

ME6030 Assignment

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1. Gauss elimination

Write a code to take a matrix A as input, and print the following as output:

- a. Row echelon form of matrix
- b. Elementary matrices used in each step
- c. Rank of the given matrix

Note that you are not to use any of the internal commands for the above steps. Then use the *timeit* function to compare the time to generate the row reduced echelon form against the builtin command *rref* for the A matrix in 2.d.

Solution

Algorithm 1 Gauss–Jordan Elimination for RREF

Require: $A \in \mathbb{R}^{m \times n}$

Ensure: R : reduced row echelon form of A , rank r , elementary matrices $\{E_k\}$

- 1: Initialize $R \leftarrow A$, $pivot_row \leftarrow 1$
 - 2: Initialize empty lists for pivot columns and elementary matrices
 - 3: **for** each column $c = 1, \dots, n$ **do**
 - 4: **if** $pivot_row > m$ **then**
 - 5: **break**
 - 6: **end if**
 - 7: Select a nonzero pivot in column c using partial pivoting
 - 8: Swap pivot row into position $pivot_row$
 - 9: Scale pivot row to make the pivot equal to 1
 - 10: Eliminate all entries below the pivot
 - 11: Record pivot column and increment $pivot_row$
 - 12: **end for**
 - 13: Eliminate all entries above each pivot (backward elimination)
 - 14: Compute rank r as the number of nonzero rows of R
 - 15: Return R , r , $\{E_k\}$
-

The above algorithm is used to calculate the Row Reduced echelon form of the matrix in the code. It is tested on the following matrix-

$$\begin{bmatrix} 2 & 1 & -1 & 8 \\ 3 & -1 & 2 & -11 \\ -2 & 1 & 2 & -3 \end{bmatrix}$$

Both the inbuilt function and the custom function provided the same solution and the results including time are as follows-

```
RREF:
    1.0000      0      0      2.0000
      0      1.0000      0      3.0000
      0      0      1.0000     -1.0000

Rank = 3
My time: 0.001625 sec
Built-in time: 0.020758 sec
Built-in result:
    1      0      0      2
    0      1      0      3
    0      0      1     -1
```

2.Geometry of linear equations

Graphically generate the row images and column images of the following set of linear equations. Identify and describe the nature of solutions.

- $u + v + w = 2; u + 2v + 3w = 1; v + 2w = 0$
- $x + y + z = 2; x + 2y + z = 3; 2x + 3y + 2z = 5$
- $x + y + z = 2; x + 2y + z = 3; 2x + 3y + 2z = 9$
- $x + y + z = 2; x + 2y + z = 3; 2x + 9y + 5z = 12$

Find the row reduced echelon form of each system using the code from previous steps and correlate with the nature of solutions.

Solution

Plots of the system of equations

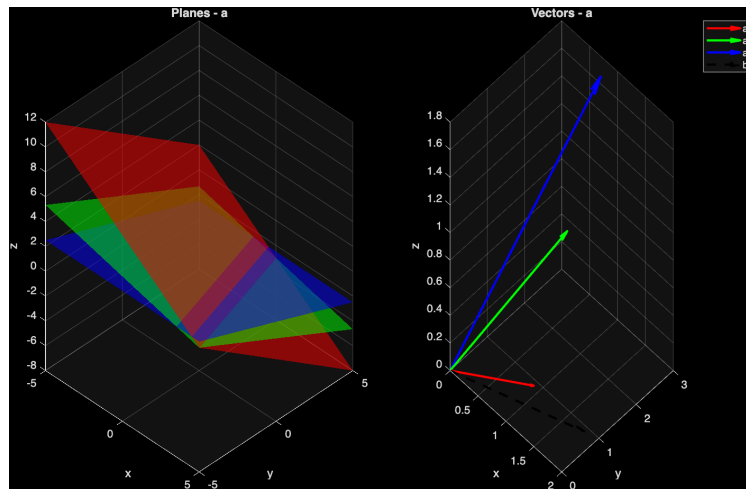


Figure 1: Equations 1

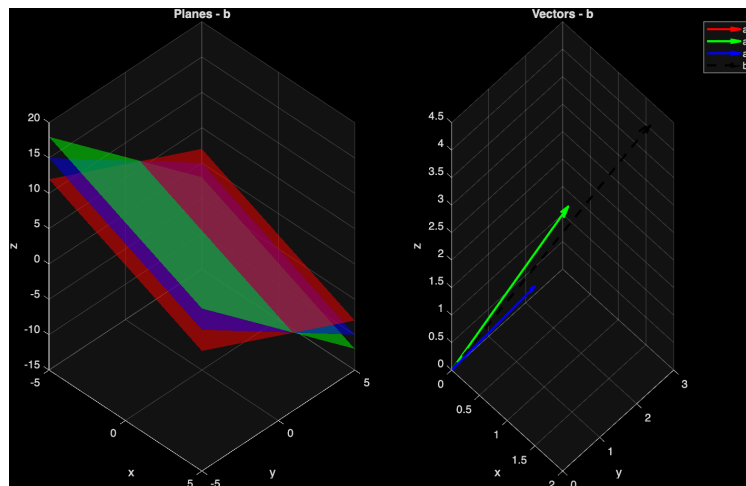


Figure 2: Equations 2

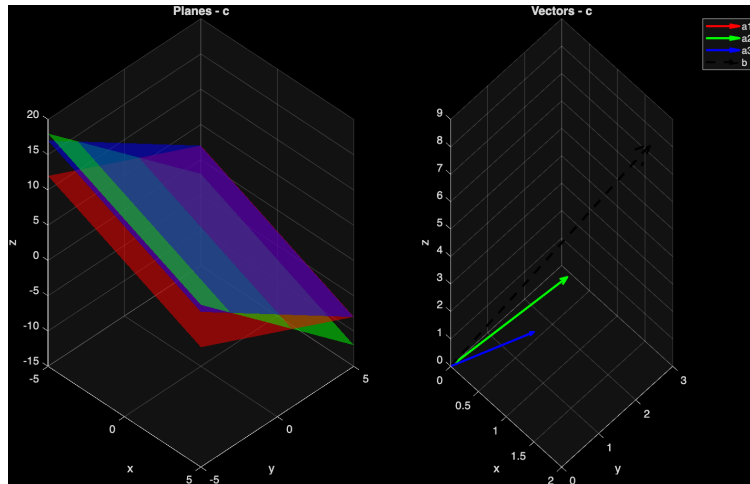


Figure 3: Equations 3

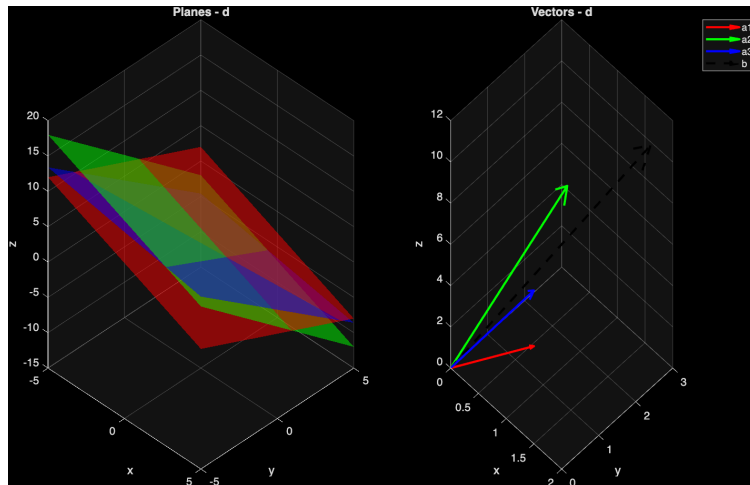


Figure 4: Equations 4

Solutions and RREF

```
Rank = 3

Final transformation matrix E_total:
  -1    2   -3
   0    0    1
   1   -1    1

RREF of augmented:
   1    0   -1    0
   0    1    2    0
   0    0    0    1

rank(A) = 2, rank([A|b]) = 3
Solution: No solution
```

Figure 5: Equations 1

```
Rank = 2

Final transformation matrix E_total:
   0   -3    2
   0    2   -1
   1    1   -1

RREF of augmented:
   1    0    1    1
   0    1    0    1
   0    0    0    0

rank(A) = 2, rank([A|b]) = 2
Solution: 1 free variables
```

Figure 6: Equations 2

```

Rank = 3

Final transformation matrix E_total:
    2.2500    -0.7500    -0.2500
   -0.7500     1.2500    -0.2500
   -0.2500    -0.2500     0.2500

RREF of augmented:
    1     0     1     0
    0     1     0     0
    0     0     0     1

rank(A) = 2, rank([A|b]) = 3
Solution: No solution

```

Figure 7: Equations 3

```

Rank = 3

Final transformation matrix E_total:
    0.3333    1.3333   -0.3333
   -1.0000    1.0000     0.0000
    1.6667   -2.3333    0.3333

RREF of augmented:
    1.0000     0.0000     0.0000    0.6667
     0.0000    1.0000     0.0000    1.0000
     0.0000     0.0000    1.0000    0.3333

rank(A) = 3, rank([A|b]) = 3
Solution: Unique solution

```

Figure 8: Equations 4

3. Back substitution

Take in any $n \times n$ system of equations $\mathbf{Ax} = \mathbf{b}$ as input with the last column of the input being the output $[\mathbf{A} \mid \mathbf{b}]$. Augment the code from part 1 to generate the row echelon form, with a n algorithm for back substitution to find \mathbf{x} . Using *timeit* compare the time to generate the solutions for the system in 2.d against the inbuilt solution method. You can solve the system of equations directly in Matlab using `inv(A)*b` or using the `/` operator.

Solution

Based on the implementation of the RREF and back substitution algorithm, the system of equations from 2.d was solved. The input system is defined as:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 9 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 12 \end{bmatrix}$$

1. Augmented Matrix and RREF

The augmented matrix $[A \mid \mathbf{b}]$ was processed to reach the Row Reduced Echelon Form (RREF):

$$\text{RREF}([A \mid \mathbf{b}]) = \begin{bmatrix} 1.0000 & 0 & 0 & 0.6667 \\ 0 & 1.0000 & 0 & 1.0000 \\ 0 & 0 & 1.0000 & 0.3333 \end{bmatrix}$$

The rank of the matrix is calculated as $r = 3$, confirming that the system has a unique solution.

2. Computed Solution

By extracting the final column of the RREF (performing the equivalent of back substitution), the solution vector \mathbf{x} is:

$$\mathbf{x} = \begin{bmatrix} 0.6667 \\ 1.0000 \\ 0.3333 \end{bmatrix}$$

3. Performance Comparison

The execution time was recorded to compare the custom implementation against MATLAB's built-in methods. The results are summarized below:

Method	Time (sec)
Custom (My solution)	0.000908
Built-in $\text{inv}(A)*\mathbf{b}$	0.002221
Built-in $A \setminus \mathbf{b}$	0.000029

Code to the Assignment can be found here

https://github.com/rogue-infinity/ME6030/tree/main/Assignment_1