Dialogue Semantics for Bilateralism: Towards a Multi-Agent Account

Rohan French
University of Groningen

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Bilateralism

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- The bilateralist proposes to do this by explaining them in terms of their role in conversation.

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From Positions to Consequence

- We can then define logical consequence in terms of positions by saying that

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whenever the position $[X : Y]$ *involves a clash*.

- This gives us a novel interpretation in social terms of Gentzen's multiple conclusion sequent calculus.

Positions and Consequence: Example

$$\frac{\Gamma \vdash \Delta, A}{\Gamma, \neg A \vdash \Delta}\ [\neg L]$$

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- This rule tells us about how the speech act of denial and the negation operator interact.
- If denying A in a position involves a clash, then asserting $\neg A$ in that same position also involves a clash.
- Similarly, if asserting $\neg A$ in a position does not involve a clash then neither does denying A.

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- **Assertions** are social acts, which involve multiple agents.

  But there's barely one agent involved in the above account.

  This is just 'assertion in the void'.
- We need an account of what's involved in a position **clashing**.

The Need for a Multi-Agent Account

What's involved in a clash?

"[T]he failure in such a conversational position, in which each member of X is asserted and each member of Y is denied, is of the same kind as the simplest failure of them all, the joint assertion and denial of the one statement. The fundamental normative force holds between assertion and denial. We take a denial of A to stand against an assertion of A, and vice versa."

–Greg Restall *Truth Values and Proof Theory, p.243.*

What's involved in a clash?

- What kind of failure is involved here? There are a variety of stories which one could tell (appealing to various norms on assertion, etc.), but there's a more direct answer available if we look at the role of assertion and denial in our conversational practices...

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- What kind of failure is involved here? There are a variety of stories which one could tell (appealing to various norms on assertion, etc.), but there's a more direct answer available if we look at the role of assertion and denial in our conversational practices...
- ... and the best model we have of such practices are dialogue games.

Agreement Games

Agreement Games: Motivation

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- So if the conversational context is a position which involves a clash then we are guaranteed to end up in disagreement.
- *Agreement Games* are designed to model situations in which a pair of agents are attempting to examine and extrapolate each others commitments in such a way as to attempt to avoid disagreement.
- If they can then there position does not involve a clash.

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- $\mathfrak{L}_{\mathrm{rb}}$ will be the language which contains, for every $A \in \mathfrak{L}$ the formulas $[+]A$, $[-]A$, $[+]A$, $[-]A$.

Agreement Games: Definition

- The game is played relative to what we'll call a *context*: a pair of sets (C_r, C_b) of $\mathfrak{L}_{rb}$-formulas; where each C_i is a set of formulas of the form $[+_i]A$ and $[-_i]A$.

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- The players take turns *challenging* formulas from the context, **Red** challenging formulas from C_b and **Black** challenging formulas from C_r.
- Players then get to respond to the challenge in a manner which depends on the shape of the fromula challenged, adding a formula to their part of the context.

Agreement Games: Challenges

Formula	Challenge	Response
$[+]A \wedge B$	$? \wedge_+ L$	$[+]A$
	$? \wedge_+ R$	$[+]B$
$[-]A \wedge B$	$?\wedge_-$	$[-]A$
		$[-]B$
$[+]\neg A$	$?\neg_+$	$[-]A$
$[-]\neg A$	$?\neg_-$	$[+]A$

- If the context ever ends up containing $[+]A$ and $[-]A$ or $[-]A$ and $[+]A$ both players lose.

Agreement Games: Winning & Losing

- If the context ever ends up containing $[+]A$ and $[-]A$ or $[-]A$ and $[+]A$ both players lose.
- If there are no further challenges which can be made and the game has not been lost then the players have won.

Agreement Games: An Example

Moves

1. Black: $?[-]\neg A$

Context

$\{[-]\neg A, [+]B\}$

$\{[+]\neg(A \wedge B)\}$

Agreement Games: An Example

Moves

1. Black: $?[-]\neg A$
2. Red: $![+]A$

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Agreement Games: An Example

Moves

1. Black: $?[-]\neg A$
2. Red: $![+]A$
3. Red: $?[+]\neg(A \wedge B)$
4. Black: $![-]A \wedge B$

Context

$\{[-]\neg A, [+]B, [+]A\}$

$\{[+]\neg(A \wedge B), [-]\mathbf{A} \wedge \mathbf{B}\}$

Agreement Games: An Example

Moves

1. Black: $?[-]\neg A$
2. Red: $![+]A$
3. Red: $?[+]\neg(A \wedge B)$
4. Black: $![-]A \wedge B$
5. Black: Pass

Context

$\{[-]\neg A, [+]B, [+]A\}$

$\{[+]\neg(A \wedge B), [-]A \wedge B\}$

Agreement Games: An Example

Moves

1. Black: $?[-]\neg A$
2. Red: $![+]A$
3. Red: $?[+]\neg(A \wedge B)$
4. Black: $![-]A \wedge B$
5. Black: Pass
6. Red: $?[-]A \wedge B$

Context

$\{[-]\neg A, [+]B, [+]A\}$

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Agreement Games: An Example

Moves

1. Black: $?[-]\neg A$
2. Red: $![+]A$
3. Red: $?[+]\neg(A \wedge B)$
4. Black: $![-]A \wedge B$
5. Black: Pass
6. Red: $?[-]A \wedge B$
7. Black: $![-]A$

Context

$\{[-]\neg A, [+]B, [+]A\}$

$\{[+]\neg(A \wedge B), [-]A \wedge B, [-]\mathbf{A}\}$

Agreement, Disagreement and Consequence

Winning Strategies

- Usually dialogue games are shown to characterize logics via the existence of *winning strategies*—complete trees of moves which, if followed, will always result in a win for that player.

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- This is usually because the moves one player makes constrains the moves which the other player is able to make.
- This rarely happens in real-world social situations, though.

Winning Strategies

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- Cooperation can't be forced!

- What we will show is that unsuccessfull proof searches in a one sided sequent calculus for $\mathfrak{L}_{\mathrm{rb}}$ can be transformed into 'cooperative strategies' of this kind.

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- Given that these correspond many-one with sequent derivations in Gentzen’s multiple conclusion sequent calculus this gives us a characterisation of when a position $[X : Y]$ involves a clash.

The 'Red-Black Calculus'

$$\frac{}{\Gamma, [+_i]A, [-_j]A}\ [Ax]$$

$$\frac{\Gamma, [+_i]A, [+_i]B}{\Gamma, [+_i]A \wedge B}\ [+_i\wedge] \qquad \frac{\Gamma, [-_i]A \quad \Gamma, [-_i]B}{\Gamma, [-_i]A \wedge B}\ [-_i\wedge]$$

$$\frac{\Gamma, [-_i]A}{\Gamma, [+_i]\neg A}\ [+_i\neg] \qquad \frac{\Gamma, [+_i]A}{\Gamma, [-_i]\neg A}\ [-_i\neg]$$

- Say that a set X of formulas from $\mathfrak{L}_{\mathrm{rb}}$ is *compatible* iff it corresponds in the natural way to a context which can be extended into one which results in a win.

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- Then what we can show is that precisely those sets X which are compatible are unprovable.
- If X is provable it is easy to see that it cannot be compatible due to a correspondance between plays of the agreement game and paths through derivations in the red-black calculus.

Theorem: If X is unprovable then X is compatible.

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- If X is unprovable then a proof search for it will result in a tree Π.
- Π will have some leaves $S_1, \ldots, S_n$ which are not instances of $[Ax]$
- Permute the rules with one another so that we alternate between r and b insertion rules.
- Players can then maintain the invariant of staying in a branch whose leaves contain one of the S_i, challenging at each step the last formula introduced.

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- In this case it also might shed some light on the status of the rule of [Cut] in Bilateralist treatments of logic.
- According to the current account, [Cut] corresponds to an ‘emergent property’ of compatible and incompatible positions.
- This suggests that [Cut] might not be constitutive of bilateralist framework.

Thank You!