Differential based Torque Vectoring Systems for Vehicle Stability Control

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INTRODUCTION

Vehicle Stability Control (VSC) is an active vehicle safety and stability system that intends to mitigate the driver's loss of vehicle control by correcting certain parameters. The goal of this stage of the project is to model and simulate a VSC controller that can optimally stabilize the vehicle using active differentials at the rear axle. VSC first needs to calculate the optimal corrective yaw moment required to stabilize the vehicle and then should split this corrective action into individual differential torques.

VEHICLE MODEL

The full vehicle model from carsim was used for simulating the behaviour of a high degree of freedom high fidelity practical vehicle. A class D sedan was chosen with a total vehicle mass (m) of 1370 Kg, wheelbase (l) of 2.78m. front wheelbase (a) of 1.11m, and inertia along the z-axis (J_z) of 2315.3 Kgm^2. For tires, a standard tire model was selected with 215/55 R17 tires used for all 4 wheels with axle cornering stiffness (C_1) as 5396.6 N/deg. Since, carsim uses a non-linear tire model, to simplify simulations, the slope of the linear region of the tire lateral force (F_y) vs tire slip angle (C_α) plot was considered[1], [2].

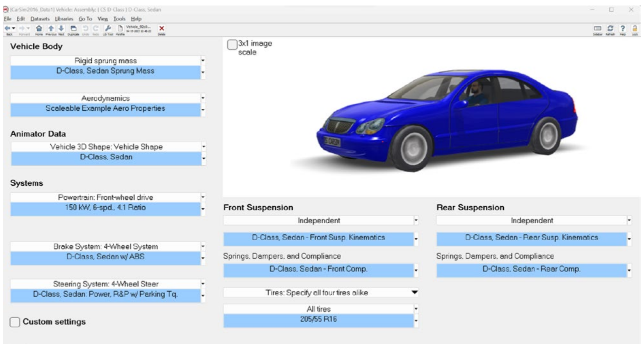

Figure 1: CARSIM workspace

TEST METHODS

For the simulations conducted in this project, certain standard vehicle tests were performed to estimate the stability and path-tracking ability of the vehicle with and without the VSC controller. Following are the standard tests conducted for the simulations:

1. **Sine with Dwell Test**
   The first test conducted is a standard Sine with Dwell test as per the FMVSS 126 ESC Test standards. In this test, a specific steering angle input is given to the vehicle in the form of a sine wave of 0.7 Hz frequency, but there is a 400 ms dwell period added at the end of the second peak amplitude (negative amplitude). For the ease of simulations and a better understanding of the controller, this test has been modified a bit to be consistent with the model[1], [3]. A left-hand turn with a peak steering angle (road wheel angle) of 4 deg was chosen for the simulations with the following waveform:

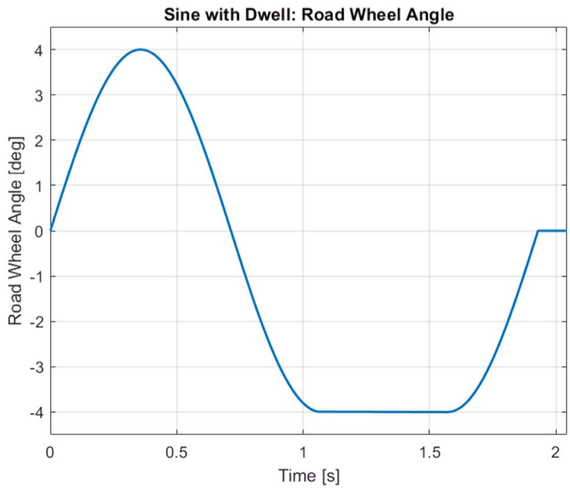

Figure 2: Sine with Dwell test road wheel angle waveform

2. **J-Turn Test**
   The second test that was conducted was the standard J-turn test which can be easily simulated by giving a step steer input to the vehicle. In this test, a left-hand turn with a step steer input (road wheel angle) is applied at 4 seconds with a magnitude of 2 degrees and the following waveform[4]:

Figure 3: J-turn test road wheel angle waveform

VEHICLE STABILITY CONTROL

The vehicle stability control system aims to minimize the deviation between the actual vehicle states and the desired or reference vehicle states as mentioned above. As seen in fig. 4 the Vehicle Stability control consists of two hierarchical controllers viz. Upper-level control and lower-level control. The upper-level control calculates the corrective yaw moment (M_{ze}) that needs to be generated so that the actual vehicle trajectories and the desired vehicle trajectories match. It then passes this M_{ze} value to the lower level controller that uses optimal controller allocation technique to distribute the required yaw moment into left and right axle torques of differential (T_L and T_r), subject to certain constraints.

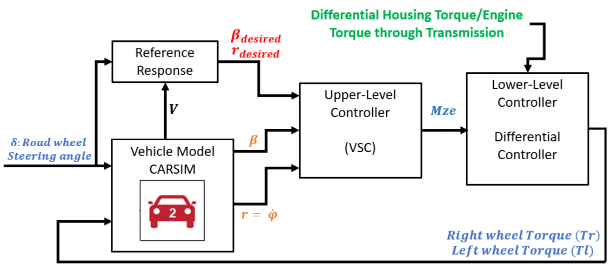

Figure 4: VSC control flowchart

REFERENCE DESIRED TRAJECTORY

The reference desired trajectory is obtained from a basic 2 DOF vehicle model or bicycle model with only lateral and yaw degrees of freedom. This gives an idea of the desired behavior of the vehicle. For a given road wheel steering angle (δ) following equations give the desired yaw rate (r_d), desired side slip angle (β_d) and understeering gradient (K_{us}) expressions for a stable maneuver:

$$r_d = \frac{V}{L + (K_{us} * V^2)} * \delta \tag{1}$$

$$\beta_d = \frac{a*b*\left(\frac{L}{a} - \frac{m*V^2}{b*C_2}\right)* \delta}{L^2 - m*\left(\frac{a}{C2} - \frac{b}{C1}\right)*V^2} \tag{2}$$

$$K_{us} = \frac{m * ((C_2 * b) - (C_1 * a))}{L * C_2 * C_1} \tag{3}$$

The velocity (V) and the road wheel steer angle (δ) are outputs of the car sim block and that is added as a gain to the other vehicle parameters [5], [6].

SATURATIONS

Certain saturations must be considered for the state trajectories (β and r) as well as the M_{ze} to ensure that there is no over-actuation of the differential clutches as well as there is no over-the-limit desired trajectory estimation[7]. Following are the saturation limits considered for the β and r:

$$r \leq \left|\frac{\mu * g}{V}\right| \tag{4}$$

$$\beta \leq |\tan^{-1}(0.02 * \mu * g)| \tag{5}$$

where μ is the coefficient of friction which is considered to be 0.85 as per car sim simulations. For limiting the M_{ze} values, a M_{ze} torque magnitude limit of 3000 Nm was given[8]. This value was obtained through market research.

UPPER-LEVEL CONTROLLER

The main function of the upper-level controller is to calculate the optimal external yaw moment that the vehicle requires to become stable so that the reference trajectories and actual trajectories match. As seen in fig. 6, a Simulink controller model co-simulated with the carsim model was created to simulate the upper-level controller behavior.

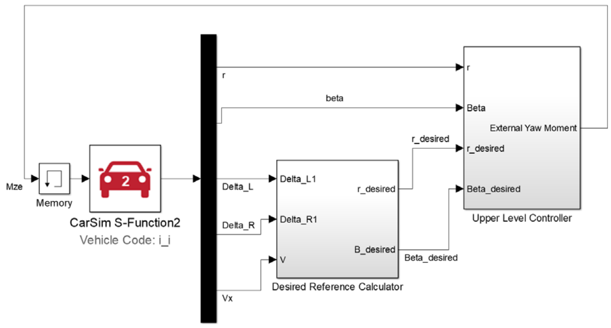

Figure 5: CarSim - Simulink co-simulation

Since, the car sim model is a high fidelity, high degree of freedom model, a Proportional derivative integral (PID) controller was used to find the optimal M_{ze} values[4]. The input to the PID was selected to be the error in yaw rate (e_r) of the vehicle and the control gains were adjusted in such a way that required M_{ze} values were obtained without exceeding the saturation constraints of parameters mentioned above.

$$e_r = r_d - r_{actual} \tag{6}$$

$$M_{ze} = K_p * e_r + K_i * \int_0^t e_r . dt + K_d * \frac{de_r}{dt} \tag{7}$$

Where Kp, Ki, and Kd are the proportional, integral, and derivative gains of the PID controller. Following are the PID gains finalized for the two standard tests along with the average magnitude of M_{ze} values:

Test	Kp	Ki	Kd	Average M_{ze}
Sine with Dwell Test	10000	0	0	515.75 Nm
J- Turn Test	45000	40000	200	548.50 Nm

Table 1:PID Gains and Average Yaw moment value

LOWER-LEVEL CONTROLLER

The lower-level torque distribution is done by the torque vectoring differential[9]–[11]. The input to the differential is T_i which is the differential input torque. T_L and T_r are the differential output shaft torques and T_{cr} and T_{cl} are the clutch torques. N_l and N_r are the shaft gear ratios. The following are the differential equations[12]:

$$T_l = \frac{T_i}{2} - \frac{N_r}{2} * T_{cr} + \frac{N_l}{2} * T_{cl} \tag{8}$$

$$T_r = \frac{T_i}{2} + (1 - \frac{N_r}{2}) * T_{cr} - (1 - \frac{N_l}{2}) * T_{cl} \tag{9}$$

The external moment M_{ze} is equal to the torque difference. Which means that $M_{ze} = T_r - T_l$. And from the above equations, we can see that

$$M_{ze} = T_r - T_l = T_{cr} - T_{cl} \tag{10}$$

For the lower-level controller, Model Predictive Control (MPC) was used to find the optimal torque distribution between the left and right clutch. Since MPC requires, vehicle model equations to work, which are not available from carsim, a separate 4-wheel vehicle model was used as a vehicle plant model in MPC. This vehicle model was tuned so that the results approximately match the carsim model. The M_{ze} values obtained from the upper-level carsim-simulink co-simulation model were saved in a lookup table and this lookup table was called in a separate matlab MPC model. Following is the MPC formulation model:

$$\min_U J = \sum_{i=0}^{N+1} x(i)^T Q x(i) + \sum_{i=0}^{N} U(i)^T R U(i) \tag{11}$$

subject to:

$$X_{min} \leq X(i) \leq X_{max} \tag{12}$$

$$U_{min} \leq U \leq U_{max} \tag{13}$$

Car Plant Model:

$$U_1(i) - U_2(i) = M_{ze}(i) \tag{14}$$

Where:

$$U = [U_1 \quad U_2]^T = [T_R \quad T_L]^T \tag{15}$$

$$X = [\beta \quad \varphi \quad r]^T \tag{16}$$

subject to:

$$Mze_{min} \leq T_L \leq Mze_{max} \tag{17}$$

$$Mze_{min} \leq T_R \leq Mze_{max} \tag{18}$$

$$-30° \leq \beta, \varphi \leq 30° \tag{19}$$

$$-30\,°/s \leq r \leq 30\,°/s \tag{20}$$

Q and R are identity matrices of 3x3 and 2x2 size respectively. This optimization problem is changed to optimal control problem using single collocation. The sampling time is 0.1s and the horizon size (N) is 10.

RESULTS

1. Sine with Dwell Test

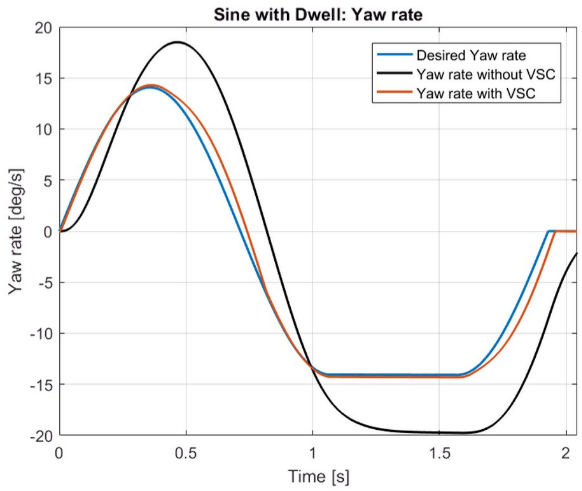

Figure 6: Sine with Dwell Yaw rate response

The vehicle speed was chosen to be 60 km/hr for this test. The PID was tuned to get the best tracking of the yaw rate. There is a slight overshoot in the yaw rate. During around 0.3 s, the vehicle yaw rate is more than desired yaw rate meaning the vehicle is oversteering and from fig. 2 we know this is a left turn. This means that there needs to be torque added left wheel at this moment. A similar trend can be explained for every time step. The actual yaw rate is lagging the desired yaw rate. But as seen from fig 6 the vehicle without VSC is overshooting too much and is not at all settling resulting in instability.

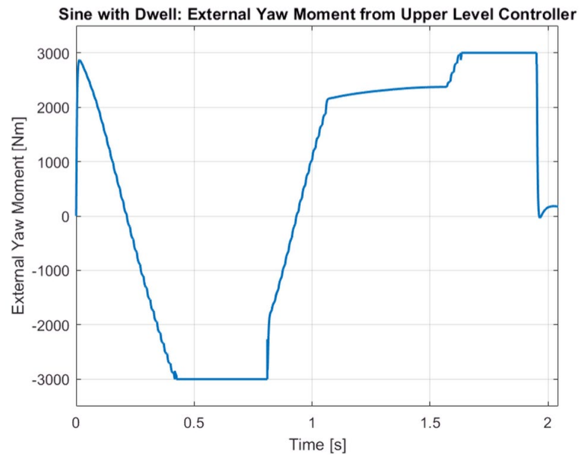

Figure 7: Sine with Dwell External Yaw Moment plot

In the case of the required external moment to control the vehicle, since $M_{ze} = T_R - T_L$, positive M_{ze} values mean that the right clutch is activating, and torque is added on the rear right wheel and negative M_{ze} values mean that the left clutch is activating and torque is added on the rear left wheel. At around 0.3 s it can be seen that the M_{ze} values become negative signifying that the left clutch is activating as the vehicle is oversteering.

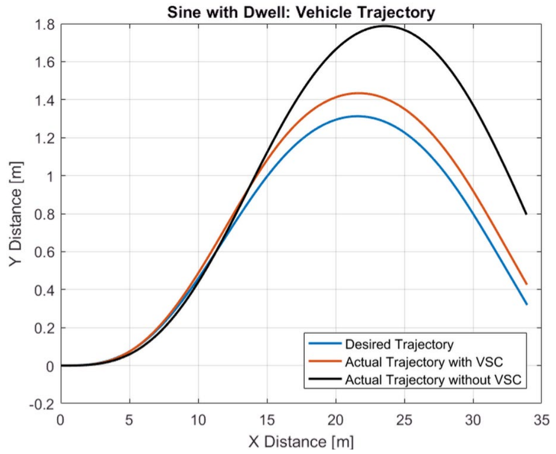

Figure 8: Sine with Dwell Vehicle Trajectory

The vehicle trajectory can be seen in fig 8. The actual vehicle trajectory is very similar to the desired vehicle trajectory, there is a light overshoot in the lateral distance (Y). But the vehicle without VSC has a large overshoot and is not following the desired trajectory at all.

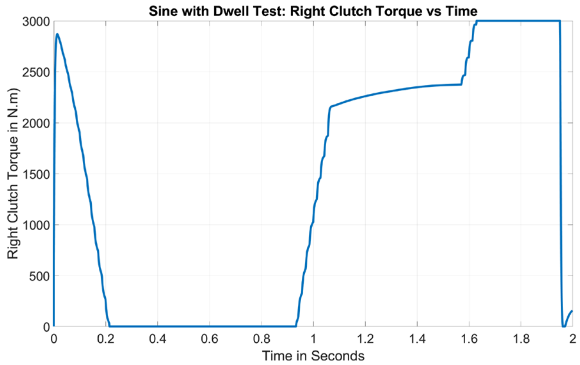

Figure 9: Sine with Dwell test right Clutch torque

From fig. 9 it can be seen that at first the right clutch is activated and then deactivated for a short instance of time and then after almost 1 sec the right clutch activates again.

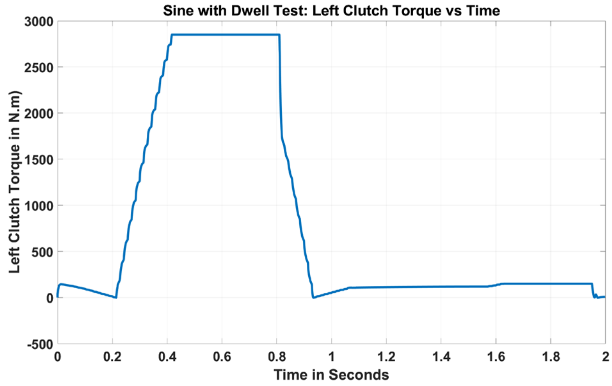

Figure 10: Sine with dwell - Left clutch torque

From fig 10 it can be seen that at about 0.2 seconds first the left clutch is activated. Left clutch remains activated for most of the simulation time during the peak turns.

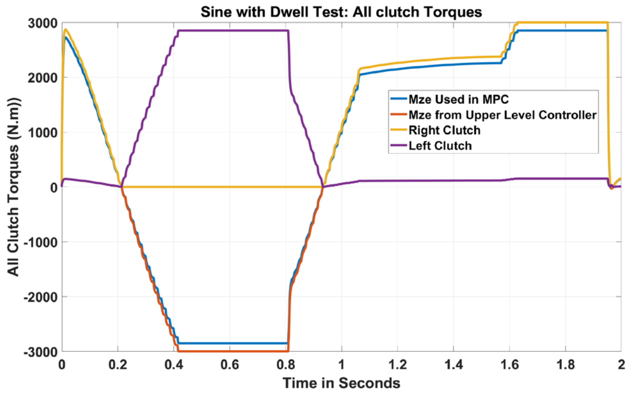

Figure 11: Sine with dwell - All clutch torques

Fig 11 shows both left and right clutch activation superimposed with the M_{ze} values obtained from the upper-level controller as well as the lower-level controller of MPC. It can be seen that sometimes, MPC uses slightly different M_{ze} values since the vehicle model is different from the carsim model.

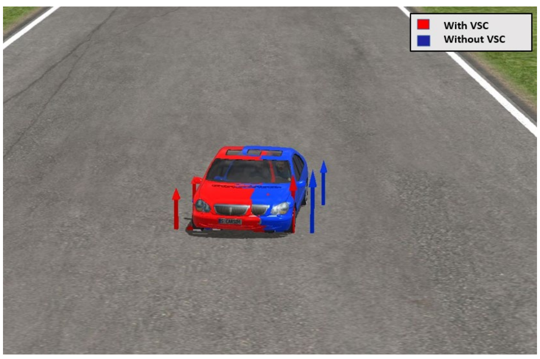

Figure 12: Sine with dwell - Car sim simulation

Fig 9 shows the superimposed car sim simulation video screenshot. It can be seen that the vehicle without VSC has a deviation in trajectory.

2. J Turn Test

The vehicle speed chosen for this test was 60 km/h. The PID was tuned to get the best tracking of the desired yaw rate. As seen in fig 13 the vehicle with VSC has a slight overshoot initially at around 4.2 s, but the yaw rate stabilizes within 2.5 seconds. Overshoot signifies that the vehicle is oversteering. And from fig 3 we know it is a left-hand turn which means that the right clutch should be activated.

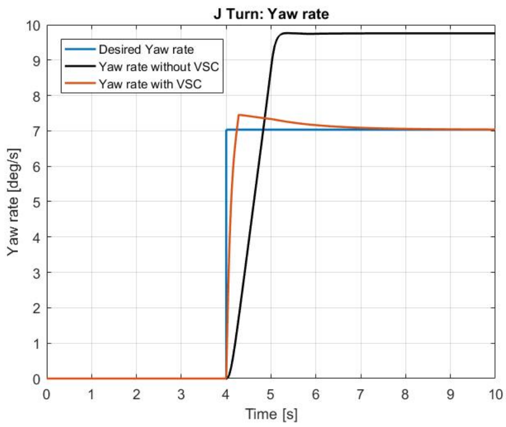

Figure 13: J turn - Yaw rate

As the vehicle is steered to make a left turn, the vehicle understeers initially. This causes a positive yaw error, which directly correlates to a required positive or anticlockwise moment to the vehicle. As seen in the fig 14 the right clutch is then activated which sends more torque to the rear right wheel and it provides an anticlockwise moment to the vehicle. Due to this applied torque, the yaw error becomes negative, or the vehicle oversteers. The left clutch is then activated to send more torque to the rear left wheel, in turn giving a negative yaw moment to the vehicle to stabilize it.

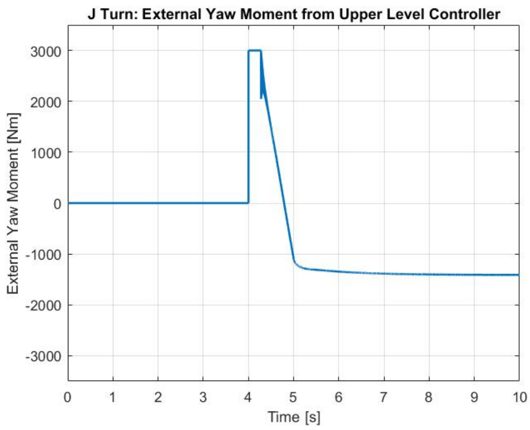

Figure 14: J turn- External moment

As seen from fig 15, it can be seen that the trajectory of the vehicle with VSC tracks the desired trajectory perfectly, as compared to the vehicle without VSC which understeers.

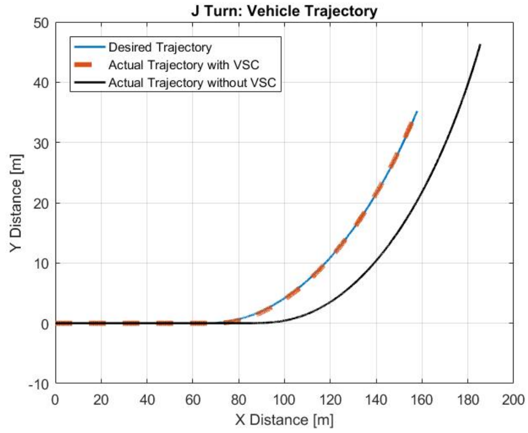

Figure 15: J turn - Vehicle Trajectory

Fig 16 shows the activation of the right clutch that is obtained from the lower-level MPC controller. At around 4.2s, the right clutch gets activated and torque is added to the right axle to control the vehicle.

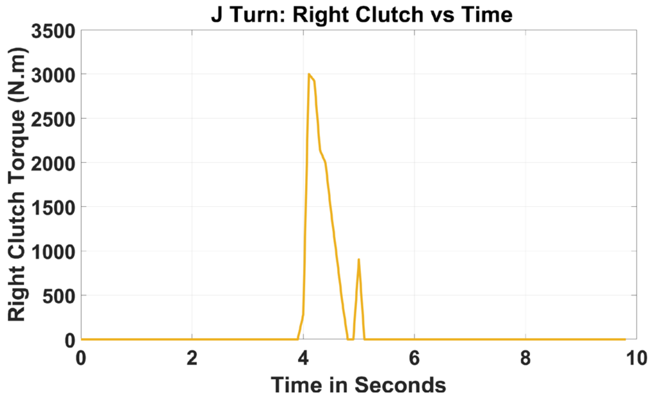

Figure 16: J turn - Right clutch torque

Fig 17 shows the activation of the left clutch that is obtained from the lower-level MPC controller. After the right clutch is activated and the vehicle becomes oversteer, there is a need for the left wheel torque to be increased, and the left clutch is activated.

Figure 17: J turn - Left clutch torque

Fig 18 shows both left and right clutch activation superimposed with the M_{ze} values obtained from the upper-level controller as well as the lower-level controller of MPC. It can be seen that sometimes, MPC uses slightly different M_{ze} values since the vehicle model is different from the carsim model.

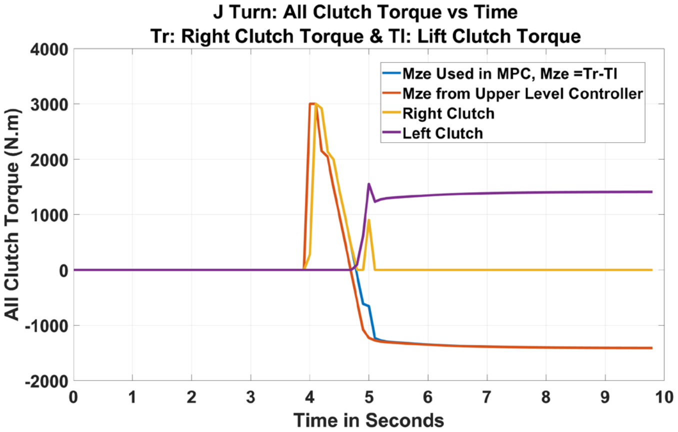

Figure 18: J turn - All torques

Fig 19 shows the superimposed car sim simulation video screenshot. The road is modeled in carsim to replicate the desired trajectory.

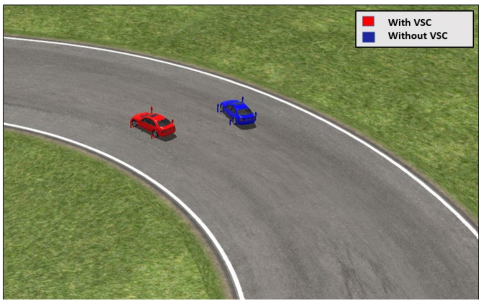

Figure 19: J-Turn - Carsim simulation

CONCLUSION

The vehicle can be effectively controlled by a Vehicle stability controller. Using a PID controller for the upper level to calculate the external yaw moment, with a Model Predictive control approach for the lower-level controller to optimally distribute torques on left and right rear wheels, proved to be a successful approach. The graphs and outputs show that this strategy is successful in reducing the yaw rate error, while efficiently distributing the torques between the right and left wheels.

ACCOUNTABILITY STATEMENT

For the course of this project, Ankit Kulkarni and Rohit Ravikumar were responsible for upper-level controller design and tuning along with car sim vehicle modeling and co-simulations. They were also responsible for selecting and modeling standard tests. Hayleyesus Alemayehu was responsible for modeling, tuning, and simulation of the lower-level controller i.e. MPC. Report was written by all three collectively.

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