STRUCTURAL REFLEXIVITY AND THE SEMANTIC PARADOXES

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THE CRIME SCENE

Structure

The Crime Scene

Reasons to Suspect Reflexivity

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Wither Logic?

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Understanding Non-Reflexive Consequence

Recapturing Classical Reasoning

The Crime Scene

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The Semantic Paradoxes

- Trying to find a uniform solution to the semantic paradoxes is much like attempting to solve a crime spree.
- We're relatively certain that there's a single (or at least a small number) of culprits to blame.
- The question is who do we pin the crimes on...

Connectives seem to be innocent...

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- But while negation is involved in the liar and Russell's paradox, it isn't involved in Curry's paradox.
- So maybe both negation and the conditional are to blame (and we need to find a 'reasonable' conditional)...
- But neither of these are involved in connective-free Hinnion-Libert style paradoxes...

Reviewing the Crime Scenes: Structural Rules

$$
\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{\mathsf{T}\langle\lambda\rangle \succ \mathsf{T}\langle\lambda\rangle}{\succ \mathsf{T}\langle\lambda\rangle, \neg\, \mathsf{T}\langle\lambda\rangle}}{\succ \mathsf{T}\langle\lambda\rangle, \lambda}}{\succ \mathsf{T}\langle\lambda\rangle,\ \mathsf{T}\langle\lambda\rangle}}{\succ \mathsf{T}\langle\lambda\rangle}
\qquad\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\mathsf{T}\langle\lambda\rangle \succ \mathsf{T}\langle\lambda\rangle}{\neg\, \mathsf{T}\langle\lambda\rangle,\ \mathsf{T}\langle\lambda\rangle \succ}}{\lambda,\ \mathsf{T}\langle\lambda\rangle \succ}}{\mathsf{T}\langle\lambda\rangle,\ \mathsf{T}\langle\lambda\rangle \succ}}{\mathsf{T}\langle\lambda\rangle \succ}
}{\succ}
$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathsf{T}\langle\lambda\rangle \succ \mathsf{T}\langle\lambda\rangle}{\succ \mathsf{T}\langle\lambda\rangle, \neg \mathsf{T}\langle\lambda\rangle}}{\succ \mathsf{T}\langle\lambda\rangle, \lambda}}{\succ \mathsf{T}\langle\lambda\rangle, \mathsf{T}\langle\lambda\rangle}}{\succ \mathsf{T}\langle\lambda\rangle} \quad \textsc{Contraction} \quad \dfrac{\dfrac{\dfrac{\dfrac{\mathsf{T}\langle\lambda\rangle \succ \mathsf{T}\langle\lambda\rangle}{\neg \mathsf{T}\langle\lambda\rangle, \mathsf{T}\langle\lambda\rangle \succ}}{\lambda, \mathsf{T}\langle\lambda\rangle \succ}}{\mathsf{T}\langle\lambda\rangle, \mathsf{T}\langle\lambda\rangle \succ}}{\mathsf{T}\langle\lambda\rangle \succ}}{\succ}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathsf{T}\langle\lambda\rangle \succ \mathsf{T}\langle\lambda\rangle}{\succ \mathsf{T}\langle\lambda\rangle, \neg\,\mathsf{T}\langle\lambda\rangle}}{\succ \mathsf{T}\langle\lambda\rangle, \lambda}}{\succ \mathsf{T}\langle\lambda\rangle,\ \mathsf{T}\langle\lambda\rangle}}{\succ \mathsf{T}\langle\lambda\rangle}\ \textsc{Contraction} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\mathsf{T}\langle\lambda\rangle \succ \mathsf{T}\langle\lambda\rangle}{\neg\,\mathsf{T}\langle\lambda\rangle,\ \mathsf{T}\langle\lambda\rangle \succ}}{\lambda,\ \mathsf{T}\langle\lambda\rangle \succ}}{\mathsf{T}\langle\lambda\rangle,\ \mathsf{T}\langle\lambda\rangle \succ}}{\mathsf{T}\langle\lambda\rangle \succ}}{\succ}\ \textsc{Cut}$$

$$
\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{T\langle\kappa\rangle \succ T\langle\kappa\rangle \qquad p \succ p}{T\langle\kappa\rangle \rightarrow p,\ T\langle\kappa\rangle \succ p}}{\kappa,\ T\langle\kappa\rangle \succ p}}{T\langle\kappa\rangle,\ T\langle\kappa\rangle \succ p}}{T\langle\kappa\rangle \succ p}}{\succ T\langle\kappa\rangle \rightarrow p}}{\succ \kappa}}{\succ T\langle\kappa\rangle}
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{T\langle\kappa\rangle \succ T\langle\kappa\rangle \qquad p \succ p}{T\langle\kappa\rangle,\ T\langle\kappa\rangle \rightarrow p \succ p}}{T\langle\kappa\rangle, \kappa \succ p}}{T\langle\kappa\rangle,\ T\langle\kappa\rangle \succ p}}{T\langle\kappa\rangle \succ p}}{T\langle\kappa\rangle \succ p}
}{\succ p}
$$

$$
\begin{prooftree}
\AxiomC{$T\langle\kappa\rangle \succ T\langle\kappa\rangle$}
\AxiomC{$p \succ p$}
\BinaryInfC{$T\langle\kappa\rangle \to p,\ T\langle\kappa\rangle \succ p$}
\UnaryInfC{$\kappa,\ T\langle\kappa\rangle \succ p$}
\UnaryInfC{$T\langle\kappa\rangle,\ T\langle\kappa\rangle \succ p$}
\RightLabel{\textsc{Contraction}}
\UnaryInfC{$T\langle\kappa\rangle \succ p$}
\UnaryInfC{$\succ T\langle\kappa\rangle \to p$}
\UnaryInfC{$\succ \kappa$}
\UnaryInfC{$\succ T\langle\kappa\rangle$}
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\qquad
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}{\succ p}\ \textsc{Cut}
$$

[CUT]

$$\frac{\Gamma \succ \Delta, A \quad \Gamma, A \succ \Delta}{\Gamma \succ \Delta}$$

[CONTRACTION]

$$\frac{\Gamma, A, A \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ \Delta, A, A}{\Gamma \succ \Delta, A}$$

[Cut]

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[Reflexivity]

$$\frac{}{\Gamma, A \succ A, \Delta}$$

Reasons to Suspect Reflexivity

Is identity really innocent?

- We have reasons to doubt Reflexivity which are independent to semantic paradox.

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- Aristotle, for example, thought that (Syllogistic) consequence was irreflexive for reasons of epistemic-gain.

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- Suppose we think of statements of consequence as telling us that the conclusion(s) are *true in virtue* of the premises.

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- Then on this account consequence will inherit irreflexivity from the *true in virtue of* relation.

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  The conclusion of this inference is false; therefore the conclusion of this inference is false.

Wither Logic?

A Premature Reaction

- Dropping Reflexivity from our sequent calculus means that there are no derivable sequents.

A Premature Reaction

- Dropping Reflexivity from our sequent calculus means that there are no derivable sequents.
- So it seems like we are presented with a choice of keeping Reflexivity or abandoning logic.

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$\wedge$	0	i	1
0	0	0	0
i	0	i	i
1	0	i	1

$\vee$	0	i	1
0	0	i	1
i	i	i	1
1	1	1	1

$\neg$	
0	1
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1	0

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- Then one would be forced to claim that K_3 is not a logic, as it has no theorems.
- But we know that K_3 does have a great many valid sequents.

- So similarly, we know that an logic without REFLEXIVITY has no derivable sequents…

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- … but it does have a great many derivable *metasequents*.

Understanding Non-Reflexive Consequence

Metasequents

- A metasequent is a structure $S \Rightarrow s$ consisting of a set of sequents S and a sequent s.

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- Say that a metasequent is derivable if s has a derivation all of whose initial sequents are contained in S.

- Suppose that we understand a sequent $\Gamma \succ \Delta$ as telling us that if we don't *reject* some members of Γ then we should *accept* some members of Δ.

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- So sequents take us from non-rejected premises to accepted conclusions.
- So for $A \succ A$ to be invalid we would have to neither reject A nor accept it.

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Acceptance, Rejection and Metasequents

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- In particular, consider the case where all the members of S are instances of Reflexivity (with $\Gamma = \Delta = \emptyset$.
- Then a metasequent of the form

$$P_1 \succ P_1, \ldots, P_n \succ P_n \Rightarrow \Gamma \succ \Delta$$

can be read as telling us that if we either accept or reject each of the P_is then if we don't reject all the members of Γ then we should accept some member of Δ.

Recapturing Classical Reasoning

If every formula in $\Gamma \cup \Delta$ is either accepted or rejected then we can read a sequent $\Gamma \succ \Delta$ as telling us that if we accept all the members of Γ then we should accept some member of Δ

- Suppose $\{P_1, \ldots, P_n\}$ are all the atomic expressions occurring in $\Gamma \cup \Delta$.

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- Then the metasequent

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means that if we accept or reject each of the P_is then if we accept all the members of Γ then we should accept some member of Δ.

Classical Logic as Enthymematic

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Classical Logic as Enthymematic

- We can now recapture all of classical logic as a kind of enthymematic reasoning.
- While sequents like $p, q \succ p \wedge q$ are not derivable in a non-reflexive logic...
- The metasequent $p \succ p, q \succ q \Rightarrow p, q \succ p \wedge q$ is derivable.
- So classical reasoning involves a suppressed assumption that we have accepted or rejected each of the atomic expressions involved in a sequent.

Diagnosis

What does Paradoxical Reasoning Show us?

- On this account what paradoxical reasoning shows us is that if we accept or reject sentences like λ and κ then we are forced to accept everything!

What does Paradoxical Reasoning Show us?

- On this account what paradoxical reasoning shows us is that if we accept or reject sentences like λ and κ then we are forced to accept everything!
- So we ought not accept or reject such sentences, and instead remain silent.

Thank You!