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Balancing Input-Output tables with Bayesian slave-raiding ants

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Abstract. Input-Output (I-O) tables are produced by statistical offices to estimate the relationships between the sectors of an economy. I-O tables can be unbalanced if the sum of its columns (total input destination) does not equal the sum of its rows (total output). An artificial Ant Colony (ACO) algorithm based on Bayesian slave-making polymorphus ants is proposed for balancing an I-O matrix. The approach is inspired on the behavior of *Rossomyrmex minuchae*, a parasite ant that enslaves other species of ants (*Proformica*) which in turn choose an optimal path between their colony and the source of food by leaving a trace of pheromones. In the algorithm, an improvement in the balance of I-O accounts increase the pheromones, thus raising the probability of ants moving towards the equilibrium of the matrix. An application to a real I-O matrix and Monte Carlo experiments were performed to evaluate the proposed ACO algorithm. The results showed that slave-raiding ACO can be used by statistical offices as an automated algorithm to produce more timely and reliable I-O tables.

Keywords: Input-Output models, artificial Ant Colony optimization, Bayesian analysis

JEL codes: C67, C61, C11

1. Introduction

Input-Output (I-O) tables are produced by statistical offices with the purpose of measuring the transactions of an economy. I-O tables can be unbalanced if the sum of its columns (total input destination) does not equal the sum of its rows (total output of the sectors). In this paper, a slave-raiding Ant Colony (henceforth, ACO) algorithm is proposed for balancing an I-O table.

An ACO algorithm is a metaheuristic optimization technique based on the swarm behavior of ants, which choose and optimal path between their colony and the source of food by leaving a trace of pheromones. In the ACO algorithm of this paper, the "distance" between the unbalanced and the balanced matrix is the Euclidean norm between the row-sums and the column-sums of the I-O matrix. Artificial *Proformica* ants randomly explore modifications of the elements of the matrix. An improvement in the balance of the I-O matrix – i.e. a reduction of the "distance" between the rows and columns – increases the amount of pheromones, thus increasing the probability of choosing the path towards the equilibrium of the matrix. The slave-making behavior of *Rossomyrmex minuchae* is used to decide both the polymorphism and the number of *Proformica* ants in the enslaved colony. Polymorphism – variations in size and morphology of the artificial ants – facilitates the allocation and partitioning of the optimization task, since "big" artificial ants look for rough solutions for a balanced matrix and then "small" artificial ants improve the precision of the differences between the row sums and column sums of the I-O matrix.

Section 2 briefly explains the biological inspiration behind the ACO algorithm. Section 3 describes the slave-raiding ACO algorithm. Section 4 shows a simulation exercise, a comparison with RAS and an empirical application to a real I-O table. Section 5 concludes.

2. Ant Colony optimization

The communication among ants is based on chemicals, called pheromones, which are produced by the ants themselves. Some ant species (as *Proformica ferreri*) mark a path between food sources and the nest leaving a pheromone trail on the ground. Ants

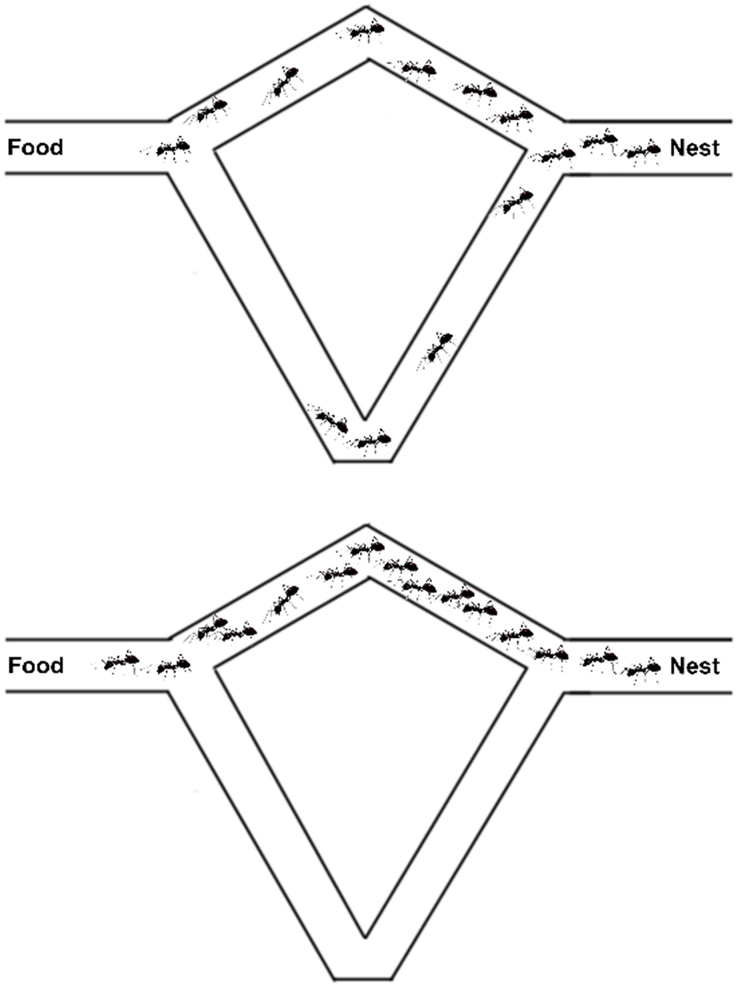

Fig. 1. Illustration of the double bridge experiment.

can smell the trail with their antennae and they tend to choose, probabilistically, paths marked by strong pheromone concentrations.

In a controlled experiment, Deneubourg et al. [4] used a double bridge connecting a food source and the nest of *Linepithema humile* (formerly *Iridomyrmex humilis*) ants. The bridge had two branches of unequal length, one being larger than the other. Ants were left free to move between the nest and the food source. Although initially ants randomly explore the two branches, eventually all the ants used the same, shortest branch (Fig. 1). The result of Deneubourg et al. [4] can be explained as follows: initially, there is no pheromone on the two branches; hence, ants do not have a favorite path and they select with the same probability any of the branches. Because one branch is shorter than the other, the ants choosing the short branch are the first to reach the food and to start their return to the nest. Therefore, pheromone starts to accumulate faster on the shortest branch. This mechanism, called stigmergy, leads to a complex, seemingly intelligent structure in the colony [7].

In terms of polymorphism, some species of ants, as *Pheidologeton diversus*, have continuous allometric variations in size and morphology to facilitate task allocation and partitioning: as in other social insect colonies, specific workers are engaged in specific tasks (task-allocation) or a single task is performed by different workers (task-partitioning) – see Gordon [9] and Ratnieks and Anderson [13]. This strategy allows minor or middle ants to act as workers, while often ants with disproportionately larger heads and stronger mandibles act as soldiers in the colony.

Slave-raiding is a behavior exhibited by slave-making ants, as e.g. *Rossomyrmex minuchae*, a parasite ants that captures and enslaves other species of ants

Fig. 2. *Proformica ferreri* (top) and *Rossomyrmex minuchae* (bottom).

(specifically *Proformica ferreri*) to increase the work force of their own colony (Fig. 2). In contrast to other species of slave-making ants, *R. minuchae* does not use pheromones during raiding [18]. See Deneubourg et al. [4], Dorigo et al. [6] or Attiratanasunthron and Fakcharoenpho [1] for a detailed discussion about the biological inspiration behind ACO algorithms.

3. Artificial Ant Colony algorithm for balancing Input-Output matrices

The raiding-slaving behavior of *R. minuchae* was used as an inspiration to automatically choose the optimal number of ants and the optimal size (polymorphism) of the ants in the artificial enslaved colony. The optimal enslaved colony solves the task of balancing the I-O matrix. The following is the definition of a balanced matrix used in this study:

Definition 1. Let $\mathbf{M}$ be a $n \times n$ matrix with m_{ij} elements $(i = 1, 2, \ldots, , n, j = 1, 2, \ldots, n), \mathbf{f} = (f_1, f_2, \ldots, f_n)$ is a vector with the sums of the n-rows of $\mathbf{M}$ and $\mathbf{c} = (c_1, c_2, \ldots, c_n)$ is a vector with the sums of the n-columns of $\mathbf{M}$. If $\mathbf{M}$ is not balanced, all or at least one $f_i \neq c_j$ for $ij \in \{1, 2, \ldots, n\}$; in turn, $\mathbf{M}$ is a balanced matrix if $f_i = c_j$ for all $ij \in \{1, 2, \ldots, n\}$, where, in the case of a square matrix, $i = j$.

In the I-O context, this definition implies that an I-O table is balanced if the sum of its columns (total input destination) is equal the sum of its rows (total output).

The problem of balancing the matrix $\mathbf{M}$ can be solved minimizing the Euclidean norm of a distance metric of the form $d\,(f_i, c_j) = (f_1 - c_1, f_2 - c_2, \ldots, f_n - c_n)$,

$$\min_{m_{ij} \in \mathbb{R}^{0,+}} ||d||$$

$$\text{for } ||d|| := \sqrt{(f_1 - c_1)^2 + (f_2 - c_2)^2 + \cdots + (f_n - c_n)^2}, \quad = \sqrt{\mathbf{d} \cdot \mathbf{d}}$$

The following is a slave-raiding ant colony algorithm design to minimize $||d||$.

Definition 2. Let $\mathbf{M}^{(s)}$ be the matrix $\mathbf{M}$ in a s-iteration and $\mathbf{M}^{(s+1)}$ the matrix in iteration $s + 1, \mathbf{M}^{(s+1)} = \mathbf{M}^{(s)} + \mathbf{R} \odot \left(\frac{\mathbf{M}^{(s)}}{\phi^{(s)}}\right)$, for $\mathbf{R} \in \mathbb{R}^{n \times n}$ a random stochastic matrix with a multivariate discrete uniform distribution $\mathcal{U}\,(-1, 1)$, $\odot$ the Hadamard operator and $\phi^{(s)}$ a scale parameter that controls ant's polymorphism.

Definition 3. Let $\theta^{(s)}$ be the value of the Euclidean norm $\|d\|$ in a s-iteration, and $\theta^{(s+1)}$ the norm in the iteration $s + 1$.

Definition 4. Let $\mathcal{P}\,(x|\alpha, \beta)$ be a transition probability from iteration s to iteration $s+1$ based on pheromones $x = 1 - \frac{\theta^{(s+1)}}{\theta^{(s)} + \theta^{(s+1)}}$ measured with a Beta cumulative distribution function,

$$x\mathcal{P}(x|\alpha, \beta) := F(x|\alpha, \beta)$$

$$= \begin{cases} 0 & \text{for } x < 0, \\ \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)+\Gamma(\beta)} \int_0^x u^{\alpha-1} (1-u)^{\beta-1} du, & \text{for } 0 \leqslant x \leqslant 1 \\ 1 & \text{for } x > 0. \end{cases}$$

Definition 5. Let $d_1 d_2 d_3, \ldots, d_s = \{d_k\}_{k=1}^{s}$ be a series of $d(f_i, c_j)$ divergence measures during s-iterations, where each divergence measures the degree of matrix imbalance. A $\mathcal{P}_{s,s+1}$ probability of $\{d_k\}_{k=1}^{s}$ not leading to a balanced matrix between iterations s and $s+1$ (i.e. the probability that the current $\phi^{(s)}$-colony is not balancing the I-O matrix) can be calculated with the Bayesian posterior odds of a hypothesis of balancing improvement $\mathbb{H}_1 : d_1 > d_2 > d_3 > \cdots > d_s$,

$$\mathcal{P}_{s,s+1} := \frac{P(\mathbb{H}_1)P(\mathit{data}|\mathbb{H}_1)}{P(\mathbb{H}_1)P(\mathit{data}|\mathbb{H}_1) + P(\mathbb{H}_2)P(\mathit{data}|\mathbb{H}_2)},$$

$$= \frac{\sqrt{\pi}\left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi}\left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)}},$$

with $F = \frac{(s-\xi)\sum_{k=1}^{s}\left(\hat{d}_k - \bar{d}\right)^2}{(\xi-1)\sum_{k=1}^{s}\left(d_k - \hat{d}_k\right)^2}$ a Fisher statistic of an $AR(\xi)$-autoregressive equation for $\{d_k\}_{k=1}^{s}$. See Koop [10].

Based on Definitions 1 to 5, Box 1 below shows the two-step slave-raiding ACO algorithm for balancing an I-O matrix:

Box 1: Slave-raiding ACO algorithm

Step 1 (Slave-raiding)

1.1. Optimal number of ants

while $p_{s,s+1} = 1$

$\delta \longleftarrow \delta_0 + 1$

$s \longleftarrow s_0 \exp(\delta)$

$\{d_k\}_{k=1}^{s} = f(\phi_0, s)$

$$p_{s,s+1}\left(\{d_k\}_{k=1}^{s_0}\right) = \frac{\sqrt{\pi}\left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi}\left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)}},$$

1.2. Optimal size of ants (polymorphism)

while $p_{s,s+1} \geqslant$ epsilon

$\sigma \longleftarrow \sigma_0 + 1$

$\phi \longleftarrow \phi_0 \exp(\sigma)$

$\{d_k\}_{k=1}^{s} = f(\phi, s)$

$$p_{s,s+1}\left(\{d_k\}_{k=1}^{s}\right) = \frac{\sqrt{\pi}\left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi}\left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)}},$$

Step 2

For an optimal ϕ and optimal $1, 2, \ldots, s$-iterations:

i. If $\theta^{(s+1)} \leqslant \theta^{(s)}$, then,
$\mathbf{M}^{(s+1)} \longleftarrow \mathbf{M}^{(s)}$ and $\theta^{(s+1)} \longleftarrow \theta^{(s)}$

ii. If $\theta^{(s+1)} > \theta^{(s)}$, but $p(x|\alpha, \beta) > \omega, \omega \sim U(0,1)$
$\mathbf{M}^{(s+1)} \longleftarrow \left\{\lambda \mathbf{M}^{(s)} + (1-\lambda)\mathbf{M}^{(s)}\right\}, < \lambda < 1,$
$\phi^{(s+1)} \longleftarrow (1+\varrho)\,\phi^{(s)} 0 \leqslant \varrho < 1$

In the first step of Box 1 (slave-raiding), an arbitrary number of ants (calculated with $s = s_0 \exp(\delta_0 + 1)$) with an arbitrary size (equal to $\phi = \phi_0 \exp(\sigma_0 + 1)$) are chosen based on the initialization parameters δ_0, σ_0. Then, $\{d_k\}_{k=1}^{s}$ divergence measures are calculated for s-iterations; during these s-iterations, each individual element of $\mathbf{M}$ is randomly adjusted and then artificial slave-making ants look both for the optimal number of ants (i.e. the optimal number of iterations s needed to solve the optimization task) and the optimal size of these ants (ϕ), until the probability of not having a balanced matrix is close to zero ($p_{s,s+1} \approx 0$). In the second step, the optimized enslaved colony looks for the most balanced version of an I-O matrix$\mathbf{M}$ following two criteria: (i) if the value of the Euclidean norm $\|d\|$ in a $(s+1)$-iteration $\theta^{(s+1)}$ is less than or equal to the value in the previous iteration $\theta^{(s)}$ – i.e. if there as improvement towards a balanced matrix after changing the individual elements of $\mathbf{M}$– then the values of the matrix $\mathbf{M}^{(s)}$ are replaced by the candidate solution $\mathbf{M}^{(s+1)}$, while (ii) if $\theta^{(s+1)} > \theta^{(s)}$ but if the probability $p(x|\alpha, \beta)$ is greater than a random number ω from a uniform distribution $\omega \sim U(0,1)$, then the matrix $\mathbf{M}^{(s)}$ is replaced by $\lambda \mathbf{M}^{(s)} + (1-\lambda)\mathbf{M}^{(s)}$ and the size of ants is modified by $\phi^{(s+1)} = (1+\varrho)\,\phi^{(s)}$, with the parameters $\lambda \in (0,1)$ and $\varrho \in [0,1)$ controlling the degree of random exploration (λ) and random polymorphism (ϱ), respectively.

4. Applications

4.1. Application to an artificial unbalanced matrix

Let $\mathcal{M}$ be an unbalanced 4×4 matrix,

$$\mathcal{M} = \begin{pmatrix} 7.54 & 6.99 & 13.92 & 10.10 \\ 8.04 & 7.10 & 13.52 & 11.61 \\ 12.43 & 4.69 & 7.03 & 10.18 \\ 8.77 & 2.08 & 7.02 & 9.62 \end{pmatrix}$$

The differences between the row-sums and the column-sums of this matrix are,

$$\mathbf{d}_{\mathcal{M}} = [-1.77, \ \ -19.41, \ \ 7.17, \ \ 14.01],$$

which lead to an Euclidean norm of $\|\mathbf{d}_{\mathcal{M}}\| = 25.05$. The slave-raiding ACO algorithm describe in the previous section automatically choose 400 artificial ants with a polymorphism equal to 66.5 to optimize the unbalanced matrix $\mathcal{M}$. After using ACO, $\mathcal{M}$ becomes,

$$\mathcal{M}_{ACO} = \begin{pmatrix} 6.49 & 9.27 & 11.53 & 6.87 \\ 5.60 & 7.16 & 6.08 & 6.24 \\ 12.04 & 5.77 & 6.89 & 8.23 \\ 9.97 & 2.90 & 8.42 & 9.06 \end{pmatrix}$$

with $\|\mathbf{d}_{\mathcal{M}_{ACO}}\| = 0.06$, thus $\mathcal{M}_{ACO}$ is now a balanced matrix.

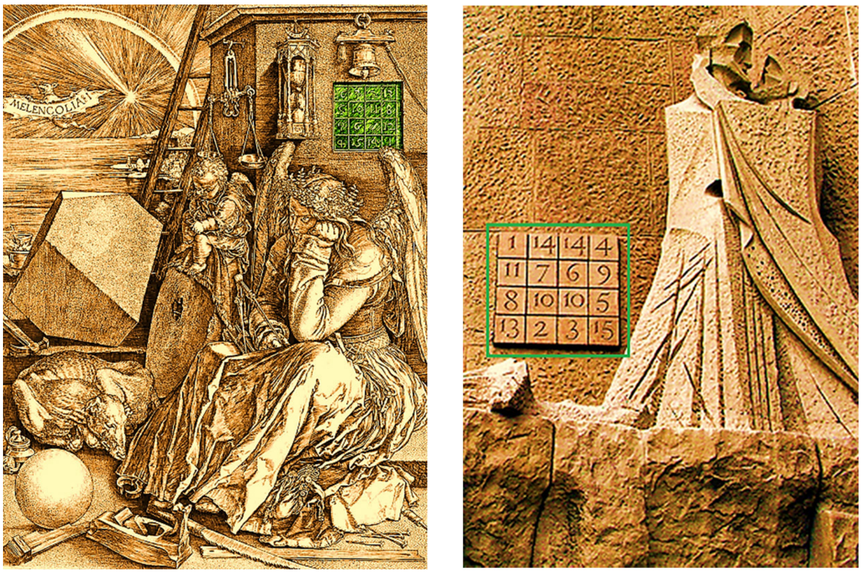

Fig. 3. Subirachs Magic Square (right) and Dürer's Magic Square (left).

Table 1
Results of the Monte Carlo experiment

Design	Method	γ Subirachs	γ Dürer
$\varepsilon = 0.5$	RAS	0.3445	0.3422
r = 100	ACO	0.2154	0.2057
$\varepsilon = 1$	RAS	0.6678	0.6587
r = 1000	ACO	0.4113	0.4031

4.2. *Monte Carlo simulation experiments: ACO vs. RAS*

RAS, also called iterative proportional fitting [14], is a popular balancing method widely used in statistical offices. Monte Carlo simulation experiments were performed to compare the balancing capacity of ACO against RAS, using two well-known balanced matrices: Subirachs Magic Square – located at the Passion façade of the Basílica i Temple Expiatori de la Sagrada Família in Barcelona (Fig. 3, right) – and Dürer's Magic Square (Fig. 3, left), part of a 1514 engraving by the German Renaissance master Albrecht Dürer.

The Passion façade of the Basílica i Temple Expiatori de la Sagrada Família in Barcelona, conceptualized by Antoni Gaudí and designed by sculptor Josep Subirachs, features a 4×4 balanced matrix:

$$\mathbf{M}_S = \begin{pmatrix} 1 & 14 & 14 & 4 \\ 11 & 7 & 6 & 9 \\ 8 & 10 & 10 & 5 \\ 13 & 2 & 3 & 15 \end{pmatrix}.$$

The rom-sums and column-sums of this matrix are equal to 33, the age of Jesus at the time of the Passion.

Melencolia I it is a 1514 engraving by the German Renaissance master Albrecht Dürer. The 4×4 square on this engraving has rom-sums and column-sums that are equal to 34; the square's four quadrants, corners and center also equal this number:

$$\mathbf{M}_D = \begin{pmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{pmatrix}.$$

These two matrices were disturbed with random noise and the ability of ACO and RAS to recover the original balanced matrix was compared using the aggregate mean absolute differences between the elements of the unbalanced ($m^0_{ij} \in \mathbf{M}_0$) and the balanced matrix ($m_{ij} \in \mathbf{M}$), using the statistic $\gamma = i^{-2} \sum_{ij} \left| m^0_{ij} - m_{ij} \right|$. A smaller value of γ indicates smaller differences between the randomly unbalanced matrix $\mathbf{M}_0$ and the original balanced matrix$\mathbf{M}$. The design of the experiment was the following one: different levels of gaussian noise were added to the balanced matrices $\mathbf{M}_D$ and $\mathbf{M}_S$, using a contamination scalar ε times a random matrix $\mathcal{R}$, i.e. $\mathbf{M}_0 = \mathbf{M} + \varepsilon\mathcal{R}$. Subsequently, ACO and RAS were used to balance these $\mathbf{M}_0$ disturbed matrices, r-times.

Table 1 and Fig. 4 show the results of the experiments. In all the cases, the differences of γ with ACO are smaller than those of RAS, thus showing that ACO recovers a balanced matrix which is much closer, element-by-element, to the original, unbalanced matrices $\mathbf{M}_D$ and $\mathbf{M}_S$.

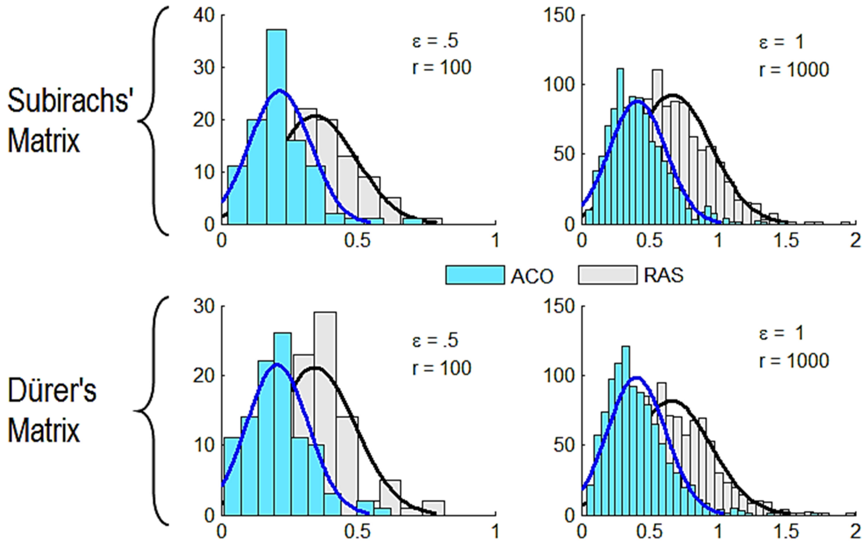

Fig. 4. Results of the Monte Carlos simulation experiments.

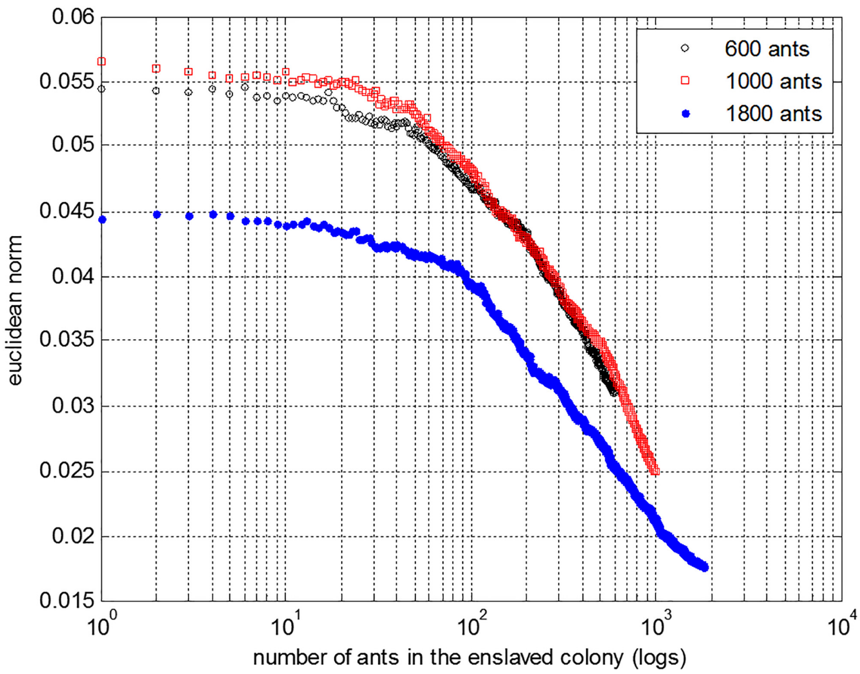

Fig. 5. Empirical application to Turkey's Input-Output table.

4.3. Empirical application

Figure 5 shows an application of the slave-raiding ACO algorithm to Turkey's Input-Output table for 2011, which is available at WIOD database. A random contamination noise of $\varepsilon \sim N(0, 1\text{E}3)$ was added to the original 35×35 I-O matrix of Turkey and the ability of the slave-raiding ACO algorithm to recover the original balanced I-O matrix was evaluated using different number of ants and polymorphism automatically choose by the artificial slave-making ants: $(i) s = 600$, $\phi = 1.96\text{E}7$, (ii) $s = 1000$, $\phi = 3.27\text{E}7$, and (iii) $s = 1800$, $\phi = 2.29\text{E}7$. In all the cases – and particularly when there was a large number of ants in the colony – the slave-raiding ACO algorithm was able to reduce the differences in the imbalance of the perturbed matrix (i.e. reduce the Euclidean norm of the differences between the row-sums and the column-sums), thus recovering the original balanced I-O matrix from Turkey.

5. Discussion

A slave-raiding ACO algorithm for balancing I-O matrices was proposed. Artificial slave-making ants were used to choose both the optimal number and the optimal polymorphism of ants in an enslaved colony; the optimal enslaved colony randomly explores modifications of the elements of an unbalanced I-O matrix until a balanced matrix is found. Monte Carlo experiments showed that the balancing capacity of ACO tends to be superior of that from RAS, even in the presence of high levels of random noise. These results show that slave-raiding ACO can be used by statistical offices as an automated algorithm to produce more timely and reliable Input-Output tables.

It is worth noting that the ACO algorithm presented in this document is a heuristic balancing algorithm which does not explicitly take into account the correlation structure of the unbalanced matrix, which is important for matrices based on collected data as I-O tables. Future studies can overcome this limitation by including a meaningful structure-preserving condition (as e.g. a correlation-preserving condition) in the loss function of the algorithm, possibly emulating the necrophoresis of *Lasius niger*, a species of ants with workers specialized in the removal of dead members from the colony, see López-Riquelme and Fanjul-Moles [11]. In computational terms, this corpse disposal outside of the colony would be equivalent to discarding solutions that do not comply with the structure-preserving conditions of the slave-raiding ACO algorithm.

References

[1] Attiratanasunthron, N. and Fakcharoenphol, J. A running time analysis of an ant colony optimization algorithm for shortest paths in directed acyclic graphs. Inf. Process. Lett. (2008); 105(3): 88–92.

[2] Chen, E. and Liu, X. Multi-colony ant algorithm. In Ant Colony Optimization – Methods and Applica-tions, pp. 3–12; In Tech Books and Journals, 2011.

[3] de Lara Peñate, C.M. and Peñate, D.R.S. New nonlinear approaches for the adjustment and updating of a SAM. Documentos de trabajo conjunto, Facultad de Ciencias Económicas de la ULPGC, 2003.;

[4] Deneubourg, J.-L. Aron, S. and Goss, S. The self-organizing exploratory pattern of the argentine ant. Journal of Insect Behavior. (1990); 3: 159–169.

[5] Deneubourg, J.L. Goss, S. Franks, N. Sendova-Franks, A. Detrain, C. and Chrétien, L. The dynamics of collective sorting robot-like ants and ant-like robots, In Proceedings of the first international conference on simulation of adaptive behavior, from animals to animats, 1990; pp. 356–363. MIT Press.

[6] Dorigo, M. Maniezzo, V. and Colorni, A. Ant system: optimization by a colony of cooperating agents. Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on. (1996); 26(1): 29–41.

[7] Dorigo, M. and Stützle, T. Ant Colony Optimization. Bradford Company, 2004.

[8] Gambardella, L. Taillard, M.E. and Agazzi, G. Macs-vrptw: A multiple ant colony system for vehicle routing problems with time windows. In New Ideas in Optimization, 1999; pp. 63–76. McGraw-Hill.

[9] Gordon, D.M. The organization of work in social insect colonies. Complex. (2002; September); 8(1): 43–46.

[10] Koop, G. Objective' Bayesian unit root tests. Journal of Applied Econometrics. (1992); 7(1): 65-82.

[11] López-Riquelme, G.O. and Fanjul-Moles, M.L. The funeral ways of social insects. Social strategies for corpse disposal. Trends Entomol. (2013); 9: 71-129.

[12] Merkle, D. Middendorf, M. and Schmeck, H. Ant colony optimization for resource-constrained project scheduling. In IEEE Transactions on Evolutionary Computation, 2000; pp. 893–900. Morgan Kaufmann.

[13] Ratnieks, F.L.W. and Anderson, C. Task partitioning in insect societies. Insectes Sociaux. (1999); 46(2): 95–108.

[14] Schneider, M.H. and Zenios, S.A. A Comparative Study of Algorithms for Matrix Balancing. Operations Research. (1990); 38(3): 439–455.

[15] Socha, K. and Dorigo, M. Ant colony optimization for continuous domains. European Journal of Oper-ational Research. (2008); 185(3): 1155–1173.

[16] Stützle, T. and Hoos, H.H. Max-min ant system. Future Gener. Comput. Syst. (2000); 16(9): 889–914.

[17] Tfaili, W. Continuous time dynamic optimization. In Ant Colony Optimization – Methods and Applications, pp. 13–24; In Tech Books and Journals, 2011.

[18] Ruano, F. and Tinaut, A. Raid process, activity pattern and influence of abiotic conditions in the slave-making ant Rossomyrmex minuchae (Hymenoptera, Formicidae) a slave-maker species, Insectes Sociaux. (1999); 46(4): 341–347.