Department of Physics
Indian Institute of Technology, Madras
ID 5841 Quantum Computing
Assignment 1

28 Jan 2026 Preliminaries: State Preparation and Entanglement

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1. Using Quantum Circuits on IBM Q, verify the following.
   (a) Show that the Fredkin gate is self-inverse.
   (b) $CX_1C = X_1X_2$
   (c) $CZ_1C = Z_1$
   (d) $CY_2C = Z_1Y_2$
   (e) $CY_1C = Y_1X_2$

   Here C is the CNOT gate with qubit 1 being the control qubit and qubit target qubit. X_1 implies Pauli X-gate on the first qubit.

2. A $|GHZ\rangle$ state is a multipartite state of the form

$$|GHZ\rangle = \frac{1}{\sqrt{2}}\left(|0\dots0\rangle + |1\dots1\rangle\right).$$

   Work out and implement the quantum circuit for the 3-qubit, 4-qubit and 5-qubit GHZ states. Give your results on the simulator as well as on any one of the IBMQ machines.

3. Obtain the guantum circuit to prepare the following states:
   (a) $\frac{1}{\sqrt{2}}\left(|001\rangle + |110\rangle\right)$
   (b) $\frac{1}{\sqrt{2}}\left(|101\rangle + |010\rangle\right)$
   (c) $\frac{1}{\sqrt{2}}\left(|100\rangle + |011\rangle\right)$

4. The standard W-state is an entangled bipartite state of the form

$$|W\rangle = \frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |100\rangle\right).$$

   For a 4-qubit system the state reads,

$$\frac{1}{\sqrt{4}}\left(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle\right).$$

   Construct the quantum circuits that can prepare these states and verify their output via quantum state tomography on any IBMQ machine.

5. The N-qubit GHZ state and W states can be defined in one of two forms:

$$
\begin{aligned}
|GHZ\rangle &= \frac{1}{\sqrt{2}}\left(|00\ldots0\rangle + |11\ldots1\rangle\right). \\
|W\rangle &= \frac{1}{\sqrt{N}}\left(|00\ldots01\rangle + |00\ldots10\rangle + \ldots + |01\ldots00\rangle + |100\ldots00\rangle\right).
\end{aligned}
$$

Work out the quantum circuit for preparing these states.