ME6030 Assignment

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1.Gauss elimination

Write a code to take a matrix A as input, and print the following as output:

a. Row echelon form of matrix

b. Elementary matrices used in each step

c. Rank of the given matrix

Note that you are not to use any of the internal commands for the above steps. Then use the *timeit* function to compare the time to generate the row reduced echelon form against the builtin command *rref* for the A matrix in 2.d.

Solution

Algorithm 1 Gauss–Jordan Elimination for RREF

Require: $A \in \mathbb{R}^{m \times n}$
Ensure: R: reduced row echelon form of A, rank r, elementary matrices $\{E_k\}$
Initialize $R \leftarrow A$, $pivot\_row \leftarrow 1$
Initialize empty lists for pivot columns and elementary matrices
for each column $c = 1, \ldots, n$ **do**
    if $pivot\_row > m$ **then**
        break
    end if
    Select a nonzero pivot in column c using partial pivoting
    Swap pivot row into position $pivot\_row$
    Scale pivot row to make the pivot equal to 1
    Eliminate all entries below the pivot
    Record pivot column and increment $pivot\_row$
end for
Eliminate all entries above each pivot (backward elimination)
Compute rank r as the number of nonzero rows of R
Return R, r, $\{E_k\}$ =0

The above algorithm is used to calculate the Row Reduced echelon form of the matrix in the code. It is tested on the following matrix-

$$\begin{bmatrix} 2 & 1 & -1 & 8 \\ 3 & -1 & 2 & -11 \\ -2 & 1 & 2 & -3 \end{bmatrix}$$

Both the inbuilt function and the custom function provided the same solution and the results including time are as follows-

```
RREF:
    1.0000         0         0    2.0000
         0    1.0000         0    3.0000
         0         0    1.0000   -1.0000

Rank = 3
My time: 0.001625 sec
Built-in time: 0.020758 sec
Built-in result:
     1     0     0     2
     0     1     0     3
     0     0     1    -1
```

2.Geometry of linear equations

Graphically generate the row images and column images of the following set of linear equations. Identify and describe the nature of solutions.

a. $u+v+w=2$; $u+2v+3w=1$; $v+2w=0$

b. $x+y+z=2$; $x+2y+z=3$; $2x+3y+2z=5$

c. $x+y+z=2$; $x+2y+z=3$; $2x+3y+2z=9$

d. $x+y+z=2$; $x+2y+z=3$; $2x+9y+5z=12$

Find the row reduced echelon form of each system using the code from previous steps and correlate with the nature of solutions.

Solution

Plots of the system of equations

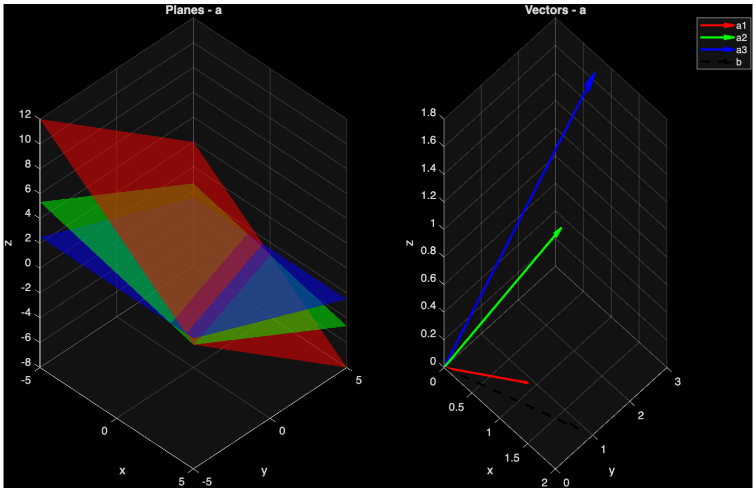

Figure 1: Equations 1

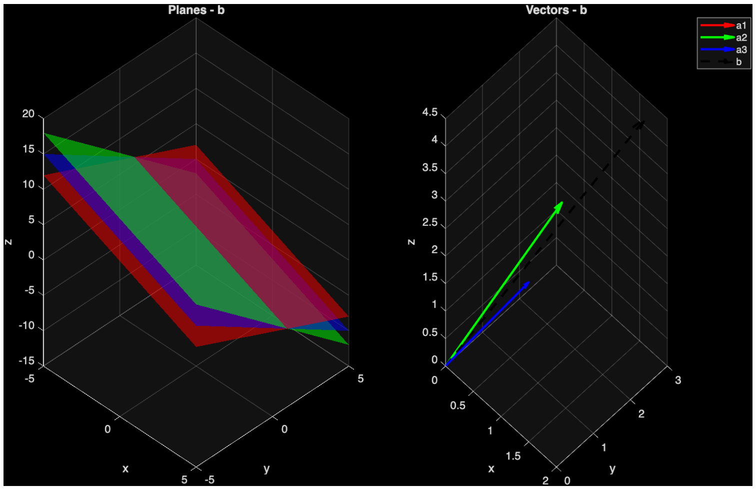

Figure 2: Equations 2

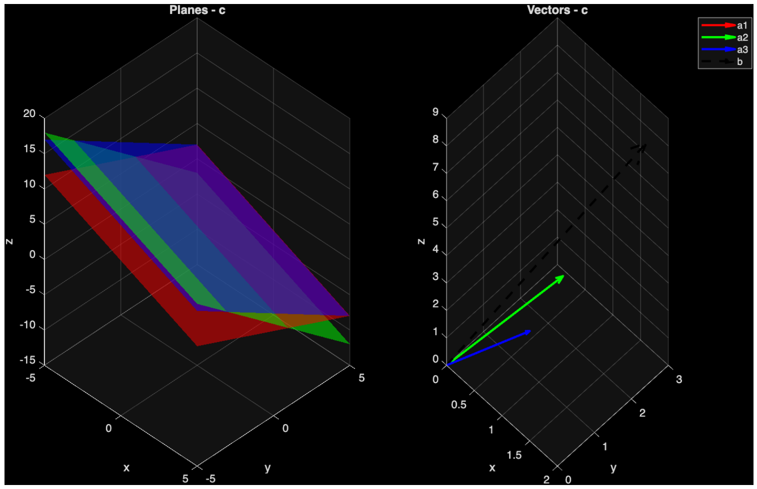

Figure 3: Equations 3

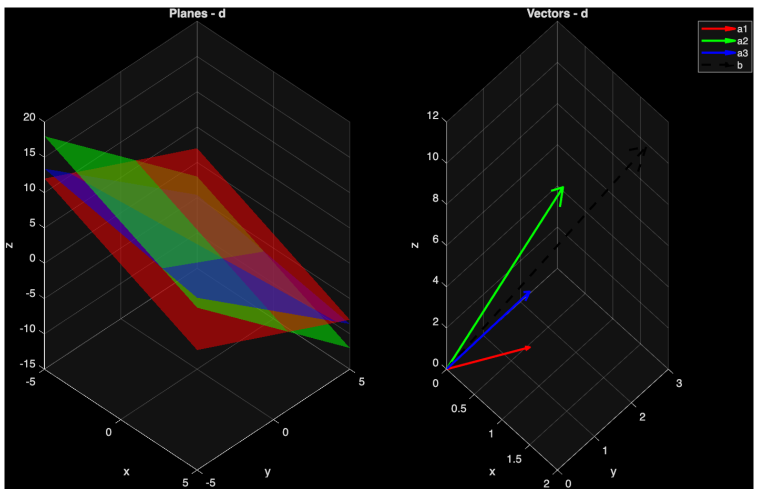

Figure 4: Equations 4

Solutions and RREF

```
Rank = 3

Final transformation matrix E_total:
    -1     2    -3
     0     0     1
     1    -1     1

RREF of augmented:
     1     0    -1     0
     0     1     2     0
     0     0     0     1

rank(A) = 2, rank([A|b]) = 3
Solution: No solution
```

Figure 5: Equations 1

```
Rank = 2

Final transformation matrix E_total:
     0    -3     2
     0     2    -1
     1     1    -1

RREF of augmented:
     1     0     1     1
     0     1     0     1
     0     0     0     0

rank(A) = 2, rank([A|b]) = 2
Solution: 1 free variables
```

Figure 6: Equations 2

```
Rank = 3

Final transformation matrix E_total:
    2.2500   -0.7500   -0.2500
   -0.7500    1.2500   -0.2500
   -0.2500   -0.2500    0.2500

RREF of augmented:
     1     0     1     0
     0     1     0     0
     0     0     0     1

rank(A) = 2, rank([A|b]) = 3
Solution: No solution
```

Figure 7: Equations 3

```
Rank = 3

Final transformation matrix E_total:
    0.3333    1.3333   -0.3333
   -1.0000    1.0000         0
    1.6667   -2.3333    0.3333

RREF of augmented:
    1.0000         0         0    0.6667
         0    1.0000         0    1.0000
         0         0    1.0000    0.3333

rank(A) = 3, rank([A|b]) = 3
Solution: Unique solution
```

Figure 8: Equations 4

3.Back substitution

Take in any $n \times n$ system of equations $\mathbf{Ax} = \mathbf{b}$ as input with the last column of the input being the output $[A \mid b]$. Augment the code from part 1 to generate the row echelon form, with a n algorithm for back substitution to find $\mathbf{x}$. Using *timeit* compare the time to generate the solutions for the system in 2.d against the inbuilt solution method. You can solve the system of equations directly in Matlab using $inv(A) * b$ or using the / operator.

Solution

Based on the implementation of the RREF and back substitution algorithm, the system of equations from 2.d was solved. The input system is defined as:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 9 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 12 \end{bmatrix}$$

1. Augmented Matrix and RREF

The augmented matrix $[A \mid \mathbf{b}]$ was processed to reach the Row Reduced Echelon Form (RREF):

$$\text{RREF}([A \mid \mathbf{b}]) = \begin{bmatrix} 1.0000 & 0 & 0 & 0.6667 \\ 0 & 1.0000 & 0 & 1.0000 \\ 0 & 0 & 1.0000 & 0.3333 \end{bmatrix}$$

The rank of the matrix is calculated as $r = 3$, confirming that the system has a unique solution.

2. Computed Solution

By extracting the final column of the RREF (performing the equivalent of back substitution), the solution vector $\mathbf{x}$ is:

$$\mathbf{x} = \begin{bmatrix} 0.6667 \\ 1.0000 \\ 0.3333 \end{bmatrix}$$

3. Performance Comparison

The execution time was recorded to compare the custom implementation against MATLAB's built-in methods. The results are summarized below:

Method	**Time (sec)**	**Speed Relative to Custom**
Custom (My solution)	0.000908	1.00x
Built-in `inv(A)*b`	0.002221	0.41x
Built-in `A\b`	0.000029	31.31x