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AFD Department

MSA Toolkit Documentation

REV. *0*

IPT MSA
INTERNAL REPORT

Skyward Experimental Rocketry
Politecnico di Milano

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Abstract

The purpose of this document is to describe the contents of the MSA-toolkit folder and to underline the mechanics involved in the simulator. A flow chart of the simulator is presented first, and then the equations used in the simulator will be discussed in the last two chapters. The simulator was created in order to predict the behavior of a sounding rocket flight during all its flight phases. All calculations (trajectory, forces, acceleration, etc.) are used to size the rockets and to build up the trajectory envelope.

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Contents

Change Log

Date	**Revision**	**Editor**	**Changes**
20/04/2015	0	R. Di Battista, A. Rivolta	First version.
14/04/2018	1	A. Ben Dia, L. Mignozzi, M. Pozzoli	Second version.
14/04/2021	2	F. Artioli, A.F. Inno, G. Pacifici, D. Rosato	Revision of the old documentation. Introduction of the 6 D.o.F. descent model.

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Chapter 1

Introduction to the MSA toolkit

The MSA toolkit is the repository in which the codes implemented by the MSA team are stored. It is composed by several folders that will be briefly introduced in the following paragraphs.

simulator. This is a code developed in MATLAB for the simulation of 6 d.o.f. rocket dynamics. The simulator predicts 3D trajectory, apogee, forces acting on the rockets, and various other aerodyanmics data. The dynamics equations used, are presented in 2 and 3.

data. Folder with the current flight data, rocket geometry and simulation parameters.

commonFunctions. In this folder, the common functions employed in the codes of the toolkit are stored.

autoMatricesProtub. This code allows an automatic computation of the rocket aerodynamic coefficients using Missile DATCOM, for different aerobrakes configuration.

aerodynamicsOptimization. This code implements an aerodynamics optimization of the rocket. The optimization variables are the fin chords and heigth, the fin shape, the ogive length and the ogive shape. The code uses the genetic algorithm to reach the aim.

apogeeAnalysis. This code implements a primary apogee analysis with different motors when the structural mass is known with a degree of uncertainty, in order to chose the best one.

sensitivityAnalysis. This code implements a sensitivity analysis on the ascent phase of the rocket. Two types of analysis are available: deterministic and stochastic.
In the deterministic analysis it is possible to vary the nominal values of the aerodynamics coefficients and the structural mass of the rocket. The relative magnitude of the variations is set by the user and it is the same for all the parameters considered in the analysis.
In the stochastic analysis several simulations are performed, in each of these the uncertain parameters assume values according to a normal distribution centered on their respective nominal values.

utils. Although the most important folders are in the main path, this folder contains work-alone tools, and some useful scripts.

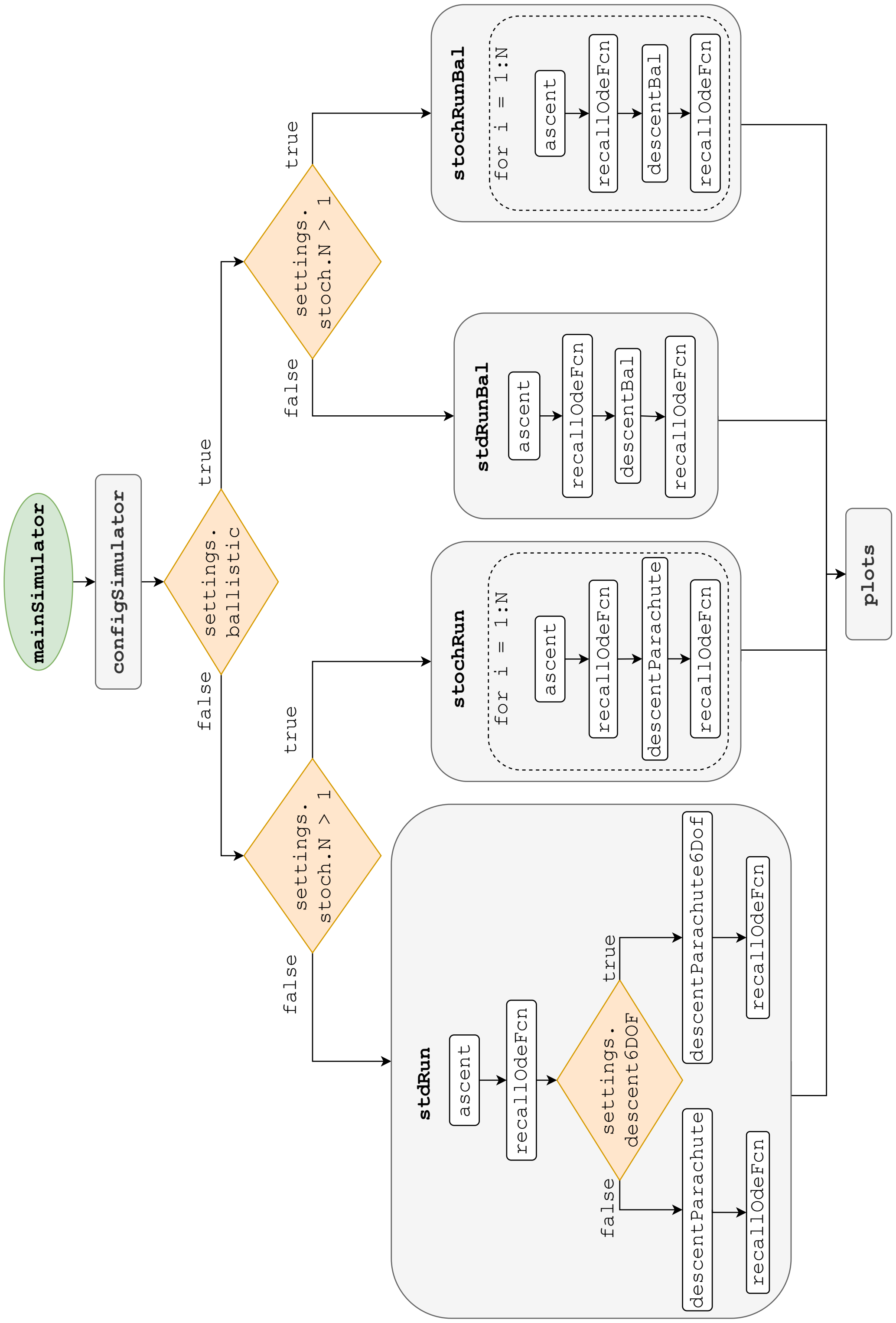

Figure 1.1: Simulator Flowchart

Chapter 2

6 D.o.F. Model of an Ascending Rocket

2.1 Assumptions

Flat Earth. The altitude and the length of the flight path are relatively small in comparison with the Earth typical lengths so the curvature has been neglected.

Earth's surface-centered frame as inertial frame. The acceleration given by the most important Earth's movement, rotation, has been neglected. This assumption has the meaning of considering the reference frame placed on the surface of the Earth as inertial or, equivalently, the Coriolis accelerations are not considered.

Axisymmetric Rocket. The body-centered frame, that is the center of mass of the rocket, has one axis (the x-axis) that gives mass symmetry, the other two axes are taken to complete the principal reference frame [1].

2.2 Reference Frames

The path flown by the rocket could be requested in different reference frames. In particular the trajectory, given in the form of spatial coordinates x, y, z, is requested in the inertial frame. The velocities u, v, w could be given in the inertial frame or in the body frame; so a distinction and explanation of the different reference frames used in the dynamics model is now exposed.

[1] In a principal reference frame the inertia tensor $\mathbf{I}$ is a diagonal matrix $\begin{bmatrix} I_x & & \\ & I_y & \\ & & I_z \end{bmatrix}$ that doesn't change with attitude variation.

2.2.1 North-East-Down Frame (NED)

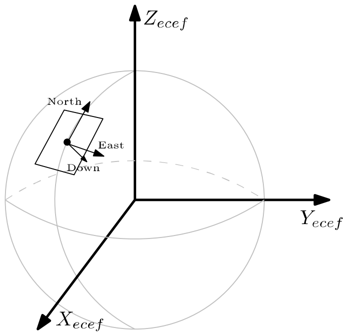

Figure 2.1: NED frame

The inertial frame on the Earth surface is intended as a North-East-Down frame. The x-axis is pointing North, the y-axis is pointing East, the z-axis is pointing downward to the Earth's center. The spatial coordinates describing the trajectory are given in this reference frame.

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{NED}} \tag{2.1}$$

2.2.2 Body Frame

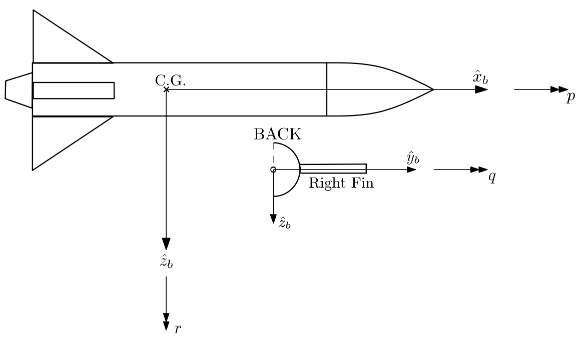

Figure 2.2: Body frame

The body frame is centered in the center of mass of the rocket, aligned with the mass symmetry axes (that, as stated in section 2.1, coincide with symmetry axes). The linear velocities could be expressed in this frame

$$\mathbf{v} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}_B \tag{2.2}$$

and the angular rates are also expressed in this reference frame

$$\boldsymbol{\omega} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_B \tag{2.3}$$

2.3 Rotations

Since some forces are easier to be written in NED frame and others in body frame, rotations from a frame to the other are needed. The rotations are achieved through the use of unit quaternions as reported in [1].

This parametrization has been chosen to avoid singularities in the integration.

2.4 Dynamics Equations

The equations are written in the body frame, that is not an inertial frame, as referred in 2.2.2.

2.4.1 Mass Properties

The rocket's mass properties $(m(t), \mathbf{I}(t))$ are functions of time: they change during the thrusted flight. The model used is a linear model thus the rates of change for the mass properties are constant:

$$\begin{cases} \dot{m} = \text{constant} = -\dfrac{m_0 - m(t_b)}{t_b} \\ \dot{\mathbf{I}} = \text{constant} = -\dfrac{\mathbf{I}_f - \mathbf{I}_e}{t_b} \end{cases} \tag{2.4}$$

where $\mathbf{I}_f$ and $\mathbf{I}_e$ are respectively the inertia tensor for the wet configuration (with all the propellant embarked) and dry configuration (all the propellant burnt), calculated with CAD softwares or experimentally.

Sometimes it is possible to retrieve information on the propellant mass variation of a certain motor. In these cases the mass can be computed at each instant of time as the sum of the structural mass and the propellant mass.

2.4.2 Translational Equation

Using Poisson's relation for the kinematics in a non-inertial frame, the momentum equation for the rocket would be:

$$\left(\frac{d\mathbf{p}}{dt}\right)_{NED} = \left(\frac{d\mathbf{p}}{dt}\right)_B + \boldsymbol{\omega}_B \times \mathbf{p}_B = \sum_i \mathbf{f}_{i,ext} \tag{2.5}$$

Where $\mathbf{p}$ is the momentum of the rocket, and $\mathbf{f}_{i,ext}$ are the external forces acting on the rocket (thrust excluded). The time derivative of the momentum $\mathbf{p}$ in body frame can be determined considering the value of $\mathbf{p}$ at the time instant $t + dt$.

$$\mathbf{p} + d\mathbf{p} = (m - \dot{m}_p \cdot dt)(\mathbf{v} + d\mathbf{v}) + \dot{m}_p \cdot dt\,(\mathbf{u}_p + \mathbf{v}) \tag{2.6}$$

Where $\dot{m}_p$ is propellant mass-flow rate, and $\mathbf{u}_p$ is the the exit velocity of the propellant.
Considering that $\mathbf{p} = m\mathbf{v}$, we can compute the time derivative of $\mathbf{p}$ as follows.

$$\frac{d\mathbf{p}}{dt} = \lim_{dt \to 0} \frac{(\mathbf{p} + d\mathbf{p}) - \mathbf{p}}{dt} = m\frac{d\mathbf{v}}{dt} + \dot{m}_p \cdot \mathbf{u}_p \tag{2.7}$$

Equation 2.5 can be rewritten as follows.

$$m\left(\frac{d\mathbf{v}}{dt}\right)_B + \boldsymbol{\omega}_B \times \mathbf{p}_B = \sum_i \mathbf{f}_{i,ext} - \dot{m}_p \cdot \mathbf{u}_p = \sum_i \mathbf{f}_i \tag{2.8}$$

Where $\sum_i \mathbf{f}_i$ is the total force acting on the rocket including thrust (that is equal to $-\dot{m}_p \cdot \mathbf{u}_p$). The previous equation can be projected along the three body axes:

$$\begin{cases} m(\dot{u} + qw - rv) & = & f_x \\ m(\dot{v} + ru - pw) & = & f_y \\ m(\dot{w} + pv - qu) & = & f_z \end{cases} \tag{2.9}$$

f_x, f_y, f_z are the external forces acting on the rocket during flight, along the body axes.

$$\begin{cases} f_x & = & \mathcal{T} - \mathcal{X} + \mathcal{W}_x \\ f_y & = & \mathcal{Y} + \mathcal{W}_y \\ f_z & = & -\mathcal{Z} + \mathcal{W}_z \end{cases} \tag{2.10}$$

where $\mathcal{T}$ is the thrust, $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ are the components of aerodynamic force in body axes, $\mathcal{W}_x, \mathcal{W}_y, \mathcal{W}_z$ are the components of weight in body axes [2].

The sign of the aerodynamic forces are selected according to the reference system depicted in page 3 of the Missile Datcom User Manual 2011 [5]. Indeed the x and z axes of this reference system are inverted with respect to the frame in 2.2.2.

[2] $\begin{bmatrix} \mathcal{W}_x \\ \mathcal{W}_y \\ \mathcal{W}_z \end{bmatrix}_B = \mathbf{R}_{NED \to B} \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}_{NED}$

2.4.3 Rotational Equation

Using Poisson's relation for the kinematics in a non-inertial frame, the rotational momentum equation for the rocket would be:

$$\left(\frac{d\mathbf{k}}{dt}\right)_{NED} = \left(\frac{d\mathbf{k}}{dt}\right)_{B} + \boldsymbol{\omega}_B \times \mathbf{p}_B = \sum_i \mathbf{m}_i \tag{2.11}$$

being

$$\mathbf{k}_B = \mathbf{I}\boldsymbol{\omega}_B = \begin{bmatrix} pI_x \\ qI_y \\ rI_z \end{bmatrix} \tag{2.12}$$

In an expanded form we get:

$$\begin{cases} \dot{p}I_x + p\dot{I}_x + qr(I_z - I_y) & = & m_x \\ \dot{q}I_y + q\dot{I}_y + pr(I_x - I_z) & = & m_y \\ \dot{r}I_z + r\dot{I}_z + pq(I_y - I_x) & = & m_z \end{cases} \tag{2.13}$$

where m_x, m_y, m_z are the external moments acting on the rocket around the body axes. So, since thrust misalignment is neglected, it follows:

$$\begin{cases} m_x & = & \mathcal{L} \\ m_y & = & \mathcal{M} \\ m_z & = & \mathcal{N} \end{cases} \tag{2.14}$$

where $\mathcal{L}$, $\mathcal{M}$, $\mathcal{N}$ are respectively the roll, pitch and yaw aerodynamic moment acting on the rocket .

$\dot{I}_i$ is the i-th rate of change of inertia tensor, calculated as:

$$\dot{\mathbf{I}} = \frac{\mathbf{I}_f - \mathbf{I}_e}{t_b} = \begin{bmatrix} \dot{I}_x & & \\ & \dot{I}_y & \\ & & \dot{I}_z \end{bmatrix} \tag{2.15}$$

2.4.4 Kinematics Relations

The attitude expressed through a quaternion $\mathbf{q}$ is related to angular rates of the rocket. The expression that relates the two quantities is (as reported in [1]) is:

$$\dot{\mathbf{q}} = \frac{1}{2} \underbrace{\begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix}}_{\Omega} \mathbf{q} \tag{2.16}$$

2.4.5 Aerodynamics Model

The model for the aerodynamic actions acting on the rocket is taken from [2] [3], and rearranged as described in [3].

$$\begin{cases} \mathcal{X} &= \frac{1}{2}\rho\,|\tilde{\mathbf{u}}|^2\, SC_A \\ \mathcal{Y} &= \frac{1}{2}\rho\,|\tilde{\mathbf{u}}|^2\, SC_Y \\ \mathcal{Z} &= \frac{1}{2}\rho\,|\tilde{\mathbf{u}}|^2\, SC_N \\ \mathcal{L} &= \frac{1}{2}\rho\,|\tilde{\mathbf{u}}|\, C\left(|\tilde{\mathbf{u}}|\, C_l + \frac{C_{lp} p\, C}{2}\right) \\ \mathcal{M} &= \frac{1}{2}\rho\,|\tilde{\mathbf{u}}|\, C\left(|\tilde{\mathbf{u}}|\, C_m + \frac{(C_{m\dot{\alpha}} + C_{mq})qC}{2}\right) \\ \mathcal{N} &= \frac{1}{2}\rho\,|\tilde{\mathbf{u}}|\, C\left(|\tilde{\mathbf{u}}|\, C_n + \frac{(C_{nr} r + C_{np} p)C}{2}\right) \end{cases} \tag{2.17}$$

Where $\tilde{\mathbf{u}}$ is the relative velocity to wind:

$$\tilde{\mathbf{u}} = \mathbf{u} - \mathbf{u}_{\text{wind}} \tag{2.18}$$

ρ is the air density at a given altitude computed with the ISA (International Standard Atmosphere) model. C is the rocket's diameter, S is the rocket's cross section.

The Aerodynamic coefficients are calculated using the Missile Datcom software version 1997 (see [4] for a complete documentation).

[3] Rewritten in a way that avoids the presence of $|\tilde{\mathbf{u}}|$ at the denominator that is a issuing situation in the first phases of simulation when $|\tilde{\mathbf{u}}| \approx \mathbf{0}$

2.4.6 Final ODE System

The final ODE system counts 13 differential equations and it's reported here below:

$$\begin{cases} \dot{x} &= u \\ \dot{y} &= v \\ \dot{z} &= w \\ \dot{u} &= \frac{\mathcal{T}-\mathcal{X}+\mathcal{W}_x-m(qw-rv)-\dot{m}u}{m} \\ \dot{v} &= \frac{\mathcal{Y}+\mathcal{W}_y-m(ru-pw)-\dot{m}v}{m} \\ \dot{w} &= \frac{-\mathcal{Z}+\mathcal{W}_z-m(pv-qu)-\dot{m}w}{m} \\ \dot{p} &= \frac{\mathcal{L}-qr(I_z-I_y)-p\dot{I}_x}{I_x} \\ \dot{q} &= \frac{\mathcal{M}-pr(I_x-I_z)-q\dot{I}_y}{I_y} \\ \dot{r} &= \frac{\mathcal{N}-pq(I_y-I_x)-r\dot{I}_z}{I_z} \\ \dot{q}_0 &= -q_1p - q_2q - q_3r \\ \dot{q}_1 &= q_0p + q_3q - q_2r \\ \dot{q}_2 &= -q_3p + q_0q + q_1r \\ \dot{q}_3 &= q_2p - q_1q + q_0r \end{cases} \tag{2.19}$$

Note that the velocity for the spatial coordinates are in NED frame, so the velocity vector must be rotated:

$$\mathbf{u}_{\text{NED}} = \mathbf{R}_{\text{B}\to\text{NED}}\mathbf{u}_{\text{B}} \tag{2.20}$$

Chapter 3

Descent models

The models of the parachutes are here presented.

3.1 3 D.o.F Model for Parachute descent

The model used for the parachute is quite simpler, compared to the rocket one, in the modelization and in the assumptions. The descending rocket body is modeled as a point with mass $\tilde{m} = m_s + m_{para}$ which has a reference surface S_{para} and drag coefficient $C_{D_{para}}$.

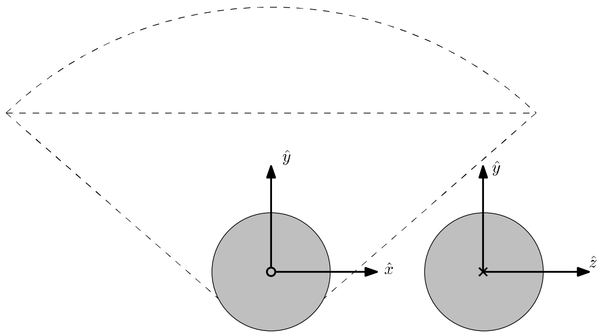

Figure 3.1: Rocket descent with parachute model.

The equations describing the descending trajectory of the body rocket+parachute are only translational; recalling 2.5 and since $\boldsymbol{\omega} = 0$ for this model, the ODE system is:

$$\left(\frac{d\mathbf{p}}{dt}\right)_{NED} = \left(\frac{d\mathbf{p}}{dt}\right)_{B} + \boldsymbol{\omega}_B \times \mathbf{p}_B = \sum_i \mathbf{f}_i \tag{3.1}$$

$$\begin{cases} \tilde{m}\dot{u} &= -\mathcal{D}_x \\ \tilde{m}\dot{v} &= -\mathcal{D}_y \\ \tilde{m}\dot{w} &= -\mathcal{D}_z + \tilde{m}g \end{cases} \tag{3.2}$$

3.1.1 Aerodynamics Model

The model adopted in this phase for the aerodynamics characteristics is simple. The aerodynamic coefficient for the parachute and its surface are assumed constant. Drag of linkers and strings is neglected.

$$\mathcal{D} = \frac{1}{2}\rho\,|\tilde{\mathbf{u}}|^2\,S_{para}C_{D_{para}} \tag{3.3}$$

$$\begin{cases} \mathcal{D}_x & = \mathcal{D}\cdot\frac{u}{|\tilde{\mathbf{u}}|} \\ \mathcal{D}_y & = \mathcal{D}\cdot\frac{v}{|\tilde{\mathbf{u}}|} \\ \mathcal{D}_z & = \mathcal{D}\cdot\frac{w}{|\tilde{\mathbf{u}}|} \end{cases} \tag{3.4}$$

3.1.2 Final ODE System

The final ODE system for rocket descent with parachute counts 6 ordinary differential equations:

$$\begin{cases} \dot{x} & = u \\ \dot{y} & = v \\ \dot{z} & = w \\ \tilde{m}\dot{u} & = -\mathcal{D}_x \\ \tilde{m}\dot{v} & = -\mathcal{D}_y \\ \tilde{m}\dot{w} & = -\mathcal{D}_z + \tilde{m}g \end{cases} \tag{3.5}$$

3.2 6 D.o.F Model for Parachute descent

3.2.1 Hypothesis and assumptions

In the simulator the following assumptions have been considered:

- **Inertial frame of reference**: the acceleration given by the Earth rotation is tiny in comparison of the flight acceleration. Furthermore the flight has a very short time span, so in first approximation the Coriolis acceleration can be neglected.
- **No aerodynamic force on rocket**: Since parachute descent flight enables low speed, it is reasonable to neglect the influence of aerodynamic forces on rocket.
- **Parachute as point mass**: all the simulated parachutes are considered as points with constant mass.
- **Drag force**: only the parachutes that are in the opening process or fully opened generate drag. The drag on closed parachutes and on the missile is neglected. The drag is always parallel to the air speed vector and in the opposite direction.

3.2.2 Reference frames

Analogously with the ascent outputs, the rocket dynamics are computed in the body frame (see 2.2.2), while the parachute dynamics are instead computed in NED frame (see 2.2.1).

3.2.3 Aerodynamic model

Since the flight envelope of the descent is strictly subsonic, we can use the basic drag formula:

$$\mathcal{D} = \frac{1}{2}\rho\,|\tilde{\mathbf{u}}|^2\,S_{para}C_{D_{para}} \tag{3.6}$$

The trajectory integration gives us all the needed elements for the drag calculation with the exception of the drag area. The algorithm for the determination of the C_dS for a normal parachute is taken from [6] and can be seen in 3.2.3.1.

Knowing all the terms of the drag formula it's possible to decompose the force along the three NED axis:

$$\mathbf{D} = -\frac{\tilde{\mathbf{u}}}{\tilde{u}}\mathcal{D} \tag{3.7}$$

3.2.3.1 Normal parachute

1. At first are calculated the opening time, the time at which the canopy reaches the nominal drag area, t_0:

$$t_0 = \frac{n_f d^0}{\tilde{u}} \tag{3.8}$$

   and the over-expansion time:

$$t_x = t_0 C_X^{\frac{1}{m}} \tag{3.9}$$

2. the opening process start when the parachute line reach the nominal length, from this moment the drag area grows until the over expansion following a curve with grade m:

$$C_D S(t) = C_D S^0 \left(\frac{t}{t_0}\right)^m \tag{3.10}$$

3. from the over-expansion the drag area reach the nominal value following an exponential curve:

$$C_D S(t) = C_D S^0 \left(1 + (C_X - 1)\exp\left(2\frac{t_x - t}{t_0}\right)\right) \tag{3.11}$$

3.2.4 Rope model

The parachutes in this simulation are connected to the missile by shock cords. All the cords are simulated by a spring damper model, so for the calculation of the tension the only quantities needed are the distance of the parachute from the rocket and the relative speed.

Thus, the equations are:

$$\begin{aligned}
\mathbf{r_{rel}} &= \mathbf{r^r} + \mathbf{r_{CG}} - \mathbf{r^p} \\
\tilde{\mathbf{u}}_{\mathbf{rel}} &= \tilde{\mathbf{u}}^{\mathbf{r}} + (\mathbf{w^r} \times \mathbf{r_{CG}}) - \tilde{\mathbf{u}}^{\mathbf{p}} \\
\mathbf{T} &= k(|||\mathbf{r_{rel}}|| - L^0) + c\,||\tilde{\mathbf{u}}_{\mathbf{rel}}||
\end{aligned} \tag{3.12}$$

Note that if the rope is not stretched (the parachute is closer to the payload than the nominal rope length) there is no tension exchanged.

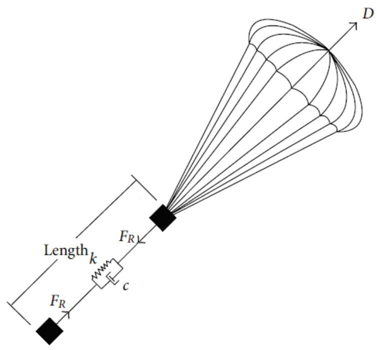

Figure 3.2: Model of the shock cord

Since rocket and parachutes dynamics are referred to different reference frames (NED for the rocket and body for the parachutes), the modulus of the force must be projected both in NED and body frame, but this time on the unit vector that connects the missile and the parachute, following the sign convention of 3.2 the following equation is obtained:

$$\mathbf{T} = \frac{\mathbf{r}_{rel}}{r_{rel}} T \tag{3.13}$$

Since the rocket is modeled as a rigid body, its rotations due to cord tension must be taken into account.

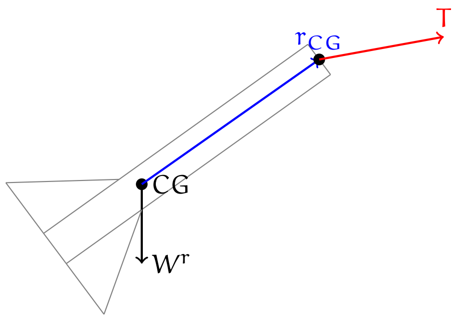

Figure 3.3: Rocket rotation due to cord Tension

Knowing the distance between rocket center of mass and the base of the nosecone, the cord tension force momentum is given by:

$$\mathbf{M} = \mathbf{r}_{CG} \times \mathbf{T} \tag{3.14}$$

3.2.5 Final ODE System

For an easier implementation of the model, the descent trajectory has been split in three phases: the first goes from apogee altitude to the altitude at which main parachute extraction is scheduled. The second from the start of the main parachute extraction to the first line stretch of the main shock cord; the last from the start of the opening of the main parachute to the landing. All the three phases are modeled by a first order differential equation problem:

$$\dot{\mathbf{y}}(t) = \mathbf{f}(t, \mathbf{y}(t))$$

The following sections exploit the dynamic model chosen for each one.

3.2.5.1 Drogue parachute descent

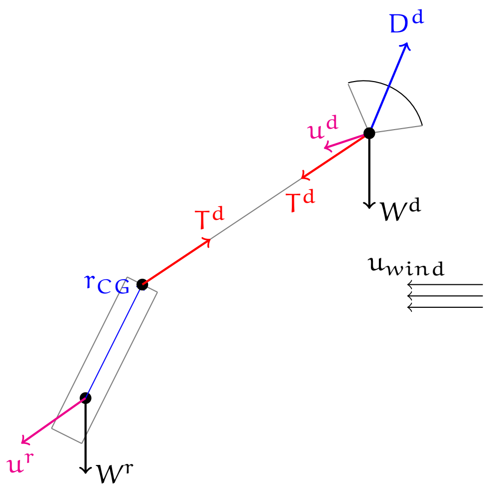

Figure 3.4: Drogue descent model

The first phase starts with the nosecone expulsion and drogue chute extraction process at apogee. In this phase there is a tension force between the rocket and the drogue, the drag force is applied only on the parachute and it's calculated by the opening model explained in section 3.2.3.1. The nosecone is modeled only as an added mass to the drogue parachute. The system solved at each time step is:

$$\begin{cases} \dot{\mathbf{r}}^r & = \mathbf{u}^r \\ \dot{\mathbf{r}}^d & = \mathbf{u}^d \\ \dot{\mathbf{u}}^r & = \mathbf{W}^r + \mathbf{T}^d \\ \dot{\mathbf{u}}^d & = \mathbf{W}^d + \mathbf{T}^d + \mathbf{D}^d \\ \dot{p} & = \frac{M_X - qr(I_z - I_y)}{I_x} \\ \dot{q} & = \frac{M_Y - pr(I_x - I_z)}{I_y} \\ \dot{r} & = \frac{M_Z - pq(I_y - I_x)}{I_z} \\ \dot{q}_0 & = -q_1 p - q_2 q - q_3 r \\ \dot{q}_1 & = q_0 p + q_3 q - q_2 r \\ \dot{q}_2 & = -q_3 p + q_0 q + q_1 r \\ \dot{q}_3 & = q_2 p - q_1 q + q_0 r \end{cases} \tag{3.15}$$

Where M_X, M_Y, M_Z are given by 3.14.

3.2.5.2 Main parachute extraction

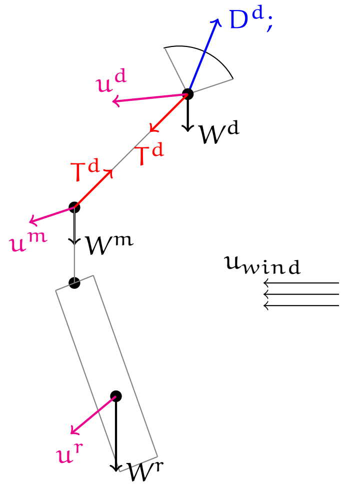

Figure 3.5: Main extraction model

This phase simulates the extraction of the deployment bag from the rocket and ends when the shock cord of the main parachute reaches the nominal length. The drogue extracts the main parachute from the rocket by applying a tension force on it. Since in this phase the shock cord of the main is never stretched, there are no forces acting on the rocket aside the gravity. This means that the payload is considered in free fall. Since the time elapsed from the opening of the drogue parachute is >> t_0, from now on we can assume its drag area constant. The system solved at each time step is:

$$\begin{cases} \dot{\mathbf{r}}^{\,r} &= \mathbf{u}^r \\ \dot{\mathbf{r}}^{\,d} &= \mathbf{u}^d \\ \dot{\mathbf{r}}^{\,m} &= \mathbf{u}^m \\ \dot{\mathbf{u}}^{\,r} &= \mathbf{W}^r \\ \dot{\mathbf{u}}^{\,d} &= \mathbf{W}^d + \mathbf{T}^d + \mathbf{D}^d \\ \dot{\mathbf{u}}^{\,m} &= \mathbf{W}^m + \mathbf{T}^{\,d} \\ \dot{p} &= \frac{-qr(I_z - I_y)}{I_x} \\ \dot{q} &= \frac{-pr(I_x - I_z)}{I_y} \\ \dot{r} &= \frac{-pq(I_y - I_x)}{I_z} \\ \dot{q}_0 &= -q_1 p - q_2 q - q_3 r \\ \dot{q}_1 &= q_0 p + q_3 q - q_2 r \\ \dot{q}_2 &= -q_3 p + q_0 q + q_1 r \\ \dot{q}_3 &= q_2 p - q_1 q + q_0 r \end{cases} \tag{3.16}$$

3.2.5.3 Main parachute descent

The last phase starts with the opening of the main chute and ends with the rocket touchdown. This time the main and the drogue parachutes are connected to the rocket and exert a force on it. The main parachute

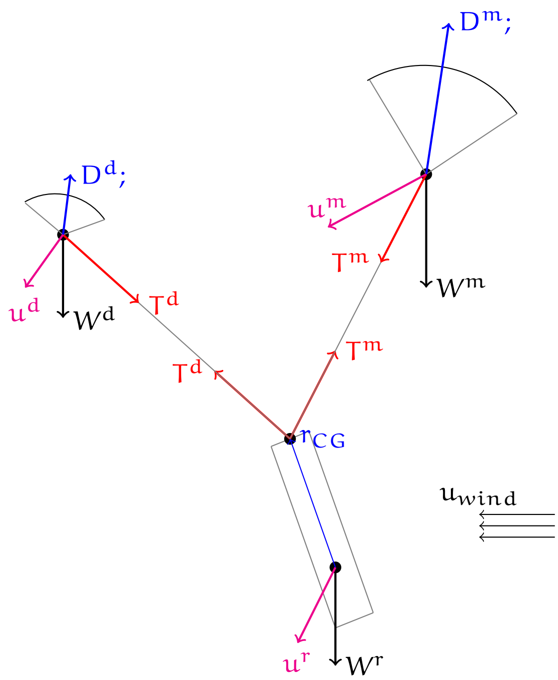

Figure 3.6: Main descent model

starts its opening process, so its drag area is calculated by the algorithm presented in section 3.2.3.1, while, as before, the drogue chute has a constant drag area.
The system solved at each time step is:

$$\begin{cases} \dot{\mathbf{r}}^{\,r} &= \mathbf{u}^r \\ \dot{\mathbf{r}}^{\,d} &= \mathbf{u}^d \\ \dot{\mathbf{r}}^{\,m} &= \mathbf{u}^m \\ \dot{\mathbf{u}}^{\,r} &= \mathbf{W}^r + \mathbf{T^d} + \mathbf{T^m} \\ \dot{\mathbf{u}}^{\,d} &= \mathbf{W}^d + \mathbf{T}^d + \mathbf{D}^d \\ \dot{\mathbf{u}}^{\,m} &= \mathbf{W}^m + \mathbf{T}^{\,m} + \mathbf{D^m} \\ \dot{p} &= \frac{M_X - qr(I_z - I_y)}{I_x} \\ \dot{q} &= \frac{M_Y - pr(I_x - I_z)}{I_y} \\ \dot{r} &= \frac{M_Z - pq(I_y - I_x)}{I_z} \\ \dot{q}_0 &= -q_1 p - q_2 q - q_3 r \\ \dot{q}_1 &= q_0 p + q_3 q - q_2 r \\ \dot{q}_2 &= -q_3 p + q_0 q + q_1 r \\ \dot{q}_3 &= q_2 p - q_1 q + q_0 r \end{cases} \tag{3.17}$$

Note that if the system is descending with a velocity which is greater than the equilibrium velocity of drogue-rocket system, the drogue parachute will dangle from the rocket and its Drag is neglected. Therefore, in that case:

$$\dot{\mathbf{u}}^{\,d} = \mathbf{W}^d + \mathbf{T}^d \tag{3.18}$$

3.3 3 D.o.F Model Validation

6 D.o.F Model for descending phase allows the computation of some parachute features (i.e. shock cords tension through a spring damper model, see 3.2.4) that the 3 D.o.F Model can not predict. However, 6 D.o.F Model requires a much bigger time to compute all those "extra" features, hence for a large number of simulations (i.e. stochastic simulation) the 3 D.o.F model is the most suitable, but a validation is needed.
Since landing point is one of the main factor when it comes to evaluate risks area, the aim of this section is to validate 3 D.o.F model basing on the errors made in the computation of landing point with respect to 6 D.o.F model.

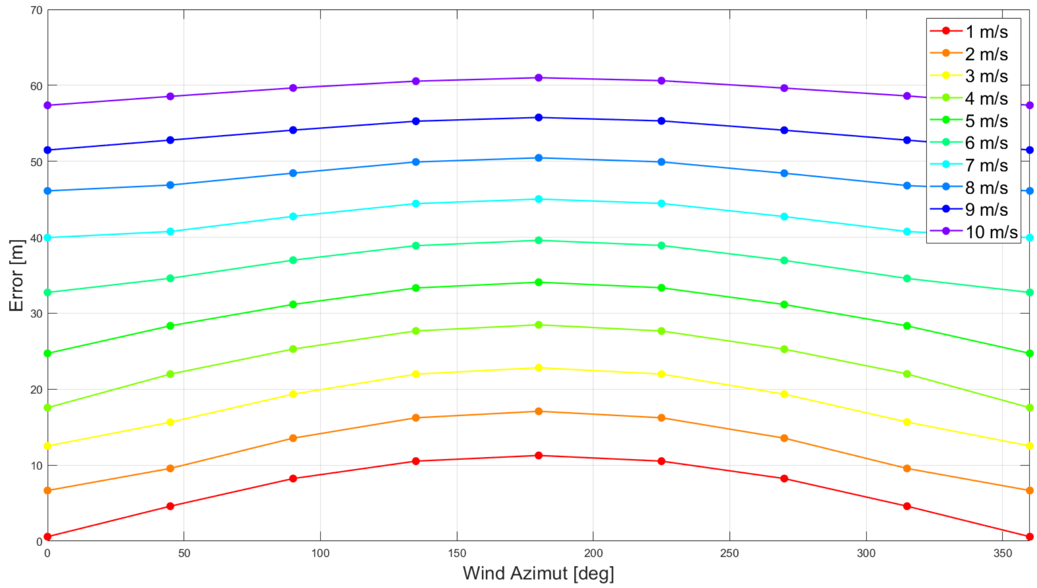

Figure 3.7: Errors made by 3 D.o.F Model

Let r_{3DoF} and r_{6DoF} be the landing point position vector with launchpad as origin, respectively computed with 3 D.o.F and 6 D.o.F Models. For each wind Azimuth and Magnitude the percentage errors can be evaluated:

$$\mathrm{error} = ||\mathbf{r_{3DoF}} - \mathbf{r_{6DoF}}|| \tag{3.19}$$

The results are presented in fig. 3.7.

The error, besides being slightly low, particularly grows as the wind Magnitude increases. In order to establish what causes such a behavior a further trajectory analysis is now conducted.

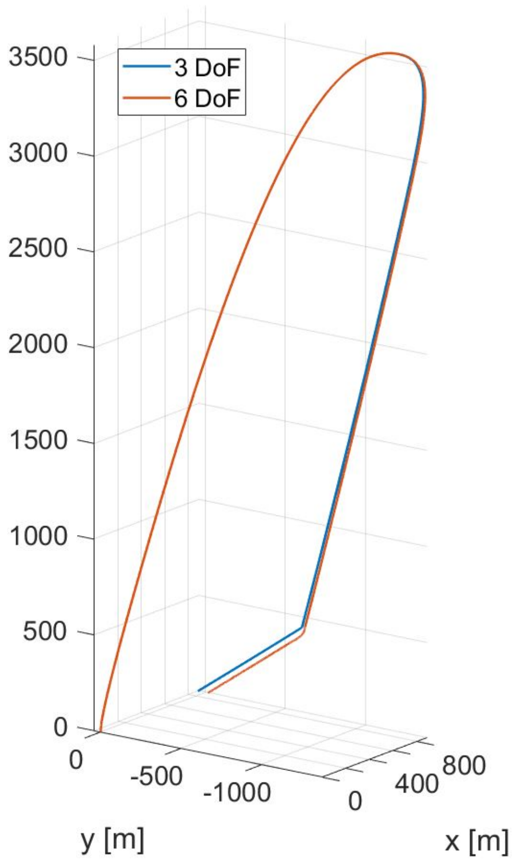

Figure 3.8: Trajectories with wind.Azimuth = 90° and wind.Magnitude = 10 m/s

Analyzing the trajectories computed by the two models (see fig. 3.8), the main source of error is clear: main parachute extraction phase. In fact, while the 3 D.o.F Model directly switches from the drogue to the main, the 6 D.o.F Model also takes into account the transitory phase in which the main chute is extracted and the rocket is considered in free fall. Dependency on wind Magnitude is then explained: the greater the magnitude is, the further the 3 D.o.F trajectory will depart from the 6 D.o.F one during main parachute extraction.

To conclude, due to its very short computational time and its very small errors, the 3 D.o.F descent model is still the most suitable to compute parachutes flight, and it is hence used in majority of the simulations.

Chapter 4

Model Limitations

The simulator is considered to be finished.

The aerodynamic model should be validated, and aerodynamic coefficients should be tested in wind gallery or validated.

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