# CPS815/CP8201 - Assignment 5

1. (50 marks) Recall the 3-Sat problem: given $n$ variables $x_1, \ldots, x_n$ and $k$ clauses $C_1, \ldots, C_k$ where each $C_i$ is a disjunction of exactly 3 variables, the problem is to decide whether there is a truth assignment for $x_1, \ldots, x_n$ that satisfies all the clauses $C_1, \ldots, C_k$. We proved in the class that 3-Sat is NP-complete. The 5-Sat problem is defined similar to the 3-Sat problem: there are $n$ variables $x_1, \ldots, x_n$ and $k$ clauses $C_1, \ldots, C_k$ where each clause $C_i$ is a disjunction of 5 variables. For example, a clause is of the form $x_2 \vee x_5 \vee \overline{x_6} \vee x_7 \vee \overline{x_9}$. The parameters $n, k$ are general parameters and are not related to the parameters $n, k$ in 3-Sat. Prove that 5-Sat is polynomial-time equivalent to 3-Sat. That means you have to show that 5-Sat $\leq_P$ 3-Sat and 3-Sat $\leq_P$ 5-Sat.

2. (50 marks) Recall the graph coloring problem: given a graph $G = (V, E)$ of $n$ vertices, decide whether $G$ is $k$-colorable. The graph $G$ is said to be $k$-colorable if, using at most $k$ colors, we can assign a color to each vertex of $G$ such that the following holds: if $(u, v) \in E$ is an edge then $u$ and $v$ must have different colors. Assume you have access to an oracle $O$ that can solve the 3-coloring problem. Write an algorithm using $O$ as a subroutine to obtain the actual coloring of the graph. The inputs and outputs of the your algorithm should be

   *Input:* a graph $G$ of $n$ vertices.

   *Output:* "no 3-coloring" if $G$ is not 3-colorable. If $G$ is 3-colorable return the actual coloring of the vertices.

   Remember that the oracle $O$ accepts a graph as an input and returns 1 or 0 if the graph is 3-colorable or not, respectively. Your algorithm should call $O$ only a polynomial number of times.

3. (Bonus, 30 marks) Generalize your proof of Question 1 to show that for any constant $\ell$, 3-Sat is polynomial-time equivalent to $\ell$-Sat.