Part of

MATERIALS SCIENCE & ENGINEERING

A Learner's Guide

AN INTRODUCTORY E-BOOK

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Understanding Stress & Strain

In these brief set of slides we try to get a grasp of tensile, compressive and shear stresses

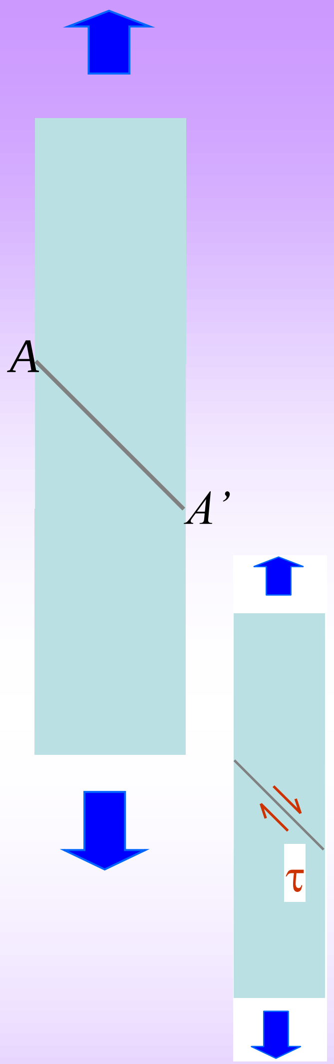
Sorry, this will not help with worldly stresses!!!

What will you learn in this chapter?

- How do forces lead to stresses and strains?
  Can stress exist without strain and can strain exist without stress?
- How to 'physically' understand stress?
- Understanding stress and strain tensors in terms of their components.
- Hydrostatic and deviatoric components of stress and strain.
- Planes of maximum shear stress (role in plasticity).
- Understanding surface stress.
- Residual stress. Microstructural origins of residual stress.

Stress and Strain

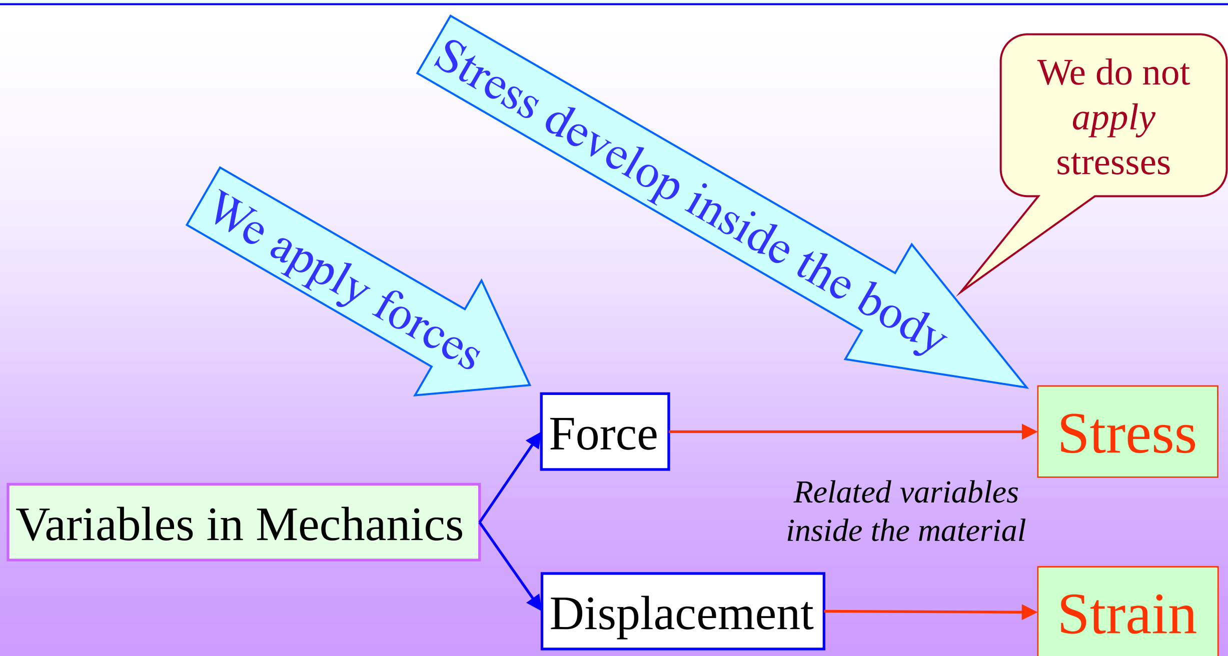
- In normal life we are accustomed to loads/forces and displacements. These are most appropriate variables when one talks about 'point masses', 'rigid bodies' or is sitting outside a body.
- Inside a body (typically a deformable body with mass and extent), one can locate other (appropriate) *'field variables'* to describe the state of the system. E.g. inside a gas kept in a cylinder, instead of tracking the velocities of the molecules, we come up with a field variable called *pressure*, which describes the momentum transferred by these molecules per unit area per unit time (pressure is a 'time averaged' macroscopic quantity).
- To understand the above point let us consider the 'pulling' of a body in tension (figure to right). Assume there is a weak plane (AA'), the two sides of which can slip past one another. We note as in the graphic that the inclined plane 'shears' even though we *applied tensile forces*. That is the plane feels shear stresses (τ).
- Hence, when we apply only tensile forces to the body (in the simple example considered), 'certain field' develops within the body, which depending on the orientation of the plane in the body (or a unit volume being considered) *can** undergo shear and/or dilatation.
- This *field* is the *stress field* and is a second order tensor with 9 components in general in 3D (4 in 2D).

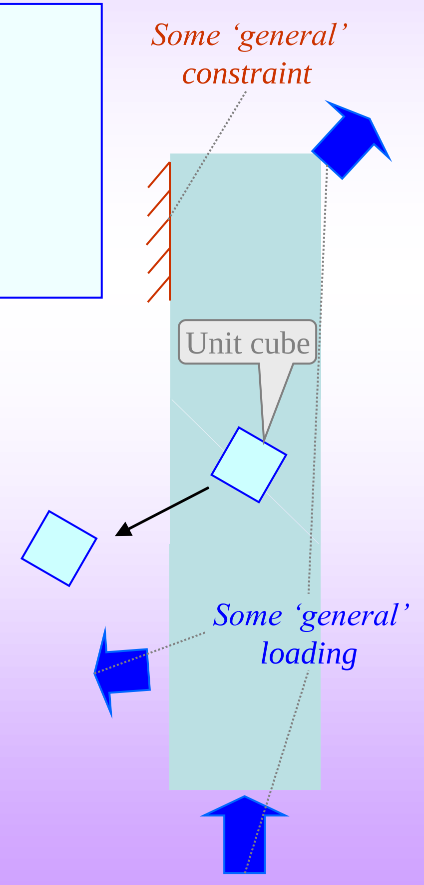


** It may so happen that some planes do not feel any shear stresses (like the horizontal and vertical planes in in figures)*

- Stress in 1D is defined as: Stress = Force/Area. This implies that in 1D stress is a scalar. Clearly, this is valid in 1D only, where a even a tensor looks like a scalar!!
- Similar to the stress field (which we noted to be the 'force dependent' term within the body), we can define a strain field, which is a *'displacement dependent'* term. Strain is also a 2^{nd} order tensor with 9 components in general in 3D. Strain in 1D is: Strain = change in length/original length (usually for small strains).

- In summary:
  External forces and constraints give rise to a stress field within a body. Depending on the orientation of a unit element (cube in the figure), the cube may stretch along one or two directions and/or may shear.





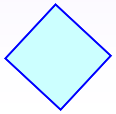
Funda Check

What can happen to a unit volume inside a body on the application of external loads/forces/constraints?

What can happen to a unit volume in a material when we apply forces/constraints to the outside of the body

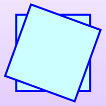
Contraction/dilation *Volume change*

Shear *Shape change*

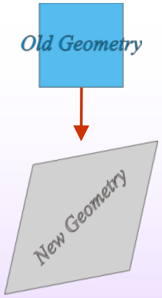


Rigid body translation/rotation

Position/Orientation change



Or a combination of these



As such no distortion to the volume element *(but affects neighbouring elements)*

Funda Check

Does stress cause strain or does strain cause stress?

- First point: stresses can exist without strains (heating a body between rigid walls) and strains can exist without stresses (heating a unconstrained/free-standing body).
- What we are asking here is which came first (something like the proverbial chicken and egg problem!).
- Both situations are possible *(at least from a perspective of easy understanding)*.
- If we load a body and this leads to stress inside the body→ this will lead to strains in a deformable body. I.e. stress gives rise to strain. Load → Stress → Strain.
- Now if a cubic phase transforms to another cubic phase with a larger lattice parameter (i.e. the transformation involves volume expansion), we can assume two situations:
  1) the transforming material is small and the whole volume transforms (Fig.1a)
  2) the transforming volume is small, but now embedded in a matrix (Fig.1b).
- In case (1) above there are no stresses.
  In case (2) above the surrounding matrix will try to constrain the expansion, leading to stresses. The primary causative agent in case (2) is strains (due to phase transformation), which further causes stresses. Phase transformation→ Strain → Stress.

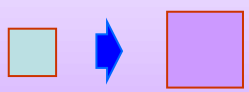


Fig.1a: Strains but no stresses (dilatation during phase transformation)

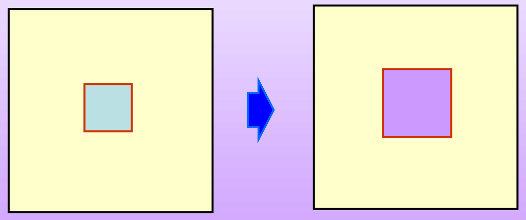


Fig.1b: Strains with stresses

Examples of some vector and tensor quantities

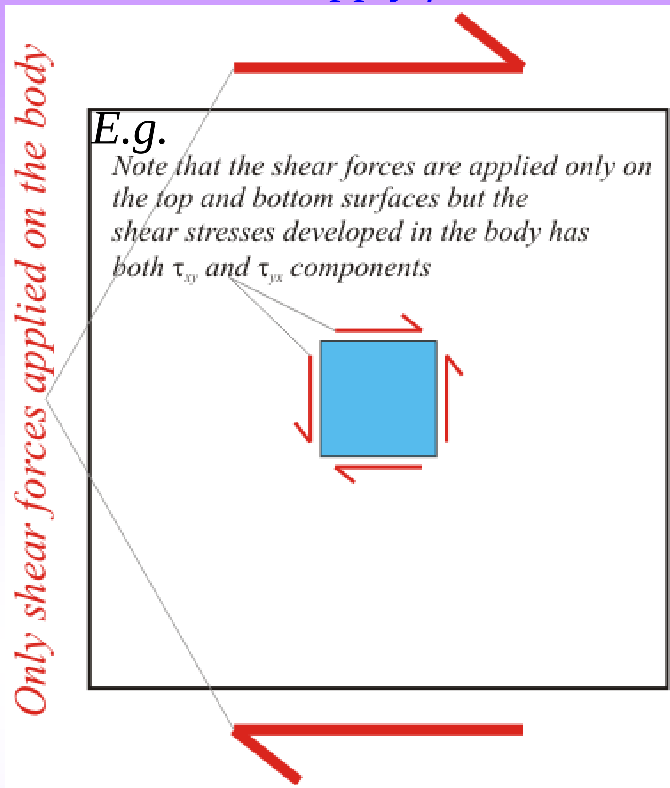
Quantity	Type	Acts
Force	(Polar) vector	At a point mass
Torque	Pseudo Vector (Axial Vector)	About an axis
Traction	(Polar) Vector	On a surface element
Stress	Tensor	Acts on a volume element

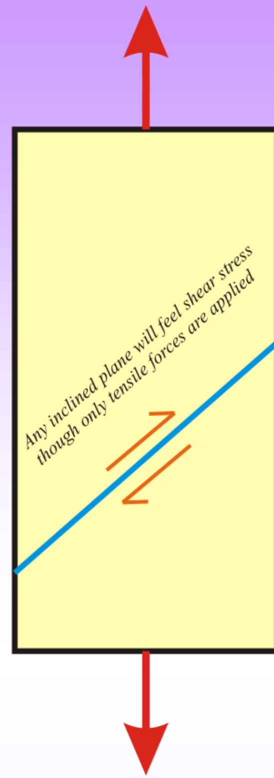
- Traction and stress may vary with position, orientation and time; i.e., are field quantities with spatial and temporal variations (next slide).
- Polar vectors reflect in a mirror, axial vectors do not reflect.

Stress

- Stress is a second order tensor and best understood in terms of its effect on a unit body (cube in 3D and square in 2D), in terms of its components.
- Stresses can be Compressive, Tensile or Shear *(in terms of specific components)*.
- We may apply forces/constraints and stresses will develop within the material (including the surface) → *we apply forces (or constraints) and not stresses.*
- The source of stress could be an external agent (forces etc.) or could be internal (dislocations, coherent precipitates etc.) → i.e. stresses can exist in a body in the absence of external agents.
- The effect of stress at a particular point in the material is not dependent on how the stress came about (i.e. could be external or internal factors) → just the components of stress matter in determining the response of the material.
- We *can* have stress without strain and strain without stress *(ideal circumstances)*
  - Strain without stress → heat a unconstrained body (it will expand and no stresses will develop)
  - Stress without Strain → heat a body constrained between rigid walls (it will not be able to expand but stresses will develop).

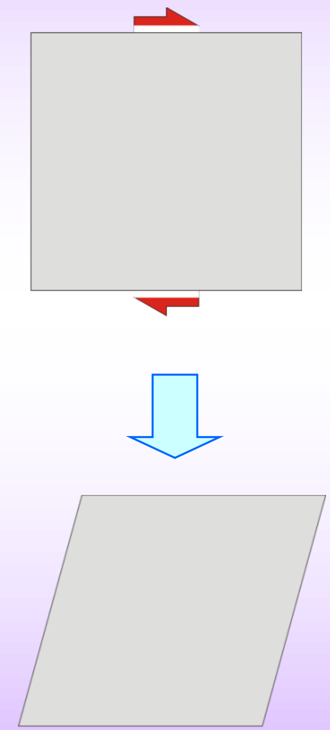
Note: We can apply forces and not stresses- stresses develop within the body

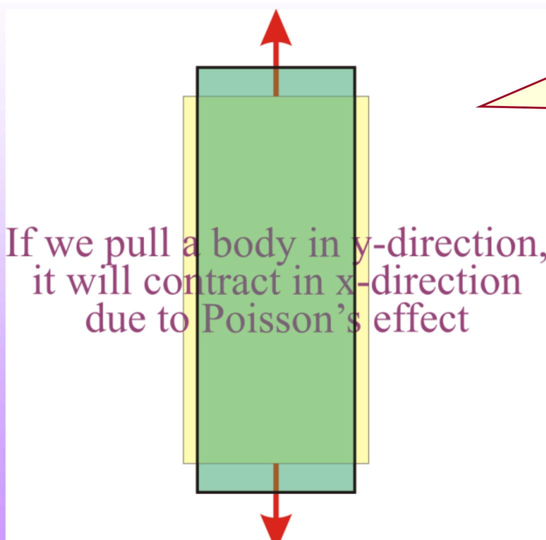




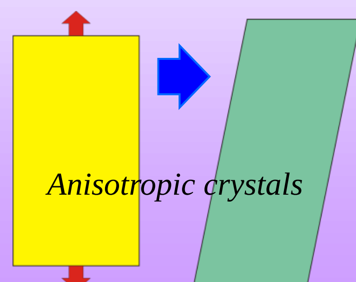
Shear

Only shear tends to change the shape of a body without changing its volume





In anisotropic crystals it may do more (may even shear the crystal)!



Note: we apply shear force and shear stresses develop in the interior of the material

'Physical' Understanding of Stress

Any of these may be used depending on the situation

Method A

Effect on points, lines, surfaces and volumes in the body

Method B

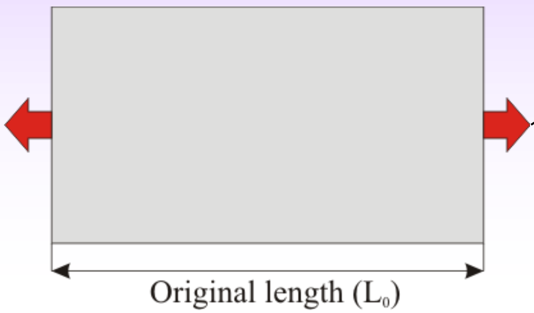
Effect on release of constraint

This visualization may or may not be easy in many situations

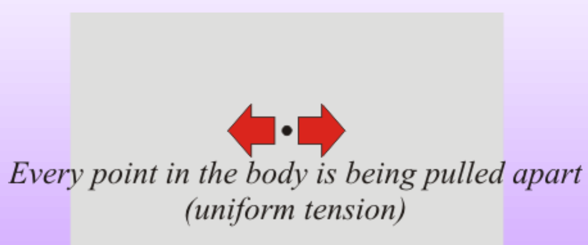
Method A

Forces on the external surface of a body

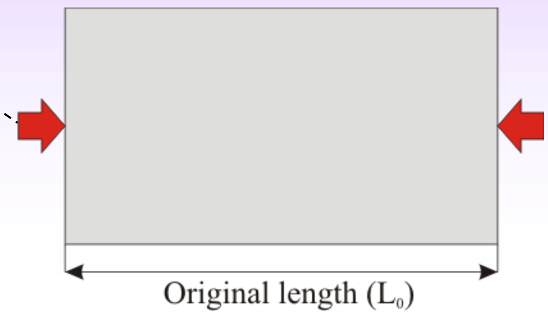
Tensile Stress



Uniaxial tensile stress tends to elongate the body



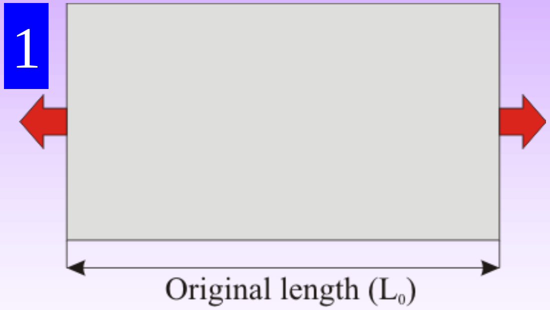
Compressive Stress



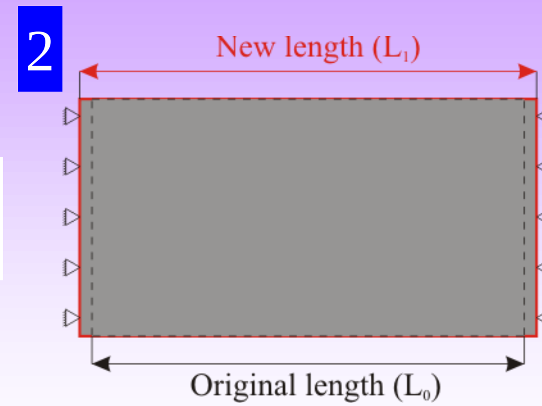
Uniaxial compressive stress tends to reduce the length of the body (shorten the body)

Method B

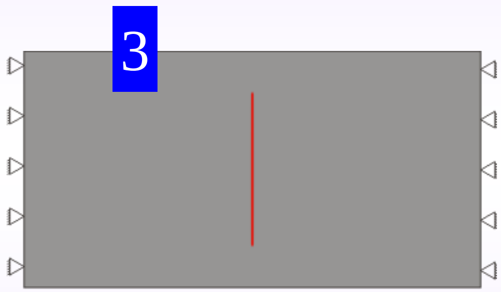
Let us get a physical feel for TENSILE STRESS

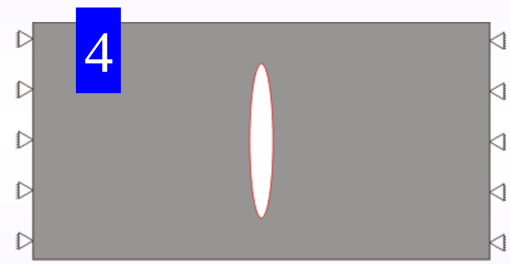


Pull a body of length L_0 to new length L_1 and hold it at this length



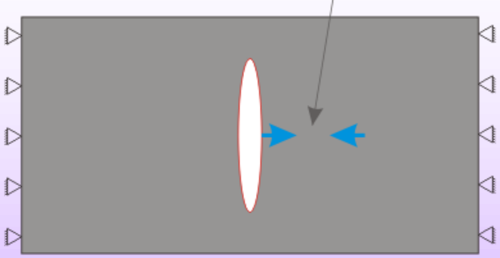
Introduce a cut (crack) in the body





The crack will open up due to the tensile stress

On release of the constraint points in the body move towards one another

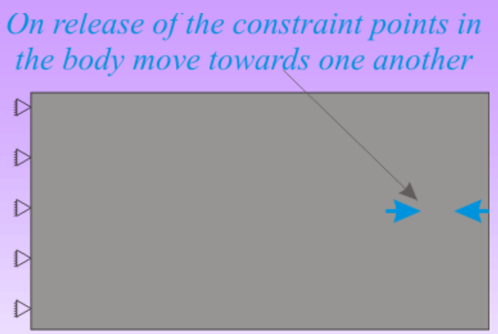


That is when the constraint is removed points in the body move towards each other

I.e. under tensile stress the points in a body tend to move towards one another (while the crack faces move apart)

!***!

This is because we have increased the interatomic distance over the equilibrium value.



Alternately if the external constraint is removed points in the body move towards each other

I.e. under tensile stress the points in the body tend to move towards one another

The reverse will happen under compressive stress:

That is when the constraint is removed points in the body move away from each other

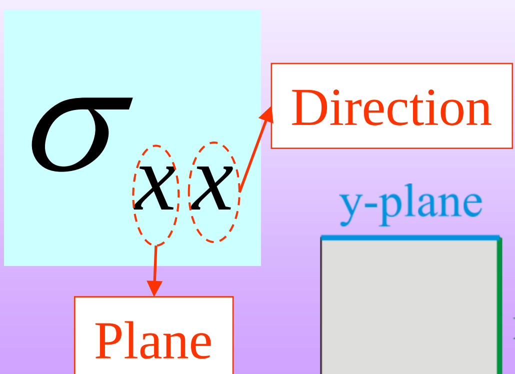
I.e. under compressive stress the points in the body tend to move away one another

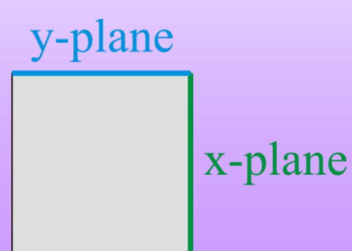
The Stress (& Strain) Tensors

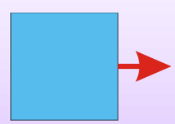
- Tensors which measure crystal properties (e.g. magnetic susceptibility) have a definite orientation within a crystal and its components are dictated by the crystal symmetry. These are Material Property Tensors or Matter Tensors.
- The stress and strain tensors can have any orientation within a crystal and can even be defined for amophous (or isotropic) materials.
- The stress tensor ‘develops’ the material in response to ‘forces’.
- The stress and strain tensors are Field Tensors.
- (Say) when forces are applied to a body, stress and strain tensor fields develop within the body.

Understanding stress in terms of its components

- Stress is a Second Order Tensor.
- It is easier to understand stress in terms of its components and the effect of the components in causing deformations to a unit body within the material.
- These components can be treated as vectors.
- Components of a stress:
  2D $\rightarrow$ 4 components [2 σ (tensile) and 2 τ (shear)]
  3D $\rightarrow$ 9 components [3 σ (tensile) and 6 τ (shear)]
- σ written with subscripts not equal implies τ (shear stress)
  e.g. $\sigma_{xy} \equiv \tau_{xy}$.
- First index refers to the plane and the second to the direction.
- Close to 2D state of stress (plane stress) can occur in thin bodies and 2D state of strain (plane strain) very thick bodies.
- Shear stresses are responsible for plastic deformation in metallic materials.





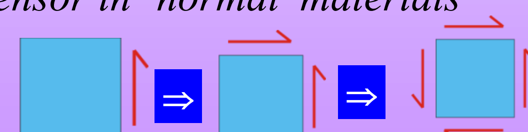


$\sigma_{xx} = \sigma_{11}$ *x-plane, x-direction*
Also sometimes written as σ_x

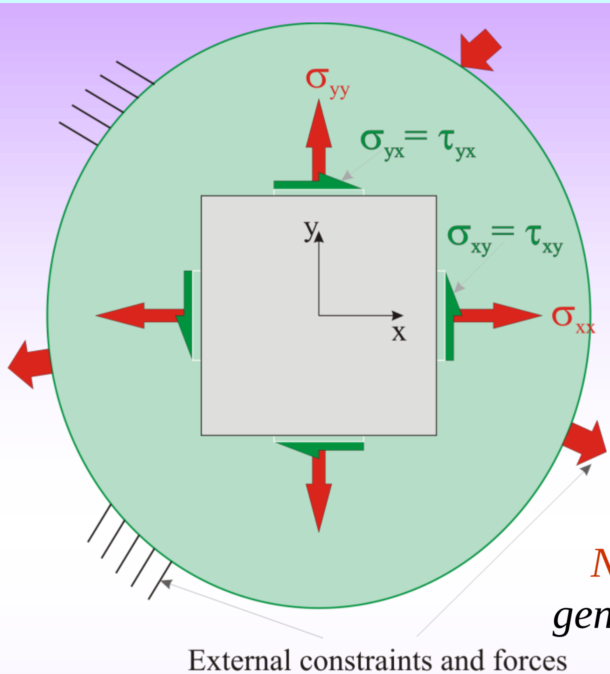
$\sigma_{xy} = \tau_{xy} = \sigma_{12}$ *x-plane, y-direction*

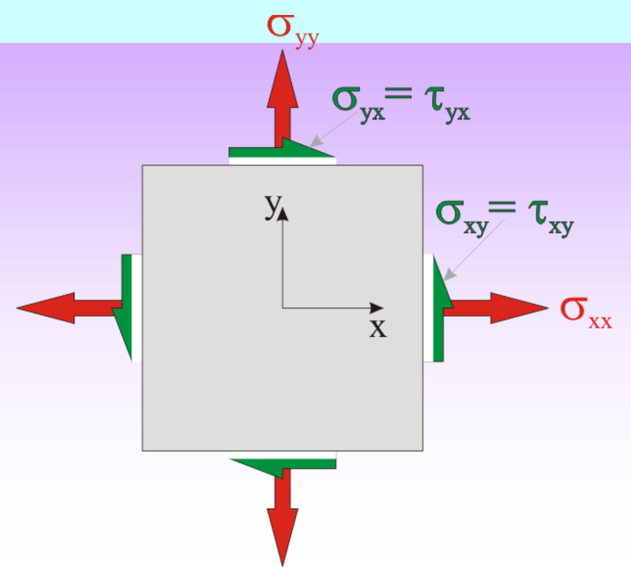
As stress is a symmetric tensor in 'normal' materials

$\sigma_{yx} = \tau_{yx} = \sigma_{21}$



Let us consider a body in the presence of external agents (constraints and forces) bringing about stresses in the body. A unit region in the body (assumed having constant stresses) is analyzed. (body forces are ignored)

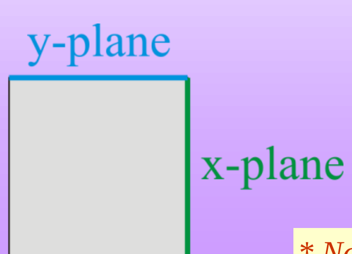




Note: the directions of the stresses shown are arbitrary (the stresses in general could be compression/tension and shear could be opposite in sign)

2D

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \quad \begin{pmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{pmatrix} \quad \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$



* *Note:* $\sigma_{xx} = \sigma_x$

Understanding how stress develops inside a material based on the load applied

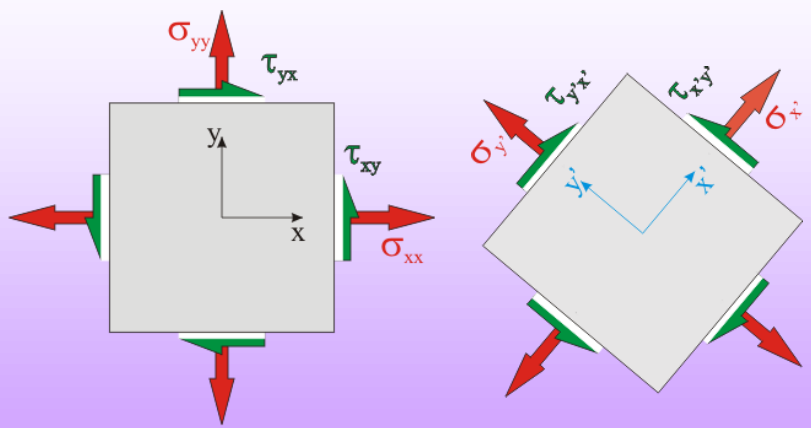
2D

- The normal stresses (σ_x & σ_y) tend to elongate the body (the square in the figure below) → this will give rise to volume changes.
- The shear stress ($\tau_{xy} = \tau_{yx}$) will tend to change the shape of the body → without changing its volume.
- Depending on the orientation of the unit volume considered, the stresses acting on its faces will change.
- A good feel for the same can be got by looking at stress in 2D (**plane stress**, with 3 independent components).
- We have already noted that even if we apply tensile/compressive forces, shear stresses can develop on inclined planes.
- Stress on one axes set can be mapped to stress on another axes set by the formulae as below.
- There will always be one unique axis set (x', y'), wherein the shear stresses are zero. The corresponding planes are the principal planes and the principal normal stresses are labeled: σ_1 and σ_2 (More about this soon).

$$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) Cos2\theta + \tau_{xy}\, Sin2\theta$$

$$\sigma_{y'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right) Cos2\theta - \tau_{xy}\, Sin2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) Sin2\theta + \tau_{xy}\, Cos2\theta$$



Points to be noted (some of these will be illustrated via figures in coming slides):

- Planes of maximum/minimum normal stress (σ) correspond to zero shear stress ($\tau_{xy} = 0$) $\rightarrow$ known as the principal planes. The corresponding stresses are the principal stresses (labeled σ_1 and σ_2)
- There exist planes where shear stress is zero. These planes also correspond to extremum in normal stresses. Planes of extremum shear stress are 45° from planes of zero shear stress (which correspond to the principal planes).
- The period of the functions is 180° (as above equations are functions of Sine and Cosine of 2θ) $\Rightarrow$ the maxima of the functions is separated from the minima by 90°. This is expected: e.g. the stress in +x (σ_{xx}) is expected to be same as stress in –x (σ_{xx}).
- Extremum in shear stress occurs midway in angle between extrema in normal stress.
- Shear stress is symmetric, i.e. $\tau_{xy} = \tau_{yx}$. Minimum value of shear stress = – (Maximum value of shear stress).

Principal stresses

$$\begin{matrix}\sigma_{\max} \\ \sigma_{\min}\end{matrix} = \begin{matrix}\sigma_1 \\ \sigma_2\end{matrix} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2\right]^{1/2}$$

$$\left(Tan2\theta\right)_{Principal\ plane} = Tan2\theta_n = \frac{2\tau_{xy}}{\left(\sigma_x - \sigma_y\right)}$$

Maximum shear stress

$$\tau_{\max} = \pm\left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2\right]^{1/2}$$

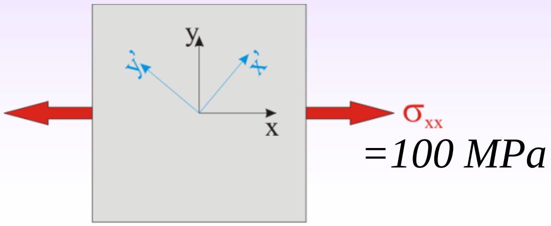
$$\left(Tan2\theta\right)_{Max\ shear\ stress\ plane} = Tan2\theta_s = \frac{-\left(\sigma_x - \sigma_y\right)}{2\tau_{xy}}$$

$$Tan2\theta_n = -\frac{1}{Tan2\theta_s}$$

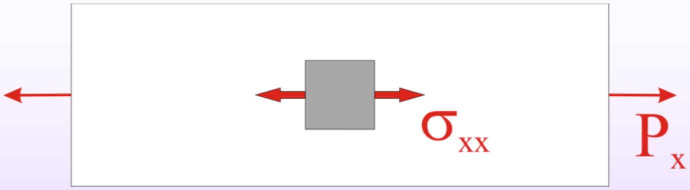
Now we will consider special cases of importance

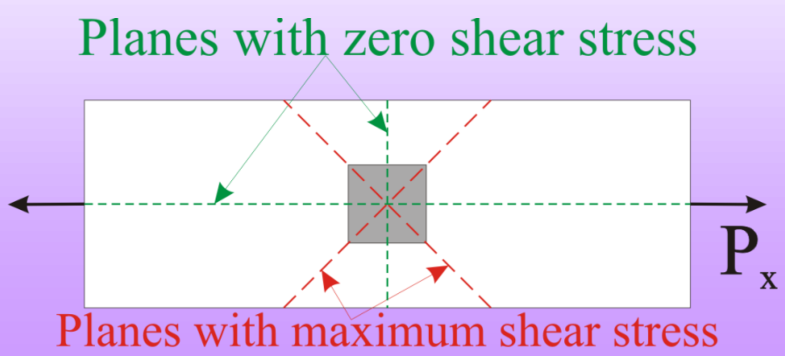
Case-1

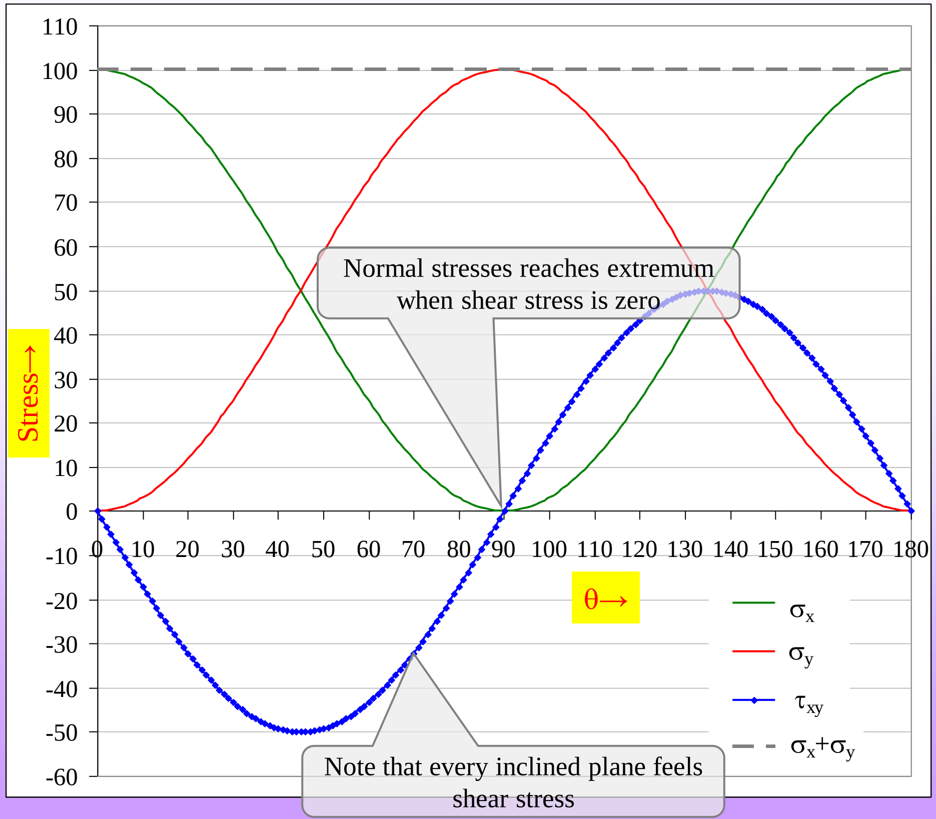
- The simplest case can be loading in uniaxial tension.
- For x and y as in the figure below only the vertical and horizontal planes feel no shear stress *(every other plane feels shear stress)*. This is in spite of the fact that we applied only a tensile force.
- Shear stress is maximum at 45°. For σ_{xx} = 100MPa, $|\tau_{max}|$ = 50 MPa.
- Rotation of 90° implies that x goes to y and y goes to –x (which is same as x).
- The principal stress is the resultant of what we applied → P_x (i.e. σ_1 = 100 MPa).



The above stress state can be thought of arising from a loading as below

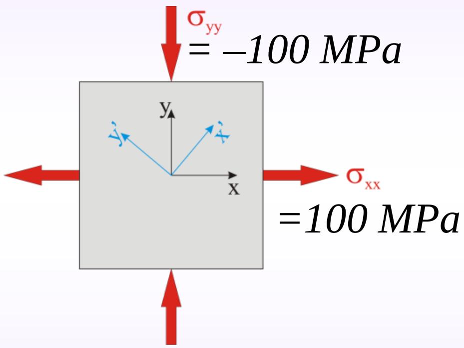




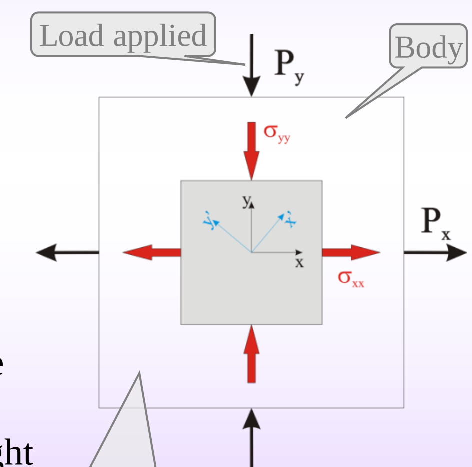


Case-2

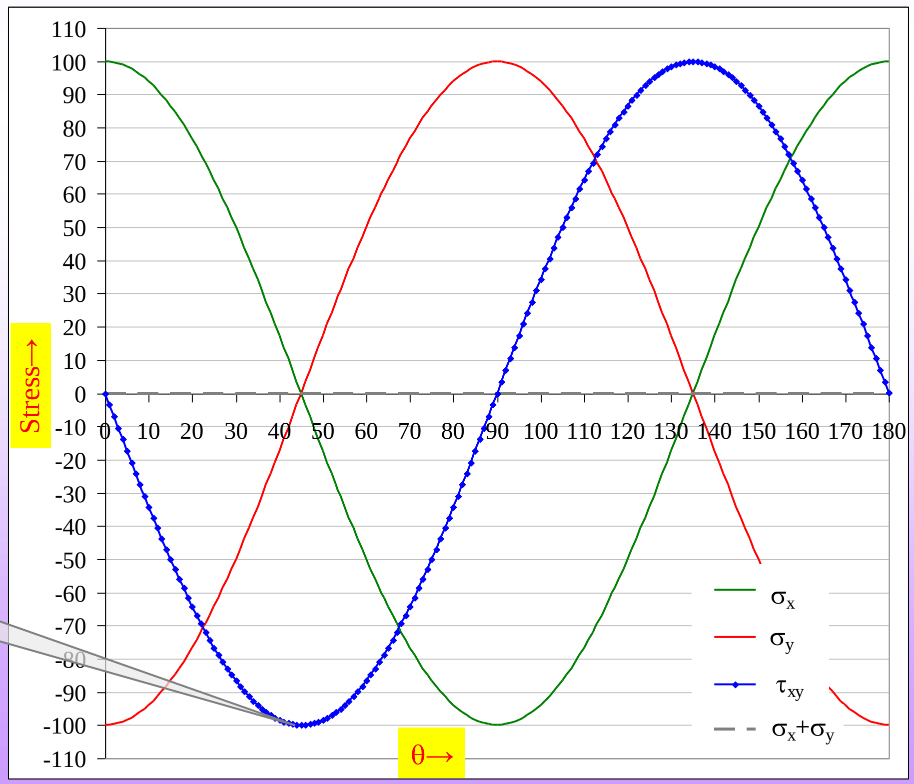
- If we push along one direction (say y) and pull along another direction (say x), with equal magnitude.
- For x and y as in the figure below the vertical and horizontal planes feel no shear stress. The are the principal planes and the principal stress are (trivially): σ_1 = 100MPa, σ_2 = –100MPa
- Shear stress is maximum at 45° (at this angle both normal stresses are zero).
  For σ_{xx} = 100MPa & σ_{yy} = –100MPa, $|\tau_{max}|$ = 100 MPa → the shear stress equals the normal stresses in magnitude (even though we did not apply shear forces)
  This implies this *'push-pull'* configuration gives rise to a higher value of shear stress. This aspect can be physically visualized as well.
- All stress functions (σ & τ) are identical and only phase shifted from each other.



The above stress state can be thought of arising from a loading as in the figure to right



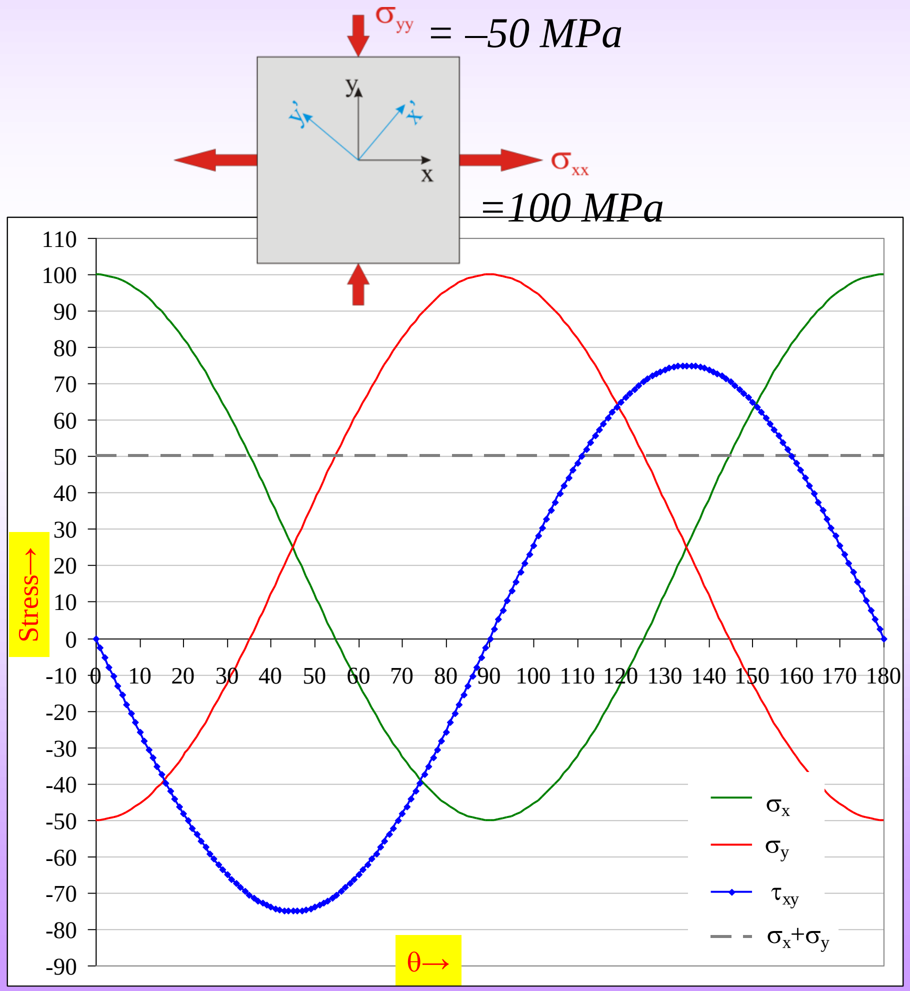
This loading is equivalent of applying shear stress at planes inclined at 45°.



Note that ($\sigma_x + \sigma_y$) = 0 for all θ

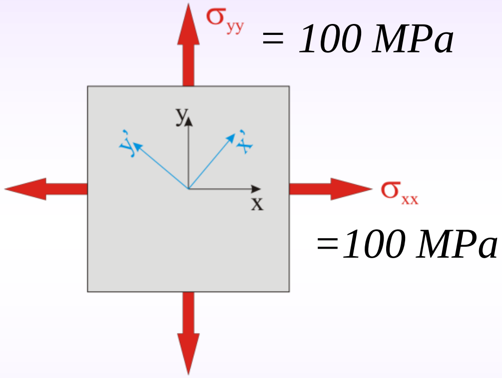
Case-3

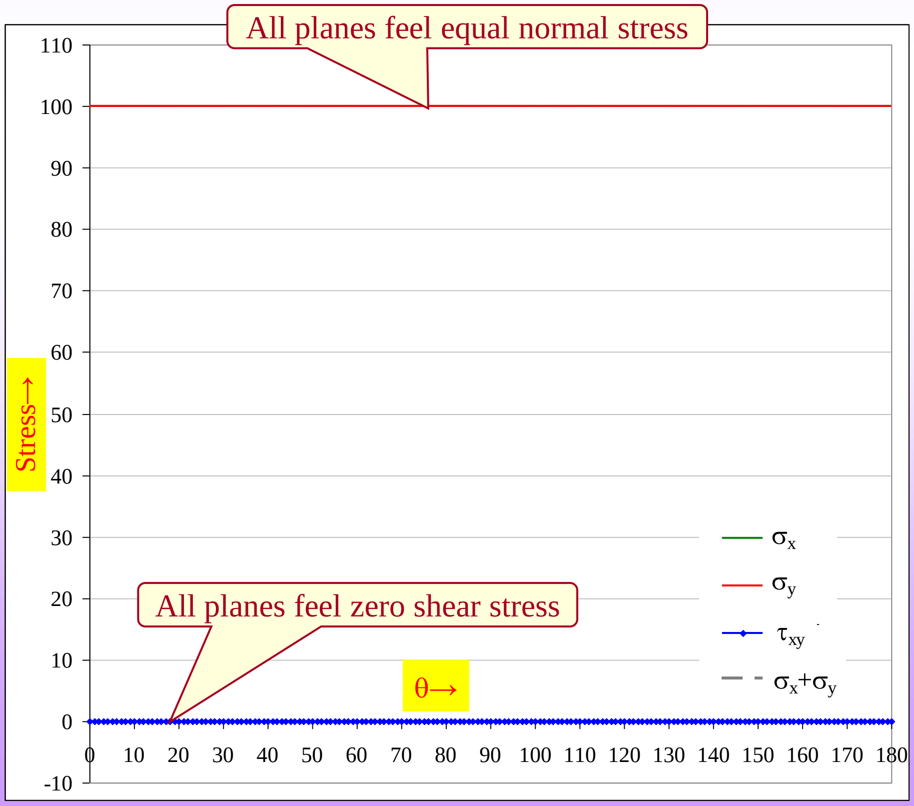
- If we pull along one direction (say x) and push along another direction (say y) with lesser force.
- For x and y as in the figure below the vertical and horizontal planes feel no shear stress.
- Shear stress is maximum at 45°. For σ_{xx} = 100MPa & σ_{yy} = –100MPa, $|\tau_{max}|$ = 75 MPa.
- There are *no* planes where both normal stresses are zero.



Case-4

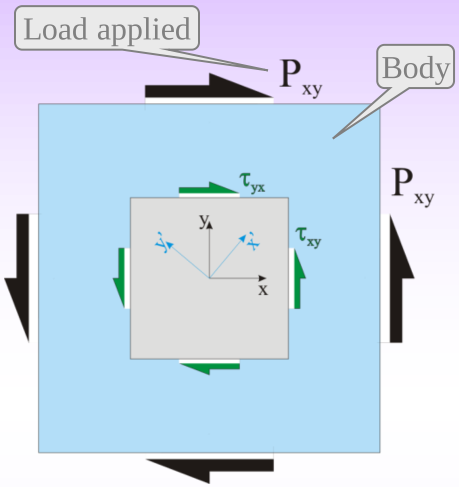
- Biaxial tension (2D hydrostatic state of stress).
- All planes feel equal normal stress.
- There is no shear stress on any plane.
- Usual materials (metallic) will not plastically deform under this state of stress.

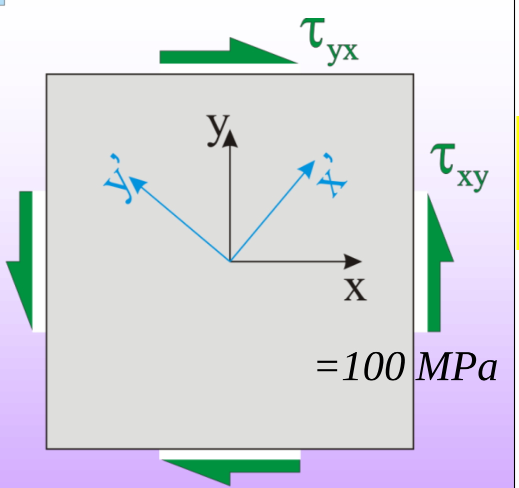


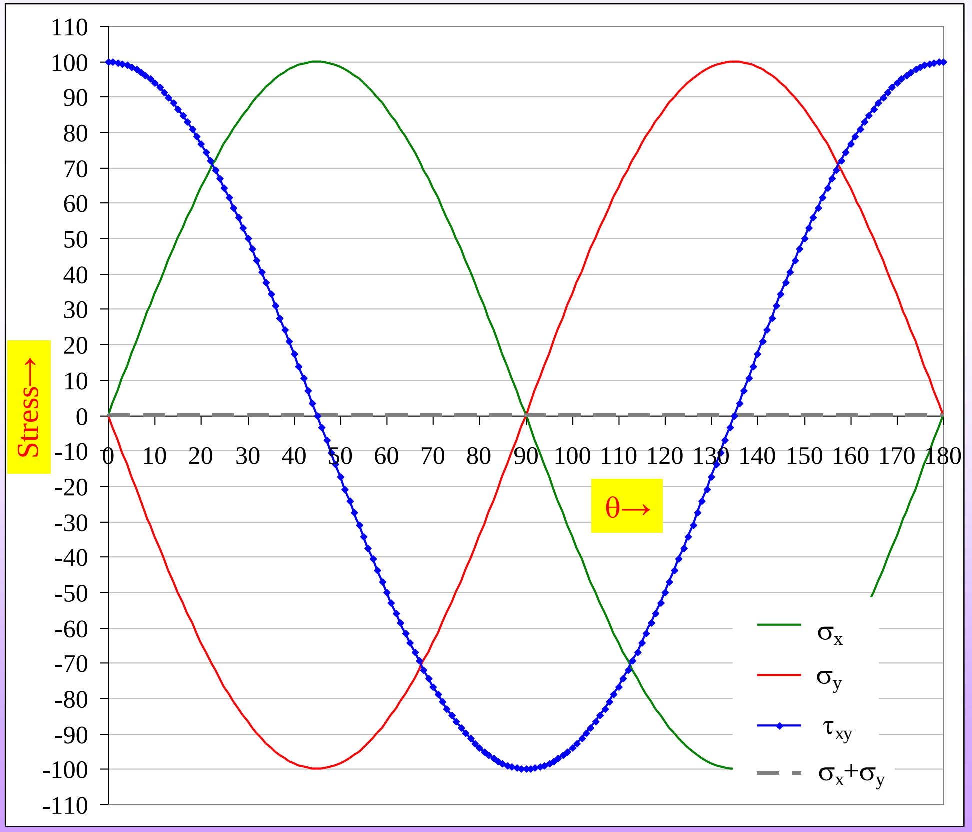


Case-5

- Only shear forces applied.
- This leads to a stress state identical to case-2, but with phase shift of 45°.
- Though we applied only shear forces, normal stresses develop in all planes except the planes where shear stresses are maximum.







Note that $(\sigma_x + \sigma_y) = 0$ for all θ

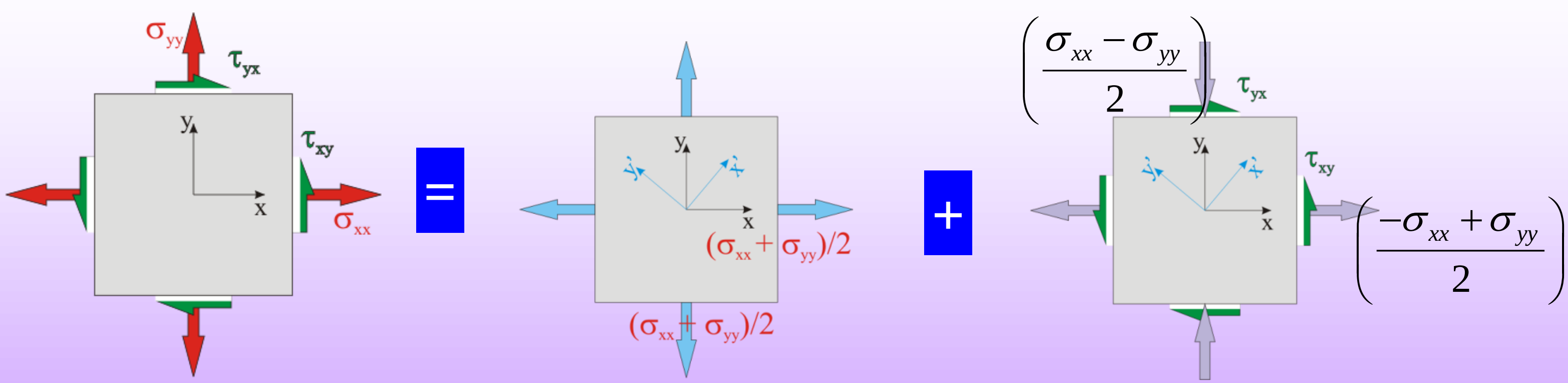
Hydrostatic and Deviatoric Components of Stress

- (In metallic materials) Hydrostatic components of stress can cause elastic volume changes and not plastic deformation.
- Yield stress (of metals) is not dependent on the hydrostatic stress. However, fracture stress (σ_f) is strongly affected by hydrostatic stress.
- We understand the concept of hydrostatic and deviatoric stress in 2D first.
- Hydrostatic stress is the average of the two normal stresses.

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{pmatrix}$$

$$\sigma^{2D}_{hydrostatic} = \sigma_m = \frac{(\sigma_{xx} + \sigma_{yy})}{2}$$

$$\sigma_{ij} = \underbrace{\begin{pmatrix} \sigma_m & 0 \\ 0 & \sigma_m \end{pmatrix}}_{Hydorstatic\ Part} + \underbrace{\begin{pmatrix} \sigma_m - \sigma_{yy} & \tau_{xy} \\ \tau_{xy} & \sigma_m - \sigma_{xx} \end{pmatrix}}_{Deviatoric\ part} = \underbrace{\begin{pmatrix} \frac{\sigma_{xx} + \sigma_{yy}}{2} & 0 \\ 0 & \frac{\sigma_{xx} + \sigma_{yy}}{2} \end{pmatrix}}_{Hydrostatic\ part} + \underbrace{\begin{pmatrix} \frac{\sigma_{xx} - \sigma_{yy}}{2} & \tau_{xy} \\ \tau_{yx} & \frac{-\sigma_{xx} + \sigma_{yy}}{2} \end{pmatrix}}_{Deviatoric\ part}$$

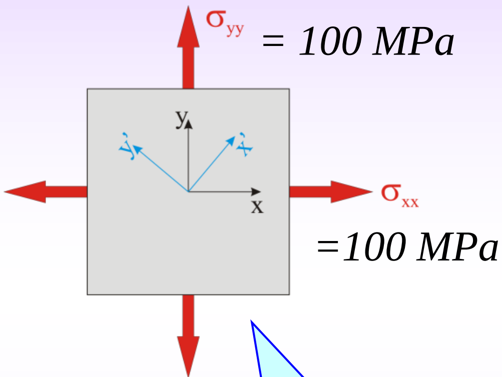


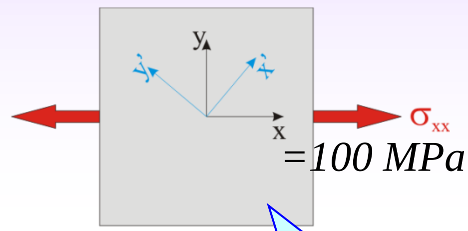
Funda Check

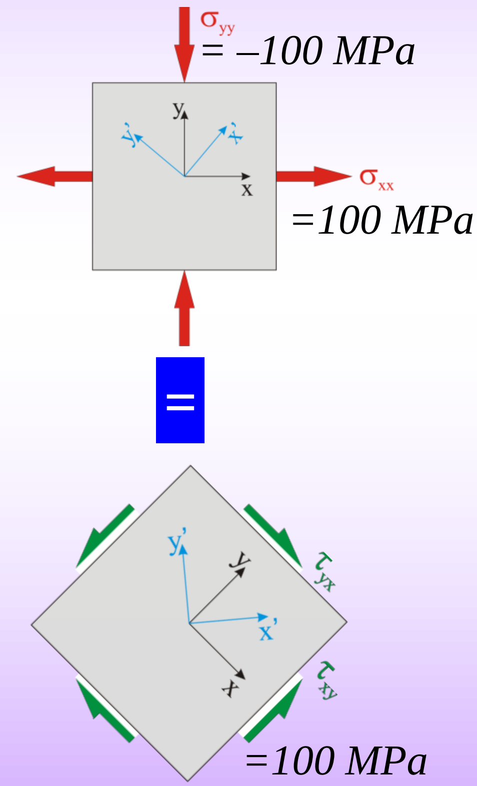
For only normal loads applied on a rectangular body (equal/zero), what is the increasing order in which there is a propensity to cause plastic deformation?

Worst for plastic deformation

Best for plastic deformation







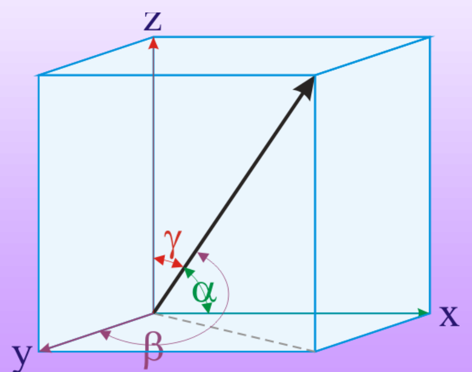
- Note: slip can still take place on the planes inclined in the 3rd dimension

- Note if we add +ve σ_{yy} to uniaxial tension this is bad for plastic deformation.
- Similarly in 3D triaxial (tensile) state of stress is bad for plastic deformation.
- Hence, triaxial state of stress 'suppresses' plastic deformation and 'promotes' fracture.

Generalized Plane Stress

3D state of stress

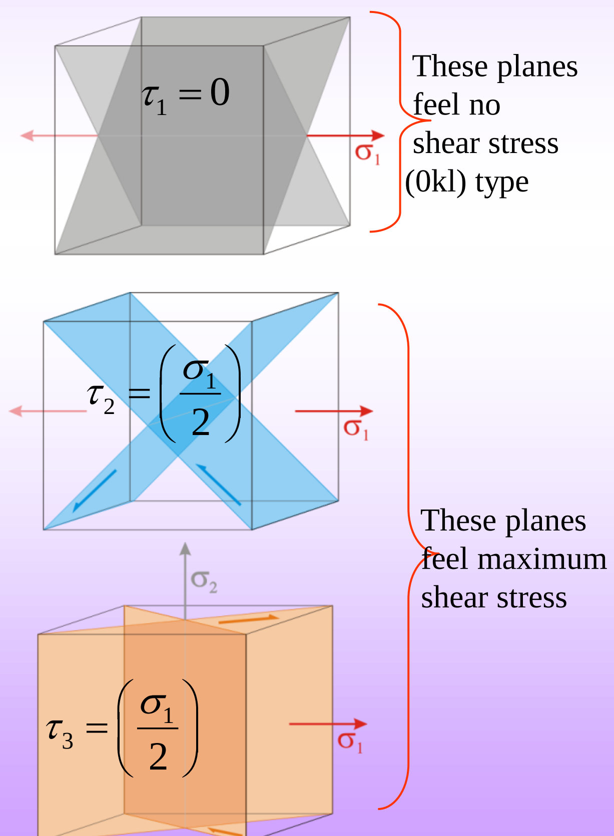
- In general, a point in a body may exist in a 3D state of stress, wherein the 3 principal stresses (σ_1, σ_2, σ_3) are not equal. The list of possibilities in this context are:
  - 3 unequal principal stresses (σ_1, σ_2, σ_3) → Triaxial state of stress
  - 2 our of the 3 principal stresses are equal (say σ_1, $\sigma_2 = \sigma_3$) → Cylindrical state of stress
  - All 3 principal stresses are equal (say $\sigma_1 = \sigma_2 = \sigma_3$) → Hydrostatic/spherical state of stress
  - One of the 3 principal stresses is zero (say σ_1, σ_2, $\sigma_3 = 0$) → Biaxial/2D state of stress
  - One of the 3 principal stresses is zero & the remaining two are equal to each other (say $\sigma_1 = \sigma_2$, $\sigma_3 = 0$) → 2D hydrostatic state of stress
  - Two of the 3 principal stresses is zero (say σ_1, $\sigma_2 = \sigma_3 = 0$) → Uniaxial state of stress.
- We can start with the state of stress on an unit cube and observe the state of stress as the orientation of the cube is changed (by rotation in 3D) or we can look at an inclined plane with direction cosines l (=Cosα), m (=Cosβ), n (=Cosγ). This is akin to the square we used in 2D and rotate it about the z-axis.



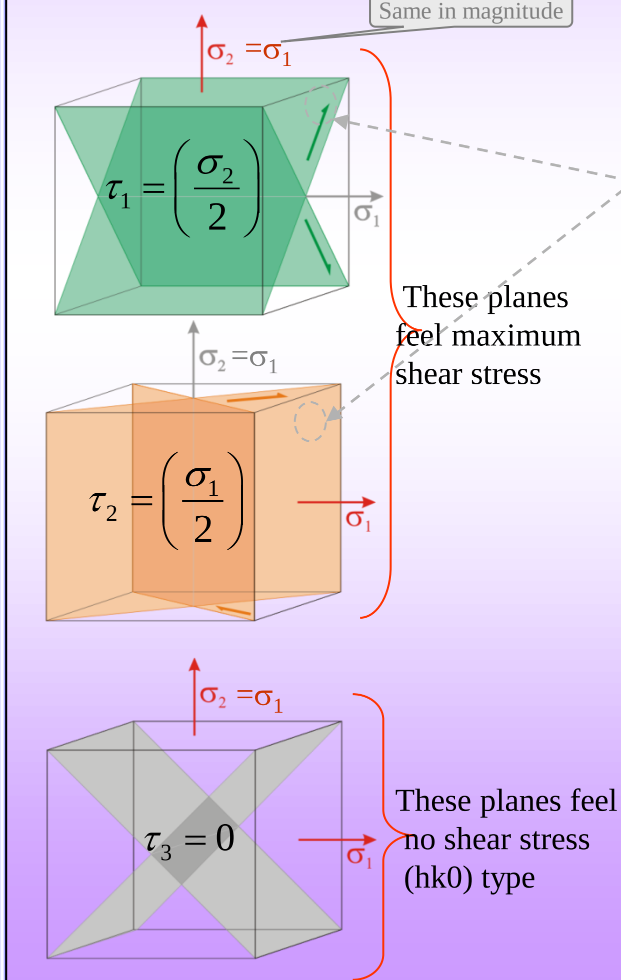
Planes which experience maximum shear stress/no shear stress

- Plastic deformation by slip is caused by shear stress (at the atomic level). Hence, we would like to identify planes of maximum shear stress.
- For uniaxial tension, biaxial hydrostatic tension, triaxial hydrostatic tension, etc., we try to identify planes experiencing maximum shear stress.

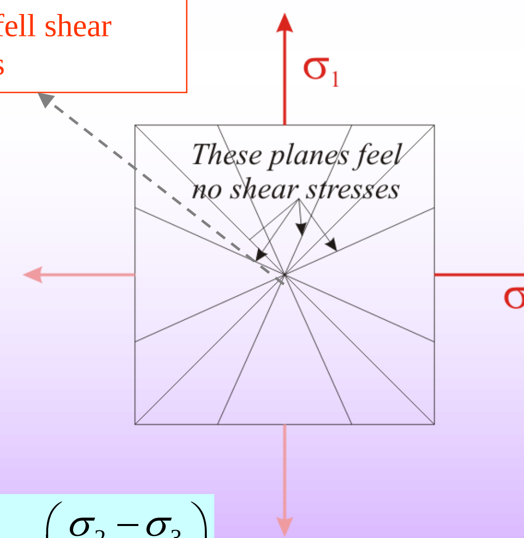
Uniaxial tension



Biaxial hydrostatic tension



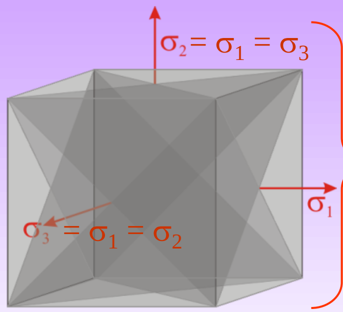
But yielding can take place due to planes inclined in the third dimension which fell shear stresses



$$\tau_1 = \left(\frac{\sigma_2 - \sigma_3}{2}\right)$$

$$\tau_2 = \left(\frac{\sigma_1 - \sigma_3}{2}\right) \qquad \tau_3 = \left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

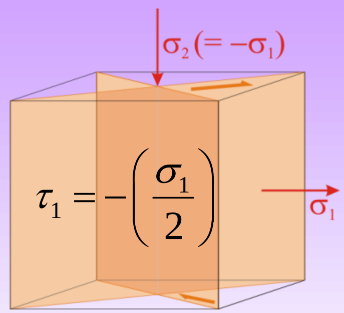
Triaxial hydrostatic tension



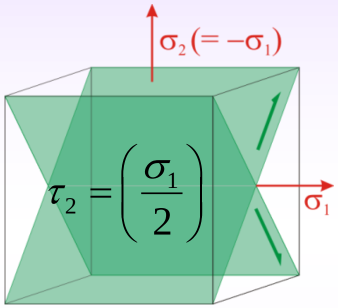
No plane feels any shear stress

$$\tau_1 = \tau_2 = \tau_3 = 0$$

'Push-pull' normal stresses

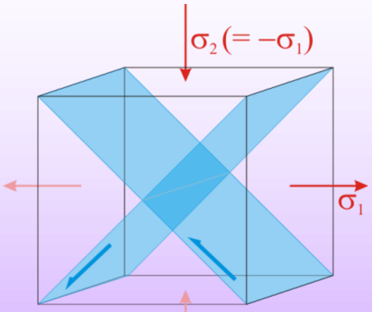


$$\tau_1 = -\left(\frac{\sigma_1}{2}\right)$$



$$\tau_2 = \left(\frac{\sigma_1}{2}\right)$$

These planes feel shear stress

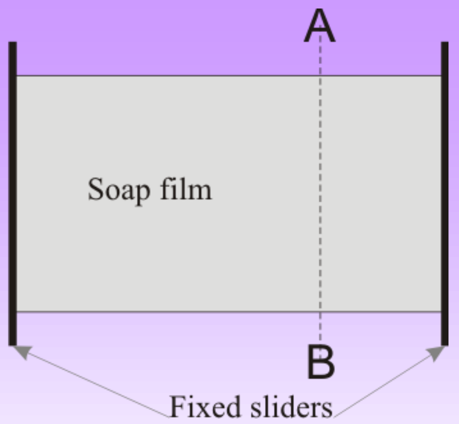


$$\tau_3 = \left(\frac{\sigma_1 - (-\sigma_1)}{2}\right) = \frac{2\sigma_1}{2}$$

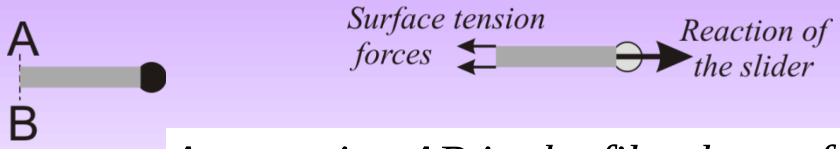
These planes feel maximum shear stress *twice the other planes (above)*

Surface Stress

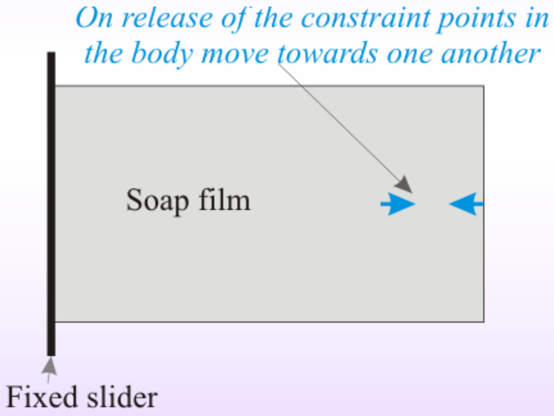
- Surface is associated with surface energy (see topic on Surface Energy and Surface Tension).
- Hence a body wants to minimizes its surface area. In the process surface atoms want to move towards each other.
- The surface of a body (say a liquid) is under tensile stress (usual surfaces are under tensile stress, under some circumstances (e.g. polar surfaces) can be under surface compression).
- As the molecules of water want to come towards one another (to minimize surface area) the stress has to be tensile.
- This can also be understood by releasing a constraint as in coming slides (as before).



Consider a soap film held between fixed sliders



At a section AB in the film the surface tension forces balance the reaction of the slider

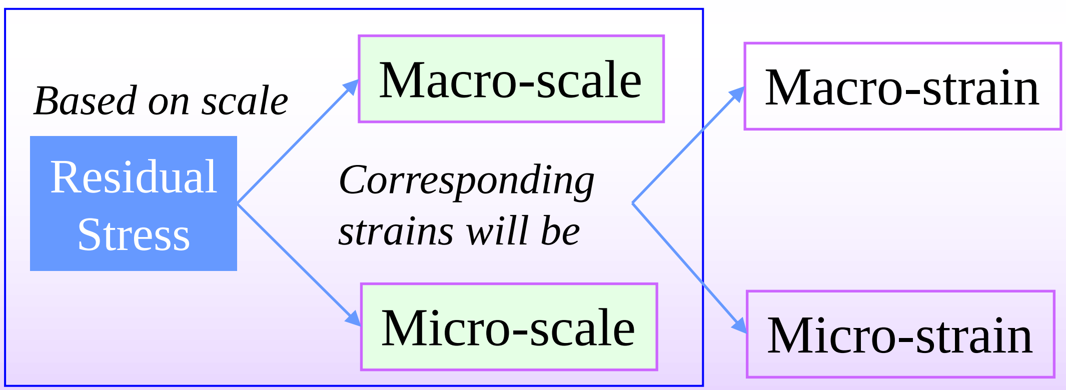


If a constraint is removed then the film will tend to shrink as the points want to move towards each other $\Rightarrow$ the surface is under tension

Residual stress

What is 'residual stress' and how can it arise in a material (/component)?

- The stress present in a material/component in the absence of external loading/forces or constraints (i.e. in a free-standing body) is called residual stress.
- Residual stress can 'be' in the macro-scale or micro-scale and can be deleterious or beneficial depending on the context (diagram below).
- Residual stress may have multiple origins as in the diagrams (next slide).
- We have already noted that residual stress is an important part of the definition of microstructure *(it can have profound impact on properties)*.



- Residual stress can be beneficial (+) or detrimental (–)
- E.g.
  - ⇨ – Stress corrosion cracking
  - ⇨ + Residual Surface Stress (e.g. in toughened glass)

Microstructure

Phases + Defects +

- Vacancies
- Dislocations
- Twins
- Stacking Faults
- Grain Boundaries
- Voids
- Cracks

Residual Stress

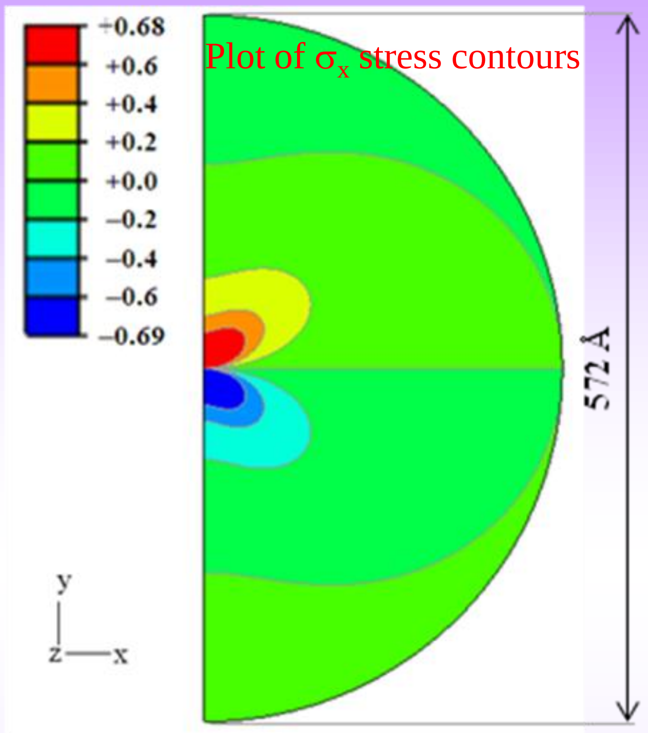
- Defects
  - Vacancies
  - Dislocations
  - Voids
  - Cracks
- Phase Transformation & reactions
- Thermal origin
  - Mismatch in coefficient of thermal expansion

Residual Stress

Origins/Related to

- Physical properties
  - Thermal
  - Magnetic
  - Ferroelectric
- Geometrical entities

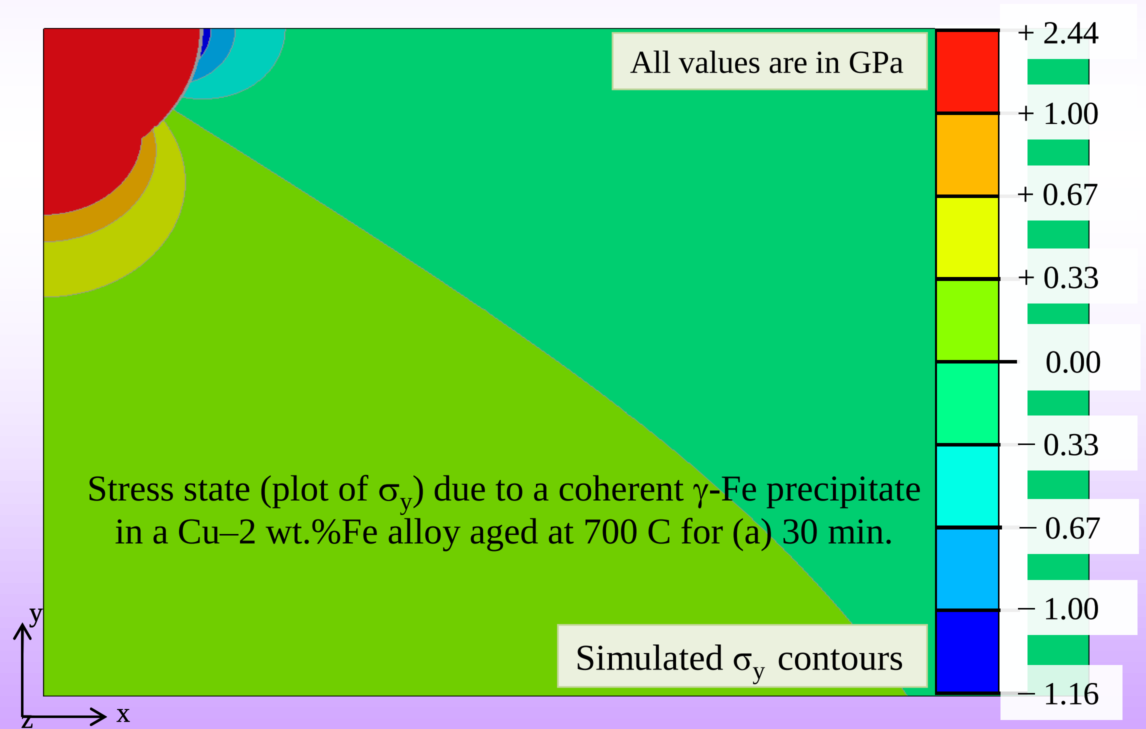
Residual stresses due to an edge dislocation in a cylindrical crystal



Due to a dislocation
(a crystallographic defect)

Due to phase transformation

Residual stresses due to an coherent precipitate



- Often one gets a feeling that residual stress is *only* harmful for a material, as it can cause warpage of the component- this is far from true.
- Residual stress can both be beneficial and deleterious to a material, depending on the context.
- Stress corrosion cracking leading to an accelerated corrosion in the presence of internal stresses in the component, is an example of the negative effect of residual stresses.
- But, there are good numbers of examples as well to illustrate the beneficial effect of residual stress; such as in transformation toughened zirconia (TTZ). In this system the crack tip stresses (which are amplified over and above the far field mean applied stress) lead to the transformation of cubic zirconia to tetragonal zirconia. The increase in volume associated with this transformation imposes a compressive stress on the crack which retards its propagation. This dynamic effect leads to an increased toughness in the material.
- Another example would be the surface compressive stress introduced in glass to toughen it (Surface of molten glass solidified by cold air, followed by solidification of the bulk → the contraction of the bulk while solidification, introduces residual compressive stresses on the surface → fracture strength can be increased 2-3 times).