A Dialogical Account of Logical Pluralism

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1 The Explanatory Problem for Logical Pluralism

- Logical pluralists holds that there are, in some sense, multiple different correct logics which we can use to assess the validity of any given argument, any of which may deliver inconsistent validity judgements concerning a particular argument.

> A characterisation of a pluralist position needs to explain what it is to endorse all of the various consequence relations the pluralist accepts and how they relate to an intuitive notion of logical consequence. Keefe (2014, p.1376)

- The logical pluralist needs to explain two things: first, they need to explain how there can be more than one correct notion of logical consequence; and secondly, why these various different notions of logical consequence are notions of *logical* consequence.
- Our strategy here is to use this explanatory challenge to both enhance our understanding of the built-in opponent conception of deduction, the issues arising in attempting to answer this explanatory challenge being of interest even to card-carrying logical monists.

2 The Built-In Opponent Conception of Deduction

- A dialogical, multi-agent conception of the nature logica and deductive proof introduced in Dutilh Novaes (2013, 2015).
- Deductive arguments correspond to a specialised kind of semi-adversarial dialogues—dialogical interactions with both an adversarial as well as a cooperative component. The two participants in these dialogues have opposed goals:
  - The Prover (=Proponent) seeks to establish that a certain conclusion follows from given premises
  - The Skeptic (=Opponent) seeks to block the establishment of this conclusion, asking pointed 'why does this follow?' questions (i.e. 'if you can show *this* i'll accept *that*')
- In essence the Skeptic asks for, and the Prover gives, reasons for their claims of logical consequence.

> Proponent's job is not only to 'beat Opponent'; she also seeks to *persuade* Opponent of the truth of the conclusion, if he has granted the truth of the premises. In fact, the goal is not only to show *that* the conclusion follows from the premises, but also *why* it does; this corresponds to the idea that deductive arguments ought to have explanatory value. In this sense, Proponent and Opponent are cooperating in a common inquiry to establish what follows from the premises, and thus to further investigate the topic in question. Dutilh Novaes (2015, p.599f)

- Skeptic's role also involves higher-order cooperation: he must say what would convince them, and not obstinately refuse to admit that the premises follow from the conclusion.
- One can view part of the tension between Achilles and the Tortoise in Carrol (1895) as arising out of the Tortoise not being a particularly cooperative Skeptic, and not being clear about what it would take to convince them that an inference via Modus Ponens is valid (resulting in them being a decidedly passive agressive Skeptic).
- The other important aspect of the present approach is the idea that over time Skeptic has been progressively 'silenced' and idealized, until they are no longer an active participant in a deductive dialogue but have instead become part of the deductive method itself. In essence their role has been internalized and is played 'offline' by Prover. In order to beat such an idealized opponent Prover needs to ensure that there are no counterexamples to the inferential steps which they provide as responses to the Skeptic's queries.

3 Introducing Prover-Skeptic Games

- *Prover-Skeptic games* are a kind of dialogue game introduced in (Sørensen and Urzyczyn, 2006, p.89–94) to game-theoretically characterise the implicational fragment of intuitionistic logic. We follow (and build on) the presentation of such games given in Shoham and Francez (2008).

In the library two logic students, ***Penelope*** *and* ***Scott*** *are arguing over the validity of the argument from* $p \to q$ *and* $q \to r$ *to* $p \to r$. *Penelope thinks that this argument is valid and is trying to convince Scott, who is skeptical.*

- **Penelope (1):** I reckon that $p \to r$ follows from $p \to q$ and $q \to r$
- **Scott (1):** Yeah? Well if that's so then suppose I grant you $p \to q$ and $q \to r$ along with p, how are you meant to get r?
- **Penelope (2):** If you grant me q I can get r from $q \to r$ (which you just granted).
- **Scott (2):** But why should I grant you q?
- **Penelope (3):** Well if you were to grant me p then I could get q from $p \to q$ which you granted at the start.
- **Scott (3):** But why should I grant you p?
- **Penelope (4):** Because you granted it to me at the start!

Penelope leaves the library triumphantly.

Figure 1: **A Dialogue** representing the kind of situation modelled by Prover-Skeptic games.

- We stick with the language $\mathcal{L}$ of implicational logic, the formulas of which are constructed out of a countable supply of propositional variables $p_0, p_1, p_2, \ldots$ using the binary connective '$\to$' of implication.
- Throughout we will be concerned with fragments of the following natural deduction system, resulting from dropping some or all of the structural rules listed.

AXIOMS

$$A \succ A$$

INTRODUCTION/ELIMINATION RULES

$$\frac{\Gamma, A \succ B}{\Gamma \succ A \to B}\ (\to I) \qquad \frac{\Gamma \succ A \quad \Delta \succ A \to B}{\Gamma, \Delta \succ B}\ (\to E)$$

STRUCTURAL RULES

$$\frac{\Gamma, A, A \succ B}{\Gamma, A \succ B}\ (W) \qquad \frac{\Gamma \succ B}{\Gamma, A \succ B}\ (K)$$

Figure 2: Our Sequent-to-Sequent Style Natural Deduction System

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- Given a multiset of sequents S and sequents β, γ let us write:
  - $\beta \Rightarrow^*_{(I)} \gamma$ whenever we can derive γ from β using zero or more applications of our introduction rules (in our case $(\to I)$)
  - $S \Rightarrow^*_{(E)} \beta$ iff we can derive β from the sequents in S via zero or more applications of our elimination and structural rules (i.e. using zero or more applications of $(\to E)$, (W) and (K)).
- Finally we will say that a sequent $\Gamma \succ A$ is *atom-focused* whenever A is a propositional variable.

Definition 3.1. *(Dialogue Game) A dialogue over (Γ, A) is a (possibly infinite) sequence $S_1, \beta_1, S_2, \beta_2, \ldots$ where each S_i is a multiset of sequents, and each β_i a sequent where:*

1. $S_1 = [\Gamma \succ A]$
2. $\beta_i \Rightarrow^*_{(I)} \sigma$ *for some non-axiomatic $\sigma \in S_i$ and some atom-focused sequent β_i.*
3. $S_{i+1} \Rightarrow^*_{(E)} \beta_i$

- We can understand a *Skeptic-move* β as doubting σ and asking Prover to show β.
- Similarly we can understand a *Prover-move* S in responce to a skeptic-move β as Prover providing the grounds for their assertion that σ.
- We will say that a Prover *wins a dialogue* if we reach a point at which Skeptic can make no further move. Further, we have that if $\Gamma \succ A$ is provable in our natural deduction system, then Prover has a *winning strategy* in the dialogue over (Γ, A).

4 INTERNALISATION & COOPERATION

- To see that Skeptic's moves are apt for being internalized it is worth noting that we can re-describe Skeptic moves in the following way. A Skeptic move challenging a sequent $\Gamma \succ B$ will result in a sequent $\beta = \Gamma, B_1, \ldots, B_n \succ p_i$ where $B = B_1 \to (\ldots \to (B_n \to p_i))$.
- Skeptic's moves do not require any form of creativity to perform. Given that their role is essentially algorithmic it is easy to see how it is apt to be performed 'offline' by a Prover who follows this method.
- We also have the right mix of adversariality and cooperation between Prover and Skeptic present in Prover-Skeptic games. The two players have different goals, and it's the interaction between these different goals which forces Prover to try to put forward only derivable sequents (and thus to try to present a valid argument). At the same time the players also have to agree on what counts as a challenge, what counts as an acceptable responce to a challenge.

> The adversarial component is what accounts for the property of necessary truth preservation; the cooperative component is what explains the ideal of perspicuity and explanatoriness. (Dutilh Novaes, 2015, p.599f)

- It is from these cooperative components that pluralism arises.

5 PROOF AND EXPLANATION

- Prover moves are meant to do more than merely show that the premises follow from the conclusion, they're meant to be *explanatory*.
- The notion of explanation which is most appropriate here is broadly 'pragmatic' in nature, thinking of explanation as answers to why-questions following Van Fraassen (1988).
- A why-question is just a request for information of a certain sort, and what information is requested can differ from context to context. (Van Fraassen, 1988, p.153).
- In the present context, the information requested is of the form 'why does this follow', with the answer intended to convince the Skeptic that it follows. But what counts as a satisfactory answer to such a question may vary from context to context. If it does in the right way, then we will have a naturally arising form of folk-relativism about logical consequence.

6 A ROUTE TO PLURALISM

- Do different contraints on what counts as an acceptable responce to a Skeptic challenge have logical repercussions? One plausible kind of constraint that might arise concerns the level of detail, or granularity, of the required explanation. Indeed we can see this in mathematical practice, where different levels of detail are requested and required in teaching as opposed to research situations (Hersh, 1993).

- There do seem to be cases where changes in what counts as acceptable answer the question of 'why does this follow' have logical repercussions.

6.1 *Foundations of Geometry*

- In Pambuccian (2004) we are told that work in the foundations of geometry at the beginning of the 20th century appears to display a concern, not just for what axioms are used in proving a particular theorem, but also in the *number of times* that such axioms are, or indeed must, be so appealed to.
- For example, G. Hessenberg in 1905 proves that Desargues axiom follows from a collection of axioms for plane projective geometry along with a threefold use of the Pappus axiom. Pambuccian notes that all known proofs both require, and explicitly mention, this threefold use.
- So this can be seen as a situation in which what counts as an acceptable answer to the question of 'why does this follow' must not just display that the challenged claim follows 'of necessity' from the claims provided, but also the extent to which different claims are used in showing this. This 'resource consciousness' suggests an abandoning of the structural rule of contraction, according to which whenever A follows from Γ and two copies of B, and when A follows from Γ and a single copy of B.

6.2 *Confirmation*

- According to Glymour (1980) Hypothetico-Deductivism is "hopeless as a method of confirmation because it cannot maintain the condition of relevance between a hypothesis and the evidence which confirms it", as Waters (1987, p.453) summarises the main thrust of Glymour's very short paper.
- Attempts to save the Hypothetico-Deductive model of confirmation from Glymour's charges (e.g. those in Schurz (1991), Gemes (1993)) all appear to put forward constraints which are variations on the theme of Relevant logic á la Anderson and Belnap (1975).
- This suggests that in contexts where we are concerned with confirmation the norms on explanation require the appropriate kind of connection of relevance between theory, hypothesis and evidence when we are trying to show that the theory and hypothesis entail the evidence. At the very least this suggests a rejection of the structural rule of weakening.

- These cases appear to give us places where the norms on explanation which govern deductive dialogues appear to be (contra (Beall and Restall, 2006, p.88)) relative to communities of inquiry, or perhaps better, relative to the particular interests of those communities.

7 Prover-Skeptic Games for Substructural Logics

- We can model this in our Prover-Skeptic games in the obvious way, by placing constraints on which structural rules can be appealed to in a Prover move.
- So, for example, in the contexts of concerns about the foundations of geometry we cannot use contraction in our Prove moves (resulting in the games characterising affine or linear logic), and in confirmation contexts we cannot use weakening (resulting in the games characterising relevant logic).

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Prover-Skeptic Games: Some Examples

Transitivity of Entailment Our furst dialogue is a formalization of the dialogue between Penelope and Scott in Figure 1. This is a dialogue over $([p \to q, q \to r], p \to r)$, and proceeds as follows.

P(1): $[p \to q, q \to r \succ p \to r]$
Prover begins by stating the claim they wish to defend.

S(1): $p \to q, q \to r, p \succ r$
Skeptic responds by challenging the only available sequent, requesting that Prover demonstrate how they can get here.

P(2): $[q \to r \succ q \to r\,;\, p \to q, p \succ q]$
Prover responds by providing a collection of sequents from which they can derive the challenge via $(\to E)$.

S(2): $p \to q, p \succ q$
Skeptic challenges the only non-axiomatic sequent in the previous Prover move.

P(3): $[p \to q \succ p \to q\,;\, p \succ p]$
Prover replies offering two axiomatic sequents.

S(3): No Available Move
As all of the sequents which Prover offered are axioms there are no further moves available to the Skeptic.

'Totality' We turn now to a fragment of a losing dialogue for Prover, which exhibits more of the back-and-forth of a standard debate. Here we consider a dialogue for the principle one might refer to as 'totality', namely $(p \to q) \to r \succ ((q \to p) \to r) \to r$. The quotes here are shudder quotes intending to indicate that this involves taking what is sometimes called the 'deductive disjunction' of the two disjuncts in the standard totality principle $(p \to q) \vee (q \to p)$.

P(1): $[(p \to q) \to r \succ ((q \to p) \to r) \to r]$
Prover begins by stating the claim they wish to defend.

S(1): $(p \to q) \to r, (q \to p) \to r \succ r$
Skeptic responds by challenging the only available sequent, requesting that Prover demonstrate how they can get here.

P(2): $[\succ ((p \to q) \to r) \to (((q \to p) \to r) \to r)\,;\, (p \to q) \to r \succ (p \to q) \to r\,;\, (q \to p) \to r \succ (q \to p) \to r)]$
Prover responds by providing a collection of sequents from which the challenge can be derived using $(\to E)$.

S(2): $(p \to q) \to r, (q \to p) \to r \succ r$
Skeptic challenges the first formula, putting us back in the position we were in at the start of the dialogue.
This Dialogue will Continue On Indefinitely

Three-Fold Contraction Finally, as an example of how the availability of structural rules effects our Prover Skeptic games, let us consider the dialogue for $p \to (p \to (p \to q)) \succ p \to q$.

P(1): $[p \to (p \to (p \to q)) \succ p \to q]$

S(1): $p \to (p \to (p \to q)), p \succ q$

Now, again, at this stage how the Prover can respond depends on the kind of dialogue we are dealing with. If we are in a standard or relevant dialogue then we can respond with:

P(2): $[p \to (p \to (p \to q)) \succ p \to (p \to (p \to q))\,;\, p \succ p\,;\, p \succ p\,;\, p \succ p]$

From these four sequents we can conclude $p, p \to p \to (p \to (p \to q)) \succ p \to (p \to q)$ by on application of $(\to E)$, then $p, p, p \to (p \to q) \succ p \to q$ by a further application of $(\to E)$, and then finally $p, p, p, p \to (p \to q) \succ q$ by a final application of $(\to E)$. The challenge then follows from this sequent by a two applications of (W). As this requires contraction, though, this strategy is unavailable in affine or linear dialogues. In fact, there is no way for Prover to respond to this challenge in an affine or linear dialogue which results in a win.