

A Dialogical Account of Logical Pluralism

Rohan French

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MY PLAN

A Problem for Logical Pluralists

The Built-In Opponent Conception of Deduction

Introducing Prover-Skeptic Games

Prover-Skeptic Games: An Example

Proofs & Explanations

A Route to Pluralism



A Problem for Logical Pluralists



THE EXPLANATORY CHALLENGE FOR LOGICAL PLURALISTS

A characterisation of a pluralist position needs to explain what it is to endorse all of the various consequence relations the pluralist accepts and how they relate to an intuitive notion of logical consequence.

—Keefe, 2014, p.1376

Two CENTRAL QUESTIONS

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- What is it to ‘endorse’ a consequence relation?

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- What is it to ‘endorse’ a consequence relation?
- Why are these consequence relations relations of logical consequence?

The background image shows an aerial view of a city or industrial area. The streets form a clear grid pattern, and there are numerous rectangular buildings, some with dark roofs and others with light-colored facades. In the foreground, there are several large, dark industrial buildings with multiple stories and various pipes and structures visible on their exteriors. The overall scene suggests a well-planned urban environment with significant industrial activity.

The Built-In Opponent Conception of Deduction

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- ... viewing **proofs** as grounded in semi-adversarial debates between a Prover/Proponent and a Skeptic/Opponent.
 - Prover seeks to establish the conclusion.
 - Skeptic forces Prover to make their reasoning explicit.
- Deductive proofs are then winning strategies for Prover in these debates.

Proponent's job is not only to 'beat Opponent'; she also seeks to persuade Opponent of the truth of the conclusion, if he has granted the truth of the premises. In fact, the goal is not only to show that the conclusion follows from the premises, but also why it does; this corresponds to the idea that deductive arguments ought to have explanatory value.

– Dutilh Novaes, 2015, p.599f

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- Similarly, Skeptic has to be clear about what would convince them, and what would count as an appropriate explanation.
- Otherwise we can end up with a ‘passive-aggressive’ Skeptic such as the Tortoise, constantly moving the goalposts regarding what Achilles must do to convince them.

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- In order to beat such an idealised opponent Prover has to ensure that there are no counterexamples to the inferential steps which they employ in responding to Skeptic's queries.
- This provides a bridge between multi-agent dialogical and mono-agent inferential practices.

A dark, atmospheric photograph of a city street at night or dusk. Bare trees are silhouetted against a bright, hazy sky. In the foreground, there's a grassy area and a paved path. Buildings are visible in the background.

Introducing Prover-Skeptic Games

A DIALOGUE

*In the library two students, **Penelope** and **Scott** are arguing over the validity of the argument from $A \rightarrow B$ and $B \rightarrow C$ to $A \rightarrow C$. Penelope thinks that this argument is valid and is trying to convince Scott, who is skeptical.*

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- **Penelope (1):** I reckon that $A \rightarrow C$ follows from $A \rightarrow B$ and $B \rightarrow C$
- **Scott (1):** Yeah? Well if that's so then suppose I grant you $A \rightarrow B$ and $B \rightarrow C$ along with A , how are you mean to get C ?

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Penelope leaves the library triumphantly.

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$$A \succ A$$

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INTRODUCTION/ELIMINATION RULES

$$\frac{\Gamma, A \succ B}{\Gamma \succ A \rightarrow B} (\rightarrow I)$$

$$\frac{\Gamma \succ A \quad \Delta \succ A \rightarrow B}{\Gamma, \Delta \succ B} (\rightarrow E)$$

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STRUCTURAL RULES

$$\frac{\Gamma, A, A \succ B}{\Gamma, A \succ B} \ (W)$$

$$\frac{\Gamma \succ B}{\Gamma, A \succ B} \ (K)$$

A NATURAL DEDUCTION SYSTEM: EXAMPLE & NOTATION

$$\frac{\frac{A \rightarrow B \succ A \rightarrow B \quad A \succ A}{A \rightarrow B, A \succ B} \text{ } (\rightarrow E) \quad B \rightarrow C \succ B \rightarrow C}{\frac{A \rightarrow B, B \rightarrow C, A \succ C}{A \rightarrow B, B \rightarrow C \succ A \rightarrow C}} \text{ } (\rightarrow I)$$

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- Let us write $\Gamma \succ A \Rightarrow_{(I)}^* \Delta \succ B$ iff we can derive $\Delta \succ B$ from $\Gamma \succ A$ using $(\rightarrow I)$.

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- $\{\Gamma_1 \succ A_1, \dots, \Gamma_n \succ A_n\} \Rightarrow_{(E)}^* \Delta \succ B$ iff we can derive $\Delta \succ B$ from the $\Gamma_i \succ A_i$ s using $(\rightarrow E)$ and the structural rules.

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» Prover begins by asserting the sequent under discussion.

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» *Skeptic challenges an (interesting) Prover assertion*

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3. $S_{i+1} \Rightarrow_{(E)}^* \beta_i$

» Prover replies, offering sequents which derive the challenge.

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- We'll say that Prover *wins* a dialogue whenever Skeptic can make no further move, ...
- ... and has a winning strategy if she has a way of playing such that, no matter what moves Skeptic makes Prover can always win the dialogue.

A BRIEF ASIDE ON ADEQUACY

Theorem

Prover has a winning strategy in the dialogue over (Γ, A) iff $\Gamma \succ A$ is provable.

- The ‘only if’ direction here is straightforward given the connection between Prover and Skeptic moves and our proof system.

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Theorem

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- The ‘only if’ direction here is straightforward given the connection between Prover and Skeptic moves and our proof system.
- The ‘if’ direction requires us to convert proofs into long normal form and then push the structural rules into the E-part of the tracks in the proof. Given that the Skeptic moves directly correspond to the I-segments of proofs, we can then read a winning strategy off the tracks in the proof.

- Note that a Skeptic move challenging a sequent $\Gamma \succ B$ will always result in a challenge

$$\beta = \Gamma, B_1, \dots, B_n \succ p_i$$

where $B = B_1 \rightarrow (\dots \rightarrow (B_n \rightarrow p_i))$

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- So Skeptic moves are mechanical and thus are apt for being performed ‘offline’ by the Prover.
- We also have the right degree of higher-order cooperation, with both parties having to agree on what counts as a challenge and what counts as an acceptable response to a challenge.

The background image shows a canal scene in a European city during autumn. The water is dark, reflecting the surrounding trees and buildings. Large, leafy trees with orange and yellow foliage line the canal. In the distance, traditional brick buildings with white-framed windows are visible. A small bridge or pier extends into the canal on the left side.

Prover-Skeptic Games: An Example

REVISITING THE PREVIOUS EXAMPLE

Consider the following dialogue over $p \rightarrow q, q \rightarrow r \succ p \rightarrow r$.

P(1) { $p \rightarrow q, q \rightarrow r \succ p \rightarrow r$ } » Prover Starts

REVISITING THE PREVIOUS EXAMPLE

Consider the following dialogue over $p \rightarrow q, q \rightarrow r \succ p \rightarrow r$.

P(1) $\{p \rightarrow q, q \rightarrow r \succ p \rightarrow r\}$

S(1) $p \rightarrow q, q \rightarrow r, p \succ r$

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A DUBIOUS EXAMPLE

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Proofs & Explanations



PROOFS & EXPLANATIONS

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- The notion of explanation which is most appropriate here is broadly 'pragmatic' in nature...
- ... with explanations being answers to why-questions—requests for information of a particular sort
- What counts as an answer to a why-question, what information is requested, can differ from context to context.

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- For example, the most common kind of constraint that arises in these contexts concerns the ‘granularity’ of the response. (Hersh)
- In teaching contexts, for example, one often needs to provide more information than in research contexts.

A wide-angle photograph of a field covered in low-growing, purple-flowered plants, likely heather or wildflowers. The foreground is filled with these purple blossoms. In the middle ground, a dense line of dark green trees marks the horizon. The sky above is a uniform, dark grey, suggesting overcast weather or dusk.

A Route to Pluralism

A ROUTE TO PLURALISM – Two EXAMPLES

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- So, given the plausibility of these examples, it looks like (pace Beall & Restall) whether an argument is valid is sometimes relative to communities of inquiry.
- We will look two examples: one in the foundations of geometry, and the other concerning qualitative Hypothetico-Deductive accounts of confirmation.

RESOURCES CONSCIOUSNESS IN THE FOUNDATIONS OF GEOMETRY

- Pambuccian (2004) recounts some work in early 20th Century foundations of Geometry where we see a concern, not just for which axioms are used in proving something, but also in *how many times* such axioms are appealed to in such a proof.

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- E.g., Hessenberg notes in 1905 proof that, against the background of other axioms of projective geometry, Desargues axioms follows from a ‘three-fold use’ of Pappus’s axiom.

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- E.g., Hessenberg notes in 1905 proof that, against the background of other axioms of projective geometry, Desargues axioms follows from a ‘three-fold use’ of Pappus’s axiom.
- This curious situation is at least suggestive of a concern, in showing that something follows, not just with necessary truth preservation, but also with the *extent* to which certain claims are used in proofs. A concern which points towards a rejection of the structural rule of CONTRACTION.

HYPOTHETICO-DEDUCTIVISM AND RELEVANCE

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HYPOTHETICO-DEDUCTIVISM AND RELEVANCE

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- The main issues pointed to by Glymour concerned the inability of HD-confirmation to “maintain the condition of relevance between a hypothesis and the evidence which confirms it” (Waters)

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- This is suggestive of the a norm on explanation which requires that in contexts where we're concerned about confirmation we require an appropriate connection of relevance between theory, hypothesis and evidence.
- At the very least this will result in a rejection of the structural rule of WEAKENING.

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- All of these notions of logical consequence arise naturally out of the built-in opponent conception of deduction—in particular out of crucial higher-order cooperation between Prover and Skeptic.

A photograph of a street in Utrecht, Netherlands, looking down a narrow street. On the left, there's a building with a sign for "Dovey Nails & Spa". In the center background, the tall, dark Gothic-style tower of the Domkerk (Cathedral) rises against a bright blue sky with scattered white clouds. The street is lined with various buildings, mostly multi-story houses with different facades and window frames.

Thank You!

The background of the image is a photograph of a city skyline at sunset. The sky is filled with warm orange and yellow clouds. In the foreground, the dark silhouette of a modern building with large windows is visible. A set of stairs leads up to the entrance of the building. The city skyline in the distance includes several recognizable buildings, such as the V&A Museum and the British Library in London.

Adequacy of Prover-Skeptic Games

FROM PROOFS TO WINNING STRATEGIES

$$\frac{\frac{p \succ p \quad p \rightarrow q \succ p \rightarrow q}{p, p \rightarrow q \succ q} (\rightarrow E) \quad \frac{p \rightarrow (q \rightarrow r) \succ p \rightarrow (q \rightarrow r) \quad p \succ p}{p, p \rightarrow (q \rightarrow r) \succ q \rightarrow r} (\rightarrow E)}{p \rightarrow (q \rightarrow r), p \rightarrow q, p, p \succ r} (\wedge)$$
$$\frac{p \rightarrow (q \rightarrow r), p \rightarrow q, p \succ r}{p \rightarrow (q \rightarrow r), p \rightarrow q \succ p \rightarrow r} (\rightarrow I)$$
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$$\frac{p \rightarrow (q \rightarrow r), p \rightarrow q \succ p \rightarrow r}{p \rightarrow (q \rightarrow r) \succ (p \rightarrow q) \rightarrow (p \rightarrow r)} (\rightarrow I)$$

- $P(1) = \left\{ p \rightarrow (q \rightarrow r) \succ (p \rightarrow q) \rightarrow (p \rightarrow r) \right\}$

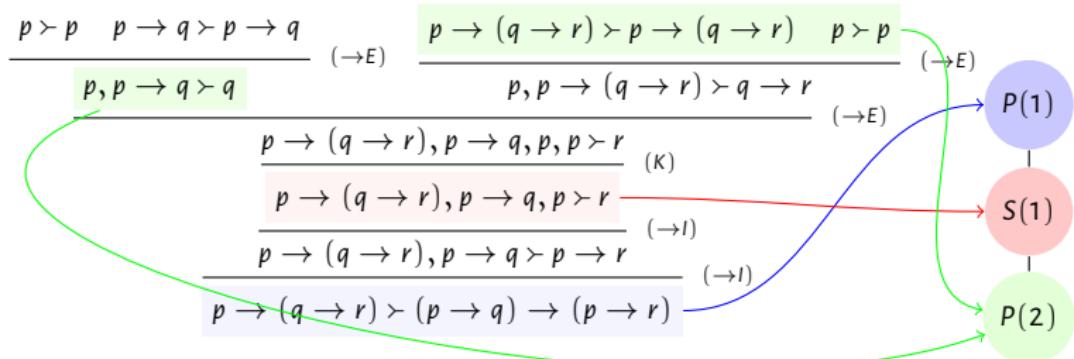
FROM PROOFS TO WINNING STRATEGIES

$$\frac{\frac{p \succ p \quad p \rightarrow q \succ p \rightarrow q}{p, p \rightarrow q \succ q} (\rightarrow E) \quad \frac{p \rightarrow (q \rightarrow r) \succ p \rightarrow (q \rightarrow r) \quad p \succ p}{p, p \rightarrow (q \rightarrow r) \succ q \rightarrow r} (\rightarrow E)}{p \rightarrow (q \rightarrow r), p \rightarrow q, p, p \succ r} (\wedge)$$
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The diagram illustrates the derivation of winning strategies. It shows a sequence of logical steps leading from the initial premises to the final formula. A blue arrow originates from the final formula and points to a blue circle labeled $P(1)$. A red arrow originates from the same final formula and points to a red circle labeled $S(1)$.

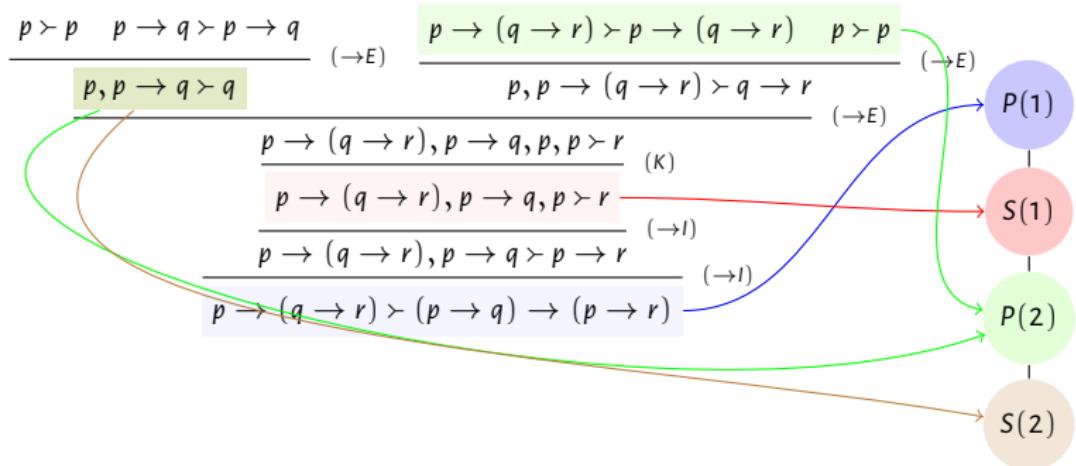
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FROM PROOFS TO WINNING STRATEGIES



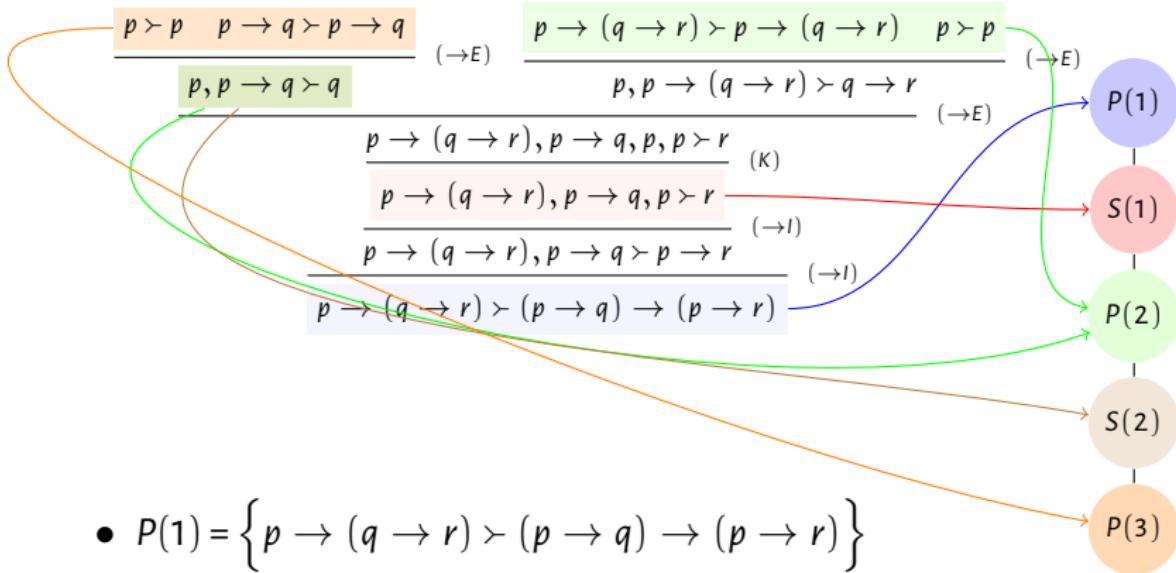
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- $P(2) = \left\{ p \rightarrow (q \rightarrow r) \succ p \rightarrow (q \rightarrow r) , p \succ p , p, p \rightarrow q \succ q \right\}$

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