Assignment 1

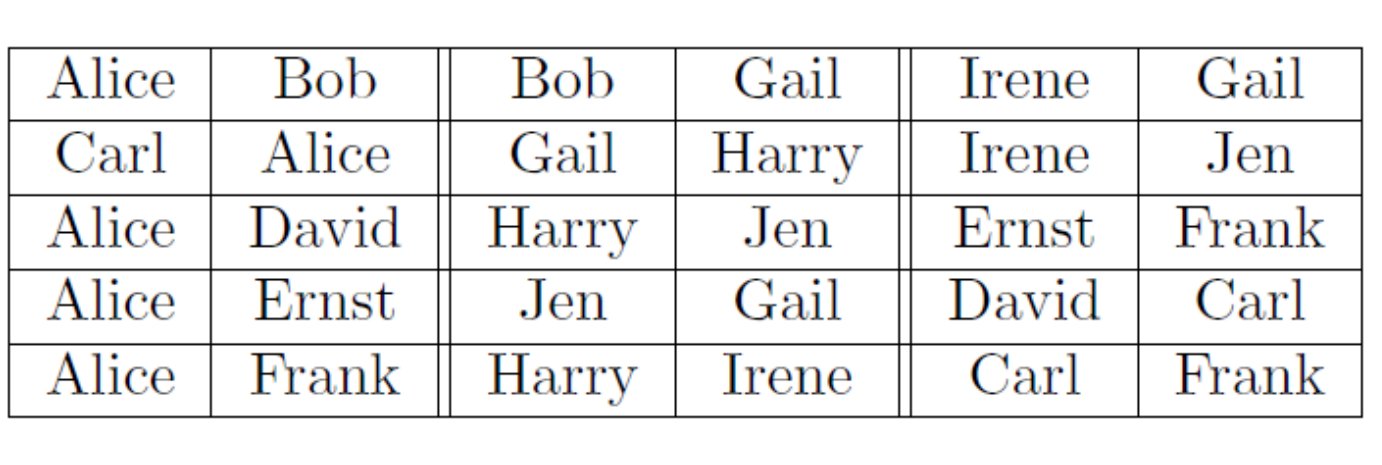
In [1]:
```
import numpy as np
import pandas as pd
import networkx as nx
import matplotlib.pyplot as plt

# displaying .png images
from IPython.display import Image
```

In [2]:
```
# turn on pretty printing
# %pprint
```

1. Consider the undirected network defined by the following set of links:

In [3]:
```
Image('data/network.png')
```

Out[3]:



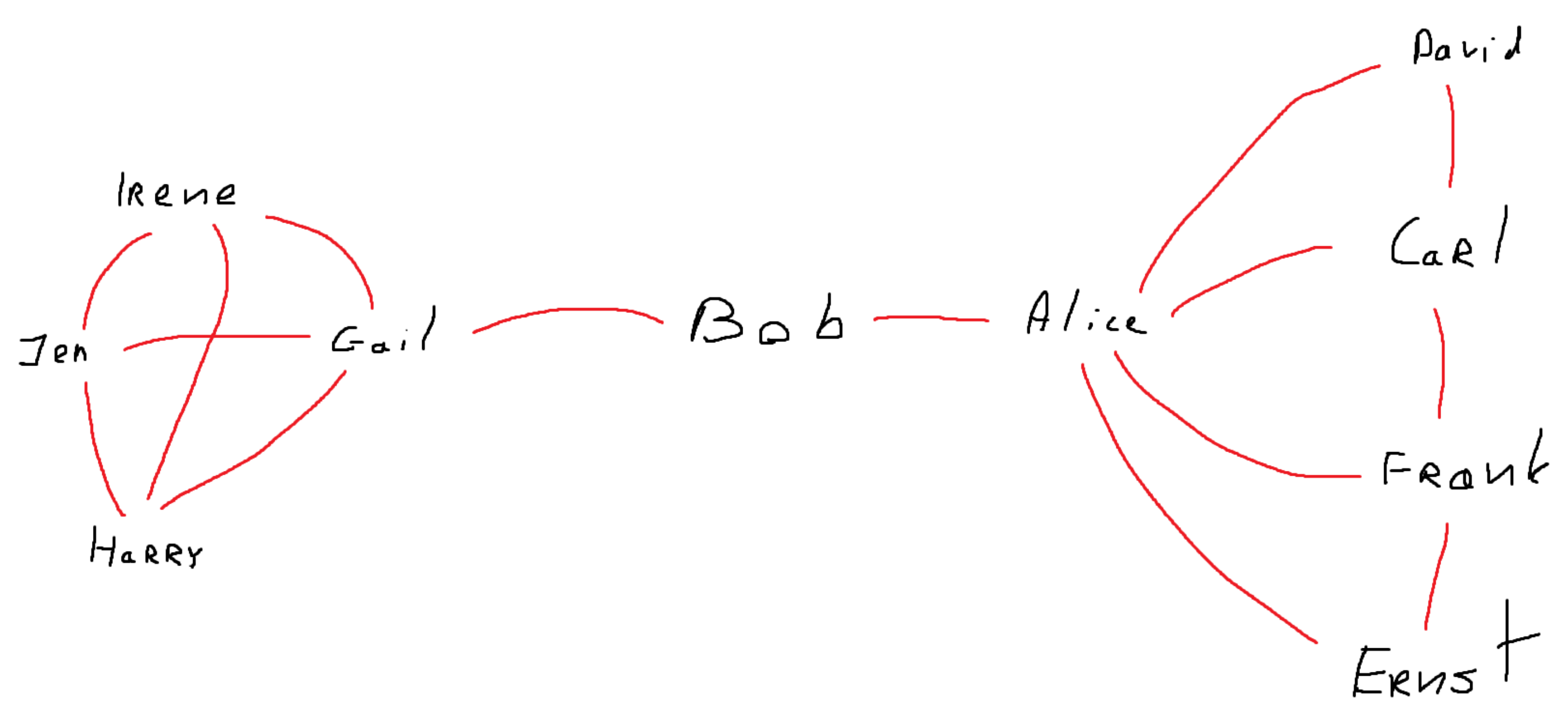
Alice	Bob	Bob	Gail	Irene	Gail
Carl	Alice	Gail	Harry	Irene	Jen
Alice	David	Harry	Jen	Ernst	Frank
Alice	Ernst	Jen	Gail	David	Carl
Alice	Frank	Harry	Irene	Carl	Frank

In [4]:
```
# creating network instance using networkx module
network_df = pd.read_csv('data/network.csv')

network = nx.Graph()

# filing the network
network.add_edges_from(network_df.values)
```

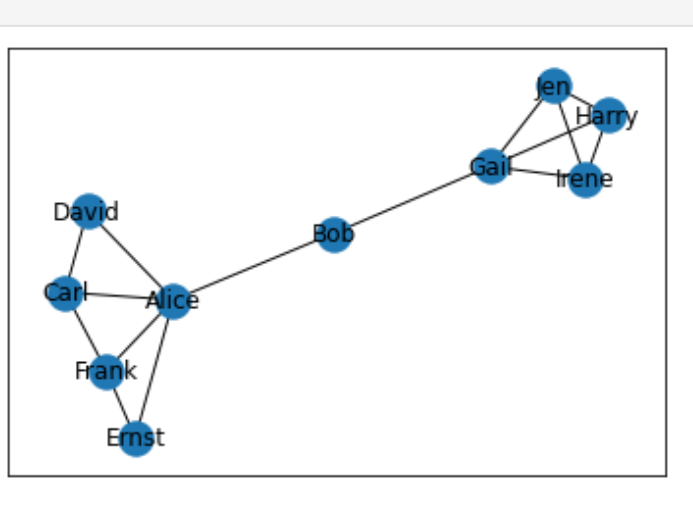
(a) Draw the network by hand.

In [5]:
```
Image('data/network_by_hand.png')
```

Out[5]:



In [6]:
```
# using Python
nx.draw_networkx(network)
```



(b) How many nodes are there?

answer: *10*

In [7]:
```
# using Python
len(network)
```

Out[7]:
```
10
```

(c) What is the density of the network?

density of the network tells us the percentage of connected nodes from possible connections. In that case it's 33.3% \'# k' stands for 'number of k'

$$\text{density} = \frac{\#\text{ actual connections}}{\#\text{ potential connections}} = \frac{15}{\frac{10*9}{2}} = \frac{15}{45} = \frac{1}{3}$$

In [8]:
```
# using Python
nx.density(network)
```

Out[8]:
```
0.3333333333333333
```

(d) Calculate the degree of each node. Who is the most central node according to this measure?

This measure shows us the most integral parts of the network. Just calculate how many edges come from each node. Ex. Alice has 5 connections and Bob 2 (Alice and Gail)

In [9]:
```
nx.degree(network)
```

Out[9]:
```
DegreeView({'Alice': 5, 'Bob': 2, 'Carl': 3, 'David': 2, 'Ernst': 2, 'Frank': 3, 'Gail': 4, 'Harry': 3, 'Jen': 3, 'Irene': 3})
```

(e) Calculate the clustering of each node and the average clustering of the network.

This measure shows us how well connected are the neighbours of given node. Ex. if statistics = 1, we are sure that we can delete the node and nothing happens to other nodes. Clusterring coefficient of a node: $C_i = \frac{2L_i}{k_i(k_i-1)}$

where

k_i - degree of node i.

L_i - number of edges between the neighbours of node i.

ex. Carl has $k_i = 3$ neighbours with $L_i = 2$ connections between them.

$$C_i = \frac{2L_i}{k_i(k_i-1)} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{2}{3}$$

In [10]:
```
# clastering
clustering_dict = nx.clustering(network)
clustering_dict
```

Out[10]:
```
{'Alice': 0.3,
 'Bob': 0,
 'Carl': 0.6666666666666666,
 'David': 1.0,
 'Ernst': 1.0,
 'Frank': 0.6666666666666666,
 'Gail': 0.5,
 'Harry': 1.0,
 'Jen': 1.0,
 'Irene': 1.0}
```

In [11]:
```
# avg clastering
# just calculate the mean
np.mean(list(clustering_dict.values()))
```

Out[11]:
```
0.7133333333333333
```

(f) Calculate the closeness centrality for each node. Who is the most central node according to this measure?

This measure shows us the centrality of a node, in average how far it is from the other nodes . the formula:

$$C(x) = \frac{n-1}{\sum_{y \neq x} d(x,y)}$$

where

n is the number of nodes in the network.

$d(x,y)$ is the shortest distance between x and y

ex. Carl $n = 10$

$$C(Carl) = 9/\left(d(Carl, Alice) + d(Carl, Bob) + \ldots\right) = 9/\left(1+2+1+2+1+3+4+4+4\right) = \frac{9}{22}$$

In [12]:
```
#  ofc it's easier in Python
nx.closeness_centrality(network)
```

Out[12]:
```
{'Alice': 0.5625,
 'Bob': 0.5625,
 'Carl': 0.4090909090909091,
 'David': 0.391304347826087,
 'Ernst': 0.391304347826087,
 'Frank': 0.4090909090909091,
 'Gail': 0.5,
 'Harry': 0.375,
 'Jen': 0.375,
 'Irene': 0.375}
```

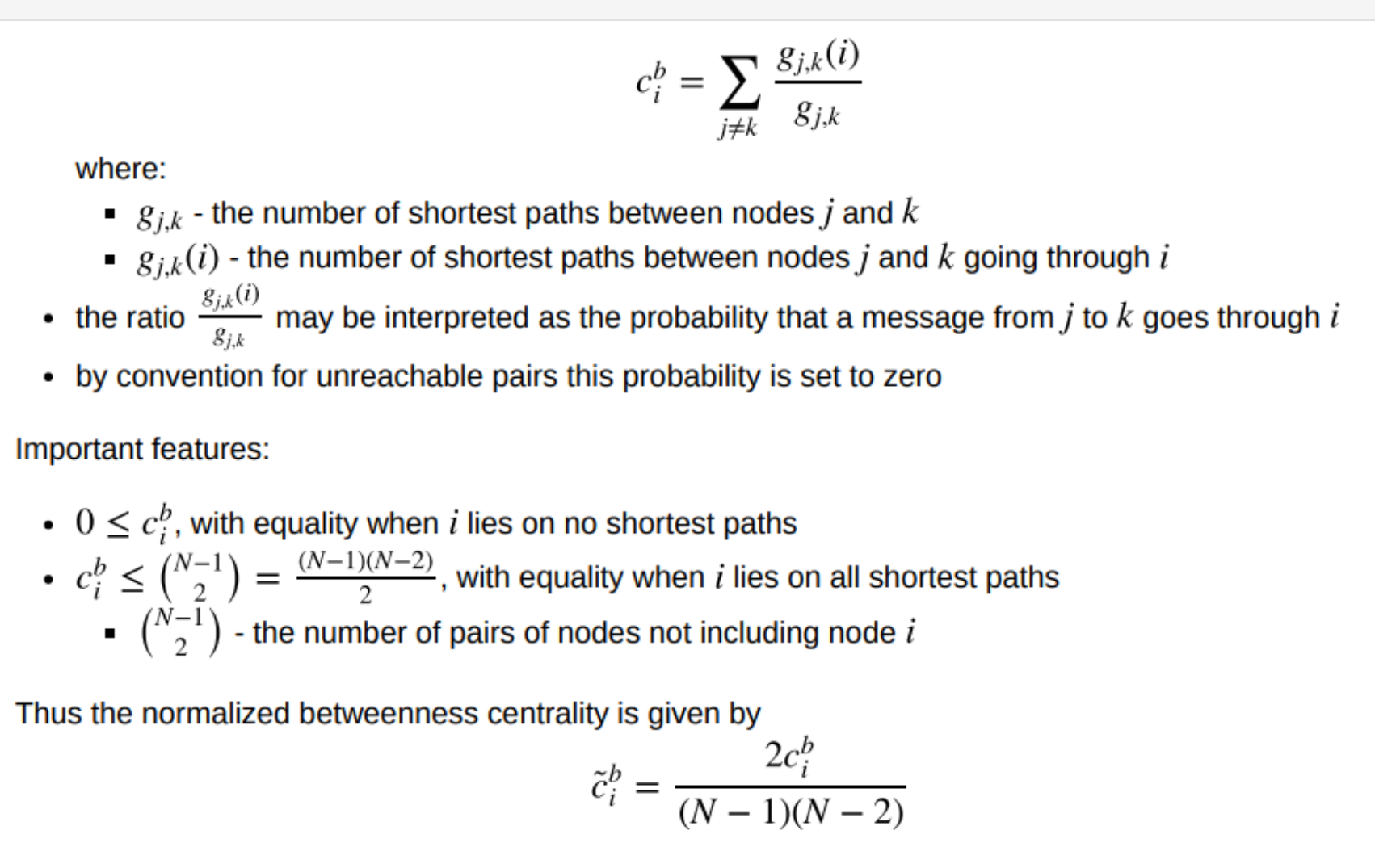
Bob and Alice are both the most central nodes

(g) Calculate the betweenness centrality of each node. Who is the most central node according to this measure?

This shows us how important the node is when the length of path matters. Ex. when it's 0, we know it' not important for other nodes to travel with the fastest way. the formula (from prof. Szawbiński lecture):

In [13]:
```
Image('data/betweenness_centrality.png')
```

Out[13]:



$$c_i^b = \sum_{j \neq k} \frac{g_{j,k}(i)}{g_{j,k}}$$

where:
  - $g_{j,k}$ - the number of shortest paths between nodes j and k
  - $g_{j,k}(i)$ - the number of shortest paths between nodes j and k going through i
- the ratio $\frac{g_{j,k}(i)}{g_{j,k}}$ may be interpreted as the probability that a message from j to k goes through i
- by convention for unreachable pairs this probability is set to zero

Important features:

- $0 \leq c_i^b$, with equality when i lies on no shortest paths
- $c_i^b \leq \binom{N-1}{2} = \frac{(N-1)(N-2)}{2}$, with equality when i lies on all shortest paths
  - $\binom{N-1}{2}$ - the number of pairs of nodes not including node i

Thus the normalized betweenness centrality is given by

$$\tilde{c}_i^b = \frac{2c_i^b}{(N-1)(N-2)}$$

ex. Frank it's 0 for almost each pairs, because the shoretest way cannot include Frank's node. We check 2 pair: Ernest-Carl and Ernest-David. For the first case there are 2 shortest ways - 1 through Alice and 1 trough Frank, so it's 0.5 for that pair. the second case Ernest-David has it's shortest way throught Alice, not Frank so it's 0 for Frank.

In [14]:
```
# normalized
nx.betweenness_centrality(network)
```

Out[14]:
```
{'Alice': 0.611111111111111,
 'Bob': 0.5555555555555556,
 'Carl': 0.013888888888888888,
 'David': 0.0,
 'Ernst': 0.0,
 'Frank': 0.013888888888888888,
 'Gail': 0.5,
 'Harry': 0.0,
 'Jen': 0.0,
 'Irene': 0.0}
```

In [15]:
```
# not normalized
nx.betweenness_centrality(network,normalized=False)
```

Out[15]:
```
{'Alice': 22.0,
 'Bob': 20.0,
 'Carl': 0.5,
 'David': 0.0,
 'Ernst': 0.0,
 'Frank': 0.5,
 'Gail': 18.0,
 'Harry': 0.0,
 'Jen': 0.0,
 'Irene': 0.0}
```

Alice is the most central node

2. For the above network:

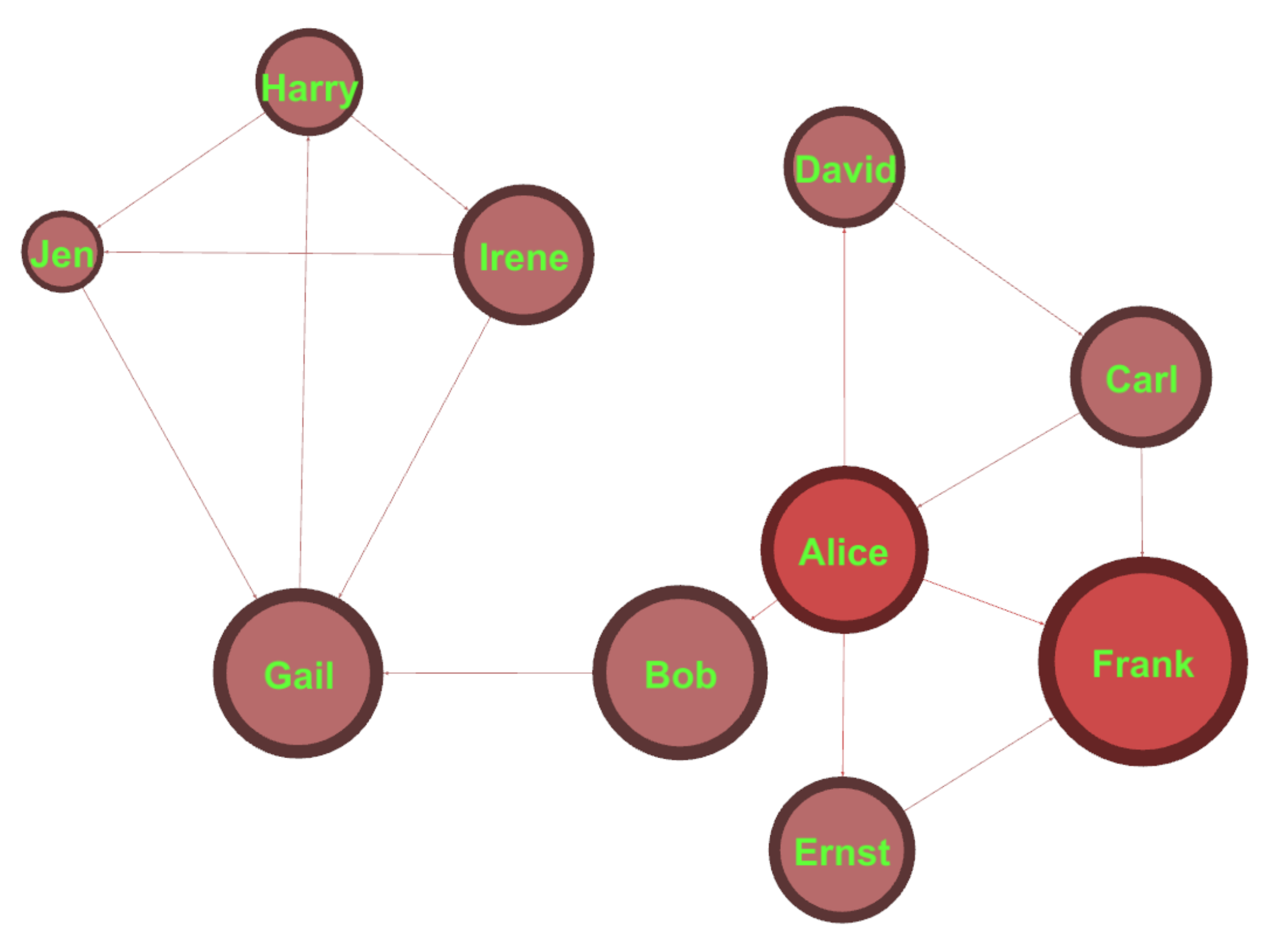
(a) prepare a CSV file with the edge list;

already done before.

(b) visualize the network by making use of the Gephi software;

In [16]:
```
Image('data/graph_gephi.png')
```

Out[16]:

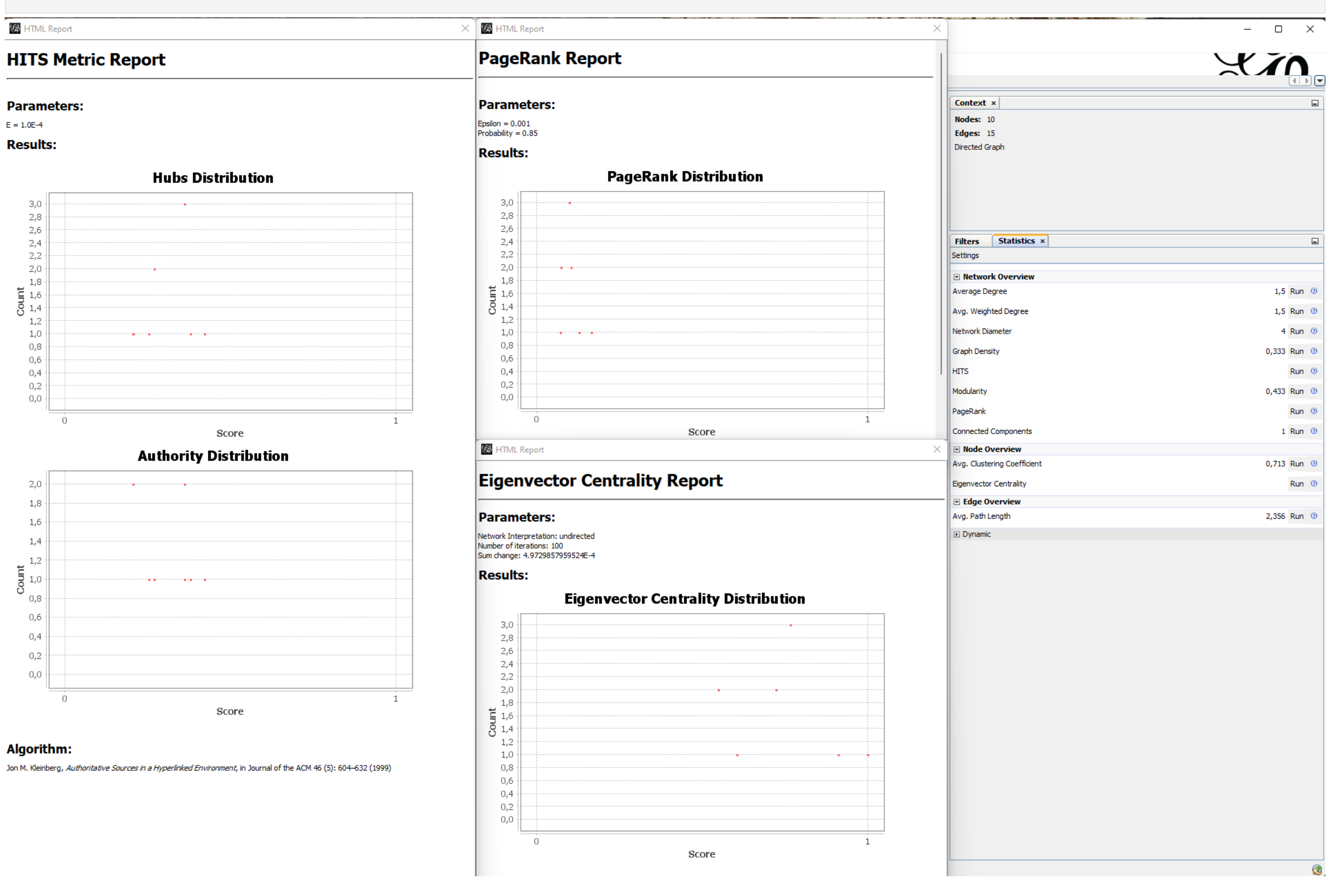


(c) calculate the basic network measures within Gephi.

I'm not going to add there all distributions done by Gephi. There are some of them and calculated averages on the right.

In [17]:
```
Image('data/gephi_measures.png')
```

Out[17]:



HITS Metric Report

Parameters:

E = 1.0E-4

Results:

Hubs Distribution

Authority Distribution

Algorithm:

Jon M. Kleinberg, *Authoritative Sources in a Hyperlinked Environment*, in Journal of the ACM 46 (5): 604–632 (1999)

PageRank Report

Parameters:

Epsilon = 0.001
Probability = 0.85

Results:

PageRank Distribution

Eigenvector Centrality Report

Parameters:

Network Interpretation: undirected
Number of iterations: 100
Sum change: 4.9729857959524E-4

Results:

Eigenvector Centrality Distribution

Context

Nodes: 10
Edges: 15
Directed Graph

Network Overview

Average Degree	1,5
Avg. Weighted Degree	1,5
Network Diameter	4
Graph Density	0,333
HITS	
Modularity	0,433
PageRank	
Connected Components	1

Node Overview

Avg. Clustering Coefficient	0,713
Eigenvector Centrality	

Edge Overview

Avg. Path Length	2,356

Exercise 3

An undirected unweighted network of size N may be represented through a symmetric adjacency matrix $A \in R^{N \times N}$, which has $a_{ij} = 1$, if nodes i and j are connected, and $a_{ij} = 0$ otherwise. We assume that $a_{ii} = 0$, so there are no self-loops in the network.

Let e be a column vector of N elements all equal to 1, i.e. $e = (1, 1, \ldots, 1)^T$, where the superscript T indicates the transposition.

Write expressions for or answer each of the following by making use of the above quantities and the matrix formalism (no sum symbol $\sum$ allowed!):

In [18]:
```
# define A and e from above
A = nx.adjacency_matrix(network).toarray()
A
```

```
C:\Users\piotr\AppData\Local\Temp\ipykernel_13368\3533083606.py:2: FutureWarning: adjacency_matrix will return a scipy.sparse array instead of a matrix in Networkx 3.0.
  A = nx.adjacency_matrix(network).toarray()
```

Out[18]:
```
array([[0, 1, 1, 1, 1, 1, 0, 0, 0, 0],
       [1, 0, 0, 0, 0, 0, 1, 0, 0, 0],
       [1, 0, 0, 1, 0, 1, 0, 0, 0, 0],
       [1, 0, 1, 0, 0, 0, 0, 0, 0, 0],
       [1, 0, 0, 0, 0, 1, 0, 0, 0, 0],
       [1, 0, 1, 0, 1, 0, 0, 0, 0, 0],
       [0, 1, 0, 0, 0, 0, 0, 1, 1, 1],
       [0, 0, 0, 0, 0, 0, 1, 0, 1, 1],
       [0, 0, 0, 0, 0, 0, 1, 1, 0, 1],
       [0, 0, 0, 0, 0, 0, 1, 1, 1, 0]], dtype=int32)
```

In [19]:
```
e = np.ones(10).transpose()
e
```

Out[19]:
```
array([1., 1., 1., 1., 1., 1., 1., 1., 1., 1.])
```

(a) the vector k whose elements are the degrees k_i of the nodes $i = 1, 2, 3, \cdot, N$;

$k = Ae$

In [20]:
```
# k = A x e
k = np.matmul(A,e)
k
```

Out[20]:
```
array([5., 2., 3., 2., 2., 3., 4., 3., 3., 3.])
```

(b) the total number L of links in the network;

$L = \frac{e^T Ae}{2}$

In [21]:
```
# we do e^T X k once again
L = np.matmul(e.transpose(),k) / 2
L
```

Out[21]:
```
15.0
```

(c) the matrix N whose element n_{ij} is equal to the number of common neighbors of nodes i and j;

$N = A^2$

In [22]:
```
N = np.matmul(A, A)
N
```

Out[22]:
```
array([[5, 0, 2, 1, 1, 2, 1, 0, 0, 0],
       [0, 2, 1, 1, 1, 1, 0, 1, 1, 1],
       [2, 1, 3, 1, 2, 1, 0, 0, 0, 0],
       [1, 1, 1, 2, 1, 2, 0, 0, 0, 0],
       [1, 1, 2, 1, 2, 1, 0, 0, 0, 0],
       [2, 1, 1, 2, 1, 3, 0, 0, 0, 0],
       [1, 0, 0, 0, 0, 0, 4, 2, 2, 2],
       [0, 1, 0, 0, 0, 0, 2, 3, 2, 2],
       [0, 1, 0, 0, 0, 0, 2, 2, 3, 2],
       [0, 1, 0, 0, 0, 0, 2, 2, 2, 3]], dtype=int32)
```

(d) find the number T of traingles present in the network.

$T = \frac{tr(A^3)}{3!}$

In [23]:
```
T=np.trace(np.linalg.matrix_power(A, 3)) / 6
T
```

Out[23]:
```
7.0
```

In [23]:
```

```

(e) how would you determine whether the network is connected only by looking at the adjacency matrix?

1. to check if there is only 1 not connected node - just check if (a) or diagonal of (c) contains any 0.

In general:

If you put all 1 on the diagonal of your adjacency matrix A, and all edge weights are positive then when you multiply A^2 you get a non-zero entry a_{ij} in A^2 if and only if there exist non-zero a_{ik} and a_{kj} in A for some k, i.e. there is a path of length 2 between i and j if $k \neq j$ and $k \neq i$ and there is a path of length 1 if $k = j$ or $k = i$. So the non-zero entries in A^2 tell you all pairs of nodes that are connected by a path of length 2.

Similarly the entries in A^k tell you all pairs of nodes that are connected by a path of length k. So if you start with A and keep squaring until you get Ak where $k \geq n$ where n is the number of nodes, then the non-zero entries in row i tell you all the nodes that are connected to node i (since two connected nodes must be connected by a path of length n or less). So if you have a row in A^k that is all non-zero, then the graph is connected. If the graph is not connected, you can similarly tell the connected components from the rows of A^k

i have it from: https://math.stackexchange.com/questions/864604/checking-connectivity-of-adjacency-matrix

example:

In [24]:
```
def square_until_k_geq_n(matrix: np.matrix) -> np.matrix:
    """ function doing exactly what is written in the cell above after "in general:".
    Args:
        matrix (np.matrix): adjacency matrix
    returns:
        (np.matrix)
    """
    k = 2
    n = len(matrix)
    matrix_wt_ones_on_diagonal = matrix + np.eye(n)

    mtx_squared = np.linalg.matrix_power(matrix_wt_ones_on_diagonal, k)
    while k < n:
        k = k ** 2
        mtx_squared = np.linalg.matrix_power(mtx_squared, k)

    return mtx_squared
```

In [25]:
```
# A_with_ones_on_diagonal = A + np.eye(len(A))
#
# np.linalg.matrix_power(A_with_ones_on_diagonal , 2)
square_until_k_geq_n(A)
```

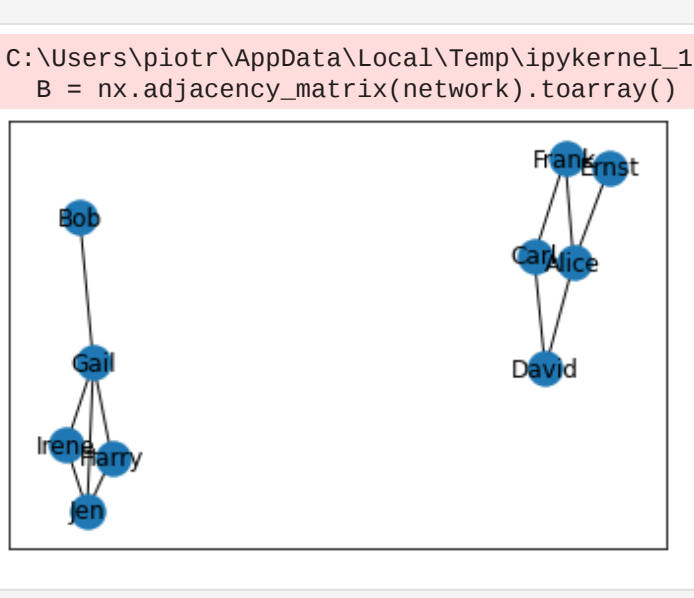
Out[25]:
```
array([[3.19215035e+78, 2.12601709e+78, 2.26833873e+78, 1.72401632e+78,
        1.72401632e+78, 2.26833873e+78, 3.54259826e+78, 3.03248969e+78,
        3.03248969e+78, 3.03248969e+78],
       [2.12601709e+78, 1.41839524e+78, 1.51027352e+78, 1.14790317e+78,
        1.14790317e+78, 1.51027352e+78, 2.36722749e+78, 2.02690816e+78,
        2.02690816e+78, 2.02690816e+78],
       [2.26833873e+78, 1.51027352e+78, 1.61197055e+78, 1.22514539e+78,
        1.22514539e+78, 1.61197055e+78, 2.51585504e+78, 2.15348469e+78,
        2.15348469e+78, 2.15348469e+78],
       [1.72401632e+78, 1.14790317e+78, 1.22514539e+78, 9.31147563e+77,
        9.31147563e+77, 1.22514539e+78, 1.91227429e+78, 1.63685005e+78,
        1.63685005e+78, 1.63685005e+78],
       [1.72401632e+78, 1.14790317e+78, 1.22514539e+78, 9.31147563e+77,
        9.31147563e+77, 1.22514539e+78, 1.91227429e+78, 1.63685005e+78,
        1.63685005e+78, 1.63685005e+78],
       [2.26833873e+78, 1.51027352e+78, 1.61197055e+78, 1.22514539e+78,
        1.22514539e+78, 1.61197055e+78, 2.51585504e+78, 2.15348469e+78,
        2.15348469e+78, 2.15348469e+78],
       [3.54259826e+78, 2.36722749e+78, 2.51585504e+78, 1.91227429e+78,
        1.91227429e+78, 2.51585504e+78, 3.95652147e+78, 3.38855413e+78,
        3.38855413e+78, 3.38855413e+78],
       [3.03248969e+78, 2.02690816e+78, 2.15348469e+78, 1.63685005e+78,
        1.63685005e+78, 2.15348469e+78, 3.38855413e+78, 2.90224052e+78,
        2.90224052e+78, 2.90224052e+78],
       [3.03248969e+78, 2.02690816e+78, 2.15348469e+78, 1.63685005e+78,
        1.63685005e+78, 2.15348469e+78, 3.38855413e+78, 2.90224052e+78,
        2.90224052e+78, 2.90224052e+78],
       [3.03248969e+78, 2.02690816e+78, 2.15348469e+78, 1.63685005e+78,
        1.63685005e+78, 2.15348469e+78, 3.38855413e+78, 2.90224052e+78,
        2.90224052e+78, 2.90224052e+78]])
```

As we can see, there are no zeros - network is connected.

Now try with different network.

In [26]:
```
# delete connection between Bob and Alice and create new network
new_network_df=network_df.drop([0],axis=0)

network = nx.Graph()
network.add_edges_from(new_network_df.values)
B = nx.adjacency_matrix(network).toarray()
nx.draw_networkx(network)
```

```
C:\Users\piotr\AppData\Local\Temp\ipykernel_13368\280792950.py:6: FutureWarning: adjacency_matrix will return a scipy.sparse array instead of a matrix in Networkx 3.0.
  B = nx.adjacency_matrix(network).toarray()
```



In [27]:
```
square_until_k_geq_n(B)
```

Out[27]:
```
array([[3.18532661e+75, 3.78905437e+75, 2.37592974e+75, 2.37592974e+75,
        3.18532661e+75, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00],
       [3.78905437e+75, 4.50720908e+75, 2.82624926e+75, 2.82624926e+75,
        3.78905437e+75, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00],
       [2.37592974e+75, 2.82624926e+75, 1.77220198e+75, 1.77220198e+75,
        2.37592974e+75, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00],
       [2.37592974e+75, 2.82624926e+75, 1.77220198e+75, 1.77220198e+75,
        2.37592974e+75, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00],
       [3.18532661e+75, 3.78905437e+75, 2.37592974e+75, 2.37592974e+75,
        3.18532661e+75, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00],
       [0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 5.09414284e+76, 1.57211880e+77, 1.44744968e+77,
        1.44744968e+77, 1.44744968e+77],
       [0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 1.57211880e+77, 4.85176332e+77, 4.46701816e+77,
        4.46701816e+77, 4.46701816e+77],
       [0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 1.44744968e+77, 4.46701816e+77, 4.11278332e+77,
        4.11278332e+77, 4.11278332e+77],
       [0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 1.44744968e+77, 4.46701816e+77, 4.11278332e+77,
        4.11278332e+77, 4.11278332e+77],
       [0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 1.44744968e+77, 4.46701816e+77, 4.11278332e+77,
        4.11278332e+77, 4.11278332e+77]])
```

As we can see, there are some 0 - network is not connected.