$$\mathrm{Im}[\chi^{\mathrm{abc}}_{e,\omega}] = \frac{\pi|e|^3}{2\hbar^2}\int\frac{d^3k}{8\pi^3}\sum_{vc}\sum_{q\neq(v,c)}\frac{1}{\omega^{\Sigma}_{cv}}\left[\frac{\mathrm{Im}[\mathcal{V}^{\Sigma,\mathrm{a}}_{qc}\{r^{\mathrm{b}}_{cv}r^{\mathrm{c}}_{vq}\}]}{(2\omega^{\Sigma}_{cv}-\omega^{\Sigma}_{cq})}-\frac{\mathrm{Im}[\mathcal{V}^{\Sigma,\mathrm{a}}_{vq}\{r^{\mathrm{c}}_{qc}r^{\mathrm{b}}_{cv}\}]}{(2\omega^{\Sigma}_{cv}-\omega^{\Sigma}_{qv})}\right]\delta(\omega^{\Sigma}_{cv}-\omega)$$

$$\mathrm{Im}[\chi^{\mathrm{abc}}_{e,2\omega}] = -\frac{\pi|e|^3}{2\hbar^2}\int\frac{d^3k}{8\pi^3}\sum_{vc}\frac{4}{\omega^{\Sigma}_{cv}}\left[\sum_{v'\neq v}\frac{\mathrm{Im}[\mathcal{V}^{\Sigma,\mathrm{a}}_{vc}\{r^{\mathrm{b}}_{cv'}r^{\mathrm{c}}_{v'v}\}]}{2\omega^{\Sigma}_{cv'}-\omega^{\Sigma}_{cv}}-\sum_{c'\neq c}\frac{\mathrm{Im}[\mathcal{V}^{\Sigma,\mathrm{a}}_{vc}\{r^{\mathrm{c}}_{cc'}r^{\mathrm{b}}_{c'v}\}]}{2\omega^{\Sigma}_{c'v}-\omega^{\Sigma}_{cv}}\right]\delta(\omega^{\Sigma}_{cv}-2\omega)$$