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Lesson Title: *Chain Rule*

Key:
Green text: Directions for teacher
Red text: Write on board
Blue text: Question and answer

Lesson Objective:

Students should be able to apply the chain rule to take the derivative of a composition of two elementary functions f(g(x))

Prerequisite skills:

- Students should be comfortable with all basic algebraic abilities
- Students should be familiar with all elementary functions
- Students should be familiar with operations on functions, specifically function composition
- Students should already understand how to take the derivatives of elementary functions.
- Students understand the various notations for derivatives

Opening

Gain Attention

T: Last class, we finished learning the trig derivatives. At the end we encountered a function which we could not take the derivative of, sin(x^2). Today, we will solve this problem, and finally finish our introduction to derivatives.

S: Students give full undivided attention.

Review

T: To warm up, let's do a practice problem.
How would we take the derivative of the function: $f(x) = sin(x) + cos(x)$?

S: $f'(x) = cos(x) - sin(x)$

T: That's right, it's f'(x)=cos(x)-sin(x)
Goal

T: This lesson we're going to be learning the chain rule for compositions of functions, when to use the rule, and how to perform it.

Relevance/Rationale
T: The chain rule allows us to take the derivative when we have one function inside of another, like our example from last class: $sin(x^2)$ or $cos(x^3+x)$. It also allows us to take the derivative of expressions like $sin^2(x)$.

Body

Model

T: If we have 2 functions, $f(x)$ and $g(x)$, the derivative of $f(g(x))$ is
$\frac{d}{dx}(f(g(x)) = f'(g(x))g'(x)$.
Write rule on chalkboard, draw a box around it, and leave it for the rest of lesson

The chain rule is called the "chain" rule because it links, or "chains," the derivative of a composite function by multiplying the derivative of the outer function with the derivative of the inner function.
When we use the chain rule to take the derivative, first we have to identify what f(x) and g(x) even are. Then, we find the derivatives of f(x) and g(x): f’(x) and g’(x). Our last step is to plug what we know back into our formula.

Model 1

T: Let's start with our $sin(x^2)$ example. First, we identify f(x) and g(x). Here, $f(x) = sin(x)$ and $g(x) = x^2$. Next, we take the derivative of $f(x)$. The derivative of $sin(x)$ is $cos(x)$, so $f'(x) = cos(x)$. Next, we take the derivative of $g(x)$. The derivative of x^2 is $2x$, so $g'(x) = 2x$. Finally, we can use our rule to put it all together. *Pointing to rule and $f'(x)$.* $f'(g(x)) = cos(x^2)$, and $g'(x)$ is $2x$. Therefore, when we multiply those together, we get $(f(g(x))' = 2x\ cos(x^2)$.

Model 2

T: I'll give another example. Say we have a function $h(x) = ln(2x + 1)$. First, we identify our inner and outer functions. Here, $f(x) = ln(x)$ and $g(x) = 2x + 1$. Next, we take the derivative of $f(x)$. The derivative of $ln(x)$ is $\frac{1}{x}$, so $f'(x) = \frac{1}{x}$. Next, we take the derivative of $g(x)$. The derivative of $2x + 1$ is 2, so $g'(x) = 2$. Finally, we can use our rule to put it all together. *Pointing to rule and $f'(x)$.* $f'(g(x))$ is $\frac{1}{2x+1}$, and $g'(x)$ is 2. Therefore, $h'(x) = \frac{2}{2x+1}$.

Model 3

T: I'll give one more example. Say we have a function $h(x) = e^{sin(x^2)}$. Our first step is to identify our inner and outer functions. This example can be tricky to identify f(x) and g(x). What we can notice is that $sin(x^2)$is nested within our exponential function e^x. So, $f(x) = e^x$ and $g(x) = sin(x^2)$. Next, we take the derivative of $f(x)$. The derivative of e^x is e^x, so $f'(x) = e^x$. Next, we take the derivative of $g(x)$. Using our first example, which we needed chain rule to solve, the derivative of $sin(x^2)$ is $2x\ cos(x^2)$, so $g'(x) = 2x\ cos(x^2)$. Finally, we can use our rule to put it all together. *Pointing to rule and $f'(x)$.* $f'(g(x))$ is $e^{sin(x^2)}$, and $g'(x)$ is $2x\ cos(x^2)$. Therefore, $h'(x) = 2x\ cos(x^2)\ e^{sin(x^2)}$.

Prompt

T: Now we will work together on a few examples. Follow along and do the problem as I do it.

Prompt 1

T: Let's do one of the functions I mentioned earlier: $h(x) = sin^2(x) = (sin(x))^2$. The first step is to identify what our outside and inside functions are. Here, I would write $f(x) = x^2$ and $g(x) = sin(x)$. The second step is to find the derivatives of our inside and outside functions. Here, I would write $f'(x) = 2x$ and $g'(x) = cos(x)$. Our third and final step is to use our rule to put it all together. Here, the final answer would be $h'(x) = 2\,sin(x)\,cos(x)$.

S: Have students write the functions

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Prompt 2

T: Let's do another example together. Say we have the function $h(x) = e^{-x^2}$. What is the first step? Yes. Here, our outside function is $f(x) = e^x$, and our inside function is $g(x) = -x^2$. What is our second step? Awesome. $f'(x) = e^x$ and $g'(x) = -2x$. What is our final step? That's right. Here that would make our final answer $h'(x) = -2x\,e^{-x^2}$.

S: The first step is to identify our inside and outside functions.
The second step is to find the derivatives of those functions.
The final step is to use the chain rule.

Prompt 3

T: Now, you guys are going to lead me through an example. Say we have the function $h(x) = tan(3x^2 + 4x + 1)$. What is the first step? Awesome. What are our functions? Good job. $f(x) = tan(x)$ $g(x) = 3x^2 + 4x + 1$. What is the second step? Yes. What are our derivatives? Correct. $f'(x) = sec^2(x)$ $g'(x) = 6x + 4$ Now put it together. What is our final answer? That's right. $h'(x) = (6x + 4)\,sec^2(3x^2 + 4x + 1)$

S: The first step is to identify our inside and outside functions.
$f(x)$ would be $tan(x)$ and $g(x)$ would be $3x^2 + 4x + 1$
The second step is to find the derivatives of those functions.
$f'(x) = sec^2(x)$ and $g'(x) = 6x + 4$
It would be $h'(x) = (6x + 4)\,sec^2(3x^2 + 4x + 1)$