Exercise 3

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$\phi: \mathbb{R} \to \mathbb{R}$

(1) $x^{(k+1)} = \phi\left(x^{(k)} + \sin\left(x^{(k)}\right)\right)$

- The fixed points of this equation are when:

$$x = \phi(x) = c\pi \quad , \forall c \in \mathbb{Z} \text{ (set of all integers } (-\infty, \infty))$$

(2) We know that the sequence converges to π since $\forall x_1, x_2 \in \left]\frac{\pi}{2}, \frac{3\pi}{2}\right[\quad \exists r$ such that

$$r = |x_1 - x_2|$$

Since our interval is strictly within $\left]\frac{\pi}{2}, \frac{3\pi}{2}\right[$, and $\left|\frac{\pi}{2} - \frac{3\pi}{2}\right| = \pi$, we know r must be strictly less than π.

Additionally, we know there is one fixed point $\pi \in \left]\frac{\pi}{2}, \frac{3\pi}{2}\right[$ where $x = \phi(x) = \pi$. Thus, there must exist only one fixed point π and thus the sequence converges. ∎

(3) $\phi'(x) = 1 + \cos(x) = 2$ Let $x = 0$

$\phi''(x) = \sin(x) = 0$

$\phi'''(x) = \cos(x) = -1$

Thus, the sequence converges with order 3.