Exercise 3

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$$A = \begin{bmatrix} 1 & -4 & 3 \\ 1 & -1 & 1 \\ -3 & 12 & -9 \end{bmatrix}$$

$R_2 - R_1 \longrightarrow R_2$

$$A = \begin{bmatrix} 1 & -4 & 3 \\ 0 & 3 & -2 \\ -3 & 12 & -9 \end{bmatrix} \qquad L_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 + 3R_1 \longrightarrow R_3$

$$A = \begin{bmatrix} 1 & -4 & 3 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{bmatrix} = U \qquad L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$L = L_2 L_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$PA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \\ 1 & -1 & 1 \\ -3 & 12 & -9 \end{bmatrix} = \begin{bmatrix} 1 & -4 & 3 \\ 1 & -1 & 1 \\ -3 & 12 & -9 \end{bmatrix}$$

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -4 & 3 \\ 1 & -1 & 1 \\ -3 & 12 & -9 \end{bmatrix}$$

✓ $PA = LU$

$$\det(A) = (-1)^s \prod_{i=1}^{n} u_{i,i} = 1 \cdot 3 \cdot 0 = 0$$

<u>Eigenvalues for A:</u>

$$A - \lambda I = \begin{bmatrix} 1 & -4 & 3 \\ 1 & -1 & 1 \\ -3 & 12 & -9 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & -4 & 3 \\ 1 & -1-\lambda & 1 \\ -3 & 12 & -9-\lambda \end{bmatrix}$$

① $(1-\lambda)\left[(-1-\lambda)(-9-\lambda) - 12\right]$

$= (1-\lambda)\left[(9+\lambda+9\lambda+\lambda^2)-12\right] = (1-\lambda)\left[\lambda^2+10\lambda-3\right]$

$= \lambda^2+10\lambda-3-\lambda^3-10\lambda^2+3\lambda = \boxed{-\lambda^3-9\lambda^2+13\lambda-3}$

$$\begin{bmatrix} 1-\lambda & -4 & 3 \\ 1 & -1-\lambda & 1 \\ -3 & 12 & -9-\lambda \end{bmatrix}$$

② $4[(-9-\lambda) - (-3)]$

$= 4[-\lambda - 6] = -4\lambda - 24$

③ $3[12 - ((-1-\lambda)(-3))]$

$= 3[12 - (3 + 3\lambda)] = 3[12 - 3 - 3\lambda]$

$= 3[-3\lambda + 9] = -9\lambda + 27$

$$-\lambda^3 - 9\lambda^2 + \cancel{13\lambda} - \cancel{3} - \cancel{4\lambda} - \cancel{24} - \cancel{9\lambda} + \cancel{27} = 0$$

$\Rightarrow -\lambda^3 - 9\lambda^2 = 0$

$\Rightarrow -\lambda^2(\lambda + 9) = 0$

$\boxed{\lambda = -9,\ 0}$

$\therefore$ No, the eigenvalues of A ($\lambda = -9, 0$) are not the same as the diagonal of a $(1, 3, 0)$

$\blacksquare$