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1. Variables

1.1. Nomenclature

m	Vehicle Total Mass	$P_{e,max}$	Max. Engine Power	θ	Inclination angle
m_c	Vehicle Curb Mass (mass without driver)	T_e	Engine Torque	r	Tire Radius (Static)
m_d	Driver's Mass	$T_{e,max}$	Max. Engine Torque	V	Vehicle Speed
m_r	Equivalent Rotational Mass	T_0	Torque Interpolation Polynomial Coefficients	i	Drivetrain Slip
F_g	Grade Load	T_1		ρ	Air Density
F_w	Vehicle Weight on Inclination	T_2		A	Vehicle Frontal Area
F_d	Drag Force	w_e	Engine Speed	C_D	Vehicle Drag Coefficient
F_i	Drivetrain Inertial Resistance	$w_{e,max}$	Max. Engine Speed	δ	Rotational Inertial Mass Factor
F_{rr}	Rear Rolling Resistance	$w_{e,tmax}$	Engine Speed at Max. Torque	η_{tf}	Drivetrain Efficiency
F_{rf}	Front Rolling Resistance	F	Net Force	N_t	Transmission Reduction Ratio
F_r	Total Rolling Resistance	$F_{max,eng}$	Max. Net Force (Engine Limit)	N_f	Final Reduction Ratio (non-transmission)
F_{tr}	Rear Traction Force	$F_{max,fr}$	Max. Net Force (Friction Limit)	N_{tf}	Drivetrain Reduction Ratio
F_{tf}	Front Traction Force	a	Acceleration	$N_{tf,low}$	Drivetrain Low Reduction Ratio
F_t	Total Traction Force	$a_{max,eng}$	Max. Acceleration (Engine Limit)	$N_{tf,high}$	Drivetrain High Reduction Ratio
$F_{t,max,eng}$	Max. Total Traction Force (Engine Limit)	$a_{max,fr}$	Max. Acceleration (Friction Limit)	$N_{overall}$	Drivetrain Overall Ratio (spread)
$F_{t,max,fr}$	Max. Total Traction Force (Friction Limit)	b	Longt. Distance from front axle to COG	μ_{Rr}	Rear Roll. Resistance Coefficient
W_r	Rear Axle Normal Load	c	Longt. Distance from rear axle to COG	μ_{Rf}	Front Roll. Resistance Coeff.
W_f	Front Axle Normal Load	L	Wheelbase (Longt. Distance b/w axles)	μ_R	Rolling Resistance Coefficient
h_a	Height of center of frontal area	h	Height of COG	μ	Friction Coefficient

1.2. Values

Variable	Value	Units	Reasoning
T_0	9.2260636364	-	Estimated using interpolation from torque curve
T_1	0.0074237652	-	
T_2	-0.0000014089	-	
η_{tf}	0.85	-	Transmission = 90%, FNR = 98%, Chain Drive = 99%, Differential = 98%, Axle = 99% Total = 0.99*0.9*0.98*0.98*0.99 = 0.85 [Ref]
r	0.3175	m	Available tire's radius
m_d	80	kg	Rulebook requirement (95th percentile male)
m_c	260	kg	Based on previous years' vehicle data
μ_R	0.05	-	Value for tractor type tires on fields [Ref]
μ	0.68	-	Value for earth/dirt roads [Ref]
ρ	1.225	kg/m^3	Standard Air density
A	1.0	m^2	Estimated from CAD model
C_D	1.0	-	For a flat plate it is 1.28. Usually cars have values < 1, but for simplicity and conservativeness, 1 is used
i	0.02	-	Drivetrain slip of 2-5% is assumed [Ref]
g	9.81	m/s^2	Standard value
h	1.0	m	Estimated from CAD model
h_a	1.0	m	Assumed to be the same as h
b	0.88	m	Estimated from CAD model, initial assumption based on 60:40 Rear/Front weight distribution commonly found.
c	0.58	m	
L	1.46	m	Estimated from the CAD model, initial assumption
$P_{e,max}$	6.7	kW	From the engine's performance curves
$T_{e,max}$	19	Nm	From the engine's performance curves
$w_{e,max}$	3800	RPM	From the engine's performance curves
$w_{e,t,max}$	2700	RPM	From the engine's performance curves

2. Driveline Dynamics

Consider a vehicle climbing an arbitrary inclination θ with an acceleration a, as shown in **Figure 1**. For a two-dimensional simplification, any lateral forces or moments can be assumed to be zero. Thus, the forces acting on the vehicle can be resolved along two axes: longitudinal (i.e., the along the direction of motion of the vehicle) and vertical (perpendicular to the longitudinal axis).

The net force can be represented by an equivalent inertial force in the opposing direction (so that the net acceleration is zero) called d'Alembert force (F).

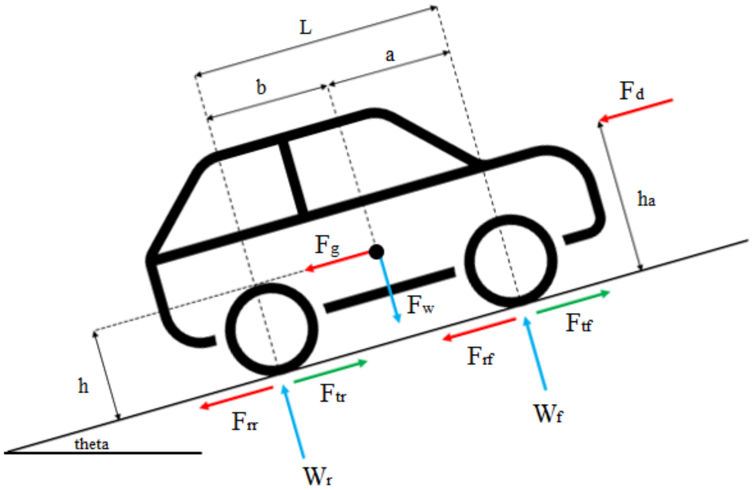

Figure 1: *Free Body Diagram of the Vehicle*

2.1. Engine

The power and torque provided by an IC engine are not constant, and instead vary with the speed. For spark ignition engines, the torque provided reaches a peak at a certain RPM range before decreasing. This is because as the engine speed is increased, the combustion process inside the piston cylinders adds on to the rotational inertia from the previous cycles and thus increases the torque.

However, increasing the speed also increases the friction losses in the piston cylinders. Furthermore, as the speed increases, the time between opening and closing of the valves also significantly decreases, to the point that the air-fuel mixture input is limited by this limitation of the valves. Therefore, torque decreases.

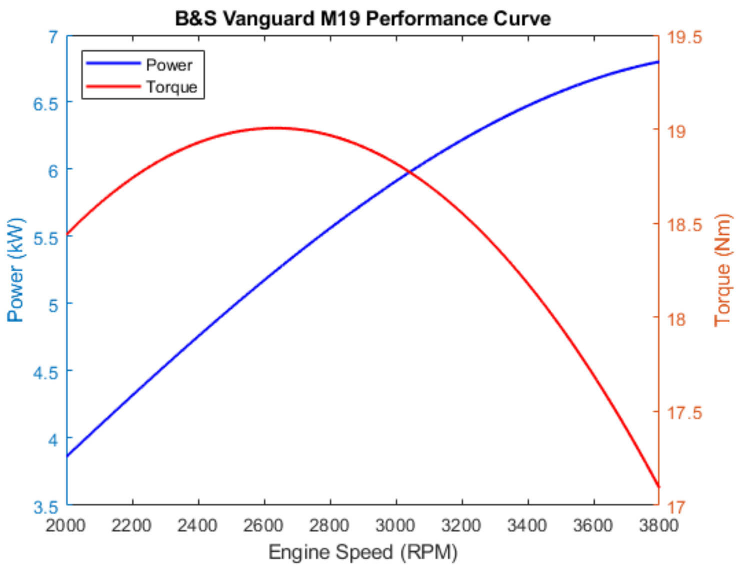

Figure 2: *Performance curves of Baja SAE Engine*

The engine power on the other hand, continues to increase with speed, eventually decreasing at very high speeds. This is because power is product of torque and rotational speed, and even though torque decreases, speed is still increasing. Only when the speed is very high, the torque falls so much that speed increase is unable to increase power.

The power and torque provided by the engine are related to its speed through its performance curves, which are obtained by noting torque determined by a dynamometer. The relationship of engine torque and speed can be approximated by a second-order polynomial from its torque curve data.

$$T_e = T_0 + T_1 w_e + T_2 w_e^2$$

Equation 1: *Engine torque as a function of its speed*

2.2. Resistive Forces

2.2.1. Rolling Resistance

The rolling resistance stems from the distortion of the tire's circular shape at the contact patch, and/or the distortion of the surface on which the tire is rolling. This distortion resists the rolling motion of the tire and is linearly related to the normal axle load via coefficient of rolling resistance. Total rolling resistance load is therefore:

$$F_r = F_{rf} + F_{rr} = \mu_{Rf} W_f + \mu_{Rr} W_r$$

The coefficient of rolling resistance depends on several factors, such as axle load, tire pressure, contact surface types, linear velocity, etc. However, for simplicity, it can be assumed to vary with contact surfaces only. Therefore, if both tires are on same type of surface, then the rolling resistance coefficient for both is equal.

$$F_r = \mu_R W_f + \mu_R W_r = \mu_R \left(W_f + W_r\right)$$

$$F_r = \mu_R \mathrm{mg}\, cos\, \theta$$

2.2.2. Drag

Drag is caused by the friction of the moving vehicle with air and depends on the relative velocity of air with respect to the vehicle. If the air is assumed to be static, this relative velocity is equal to the vehicle velocity.

$$F_d = \frac{1}{2} \rho A C_D V^2$$

2.2.3. Grade Load

Grade load is essentially the component of the vehicle's weight along the longitudinal axis.

$$F_g = \mathrm{mg}\, sin\, \theta$$

2.3. Normal Axle Loads

The axle loads are the normal forces on front and rear axles which balance out the vertical component of the vehicle's weight. When the vehicle is stationary, the axle loads only depend on the weight distribution of the vehicle. When it is accelerating, the load will shift from front to rear axle (or vice versa in case of deceleration).

Applying translational equilibrium condition along the vertical axis results in the following equation:

$$W_f + W_r = F_g = \mathrm{mg}\, cos\, \theta$$

Since the vehicle is in rotational equilibrium, net moment about any point on it must be zero. Applying this condition on the front and rear tire contact patches provides the axle loads.

$$W_f = \frac{mgccos\theta - mah - F_d h_a - \mathrm{mgh} sin\theta}{L}$$

$$W_r = \frac{mgb\, cos\, \theta + mah + F_d h_a + \mathrm{mgh}\, sin\, \theta}{L}$$

The axle load shift is not caused by the vehicle's suspension only; even stiff/rigid vehicles will display this behavior to maintain a net zero moment, as observable from the above equations.

2.4. Vehicle Speed

The vehicle speed will be equal to the linear speed of the tires at the contact patch with the road. If the linear speed at any tire's contact patch is different than the others, the vehicle speed will be equal to the lowest linear speed, while the other tires will slip.

For FWDs/RWDs, only two tires are powered by the engine, and so the speed difference generally only occurs when turning (outer wheels spinning faster than inner ones) or with an open differential. For 4WDs/AWDs however, it is possible that front and rear tires produce different linear speeds due to an open central differential or different tire radii.

Here, it can be assumed that all wheels have same radii, and the central differential is a locked one, so that all wheels always produce same linear speed. The linear speed (and thus the vehicle speed) at any tire's contact patch is equal to its radius times the rotational speed (which is essentially engine speed reduced by the drivetrain ratio). If the net slip occurring in the drivetrain is i%, then the rotational speed at the wheels would decrease by this factor.

$$\mathrm{V} = \mathrm{r} \cdot \frac{2\pi}{60} \cdot (1 - \mathrm{i}) \frac{w_e}{N_{tf}}$$

$$V = \frac{\pi r w_e (1 - i)}{30 N_{tf}}$$

Equation 2: *Vehicle speed relationship with engine speed*

2.5. Tractive Force

The tractive force is the one generated by the drivetrain on the tire contact patches, which essentially pushes the vehicle forward. Since both front and rear tires produce traction in our case, and the central differential is locked, the torque supplied to both front and rear axles is the same (and thus the traction force at each axle is one-half of the total tractive effort). So, the total tractive effort provided is the sum of tractive efforts on front and rear tires.

$$F_t = F_{tf} + F_{tr}$$

The actual tractive force results from the engine's actual torque (varying with engine speed as obtained from **Equation 1**) amplified by the drivetrain (with frictional losses, represented by drivetrain efficiency).

$$F_t = \frac{T_e N_{tf} \eta_{tf}}{r} = \frac{(T_0 + T_1 w_e + T_2 w_e^2) N_{tf} \eta_{tf}}{r}$$

Some of the torque coming from the engine is lost in accelerating the drivetrain components (rotational inertial losses). As torque is the product of moment of inertia and angular acceleration, it follows that to determine these losses, moment of inertias of all components of the drivetrain are required.

Since this approach is quite cumbersome and not useful during early parts of design (when components are not even finalized), the inertia of all these components is usually lumped together in the form of an equivalent mass. This equivalent rotational mass is generally expressed in terms of vehicle curb mass called inertial equivalent mass factor.

$$\frac{m_r}{m_c} = \delta \quad \rightarrow \quad m_r = \delta m_c$$

This transforms the inertial losses into a resistive force, defined as follows:

$$F_i = m_r \mathrm{a} \quad \rightarrow \quad F_i = \delta m_c \mathrm{a}$$

The traction force available at the tires at any engine speed thus becomes:

$$F_t = \frac{(T_0 + T_1 w_e + T_2 w_e^2) N_{tf} \eta_{tf}}{r} - \delta m_c \mathrm{a}$$

2.6. Rotational Inertia equivalent mass factor

The inertial equivalent mass factor is generally estimated by the following equation for the passenger vehicles.

$$\delta = 0.04 + 0.0025 N_{tf}^2$$

However, for very large drivetrain ratios, the above equation gives very large values of mass factor (for example, ~5 for reduction ratio of 40). To solve this procedure, the inertias of the vehicle drivetrain components (tentative) were estimated using the CAD software. From these, the mass factor for several gear ratios were noted. These were then used to find an approximating polynomial as follows.

$$\delta = 0.04 + 0.025 N_{tf} + 0.0000004 N_{tf}^2$$

The mass factors obtained by both equations can be found in **Figure 3**. As is evident, the general equation gives extremely high, improbable values. At reduction ratio of 40 for instance, the equivalent rotational mass is 4 times that of vehicle mass alone. This possibly is because the equation is generally used for passenger vehicles which have relatively powerful engines and thus lower transmission ratios, where it gives more accurate results.

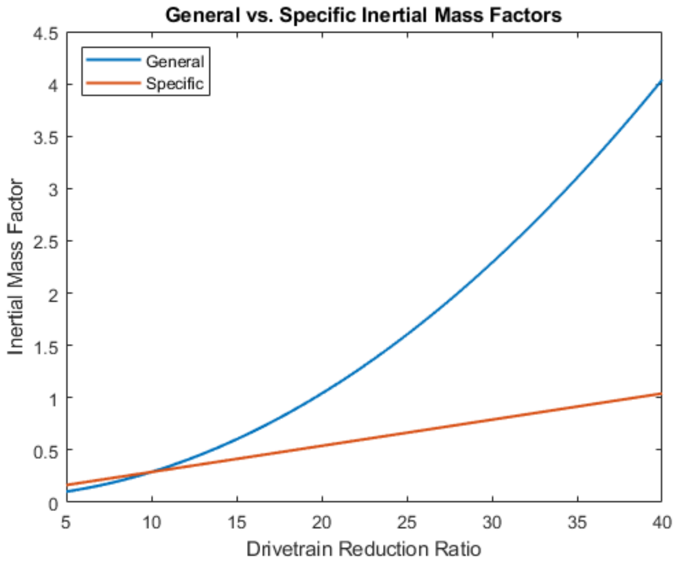

Figure 3: *Rotational mass factors as obtained from general and specific equations*

2.7. Net Acceleration

The net longitudinal force on the vehicle is the sum of all longitudinal forces:

$$F = F_t - F_r - F_d - F_g$$

$$a = \frac{\dfrac{(T_0 + T_1 w_e + T_2 w_e^2) N_{tf} \eta_{tf}}{r} - \mathrm{mg}(\mu_R \cos\theta + \sin\theta) - \dfrac{1}{2}\rho A C_D \left(\dfrac{\pi r w_e (1-i)}{30 N_{tf}}\right)^2}{\mathrm{m_d} + \left(1.04 + 0.025 N_{tf} + 0.0000004 N_{tf}^2\right) m_c}$$

Equation 3: *Vehicle Acceleration at any engine speed, with any drivetrain ratio*

3. Vehicle Performance Limits

3.1. Tractive Force Limit

The tractive force is limited by the maximum power that the engine can supply, and the frictional force between tires and road surface. At any speed, the lower of these limits acts as a boundary for the traction.

3.1.1. Engine Limited

The engine limited maximum tractive force is the upper limit set by the maximum power that the engine can provide. Consider the ideal case, where the engine always provides its maximum power (instead of it varying with speed) and there are no inertial or frictional losses inside the drivetrain. As power is product of force and velocity, it thus follows:

$$P_{e,max} = F_{t,max,eng} \cdot V \quad \rightarrow \quad F_{t,max,eng} = \frac{P_{e,max}}{V}$$

As is evident from the above equation, the ideal maximum tractive force decreases with the speed.

3.1.2. Friction Limited

The friction limited maximum tractive force is the upper limit set by the frictional force between the tires and road surface (which, when overcome, will cause the tires to lose traction and slip). Since both axles are powered by the engine, the traction force is available on all four tires, and this frictional force at both axles will be considered.

$$F_{t,max,fr} = \mu\left(W_f + W_r\right) \quad \rightarrow \quad F_{t,max,fr} = \mu mg\, cos\, \theta$$

3.2. Acceleration Limit

The limit of acceleration set by the maximum traction force (engine limited) can be found as follows:

$$F_{max,eng} = F_{t,max,eng} - F_r - F_d - F_g$$

$$\mathrm{a_{max,eng}} = \frac{\frac{P_{e,max}}{V} - \mathrm{mg}(\mu_R\, cos\, \theta + sin\, \theta) - \frac{1}{2}\rho A C_D V^2}{\mathrm{m}}$$

Equation 4: *Acceleration Limit (Engine)*

The limit of acceleration set by the maximum traction force (friction limited) can be found as follows:

$$F_{max,fr} = F_{t,max,fr} - F_r - F_d - F_g$$

$$\mathrm{a_{max,fr}} = \frac{\mu \mathrm{mg} \cos\theta - \mathrm{mg}(\mu_R\, cos\, \theta + sin\, \theta) - \frac{1}{2}\rho A C_D V^2}{\mathrm{m}}$$

Equation 5: *Acceleration Limit (Friction)*

Equation 4 and **Equation 5** can be plotted against the vehicle speed, for a certain road inclination. At each vehicle speed, the lower limit of the two will constitute the maximum achievable acceleration. The highest point on the combined curve will be the maximum acceleration the vehicle can achieve, regardless of the transmission or reduction ratio used.

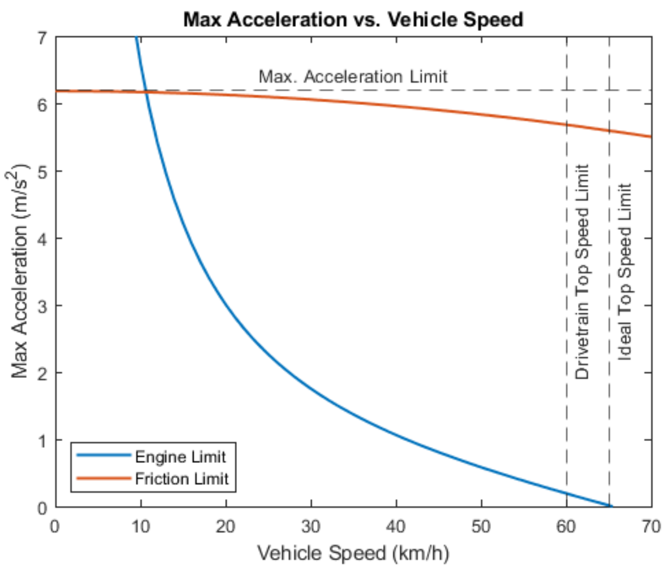

Figure 4: *Maximum Acceleration & Top Speed Limits*

The plots of these equations for an inclination of zero are shown in **Figure 4**. It can be noted from the plots that at very low speeds, the maximum acceleration is limited by the friction, while at higher speeds engine is the deciding factor. The maximum possible acceleration with specified parameters is $6.2 m/s^2$.

3.3. Top Speed Limit

In both limits (engine and friction), the **top speed limits** will be the one where acceleration becomes zero. The smaller one of these is the absolute maximum speed that the vehicle can reach for given conditions, regardless of the transmission or drivetrain reduction ratio used.

From **Figure 4**, the maximum attainable top speed (where acceleration becomes zero) is $65\ km/h$.

For an actual drivetrain, the top speed will be achieved under the highest reduction ratio. Following conditions will be applicable:

- The engine is at its maximum speed.
- The vehicle is not accelerating any further.
- No inclination, the vehicle is on flat surface

With these, **Equation 3** becomes as noted below:

$$\frac{\left(T_0 + T_1 w_{e,max} + T_2 w_{e,max}^2\right) N_{tf,high} \eta_{tf}}{r} - \mu_R mg - \frac{1}{2}\rho A C_D \left(\frac{\pi r w_{e,max}(1-i)}{30 N_{tf,high}}\right)^2 = 0$$

$$\frac{\left(T_0 + T_1 w_{e,max} + T_2 w_{e,max}^2\right)\left(\frac{\pi r w_{e,max}(1-i)}{30 V_{top,max}}\right)\eta_{tf}}{r} - \mu_R mg - \frac{1}{2}\rho A C_D {V_{top,max}}^2 = 0$$

Solving this implicit equation via graphing gives the top speed limit as $60\ km/h$.

If the drivetrain ratio is set lower than that for this speed, the top speed will not increase. This is because engine will have not yet reached its maximum speed, but the acceleration has become zero. As such, it cannot provide more force to the vehicle and thus reach its top rotational speed.

3.4. Gradeability Limit

The maximum inclination climbable by the vehicle depends on the coefficient of friction between the road and tires, and for a 4WD/AWD is given by the following equation:

$$tan(\theta_{max}) = \mu$$

The limit is independent of engine, transmission, or vehicle, as the tires WILL slip at this inclination and lose traction. For the considered vehicle and conditions, the maximum gradeability achievable is 68% since the value of friction coefficient for the road surface is 0.68.

$$tan\,\theta_{max} = 0.68 \quad \rightarrow \quad \theta_{max} = 34^\circ$$

4. Drivetrain Calculations

4.1. Drivetrain Requirements

The drivetrain requirements are set by studying the goals and performance of previous years' teams, and vehicle's ideal limits. They are described as follows:

- A top speed of 60 km/h
- Maximum gradeability of 30 degrees (57%)

4.2. Reduction Ratios

4.2.1. High Reduction Ratio

The high reduction ratio can be obtained from **Equation 2** with vehicle speed set at the desired top speed of 60 km/h, and the engine running at its maximum speed.

$$\mathrm{V_{top}} = \frac{\pi r w_{e,max}(1-i)}{30N_{tf,high}}$$

$$N_{tf,high} = 7.5$$

4.2.2. Low Reduction Ratio (Climbing)

The lowest gear ratio will be used for highest torque requirement. This will occur when the vehicle is climbing its maximum gradeability or when starting (for maximum acceleration).

Following conditions will be applicable when the vehicle is climbing maximum possible inclination.

- The engine is producing its maximum torque.
- So, the engine speed would be that at maximum torque conditions.
- The vehicle is climbing an inclination.
- The vehicle is not accelerating any further (as the sum of all forces on it is zero)

With these conditions, **Equation 3** becomes:

$$\frac{T_{e,max}N_{tf,low}\eta_{tf}}{r} - \mathrm{mg}(\mu_R \cos\theta + \sin\theta) - \frac{1}{2}\rho A C_D\left(\frac{\pi r w_{e,t,max}(1-i)}{30N_{tf}}\right)^2 = 0$$

Equation 6: *Climbable inclination relationship with low gear ratio*

Solving this implicit equation with an inclination of 30 degrees gives a low gear ratio of:

$$N_{tf,low} = 35$$

4.2.3. Low Reduction Ratio (Max. Acceleration)

The vehicle will produce maximum acceleration at the obtained low reduction ratio, and following conditions will be applicable:

- The engine is producing its maximum torque.
- So, the engine speed would be that at maximum torque conditions.
- The vehicle is on a flat terrain.

Applying these conditions to **Equation 3** transforms it into following equation:

$$\mathrm{a_{max}} = \frac{\frac{\mathrm{T_{e,max}}N_{tf,low}\eta_{tf}}{r} - \mu_R \mathrm{g}(\mathrm{m_c}+\mathrm{m_d}) - \frac{1}{2}\rho A C_D\left(\frac{\pi r w_{e,tmax}(1-i)}{30N_{tf,low}}\right)^2}{\mathrm{m_d} + \left(1.04 + 0.025N_{tf,low} + 0.0000004N_{tf,low}^2\right)m_c}$$

Equation 7*: Maximum Vehicle Acceleration for a given low ratio*

Solving this equation with the obtained value of low ratio gives the maximum acceleration as follows:

$$a = 2.9m/s^2$$

Assuming a constant acceleration (which is not the case in real life), using the second equation of motion with initial velocity set to zero, we can note that with this acceleration, the vehicle will traverse 100m in ~9 seconds.

4.2.4. Overall Ratio

The reduction ratio spread (or overall ratio) can be found by dividing the lower ratio with the higher ratio.

$$N_{overall} = \frac{N_{tf,low}}{N_{tf,high}} = 4.67$$

4.3. Manual Transmission Performance

For a manual transmission, the available gearbox has reduction ratios defined as follows:

- 1st Gear: 10, 2nd Gear: 6
- 3rd Gear: 4.5, 4th Gear: 3.5

The overall ratio of the gearbox is therefore 10/3.5 = 2.857, which is lower than the required overall ratio of 4.67. This means that both design conditions (top speed and acceleration/gradeability) cannot be satisfied. Either one of the conditions will have to be compromised by a large margin, or both must be compromised with a smaller margin by choosing values in between the desired range of reduction ratios (7.4 and 35). Consider the velocity equation for top speed again. The high gear ratio can be expressed in terms of low gear ratio and overall ratio as follows:

$$V_{top} = \frac{\pi r w_{e,max}(1-i)}{30 N_{tf,high}}$$

$$V_{top} = \frac{\pi r w_{e,max}(1-i)}{30 \dfrac{N_{tf,low}}{N_{overall}}}$$

Equation 8: *Vehicle Top Speed as a function of low gear ratio*

Equation 6, **Equation 7**, and **Equation 8** can be plotted to get the maximum climbable inclination, maximum acceleration and top speed variation with low gear ratio. Note that **inclination is zero**.

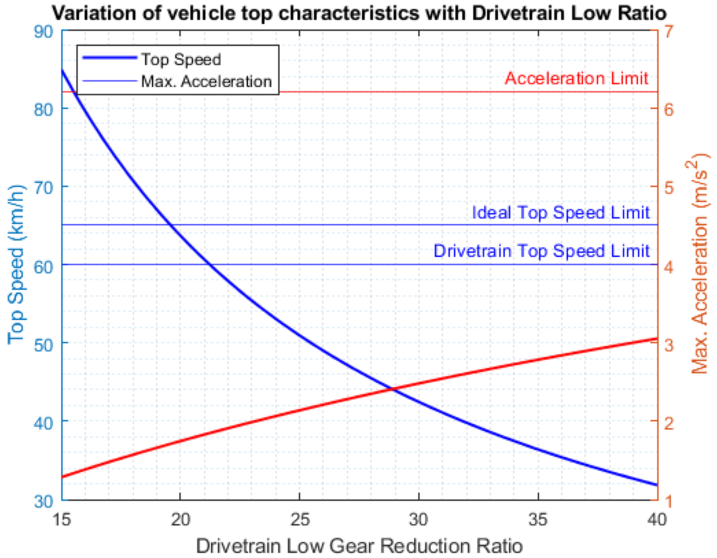

Figure 5: *Variation of top speed and maximum acceleration with the given gearbox*

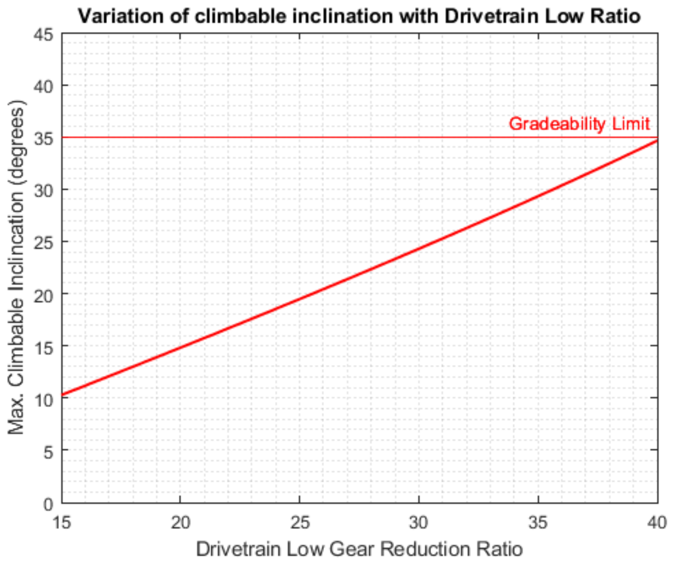

Figure 6: *Variation of climbable inclination with the given gearbox*

Since endurance race is scored much higher in the competition, top speed requirement is considered more important. So, the reduction ratio which gives the target top speed of 60 km/h is chosen from the graph:

$$N_{tf,low} = 20$$

Since the low ratio of transmission gearbox is 10 (that is, first gear), the required final drive ratio is:

$$10 \cdot N_f = 20 \quad \rightarrow \quad N_f = 2$$

With this final drive ratio, the drivetrain reduction ratios in all four gears will be:

- 1^{st} Gear: 20
- 2^{nd} Gear: 12
- 3^{rd} Gear: 9
- 4^{th} Gear: 7

Using these values in the **Equation 2** and **Equation 3**, we can plot the graphs of acceleration vs. velocity and velocity vs. engine speed as follows. Note that **inclination is zero**.

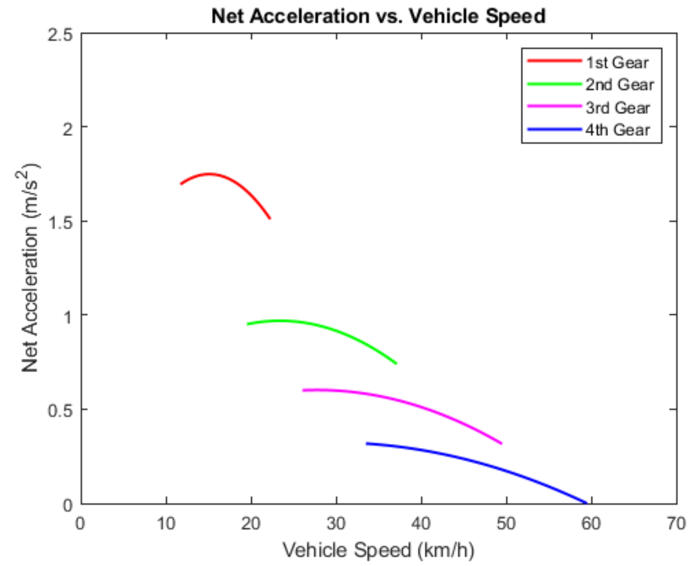

Figure 7: *Acceleration vs Speed Graph with given gearbox*

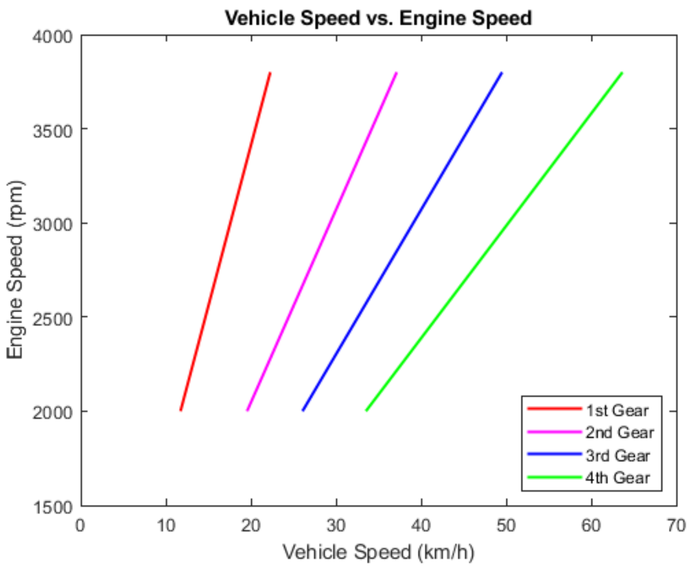

Figure 8: *Vehicle speed vs Engine Speed graph with given gearbox*

Since the tuk-tuk gearbox used is designed for high RPM ranges (around 8000), it would be preferable to use a gear ratio between it and engine so that the engine's smaller RPM range is effectively increased. To counter this torque reduction effect in overall ratio, the subsequent central chain drive's ratio will be kept such that it nullifies this reduction.

The drivetrain constant ratios are:

- **Engine-Gearbox: 0.577**
- **FNR Gearbox: 1.25**
- **Central Chain Drive: 1.11**
- **Differential: 2.5**

The final characteristics of the drivetrain are:

- **Max. Acceleration: 1.8 m/s^2**
- **Top Speed: 60 km/h**
- **Gradeability: 15 degrees (27%)**
- **Layout: 4WD with locked center-differential and open end-differentials**
- **Four speed manual transmission (250cc Tuk-Tuk) with reverse gear**

Other stuff

Net Inertial Losses in the drivetrain for calculating mass factor (refer to Gillespie's book for more information on how this equation is formed)

$$I_{net} = I_e a_e - I_{sp1}a_{sp1} - I_{sp2}a_{sp2} - I_{gi}a_{gi} - I_{go}a_{go} - I_{fnr} - a_{fnr} - I_{sp3}a_{sp3} - I_{sp4}a_{sp4} - I_d a_d - I_{df}a_{df} - I_{ax}a_{ax} - I_w a_w$$

$$I_{net} = I_e N_{df}N_{cd2}N_{fnr}N_t N_{cd1}a_w - I_{sp1}N_{df}N_{cd2}N_{fnr}N_t N_{cd1}a_w - I_{sp2}N_{df}N_{cd2}N_{fnr}N_t a_w - I_{gi}N_{df}N_{cd2}N_{fnr}N_t a_w - I_{go}N_{df}N_{cd2}N_{fnr}a_w - I_{fnr}N_{df}N_{cd2}a_w - I_{sp3}N_{df}N_{cd2}a_w - I_{sp4}N_{df}a_w - I_d N_{df}a_w - I_{df}a_w - I_{ax}a_w - I_w a_w$$

$$m_r = \left(I_e + I_{sp1}\right)N_{df}N_{cd2}N_{fnr}N_t N_{cd1} + \left(I_{sp2} + I_{gi}\right)N_{df}N_{cd2}N_{fnr}N_t + I_{go}N_{df}N_{cd2}N_{fnr} + \left(I_{fnr} + I_{sp3}\right)N_{df}N_{cd2} + \left(I_{sp4} + I_d\right)N_{df} + \left(I_{df} + I_{ax} + I_w\right)/r^2$$