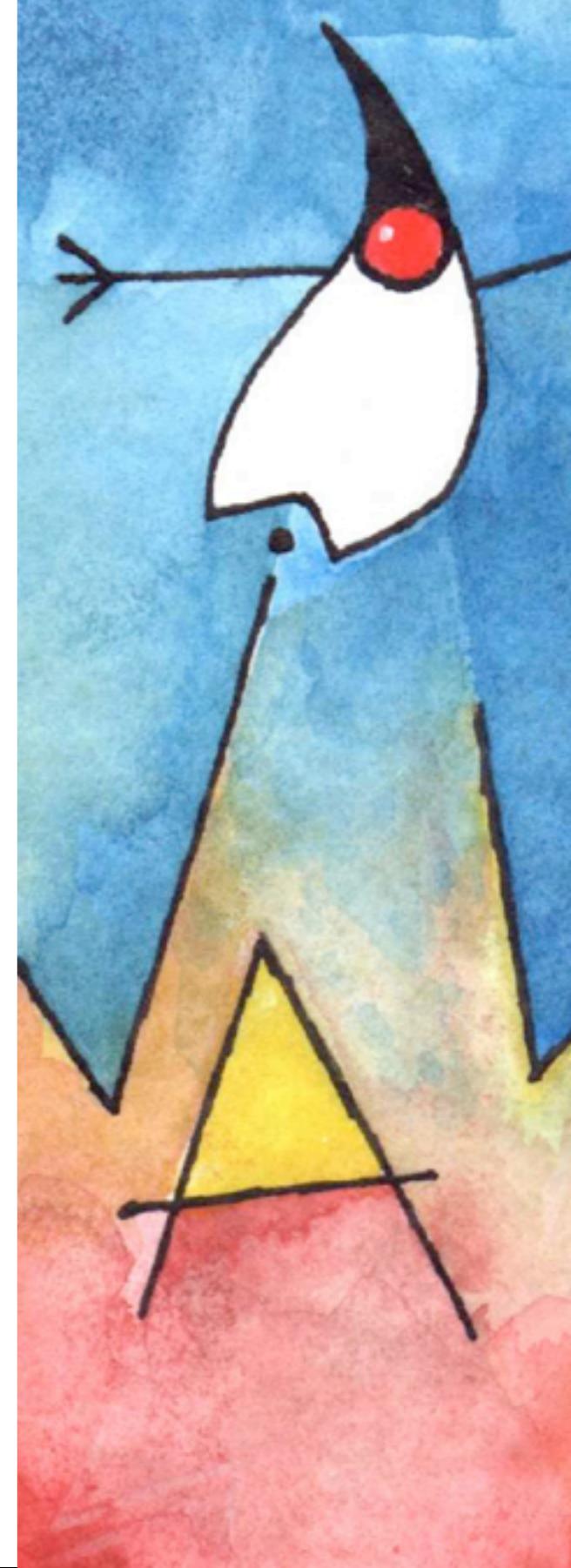
→ "(" Exp ")"



MiniJava Expressions

```
new int[42]
x[42]
x.length
new Class()
this
x.m(y, z)
```

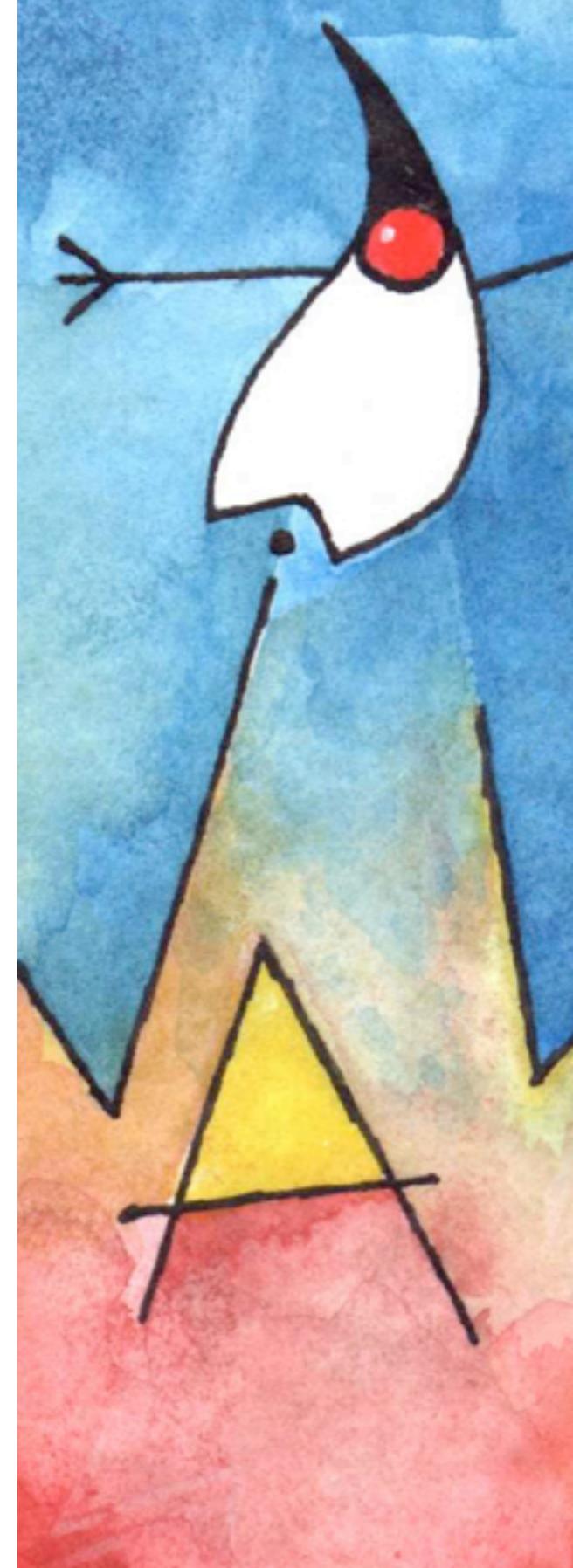
```
Exp → "new" "int" "[" Exp "]"
      → Exp "[" Exp "]"
      → Exp "." "length"
      → "new" Id "(" ")"
      → "this"
      → Exp "." Id "(" ExpList ")"
```



MiniJava Expressions

```
new int[42]
x[42]
x.length
new Class()
this
x.m(y, z)
```

```
ExpList → Exp ExpRest*
          →
ExpRest → "," Exp
```



MiniJava Statements

```
x = y ;  
x[z] = y ;  
if (b) x = y ; else y = x ;  
while (x < 42) x = x+1 ;  
System.out.println(x) ;  
{ x = y ; y = z ; }
```

Statement → Id "=" Exp ";"
→ Id "[" Exp "] "=" Exp ";"
→ "if" "(" Exp ")" Statement
 "else" Statement
→ "while" "(" Exp ")" Statement
→ "System.out.println" "(" Exp ")" ";"
→ "{" Statement* "}"

MiniJava Class Declarations

```
class Main {  
    public static void main(String[] args) {  
        System.out.println(new Fac().fac(10));  
    }  
}  
  
class Fac { ... }
```

Program → MainClass ClassDecl*

MainClass → "class" Id "{" "public" "static" "void"
 "main" "(" "String" "[" "]" Id ")"
 "{" Statement "}" "}"

ClassDecl → "class" Id "{" VarDecl* MethodDecl* "}"
 → "class" Id "extends" Id
 "{" VarDecl* MethodDecl* "}"

MiniJava Declarations

```
int x ;  
public int id(int p) {int r ; r = p ; return r ; }  
int  
boolean  
int[]  
Class
```

```
VarDecl      → Type Id ";"  
MethodDecl   → "public" Type Id "(" FormalList ")" "{"  
                           VarDecl* Statement* "return" Exp ";" "}"  
Type          → "int"  
                  → "boolean"  
                  → "int" "[" "]"  
                  → Id
```

MiniJava Declarations

```
int x ;  
public int id(int p) {int r ; r = p ; return r ; }  
int  
boolean  
int[]  
Class
```

FormalList \rightarrow Type Id FormalRest*
 \rightarrow
FormalRest \rightarrow "," Type Id

V

summary

Summary lessons learned

What is a formal language?

- set of strings over a vocabulary

What is a formal grammar?

- quadruple of sets of nonterminal and terminal symbols, a set of productions, and a start symbol

What is a context-free grammar and how can you read it?

- formal grammar with a single nonterminal symbol at left-hand sides of production rules
- productive form, reductive form, tree construction rules

What is the syntax of MiniJava programs?

Literature

[learn more](#)

formal languages

Noam Chomsky: Three models for the description of language. 1956

Noam Chomsky: Syntactic Structures. 1957

J. E. Hopcroft, R. Motwani, J. D. Ullman: Introduction to Automata Theory, Languages, and Computation. 2006

MiniJava

Andrew W. Appel, Jens Palsberg: Modern Compiler Implementation in Java, 2nd edition. 2002

Outlook

coming next

declarative syntax definition

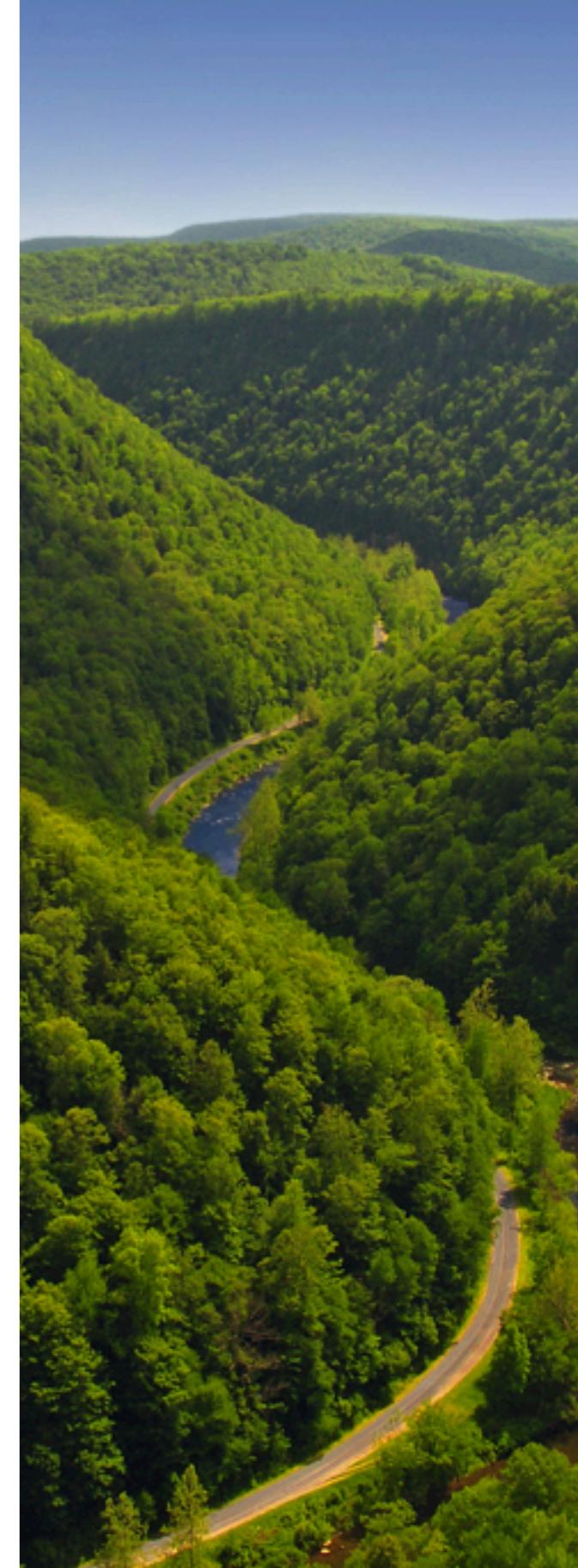
- Lecture 2: SDF and ATerms

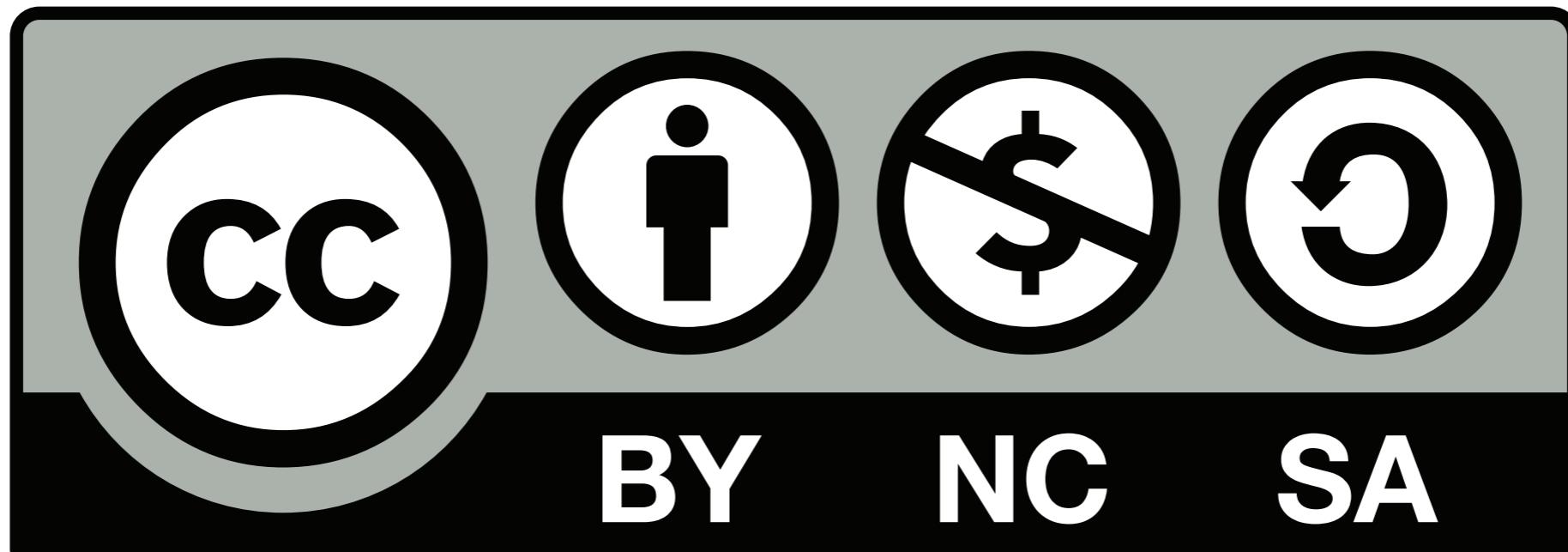
declarative semantics definition

- Lecture 3: Name Binding and Type Systems
- Lecture 4: Term Rewriting
- Lecture 5: Static Analysis and Error Checking
- Lecture 6: Code Generation

next 2 Labs

- install minimal MiniJava project
- write positive and negative test cases





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