$$\text{Im}[\chi_{e,\omega}^{\text{abc}}] = \frac{\pi|e|^3}{2\hbar^2} \int \frac{d^3k}{8\pi^3} \sum_{vc} \sum_{q\neq(v,c)} \frac{1}{\omega_{cv}^{\Sigma}} \left[ \frac{\text{Im}[\mathcal{V}_{qc}^{\Sigma,\text{a}}\{r_{cv}^{\text{b}}r_{vq}^{\text{c}}\}]}{(2\omega_{cv}^{\Sigma} - \omega_{cq}^{\Sigma})} - \frac{\text{Im}[\mathcal{V}_{vq}^{\Sigma,\text{a}}\{r_{qc}^{\text{c}}r_{cv}^{\text{b}}\}]}{(2\omega_{cv}^{\Sigma} - \omega_{qv}^{\Sigma})} \right] \delta(\omega_{cv}^{\Sigma} - \omega)$$

$$\text{Im}[\chi_{e,2\omega}^{\text{abc}}] = -\frac{\pi|e|^3}{2\hbar^2} \int \frac{d^3k}{8\pi^3} \sum_{vc} \frac{4}{\omega_{cv}^{\Sigma}} \left[ \sum_{v'\neq v} \frac{\text{Im}[\mathcal{V}_{vc}^{\Sigma,\text{a}}\{r_{cv'}^{\text{b}}r_{v'v}^{\text{c}}\}]}{2\omega_{cv'}^{\Sigma} - \omega_{cv}^{\Sigma}} - \sum_{c'\neq c} \frac{\text{Im}[\mathcal{V}_{vc}^{\Sigma,\text{a}}\{r_{cc'}^{\text{c}}r_{c'v}^{\text{b}}\}]}{2\omega_{c'v}^{\Sigma} - \omega_{cv}^{\Sigma}} \right] \delta(\omega_{cv}^{\Sigma} - 2\omega)$$