$\tan\psi = \frac{y+(t)}{h}$

If is a particale is droped from a high 100 meter from a pt. about the earth whose longitude and latitud θ, and ϕ.
$\theta = 45^\circ$, $\phi = 100^\circ$ How find out the lin linear and lateral angular deflation when hits the down.

Harmonic Oscilator:-

Harmonic → which has sinusoidal variation.

displacement

↳ Harmonic motion ↗ Harmonic oscillator

Harmonic traslation.

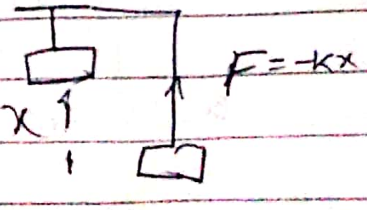
Harmonic Oscilation.
$f(t)$

wave; Harmonic oscilator.
$f(x, t)$

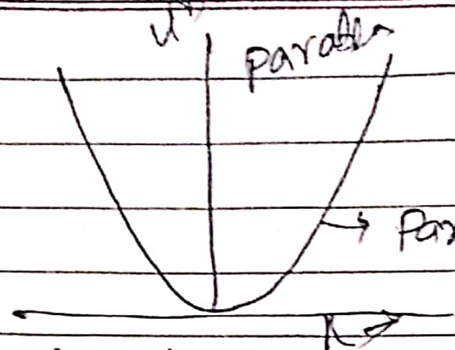
Spring mass:-
Restoring force.

$F = -k \cdot x$

$v(x) = \int_0^x F(x)\,dx$
$= \frac{1}{2} k x^2$



Ideal spring

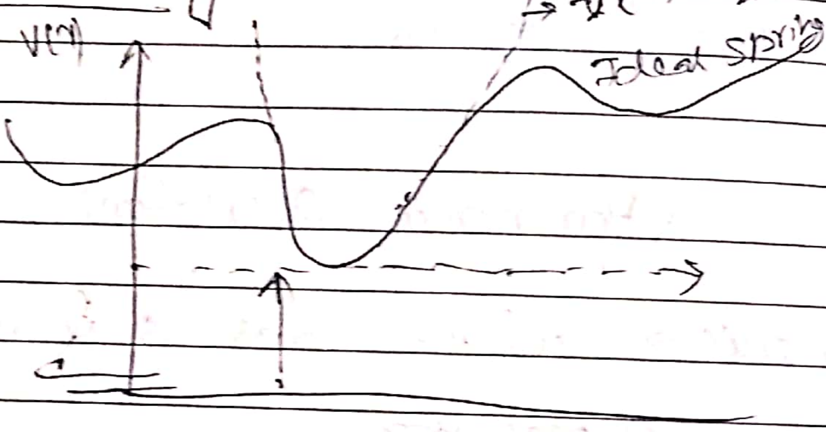


P.E of spring mass

$V(x) = \frac{1}{2} kx^2$

Simple Harmonic oscilator.

Real spring:



$V \neq V(x)$

$\downarrow$ expand $V(x)$ in terms of about $x = x_0$

$$V(x) = V_0 + V'(x_0)(x-x_0) + \frac{1}{2!} V''(x_0)(x-x_0)^2 + \frac{1}{3!} V'''(x_0)(x-x_0)^3 + \cdots$$

$$V(x) \simeq \frac{1}{2!} V''(x_0)(x-x_0)^2$$

$$V(x) = \frac{1}{2} k (x-x_0)^2$$

$V''(x_0) = k$ = constant

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{V''(x_0)}{m}}$$

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$$F = -kx.$$

$$f = -\frac{dV}{dx}$$

$$dV = -\int_0^x (-kx)\,dx$$

$$= \frac{1}{2}kx^2$$

$$V(x) = \frac{1}{2}kx^2$$

for extrime

$$\frac{dV}{dx} = 0.$$

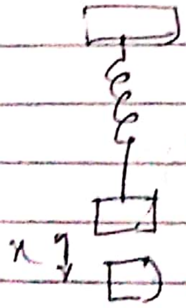
$$kx = 0.$$

$$x = 0.$$

$$\left[\frac{d^2V}{dx^2}\right]_{x=0} = k > 0.$$

at $x = 0$ we have $V = V_{min} = V_0$

$$\omega = \sqrt{\frac{\left(\frac{d^2V}{dx^2}\right)_{x=0}}{m}} = \sqrt{\frac{k}{m}}$$



$$f = -kx$$

$$m\frac{d^2x}{dt^2} = -kx.$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

solu^n will be

$$x(t) = A\cos(\omega t + \phi)$$

arbitcast arbitary cost

$\cancel{A}$

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$x(t) = x(t+T)$
where T = Time period

$A \cdot \cos(\omega t + \phi) = A \cdot \cos[\omega(t+T) + \phi]$

$\omega t + \phi = \omega(t+T) + \phi$

$\omega t + \phi = \omega t + \omega$

$x = A\cos(\omega t + \phi)$

$x(t) = A\cos\left(\frac{2\pi}{T} t + \phi\right)$

$x(t+T) = A\cos\left[\frac{2\pi}{T}(t+T) + \phi\right]$

$x(t) = x(t+T)$

$A \cdot \cos\left(\frac{2\pi}{T} t + \phi\right) = \cancel{A \cdot \cos}$

$= A\cos\left[\frac{2\pi}{T} t + 2\pi + \phi\right]$

$x(t+T) = A \cdot \cos\left[\frac{2\pi}{T} \cdot t + \phi\right]$

$x(t+T) = x(t+T)$

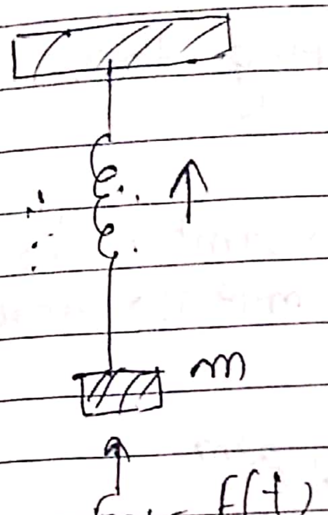
$= A \cdot \cos\left(\frac{2\pi}{T} t + \phi\right) = x(t)$

$\omega = \frac{2\pi}{T}$

$f = \frac{1}{T}$ Hz. $\omega = 2\pi f$ ↳ angular freq.

angular freq

Real Harmonic oscilator



Restoring force.

$F_{ROS} = -Kx$ → directed towards the root

damping force $F_{dam} \propto \dot{x}$

$F_{damp} = -b\dot{x}$ → damping const.

(Resistive force)

it is always opposite to the velocity of the mass

External force $F_{ext} = f(t)$

equation of motion.

$$m\ddot{x} = F_{\cancel{ext}\,total}$$

$$m\ddot{x} = -Kx - b\dot{x} + f(t)$$

$$m\ddot{x} + b\dot{x} + Kx = f(t)$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{K}{m}x = \frac{f(t)}{m}$$

Let, $\frac{b}{m} = 2\gamma$

and $\sqrt{\frac{K}{m}} = \omega$

$$\ddot{x} + 2\gamma\dot{x} + \omega^2 x = \frac{f(t)}{m}$$

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega^2 x = \frac{f}{m}(t)$$

→ second order nonhomogenious differential equation of ~~coss~~ with const coefficients

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undamped oscilation:-

$b = 0 \rightarrow$ no damping force

$F_{ext} = f(t) = 0$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$ $\rightarrow$ simple harmonic motion $\rightarrow$ Ideal sta

Trial soln.

$$x = Ae^{mt}$$

$$\frac{d^2x}{dt^2} = m^2 \cdot A \cdot e^{mt}$$

$$m^2 + \omega^2 = 0$$

$$m = \pm i\omega \rightarrow \text{to real roots}$$

$$x_1 = c_1 e^{i\omega t}$$

$$x_2 = c_2 e^{-i\omega t}$$

$$x(t) = c_1 e^{i\omega t} + c_2 e^{-i\omega t}$$

$\downarrow$

Imaginary soln. (X) not possible.

To have real soln

$$c_1 = c_2^*$$

$$c_1 = c e^{i\phi}$$

$$c_2 = c_1^* = c \cdot e^{-i\phi}$$

$$x(t) = 2.c\left[\frac{e^{i(\omega t+\phi)} + e^{-i(\omega t+\phi)}}{2}\right]$$

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$x(t) = 2C \cdot \cos(\omega t + \phi)$

$x(t) = A \cdot \cos(\omega t + \phi)$

Forced vibration / oscillation. (No damping)

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega^2 x = \frac{f(t)}{m}$$

$$\gamma = \frac{b}{m} \qquad \omega = \sqrt{\frac{k}{m}}$$

$\Rightarrow$ $b = 0$, and $F(t)$ = periodic force.

$$F(t) = A\sin\omega_0 t$$

where ω_0 is the frequency of the periodic force.

$$\frac{d^2x}{dt^2} + \omega^2 x = \frac{A}{m}\sin\omega_0 t$$

The Trial solution $\cdot$ $x(t) = E\sin\omega_0 t$

$$-E\omega_0^2\sin\omega_0 t + E\omega^2\sin\omega_0 t = \frac{A}{m}\sin\omega_0 t$$

$$E = \frac{A}{m(\omega^2 - \omega_0^2)}$$ $\Rightarrow$ Undetermined co-efficient.

The soln $x(t) = \frac{A}{m(\omega^2 - \omega_0^2)}\sin\omega t$

$\therefore (\omega \neq \omega_0)$

Particular solution:

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Charecterstics equn

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

General eqn

$$x(t) = x_{cf}(t) + x_{pi}(t)$$

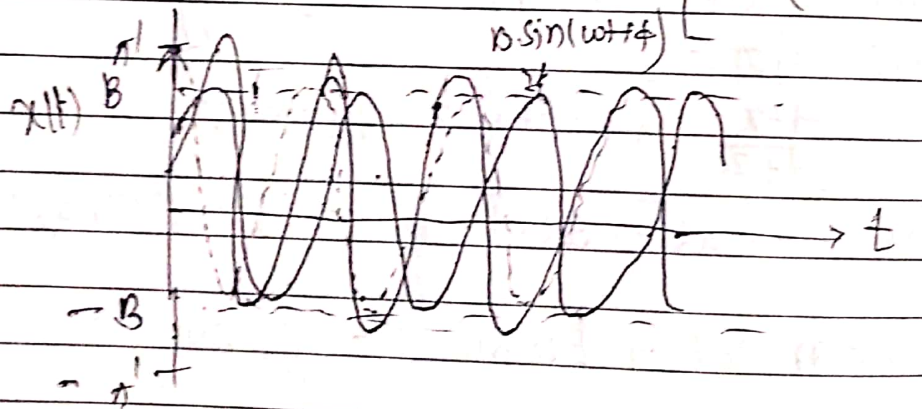
$$x_{cf}(t) = C_1 \cos\omega t + C_2 \cos\omega t$$

$$x_{pi}(t) = B.\sin(\omega t + \phi)$$

General soln

Let A'

$$x(t) = B\sin(\omega t + \phi) + \left[\frac{A}{m(\omega^2 - \omega_0^2)}\right]\sin\omega$$



Force vibration / oscilation (No damping force)

BUT $\cdot\ \omega = \omega_0$

$$\frac{d^2x}{dt} + 2\gamma \underbrace{\frac{dx}{dt}}_{0} + \omega^2 x = \frac{F(t)}{m} = \frac{A}{m}\sin$$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = \frac{A}{m}\sin\omega t}$$

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$x_{CF}(t) = B\sin(\omega t + \phi)$

$x_{PI}(t) = ?$

Let us consider the trial soln.

$x(t) = Pt\sin\omega t + Qt\cos\omega t$

P & Q are undetermined coeff-cient

$\frac{dx}{dt} = P\omega t\cos\omega t + P\sin\omega t - Q\omega t\sin\omega t + Q\cos\omega t$

$\frac{d^2x}{dt^2} = -P\omega^2 t\sin\omega t + \cancel{P\cos\omega t} + P\omega\cos\omega t - Q\omega^2 t\cos\omega t - Q\omega\sin\omega t - Q\omega\sin\omega t$

$2P\omega\cos\omega t - 2Q\omega\sin\omega t = \frac{A}{m}\sin\omega t$

Equating co-efficient of $\cos\omega t$ and $\sin\omega t$ from both sides

$P = 0 \qquad Q = -\frac{A}{2m\omega}\cos\omega t$

$x_{PI} = -\frac{At}{2m\omega}\cos\omega t$

General soln

$x(t) = \underbrace{A\sin(\omega t + \phi)}_{SHM} - \left(\frac{A}{2m\omega}\right) t\cos\omega t$

$= B\sin(\omega t + \phi) - B(t)\cos\omega t.$

→ Amplitude of the resultant vibration is a function time → Resonance phenomena

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Force Free

Damping oscilation :-

$F_{ext} = F(t) = 0$

$b \neq 0$

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega^2 x = \frac{F(t)}{m}$$

here $F(t) = 0$

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega^2 x = 0$$

Trial soln $y = e^{mx}$

charecterstics $m^2 + 2\gamma m + \omega^2 = 0$

$$m = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\omega^2}}{2}$$

$$= -\gamma \pm \sqrt{\gamma^2 - \omega^2}$$

$m_1 = -\gamma + \sqrt{\gamma^2 - \omega^2}$

$m_2 = -\gamma - \sqrt{\gamma^2 - \omega^2}$ $\left[\gamma = \frac{b}{2m}\right]$

Underdamping :-

$\gamma < \omega \Rightarrow \frac{b}{2m} < \omega$. $b < 2\omega m$

$m_1 = -\gamma \pm i\sqrt{\omega^2 - \gamma^2}$ when $\omega^2 - \gamma^2 > 0$.

$m = -\gamma \pm i\alpha$ $\therefore \alpha > 0$

$m_1 = -\gamma + i\alpha$

$m_2 = -\gamma - i\alpha$

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$$x = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$

$$x(t) = e^{-\gamma t}\left[C_1 e^{i\alpha t} + C_2 e^{-i\alpha t}\right)$$

$$C_1 = C e^{i\theta}$$

$$C_2 = (C_1)^* = C e^{-i\theta}$$

$$x(t) = 2 e^{-\gamma t} . C \left[\frac{e^{i(\alpha t+\theta)} + e^{-i(\alpha t+\theta)}}{2}\right)$$

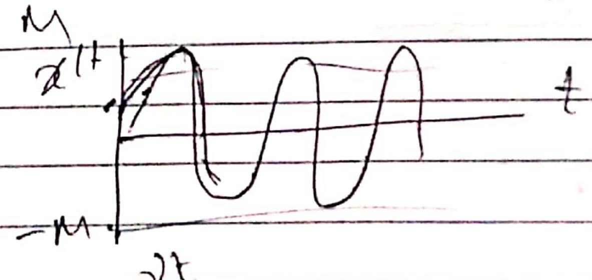
$$= \left[2 . e^{-\gamma t} . C\right] \cos(\alpha t+\theta)$$

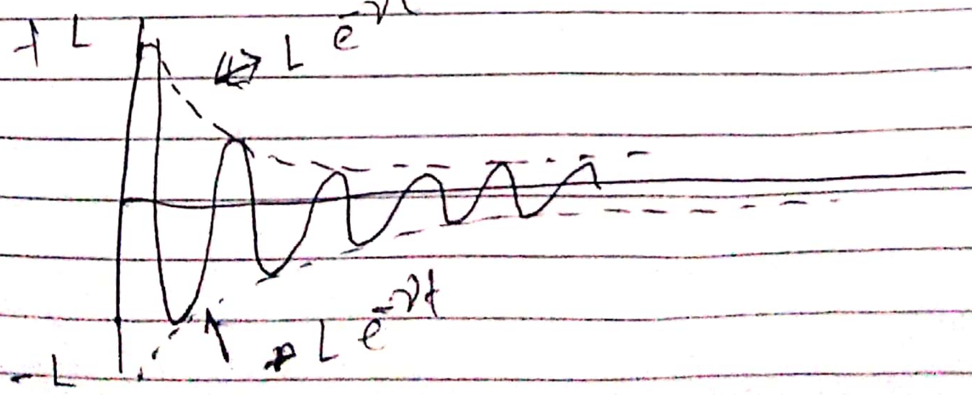
$$= L e^{-\gamma t} \cos(\alpha t + \theta)$$

↳ S.H.M with exponential decaying amplitude.

<u>SHM</u>

$$x(t) = M \cos(\alpha t+\theta)$$





Overdamping oscilation :-

$\gamma > \omega \Rightarrow b > 2m\omega$

$m = -\gamma \pm \sqrt{\gamma^2 - \omega^2}$

$m = -\gamma \pm \alpha$ $\quad [\gamma^2 - \omega^2 = \alpha]$

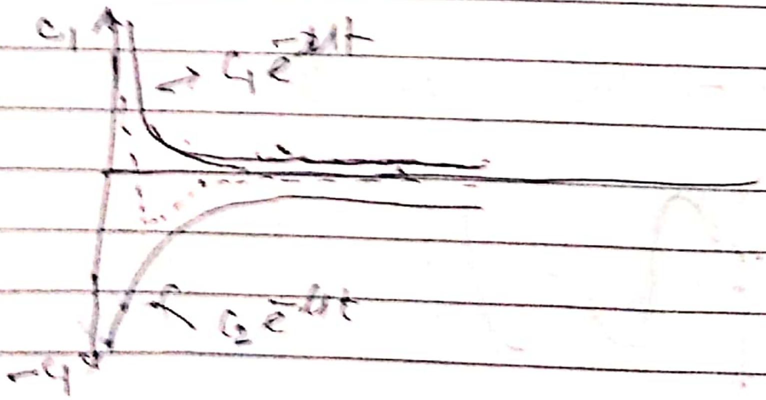
$m_1 = -\gamma + \alpha = \mu_1$

$m_2 = -\gamma - \alpha = \mu_2$

Sol^n $\quad x(t) = c_1 e^{-\mu_1 t} + c_2 e^{-\mu_2 t}$

decay term in +ve half $\qquad$ decay term in (-ve) half.

$\mu_1 >> \mu_2$



Critically damped oscilation:-

$\gamma = \omega$. $\quad b = 2m\gamma$

$m = -\gamma \pm \sqrt{0} = -\gamma, -\gamma$

solun will be

$x(t) = (C_1 + C_2 t) e^{-\gamma t}$

exponential decay of the amplitude.