PRINCIPLES OF METAL FORMING TECHNOLOGY

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INDEX

WEEK- 6

Principles of Metal Forming Technology

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Lecture – 01
Introduction to Metal Forming Technology

Keywords: Metal forming, Plastic deformation, Casting, Grain refinement, Mechanical properties

Welcome to the lecture on Introduction to Metal Forming Technology. So, hi friends I am Dr. Pradeep Kumar Jha, I am associate professor in the Department of Mechanical and Industrial Engineering IIT Roorkee and, I affiliated this course for you that is Principles of Metal Forming Technology.

So, this is the introductory lecture of this course this is lecture number 1. And in this course we are going to talk about the different aspects, related to metal forming technology. We will talk about the theories behind it, different types of metal forming processes, then you have different types of relationship, forming defect all that during this course.

So, we will start with the introductory lecture of this course. So, metal forming process as you know they are used for plastically deforming work piece to desirable shape and obtaining optimum mechanical properties. So, why there is need of forming, we have been knowing about different types of manufacturing processes and forming is one of them. The other forming processes or other manufacturing processes are like casting welding machining all that and there are other varieties.

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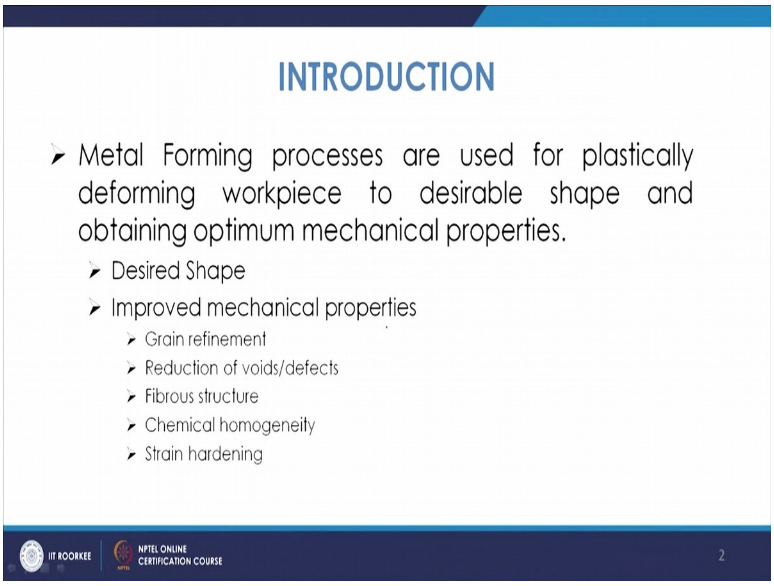

Where we use powder metallurgy and other things, but why forming is so, important. The thing is that first of all we must know that what is that forming. So, that is nothing, but you are shaping the material to a particular desired shape. Now, can that be possible by other ways. So, when we talk about any engineering process, we talk about the alternate routes, we talk about the economy of the process.

So, first of all talking about the alternate routes so, if suppose you have to form any material and, for suppose you have to reduce its dimension. So, that can be possible by many ways first of all the casting so, you can cast it of smaller size. But for that you have to further go and replicate all the processes like you have to make a smaller pattern, you have to make it smaller mould, then further you have to melt and then you have to put it.

The further easier process, or easier mode may be machining, where you are keeping the job on a machine on a lathe machine, or any other kind of machine and, then you are removing the extra materials so by that also you can think of removing the extra materials.

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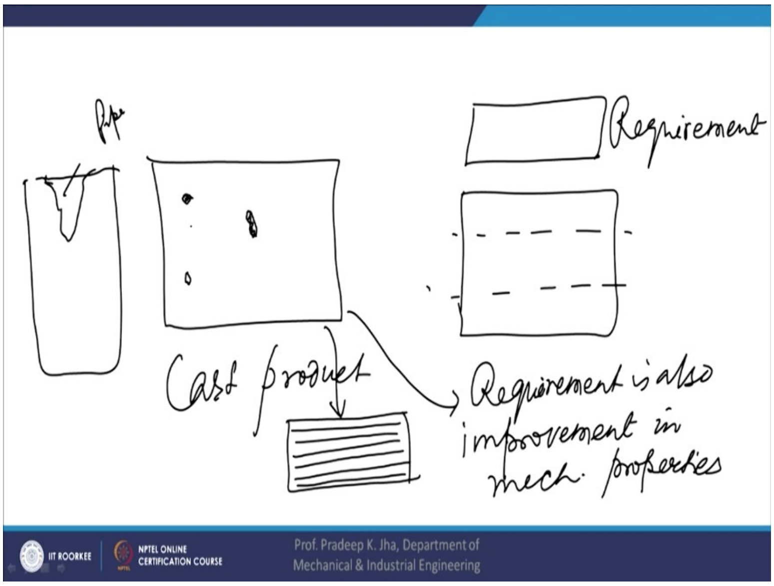

So, suppose you made one piece by casting of this dimension. So, this is your cast product. Now, you need a product of this type so, you need it. So, requirement is about this size of product. So, you can you can go for casting as we discussed that you can go for further the process of casting and, then you can make a smaller mould, you can have a smaller pattern and then you can make it.

The next process may be by machining. So, you can remove this you have larger size of product, you remove this much of portion from top and bottom. And then you can get thethis size of the product. So, that is by machining process, but when only thing is I mean in these cases you are converting the size of the product to the final shape, but what happens that when you require that its property should also be modified you require that this product requirement may be. So, your requirement is also improvement in mechanical properties improvement in mechanical properties.

Now, the thing is that this also can be done by some of the other methods you can go for the casting of this sample of smaller shape and then further you can go for the mechanical property enhancement by other routes. Like you can go for heat treatment and other ways, but the thing is that again heat treatment may not give you that kind of property enhancement.

So, the thing is that when you need a specific type of properties, specific type of you know good qualities maybe some properties in certain direction it is quite high or in fact, you want to alter the structure. So, you want to alter the structure to a fibrous type of structure, which has a very very high strength very improved property which has you know minimum of the defects in that.

So, in those cases you may have to go for one such process, where you can think of minimizing the defects having some properties which is very very good. So, what I mean to say that if we do the casting further the structure is not going to change much, the same type of microstructure you are going to get with some modification certainly. Because, if you are changing the size there may be some change in the microstructure may be at the surface, because of the different cooling rates experienced. But then overall the property may be identical. Similarly if we do by machining then also you are not going to change much may be wherever whatever properties was there in the middle it will be there.

But then if suppose you wish that you should have the material with very high property mechanical property, you should have a fibrous structure you should have very good strength in those cases you will have to apply the forces, first of all this dimension is short. So, you have the shorter dimension now how to short in the dimension. So, we discussed about the different methods. So, in this case you apply the forces, you try to convert the size to smaller one. So, in that process you apply the force, then if you know control the dimension in one direction, then in other direction than it will be changing. So, if you are so, because of the volumetric constant rule. So, in that case you can alter the you know dimension.

The thing is beneficiary part you have in your cast sample there are the chances of certain type of defects. So, you may have something like blowholes, you may have something like shrinkage cavity cavities. So, you have may have pipe formation, where we talk about the larger ingot structures you might have seen the large ingot type of casting, then what we see is that in the middle you have a pipe type of structure. So, this is these are the pipes. So, pipes are formed in the casting because of the shrinkage which occurs so, mainly because of the center line and shrinkage.

So, the thing is that when we talk about other routes, you have our especially the casting which is the mother of the manufacturing processes, you have the chance of having such defects. But then when you are forming when you are applying the pressure from both the sides, in that case what happens and when because of that you this dimension goes on decreasing in z direction, then what happens that these points of discontinuities that is defect, or the pipe or so, or shrinkage they get welded. So, they will be slowly disturbed they will be slowly disappeared.

So, in that case the property of the material gets improved. Also on the process I mean these beneficial in the sense, that when you are strain hardening the material, when you are straining the material at lower temperature side. In that case because of the strain hardening effect the materials you know strength is increasing. So, this is again I mean that is why it is important to know that what kind of property you want, if you simply want to just decrease the dimension you can go for the routes, but if you want specific properties, if you want very less porosities very good quality you will have to have alternate route and that is achieved by the forming process.

So, what we say is that it is used for plastically deforming work piece to desirable shape and obtaining optimum mechanical properties. Now, plastically deforming so, as we know that whenever we apply force there will be deformation. So, force is applied will call deformation, but every deformation is not permanent, deformation is either you know recoverable or non recoverable. So, when we are applying the force, there will be certainly some deformation, but that deformation may not be permanent so, that is elastic deformation.

Then there is a limit that is elastic limit and when we deform after that limit, then that becomes the permanent deformation and that is a plastic range of deformation. So, in that case you have a permanent set, or permanent deformation which is there in the material.

And in the case of metal forming we are deforming in the plastic range. So, that the deformation what we get is permanent. So, one thing is that so, what we say is that there are two things one is desirable shape. Now, the shape we discussed if we have to have certain shape, that we have to keep that in mind and we have to move our processes in such a manner that finally, we get that shape.

But along with that you have the improved mechanical properties. Now, improved mechanical properties means the properties which we get there are the defects, which we get normally in the cast samples, now these defects are not at all desired in certain components. Suppose, we are taking the example of the automobile, or you are taking the example of you know crane hook or so, so in such cases you require a very good high strength with minimum chances of failure, because of any kind of structural discontinuity or defect or so. So in those cases you cannot take any you know chance and, the most material must have very very high strength.

So, in those cases you need to have the process in such a manner that those I mean chances of defects are minimum. Now, the properties can be improved by many ways by so, what are those methods by which these properties are improved. So, the forming process in general, what it does it may go for grain refinement, what happens that when we form the material and especially at the higher temperature range.

In those cases what happens that after certain temperature after recrystallization temperature, when we are finishing the forming process. So, after that temperature your birth of new grains starts. So, that basically gives in the grain refine structure. Many times when we fabricate material, it may have lot of stresses, or once it is cold worked material lots of dislocation density is increased you have dislocation sites are there, imperfection sites are there.

So, basically these are the points where form the nucleation starts and, if we go to the hot working range in those cases normally you have formation of new grains. So, decrystallization crystals reappearance so, that is your that leads to the grain refinement.

Similarly, you have a reduction of voids or defects. So, that is what we discussed that when we do the material, when we fabricate the material by normal casting process. In those cases it is likely that normally in the casting processes, slightly that you have the defects like blow holes or pipe formation, or shrinkage cavities or so.

So, now these are basically the source of defects, which alter or which hamper the mechanical properties. In those cases basically when we go for when our aim is that, we should not have such defects in the forged product; So, when we use the forming process, when we apply the pressure from both the sides in those cases these voids or defects they are minimized.

So, the position, where there was no metal because of the shrinkage. Now, because of a the proximity of the grains from both the sides approach each other then ultimately they meet. So, this way wherever you have the defect they are disappeared. So, wherever you have voids they get disappeared.

So, this way once your voids and defects are minimized once your blow holes are welded together, in those cases ultimately the property gets improved. So, this is one of the main advantage of the this metal forming process, then the fibrosis structure. Now, this is also one of the important point, which is there in the case of forming.

Now, you can see the fibrosis structures, if you look at many of the materials used has fibrosis structures and, what is seen is that it is very difficult to cut the fibers difficult to cut the fibers rather than to tear it. So, what happens in many cases you prefer to have a fibers.

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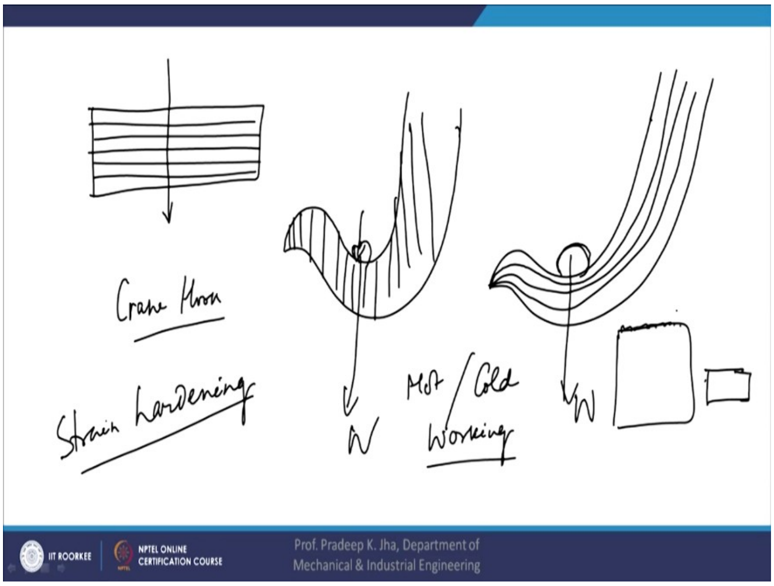

So, once you have fibrosis structure or grains oriented along certain direction. Then when you are trying to have pull it like this, it will be certainly resisting you know large forces, or many a times when you are trying to cut across the fibers, that is also very difficult.

So, you need to have these fibrosis structures to have more and more strength, a typical example is the example of crane hook. So, what happens that you must be recalling that the crane hook is of this type.

Now, in this crane hook there may be two structures, one crane hook is like this and, then you may have the crane hook like this. Now, suppose why in one case your structure fibrosis structure is all fibers or grain so, you once you have made it is like this. And in another case you may have the fibers go like that. So, now, the thing is that when we are using these grains what happens that your load is from here, and this way the load is being applied.

Now, the thing is that in this case if your fibers are oriented like this and, if you are applying large force here, then there are quite a good chance of basically that fibers getting teared a part. And in that case there will be failure of this part and that will lead to catastrophic.

So, then this may be so, this structure may be because of the cast component, or improper fiber direction and, or you can cut from a cast sample you can cut it in that case there will not be fibers and you will have you know no continuity of the fibrous line and, the fibers orientation may be like this.

In that case there is chance of failure while, if you apply the force in this type of fibrous appearance, then it is likely to resist more and more to the forces; So, normally it is kept in mind that what way your material is going to be applied.

And accordingly we adjust our processes of forging. So, that we try to have a fibrous type of a structure. So, in that case once you have a fibrosis structure and, also we see that there is no discontinuity of the fibers, fibers are not cut in between in that case your the continuity is broken and, the property will certainly be you know hampered.

So, those are the you know situations which are to be looked into and, this way once you have the fibrous structure, this is achieved by the forming process when you are forming it by applying the compressive forces, or the type of forces and you are trying to orient the grains in particular direction and then you have fibrous appearance.

Then this way the property is improved, then you have chemical homogeneity. Now, many a times when we do the forging, you will have redistribution of the chemical elements and, in that process the chemical inhomogeneity which is there. Suppose you have a many a times your certain you know impurity elements are there somewhere segregated, some something some of the thing things are not properly distributors of the alloying elements or so, or many a times you have also the residual stresses which are allowed inside.

Then when you go for forming in your heating at higher temperature that is removed, then the there will be proper diffusion at that higher temperature side. And then in that case you will have redistribution of these stresses, especially when if you talk about the iron based alloys. So, when you go to that range you go to austenitic range for the forming process basically hot forming process.

And in that case you will have a the redistribution of these chemical elements, if any or in that case your chemical homogeneity is achieved in the forming process. The point which we discussed about the forming is, one is the desired shape and other is the mechanical properties and, one of the mechanical property which is normally of importance is it is strength ductility or so.

Now, the thing is that certainly we can get the improvement in the properties, because of other type of you know events also like grain refinement reduction of voids fiber structure or chemical homogeneity, but then strain hardening is a process, in which because of straining the material becomes more and more you know a strong.

So, in that what happens that, when we increase the strain on the material in that case so, you must have heard about the dislocation density is increasing, because of the cold work. So, once you increase the cold work, when we are forming at I mean below certain temperature in that case when we are increasing the amount of cold work the dislocation density gets increased and, the material becomes harder and harder.

So, basically when we form the material especially cold forming, the material's strength becomes higher and higher. So, that is another you know purpose which is a very important factor, which is adopted for the forming process. So, this way when we so, strain hardening is the mechanism and, in strain hardening when we increase the amount of cold work. The material gets more and more hard.

But certainly we have to know that what should be the amount of cold work, because as you go on increasing the cold work, then the materials hardness will increase. And if it is too high then that will turn towards brittleness. So, the material may be extremely brittle so, the ductility may be too less the metal material may be brittle and may be of very less use for engineering applications.

So, strain hardening is experienced mostly in the case of cold forming and, you must have seen all the smaller you know thin sheets of the metals like iron sheets, or other corrugated sheets. So, they all are cold work even after I mean when there. So, think they have a good strength its all because, they are strain harden and then in that case the hardness is improved. they are many forming processes like wire drawing or so, where this principle is used, when we try to have the wires of different types of hardness.

So, basically what we do is we control the cold hot working and cold working, to achieve the you know degree of you know hardness either it will be whether you want stuff or soft type of material all you want extremely hard material. So, that depends upon the temperature and because of a temperature basically you differentiate as hot working as well as cold working.

So, when we talk about hot working, then we talk about forming to a temperature which is more, than recrystallization temperature and when we talk about cold working, then it is nothing, but forming below certain temperature that is recrystallization temperature. So, what happens that again depending upon the need, we can go either for hot or for cold working; so, hot or cold working.

Now, the thing is that you have two objective, when you have to deform to a larger extent, you will have to go for hot working. So, the material from here to hear it cannot be done by cold working.

So, it has to be done by hot working in stages because, when we heat the material its ductility is becoming more, its you know its strength becomes less and you know that way you are trying to reduce the dimension of the material, but as the temperature goes on decreasing, in that case it is a strength becomes again further higher and you require very large, or capacity equipment to further reduce the dimension.

So, when you have to decrease the dimension to a larger extent, you go for hot working, or else you go for cold working. There are limitations basically of the hot working as well as the cold working. So, they are basically since being of based on temperature, you have the different you know the parameters, basically to define them whether you should go for hot working, or cold working like when you go for hot working since you are working at very high temperature, there are chances of scale formation.

And because of that the surface appearance may be may be that, once you are surface finish may not be appropriate, you cannot control the dimension, And then another disadvantage is that, when you are further going putting it into the in between the dies, then there maybe impressions on the surface.

So, this way scale pits are formed but then when you have so, for that also there are ways like you can do in the control environment, but then if you go want to deform the cold working, there is certainly limitation up to what degree you can do the cold work, because if you do more and more then material may get, there may be in brittleness induced in the material and the material may break in between.

Similarly, when we talk about hot working so, in hot working basically recovery and recrystallization; so, recrystallization temperature is the you know boundary above, which the hot working is defined and below which the cold working is there. So, in the case of hot working there is recovery. So, what happens that when the strain grains or a strained, further that that strain is released because of the higher temperature. So, there is recovery taking place in the case of hot working whereas, in the case of cold working recovery is not there. So, every time you are deforming the material get is deformed and larger amount of stresses are generated material is get is strain hardened.

So, this is how I mean depending upon the nature depending upon the final product what you want, you would like to go for hot working as well as cold working, for materials for every material you have different types of different temperatures at which they have to be formed. And normally this temperature is a function of the melting temperature. So, that way you do the hot and cold working, apart from that when we talk about the different kind of forming processes, you have different sets of forming processes depending upon what you want what kind of product you need.

And today when we have you know good requirement of materials, which must have very very good you know reliability, which has minimum where your minimum chances of error, you must use the optimum forming process to get the product with minimum defect. So, we will discuss about the different you know types of forming processes and, and also the fundamentals of the forming processes in the coming lectures.

Thank you very much.

Principles of Metal Forming Technology
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Lecture - 02
Classification of Metal Working Processes

Keywords: Metal working processes, Plastic forming, Forging, Rolling, Compression, Bending and shearing processes.

Welcome to the lecture on Classification of Metal Working Processes. So in this lecture we will talk about the different types of metal working processes the different ways by which they are classified. So, the metal working process creates useful shapes by plastic forming processes and control mechanical properties.

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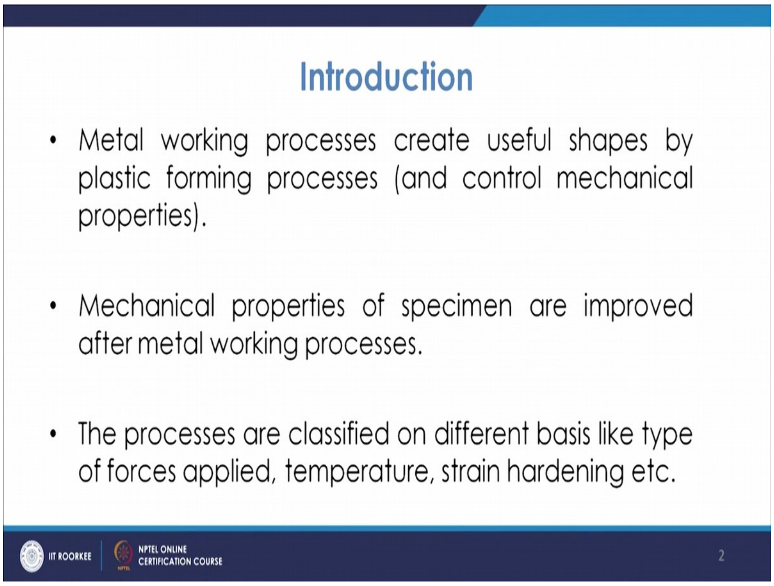

So, we have already you know studied about it that we use the plastic forming, we go the forming in the plastic state and then do the forming. So, mechanical property of the specimen are improved and certainly they are classified on different bases like type of forces applied temperature strain hardening etc.

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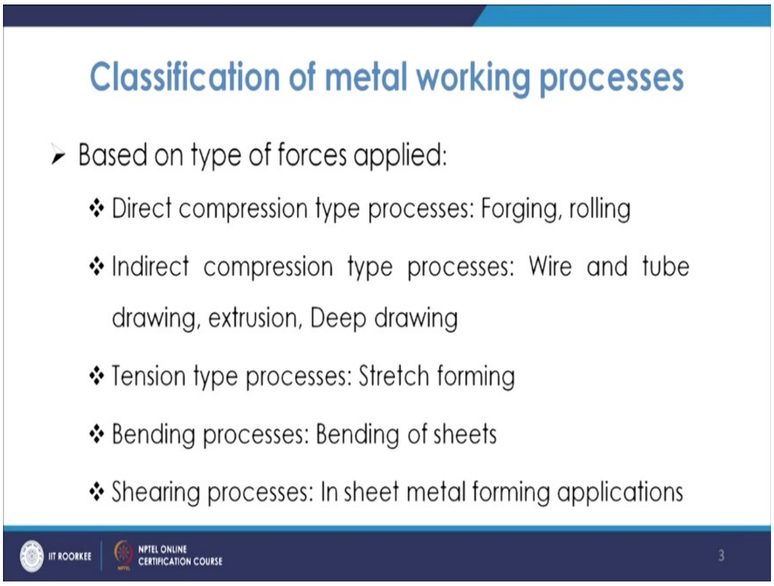

So, let us see that how they are classified? Now as we know that in the case of metal forming there are many things like the force we apply, the force may be applied in different way and its basically working on the specimen in different way, different in the different ways the stresses are generated and because of that the deformation takes place, So depending upon the type of forces which are applied you have one is direct compression type processes. So, under that basically you are applying the compressive forces because of the direct compression type stresses developed you have the deformation of the material taking place and forging and rolling is its example.

Similarly, indirect compression type process, so in this case normally what we do is we apply the force normally in a tensile way and what happens that at the point where you have suppose, die and the work piece it has certain you know reason where the interact. So, at its interface so they are basically indirectly compressive type of stress develop and then that basically does the failure of the material at that particular point and the deformation takes place.

So, basically we are not directly applying the compressive force we are indirectly basically the compressive force is applied and that basically creates the deformation and its example is wire drawing or tube drawing. So, as you know that in wire drawing you are pulling and then at the die exit you know and there when where the bar or the billet which is in contact at that point the compressive stresses are developed, compressive forces are developed.

And because of that the local failure takes place and then the material comes out, so that is your in direct compression type. Similarly tube drawing extrusion also the same thing and deep drawing, so will discuss about them. Then tension type processes where normally because of the tension force this deformation takes place an example is stress forming, where you form block and you are applying you are you are stretching a sheet in that case so it will due to the tensile stress is developed that deformation takes place, so that is stretch forming.

Bending processes where we apply the bending moment to bend the specimen and in that case the material bends, so that is bending process. Similarly shearing process an shearing process you will have blades, so you will have the from the top and bottom you have two blades and they will be shearing or cutting. So, because of that the failure takes place and then the deformation takes place, so that these are basically the different types of forces, because of which the deformation takes place and we will talk about them one by one.

So, first is the direct compression type forces, now in this the force is applied to surface of work piece and then and metal show takes place at right angles to the direction of compression. So, in this case the example is as we discussed it is forging and rolling so if you see this is forging and this is the rolling.

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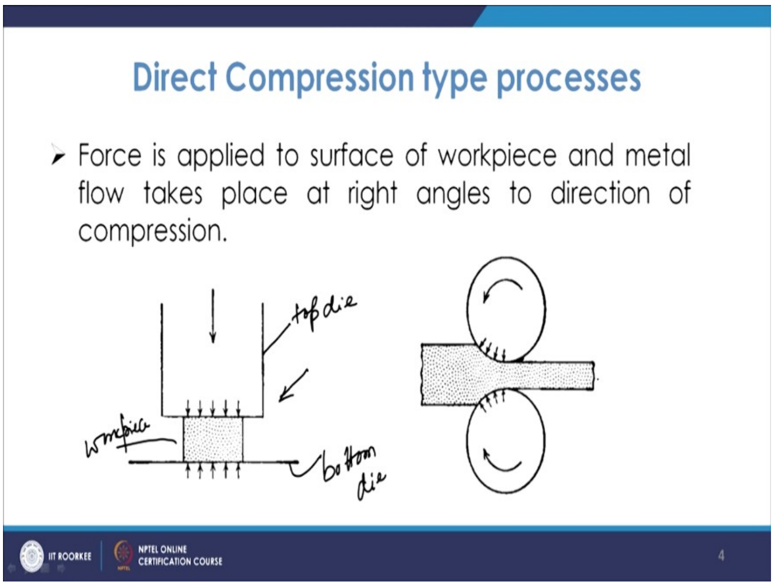

Now, in this case as you see in forging you this is the top die and this is the bottom die. So, you apply the pressure from here you apply the compressive force as you see and this job is subjected to the compressive load and then because of the compressive load the material deforms and it goes. So you are applying the you know force in the vertical direction or z direction and the material if you are keeping one of the dimension constant from both the sides then the material will be flow in x direction. So this way the flow will take place perpendicular to the direction of the applied compressive force so this is the normally case in the case of direct compression type process.

So, coming to the direct compression type processes, now as we discussed that we apply the force to the surface of the workpiece and metal flow takes place at right angles to the direction of compression. So the example is forging and rolling, so what we see this is this is the forging process and this is the forging process and this is the rolling process. So what we see is that in this process in this forging process this is the top die and this is bottom die and the workpiece is kept in between and so this will be your workpiece, so this is your workpiece. Now, the workpiece basically will be subjected to this compressive force from the top from the top so you have the reaction, so you will have this is subjected to the compressive stress.

Now, because of that what happens that this material will deform, now the thing is that it with since it is it is constraint from both these sides. So, it cannot expand in the vertical direction. Now, the further thing is that it will be expanding either in the x direction or the y direction, if it is the z direction in that case it will be changing in x or in the y direction. So, that way basically you are applying the compressive force in the z direction and it is able to flow in x or y directions is perpendicular to z direction. So this is because of the direct compression type of forces and these deformation takes place here.

Similarly, direct compression type force is also applied in the case of rolling, so in the this rolling process is very common in the case of industries where you are trying to decrease the cross section of the larger you know your products like you have (Refer Time: 08:43) or plumes or so. So in that case you are passing it through the two rolls, the rolls are extremely hard and they will be rotating in opposite direction. So, what happens when they come in contact with this job then they are applying the compressive stress and in that the material will be compressed and then they try to flow through the exit.

So, if you look at the compression here so it will be compressor in the vertical direction and then material flows in the in horizontal direction or so. So basically these are the examples of the direct compression type of forces and the forging and rolling is the example of such process. Then you have indirect compression type of forces, now in the case of indirect compression type forces normally you primarily apply the forces which are tensile, but then in the direct compressive forces are developed because of the reaction of the workpiece to the die and they reach very very high value.

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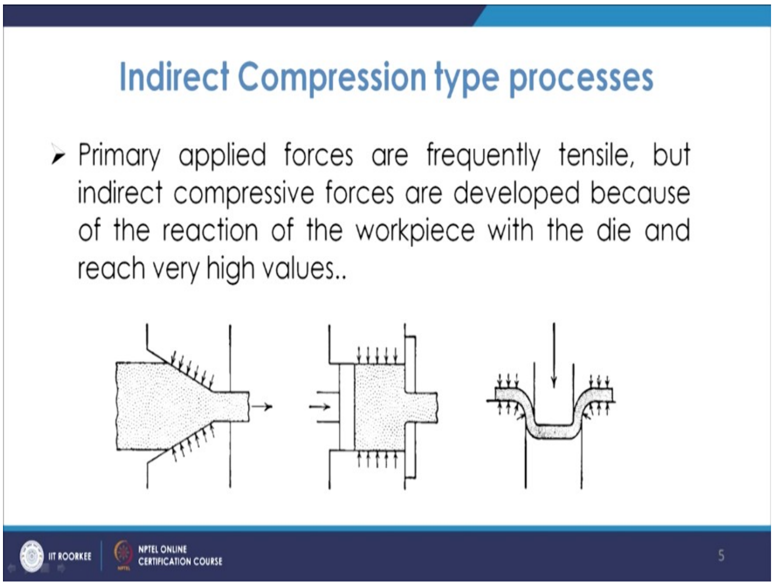

Now what will be happens in these cases like this is your tube drawing I mean this is wire drawing, so in the case of wire drawing what we see is you apply the force in this direction you are basically pulling. So, you are primarily you are taking the tensile type of primary primarily you are applying the force.

But then what happens this is the die, so this material billet or so. Now, it is in touch here, so once you pull here from here once there in touch there will be reaction forces generated. Now, the reaction force which is generated at the workpiece and the die interface here so because of the basically what happens that force is normally compressive in nature and because of that when the stresses reach more than the yield limit in that case the material deform starts deforming and then the material will flow through this orifice, so that you can draw this rod.

So this way where you can where you can draw, so wire driving is an example of indirect so ultimately you are pulling like this, but then the stress because of which the failure is taking place is basically compressive stress, so that is why we are indirectly basically able to generate such stress because by the tensile forces, so that is why it is indirect compression type. Another example is also the you know extrusion, so in the extrusion as you are applying the force like this and then because of this reaction here you have compressive type of stress developed and then it material fails and goes to this direction.

So this is also example of indirect compression type of process the other example you may be looking at the that is the drawing of the cup and if you look at here, here also you are applying the force like this. And then you see that here you have the reaction forces acting and this is basically compressive in nature and because of that the deformation is taking place and it moves towards the bottom. So these are the example of indirect compression type processes. Moving further you have tension type.

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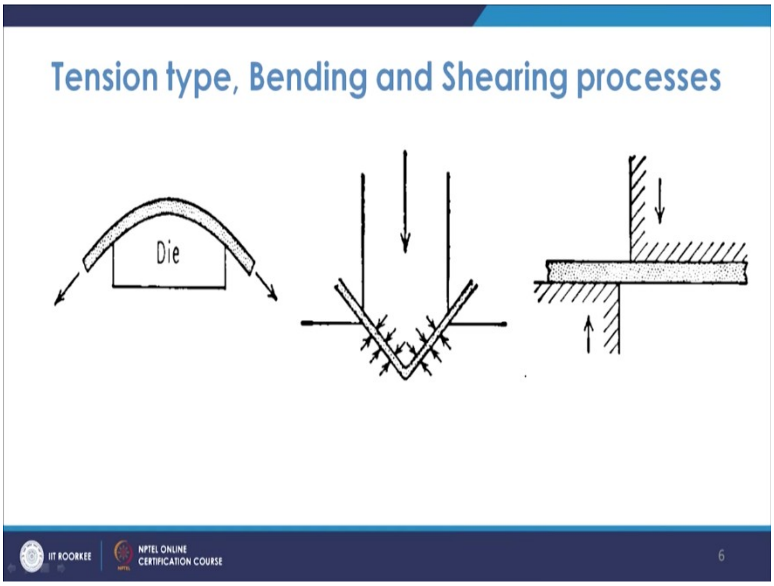

So, the stress forming is the example of tension type of forming process, where are you have these being stressed from both the sides and you have form block die. So, it will be stressed because of the tensile type of force which is there primarily and then that is why it is tension type of forming process. Similarly where you apply the bending moment in these cases and you see that with the help of this punch you induce these bending at these two places because of the bending moment created and so this is under the bending type of forces.

Similarly, you have the shearing process where you have these blades so top and bottom and once they will move in the bottom end the reaction will be there at the top. So at this point there will be shearing taking place are shearing taking place and cutting. So basically bending and shearing process normally they are applied in the case of the sheet metal forming processes and in the case of sheet metal forming processes typically you go for such process like you have punching, you have blanking, that that bending of the seats and all that so they are these processes are basically utilized more and more.

So then another classification is based on you know what kind of you are doing, so what happens that sometimes you are reducing the larger dimension to smaller dimension not the final dimension you are achieving final shape you are achieving. So when you are reducing the larger dimension product like ingot or billet to standard mill product of simple shape such as sheet, plate and bar so that is basically the primary metal working processes.

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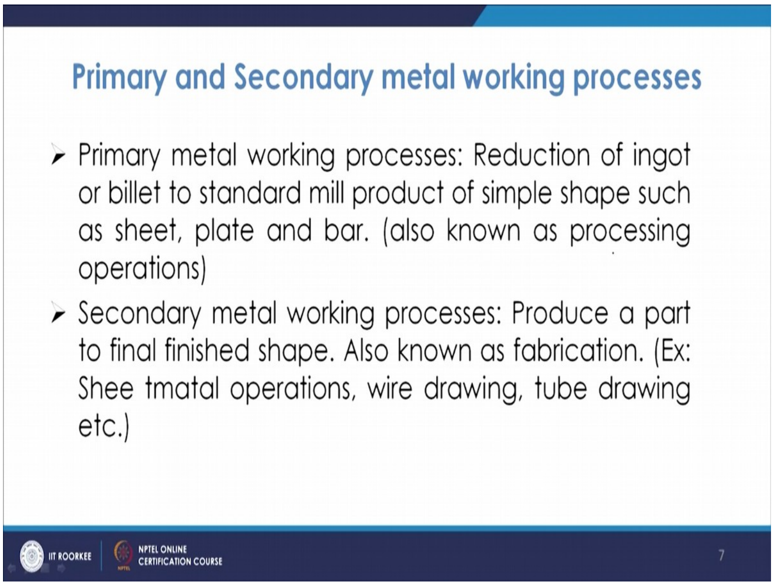

So, you are you have just like you do the rolling, forging or so intermediate one rolling basically. So those processes are where you are basically reducing the larger dimension of the product to the smaller dimension simple shapes than that is your primary metal working process and then when you are going for the final finished product, so they are known as secondary metal working processes.

So, in this case what you do is you have to finally, get the final shape so the finishing is there like the sheet metal operations, wire drawing, tube drawing all these are the one which are the example of such processes. So in such case you see that finally, you get the final shape and the example is like wire drawing, tube drawing etcetera also these primary metal working processes where are you reduce the ingot or billet to simple shapes they are known as processing operations and this is known as the fabrication operations. So, normally it will be done on the larger size products larger size you know sample like billiards or the ingots and here it will be done on the sheet metals, so that is known as fabrication. So, we talk about fabrication operations that is secondary metal working operation.

Then we already discussed that you have the classification also based on the temperature of deformation and the temperature on the basis of temperature it is defined as either hot working or cold working. So, as you know that there are two main purpose of doing the deformation one is the reduction in dimension as well as the improvement in mechanical properties and many a times the improvement in the strength. So, the deformation when we do at high temperature for getting larger deformation then it is known at as hard working or hot forming processes and it is done at sufficiently high temperature.

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Deformation Temperature	Typical Processes
1. Hot Working Processes	Done at a sufficiently high temperature such that continued deformation does not result in increased hardness. Example: Hot rolled steel. Surface finish and tolerances are inferior to cold working processes because of surface reactions (oxidation scale etc). Deformation forces are low.
2. Cold Working Processes	Strain hardening effects predominate over thermal recovery effects. With continued deformation, the hardness and strength increase and the ductility decrease. It results in an elongated grain structure and can be used to harden metals and alloys that do not respond to heat treatment. Example: Cold rolled steel. Excellent surface finish and tolerances. Deformation forces are high.

So, in that the deformation will be more, but the hardness will not be more hardness will not be increased, so also the surface finish and the tolerances are inferior. So we have discuss that in the case of hot working since we are dealing with high temperature, so that will be at high temperature and our chances of the scale formation and you cannot control the surface finish and also the dimension to a very precise value. So, basically in that your surface finish and tolerances are inferior as compared to cold working because in cold working it is done at lower temperature, so they are the finishes better also there are surface reactions like oxidation scale and all that, so that is that.

Another hot working is that, the deformation forces are low in such cases when you increase the temperature the material becomes its strength becomes less. So, the force which is required to deform the workpiece is low so you can go for larger degree of deformation. Whereas, when you go for the cold working at as the temperature comes down the strength becomes higher. So in that case the requirement of the load becomes larger, so that there is a limitation up to what degree of cold work you can go for. So in the case of cold working process the strain hardening effects predominate over thermal recovery facts.

So, as we discussed that the main difference between the hot working and the cold working is that is this recovery effect. So what we see so this is the this recovery effect, now this recovery effect is dominant in the case of hot working. What is this recovery effect, because what happens that when you are applying the forces on the body or on the sample then the grains are started the grains will experience the strain and then there will be a strained crystals. Now, when you are heating when you are heating to higher temperature and when you are forming, now at that particular high temperature the those any amount of you know strain field which is generated or the stress which is generated so that is relieved basically and thats why the crystals are strain free.

So, that is nothing, but the recovery fact so the recovery will be there in the case of hot working and in the case of cold working this recovery does not take place. So the thing is that when you do the cold working the material gets strained every further you grow a cold working it will be further strained, so the stress which is generated in the material hardness will go on increasing. Whereas, in the case hot working that does not increase because it is done at higher temperature, so that there you have strain free crystals every time as the temperature is higher than that recrystallization temperature, so this is main deform I mean difference between the hot working and cold working.

So, with continued deformation the hardness and strength increase and the ductility decrease. So in case of cold working when you are going for the cold work every time the strength will increase, hardness will increase, but that will cost the ductility to certain extent. So ductility will be decreasing so it results in an elongated grain structure and can be used to harden metals and alloys that do not respond to heat treatment. So many a times those materials which do not respond to heat treatment you can go for such treatment to such metals such materials and get their properties improved, example cold rolled steel or so.

Excellent surface finish and tolerances and deformation forces are high, so as we discussed that when we are dealing with the material at higher temperature as well as at low temperature at lower temperature the yield strength is higher. So, for deforming you require larger and larger forces and as you go on cold working every time you do the cold working the strength will be increased.

So, you require larger stress further to deform so deformation force will go on increasing you know as we go continuously deforming the material. So, there is a limit of the degree of the deformation in the case of cold work because you need very very high powered equipment. Also if you try to do the you know cold forming cold working beyond certain degree then the material may be broken, it may be it may result into fracture, because the brittleness is induced in the material.

So this is the basically difference between the hot working and the cold working, now that is basically depending upon the temperature and this temperature is a recrystallization temperature as we discussed and this recrystallization temperature is normally 0.5 to 0.6 times the melting temperature of the material in Kelvin. So if you take about the iron or so about 1000 degree or so in case of steel and low carbon and medium carbon steel, in case if it is the formed above that than it is the hot forming and below that it is a low which will low I mean hot working or it is below that it is cold working.

Now, the thing is that many for many metals if you look at the recrystallization temperature it is not very high value. So, even the room temperature may be the hot working temperature, so you have to keep in mind that so in those cases that is cold working itself or hot working itself if you do at the room temperature. Whereas, in the case of iron you have to go for very high temperature and even if you go at 600 degrees centigrade its type of cold working, so that depends upon the material properties. Now, we need to know about the deformation processing system.

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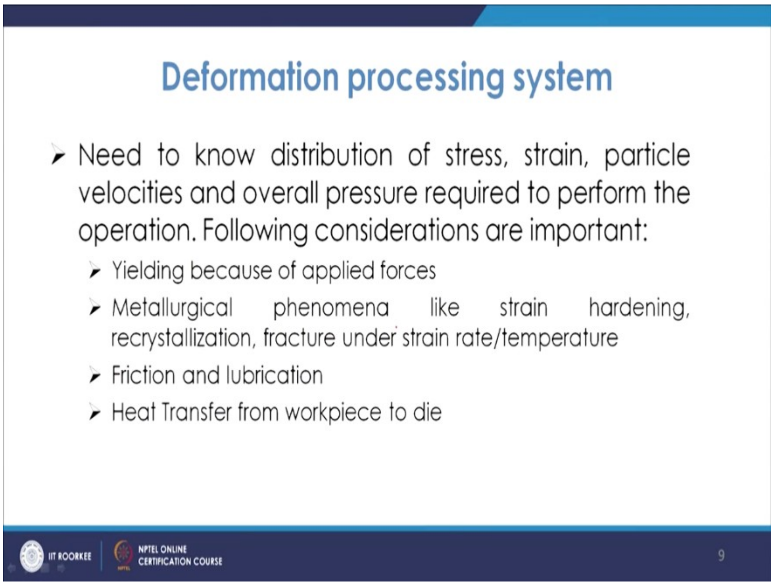

So, what is the deformation processing system? So when we talk about the deformation processing system I mean it is a domain where your you have to pin point that what you have to study. Now when we talk about any deformation process what is typically there in a typical deformation process.

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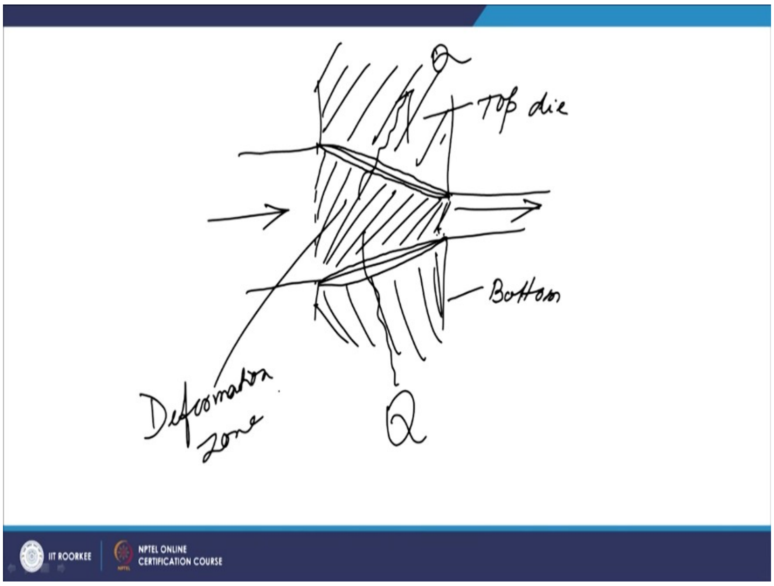

So what happens that you have suppose such is the material such is the this is the die, so this is your top die and this is your bottom die and you are allowing some material to suppose go there and then it will go through it.

So, the thing is that what happens that this is the die and surface workpiece, so basically this zone is the lubrication zone, lubrication zone is developed here and similarly you will have lubrication working here itself. Similarly you have you know if you look at this zone this is this zone is basically the deformation zones. So this zone is known as the formation zone.

So this is the zone known as the deformation zone and this is your top die, similarly this is your top die and from here is a material is pushed and it comes like this. Now, the thing is that you need to concentrate on this zone and you want to find what is the stress, strain value velocity, what is the pressure required to deform all that you need to know.

Now, the thing is that when the material which you put is heated here so what it will do it will be releasing the work heat to this side. So basically there are many things happening in this zone and this zone is known as deformation zone.

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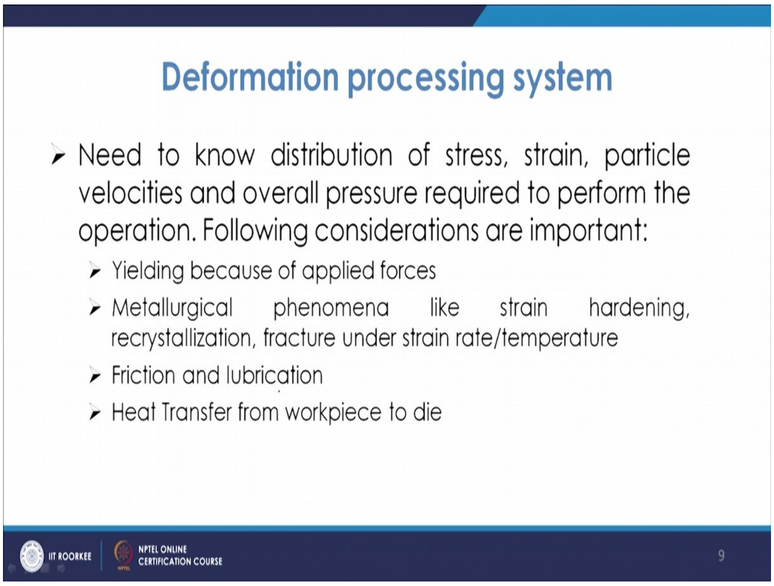

So, in this is what we do in the case of deformation processing system, we need to know the distribution of stress strain particle velocity and overall pressure required to perform the operations. So, that is what we need we need to find the stress strain velocity and also the what is the pressure required to perform the operation, so based on that you can design the equipment so that you will be doing the successful forming operation.

Now, for that there are many things many consideration which are important there will be yielding, because of the applied forces in the deformation zone. When you are applying the forces then the materials yields when the stress value will reach beyond or equal to the yield point value then the material will yield. Similarly there are many metallurgical phenomena like you have strain hardening recrystallization fracture under strain rate and temperature.

So you have there are many parameters which are to be taken into account like depending upon the temperature condition you may have these strain hardening effects, you may have the recrystallization effects, also the fracture will be there under the strain and strain rate and temperature. So, they are strain is there, strain rate is there and temperature these parameters affect basically the fractured behavior in one sense. So, you need to have a I mean consideration of all these effects you have friction and lubrication going on so as we discussed that at this place when it enters into it, there will be friction forces and then you are applying the lubrication system here.

So, this also need to be and tackled when we talk about the deformation processing system, we need to see that how much friction is generated friction forces are required so that the material goes into this and then the slowly it will come at and then it comes out here. So, basically when we talk about the different type of processor we will see that how this friction forces play the role in pushing the system into in between the rolls and between the dies and then how the friction is you know generated. What will be the frictional forces amount because depending upon the reaction amount you can have the frictional amount, what is will be the maximum friction or minimum friction which is required. So, that is another consideration which has to be kept in mind in such cases.

Then there will be heat transfer from the workpiece to the die, so that is what we have seen that your heat will be transferred from this side and it will going into the die. So from the workpiece it will go because it is intimate contact with this, so heat will be transferred so all these considerations are required to study about the deformation processing system.

So, as we discussed about the different processes we will see that how the deformation goes, which zone you have to confined yourself to study about the processes and to find further the value of pressure or stresses required to deform, velocities at different points, strain or stress at different points or so. So this is how we analyze the different type of forming processes that we will do in our coming lectures.

Thank you very much.

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Lecture-03
Behavior of Materials

Keywords: Material's behaviour, Hook's Law, Strain, Elastic and plastic deformation

Welcome to the lecture on Behavior of Materials. So, in this lecture we are going to discuss about the different behavior of materials because ultimately it is the material which is subjected to the different types of forces, especially when we talk about the metal forming processes, we have a mechanical I mean forces like compressive tensile or shear or bending or so. So, when you to know that, when the forces are acting on the body, how it behaves

Now, materials are of different type and it will behave differently. So, they are to be analyzed. Basically when we go to analyze the forces because once the force is acting on the body, then the body undergoes certain formation. At the same time you know, if you look at there will be internal resistance against the deformation are against the forces. So, at different place you will have the different you know the forces of different nature and in that way you have the definition of stress. And then that is measured in terms of strain or so, depending upon the materials property. So, in this lecture we are going to discussed about those issues.

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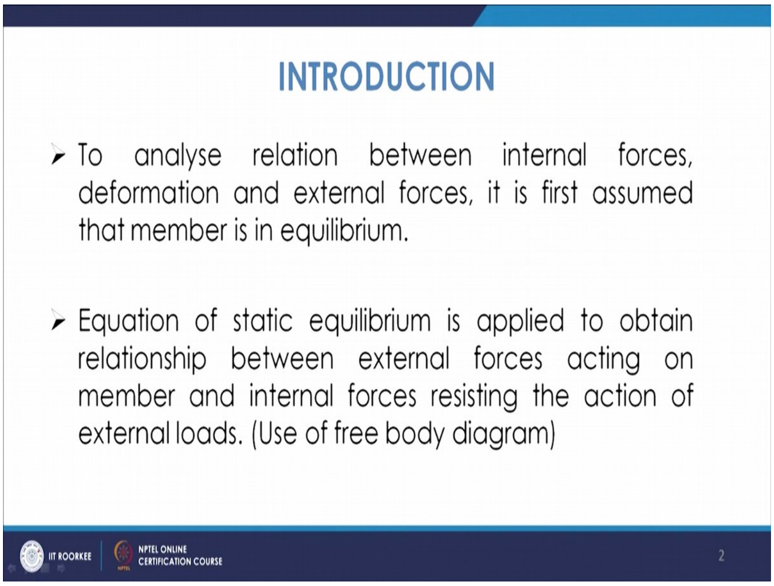

So, to analyze the relation between internal forces deformation and external forces it is first assumed that is the member is in equilibrium. So, that is what I mean the purpose is to say that when we are going to analyze the forces, analyze the member which is under the action of various type of forces like internal forces; then deformation and external forces. Then first of all you assume that the member is in equilibrium. And for that equation of static equilibrium is applied to obtain the relationship between external forces acting on the member and internal forces resisting the action of external loads.

So, that is how basically when you have a body and when you apply the loads, in that case you have; in the inside of the body there will be resistance to it. So, if you look at if you remove certain part and if you look that how that is represented by, you know you have other system of forces because that is being basically you know resistant.

So, in that case that is how you see that equilibrium conditions are maintained. In that case you see that this is the set of equation which is or the you know force balanced equations or that way. So, that that's what it is. So, ultimately you draw a free body diagram. You isolate certain part of the diagram. You see that what are the forces acting on one part and from the other part you have the balancing of and then that way there is a free body diagram concepts. That is how the equilibrium concept is kept into the picture and basically thats how you analyze the situation.

So, do you must know that what is the distribution of stress. Basically we know that in normal terms, we are mostly concerned with stress. How to measure the stress? So, because everything we quantify in terms of stress values, we say that this much of stress is generated. And the material can sustained up to this value of a stress. If the stress value reaches beyond certain limit, then the material will may fail. So, the stress is nothing, but that is basically in a very rude sense and basically the internal resistance which is acting that on a certain area, when we defined that's the stress. Although stress is defined more appropriately in terms of tensor and that is defined at a point.

So, normally what we mean to say that when you apply the forces, then there will be internal resistance is acting and you know because of the forces, you will have deformation or you will have the strains acting. And once you a strain measured then basically you calculate the value of a stress depending upon where you are. If you are in the elastic region, in that elastic region basically the Hooke's law is valid. If you go into the plastic joint Hooke's law is no longer valid. So, you have other type of relations which talk about

So, basically what we do is normally, you measure the strain. Strain is the measurable quantity not the stress. So, strain is measured and once you get the strain distribution or strain at the points, then depending upon the I mean other relationships like if you are in the elastic region, then we know that stress is proportional to strain by a constant known as Young's modulus of elasticity

So, once you know that strain once you know the Young's modulus of elasticity, you can find the values of the stress. So, also the expression for the stress will be substituted into the equation of equilibrium that's what we discussed in the last slide that you have to have the member to be in equilibrium. So, once you get that expression then you put that into the equation of equilibrium and that way once you solve these equations you get, the quantity which you are interested into.

So, for the analysis of these conditions, you need to have certain set of assumptions. And the assumptions are like the body should be continuous, homogeneous and isotropic. So, what is what you what means bodies continuous? So, we can write that when we talk about the continuous bodies.

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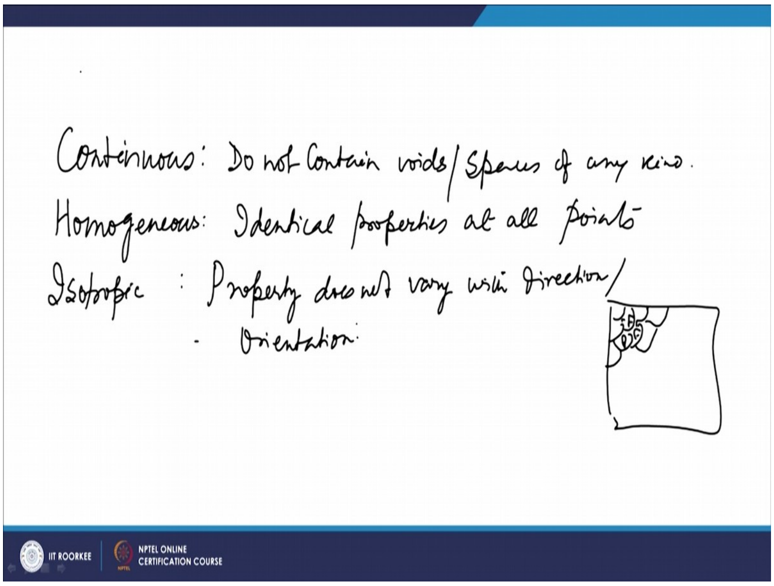

So, the conditions are the body should be continuous. So, what are the continuous bodies? The continuous bodies are the once which does not contain any voids or spaces of any type. So, they are they do not contain voids, or spaces of any kind.

Now, what is what does that mean when we try to analyze? When we are trying to analyze the equations of equilibrium and we try to analyze the behavior of the material? We assume that the material is does not contain any void. Now this is not a very practically you know correct assumption, but for the analysis point of view you have to assume it; because when we cast the material or any material there are chances that there will be voids. There may be voids in terms of casting defects or there may be any space in between.

So, basically when you apply the stresses, you cannot assume; [vocalized- noise] if suppose the crosses an area is same or. So, you assume that there is no void. The property is completely so, that matter will come later. So, you assume that the material is continuous. There is no void. So, that's because once there will be void and that part will not support any load. So, in fact, if it has void, then the measure value of a stress has to be more because the void will not contribute in bearing that you know resistance loading or resistance to the deformation. So, ultimately the effective area will be lesser. So, your stress value will be larger in other cases in other areas. So, in that case, we will have to have that kind of assumption. So, we assume that the material is continuous.

Next is that the material is considered to be homogeneous. So, what is homogeneous? So, homogeneous means it has identical properties at all the points. So, when the material is supposed to be having identical properties at all the points, then it is said to be homogeneous. Otherwise it will not be homogeneous. So, you assume that now this again is not consider to be practically completely true because when we talk about the materials at microscopic level. It is composed of a large number of constituent phases and they have different properties

Suppose ferrite and you know austenite or ferrite and perlite or so. So, everything has the different properties, but you cannot take it. If you if you do them the macroscopic analysis, then it is possible to see that, but otherwise when we do the metal forming analysis in that case, you have to assume because we are talking on the macro scale. So, and that scale you have to assume that because once you take certain volume. Suppose you are talking about certain part. So, you have a all the different type of phases. So, you may have ferrite perlite and all that. So, in this so, this may be ferrite, this may perlite, this may be ferrite or so.

Now, when we talk about the macroscopic deformation, then this reason is nothing. You might have, if you take about certain point even point range or so, you may have very large number of these you know phases or grains. So, that's why we do not take these you know properties of the different individual you know phases or the grains into consideration and in that case we take it as I am you know the homogeneous one because if you take the macroscopic images if you take certain regions. So, you will have identical type of you know properties. So, that's why you take the identical properties at all the points because we are analyzing at the macroscopic level.

Then the third point which is important which all the assumption, which is important is the isotropic; the isotropic nature means it has you know, the property does not basically vary with the direction or orientation. So, property does not vary with direction or orientation. So, what happens that many a times as we discussed that the you know anisotropy develops in the case of larger you know when we do the forming to a larger extent. Now we assume that the property does not vary with the direction or orientation and based on that we analyze. So, end this assumption has to be taken because otherwise you will have to an isotropic taken into consideration.

Now an anisotropic develops in certain cases because when we try to orient the grains in a particular direction. So, certainly the isotropic properties are little bit you know changed. But then we assume that the material has the isotropic nature. So, these are the three assumptions and do this we keep in mind while dealing with these situations, while dealing with the metal forming analysis and while, we deal with the properties because we have the many constitutive relationships and you have to have the you know use of these relationships at different points or in the different conditions.

So, these assumptions are basically require to be valid, as we told that you have different; you know grains the orientations may be they different crystallographic orientation may be different. So, practically it may hamper, but when we talk about the macroscopic scale. We assume that these properties are basically you know these assumptions are required to be valid.

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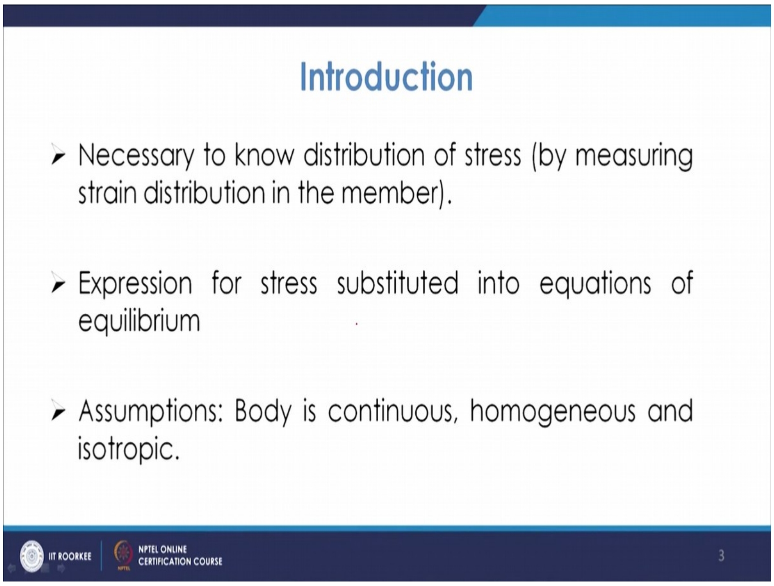

So, this is the assumption when we deal with the metal forming technology.

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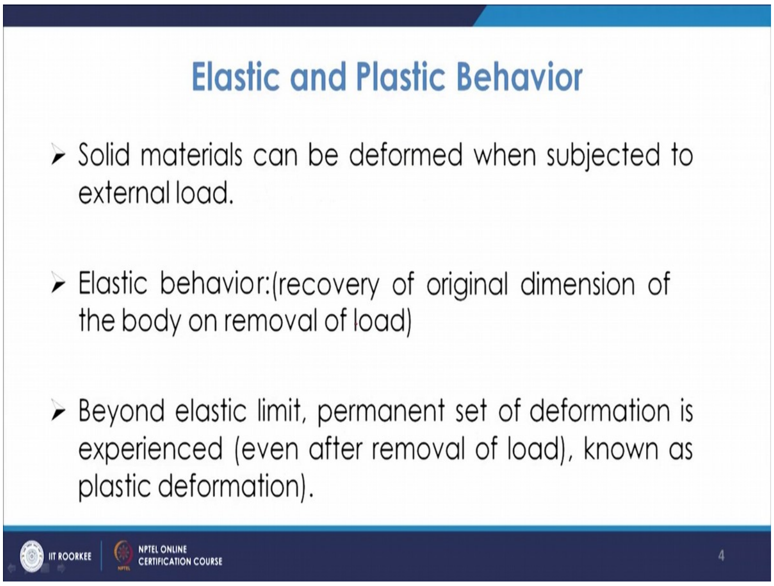

Now, we talk about the elastic and plastic behavior of the material. So, as we know that the material in this case, in the case of metal forming technology, we talk about the forces which are applied on the body and the force will create or cause the deformation in the body.

Now when we apply the load or the force, then the deformation may be in the elastic range or it may go in to the plastic range. So, the two way the material behaves; one is the elastic behavior and another is the plastic behavior.

So, we all know that what is elastic behavior. Basically when we are going up to the elastic limit and I mean in a brutalist or in a rude way, we tell that because in the elastic when we go to certain range or when you go to a stage where there is deformation and when we remove the load; in that case the material comes again or specimen comes to comes back to its original position.

So, that is the elastic condition. So, that is what recovery of original dimension of the body on removal of load. So, that is your elastic condition and elastic behavior of the material. And then and once you go beyond certain limit, then there will be permanent set or deformation is experienced and that is even after the removal of load. So, even if you remove the load then it is still that permanent set or formation is remaining. So, that is known as the plastic deformation.

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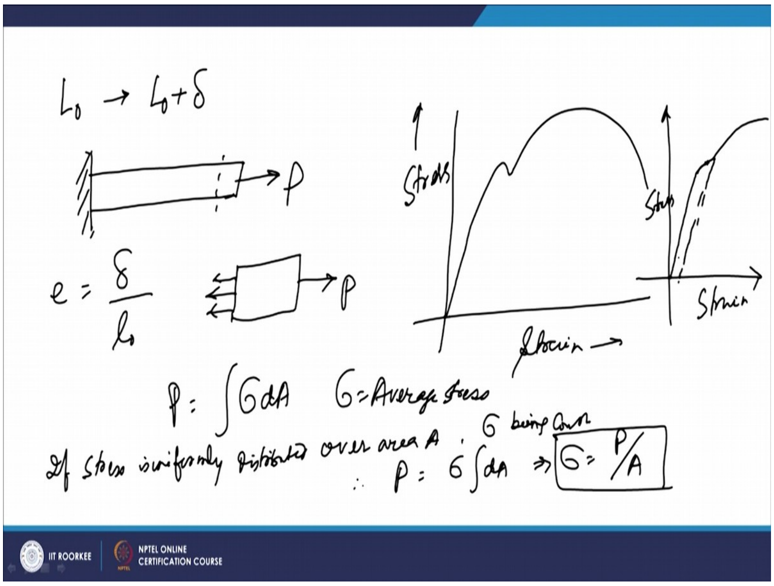

So, basically when we talk about the elastic ranges. In those cases, what happens that you know you to talk about different you know tensile stress strain curve. By this you can have the you know elastic and plastic behaviors representation; so normal typical tensile stress strain curve. So, this is your stress and this is your strain. So, what happens that we know that once you go up to this point; then if you leave, then it will be coming back to its original position.

So, there will not be any permanent strain. Otherwise once you go beyond that, then there will be permanent set. So, depending upon that you have permanent offset, so some percentages kept and then after that if you go in these reason, then there will be there will always be some permanent deformation remaining. So, so that is your plastic range and one is your elastic range

Now, when we talk about the value of stresses or strains, the average stress and average strain is normally calculated. And as we discussed that we have elastic limit so up to elastic limit, you have elastic behavior and beyond that you have the plastic behavior up to; in the plastic region you have you know wherever the stress is proportional to strain you have the Hooke's law valid and after that Hooke's law is not valid because stress is not linearly proportional to strain. So, the slope of the curve will be the Young's modulus of elasticity like that we proceed and otherwise you have the you know. Once you go beyond that you will have different set of relationship which basically come into picture.

So, when we talk about these stress or strain. So, in those cases what we see normally stress is nothing, but when we apply the load, then at any section you have the resistance acting. So, so for example, suppose you have a member and this is the member and then you apply a load P to it. So, then what happens earlier you had the length of initial length was ❑□ and then that changed to do ❑□ + .

❑□❑□ .

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So, that is your change in length. So, you had the initial length. Suppose of up to this and then this is a length that is delta which is removed. So, basically delta is the change in length and then, we define the engineering strain e as change in length upon original length. So, that is what it is. So, this is your engineering strain what we defined that to be.

Now, what happens? As we discussed that when we talk about the calculation of stresses, then what we do is you have the element and this element we are stretching with the load P and then, now what we see is that there will be resistance against these forces at this particular section. Now this is the, here the resistance which is acting on this area. So, that the load is per unit area that is; so basically this which is acting. Now the average stress multiplied by this area. Basically average stress is acting on this area. Now stress may be different on all the different you know points

Now, if the stress is normal to the cutting plane and so in both cases, if sigma be the stress. In that case you can write that P will be summation of d A.

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Now if so here sigma is the average stress. We call the sigma as the average stress and if the so, sigma is average stress and that is you know integrated to that area that should be equal to the P and if stress is uniformly distributed over area A.

So, in that case and if say sigma is basically constant in that case, so what will happen? And if sigma constant, so what will happen? P will be sigma d A; so, sigma being constant, so sigma will be . So, that is what we normally tell

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Now, stress normally will not be uniform over the area A. So, basically this is the representation for the average stress as I told that when we talk about the stress with that we will see in the latter classes. So, it will be defined at the points. Now if you we assume here, this is normally distributed over the area A and in that case we have taken dA as a as sigma being constant, but then if you have the different points where you calculate the stress at all these different points and depending upon its orientation or so, the stress is calculated.

So, what we see that you have the for the stress to be uniform absolutely. Your every longitudinal element must be you know strained in the same amount because as we know that we measure the strain and based on the strain you calculate the stress. So, there will have to experience the same amount of strain and then you use the Hooke's law to find the amount of stress what we get.

Now, we know that in the range of the elastic limit, your Hooke's law is valid; I mean up to the proportionality point and then in that case you calculate the value of these stress by using the you know the value of the you know Young's modulus of elasticity, the property of the material and that way stress is calculated.

Now if you talk about typical stress strain curve, now you have different results as we told. You have the elastic region where the Hooke's law is valid, then you have you know, further once you go beyond the elastic region. In fact, when we look at the you know stress strain diagram, you will go up to certain point where it is the linear variation of the stress and strain. So, this is where the Hook's law is valid and then there will be some non-linear variation

So, up to here also if you leave, then the material will come back to its original position. And then after that you know after that when it goes, so then once you leave from this point latter on then it goes into the plastic zone. So, you have a zone of linear one where the stresses so, this linear zone on when we talk about. Now here the slope of the this curve in this zone that will tell us the value of the Young's modulus and then you have a zone where still it is elastic, but you know that that is not the linear. This linear region is valid and then you reached at a point.

So, then you reached at the point and here if you go, then you have you are going into the plastic range. So, you have some plastic deformation which is known as the yield point where you have some you specify certain strain and so, that to you say that now it has it has yielded and after that it is going into the plastic region. So, that way you have elastic as well as plastic deformations defer plastic behavior of the material under the different conditions.

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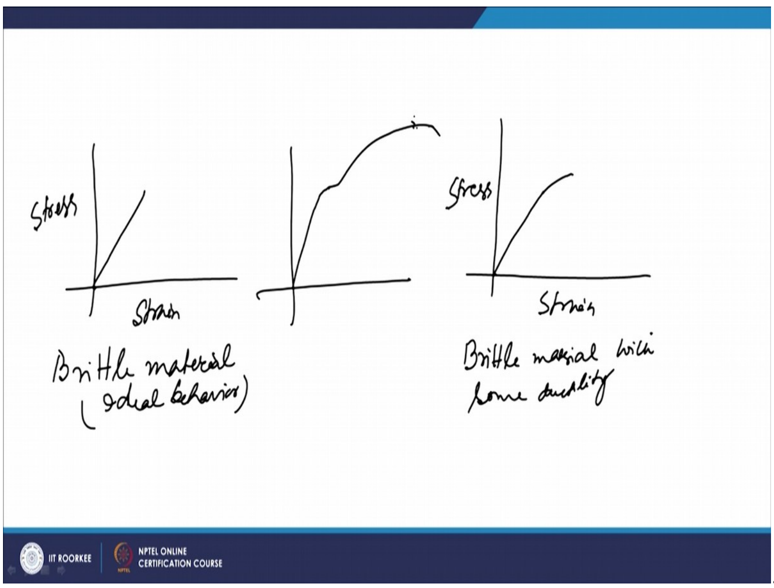

Now, what happens this is normally typically the stress strain diagram of a ductile material. So, if you talk about the different behavior of materials, now the materials may be different and it will behave in a different manner. Now if you talk about the ductile materials, as we see that in the case of ductile materials, you will have appreciable deformation whereas, if you talk about the brittle material then in the case of brittle material, the it will go like this and then it will break. So, ideally brittle material will behave like this.

So, if you have stress if you have stress and strain like this and the material will fracture here. It we will not have any appreciable deformation in such cases. So, this is brittle material. This is ideal behavior. Now if the brittle material may have some amount of ductility. So, in those cases, you will it will go like this and then it will be somewhat showing something. So, it will be the brittle material with slight amount of ductility.

Now, the thing is that in case of engineering applications, you need the ductility because when we talk about the brittle materials, normally they are said to be failing in a catastrophic way. So, as you see that in these cases, as you increase the load the stress value is increasing and at this point, it is simply going under fracture. Whereas, in the case of ductile materials, there will be yielding and then the cross-section will go on decreasing and then slowly the material will fail.

Now, in the case of you know brittle materials, what happens that normally you have the presence of cracks or notches or so. Discontinuities are there because of which the cracks are there and when you are putting the load; when the load is increased, then the stress will be increasing and at one point of time, it will fracture. So, it does not you no give any indication.

Now in most of the cases whenever there is failure, whenever you have the presence of notch or stress concentration regions then in both cases at that localized point. The stress value becomes higher the localized build up of this stress will be increasing. And then wherever you have stress concentration the crack may be formed and then, the crack will try to propagate and basically in the case of brittle materials.

When the presence of crack is not that way even essential because if these cases the yield strength and tensile strength of the material are normally same. If you look at this stress strain diagram, here in these case says the yield strength and tensile strength is same whereas, in the case of which you look at the normal ductile materials which will go like this. So, this will be your yield strength whereas, this will this will be your tensile strength.

So, in these cases yield strength and tensile strength is different whereas, in the case of brittle material your yield strength and tensile strength they are same normally same because the material starts you know break their itself.

So, you know brittleness otherwise is you cannot say that any materials in terms of absolute brittleness or ductile you know ductility because again the material behaves in a brittle manner or a ductile manner that depends upon some other conditions also and one of the one condition is the temperature. So, if you see the, if you increase the temperature and sometimes the brittle materials also behave in a ductile manner or if you decrease the temperature the ductile material behave in a brittle manner.

So, so that's why there is transition of these properties also ductile to brittle that is known as ductile to brittle transition behavior. So again that is one property because you need to you need that the material should behave, now material should fail in ductile manner. And if the temperature is very less in that case the material may fail in a brittle manner

So, if it is fail in a catastrophic manner, then many A times there may it may lead some dangerous consequences. So, that is normally seen in the case of the places where the sub zero temperature, suppose you go to the region of Kashmir or in the very high altitude area where the temperature is very very less. And if the material properties not altered in that case, the normal materials may which are otherwise ductile. They may behave in a brittle manner and they may not so appreciable deformation.

So, basically the main thing is that when you have a appreciable deformation, you know that it has yielded. So, so you can you know take precaution that itself, but in the case of brittle materials; you cannot take precaution because its yield strength as well as tensile strength both are the same. So, so that's how you must be knowing. So, that's how these two materials behave while they are being subjected to the loading conditions and while they are way being failed.

So, again the material, I mean you know they may one is which is you know ductile or which is brittle in the tension, that may be ductile under the hydrostatic condition. So, hydrostatic condition is another condition which may be you know many times may be applied or maybe you can say that this is hydrostatic condition where because. So, so that way you have to analyze the situation you have to see that how the material has to be behaved.

Thank you very much.

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Lecture – 04
Failure of Materials

Keywords: Elastic deformation, Plastic deformation and Fracture

Welcome, to the lecture on Failure of Materials. So, as we talk about the materials in the case of metal forming they are subjected to loads and under that action they are being deformed, there is plastic deformation, there is failure. So, the failure maybe in different ways and in this lecture we are going to discuss about the different modes of failure under which the material may fail and we will be talking about them in a little bit of detailed manner.

So, the machine elements which are under the action of loads; They are subjected to you know various kinds of you know stresses and in that process if they fail or there is break down so in the following manners there are basically categorization of this type of fail in three ways.

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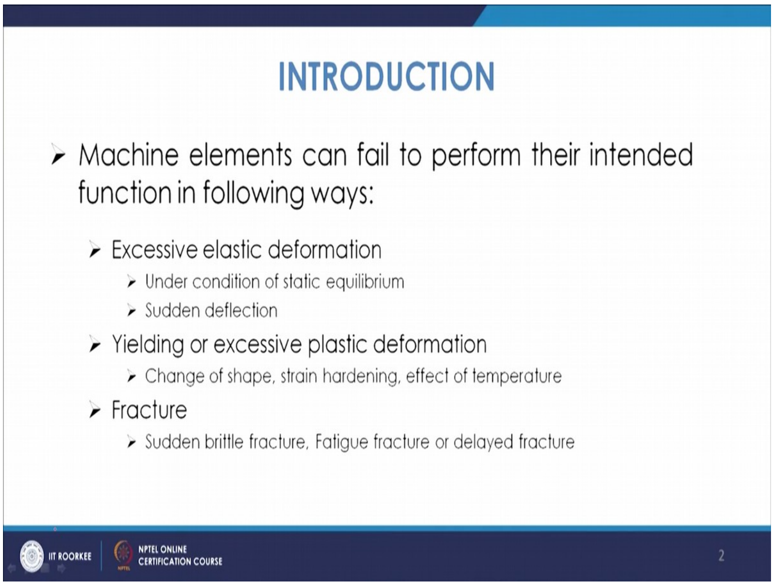

One is excessive elastic deformation So, when you have excessive elastic deformation you will have the two conditions that is under the condition of the static equilibrium you have and then one is the sudden deflection that is when we put a load suddenly impact load or so, buckling is the example of such type of failure.

Then you have also the yielding or excessive plastic deformation because here when we are putting the load and then it when it goes in the stress value reaches beyond the yield point, then you have the plastic deformation starts and in that case that is also a kind of failure of the material. And in that you have the effects like you have the there is change of the shape of the material, the material you know the element of the material changes its shape.

Similarly, you have the parameters which are important are the strain hardening and then you have the effect of temperature all these are there in that. So, that we will discuss and then you have the fracture also so, the fracture again that is categorized under the sudden brittle type of fracture, then fatigue fracture or the delayed fracture.

So, these are normally the different you know ways by which the machine elements fail and we will see that how you you will discuss about it.

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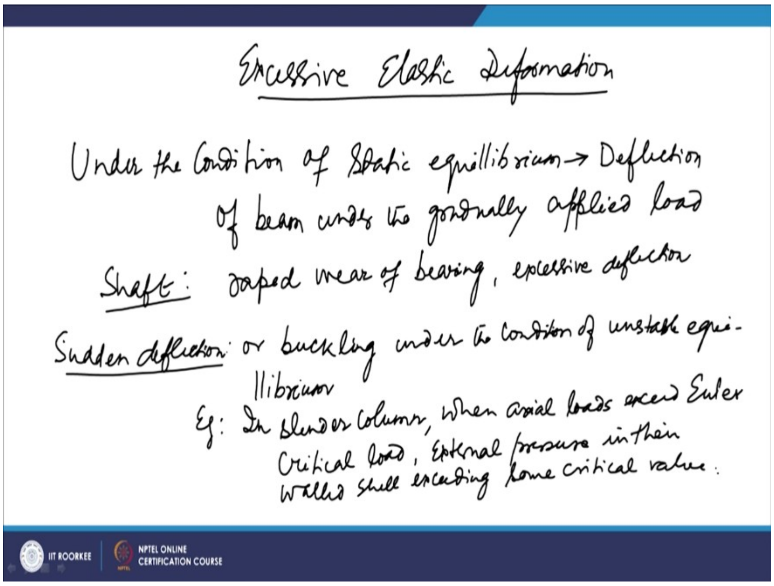

So, the first case is the you know excessive elastic deformation and in that there are two ways. So, if you talk about excessive elastic deformation so, in that there are two things one is under the condition of static equilibrium. So, that is under the condition of static equilibrium and you can have such you know cases, where if suppose there is a beam and you have the load. So, so there will be deflection in the beam under the gradual applied load so, that this is like deflection of beam under the gradually applied load.

So, in such cases what happens that you have to certainly set a criteria that what should be the maximum deflection at which you assume that the material has failed, because here the load is basically gradual and then you have for this actually there may be many conditions under which such fails occur. And suppose you have a shaft and so, shaft is an example of such kind of deflection and under that you have so, that is under certain load and the reason may be suppose you have a shaft.

So, the shaft may be you know too flexible. So, in that case if it is too flexible then there will be a rapid rate of you know wear. So, that is because of the excessive deflections. So, you will have rapid wear of you know bearing, ok. So, I mean if the shaft is there and if you have the deflection of the beam in those cases you will have the rapid wear of bearing. And then excessive deflection and in that case the making parts which are closely making parts so, that may be by you know interacting with each other and there may be damage to the shaft also. So, this is the example of the condition of static equilibrium where you have gradually applied load and there is deflection.

Then you have also the sudden type of sudden deflection. So, one is one of the cases are the cases where you have gradual deflection another cases are the sudden deflection. Now, the sudden deflection you can take the example of buckling or so, where in the so or buckling under the case of under the condition of unstable equilibrium. So, you know normally it happens in the case of slender columns where the length is too large and when the axial loads are basically exceeding the Euler critical load or the external pressure against a thin walled shell which is exceeding a critical value.

So, normally you can say that you can in slender column when axial loads exceed Euler critical load or, you may have also the example of suppose if you talk about the thin walled shell. So, in that if you see that the external pressure if you have thin walled shell and you apply the external pressure then that also exceeding certain critical value; So, external pressure in thin walled shell exceeding some critical value.

So, these are the examples of you know excessive elastic deformation and they are categorized under the you know under the you know condition of this elastic deformation and in these cases the failure will be controlled by the moral of electricity because you are in the electric elastic range. So, the failure will be controlled by the modulus of electricity not by the strength of the material. So, normally what we do is in such cases you try to increase the stiffness of the material and that can be done by changing the shape or by the cross section. So, you must have you know seen the cases like I section or channel section you when you study the strength of material in those cases we talk about these properties, in that we know that by taking the different you know sections where you increase you know its resistance to buckling or so, so that can be the you know remedy under such cases.

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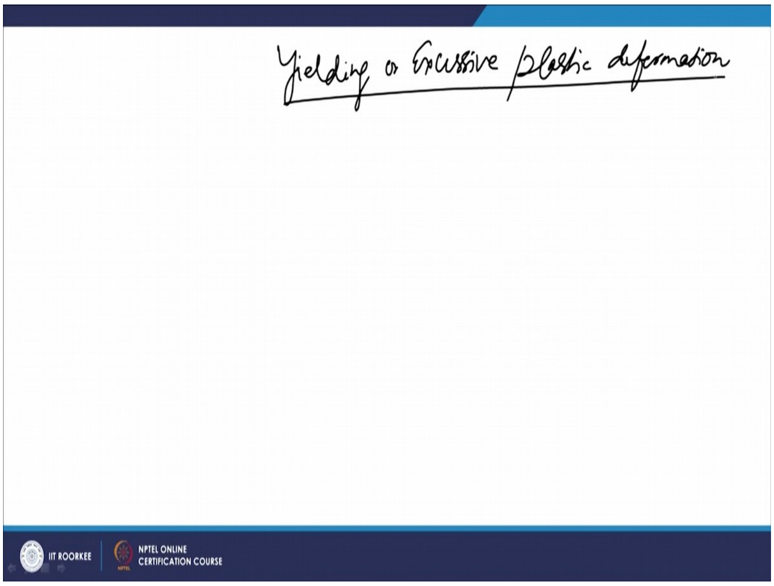

Now, the next the second point is the yielding under excessive yielding or excessive plastic deformation. So, that's what I mean we are going to discuss mostly in the case of material metal forming, normally we are concerned in the this you know type of failure where the material yields and goes under the plastic deformation.

Now, what is yielding? Basically, once you have yielding then that will lead to the permanent shape change. So, now, what happens that in such cases you have the metal for the metal if you if it is done at the room temperature then the metal undergoes strain hardening. So, metal strain hardens as the it defends deforms.

So, you have basically complex state of loading and you have suitable failure criterion is basically set in those cases because you are in the plastic region, you have a set of stress to which has to be you know formed which has to be known and then you have to choose the different failure criterion in those cases.

Now, in such cases basically temperature is also you know important. So, when the temperature is basically you know larger in those cases the strain hardening is no longer exhibited and many a times the metal will continuously deform at constant stress. So, you might have seen certain you know cases like in the case of turbine blades or so, when your temperature is quite high then this strain hardening concept is not valid.

So, in those cases what happens that the material will constantly deform even at constant value of a stress and in that type of deformation is a time dependent type of yielding and that is you know that is a phenomena known as the creep. So, in those cases the stress is not proportional to the strain. So, that is what normally you have the cases in the case of yielding or excessive plastic deformation.

So, once you may have the effect of you if you do it below and also you may we have studied about the hot type of hot forming or you have cold forming. Now, in the case of the temperature being smaller one you have the strain hardening, when you go to the temperature to too high in those cases the creep phenomena comes into picture.

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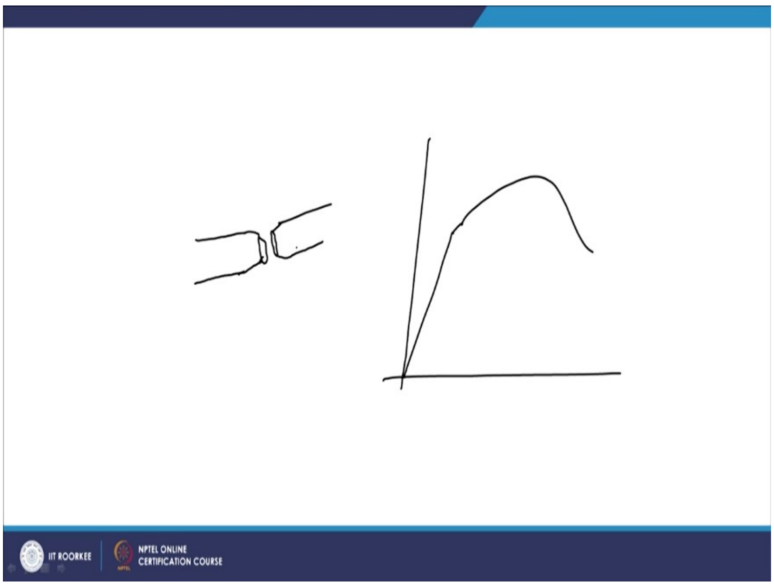

So, we already discussed about the yielding in such cases and for typically ductile materials as we know that in normally you have a the stress strain curve if you look at so, it goes and then it comes like this. So, what happens that this is up to the this point of proportionality and then you wants to you come here this is up to this you have the elastic limit.

Now, the thing is that in the case of the yielding under this plastic deformation if you have the ductile materials and if you are seeing the cross section, the cross section will start changing after the yield points. So, there will be deformation started and in this cases you have cup and cone type of structure fracture because at the point of yielding. Once you look at this specimen so, it will somewhat you know it will be decrease in the dimension and then so, the this way the two portions will be look looking at there will be decrease in the dimension at the point and then slowly the they will be getting a part and then they will fail.

So, such is the mode of the failure in the case of ductile materials. We already discussed that in the case of brittle materials there is no such warning. So, that basically you don't get any warning and the cark crack initiates and then it fractures. So, that is the case in the case of brittle materials. The thing is that when you have the temperature changes when I have or you have the loading rate of loading conditions changes the mode of the failure becomes different.

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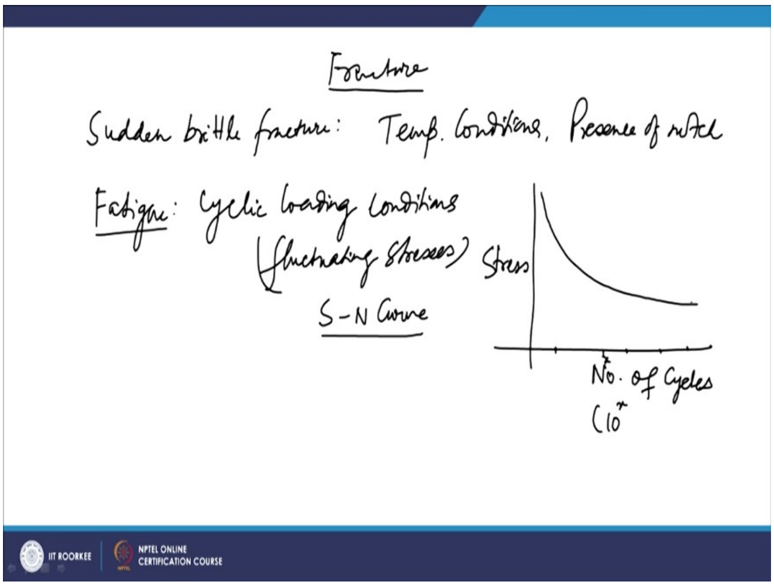

So, the third type of failure I mean third case which we were discussing it was so, if you look at we were discussing about the fracture and the fracture can be you know seen in three ways. So, if you talk about the. Fracture mode now, the metal fractures in three general ways one is the sudden middle brittle fracture. So, that we know that when we talk about the brittle materials they do not give you any indication. So, did not give you any indication and there with without any appreciable deformation the fracture.

Now, this is also true for some of the ductile materials also. As we discussed that it's not a you know you cannot say the concretely something is brittle or ductile depending upon the conditions it the failure mode may be brittle or the ductile. Now, even the ductile material may behave in a brittle manner there may not be any appreciable deformation prior to fracture. So, the case may be because of the you know the conditions of temperature. It may be because of the presence of complex state of stress.

So, in those cases you know there may be a cases where if you have suppose a notch so, may be because of the temperature conditions or presence of notch. So, even presence of notch that leads to you know presence of complex state of stress and in those cases also your sudden brittle type of fracture you know mode appears then another mode of failure under these fracture is the fatigue.

So, many machine parts are failing because of the fatigue. Now, what is fatigue? Fatigue type of failure is the one where the machine basically element is subjected to the fluctuating type of load. So, when the failure is because of the fluctuating type of float load that have normally happens in the machine shaft source or. So, which are moving as a load acting on them. So, on the cyclic cycling I mean there is a cyclic type of a load or alternating cyclic loading conditions. So, under that if the machine fails if the machine element fails then we tell or we also tell that there are fluctuating stresses or alternating stresses.

Now, because of that when there is you know fail in the element then we call the it to be the fatigue failure. So, what happens in these cases normally the stress is in the term of cycle, you will have in one cycle it will be under the you know in the positive side in the in the cycle it will be under the negative side. So, it has the alternating type of a stress development in the material.

Now, in these cases what happens that a minute crack will be normally so, you know starts at a localized point and normally that is in the form of notch or so, at the point of the notch or wherever the stress concentration is larger at those point say minute point crack will initiate and that crack will basically propagate as the cycle progresses. So, that will be basically progressing gradually on the over the cross section of the member. So, so that way once it progresses and then it may go over a whole cross section and then material fails.

So, in these cases in the case of fatigue failure there will be no visible sign of yielding. In fact, at the average stress so, in those cases and these stress values at which the they fail they are well below the you know the entire tensile strength of the material. So, that is why when you do not expect that they will be failing at these because you feel that it will go up to the entire strain tensile strength of the material, but they actually fail well below that in the case of you know fatigue.

So, in these cases basically you designed based on the empirical you know relationships or so and these failures basically for that normally what you do is they are represented by you know S-N curve. So, they talk about the stress and the number of cycles. So, there will be this will be the stress value and this will be the number of cycles. So, this number of cycle will be you know 10^x. So, this will be x basically 10 raised to the power. So, you know this will be something like 10^1, $10^{1,2,3,4,5,\vee so}$. So, like that you know you will have cycle up to which the material can sustain and this value you have stress.

So, if the stress value is larger, then in that case the material may sustain very less number of cycles and if the stress value is the smaller then the material may sustained for larger number of cycles. So, in that case the endurance strength is basically you know set. So, this is known as the endurance limit, the stress value at which it can go for finite infinite number of cycles.

So, basically it is where I mean it is the large or lesser value and if the stress value is not going beyond these then material has very high you know life and if it goes more than that then you can see that they have they will sustain for lesser number of cycles. So, this way the that the fatigue failures are they are they are characterized or they are basically presented they are interpreted for the materials which undergo such kind of failure.

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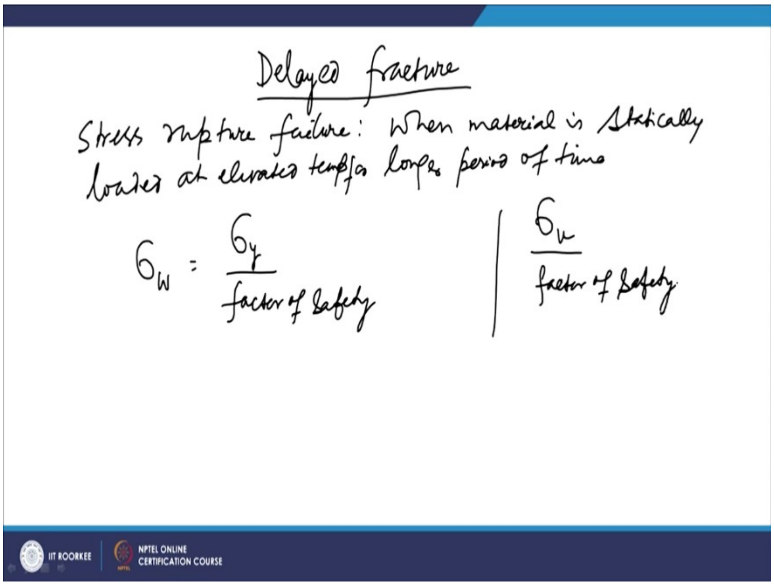

Now, there is another kind of fracture and that is delayed fracture. So, you know I mean normally the stress rupture failure is the example of such failure. So, stress rupture failure; now, in this what happens when the material is you know statically loaded at elevated temperature and for longer period of time.

So, when material is statically loaded you know, once you are statically loading the material at elevated temperature or longer period of time so, in those cases I mean for a longer period of time. So, in those cases these stress rapture failure occurs and depending upon the value of the stress and temperature there may be yielding prior to fracture.

So, if the material has the stress the material under the stress conditions stresses in those cases because when you make the material the stresses are there inside the material and in that when you are deforming the material under statically, under the limited elevated temperature for longer period of time. So, in those cases these failure occurs even at a smaller value of the stress value.

So, that is why what we see is that the material fails at the lower value of its stresses. So, such failure is known as delayed fracture. So, in those cases you have to see that the material does not contain more value of residual stresses. So, that may lead to the early failure of the material because of the stress rapture failure.

Now, the thing is that we need to design these materials to you know to safe guard against these failures. So, normally what we do is we normally define I mean design based on the factor of safety. So, you have working stress or workable stress and workable stress will be based on the what is the yield stress and then you may you may have the factor of safety.

So, this is based on you know the factor of safety you have to have the factor of safety, so that the workable stress which you which is a you know which is coming it will be will yield strength divided by the factor of safety

$$\sigma_W = \frac{\sigma_y}{Factor\ of\ Safety}$$

or you may have the that may be based on the yield stress or that may be based on the tensile strength. So, ultimate tensile strength and then you may have the factor of safety.

$$\frac{\sigma_u}{factor\ of\ safety}$$

So, these things are required to be you know understood that you have to design the material you have to have the material and you have to see that they fail in what conditions and how they will be behaving whether you will have what is the condition of the temperature, what is the condition of the you know rate of loading. So, that also basically affects the type of loading of type of you know failure of the material.

These type of delayed fracture or a stress rupture fracture failure they are also occur n in the presence of the hydrogen sometimes in the case of steel, when there is hydrogen that time also such fracture occurs. So, that needs to be kept in mind that sometimes you we control the hydrogen, so that the stress rupture failure can be prevented. So, that is about these cases.

Now, coming to the factor of safety; so, we understood that you provide the factor of safety because when you are going to use for the critical applications so, in those cases you have to have adequate value of the factor of safety. So, that the stress value does not reach you know to that value to that alarming value where the material fails and in that manner you take larger value of the factor of safety for such critical you know applications.

So, what we discussed in this lecture that the material may fail in different way when we talk about the you know deformation modes you have different deformation modes, you have many parameters which affect these mode of failures. You have a parameters like how you are how the materials subjected to the loads, what is the temperature conditions, what is the strain rate conditions. So, that basically affects you know the more of failure. So, how the material will fail in that you know such situations and then the brittle materials are there they also can behave in a you know ductile manner, if the temperature is increased.

So, even we know that many brittle materials are also formed when you go for may be increasing by increasing the temperature or if you take the application of the forces in hydrostatic fashion even the brittle materials can be formed by using the these hydrostatic type of forming techniques. So, when we talk about certain type of extrusion hydrostatic extrusion that is based on this concept itself. Then, you have you must know that when you are using same material in the different locality where the temperature conditions are different then they will they may there are chances of failure in a different way.

Then, when you are manufacturing the materials and if there are points of if there are points of cracks or sharp discontinuities that may affect you know the life of the component that may you know impede basically the chances of the failure of the material. So, these are the you know parameters which need to be looked into while discussing about the behavior of the materials, so that the failure can be predicted and ultimately this failure mode.

Basically, why it is important because when we talk in the material metal forming context so in that we basically we apply the stress is stresses and under that application of stress, what is the value of I mean stress state of system how the material will fail what way it has to go and fill the cavities. So, these things are you know utilized while dealing with such you know systems of materials.

Thank you very much.

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Lecture – 05
Concept of Stress and Strain

Keywords: Stress and strain, True stress and true strain, Plasticity theory

Welcome, to the lecture on Concept of Stress and Strain. So, we will be discussing about the concept about the stress and strain. So, in the metal forming aspect as we know that here we deal with the forces which act on the body, resulting into the generation of stresses and strains and so, we must to know that how these stresses are basically represented how these stress is are generated, how you know you nominate them or how you name them like that or even the different types of strains or so.

So, when we talk about the forces which are acting on the body they are normally the external forces which are acting and there are two types of external forces surface force and body force.

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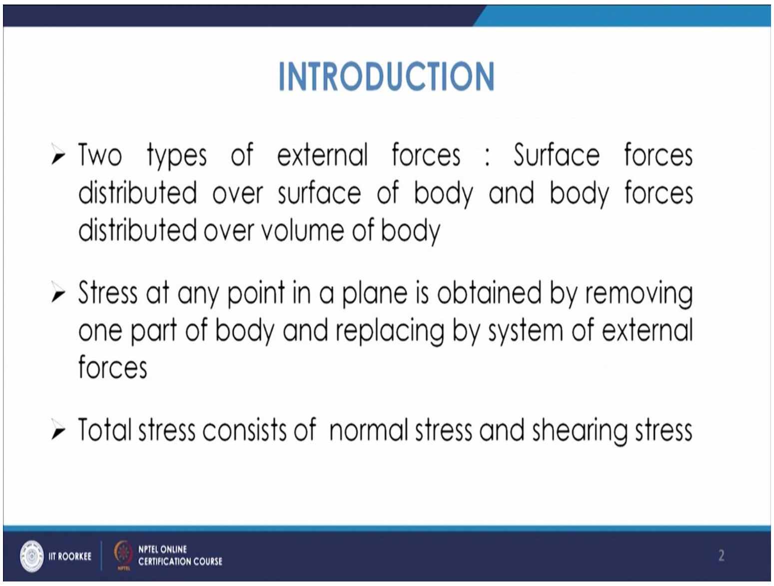

So, when the forces are distributed over the surface of the body, then they are known as surface forces and then when they are distributed over the volume of the body, then it is known as the body forces like gravitational pull or so, they are the example of body forces. Now, the static forces which are occurring they are known as the surface forces.

Now, when these forces are implied on the body and then as we know that for representing the stress what we do is that, we cut certain portion of the body and then we find the free body diagram and from there you find the force, which is you know resultant force which is acting at certain point and in that we try to find the or define the stress in that case.

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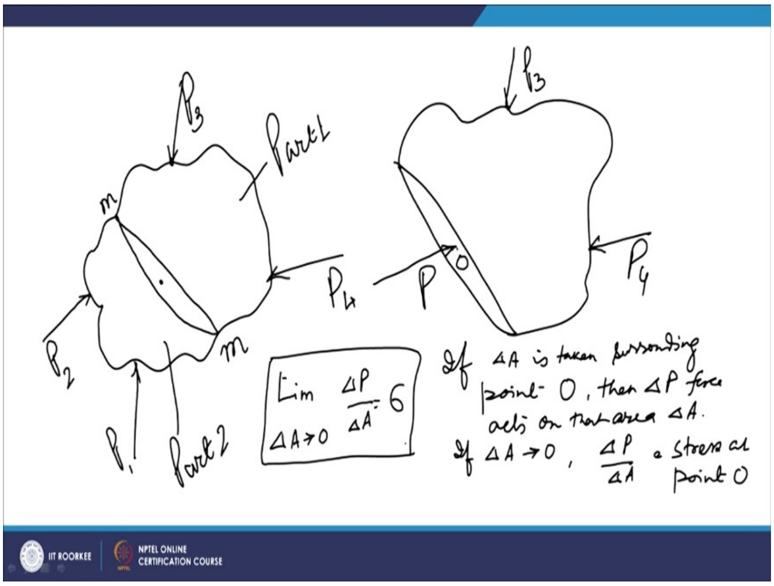

So, what we do is that, if suppose you assume that you have a body on which you have different type of forces acting. So, suppose you have a body like this. So, suppose this is a body and you have different types of forces which are acting on the body suppose this is P_1, then there may be force like P_2, then you have force like P_3, then you have force like P_4.

So, suppose the body on the body you have on the surface of the body you have different forces, which are acting and the body is in the condition of static equilibrium, so, in that case you know you have to find that, what is the stress value at any point on any plane. So, suppose you are defining a plane like this; suppose you define a plane which is suppose m, m and on this plane you want to measure the stress at suppose any point. So, there is a point on any plane inside the body and on that you have to find the stress in that case.

Now, you know that this body which is subjected to different types of forces in the equilibrium so, what you do is, you basically take this part out and try to find the stress at that particular point now if you look at. So, if you take this portion out. So, it will be looking like this and here you have this plane. So, that is what I mean normally it will be like this. So, this is this part which you we have taken so, you have this as P_3 and then you have this as P_4. So, now, this is your, this is the part 1 of the body and this is your part 2 of the body this is part 1 and this is part 2.

Now, what we do is so, you remove this part 1 of the body. So, you are taking this part to this portion. So, this is your part 1 and if you see that this P_3 and this is P_4 which is acting like this and then you are replacing all that force with the system of forces. So, you have you have to find the stress at this point. So, you will have resultant force here and then this is your P which is acting and basically you are representing this P. So, that you know the body is still in equilibrium.

So, it means that whichever point was at whichever location there is no at all any change. So, that way you have taken the free body diagram of this part 1 and you are basically representing by this set of forces so that it will retain all the you know each point in part 2 of the body in the same positions. So, that is how you are trying to have an isolation of this part 1 and in that.

Now, the thing is that if you take this point around this point if you take the very small area . So, in that case and if they are you this ΔP force is working on it is. So, suppose if actually ΔA is taken surrounding point O. So, we are taking this. So, this is your point O at which you want to find the stress. So, and you know that so, so ΔP force acts on that area that area .

Now, when we try to define the stress basically stress is defined at a point. So, when this ΔA will be limiting towards 0 in that case the limiting value of the load when the ΔA is turning into 0, that is known as the stress value at that particular point. So, what we do is if ΔA is limiting towards 0 in that case

$$\frac{\Delta P}{\Delta A} = stress\ at\ point\ O$$

So, that is how you define the stress at a particular point. So, in an nutshell what we see is that you get.

$$\lim_{\Delta A \to 0} \frac{\Delta P}{\Delta A} \sigma$$

So, that is how you calculate the stress at a particular point.

Now, the thing is that when we are in this case we are assuming this plane to be m; m now if the same point is there, but if the plane orientation is changing in that case the stress will change.

If you have any other plane from here passing in a different orientation in that case the stress you know will be different at that particular point. So, what we do is normally we it's convenient to use that stress which is inclined to area over which it is acting. So, that way you can present in a better way you can have the explanation of the stress. Now, that can be further understood by looking at this particular figure. So, what happens that what we see that when you have a force component working now in that case you will have a two types of forces.

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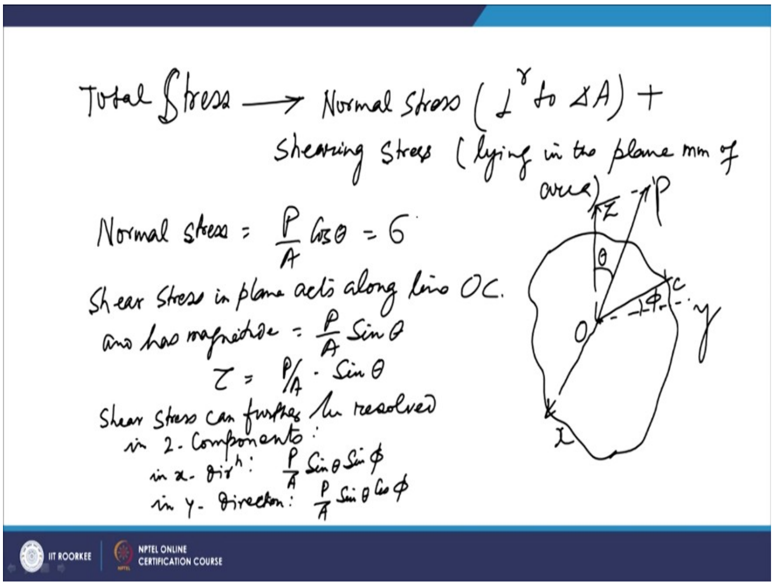

So, if you have total force. So, total force or you can say total stress, now this total stress which is acting you have a two components. Now, one is the normal stress so, that will be working perpendicular to this area. If you look at that you know stress which is generated at that point so, one will be acting perpendicular to that area and another will be acting you know in lying in that same plane. So, so what will happens. So, that will be known as the shearing stress.

Now, shearing stress it is lying in that plane. So, now the thing is that when there is a force which is acting. Now, it will have two components if it has inclination then it will have two components as we know one will be perpendicular to the plane another will be in the plane. Now, again that can be understood by referring to the figures. So, suppose you have such kind of you know body now in that suppose, this is your P which is acting now in that case what happens that, this P if suppose it makes angle theta with the z axis. So, what will happen now in that case $P\cos\theta$ will be the component along the z direction. So, $\frac{P}{A}\cos\theta$that will be basically the stress in the vertical direction or or in the direction perpendicular to the plane and that is known as the normal stress.

So, normal stress so, this is the normal component $P\cos\theta$and then the area is A so, $Normal\,stress = \frac{P}{A}\cos\theta$

So, this is your normal stress. Now, again we the one is the normal component another component will be in the plane basically. So, it will have two components one is here and another will be like this now the thing is that so, so, it will be it is resultant, right. So, now, the thing is that if this. So, basically you will have this how this is how you will have it is resultant. So, this will be the resultant of so, you will have now the thing is this is your this is your in the plane it is working and here you have other you know directions. So, suppose this is your x axis, this is your y axis and this is the component of this P in the plane.

Now, if. So, suppose this is C, now if this direction this component is making angle phi with the y axis in that case now you will have no. So, it is a basically xy plane is a vertical plane is. So, the horizontal plug is vertical plane is the z plane and then you have horizontal plane is the xy plane. So, suppose this components is making angle phi with the y axis. Now, the thing is that this component will be if it is $P.\cos\theta$ this will be $P.\sin\theta$. Now, it has two components it is making $\varnothing$with the y axis. So, $P.\sin\theta \times \cos\varnothing$will be the stress in the y direction and that will be a type of shear stress.

So, if you look at shear stress so, shear stress in plane acts along line OC. So, that is your shear stress so, this is a shear stress this is your normal stress and it is resultant is your total stress. Now, this shear stress has again two components and it has magnitude and has magnitude. So, if the normal stress have magnitude of $\frac{P}{A}\cos\theta$ it has a$Magnitude=\frac{P}{A}\sin\theta$

So, we write shear stress we define as tau and tau will be $\tau=\frac{P}{A}\sin\theta$

So, this is your shear stress. Now, this shear stress again has two components; one will be in the x direction, another will be in the y direction. So, so again for that we have to find those components. So, shear stress can further be resolved into two components. So, it will have two components; one is along the y direction, another is along the x direction. So, along the y direction since this $P.\sin\theta$ is making angle ϕ with that so, in the y direction you will have $P.\sin\theta\times\cos\varnothing$ and in the

$x\,direction:\frac{P}{A}\sin\theta\sin\phi$

and in

$y\,direction:\frac{P}{A}\sin\theta\cos\phi$

So, what we see in general that we see that when we talk about the stresses when we try to define the stress is what we see that you know on any given plane you have one normal stress that you see that this is your normal stress and then you will have two set of you know shear stresses along the x and the y axis. So, this is how you represent the stresses at any point. So, normally what when we talk about normal stresses they are normally denoted by this sine and when we talk about the shear stresses this is so, that is tau and as we know that it is working in the along the plane.

So, in that you have two directions attached normally it is two dimensional plane and then in that case it will be represented by τ and both this directions like x, y or so, it will be coming like since it has it is working you know in a particular direction and you know, it is perpendicular to some plane so, that way you give one you know terminology to it and in that case tau will be have the two you know directions attached.

So, you have σ_{xx}or σ_{yy}or σ_{zz} and tau will have two components with two different directions τ_{xy}or τ_{yz} or τ_{xz}. So, basically there are two subscripts which are required for these shear stresses and for the normal stress you can come with only one subscript like σ_x or σ_y or σ_z. Again, this there is nature also defined to it and it may be positive or negative. So, normally when we talk about the normal stresses they are either you know tension or compression. So, when we talk about the tension force they are positive and when we talk about compression forces they are negative.

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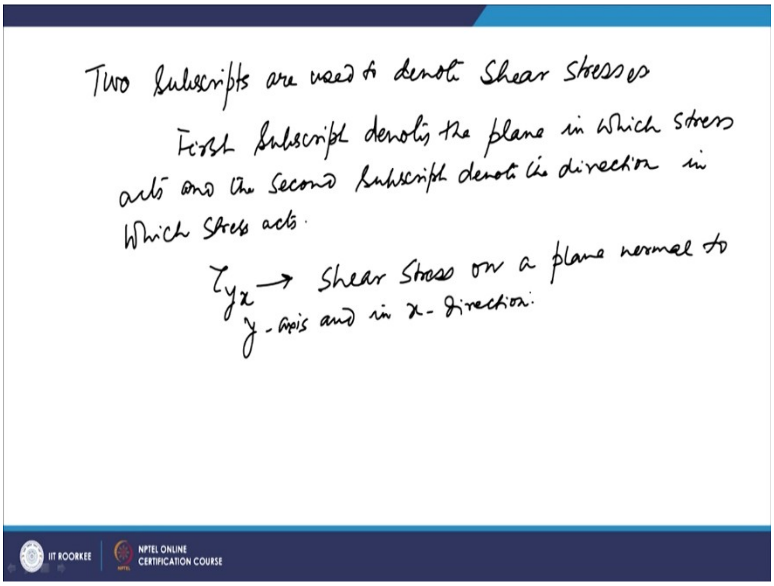

Similarly, when we talk about the you know this shear forces here you use two subscripts. So, two subscripts are used to denote shear stresses. So, what we do is the first subscript now, in this case what happens that first subscript the first subscript basically, it will be required for you know the plane it is going to discuss about the plane in which the stress act. So, first subscript denotes the plane in which stress acts and the second subscript will denote the direction, in which this stress acts and the second subscript denote the direction in which stress acts so, this is how.

So, if suppose you have τ_{yz}so, basically stress is working on a y plane normal and I mean this is working in a plane, which is normal to the y axis and then it is working in the z axis. So, τ_y axis τ_{yx}if you look at so, it will be shear stress on a plane normal to y axis and in the direction of x direction so, in and in x direction, ok. So, this is how you try to denote the stresses.

Now, coming to the so, in the case of shear stress also you have some denotations that whether it will be positive or negative. So, if they denote basically if the points in the are basically in the positive direction on positive phase then they are the positive and if in a negative direction then on the positive face then it is told as negative. So, in that way we denote the we give the value or the sine to this shear stresses.

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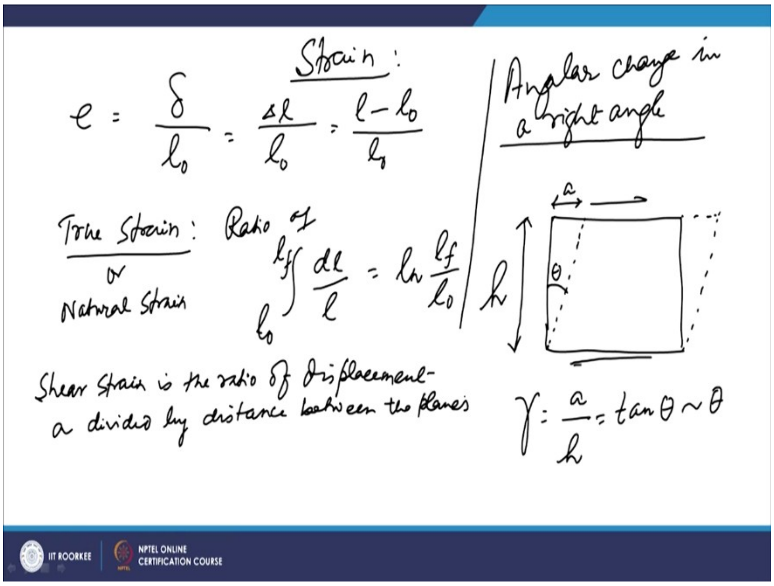

Now, coming to the concept of a strain and type of strain, when we talk about the strain now, we define the strain by the change in length to the original length of the body. So, what we define e E is defined as the strain and this is defined as delta that is your change in the length and divided by the original length. So, basically it will be change in length we call it as Δl and then it will be like l_0. So, if the final length is l and initial length is l_0 in that case it comes like this. So, it will be basically l_0 and so, that is how the strain is computed because this is change in length to the original length.

$$e = \frac{\delta}{l_0} = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0}$$

So, now strain at a point is the ratio of the formation to gauge length as the gauge length is approximating towards the towards 0. So, that is how the strain at a point is defined. Now, this is known as normally in terms of metal forming we call it as engineering strain when we are basically finding the change in length of the material and then we are dividing with the original lengths. So, that will be find in engineering strain. Now, in the case of metal forming there is another type of strain which is more realistic which is defined is the true strain. So, true strain basically it is defined as the change in the linear deformation divided by instantaneous value of the dimensions.

So, this is ratio of so, it will be you know it will be

$$\int_{l_0}^{l_f} \frac{dl}{l}$$

So, it will be you know change in the length divided by instantaneous length. So, that way we calculate the this true strain and then in that case if you look at if you take this integration that will be

$$\int_{l_0}^{l_f} \frac{dl}{l} = \ln \frac{l_f}{l_0}$$

So, this is how you compute the true strain. So, basically what happens true strain has more realistic meaning in case of metal forming and that is also known as natural strain or true strain or you can also call it as natural strain we will more discuss more about it when we start discussing about the plasticity theory.

Now, there is another type of strain that because in the case of elastic deformation it may result also in the change in the initial angle between any two lines. So, in that case a different type of strain is also defined and so, the angular change so, angular change in a right angle now, what happens that you know you have suppose one such type of element and then if you apply the stress in this case and because of this stress if suppose this the member takes this shape.

If you apply the stresses on this you know surface now in that case, in that case what happens, that this element which is there it has a height suppose of h in that case what happens that there will be you know change in the angle between the two lines. So, this line which is vertical it has inclined to certain angle theta.

Now, in this cases what we define the strain as the you know shear strain and what you see is that if suppose this is the change now in this case what we define the shear strain as a upon h, now this is the h that is height of this plane now this dimension of the plane. Now, this

$$\gamma = \frac{a}{h} = \tan \theta$$

Now, what we see is that normally this theta value is very very small and in those cases you can approximately take it as theta if theta is taken in terms of radian.

So, what happens that in this case you get the different type of strain and that is known as you know the shear strain and this shear strain is equal to that displacement a so, what we see is that the shear strain you define as the displacement that is a and divided by the distance of the plane h. So, shear strain is the ratio of displacement a and that is divided by the distance between the planes divided by distance between the planes.

So, this is how you get this shear strain values and so, basically you get this shear stress normal stress and then you have shear stress. In the case of shear stress you have the engineering strain and then you have the shear strain you and also you have alongside the engineering strain and you have the true strain. We will discuss more about this in the coming lectures.

Thank you very much.

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Lecture – 06
Description of Stress

Keywords: Two dimensional stresses, Normal and shear Stress, Principal plane.

Welcome, to the lecture on Description of Stress. So, in the last lecture we had discussion about the concept of stresses and also strains. So, now, we will discuss about the stresses how they are described specially how what are the notations for the stresses and we will also discuss about the state of stress in two dimensional and we will also see that how we are finding the value of a stress, when you have the case of two dimensions so, that we will discuss later.

So, what we discussed in the last lecture that stress had a point it can be resolved into normal component as well as the shear component. So, that is what we discussed about it.

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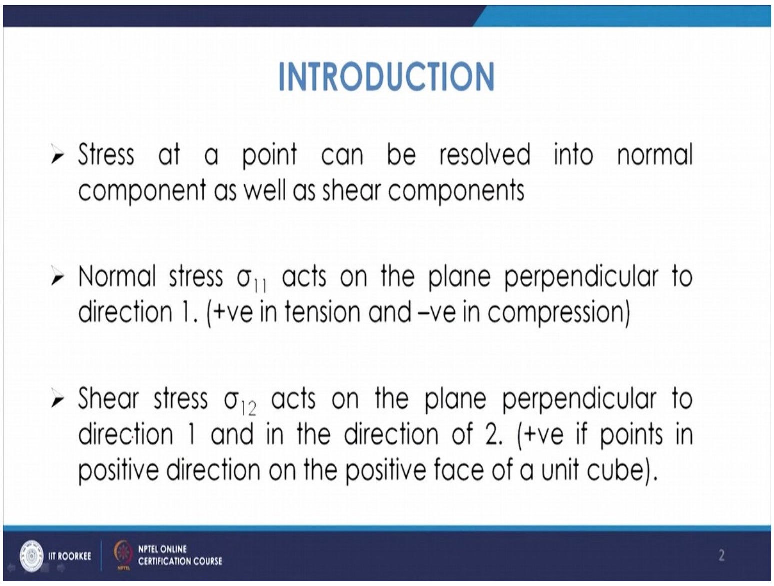

Now, normal stress that is σ_{11} or σ_{xx}, that will be acting on the plane perpendicular to the direction 1. So, if suppose you have a cube in that case you have the x directions. So, that will be if suppose you have y, z plane and then. So, x direction will be normal to it and the stress which will be acting in the perpendicular direction to that plane that will be the normal stress and similarly, you have the σ_y and σ_z then you have its nomenclature, its sign so, we discussed that if it is tension we normally take it as positive and if it is compression we take it as negative.

Then shear stress; now, shear stress normally will be denoted by σ_{12} or τ_{xy} or so and it will be acting on the plane perpendicular to direction 1 and in the direction 2. So, σ_{12} or τ_{12} or τ_{xy} normally we denote as τ. So, it will be you have one is your plane perpendicular to the direction and then another is subscript will be talking about the direction in which it is acting.

So, again it will be positive if it points in the positive direction on the positive face and otherwise it will have the, you know negative type of you know notation.

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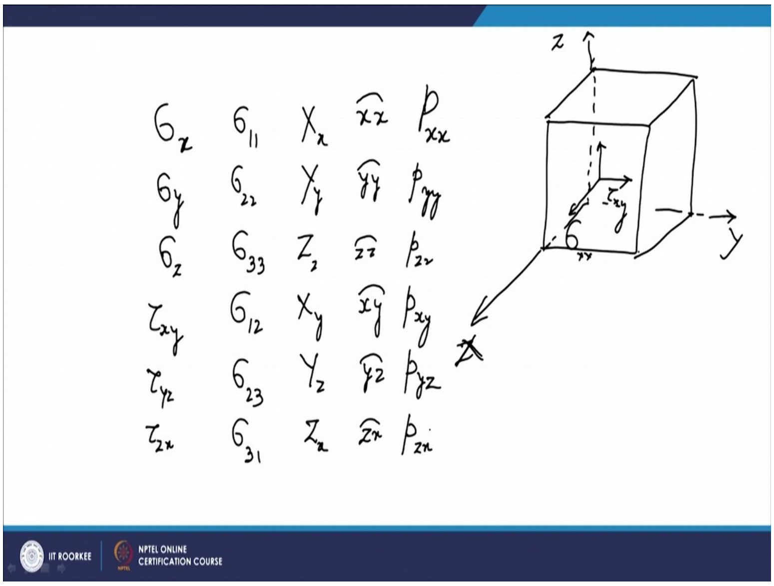

Now, the thing is that if you to talk about how this stresses are represented. So, what we see that suppose you have when we try to denote the, you know stress. So, in that case what we see is that suppose you have a cube and so, you will have so, this is how you have this is one is your suppose this is z axis and this is so, so this is x axis basically this is x axis, this is y axis and this is your z axis.

Now, in that case if you look at you will have the notations for the stresses in in all the directions and if you look at so, if this is the planes this is plane is the yz plane and in that basically you will have one stress which is in this direction. So, this will be your σ_{xx}. So, because this normal stress is parallel to I mean perpendicular to this x plane. So, I mean this is perpendicular in this plane which is and this x direction is perpendicular to this plane. So, you have σ_{xx} or which is also known as σ_1 or you can call it as σ_x.

Now, you have two type of stresses. So, you will have to basically shear stresses. Now, if you look at that so, you call it this. So, because this is perpendicular to this x direction this plane so, the first subscript will be x and since it is working in the y direction so, you will have here y. So, this way you will have the value of the notation of the normal stress as well as the shear stresses and the same way it will be σ_x it is will be τ_x and then it is in the that direction of τ_{xy}. So, this way on any plane you will have the you know value of normal stress as well as the shear stress.

As far as we denote notation we denote it as suppose σ_x or we also denote as σ_{11} we some time also denote as

$$X_x$$

then we also many a places you see the notation as like this or we also denote as xx. So, this way you have different types of notations for these normal stresses. Similarly, you have you have σ_y or this will be again then σ_{22}, you will have you have you have.

So, this will be your Y_y and similarly this will be yy and then this will be p_{yy}. So, this way you will have the normal stress in the z direction that will be σ_z and you can also call it as 3 3 then you will have this is Z_z so, this will be zz like this and this will be p_{zz}. So, these are the normal stress components and in the different situations you denote in a different manner.

Coming to the shear stresses, they are denoted as τ_{xy}. So, that is what we discussed that we denote it as τ_{xy} we can also denote them as σ_{12}, then we also denote them as X_y. So, this will be y will be the direction we also denote as like this and then we also denote as p_{xy}. So, these are the different way you denote these stresses and similarly, you will have τ_{yz} and then τ_{zx}. So, this is how. So, τ_{yz} will be again it will be σ_{23} and this will be Y_z this will be yz and this is p_{yz}. So, this way it will be 3 1 and this will be Z_x will be zx and then this is p_{zx}.

So, this is the notation of the stresses in such cases now this is about the notation of stress.

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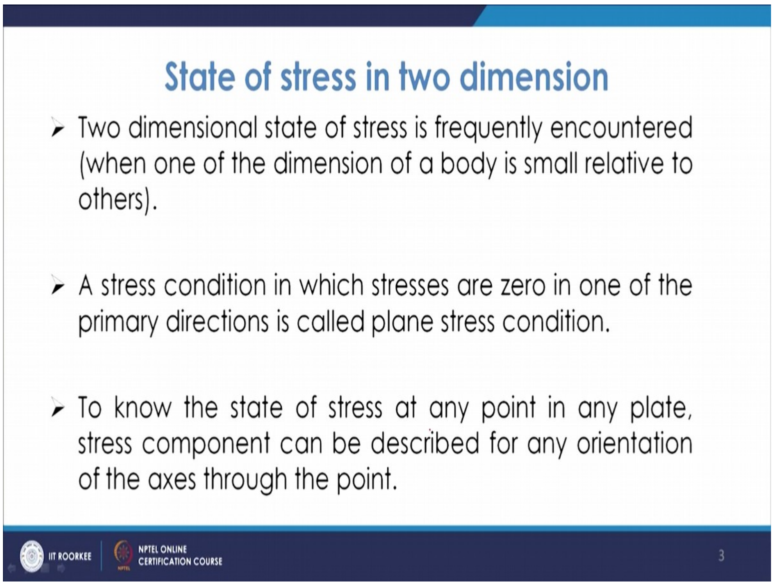

Now, we will discuss about the state of stress. Now, state of a stress in two dimension. So, when we talk about the stresses this two dimension two dimensional state of stress is a frequently encountered when one of the dimension of a body is small relative to other. Many a times we see that you are you are facing those situations where one of the dimension is very small as compare to the other two, like a very thin sheet if you look at. So, in that case the stresses are in that in that plane itself there will not be stress in the in the direction perpendicular that plane of the sheet. So, so, this is a case this is a case known as the plane stress condition where you have you know stresses are zero in one of the primary directions when the stress value is 0, then they are known as the plane stress condition.

Now, to know the stress that state of stress at any point in any plate the stress component has to be described for any orientation of the axis through the points. So, as we know that to if you have to find the state of stress at any point in the you know in the plate. So, what we do is that you find the stress component you will because that may be oriented at any angle with one of the axis and in that case you try to find what will be the, you know component of that in those inclination angles. So, that can be found out by coming to such example.

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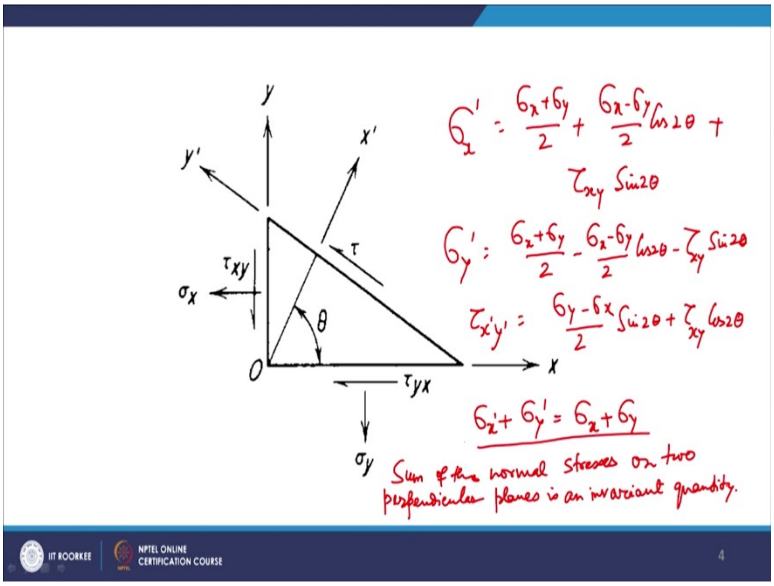

Now, what happens that if you look at such situation now, this is basically it is showing you know thin plate. So, now in this thin plate you have its thickness is in the normal to the paper. So, its thickness is like in this direction. So, its thickness is very very small. Now, in this case as we discussed that we need to find the state of stress at point O. So, for that we are we should able to describe the stress components at O you know for any orientation of the axis.

Now, what we do here is we are defining basically we are considering one oblique plane this plane which is there and normal to this plane basically is making angle θ with this x axis. So, this plane is basically you know taken and this oblique plane is normal to the plane of the paper and it is at an angle θ between the x axis and the outward normal to this plane. So, this outward normal is this x' and this x axis this direction is making angle θ with this you know normal.

So, for that now what the purpose is that you have two directions; one is x' and another is you know in the in the direction of the plane that is y' and we have to find basically you have to find this stress components in those directions x' and you have y' directions and that can be basically you know measured. So, now what we is do here for that if you know revise the concept of your strength of material or so, you must have discussed about it and what we do here is basically what we have the direction cosines defined and this direction cosine basically is represented by l, m and n and this l, m and n, l will be so, that is a function of the cosine of this angle θ.

Now, for that what we do is you will have the component of the area in that particular direction. So, the direction cosine between x' and x and y axis are basically l and m. So, l will be $\cos\theta$ and m will be $\sin\theta$. So, basically what we have to do is we have to take the summation of the forces and based on that you further find the expression for σ'_x and σ'_y and also the shear stress which is acting in that particular direction.

Now, if you so, what we discussed that you must have studied about that. So, if for the, this is the angle θ in that case

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

Similarly, σ'_ysigma y prime is also calculated and that comes as

$$\sigma'_y = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

So, this is the expression for σ'_x and σ'_y this x' and y' we know that this is x' is the direction this is normal to that oblique plane which is making angle θwith the x axis and

y' is the you know the direction in that along that oblique plane and then the expression

$$\tau_{x'y'} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

So, this is how you try to find the you know component of the stress is σ_x is this is a normal component in the x' and y prime directions and similarly, the shear stress component in that $x'y'$plane.

Now, if you see this value σ'_x, $\sigma'_y$and if you add them if you look at the addition of these two $\sigma'_x + \sigma'_y$ if you add them. So, what you see is that these two terms are cancelling. So, what you see is that this is coming as $\sigma_x + \sigma_y$.

$$\sigma'_x + \sigma'_y = \sigma_x + \sigma_y$$

It means if you are adding these two normal stresses ultimately you are getting one quantity, which is invariant which is not a function of θ. So, this is known as an invariant function. So, that is why the, we also write that the sum of the normal stresses on two perpendicular planes is an invariant quantity. So, this is known as the invariant of the stress.

Now, they are certainly some facts which can be you know found out by looking at these values. Now, the thing is if you take certain value of σ_x, σ_y and also the value of τ_{xy} if that is given to you and if you try to see that, how they are varying with the value of θ then graphically you can have a feel and you can plot the value of σ'_x, σ'_y and also $\tau_{x'y'}$ and from there basically you will have certain findings.

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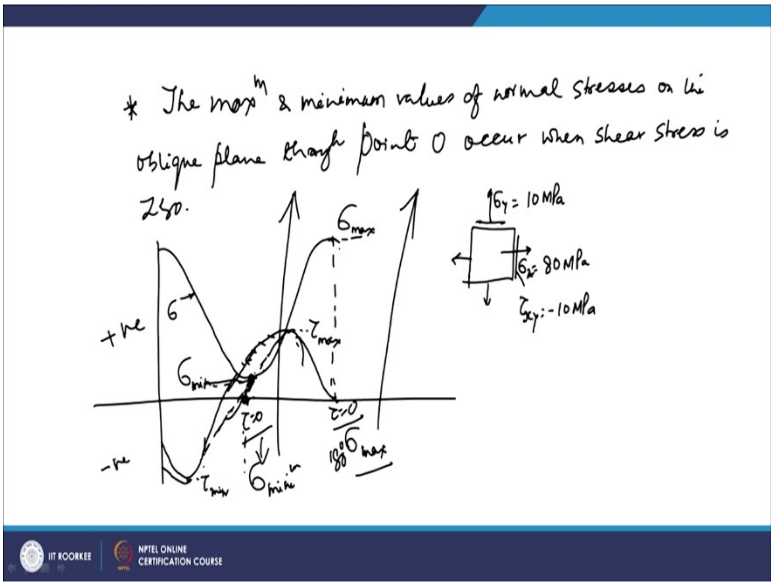

So, that finding you can have further though the findings are that the maximum and minimum values of normal stresses on the oblique plane through point O occur when shear stress is 0. Now, this can be found as we discussed that if you try to draw suppose you are taking one example suppose you have a case that you have a case where the σ_x, σ_y and τ_{xy} is given suppose σ_x is given as 80 MPa and σ_y also is given as you know 10 MPa and then you have τ_{yx}.

So, if the τ_{yx} is or τ_{xy} is to this will τ_{yx} here it will have τ_{xy}. So, τ_{xy} is given as minus 10 MPa. So, again here you will have σ_y, here you will have σ_x now for this case. So, as we see if you refer to the expression for σ'_x and σ'_y, then in that expression σ'_x will be

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \cdot$$

Similarly,

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \cdot$$

So, in that if you change the θ so, in that case what you see is you will get certain variation of the you know values and this you can try and you can draw it and if you draw you will get some curve like this and similarly, you will have the variation of the further the so, this is your sign of positive and this side you have negative, this is the value of σ.

Now, you have another curve which is coming out to be. So, that will be here and then it goes like this and further it comes, comes and it comes the maximum point. So, that will be you have to you just have to delete this points this comes the minimum here at this points where it is the minimum one.

Now, what we see that in such cases this is your minimum value this is r_{min} and this is your σ_{min} this becomes as σ_{min} and this is your σ_{max}. Now, from the here you can have the you know and this line is for the tau. So, this your σ_{min} and similarly you will have the τ_{max}. Now, what you see is the first point which we have written that the maximum and minimum value of the normal stresses on the oblique plane through point O will occur when your this shear stress is 0. So, what you see is the shear stress is 0 when you get the maximum value, ok.

And, similarly the, this part where it is minimum basically it goes like not like this it goes like this. So, like this it goes not like this. So, basically at this point where you σ is the minimum point, here also your, this line touches like this. So, here again at this point at this point the τ is 0 here also τ is 0. So, this point is your σ_{min} and this point is also σ_{max}. So, that is what this point tells that whenever you have for the maximum or minimum value of normal stress on any oblique plane through point O it has to be at that point when your shear stress is 0.

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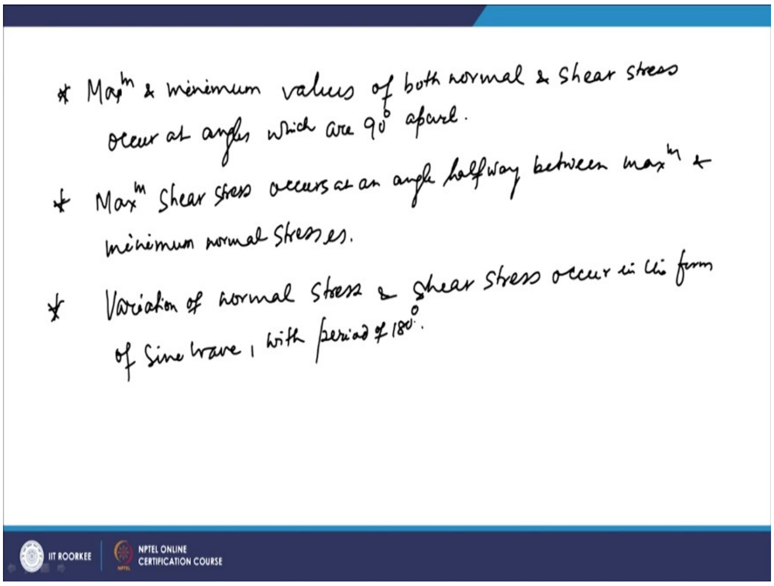

The second point which is also you know notable from this that the maximum and minimum values of both normal and shear stress occur at angles which are 90 degree apart. So, this is the second observation which is clear by looking at this you know graph that they are basically 90 degree apart. So, basically if you start from here this point is 90 degree upto this point this point is 90 degree, here where it is maximum and minimum is occurring that is 90 degree apart.

Third point is that the maximum shear stress occurs at an angle half way between maximum and minimum normal stresses. So, what we see is that the maximum shear stress value it will be occurring at an angle which is half way between the maximum and the minimum of the shear you know normal stress values. So, here if you see here or this one value or the maximum of the value it will minimum and maximum in between your this τ_{max} occurs to be.

Then also the another observation which is clear from here that the variation of normal stress and shear stress occur in the form of sine wave. So, what we see is that this is going in terms of a sine wave with period of 180º. So, basically what is happening here is that, at this point it is coming as 180º. So, here is a maximum. So, this is also maximum you are getting the period of 180º after that it will again repeat. So, that is why it goes in the period of 180º.

Now, there is a concept of the principle plane and the principle you know plane actually when we talk about these findings now the thing is that we talk about these principle stresses. So, now the another concept is about the principle planes. Now, in this plane actually these are those planes, where there is no shear stress is acting. So, in the same expression which we have found for the, you know for the value of the shear stress $\tau_{x'y'}$ now that is to be equated to 0.

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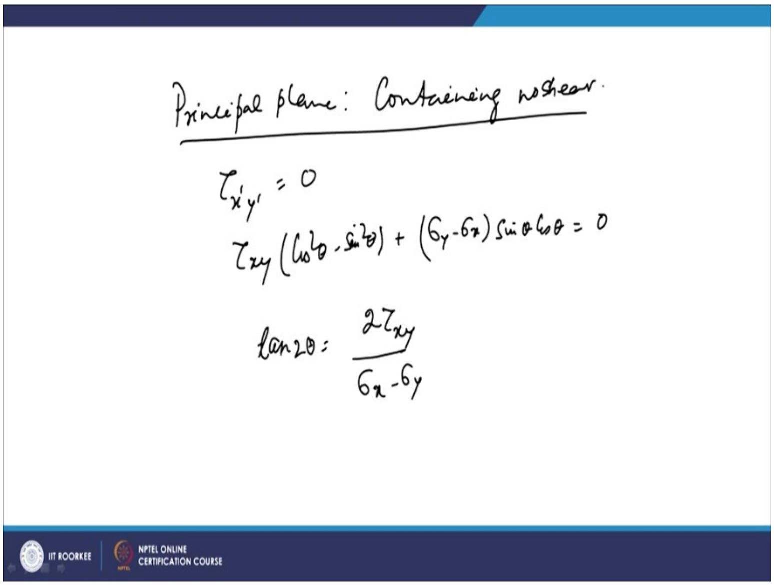

So, if you look at the principle plane that containing no shear, ok. So, basically it is about the you know principle stresses and their direction. So, as we know that in those direction there is no shear. So, now, the principle plane is defined by putting that 0 in the value of $\tau_{x'y'}$. So, in that expression we have seen that this

$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2} \sin 2\theta + \tau_{xy} \cos 2\theta = 0$ So, that will be.

$$\tau_{xy}\left((\cos\theta)^2 - (\sin\theta)^2\right) + (\sigma_y - \sigma_x)\sin\theta\cos\theta = 0$$

So, earlier it was 2. So, that 2 come goes out. So, this way you have to equate it to 0 and in that case you get the value for the theta and from you here you get basically if you do the derivation you will get

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

So, this is basically giving you the direction of this principle plane where there is no shear stress. So, that will have also certain significance when we talk about these things as you see that it is maximum value is if look at its value, the θhas to be 45ºso, then it becomes $\tan 90°$. So, that is how you have the concept of you know deformation and all that. So, this is talking about that direction of this principle planes. So, we will discuss more about these 3D stresses in our next lecture.

Thank you very much.

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Lecture – 08
Description of Strain

Welcome to the lecture on description of strain. So, in the last lecture we discussed about the state of stresses in the case of 2 dimension as well as in the 3 dimension, now we will discuss that about the strain values. So, when we talk about the you know the continuum body a concept of continuum is there in the case of analysis now in that basically you have displacement of points are there and that result from basically either from the rigid body translation or rotation or deformation.

So, you know deformation maybe again made up of either the change in volume or change in shape. So, the change in volume is you know known as dilatation or you have the change in shape. So, you will have many you know situations and when you have the situation of translation and rotation, then they are treated in the branch of mechanics that is known as dynamics and in the when we talk about the deformation to a very small you know quantity, then we that is treated in basically in the theory of elasticity and when this is you know of the larger scale larger deformations are there they are treated in the case of I mean in the theory of plasticity.

So, that basically you they are applying to all types of this continuous media now what we try to understand in the case of strain is that I mean what happens in the case of you know if you have a solid body in the fixed coordinates. So, suppose we can understand the strain like this. So, suppose you have a solid body. So, suppose you have a body.

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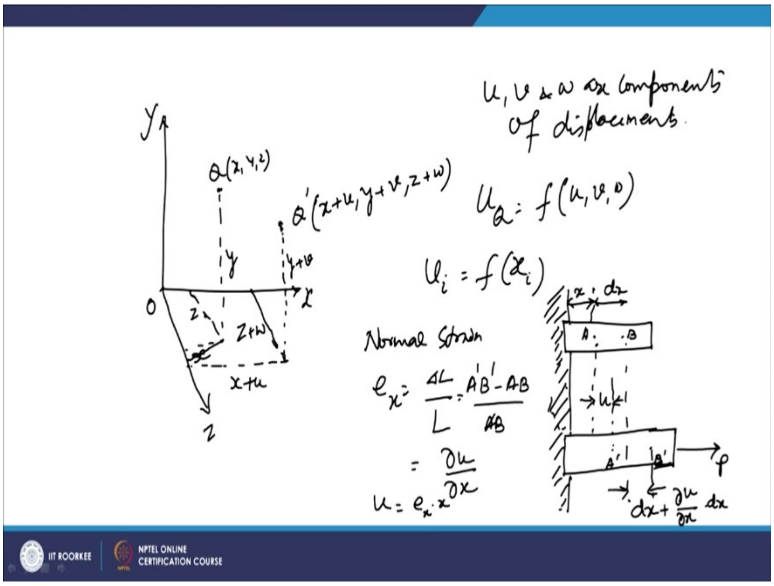

So, what happens that, you have this as the axis now you have a point that is Q and it has the coordinate that is x y z.

So, if suppose you have a solid body in fixed coordinate that is x y z. So, this is x y z and you have these are the axis. So, this is your y axis, this is your x axis and this is your z axis. So, now, if this point Q it has the position vector that is x y z. So, you will have this as the origin, this as has the coordinate axis x y and z.

Now, suppose you have a combination of deformation and movement and that basically is displacing this point Q to the another pointQ'. So, suppose this is displaced to this point Q' and this Q'has the coordinate x + u, y + v and z + w. So, basically u v and w they are basically the displacements. So, you can say that if you have if you take this point. So, it will have one point in this plane and from this plane you get further, you can have it is components in the you know y and z direction that way.

So, you will have this as z and similarly you will have this as. So, this is basically horizontal this is horizontal not, this one this is your x component this is horizontal one this is your in the z direction and this is your y component. Now this is changed to this Q' and Q' becomes x + u y + v and z + w and so, again it is if you take it is you know it is projection in this plane. So, this plane you will have this coordinate as x + u. Similarly you will have it's component if you take it is this. So, this will be you know z + w and this will be your y + v.

Now, what you see in this that you have in a body you have this point Q and under the action of deformation or stress is in the combination of movements, this point is changed to you know this point Q displaced to $Q^{'}$ and you have this these u v and w are the components of displacements.

So, what we see that you know displacement of this Q it is a u_Q and this u_Q will be the a function of u v and w. So, this is u_Q is basically the displacement of Q. So, that is the function of u v and w. Now if it is constant for all the particles of the body it means there is no strain in the body, but in general this $u_i$is different for from particle to particle and that is why this u_i will be a function of x.

So, normally u_i is a function of x_i because it does not remain the same for all the points. So, that is why u_i is taken as the function of x_i. Now in the case of the elastic deformations, now this u_i is where you have very small displacement, this u_i is a linear function of x and you have homogeneous displacements and that is why this is displacement equations are linear, but otherwise for other materials that may not be even linear.

So, we will try to have a case with the simple one dimensional analysis and if you took that analysis suppose you are we are dealing with a body that is you have this is this is the 2 point A and B. So, they have the in between they have. So, this is at a distance of you know x from the origin. So, you this is your so, similarly you will have the another body that is.

So, same body is there and it is you know under the action of this, when it is default. So, this A goes to $A^{'}$. So, and then this B goes to $B^{'}$. So, this becomes $A^{'}$ and this becomes $B^{'}$ now in this case again.

So, this is your now, what we see is that this in the case of undeformed state you have these 2 points A and B and this distance of A from this position is x and distance between A and B is dx. Now this is changed and when is the force is applied in the x direction basically. So, if this is after applying the force P in the x direction. So, there will be movement in that, and this A prime moves to A; A moves to $A^{'}$ and B moves to $B^{'}$ and since the displacement is that is u which is one dimensional case. So, it will be a function of x in such cases.

So, if you look at. So, B will be you know B will be displaced slightly larger than A and in that case if you talk about if you try to define the normal strain. So, if you try to define this normal strain and in the in the x direction. So, that is known as e_x it is nothing, but it is change in length by original length. So, it will be $\frac{\delta L}{L}$. Now the thing is that here your A B is changed to $A^{'}B^{'}$, it means the change will be $A^{'}B^{'}-AB$ and originally it was AB.

Now, the thing is this is x and this is dx and what happens for this now what is u? U is the displacement of this one. So, this part A is moving to $A^{'}$. So, this part is u. So, basically B is further moving and that is why this will be $\frac{+\partial u}{\partial x}dx$. So, this will be your between the B and the $B^{'}$ this distance this will be basically $dx+\frac{\partial u}{\partial x}dx$. So, into dx. So, this is how you find the displacement of these 2 points.

Now, if you find the normal strain in such cases. So, e_x will be $\frac{A^{'}B^{'}-AB}{AB}$ and then if you take this $A^{'}B^{'}$ as you know$dx+\frac{\partial u}{\partial x}dx$ + all these. So, once you do that you will get the value as $\frac{\partial u}{\partial x}$. So, what you see you see that you know in such cases these strain which you get now if you look at this one dimensional case you will find the displacement u to be $e_x x$ if you integrate it. So, u will be $e_x x$.

So, basically if you generalized generalize it if you generalize this to A. So, what you can write is you will have you can write in that case u as ex into x now if you generalize this to a 3 dimensional case you can write you know these component of the displacements u is basically the displacement.

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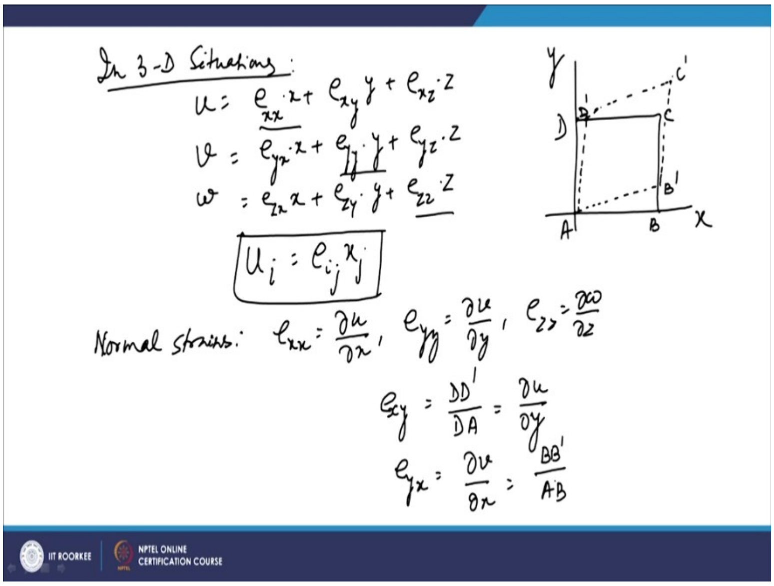

Now if you generalize in the 3 dimensional case in 3 D situations now, in the case of this 3 dimensional situations what you see that

$$u = e_{xx} x + e_{xy} y + e_{xz} z$$

So, so we saw that u is basically $e_x x$. So, if you are going to the 3 dimensional case you can represent this u as this similarly you can represent v also. So,

$$v = e_{yx} x + e_{yy} y + e_{yz} z$$

and similarly you can write.

$$w = e_{zx} x + e_{zy} y + e_{zz} z$$

So, you can write that is why you can denote it as

$$u_i = e_{ij} x_j$$

So, this way you can denote these displacement you know vector that is made of component that way to find that strain now this coefficients which are relating these displacement with the you know coordinates of the body point in the body. So, they are basically component of the relative displacement tensor. So, basically they talking about the general notations if you take all these components it will be again a 2 dimensional 2 rank tensor or so, as in the stress you will have the 9 components here also you will have the 9 components 3 raised to about 2 that is 9 you have the components to define the strain at a point.

So, what we see is that you have in this what you see that these components which have xx or yy or zz now here you see that they are basically the normal strains. So, normal strains are you have 2 types of the strains. So, you have normal strains and normal strains you have e_{xx}. So, that will be $\frac{\partial u}{\partial x}$ similarly e_{yy}. So, that will be $\frac{\partial v}{\partial y}$ and e_{zz} that will be $\frac{\partial w}{\partial z}$. So, that is what you get in the case of normal components.

$$e_{xx} = \frac{\partial u}{\partial x}$$

$$e_{yy} = \frac{\partial v}{\partial y}$$

$$e_{zz} = \frac{\partial w}{\partial z}$$

Now, you have other 6 coefficients. So, they are required to be understood what are these components, now if you talk about now if we try to find what are these 6 components you can say that if suppose you have an element in the xy plane. So, suppose you have in xy plane and this is x and this is y, now this element in this plane now this is your element. So, if suppose this is your A B C D now this you know this is distorted by shearing suppose. So, if have given in the shearing and this has gone to this type of you know shape. So, if it has taken such shape. So, this C has come to C' now this B has come to B' and now, in this case it is not it meets here not here.

So, this is your D'. So, basically this also moves like this the situation now what happens that because of the application of this shear stresses it has gone into one angular distortion type of you know situation and if you take if you see these displacement now here the displacement is parallel to x direction here similarly displacement here is parallel to the y direction or so, now in such cases you can find these e_{xy} now e_{xy} if you look at. So, e_{xy} where you see that in this case e_{xy} you have y component $\frac{\partial u}{\partial y}$.

So, $\frac{\partial u}{\partial y}$ an in the u component what is that change in that u side. So, that is why and dy. So, dy will be. So, you have divide by this. So, basically what you get is De_{xy} will be $DD^{'}$. So, that is your change in the u displacement component in the x direction.

And similarly you have to divide it divide it by DA. So, that is what you get this component divided by this component and that becomes as $\frac{\partial u}{\partial y}$. So, this you get as e_{xy} similarly you can get e_{yz} and if you see that it will be you know $\frac{\partial v}{\partial x}$. So, here you have change in the in y component. So, that is. So, that becomes basically equal to $BB^{'}$. So, you have the displacement in the y direction. So, this your displaced quantity and then it is for this AB, so, $\frac{BB^{'}}{AB}$, so this way you define this e_{xy} or e_{yx}.

So, similarly you can define the e_{yz} or e_{zy} or e_{xz} or e_{zx} and then also there is certain sign to these shear components and this shear displacements are basically positive when they are rotating from positive to one positive axis towards another positive axis then we take that shear displacements as positive and otherwise they are negative. So, if you come to the displacement tensor if you find the displacement tensor now what we have to understood we have try to understood these how we get these e_{xy} or e_{yx}.

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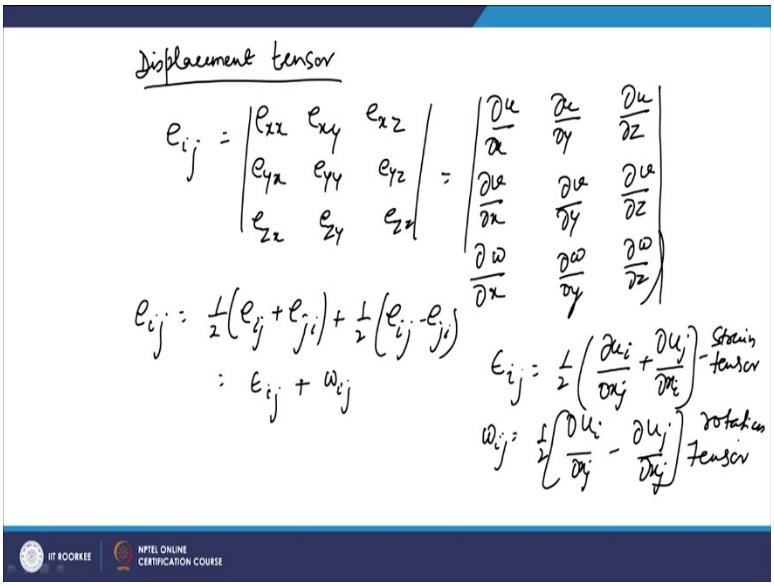

So, what we get is in the displacement tensor as. So, you have

$$e_{ij} = \begin{vmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{vmatrix} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

now what we see is this is basically known as the displacement tensor in such a cases.Now, basically these displacement components like xy e_{xy} or e_{yx} they are producing both shear strain as well as the rigid body rotation. So, what we do is basically we try to divide it into 2 components. So, one will be your strain tensor and. So, that is creating the shear strain and another part will be basically talking about the rotation part. So, that will be rotation tensor.

So, one will be talking about the strain tensor another will be for the rotation tensor and for that as we know that this matrix can be any matrix can be you know represented in terms of sum of 2 matrices one will be symmetric another will be skew symmetric. So, what we do is we are. So, this is the second rank tensor this we are basically decomposing this e_{ij} into 2 parts one is the symmetric matrix another is the skew symmetric matrix. So, one will be

$$e_{ij} = \frac{1}{2}\left(e_{ij} + e_{ji}\right) + \frac{1}{2}\left(e_{ij} - e_{ji}\right) = \varepsilon_{ij} + \omega_{ij}$$

So, this way we get one is this is your strain tensor and then the second part will be your rotation tensor that is we write it as ω_{ij}.

So, we have basically to define the strain which we have getting by this this shearing stresses since we are getting that into 2 ways we have we have to represent them in 2 ways. So, we are representing them as $\varepsilon_{ij} + \omega_{ij}$ then ε_{ij} will be basically ε_{ij} if you try to find if you you add these 2 and then divide by half. So, it will be basically you can (Refer Time: 21:05) rotate as

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

So, this is known as this known as the strain tensor.

And similarly if you take the skew symmetric part so, here ω_{ij} and this will be coming as

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_i}{\partial x_j}\right)$$

So, basically this comes as the rotation tensor now the thing is that if you now represent try to represent this ε_{ij} if if you look at this whole this strain tensor in that case. So, the strain tensor becomes we you can represent as that is ε_{ij}.

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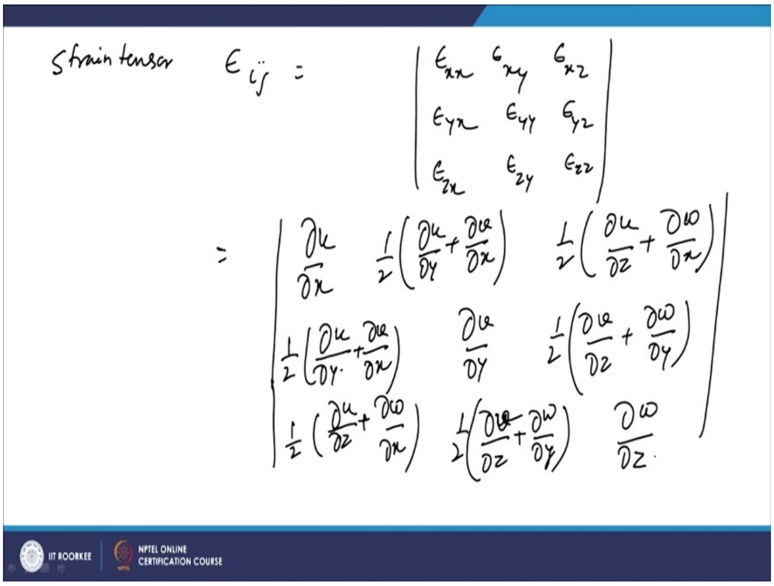

Now, this will be nothing, but you know

$$\varepsilon_{ij} = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{vmatrix} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) & \frac{\partial w}{\partial z} \end{vmatrix}$$

So, what you see is that this strain tensor is a symmetric type of matrix you can see that this and this is same similarly this and this is same this and this is same. So, this is the strain tensor if you go to find the you know rotation tensor.

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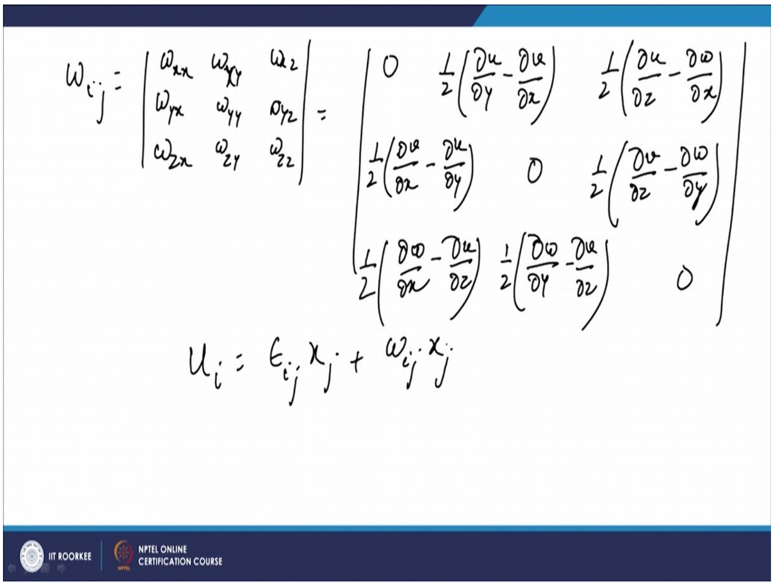

So, rotation tensor will be ω_{ij} and you will have that can further we represented as

$$\omega_{ij}=\begin{vmatrix}\omega_{xx} & \omega_{xy} & \omega_{xz}\\ \omega_{yx} & \omega_{yy} & \omega_{yz}\\ \omega_{zx} & \omega_{zy} & \omega_{zz}\end{vmatrix}=\begin{vmatrix}0 & \frac{1}{2}\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)\\ \frac{1}{2}\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right) & 0 & \frac{1}{2}\left(\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\right)\\ \frac{1}{2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\right) & 0\end{vmatrix}$$

So, this part this is known as the rotation tensor and the summation of this 2 tensors they are the displacement tensor and what we see in this case this is ω at this ω_{ij} this part is negative of this part is negative of this part is negative of this. So, this way you get these value, now if you try to generalize what we see is that you get u_i as basically

$$u_i=\varepsilon_{ij}x_j+\omega_{ij}x_j$$

because your strain tensor is $\varepsilon_{ij}+\omega_{ij}$.

Now, what we further try to we will try to further define this you know shear strain. So, if you try to see the different types of you know you know angular changes from the right angle you can see that you have many cases which are basically originated and the shear strain basically that is defined as if you talk about the shear strain.

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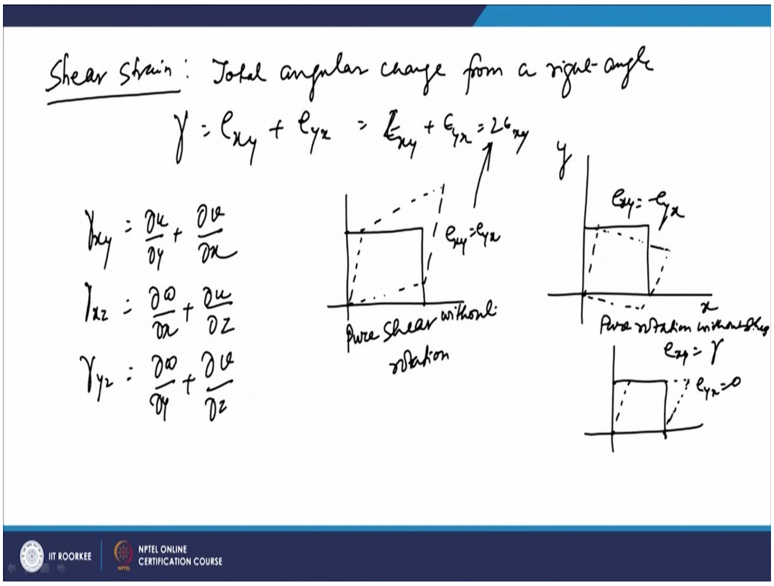

So, normally this shear strain is defined as the total angular change total angular change from a right angle. So, if you look at now the suppose γ it will be $e_{xy}+e_{yz}$.

Now, in the earlier figure what we have seen you have e_{xy} and you have e_{yx} in both the cases. So, total angular change will be your shear strain in those cases. So, basically it becomes you know

$$\gamma=e_{xy}+e_{yz}=\varepsilon_{xy}+\varepsilon_{yx}=2\varepsilon_{xy}$$

in those in that situation if you look at the situation different situation for a now in this case if you look at here where you have this is as the element and here if you look at this element goes like this, so, in such situations what you have seen previously.

In this situation e_{xy} is same as e_{yx} that is why this leads to you know $2\varepsilon_{xy}$ there is no rotation in this in such cases. So, that is why this your $2\varepsilon_{xy}$ now the thing is that. So, γ_{xy} you can you can write as

$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}$$

$$\gamma_{xz}=\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}$$

$$\gamma_{yz}=\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}$$

So, that you can refer by some other cases like you can take the example of such deformation where you have this is as an element and this has gone to such situation where it has gone like this and this has moved to such case. So, in this case what we see is if this is the y and this is your x. So, in that case you can write. So, in this case you can write that e_{xy} is minus of e_{yx}.

And if you take the example of such deformation where your this is how the deformation occurs in this case your e_{xy} will be as γ and e_{yx} is basically 0 because there is no such displacement in that y component. So, this way you have different types of you know situation this is a situation where you have this is the situation of pure shear without rotation.

So, and then there is no rotation here now in this case you have pure rotation without shear. So, that will be your pure rotation without shear and if you take the simple this is the case of simple shear that. So, here you have a shape change because of the displacement. So, this is how you try to have the expression for the shear strain as the you know in terms of u v w and x and y and z.

Further you can have the transformation of I mean transformation principle applied which we apply in the case of tensor and there also we try to have the you know expressions for the strain you have the invariance for the when the strain tensor that is also you can you know we get it like we have 3 invariance of strain tensor and similarly you have the $\sigma_x+\sigma_y+\sigma_z$ is the first strain tensor second you have.

So, this way we will have the 3 invariance of strain tensor you will have the principle shearing strains also that also can be found out and based on that you may have other strain components like you know volumetric strain can be found and you have you know for the deformation and as well as for the you know volumetric component based on that the strain deviator is also you know defined. So, basically strain is normally tensor will be basically in the form of 2 components one will be hydrostatic or mean tensor and another will be the deviatoric type of tensor which will be for these shear components it will be talking about.

So, that way you can you can you know represent this strain tensor in terms of the hydrostatic component or as well as the, you know deviatoric type of components. So, that way you can study to have a better understanding about the strains.

Thank you very much.

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Lecture – 09
Hydrostatic and deviator components of stress and strain

Welcome to the lecture on Hydrostatic and deviator components of stress and strain. So, in the last lectures we discussed about the stress as well as a strain; we had some concept about the stresses the components of stresses. So, then further we also talked about the strain different types of strain.

Now when we talk about the plastic forming technology that we need to know the components of strain in the sense that some part of the strain or some part of the stress ah, you have the components of stress in you know categorized in different ways. And it can be also categorized based on what way it performs like one thing maybe causing the change in shape, whereas the another case may cause the change in you know deformation; it may cause the deformation.

So, based on that basically we try to define this stress or strain components into 2 parts; that is hydrostatic as well are the main component and also the deviator component. So coming to the strain because we have discussed about the strain in the last class; so, if you go to the strain part what we see in the case of strain.

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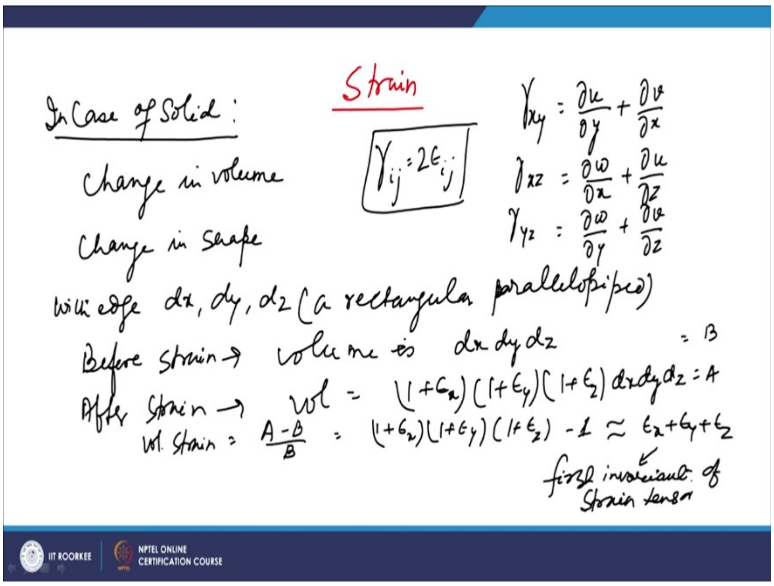

Now, in the case of strain what we see is that what we got the shear strain as γ_{xy} then γ_{xz} and γ_{yz}. So, since you have the 3 components of the shear strain and this is as we know that this is

$\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}$.

Similarly, you have this as $\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}$ and you have this as $\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}$. So, this is what we had seen in the case of in the last lecture where we found the expression for shear strain and that came from this expression we got as $\gamma_{ij}=2\varepsilon_{ij}$; so; so this way we found this is known as the shear strain. Now normally in the reason of elasticity, we discussed about this shear stress this is more common in those cases.

Further when we talk about this solid then normally you have 2 things, when we talk about the deformation is solid. So, in that you have one is your volume change and another is your change shape change of shape. So, in case of solid when we talk about the deformation in solid then you have one is change in volume and another is change in shape.

Now, the thing is that you know once you have the change in volume so, that way you will have the rising of the volumetric strain. So, you can find the expression for the volumetric strain. So, suppose how can you find these expression for volumetric strain. Suppose you have a rectangular parallelepiped and it has the edge of dx, dy and dz. So, if the with edge you know dx, dy, dz a rectangular parallelepiped is there.

So, suppose there is strain into on the sides and because of that; so, what will happen the when there will be strain there in the strain condition then their volume will be $1+\varepsilon_x$ into. So, suppose. So, before strain its volume will be before strain the volume will be volume is dx, dy and dz.

Now suppose the there is strain component in x y and z direction; so, that is ε_x. So, in that case the sites will be $(1+\varepsilon_x)$ into dx $(1+\varepsilon_y)$ into dy and $(1+\varepsilon_z)$ into dz. So, so the after straining, so after strain the

$vol=(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)dxdydz$

Now, so the volumetric strain what you get will be the, this minus this. So, if suppose this is B, and this is A; so, you can find the volumetric strain as $\frac{A-B}{B}$; so, this way you can find the volumetric strain. So, once you do this multiplication divided and subtract with this dx, dy, dz and if we be since ε_x ε_y and ε_z will be small quantities.

So, any you know multiplication of ε_x or ε_y or ε_z can be neglected in that case. So, you will have basically again dx dy dz plus that way you can have the components. So, you will have this is strain will come as $(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$.

So, this way you can have; so, if you neglect the, you know product of the 2 strain components like ε_x ε_y or so or the 3 1. So, this can be written directly as $\varepsilon_x+\varepsilon_y+\varepsilon_z$z because this 1 and 1 will be cancelled and in that case epsilon all these.

$$\frac{A-B}{B}=(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)-1\approx\varepsilon_x+\varepsilon_y+\varepsilon_z$$

So, this way what you see is this is your volumetric strain; now what we see is that as we have seen that in the case of stress or strain, you have invariants. In the case of stress also be saw that we get the invariants the and first invariant what we saw is the sum $\sigma_x+\sigma_y+\sigma_z$ or similarly in the case of strain also your first invariant of strain tensor will be $\varepsilon_x+\varepsilon_y+\varepsilon_z$ that is what it is.

So, this is basically the first invariant of strain tensor; so this is nothing, but first invariant of strain tensor. Now this component basically if it divided by 3; so, this is known as the mean strain. So, if you go to the further define that; so it is one third component will be the mean strain. So, mean strain you can defined as $\frac{\varepsilon_x+\varepsilon_y+\varepsilon_z}{3}$. So, we can further write.

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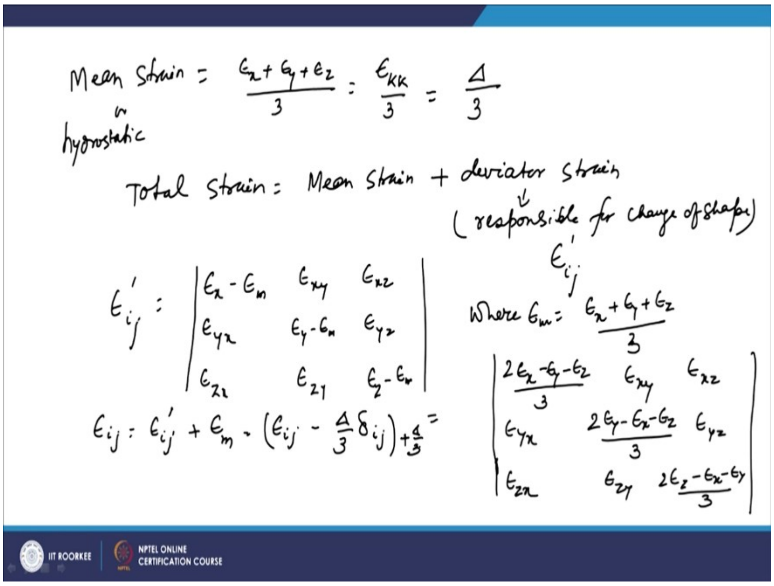

So, your mean strain or we also call it as hydrostatic strain because that is responsible for the change in the volume and that is basically $\frac{\varepsilon_x+\varepsilon_y+\varepsilon_z}{3}$.

So, this way you have the component that is known as mean component or hydrostatic component; so, or hydrostatic we also call it as hydrostatic or spherical strain. So, that way spherical component of the strain and this is divided as the $\frac{\varepsilon_x+\varepsilon_y+\varepsilon_z}{3}$. And it is also denoted you can denote it as $\frac{\varepsilon_{KK}}{3}$ because it will involving this x x parts. So, KK by 3 and it is also divided denoted as $\frac{\Delta}{3}$. So, you have the volumetric strain divided by 3; if you go now the thing is that now this is a part of the total strain.

$$Mean\,strain=\frac{\varepsilon_x+\varepsilon_y+\varepsilon_z}{3}=\frac{\varepsilon_{KK}}{3}=\frac{\Delta}{3}$$

So, it is subtracted from the total strain you will have another component of strain and that part is known as the deviator component of the strain. So, what we see is that you get this; so the deviator. So, total strain will be your mean strain and what is remaining is the deviator strain. So, that will be basically involved for the change in the shape rather than the volume change. So, one part of the strain is responsible for the volume change that is your mean strain or hydrostatic strain and then the remaining part is known as the deviator strain and that is responsible for the change in the shape.

So, this is responsible for change in shape; so, if you see the strain tensor component then in that case what we do is that this deviator strain component and this is denoted by $\varepsilon_{ij}^{'}$. So, $\varepsilon_{ij}^{'}$ if you look at now this is nothing, but from the total strain part, you are going to remove the you know mean strain part. So, mean strain part if you remove; so, what will happen it will be like this it will be

$$\varepsilon_{ij}^{'}=\begin{vmatrix} \varepsilon_x-\varepsilon_m & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y-\varepsilon_m & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z-\varepsilon_m \end{vmatrix}$$

So, this is the tensor this is known as the deviator strain tensor. So, this is a tensor which is known as the deviator component of the strain tensor and as we know that epsilon m is where epsilon m is basically you know $(\varepsilon_x+\varepsilon_y+\varepsilon_z)/2$. So, it is basically you know along the diagonal that is working. So,$\varepsilon_{xx}\varepsilon_{yy}\varepsilon_{zz}$; so, if you try to further you know see that. So, this is this strain tensor comes of the form you will get it as 2. So, your$\varepsilon_x-\frac{(\varepsilon ¿¿x+\varepsilon_y+\varepsilon_z)}{2}¿$; so, that will be.

So, this 2 will be here; so you will get it will be 3 here. So, you will get $\frac{2\varepsilon_x-\varepsilon_y-\varepsilon_z}{3}$.

Similarly, you have

$$¿\begin{vmatrix} \frac{2\varepsilon_x-\varepsilon_y-\varepsilon_z}{3} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \frac{2\varepsilon_y-\varepsilon_x-\varepsilon_z}{3} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \frac{2\varepsilon_z-\varepsilon_x-\varepsilon_y}{3} \end{vmatrix}$$

so, this is known as the deviatory component of the strain this is a strain tensor, which is responsible for the change in shape. So, what we normally write? We normally write as ε_{ij} as $\varepsilon_{ij}^{'}$ that is your deviator component of the strain plus ε mean. And we also write it as ε_{ij} and $\frac{-\Delta}{3}$ and then we Kronecker delta we put it. So, we write this Kronecker delta that is you have a tensor where you have a matrix which has one as at the you know diagonal component and that is multiplied by $\frac{\Delta}{3}$.

So, you will have a matrix which has all the you know you know elements are 0; except the diagonal elements and they are all the first diagonal this a this upper top left corner will be $\frac{\varepsilon_x}{3}$. Similarly you will have $\frac{\varepsilon_y}{3}$ and $\frac{\varepsilon_z}{3}$; so, that way. So, that is how it comes $\frac{\Delta}{3}$; so that way it comes and then further you are going to add as $\frac{\Delta}{3}\Delta_{ij}$. So, this is your, you know this part is basically what you get as the one which is the definition for the total strain part. And you will have the total strain as the deviator component as well as the, you know mean component.

Now similarly if you go to define the mean stress; so there also the stress also has 2 components. One is the mean stress component; another is the deviator component of the stress.

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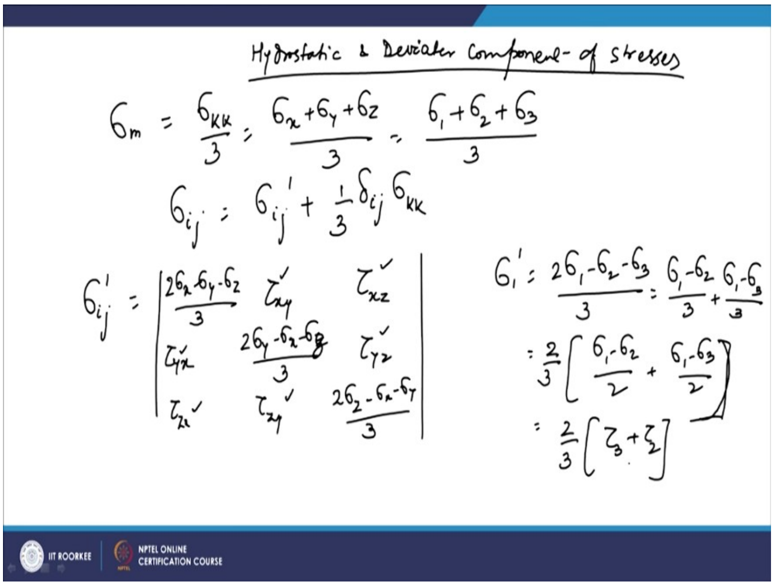

Now, so you have hydrostatic and deviator component of stresses. So, now again as we discussed that, in the case of strain, we found. Similarly in the case of stress also you have the 2 components; one stress will be required one stress, which is responsible for the change in the volume that is known as the you know hydrostatic stress or the mean stress and the stress component which is responsible for the change in the shape that is known as the deviator component of the stress.

So, again the hydrostatic component of stress that will be given as $\frac{\sigma_1 \sigma_{KK}}{3}$ or we also write it as $\frac{\sigma_x + \sigma_y + \sigma_z}{3}$.

$$\sigma_m = \frac{\sigma_{kk}}{3} = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

So, this way you write the expression for the deviator component of the, this mean component of the stress. So, further in the similar line if you try to decompose the total stress tensor into 2 parts; one will be your mean stress tensor another will be your deviator component. So, you can write it as

$$\sigma_{ij} = \sigma'_{ij} + \frac{1}{3} \delta_{ij} \delta_{kk}$$

So, this way you find this is your stress tensors decomposition and now what we see that since you have the total stress and from there you are removing the mean stress. So, you are getting the stress deviator portion; so, if you try to define the stress deviator σ'_{ij}. So, you can again write in the similar line

$$\sigma'_{ij} = \begin{vmatrix} \frac{2\sigma_x - \sigma_y - \sigma_z}{3} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \frac{2\sigma_y - \sigma_x - \sigma_z}{3} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \frac{2\sigma_z - \sigma_x - \sigma_y}{3} \end{vmatrix}$$

So, this way you get this expression for the deviator component of the stress.

So, what we have seen. Earlier we have seen that when we talk about the you know maximum value of you know shear stress that we had seen that $\frac{\sigma_1-\sigma_2}{2}$ or $\frac{\sigma_2-\sigma_3}{2}$ or $\frac{\sigma_2-\sigma_3}{2}$ and we had seen that the maximum value of the shear stress we. So, if you σ_1 is the algebraically largest value and the σ_3 is the smallest value; in that case the maximum value will be $\frac{\sigma_1-\sigma_3}{2}\frac{\sigma_1-\sigma_3}{3}$. So, that way now what you see here if you look this component we know that all these components this one, this one, this one or this one or this one or this one; they are all very explicitly you can see that they are the shear components.

But you make think of that this involves the normal components; now why they are part of this is stress deviator and the stress deviator basically is responsible for the deformation. So, it must also be a shear component; now in that case you can see that if you look at this, this can be written as if you; if you look at this you can. So, the if you it is taken as σ_1'. So, it will be $\left[\left(\sigma_x-\sigma_y\right)+\left(\sigma_x-\sigma_z\right)\right]/3$; so anyway if you can take the 3 part that side.

So, you can write $\frac{\sigma_1-\sigma_2}{3}+\frac{\sigma_x-\sigma_z}{3}$. So, so that way you see that $\sigma_x-\sigma_y$ that part again is a part which is something related to the shear stress part. Similarly $\sigma_x-\sigma_z$also is a shear stress part; so, basically you see that they are also the sum of the shear stress you know type of forces and that is why they altogether, they are basically representing a stress component which is responsible for the change in the shape of the body.

So, if you look at the

$$\sigma_1'=\frac{2\sigma_1-\sigma_2-\sigma_3}{3}=\frac{\sigma_1-\sigma_2}{3}+\frac{\sigma_1-\sigma_3}{3}$$

Now, so that you can write as 2 by 3 if you take it as common. So, you can write as

$$¿\frac{2}{3}\left[\frac{\sigma_1-\sigma_2}{2}+\frac{\sigma_1-\sigma_3}{2}\right]$$

Now, so what you see is you can write it as

$$¿\frac{2}{3}[\tau¿¿3+\tau_2]¿$$

So, where τ_3 and τ_2 they are the principal shearing stresses; so that is what we had seen earlier.

So, what we see that normally your this part this $\sigma_1^{'}$ part also has the components which has the shear components. Now this shear part which we have seen now it is also seen to be the strength deviator part, this is seen to be a second rank of tensor. So, now the principal value of these stress deviator; they are also the root of a cubic equation. So, basically if you try to find its roots. So, you have to further solve that and they will be the, you know. So, principal values of these you know stress deviator are roots of cubic equation.

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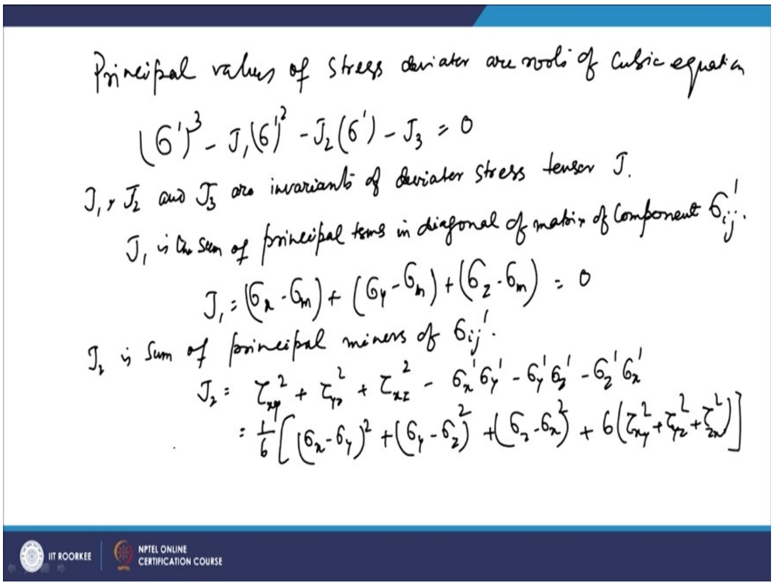

So, if you find that you will get

$$(\sigma^{'})^3 - J_1(\sigma^{'})^2 - J_1(\sigma)^{'} - J_3 = 0$$

So, what we see is that here again the σ J_1, J_2 and J_3; they will be working as the invariants of the deviator stress.

So, J_1, J_2 and J_3 are invariants of deviator stress tensor J. Now the thing is that we have seen earlier the 3 you know components of 3 deviator this 3 invariants of this stress tensor total stress tensor there. And in that we got the different first second and third invariants of the stress and first invariant as we recall it was $\sigma_x + \sigma_y + \sigma_z$ then second was again you had the expression for it.

So, similarly; so, that was the total now if you talk about this is stress deviator again it has these you know invariants and if you try to find the value of these invariants. So, what you see is that J_1 is the sum of; so, J_1 will be the sum of the principal terms in the diagonal of the matrix of the components. So, with there; you had seen that in the first case in the total case it was $\sigma_x + \sigma_y + \sigma_z$. So, here also J_1 will be basically the, this principal terms which means there along the diagonal of the term; so it will be the summation of that.

So, J_1 will be the sum of the principal terms in diagonal of matrix of component σ'_{ij}. What we saw in the earlier case that this is your σ'_{ij} and this component this component and this component. Now these 3 components its sum basically will be the J_1; so

$$J_1 = (\sigma_x - \sigma_m) + (\sigma_y - \sigma_m) + (\sigma_z - \sigma_m) = 0$$

; now you have the second component J_2 and J_2 will be again. So, there also we got that is some of the principal minors; so J_2 is sum of principal minus of σ'_{ij}.

So there also we get these second invariant in the similar fashion and if you try to find the expression for J_2; so,

$$J_2 = \tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2 - \sigma'_x \sigma'_y - \sigma'_y \sigma'_z - \sigma'_z \sigma'_x$$

And that may come if you try to simplify this will be

$$¿ \frac{1}{6} ¿$$

So, that comes as the second invariant of the strain deviator; I mean stress deviator. Then third invariant will be basically the determinant of this equation; so the third invariant comes as the determinant of the equation.

So, this way what we see that you have 3 components. So, what we have discussed that in the case of these stress stresses; totally stresses if you look at. Now the thing is if you see when we talk we will talk later on about the you know different type of theories which relate to the failure of the material; in those cases because as we know that when we talk about the deformation it is it is by shear mechanism.

So, it is because of this component of the stress that is your deviated component of the stress. And this part the second invariant of the stress component part this part will be utilized when you go to find the theory for the failure of the material or for the; you know different type of theorems you have. So, you have Von Mises theorem and you have the Tresca. So, Von Mises theorem will be used in that case basically the theorem tells that this value; the second invariant of the stress tensor of deviatoric stress tensor that must reach certain critical value; now this is used in those components.

So, basically the mean component does not take part in to these you know change in the shape. Basically it talks about the change in the volume and based on that you have certain terminologies. So, that we will discuss in the later part and this this basically this deviator part, this will be mostly utilized for because they are basically responsible for; so that mean part is normally you know, they are mostly important when we talk about the smaller deformations per small change in volume or so, so that is in the elastic range.

And when we try to talk about the plastic range that time this part is mostly of use. So, that is about this mean part, as well as the deviator part or deviator component for the stress and strain which will be utilized its concept will be utilized in our subsequent discussions.

Thank you very much.

Principles of Metal Forming Technology

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Lecture -10
Elastic stress strain relationships

Welcome to the lecture on Elastic stress strain relationships. So, we talked about the stresses and strain and basically we have to relate the stress with the strain and normally you have the constitutive equations basically which basically relate with this strain with the stress.

So, as we know that we have the measurement of strains and from there you find the value of stresses. So, you can find the stresses and the relation between these stress tensor and the strain tensor and they are basically found from the constitutive equations.

Now, we are going to discuss about the elastic solids only in this lecture. So, normally what we see, we have already we know that you have in the elastic reason you have the application of Hooke's law and ah if you talk about the Hooke's law. So, what we see in the Hooke's law basically stress is proportional to the strain.

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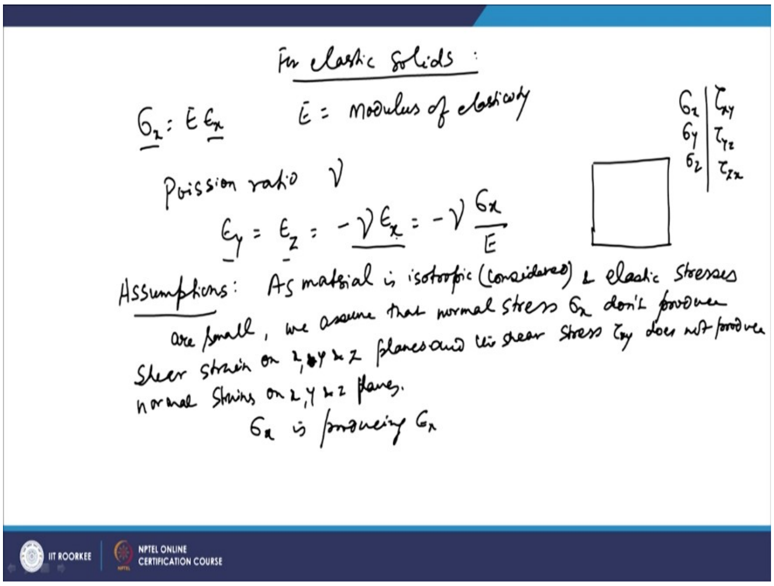

So, for elastic solids, you have the you know we know that the Hooke's law is valid in the elastic zone. So, we say that the σ_x is basically a constant times the strain. So, that is stress is proportional to this is a stress and this is a strain stress is proportional to strain and then we give a proportionality constant and this E as we know this is known as the Modulus of elasticity.

So So, this modulus of elasticity, it is either in tension or in compression and as we know that when we provide the you know tensile force, we when we apply a tensile force in the x direction. So, there will be elongation in the x direction. But it will be producing a compaction basically; a contraction in the transverse directions that is in y and z direction.

So, basically this transverse strain which you get you have one is the longitudinal strain. This is a strain in the direction of the stress and the other the strains which are there in the other directions; now they are known as the transverse strain. And it was found that this transverse strain, it is a constant fraction of the longitudinal strain. So, it was seen in most of the solids that this transverse strain which we get its a constant times the longitudinal strain and basically that way this Poissons ratio that is defined.

So, you have the Poisson ratio. So, that is defined that is nu and this nu basically it is the ratio of ah these the two types of these strains and they are normally found.

So, that is ratio of the transverse strain to the you know this longitudinal strain and it is found to be basically in the range of something like close to 0.33. It is for most of the solids. So, actually what we see is that this strain in either y or in the z directions, if you talk about the x direction as the longitudinal one. So, you will have the transverse directions as y and z.

Now, this strains in the y and z direction, they can be said to be $-\nu\varepsilon_x$. So, if there is a you know elongation in the x, you will have the you know contraction in the y and z. So, that is why you have negative sign and then it is multiplied by this ε.

So, epsilon will be nothing but the ratio of $\varepsilon_y/\varepsilon_x$and that of minus sign. Because it is contraction and this is expansion. So, so what you see is that you get $-\nu\varepsilon_x$is σ_x/E

. So, this way you have this is the co relationship which is used for finding the ε_y or ε_z when you get the ε_x.

Now, when we are talking about the, you know 3 dimensional state of stress. So, in those cases, so this is for the when we talk about. Now when we talk about the 3 dimensional state of stress and the strain, then in that case suppose you have a cube. Suppose you have a cube. So, it is it is subjected to the stresses σ_x σ_y σ_z and τ_{yz} and τ_{zx}. So, suppose it is subjected to that in those cases. Now you will have basically the elastic stresses.

So, they are normally smaller and the material being isotropic in nature. So, you have to assume. So, you have to assume that the, this normal stresses that is σ_x, they do not produce any shear strain. So, that is on x y or z plane or the shear stresses that is τ_{xy}, they are not producing any normal strain on the you know in the x y and z planes.

So, this assumption is being made and then, we apply the principle of superposition. So, you have certain assumptions you can think of and we what we assume that we assume that as the material is isotropic. So, we consider the material to be ah isotropic and the elastic stresses are small.

So, what we assume? We assume that the normal stresses basically a σ_x suppose they are not producing any shear strain. So, normal stresses normal stress σ_x donot produce you know the shear strain on x, y and z planes; x, y and z planes.

Similarly, the shear stresses which are; so, you have normal stress has σ_x, σ_y and σ_z. Similarly you have shear stresses at τ_{xy}, τ_{yz} and τ_{zx}. So, and and the shear stresses and the shear stress that is τ_{xy}; now that do not produce does not produce normal strain so, that on x y or z plane.

So, this is the assumption which is made and then, we try to find the these expressions by the principle of superposition and then we try to find the expression for ε_x, ε_y and ε_z for the you know you have σ_x, σ_y and σ_z; so, based on that you can find it.

So, what you can do is that suppose you have the stress σ_x; so, we have the stress σ_x. Now this σ_x is producing ε_x; but it is also producing ε_y as well as ε_z that is why we have seen that ε_y and ε_z that will be produced that will be minus of $-\nu\,\sigma_x$. So, this is further used. So, that can be used by referring to you can try to find the in the tabular form how can we find its a expresses suppose you have the stress, you have individual stresses and now, if you have the stress and if you have the strain in any direction.

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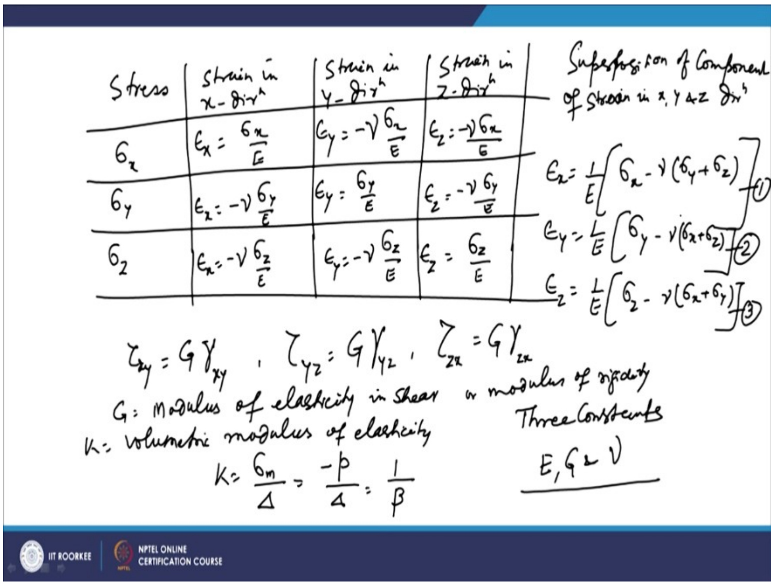

So, if you have a strain in suppose x direction. Similarly, you have a strain in y direction and you have a strain in z direction. Now, if you look at this. So, now, you have you know that you have the normal stresses are σ_x, σ_y and σ_z.

Now if you look at this, we know that σ_x will produce the stress strain in x direction as σ_x; σ_y will produce in y direction as σ_y and σ_z will produce z direction as sigma z and this is ah this you know that if this σ_z is the stress, then σ_z/E will be the strain in the z direction. Similarly, this will be σ_x/E and similarly this will be σ_x/E. So, these are the longitudinal strains the strains in the direction of the stresses.

Now, in the y and z direction; now in the y direction, we will use that property of the material that is Poisson's ratio. Now in the poisons, if the Poisson ratio is used the σ_y will be basically will be $-\nu$ times and then; that will be σ_x/E. So, that we have already defined this property of the material. Similarly σ_z will be $-\nu$ times σ_x/E.

So, similarly, here σ_x will be $-\nu$ times ε_y/E and this will be ε_z will be $-\nu$ times ε_y/E; similarly you have σ_x as $-\nu$ times ε_z/E and ε_y also will be $-\nu$ times ε_z/E. That is what we see that if you have these stresses, they are telling you they are giving you these strain components.

Now you can have the superposition of the component of so uses. So, superposition of these you know component of strain in x, y and z direction. This is strain in x, y and z direction; if you see that these three expressions. So, you have sigma x you can find.

So, you can find σ_x as $1/E$ and then, it will be

$\sigma_x - v(\sigma¿¿ y + \sigma_z)¿$

. If you look at this σ_x fine so, $\frac{\sigma_x}{E} - v\frac{\sigma_y}{E} - v\frac{\sigma_z}{E}$. So, that is what 1 by E will be common $\sigma_x - v$ times. So, the similarly you will have σ_y has $1/E$ and you will have $\sigma_y - v(\sigma¿¿ x + \sigma_z)¿$ and similarly $\sigma\varepsilon_z$ will be $1/E\ \sigma_z - v(\sigma¿¿ x + \sigma_y)¿$.

So, this way we get the expression for ε_x, ε_y and ε_z. Now these are shearing strains will also be generated and for the shearing strains, as we know that shear stresses produced the shear strain. So, that will be for the for the further unit cube, you can write for that τ_{xy} and τ_{xy} will b $G\varepsilon$ this γ_{xy}. So, γ_{xy} is the shear strain and similarly you will have τ_{yz}. So, that will be G times γ_{yz} and similarly τ_{zx}. So, that will be G times γ_{zx}. So, now here these proportionality constant which we use in the case of shear strain that is G.

So, this G is known as Modulus of elasticity in Shear. So, this you have modulus of elasticity in tensor or compression that is E and you have modulus of elasticity in shear that is defined as the G and we also call it as modulus of rigidity. So, this value of G will be basically derived from the Torsion test we find it. Now the thing is that we got these expressions and for elastic and isotropic solid, you have 3 constants. What we see you have three constants, E G and v. Now again another elastic constant is there and that is your volumetric modulus of elasticity.

So, if you go to that next type of ah elastic constant that is you know where because of the volume change, we have already seen that the, they are going to change the volume, the stresses. Now this volumetric you know modulus of elasticity is K. So, we defined the volumetric modulus of elasticity as K.

Now, this volumetric modulus of elasticity, basically here we know that these volume change is because of the hydro static component or mean component. So, you have a stress as the hydro static stress and then the volumetric strain will be the strain parts. So, that will be the ratio of K will be the ratio of the mean stress and ah. So, it is also known as the bulk modulus K. K is also known as bulk modulus and it will be a ratio of the hydro static stress that is σ_m or mean stress divided by the volumetric strain.

So, that is you know we have already discussed about it. And, so, σ_m divided by. So, this is K will be. σ_mSo, as we have a already defined the volumetric strain. So, that is σ_m divided by this and then, that will be. So, σ_m will be -p that is your hydro static pressure. And, so, this is basically defined by another parameter that is $1/\beta$. So, β is basically the compressibility. β is compressibility that will be opposite to the; so, inverse of the K, bulk modulus of elasticity or volumetric modules of elasticity.

So, as we know that it should it will be the inverse of that if we K is more, β is less or if the β is more, K will be less. So, basically p is the hydro static pressure and being compressive in nature being try to define it as -p and then, you define this β and that is why we define it as the volumetric modules of elasticity. Now from these you know constants you have E, G, v and K; here from you can have the different types of relationships between these elastic constants.

So, you can have this relationship between E, G, nu and K. So, we have already got these expressions. Now the thing is that if you try to find from the these table what we saw this table which we got ε_x, ε_y and ε_z.

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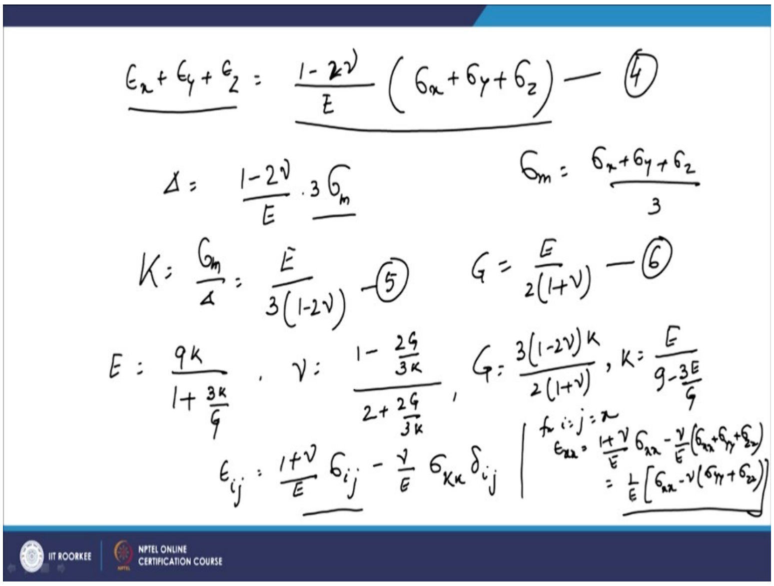

So, if you add them what we get is $\varepsilon_x+\varepsilon_y+\varepsilon_z$, you can get as $1-\nu$; $(1-2\nu)/E[$¿$\sigma_x+\sigma_y+\sigma_z]$¿.

So, this can we you know we can if we add these this is three equations, if you these if you add this 1 and this 2 and this 3, if you add these three equations what we get is $\sigma\,\varepsilon_x+\varepsilon_y+\varepsilon_z$ will be $1/E$ times and then, $(\sigma_x+\sigma_y+\sigma_z)-\nu$ times, every this is coming two times. So, that is why it will be 2ν. So, that is why $(1-2\nu)/E[$¿$\sigma_x+\sigma_y+\sigma_z]$¿. So, that is what one of the you know you are getting one of the expression for this and we can call it as equation 4.

So, if you add these 3 equations as we discussed that if you add these 3 in the the, that equation here. So, you get this particular equation. So, this is one of the equation which we get. Now the thing is that we can have other you know expressions and what we see is that you have δ.

And that will be $(1-2\nu)/E[3\sigma_m]$. So, in this case, what we see this in this expression itself this is $3\delta_m$. So, as we know that mean stress is coming as $(\sigma_x+\sigma_y+\sigma_z)/3$.

So, that is why this comes as 3 times sigma m and we get δ as $(1-2\nu)/E[3\sigma_m]$. So, what we see is we have seen that K is nothing but you have σ_m/δ and that you will get as $E/3(1-2\sigma)$. So, this is this is ah K and this σ_m/δ will be k. So, K can be put here.

So, in that case what we get is K will be $E/3(1-2\nu)$. So, this way you can have the different relationships between these stresses. You can have this is constants. You can have other you know other important expression have been found and other important expression which we get is you also get G equal to $E/2(1-\nu)$.

There are many other important equations like you have you can have E equal to $9K/$¿¿. Similarly you have nu as ¿.

So, this way you have another expression can be found you can have another expression which we get is [3(1-2ν)K]/[2(1+ν)]. Furthermore we may have a expression like K equal to

$E/[9-(\frac{3E}{G})]$.

So, this way, you get the different type of correlations or relations between theses constants and from here, if you know suppose you know here in this expression you know K and G; so, in that case you can find E or so, or you know ν. So, you have to find nu and you go know K and G. So, you can find the value of ν by knowing these K and G.

So, so that is how you find the correlation between them. Now the thing is that after this ah, we need to have basically the way how to find the stress is stresses from these elastic strains. So, before that we also see that you have you can write in the other form also Tonsorial notation an form also and there are different ways also to write these expressions in the tonsorial forms.

And some of these forms which we get suppose you want to write in the tonsorial form, you can write as ε_{ij} will be $1+(\nu/E)$ and then, $\sigma_{ij}-(\nu/E)$ and then, you can write σ_{kk} and δ_{ij}. So, this way you can write in the tensor form because you know that this will be the normal this is σ_k; this is σ_x, σ_y or σ_z will come here and this will be talking about all the i and σ_{xy} or σ_{yz} or so. So that way you may have.

So, if you do you can have this expression in the tonsorial form. Now if i or j or x, they all are you know equal; in that case you will have the expression like $\sigma\varepsilon_{ij}$. So, σ_{xx} will be actually $1+(\nu/E)$ and then this will be $\sigma_{xx}\nu$. So, this if you σ i or j or K all that we taken as x in that case you will have $1+(\nu/E)\ \sigma_{xx}$ and then again $-(\nu/E)$ and then σ_{kk}. So, you will have $\sigma_{xx}+\sigma_{yy}+\sigma_{zz}$ and then z that can be taken.

So, this way you will have again, so, $1/E$ will be coming up and you will have a expression like $\sigma_x-\sigma_{xx}-\nu(\sigma¿¿yy+\sigma_{zz})¿$. So, this type of expression; so, you can write it here. So, suppose you have for $i=j=x$ you can write σ_{xx} as $1+¿¿$. So, you can write put here and then, this will be $\sigma_{xx}-¿¿$ of $\sigma_{xx}+\sigma_{yy}+\sigma_{zz}$.

So, that can further we written as $1/E$ and then that that σ_{xx} will be ah. So, which is coming out and you will have $\sigma_{xx}-\nu(\sigma¿¿yy+\sigma_{zz})¿$. So, because this $-\nu\sigma_x$ and $+\nu\sigma_x$, xx that is cut down. So, you will have this expression$\sigma\varepsilon_{xx}$, you can find if you in the terms of E σ_{xx} σ_{yy} and σ_{zz} and ν.

So, ah further for suppose i is x and j is y in those cases also you can have the expressions; where the z component will not be there. So, this way you have different type of expression that can be found. Now the thing is that once you have these strains found; then, from these strains you have to calculate the value of these stresses and for that you can basically add so we have already seen.

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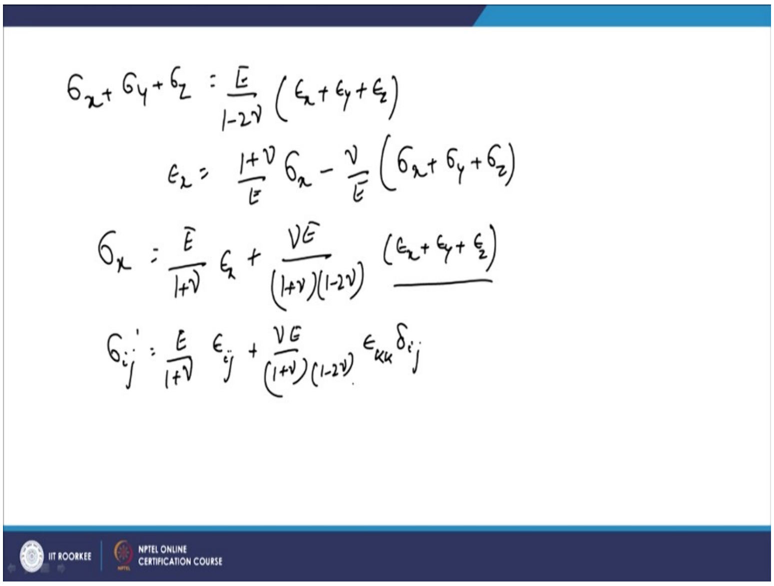

So, we have seen that what we have seen is $\sigma_x+\sigma_y+\sigma_z$. This we have seen from here. So, if you add this $\sigma_x+\sigma_y+\sigma_z$ and that can we seen as$E/(1-2v)[\varepsilon_x+\varepsilon_y+\varepsilon_z]$.

So,. So, this way you can find the value of ε_x and ah now you have another equation on from there, you can have the expression for ε_x and that will be $1+(v/E)$ and then you have

$\sigma_x-\frac{v}{E}[\sigma_x+\sigma_y+\sigma_z]$.

So, further you can have you have we have discussed about these two equations and from these two equations, if you try to substitute the value from this into this. So, you can have the expression for σ_x and that you can get as $E/(1+v)[\varepsilon_x]+vE/(1+v)(1-2v)[¿\varepsilon_x+\varepsilon_y+\varepsilon_z]¿$.

So, that will be again you will have the mean strain. So, you can have this will be this can be further you know expressed in terms of mean strain. So, you can again further denote them in the tonsorial form and you can denote them as in the tonsorial form as $E/(1+v)[\varepsilon_{ij}]+vE/(1+v)(1-2v)[¿\varepsilon_{kk}\delta_{ij}]¿$.

So, that is your mean strain. As we know this is the mean strain this can be further expression terms of immune strain. So, you can write them as that and basically this, this part this part is known as $\varepsilon v E/(1+v)$ into this that is known as a Len's constant. So, we can further interpret it in other terms. So, this way you we have seen that the stress and the strain which are basically converted into the mean as well as stress mean and that deviator parts and from there you can find the different you know values depending upon the different values given.

So, this way I mean this is about you can further study about it more and more and you have two conditions which will be coming to us of for the considerations and they are the plane stress and the plane strain conditions when in one of the directions, the you know stress is 0; σ_3 is 0 suppose then, it is a case of the plane strains and similarly you have the plane strain conditions also. So, that way you have plane stress condition is normally example of thin loaded sheet when the it is loaded in the you know thin sheet loaded in the plane of sheet.

So, that is example of the you know plane stress condition and also when we talk about the thin strain I mean plane strain condition. So, just like a long rod or cylinder you, so, where one dimension is quiet larger; then, the other two. So, that is example of the plane strain conditions you know cylinder which is has restrained ends.

So, that way you have the plane strain as well as plane strain conditions where either stress value or strain value in one of the direction is taken into 0. So, these are the cases which will we coming up for discussions or reference when we talk in our analysis in the of the forming parts. So, so that is about these you know this lecture.

Thank you very much.

Principles of Metal Forming Technology
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Lecture – 11
Introduction to theory of plasticity and flow curve

Welcome to the lecture on introduction to theory of plasticity and flow curve. So, we had seen about the elastic theory. In the last lecture, we discussed about some of the relationships where the stress and strain is related and we got certain relationships between some constants like e g k and ν and all.

So, now, will move towards the theory of plasticity because in the case of metal forming, this theory is important you know for knowledge; because when we are going for plastic deformation, it’s beyond the, you know yield limit. So, basically it is not in the elastic range. Basically we are going into the plastic range.

So, basically the deformation is to a large extent in those cases and so, there is need to know about it because and the case of elastic deformation we know that, there is there are rules like Hooke’s law. Once you know the strain, then you can predict the stress by using these Hooke’s law and others, but that rule is no longer valid, when you are going to the plastic range.

So, what how these, what are the, you know theories in plasticity; how plasticity, what is the plastic flow curve, how the material behaves when it is elongated or it is deformed. So, all that we will have some introduction about its theories.

So, coming to the, you know; first of all we will talk about the flow curve. So, if you talk about the flow curve, actually the flow is related to the plastic flow of material.

So, when we talk about the stresses, we normally talked about the stress that is engineering stress in earlier case; when we talk about the elastic reason. For once we go to the plastic reason, then the engineering stress does not have much of the significance because once the deformation starts, in that case the area is basically reducing and you will have not you cannot find the stress by dividing with the original area. So, your area is basically changing. Instantaneous area is coming into picture and in that case, you have a different type of curve that is known as the true stress, true strain curve.

So, this curve basically is known as the you know flow curve. So, flow curve basically the, it is the curve between the true stress and true strain and that is known as the in the flow curve. So, what is seen that if you draw the typical you know flow curve; if you see. You talk about the normal engineering stress and then you talk about the you know true stress and true strain.

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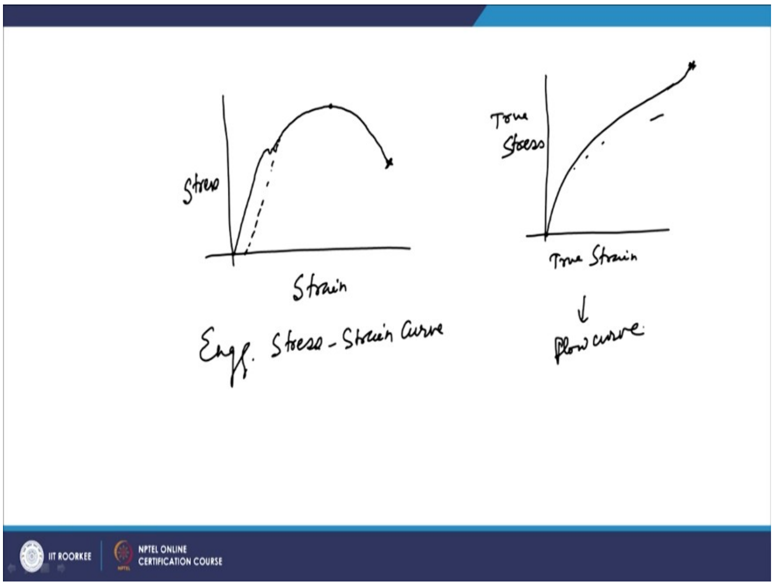

Now in the case of normal engineering stress is what happens that you have, this is the stress and this is strain. Now in this case, we assume so in that case, we assume that the area is same. So, what happens that you, the curve goes like this and then there is deep and then it comes and then it comes like this? And here is the point of facture. Now in this case, this is the point which is the maximum you know, you get the maximum point here; this is pointing related to the ultimate tensile strength of the material.

Now, in this case this is the yield point, you have certain region up to which you have the proportionality holds good and then, this is the in this is a reason where the strain hardening is taking place. And ultimately at this point, the material will be you know after that and this is will be the fracture point after that, we in fact, the stress value is coming down. And ultimately your this because the load which is there load will be basically not required, load will not be more. So, basically in this case you are getting the stress as load by original area.

So, what you sees that in the case of engineering stress strain curves. So, this is your engineering stress strain curve. Now in this case, you see that this is the maximum, but after that the load is decreasing and then ultimately, this is the point of fracture. Now thing is that when we talk about the, actual stress which is calculated; what we see is that after this point, actually there will be necking.

Now the thing is that I mean there will be yielding. So, this is the point of yield and after that as we know that, since it goes into the plastic range. So, before these if you are leaving the material, you are not you are removing the loading on the material; in that case the material will come to its original position ah. So, there will not be any strain, but once you are going beyond this point. So, suppose had here, so it will come like this. So, there will be certain strain which is leftover.

So, that is basically the plastic strain, but the thing is that after this, there is this is not actually the actual stress because your, once there is a elongation in that case; as we know that once there is elongation, you have the increase in length in certain direction. Then in the other two transfer directions, you have the contraction. You have the change in the dimension depending upon the property of the material.

So, basically the cross sectional area does not remain the same. So in fact, this is not the true representation of the value of the stress which is there at a particular instant. So, what we do in that case is that you try to find this and that the stress which is based on the actual area at particular instant. And in that case, that is done by separate curve and this curve, this curve is also the stress versus strain curve, but this is true stress and true strain curve.

Now what happens in this case as you see that it will go and then it will go and somewhere it will fracture? So, this is the fracture point in this case you see that it will go up to certain points. There are stress is propositional to strain. So, with the before that range, it will be behave in the similar fashion, but after that it goes like these and then it deeps, but here is does not deep. So, basically after this reason; if you go in this reason now this reason is basically the strain hardening zone. So, here the because the material is strain hardened and that is why stress value goes on increasing and ultimately it will go till the point where the fracture is initiated.

So, basically you need require the stress which is required to plastically deform the material and that stress value is known as the flow stress of the material. Now in this case this true stress true strain curve, this is basically known as the flow curve. Now this curve basically, it is giving you actually the stress which is required to cause the metal to flow plastically. So, that is why that is known as flow stress and this curve is known as the flow curve. Now what are the characteristics of these flow curve or the true stress true strain curve for a typical ductile materials.

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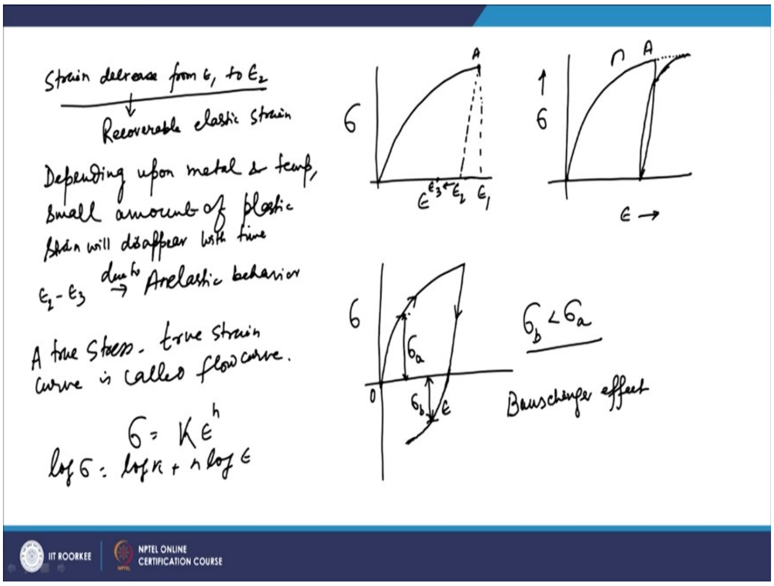

So, now let us that we have to see, how this true stress true strain curve or flow curve looks like for different type of materials. Now suppose you have a material which is ah, so we are talking about it ductile material. But you see that if you look at the typical flow stress curve. So, you so, flow stress true stress is denoted by this and this εσ and ε.

Now in this case, this is the typical flow stress of the curve. But the thing is that suppose we have gone up to point a up to that it is loaded. Now from here ah, when the load is released in that case the total strain, mean we have reached up to this point; but then once we release the load in that case immediately so you this suppose this is ε_1. Now once we release that the load is released, the total strain will immediately decrease. So, it will immediately decrease from ε_1 and it will come to ε_2. So, this is upon the immediate release of the (Refer Time: 10:45).

So, you are here at this point and at this point the total strain is ε_1, but once you release that load immediately, the strain comes back to ε_1 to ε_2. Now this is because of the elastic nature, elastic property and you know this $\varepsilon - 1$ ah. So, this basically will be nothing, but it will be equal to σ / e. So, as we know that this is strain which is elastic strain this is nothing, but the stress value divided by the module of so, rigidity. And that value σ_1 to σ_2, this is known as the recoverable elastic strain. So, strain decrease from ε_1 to ε_2 and this is known as recoverable elastic strain because this amount of the strain is anyway recovered. So, that is why it is known as the recoverable elastic strain.

Now still the strain which is here, now that is ε_2 is still that is not the permanent one. So, basically what happens, the depending upon the metal and that temperature conditions; a small amount of even the plastic strain will disappear with time. So, depending upon metal and temperature, the small amount of plastic strain will also disappear with time.

So, now what happens that we saw that ε_1 to ε_2 that is your recoverable elastic strain. And then it will be you know, somewhat going towards this with time and then ultimately it may come to this point that is ε_3. So, this $\varepsilon_2-\varepsilon_3$; so this is basically the because of the anelastic behavior. So, this is known as an elastic behavior of, this is due to anelastic behavior.

So, normally because of the complexity, we normally assume that this is there is no such phenomena which occurred in the in this figure. Now what we see that in normally, it will be coming to ε_1. This is total strain that will become going to strain to ε_2 because of the recoverable elastic strain and then ultimately it will be coming towards ε_3 because of these anelastic nature.

I think is that suppose, now this we are further loading it. So, suppose you are further loading. So, you are coming back and then you so, your flow curve is like this A. So, this is your point A. Now from here, what you did is so, now it goes it; it comes down and then it further if you are further loading it, then how it will behave. So, actually when you unloading the you know this specimen from here; so unloading will not be actually completely parallel to the elastic reason of this curve. Then now once you are unloading, so it will come to this point.

And if you are further loading it, so now, if you further load it will go like this. So, it will come to a smaller value and then if that that goes so, it will be going like this. Now what does that mean? So, this means so if you look at these curve, you have unloaded so, this is the cycle in which it has come down. So, this is your true strain and this is your true stress.

Now the thing is that when you have unloaded, then if you (Refer Time: 15:33) it, what you see that further it will be an extension to this curve only with some more plastic strain. At this point what you sees that this becomes an extension of the same curve which was coming. So, what you see that the reloading curve, it will be basically bending over and as the stress will approves the linear value have been the original value and from where it was unloaded and some additional plastic strain is experienced and after that we what we see is you see the same curve which you have got as then extension of the original you know flow curve which was seen by us.

So, what you see is that there is a hysteresis behavior in such cases, in the case of true stress to strain, when you are loading and unloading then what you see that there is a hysteresis type of loop which is formed and this is terraces behavior normally we are neglecting, in the case of these are true stress true strain analysis.

Further going to the another aspect, if you see this is this specimen is basically strained in tension. Now the thing is that if the, you are doing this. So, once you have suppose, you have deform the material beyond the yield point plastically in one direction. And then, suppose in the tension and then you are further unloading it to 0 and then further you are trying to reload in the opposite direction, then how it will behave?

So, if you look at the flow curve it goes like this. So, it will come here and you are further unloading it coming to 0. So, here you have, this is as σ and ε. Now the thing is that this is suppose your yield, yield stress in tension and so, you have gone loaded in the direction and further you have reloaded when unloaded and coming to the zero stress state. And further you are loading it in the opposite direction that is in the compression.

So, if you are going in compression. So, it behaves like this. So, suppose in the compression what you see you get the, this is the yield point yield stress in the back side, in the opposite side. And what you see that normally; when you go in the opposite direction, you do load in the opposite direction. In that case, the σ_b which you get and if this is your σ_a basically in that case what you see is that σ_b will be less than σ_a.

So, basically what you see that your yield stress. Yield strength is dependent upon the nature of loading and loading path and the direction. So, this type of effect, this type of behavior is known as the Bauschinger effect. This type effect, which is there because of which the yield stress strength will vary in one particular direction, it will be lesser in one direction; it will be more in other direction.

So, this is because of the loading path direction and this is known as the Bauschinger effect and the Bauschinger effect is basically neglected again in the case of the flow curve. So, it is neglected in the theory of plasticity and what we assume that yield strength in the tension as well as in the compression yield stress value is same. So, this is known as the Bauschinger effect.

Now the value as we already knew that the you know the graph for the curve between the true stress and the true stain is known as the flow curve. So, the flow curve is it will be giving you the stress. So, the we can write that it true stress- true strain curve is called a is called flow curve.

Now flow curve, it comes from the fact that flow stress; it gives you the flow stress value that value of stress which is required for the plastic deformation of the material at a particular given value of strain so, that that can be found from this curve and it has been tried to give a mathematical correlation to this mathematical relationship to this. And most common mathematical relation which describes this flow curve or plastic behavior of the material is σ equal to $K\varepsilon_n$. So, σ is the true stress value flow stress value.

ε is the strain true strain value, K is known as the strength coefficient; K is nothing, but if you take and as basically ε as 1. So, σ will be K. So, will be nothing, but it is stress at a ε equal to 1. So, that way you can find the value of K and n is known as the strain hardening exponent. So, you can have the, if you take the log on both the side $\log\sigma$ will be $\log K + n\log\varepsilon$. So, that way you can have log σ will be $\log K + n\log\varepsilon$.

So, you can get n as $\log\sigma - (\log K/\log\varepsilon)$. So, that way you can find the you know this n. So, that is basically if you have a log curve, so it will be the slope y = mx + c. So, if you take as y = m * x + c. So, if you take $\log\sigma$ and $\log\varepsilon$ $\log\sigma$ as the y axis and $\log\varepsilon$ is the x axis. So, this n will be the slope of that linear curve that will be talking the log log curve for the σ and ε and it will be the strain hardening exponent. So, this way we this n value will be varying for the materials.

So, now the this flow curve will be different for different types materials and materials maybe you know ductile or brittle and the flow curve basically will be giving you the flow stress which is required for deforming the material plastically at any particular value of the strain.

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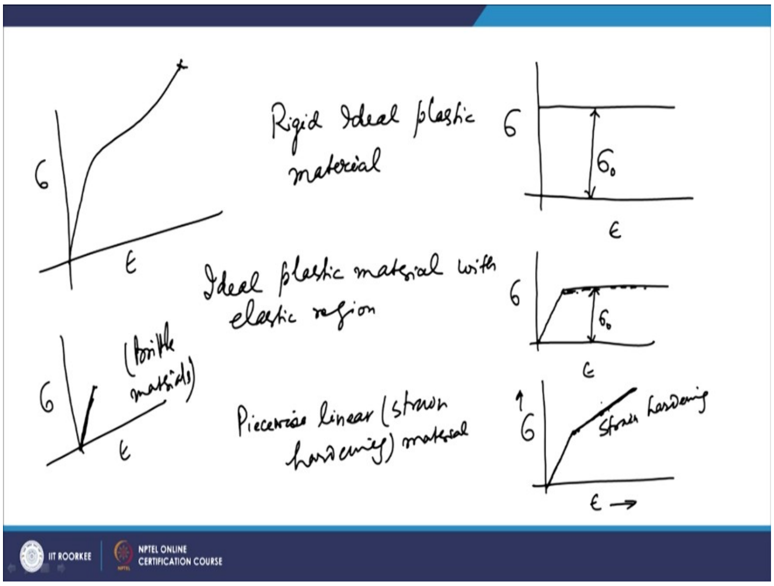

So, you can we can have the glimpse of the different types of flow curves. Suppose you are drawing the flow curve for the idealized, you know rigid ideal plastic materials. So, if you are trying to draw the rigid, I mean idealized flow curve for a rigid ideal plastic material.

Now if you look at this rigid ideal plastic material, now in this case what happens that the flow curve which is σ and ε. Now it will start like this. So, you will have this as the σ_0 value. So, this is actually the rigid ideal you know plastic material. Now in that case, you have a constant flow stress value and you will be having the deformation this way.

Now if you talk about the ideal plastic material with elastic reason. So, as we have seen that once you have the elastic reason, in that basically when you are going to deform the them so, up to certain point, your Hooke's law is valid. So, in that actually stress is linearly proportional to strain. So, if you try to draw the idealized flow curve for that, for a material which is ideal plastic material with some elastic reason. Now in this case, so in the elastic range; it will go like this and then since it is a ideal plastic material; so it will go like that. So, this will be your σ_0.

So, this curve tells that initially when you are trying to deform it. So, it will be going in the elastic range and the elastic range basically stress will be proportional to strain. So, that way it will be going and then add the ones that yield point is reached, then is being the ideal plastic material; so with constant stress value the material will flow. So, this is the idealized flow curve for the ideal plastic material with elastic reason.

Now more importantly we required to know the, you know idealized curve for a material which is piecewise linear. So, what is there? In suppose you have a flow curve which is of these states, what we see? Now what we see? That in this case you have this is the linear one and this is basically the at this point the flow stress remains constant and the material is getting deformed at constant flow stress value. Now what happens that when you have the strain hardening material as we see in normal typical cases in the ductile materials. So, what happens that in there up to the yield point where we reach? Now up to that you have so, in the elastic reason you have the you know straight you know you have a straight line with slope.

Now after that still you have you know an increasing value of a stress. So, what if you recall in earlier side, we had seen that your topical goes like this. Now this is not basically linear. So, this is a reason which is basically the reason of strain hardening. Now in this reason when basically when the plastic deformation starts, if you try to see it metallurgical after that you know you your you require larger stress to remove to move the this location. So, in that case the true stress value basically increases as the strain is increasing. So, if you look for a material with one idealized; suppose you have a piecewise linear material.

So, here you have the strain hardening zone. Now what we see is this will normally we seen encountered in typical ductile materials. In those cases you have the up to this reason, this is linear and again this is a linear, but here it is further increasing and this is the reason of strain hardening because there is straining taking place and as the plastic strain is increasing, the material gets hardened and hardened. So, hardness is going on increasing.

So, that is why this is the strain hardening zone; strain hardening is taking place during this process. So, that is when you know clear in many of the ductile materials, in most of the typically ductile materials which have which are formed. So, when you cold form them, in those cases you can see that you require. So, larger stress every time you wants to cold form, then you further try to cold form then you required larger stress value for the same incremental strain.

So, this is because of the strain hardening effect because of this strain hardening effect, the amount of stress required for the deformation for the strain becomes larger and larger. So, this is for the typical of for typical you know materials for suppose a brittle material is there or for a ductile material is there, for ductile material; it will move like this for brittle material, it will come and then it will fracture somewhere. So, and for brittle material it will come and stop somewhere. So, for an ideally brittle material, it will come and stop here because there is no plastic deformation in the case of the brittle materials.

So, this is normally for brittle materials and for the ductile materials, it will move and go towards you know increasing and then ultimately it will fracture at some point. So, this way you have and maybe for some materials you may have some degree of you know ductility. So, in those cases it will come and then further it will move towards this side and then it will fracture.

So, with the limited ductility, it will further extend towards the strain hardening zone and then there it will basically you know fracture. In case of brittle materials, what happens that you have cracks? So, it will go up to with without much of the appreciable deformation, much of the these strain it will go and then there itself, here it will be fracturing.

So, this way the flow curve for different types of materials can be seen and can be visualized and from they are you can have this value of the so, strain hardening exponent can be found for these material.

So, as the slop will be larger and larger, it will be more you know you know n value will be more and more. So, that is how the slope will be changing for the different you know material. So, that way you can analyze the different materials stress strain true stress true strain curve that is flow curve.

Thank you very much.

Principles of Metal Forming Technology

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Lecture - 12
True stress and true strain

Welcome to the lecture on True Stress and True Strain. So, we already know that when we talk about the forming technology, in that case the engineering stress and engineering strain are not the very relevant, you know, terms we have actually to refer to the you know, stress and strain term in terms of true value that is true stress and true strain. So, we will try to see more about this true stress and true strain how they are related with the engineering stress Engineering strain values in this lecture.

So, when we talk about you know, in case of, you know, linear strain when we what is the strain basically? So, normally when we talk about the strain, so, you define the strain as change in length upon original length.

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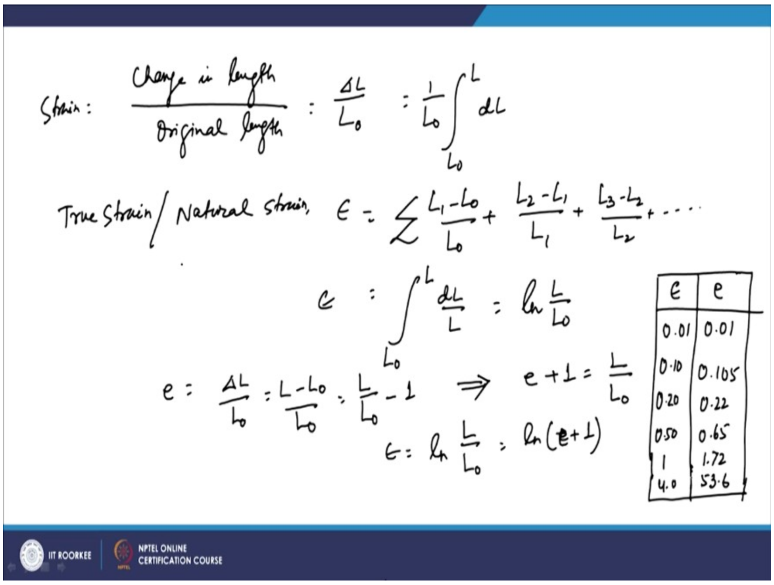

So, this is what the strain is defined as in conventional waste. Now the thing is that when this ΔL is very very small so, we call it as $\Delta L/L$. So, if L_0 is the original length, in that case this $\Delta L/L_0$ that will be giving you the, you know, actually the strain, and if you look at it so, you what do you see that in the case of conventional strain it is nothing but L_0 to L and it will be dL.

So, normally we find the conventional strain the conventional concept of train is defined in this fashion. Now this is valid only and this dL is very small. Now in the case of plastic deformation the strings are very very large, and also we have already discussed that when there is plastic deformation going on, then the gauge length is change considerably. So, what is happening? So, the Ludwig had proposed the concept of the true strain or natural strain.

So, you have the value of true strain or natural strain. So, true strain is also known as the natural strain, and it is represented by the ε, and this removes this difficulty in expressing in the case of the large plastic strains. Now it is basically it is with respect to the instantaneous gauge length. Now in this case we are taking the gauge length as the fixed one.

Now, in the in such cases basically, in this definition of the true strain basically the change in length is the referred to as the instantaneous gauge length rather to the original gauge length. So, your this is defined as so,.

$$\varepsilon = \sum \frac{L_1 - L_0}{L_0} + \frac{L_2 - L_1}{L_1} + \frac{L_3 - L_2}{L_2} + \ldots$$

So, it will go like this, and it can be further expressed like it will be L_0 to L and this will be dL/L. So, in this case this gauge length is now in this case that is fixed $1/L_0$, but that is not fixed; so from every increment if there is increment in steps.

In those cases, you have L naught here L_1. So, earlier you had L_1 before that it was L_2 or so. So, so that way so, that is from L_0 to L_1 then L_1 L_2 or L_2 to L_3, in those cases every time you have this way so, that is the definition.

So, you get the true strain value calculated in this fashion, and if you calculate that if you see that it will be $\ln\frac{L}{L_0}$. So, basically this ln by L_0, it is the true strain value. And you have engineering strain as $\Delta L/L_0$. Now if you see how can you define the relationship how can you maintain the relationship between the engineering strain and the true strain.

So, what we found that engineering strain is $\Delta L/L_0$. So, ΔL is nothing but $\frac{L-L_0}{L_0}$ which will be $\frac{L}{L_0}-1$. Now if you look at so, what will happen? You can see that from here you get e+1 it will be $\frac{L}{L_0}$. So, what you see is, you see that εis coming as $\ln\frac{L}{L_0}$.

So, $\frac{L}{L_0}$ $(e+1)$ so, it will be $\ln(e+1)$. So, you see that once, you know, the is conventional engineering strain, then add it to 1 and then you can get the true strain value and that can be compared. So, what you see is that when you have suppose engineering strain of very low value, like you have 0.01 at that point of time if you take this true strain, it will becoming as point 0, when it is low value of the engineering strain you will have the true strain as the similar one.

Similarly, so, if you have suppose if you temporarily you can calculate and you can compare the values of the true strain and the engineering strain. So, true strain is this is an engineering conventional strain is like this. So, for smaller values like if you have true strain value as 0.01. So, if you take ln of 1 + is equal to 0, once it will be coming as same as 0.01. So, similarly if you take 0.10, it will be 0.105. Similarly, if you increase further 0.20 when it will be coming as 0.22, and you have 0.50, in that case it will be 0.65 1 1.72 and 2 and 4 as 53.6, what you see is that for this values very large increase in this value.

So, this way once you have the known value of e you can find εonce, you know, the ε, in that case you can find the value of e. Now the thing is that why we use this true strain, now the use of true strain basically has many, you know, in many way you can interpret that it has more realistic, you know, interpretation.

So, suppose if you if you talk about the strain suppose you have a cylinder, and which is extended to twice it is length. Now what will happen so if you if you if the cylinder is if a specimen if any cylindrical as specimen is, you know, extended to twice it is length, 2 times it is length.

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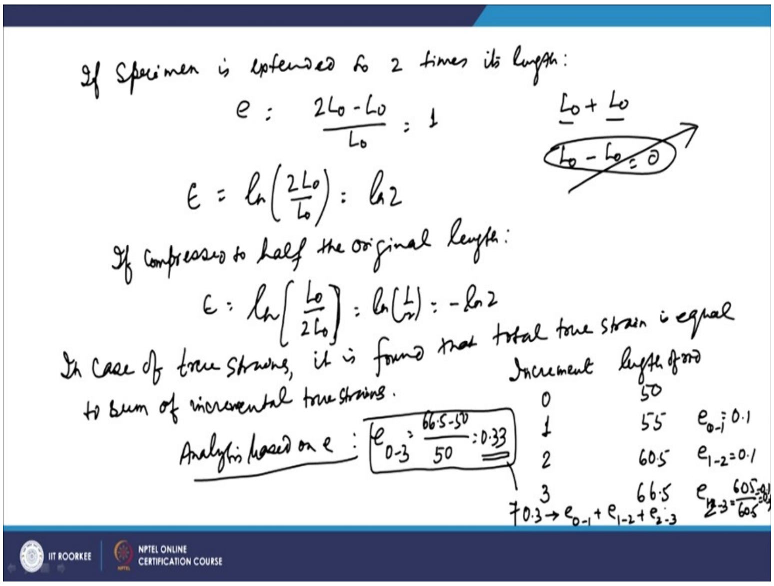

In that case if you find you know, engineering strain. So, it will be engineering strain L_0 is the actual length. So, the final length will be $\frac{2L_0 - L_0}{L_0}$ so, it will be one. So, engineering strain of 1 means it is increased to double it is length, now if you that is strain of 100 percent. Now suppose if you are thinking of the same strain in the compression side. So, in that case what will be happening? So, L_0 has gone so, this is nothing but $L_0 + L_0$

.

So, that is 100 percent of train. Now similarly if you try to interpret this in terms of compression, it means virtually it should be coming from L_0 to L_0, if it is the reduction in it is length to talk about so, it is coming to 0. And this basically, you know, you cannot interpret it anyway, because anyway you are strain in the material, in the compression side there must be the value of strain, but then if you look at the engineering that way so, you cannot go from for this 100 percent compression like that.

So, there will be no length, but that is that is, you know, that you cannot say. So, intuitively by this seems to be correct. So, but if you try to use the concept of, you know, the true strain what is true strain. So, if you look at the true strain if you see in this case

L from L_0 it has gone to 2 L_0; so it will be ln $2L_0/L_0$ ln L/L_0 we have seen that. So, L become $2L_0$ so, that is how it became ln 2. Now what does it mean? Now if you are compressing to half the length; similarly, so, if you are compressing so, if compressed to half the original length, now if you look at this, if you try to find the, you know, true strain value. So, ln of final length will be $L_0/2$ and then that is divided by L_0.

So, it will be $\ln\frac{1}{2}$ and that is why it will be -ln 2.

Now what you see is that this is the same value, but it is positive and it is come negative. So, basically that the amount of strain which you see normally is the same and it makes sense that if you are increasing to twice it is length of decreasing to half it is length. Rather than if you take the convention of engineering strain value, you see that it should go to the 0 length which is not possible.

So, that is why this true strain concept is basically used in the case of plastic forming. The next advantage what you see in the case of the use of this is a true strain is that that total true strain basically is the sum of the incremental true strains.

So, in case of true strain analysis, it is found that total true strain is equal to sum of incremental true strains. Now this can be found because the normally what we do in plastic forming. You never strain in one space from one height or one length to the final length. So, you are doing in the case of, you know, increments. Now the thing is that in the case of true strain it is basically found, that the total true strain will be equal to the sum of the incremental true strains. So, if you see that if you have a rod of suppose certain length suppose you have a rod of 50 mm length, now, in the first increment you are increasing to 55 mm length.

So, you will have the, you know, engineering strain as 0.1, similarly that time again you have to go for the, you know, length of 2 60.5, then it will have again engineering strain of 0.1. And you will have further to from 60.5 you go to 66.5 and what way you get the another engineering strain value of 0.1. So, that is what you see when you see the concept of engineering strain.

Now, if you go to the true strain concept, then if you look at the true strain concept, you will see that true strain for 1 0 2 1 to 1 to 2 and 2 to 3. You can find as you can find its value, and what you see that you will see that the true strain for the total will be some of the engineering strain in the incremental part.

So, that can be checked suppose you have one increment and then you have the length of rod. So, suppose you have increment 0 at that time you have the actual length is 50. Now you are in the first in increment it goes to 55. So, basically the value of e from 0 to 1 will be 55 - 50 by 50 so, 5 by 50 that is 0 0.1.

Similarly, if you go to the second increment; so for getting e 0 e 1 to 2 for 0.1 it will come as 60.5, because from 55 it will go to 60.5. So, 5.5 is the increment change in length divided by 55 second it will come to 0.1 and in the third, you know, situation it has to go to 66.5. And then in that case your engineering strain from 2 to 3 will be further 6.05/60.5 so, again it is 0.1; so this again coming out to be 0.1. So, in this cases, you get this sum as 0.1 + 0.1 + 0.1; so if you do the analysis based on the engineering strain. So, analysis based on conventional strain, what you see is, if you see the finals train, it will be (66.5 – 50)/50.

So, basically e 0 to 3 up to 3 total, it will be (66.5 – 50)/50. So, it will be 16.5/50 so, it is coming out as 0.33. And what you see is that if you add this 3, it is coming as only 0.3, whereas, the total is coming as 0.33. Now this 0.33 is ≠ 0.3.

0.3 is nothing but the sum of (e_{0-1}+e_{1-2}+e_{2-3}). Now, if you try to see, if you try to analyze the situation based on the true strain concept.

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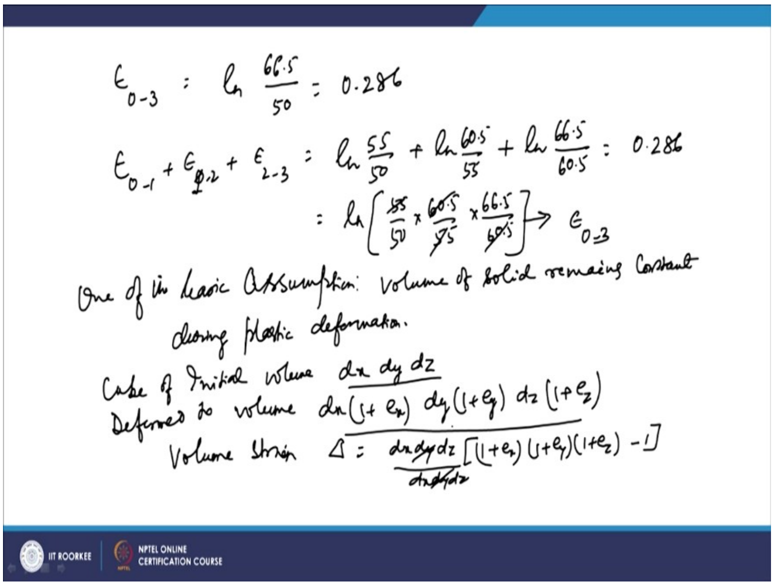

Now, in the case of true strain concept if you look so, ε_{0-3} will be ln 66.5/50.

So, this way you get, this is 0.286, now if you look at if you see go for $\varepsilon_{0-1}+\varepsilon_{1-2}+\varepsilon_{2-3}$. So, this is the, you know, increment in steps now this will be; so, basically you are going from initially

$$=\ln\frac{55}{50}+\ln\frac{60.5}{55}+\ln\frac{66.5}{60.5}$$

So, this way you can get it, and this you will also get this will be same as ln of 66.5 as, so, this will be same because once you do the logarithm, it will be nothing but the

$$¿\ln\left[\frac{55}{50}+\frac{60.5}{55}+\frac{66.5}{60.5}\right]$$

So, this will be cutting and ultimately you are getting, same as ε_{0-3} . So, what you see is that in the case of true strain analysis, if you look at incremental strains the total strain will be the sum of the incremental strains which you get in those cases.

Now, we will move to another concept in the case of plastic deformation, when we deal with the plastic deformation. One of the basic characteristic of plastic deformation is that the metal is essentially, you know, in compressible. So, basically density will be whatever density there is changing after large plastic strains they are very very small they are less than point one percent.

So, normally what we consider that the volume of the solid remains constant during plastic deformation. So, one of the basic assumption is that volume is volume of solid remains constant during plastic deformation. So, this way we if we talk about the cube of the initial volume suppose you take a cube of initial volume dx, dy and dz. Now suppose you this is the volume of the cube and it is deformed and it is volume after deformation.

So, deformed to volume

$$dx(1+e_x)dy(1+e_y)dz(1+e_z)$$

if e_x e_y and e_z, they are the strain. So, in that case this will be it is volume. So, if you try to find the volumetric strain which we have done in our earlier lectures. So, if you find the volumetric strain volume strain and that is represented by Δ. Now this will be nothing but the final volume minus so, this is the final volume this is the original volume divided by original volume.

So, it will be

$$\Delta=\frac{dxdydz}{dxdydz}[(1+e_x)(1+e_y)(1+e_z)-1]$$

And that way so, what we see is that normally we have done this analysis for the elastic, you know, materials in the case of elastic strains your this the elastic strain where it is less.

So, in that case we neglected the product of these strains because they are smaller. So, we neglected the product of e_x e_y or e_y e_z or ex e_y e_z. But we cannot neglect this when we are doing the analysis for the plastic strain, because in those cases this strain values are quite high in the last in case of elastic analysis you can neglect them because this values are quite small.

So, and also the volume change is 0 for the plastic deformation. So, what we see is that you can go further you can take this 1 to this side and this way the expiration becomes like this, so, Δ+1.

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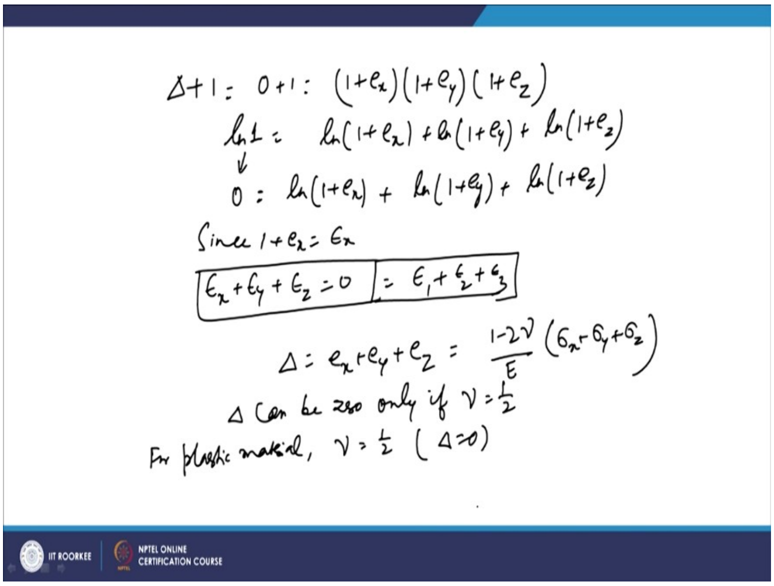

It will be coming as so, since this volume change will be 0. So, it will be

$$\Delta+1=0+1=(1+e_x)(1+e_y)(1+e_z)$$

so, this is what we get.

So, this is one in this side and once you take ln to both the sides so, it will be

$$\ln 1=\ln(1+e_x)+\ln(1+e_y)+\ln(1+e_z)$$

So, this is what is there in the case of plastic deformation, now ln one is basically 0, you get

$$0=\ln(1+e_x)+\ln(1+e_y)+\ln(1+e_z)$$

So, we have seen that 1 plus ex is nothing but epsilon x the true strain value of the x; so

$$Since\,1+e_x=\varepsilon_x$$

So, what we get from here that

$\varepsilon_x + \varepsilon_y + \varepsilon_z = 0$

So, this is what a relation we get in the case of plastic deformation and this can also further we written as.

$\varepsilon_x + \varepsilon_y + \varepsilon_z = 0 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$

So, this way this is what you get in the case of the plastic deformation, and this is nothing but the first invariant of the strain tensor we have already seen that, and we have already proved that. So, this is the sum of all this value epsilon xy and z they are to be, you know, their sum is 0.

So, what we see that this equation is not valid for the elastic reason, because, you know, there is in the case of ballistic strain there is appraisable volume change. Whereas, in the case the, you know, plastic analysis, the volume change is normally neglected it is 0. So, what we see that if you use if you try to use this, you know, if you try to add the hookes law, 3 hooks law we get we have already seen that we get this $e_x + e_y \, e_z$.

So, we have seen that it is coming as $2\mu/e$ and $\sigma_x + \sigma_y + \sigma_z$. So, what we see that in the case of, you know, plastic deformation the Δ has to be 0. So, if the Δ has to be 0 in the case of plastic deformation the volume we know that there is no change in volume. So, if this has to be 0, in those cases this has to be 0, and that is why the nu will come as 1 by 2. So, delta can be 0 for so only if μ is 1/2. So, then only the ν, for the nu value of 1/2 the this Δ will be 0 for the plastic deformation case. So, what we see that the poissons ratio is equal to 1/2 for a plastic material, for plastic material ν = 1/2.

So, because the Δ is 0 for plastic material for which the volumatric strain that is 0. So, for that the ν = 1/2. Now another thing which we can we can get from these values is the value of the true strain in another way.

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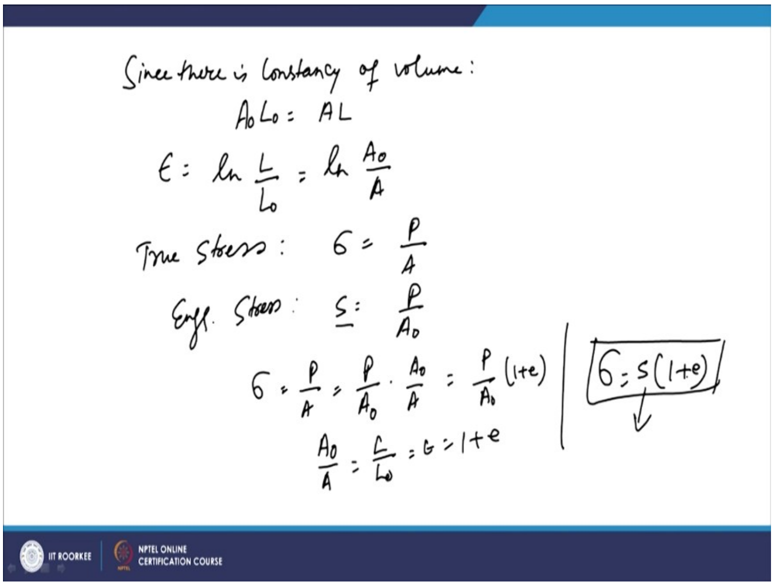

And in another way you can represent so, since there is constancy of volume. So, as we see that there is constancy of volume in the case of the plastic deformation

So, length multiplied by cross section area will be same. So, we see is you can write

$A_0 L_0 = AL$

So, so, what we have seen that

$$\varepsilon = \ln \frac{L}{L_0}$$

And so, you can write it as

$$\varepsilon = \ln \frac{L}{L_0} = \ln \frac{A_0}{A}$$

so, you can write $\frac{A_0}{A}$. So, $\ln \frac{A_0}{A}$ is also known has the and true strain.

So, either $\frac{A_0}{A}$ or $\frac{L}{L_0}$ you can find these, you know, true strain. Now true stress when we talk about true stress is basically in that case we are dividing the load with the instantaneous area, you know, in that case in the conventional analysis, the stress is by the original area, whereas, in the case of true stress, it is the load divided by the instantaneous area so, true stress when we try to define.

So, $\sigma = \frac{P}{A}$ and engineering stress if you see it is defined as $S = \frac{P}{A_0}$. A_0 is the original area. So, what we see again? So, sigma since it is $\frac{P}{A}$ so, we can write $\frac{P}{A_0} . \frac{A_0}{A}$. So, again $\frac{A_0}{A}$ is if you look at $\frac{A_0}{A}$ is nothing but $\frac{L}{L_0}$. So, which is nothing but ε so, (1 + e). So, what we see is, you can write $\frac{P}{A_0}$ and then so, and $\frac{A_0}{A}$ is 1+e.

$$\sigma = \frac{P}{A} = \frac{P}{A_0} . \frac{A_0}{A} = \frac{P}{A_0}(1+e)$$

So, what we see is $A_0 \times (1+e)$. So, what we see is, that you get σ

$$\sigma ¿ \frac{P}{A_0}$$

is S so, this is $S \times (1+e)$. So, since you have the value of engineering stress known, then and if you know the engineering strain in those cases the true stress is I mean related with these 2 values.

So this, we must solved some problems based on this true stress true strain engineering and try to familiarize with these terms. And that will be helpful in our future calculations.

Thank you very much.

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Lecture – 13
Yield criteria for ductile materials

Welcome to the lecture on Yield criteria for ductile materials. So, as we know that in the case of metal forming, you have to visualize the cases where the yielding occurs. Actually when the material is subjected to stresses, then under that you know condition of stress the material yields. And there must be certain relationships, certain correlations which must say that under the action of different type of stresses, how the material will fail or when the material will fail.

So, we will have you know we will have to define certain type of cases or certain; you know the criteria by which we can say that this way the yielding will occur in the material. So, normally in the case of the you know when the material is subjected to many stress is combined stress conditions or so; in that case the yielding may occur because of all the presence of all these stress conditions. Otherwise, you can say that when we talk about the yielding in you know uniaxial direction, then we say that it in this yielding will occur when the stress value will reach at the yield stress value or so.

There are certain basically criterion which must be you know fulfilled and there are certain conditions. One is that the hydrostatic pressure or hydrostatic stress is does not cause yielding. We have already seen that the hydrostatic component of the stress they are responsible for not responsible for the basically yielding or the change in the shape or deformation. So, we are getting one conclusion from here that among the stress part the; if the hydrostatic part is not responsible for causing the yielding, then there must be certain part by the stress deviator part.

Because we know that the total stress is the summation of the hydrostatic stress and the deviator stress. So, if the hydrostatic stresses are not causing the yielding; in those cases certainly the stress deviator the deviatory component of the stress, they must be responsible for finding for yielding actually.

So, basically we will see that based on this there are some you know criteria which are one criteria, which is proposed and that is proposed by the Von Mises and that is why it is known as the Von Mises criteria.

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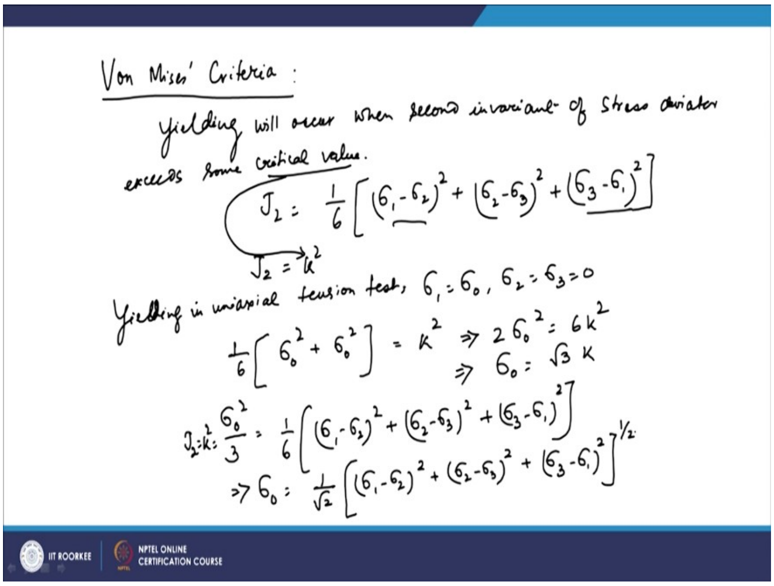

So, first criteria is Von Mises criteria; now Von Mises criteria tells that you know yielding will occur because we have when we discussed about this deviator stresses. So, we found the; we had found that deviators stress deviators, first deviator, second and third deviator in the invariants.

So, what it tells that the yielding will occur when second in variant of the stress deviator that is basically exceeds some critical value. So, in the earlier lectures we have already talked about this stress deviators; J_1, J_2 and J_3. So J_1 we know that it is a summation of the stress values and then J_2 is you know

$$J_2 = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]$$

So, basically and then J_3 also determinant of that, determinant value that was J_3. Now this values at this J_2; so J_2 must reach certain critical value. So, this J_2 is defined as; so, we had also defined in terms of σ_x, σ_y and σ_z or else we can also define it in terms of σ_1 σ_2 and σ_3. So, this will be:

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2$$

Now, this value must reach a certain critical value when it is reaching a certain critical value; then yielding will occur that is what it is proposed by the Von Mises. And it is basically related to the stresses of deviatoric nature because we know that the hydrostatic component does not cause yielding. So, in this basically now this J_2 must reach a critical value that is K^2. So, J_2 must reach this critical value K^2. So, when this J_2 is reaching this critical value K^2 in that case the yielding will occur.

So, we have to find this critical value constant that is that constant K we have to find and for that we can have you know we can find this K by relating it to a yielding in the tension test. So, suppose we are going for yielding; so for yielding in uniaxial tension test, now if you take the uniaxial tension test case in that case you have σ_1 will be σ_0 and σ_2 and σ_3 will be 0.

So in the case of uniaxial tension test they have you will have the failure when they. So, you will point it reached and in that case σ_1 will be sigma naught that is yield stress value and σ_2 and σ_3 will be 0. Now in this case if you put this value into this expression. So, this will be σ_0^2 and then further you have σ_0^2 here also. So, what will be happening $\frac{1}{6}[\sigma_0^2+\sigma_0^2]$, it will be coming from these 2 these 2 terms and in this term σ_2 and σ_3 both are 0; so, that term is anyway cancelled.

So, $\sigma_0^2+\sigma_0^2$ and that will be actually K^2. So, so that will lead to $2\sigma_0^2$ will be $6K^2$ and in that case you can get σ_0 will be $\sqrt{3}K$. So, we get σ_0 as the $\sqrt{3}K$ now from there you can have. So, K value will be $\sigma_0/\sqrt{3}$; so, if you place this value into this equation. So, what you see you can have this equation; so what you see is J_2 that is K^2 will be $\sigma_0^2/3$.

So, $\sigma_0^2/3$ that is your J_2 that is K^2. So, this will be equal to

$$\frac{1}{6}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]$$

So, this is what you get from there and so, sigma naught if you keep this side. So, this 3 will go here and 1 by it will come at $3/6$. So, $1/2$ and if you take that; so which will be 1 by root if you are taking the square root of it and in this bracket, you will have again

$1/\sqrt{2}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]^{1/2}$

So, this is how you can get this condition if because of the conditions of stresses; if this σ_1 because of the presence of σ_1 σ_2 and σ_3. If you find this value; now this σ_0 will be being. So, this condition you have got and this condition is known as the Von Mises theorem. So, you can further you can express it in the form of σ_x, σ_y and σ_z.

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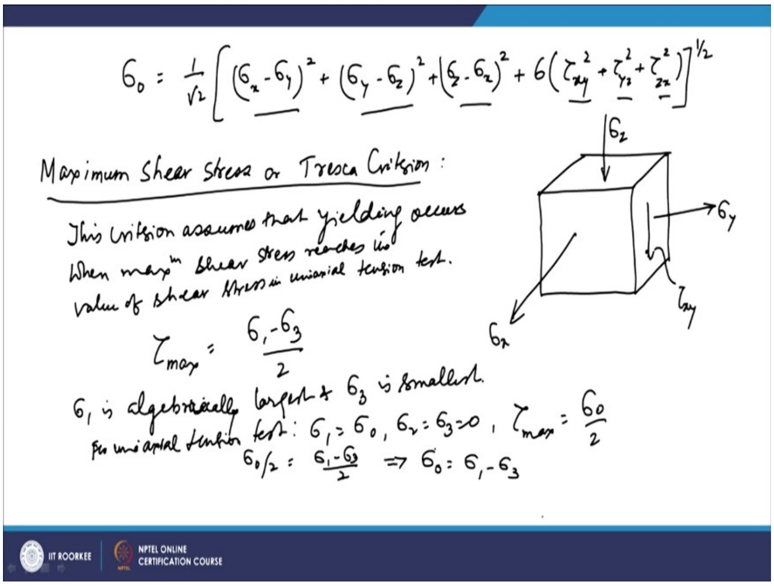

So, if you put that you can write. So, this we can be write written as

$1/\sqrt{2}[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)]^{1/2}$

So, you can further write the expression in terms of a σ_x, σ_y and the σ_z. Now the condition tells that the yielding will occur when the difference of the stresses on the right hand side that will be exceeding the yield stress in the uniaxial tension test; this is a limiting condition. If this value if the right hand side value is increasing its value is exceeding this yield stress value in uniaxial tension test in those cases the yielding will occur. So, this is known as the Von Mises criteria.

Now, criteria now when this criteria has you know many you know values or many interpretations what you see is that it involves the use of the all these 3 principal stresses or or the all the 3 components of a stresses in the 3 directions. So, so that way you can you can we will say that in most of the cases wherever you have the use of you have been given this with the problem; you can use this formula and come to its you know the prediction of the yield stress.

Now, I mean prediction of the failure conditions now you can have many of the problems in such cases. For example, if you are having a problem of a suppose you know a cube is there and in the cube you may be given certain set of stresses to which this cube is acting, you may be given sum and normal stress, you may be given some conditions like this. So, if suppose this is your σ_x is given then similarly you have σ_z is given and suppose you have a σ_y also given and some shear stress part also τ_{xy} is given.

Now, in this cases what you can do is you can do the analysis of these conditions; you may have the shear stress components on this side or that side also. So, so in those cases you can use this formula and what will be happening in this case is the σ_0 value will be given to you and once you find this side. So, suppose you are given the values of σ_x, σ_y and σ_z as well as τ_{xy} you may have τ_{yz} τ_{xz}.

So, what you will do is you will find this you know σ this right hand side value from this side. And if this is exceeding the value of the yield stress you know for that you know component; it means in that case the yielding will occur otherwise the yielding will not occur. So, so that way you try to calculate to try to analyze those situations wherever you have such conditions given.

Now, what we discussed that this criteria they are they are they involved all the you know the J_3 values of the principal shearing stresses and that way this has a lot of implication. So, as we see that we have seen in the expression that this is this is a term $\sigma_1-\sigma_2$ or $\sigma_2-\sigma_3$ or $\sigma_3-\sigma_1$. So, they are basically the values of the values of the principal shearing stress.

So, what we can make out from this analysis is that it involves all the 3 principal shearing stresses. And this all this principal shearing stresses are taken into account for the analysis for the definition of such yielding criteria. Also you see that there is no hydrostatic component involved; so, we have already seen that you know that we have defined that hydrostatic stress is does not I mean give the yielding or they are responsible for this the formation of changing shapes.

So, that also is clear from these pictures also the another thing is that all the terms are squared one $[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]$. Even here you have all these terms $(\sigma_x-\sigma_y)^2$, then you have $(\sigma_y-\sigma_z)^2$ all these terms are square.

So, basically they are plus or minus sign does not matter because they are all basically squared terms. So, this is also that also tells us that and especially to that earlier expression where we see the you know the values of the stresses which does not a matter which is the largest or which is the smallest type of principal stresses because that may be required in another case of another type of criteria for the failure analysis. So, this is the you know condition for the you know yielding of the trial materials based on the stress deviator second invariant of the stress deviator that is Von Mises theorem; this is also known as the distortion energy theorem.

This is known as distortion energy theorem because the distortion energy is defined as $1/6\,g$ $1/6\,J$ and then $\sigma_1-¿$. So, $\sigma_1^2+\sigma_2^2+\sigma_3^2-\sigma_1\sigma_2-\sigma_2\sigma_3-\sigma_3\sigma_1$. So, using that distortion energy theorem also you can come to the same expression that is σ_0 will be equal to $1/\sqrt{2}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]^{1/2}$

So, this way basically you can have; so, similar expression and also known as the distortion energy theorem. The next type of the criteria which is defined as the Von Mises cri I mean the Tresca’s criteria. So, we have seen that in the Von Mises criteria we are taking all the 3 principal stresses into account; shearing stresses into account and this is why the second invariant invariant of the stress deviator is brought into consideration and that is being equated to one critical value that will define the yielding condition.

Now, coming to the second you know criteria that is known as the maximum shear stress criteria or this is also known as Tresca criteria. So, this again we know that yield you know occurs because of the, you know shear stresses. So, this criteria is based on the assumption that yielding will occur when the maximum shear stress which is you know generated that reaches the value of the shear stress in uniaxial torsion test.

So, basically we have the torsion test which we do there from you get the yielding in this torsion because of the shear. And then the maximum shear stress which is you know calculated based on the si σ_1 and σ_2 and σ_3 we have seen that the maximum value of the shear stress is $\frac{\sigma_1-\sigma_2}{2}$ or $\frac{\sigma_2-\sigma_3}{2}$ or $\frac{\sigma_1-\sigma_3}{2}$.

So, depending upon the value of σ_1 or σ_2 or σ_3 which one is the largest and which one is the smallest you can have the maximum value of the principle shearing stress that will be $\frac{\sigma_1-\sigma_3}{2}$ if σ_1 is the largest and σ_3 is the smallest. So, so the criterion assumes; so this criterion assumes that yielding occurs when maximum shear stress reaches the value of shear stress in uniaxial tension test. So, this is nothing, but the yield shear stress. So, we are talking about the yield shear stress when it is we are undergoing the, we are putting the specimen under the torsion test. So, in that uniaxial it is not uniaxial is tension test.

So, in that basically the shear yield stress which is achieved which is calculated which is found that should be equal to the maximum shear stress which is found out. Now if you look at the condition it tells that τ_{max}; so, that is must be equal to $\frac{\sigma_1-\sigma_3}{2}$. So, if you assume the σ_1 to be the largest one and σ_3 to be the smallest one in that case. So, here σ_1 is algebraically largest and σ_3 is smallest. So, in that case $\frac{\sigma_1-\sigma_3}{2}$ will be the maximum of all the shearing stresses. So, σ this must be reaching to that and that should be the τ_{max}.

Now, if you take for example, for uniaxial tension test; now in the uniaxial torsion test we have already seen that you have σ_1 as σ_0 and σ_2 and σ_3 as 0. So, in this case the τ_{max} will be equal to; so if you see that tau max is $\frac{\sigma_1-\sigma_3}{2}$. So, that will be equal to $\sigma_0/2$; so, you can write $\sigma_0/2=\sigma_1-\sigma_3$. So, that yield; so $\sigma_0/2$ will be $\frac{\sigma_1-\sigma_3}{2}$. So, this gives sigma naught will be $\sigma_1-\sigma_3$.

So, this condition you know is this is known as the this is given by the maximum shear stress or the Tresca criteria; now if you talk about the condition of pure shear.

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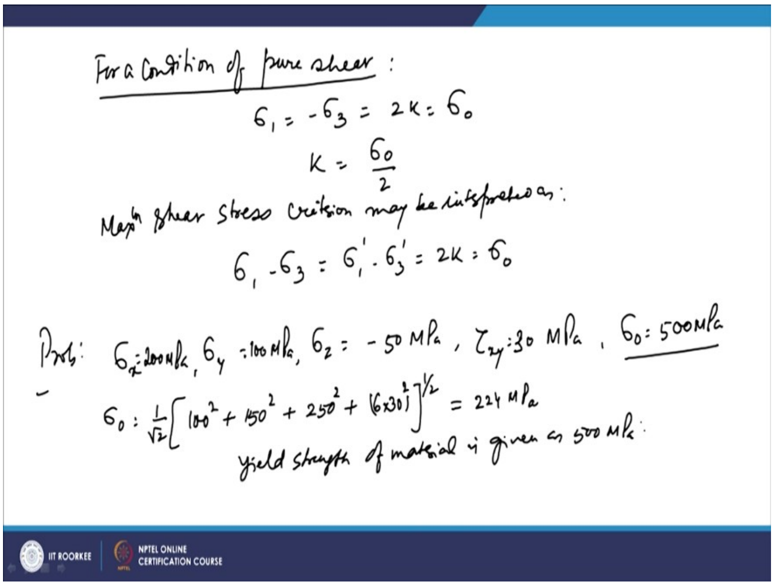

So, far a condition of pure shear trial in that case basically $\sigma_1 = -\sigma_3$. So, it will be 2K that is σ_0; so, what you see is K will be $\sigma_0/2$. So, what we see that we can interpret the maximum shear stress criteria. So, maximum shear stress criteria can be interpreted or can be written as; so, we can write as $\sigma_1 - \sigma_3$ or else you can have $\sigma_1' - \sigma_3'$ that will be 2K.

So, 2K that is σ_0; so, what we see that if you compare these 2 criterion that is maximum shear stress criterion and the Von Mises criterion; you see that this is quite simple type of a relationship where which involves only these 2 stress value σ_1 and the σ_3 does not involve the σ_2.

So, this is basically the major difficulty that you know here the problem is that you have to know that what is the maximum and what is the minimum value of the principle stresses? Then only you can find this you know expression σ_1 and σ_3; if you do not know then you cannot use this theorem. So, this criterion that way lacks or it has that you know demerit that you must know which of the fun I mean stress is intermediate in nature I mean it is not either the largest or the smallest.

So, you know that way it has that and also that it does not involved 3 stresses you know. So, that is why although it is you know moreover self less complicated, but most commonly if we go and use the Von Mises criterion because they are always have seen that it takes into account, it has more meaning also it takes into account the square term signs, signs do not that way you know play that much in that case. So, this is about the 2 type of the criterion.

Now, if you if you are given certain problems as we discussed we were discussing about some problem suppose in the earlier case. So, suppose in this case if suppose for this problem if you are given some suppose example to solve such problems. And it is given that suppose for a problem you are given that σ_x and σ_y and σ_z is given. So, suppose σ_x is given as 200 MPa and sigma now y is given as suppose σ_y is again given as 100 MPa and σ_z is negative that is minus 50 MPa. And suppose τ_{xy} is given as you know you can say τ_{xy} taken as 30 MPa.

Now, if you are told that this for a particular structural member this is the stress to which it is subjected to and it has the sigma naught value of 500 MPa. Now suppose you are told that whether you tell us whether this material will yield or not. So, if you look at that you will see that from the, you can use the since this 3 component of stresses are involved you have to use the you know condition of Von Mises criteria.

So, if you use the Von Mises criteria using these 3 conditions what you get is σ_0 and then further you can use $1/\sqrt{2}(\sigma_x-\sigma_y)^2$. So, this will be $200-100^2$; so, this will be $100^2+(\sigma_y-\sigma_z)^2$. So, 100 - 50; so, 150^2 and then $(\sigma_x-\sigma_z)^2$; so 200 + 50 to 250^2 and then you have $6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)¿$.

So, 6×30^2; now this is you know squared square root is calculated. If you find this value this comes out to be 224 MPa. Now this yield strength of material is given now yield strength of material is given as 500 makes MPa. Now, what you see is that under the combination of these stresses using the Von Mises criteria you got 244 minimum 4 MPa and it is your this σ_0 value is given.

Whereas, the materials yield strength is 500 MPa; it means the material will not failed under these conditions. So, so when we may have a problem where only σ_1 and sigma 2 or σ or 1 of the 2 stresses are given or when the 3 stresses are given. So, you can have even the and the calculation by the shearing maximum shearing stress criteria or Tresca criteria and you can come to a conclusion whether the material will fail or not. So, this is about the yield criteria of ductile materials.

Thank you very much.

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Lecture - 15
Plastic Stress Strain Relationships

Welcome to the lecture on Plastic Stress Strain Relationships. So, we have discussed about many kind of relationship in the earlier lectures, now we will go to discuss about the stress strain relationship in the case of plastic deformations; so in the case of plastic deformations. So, in the case of elastic deformation as we a nu that you have certainly up to certain limit the out slow valid, and then you have the posies ratio is there and that way you have many type of relationships by which you can relates the strain to stresses and can find the value of stresses.

But we where we go into the plastic reason, then this linear relationship is no longer valid and you will have to find basically the find the value of stresses. And basically in this case the stresses will be depending upon the entire history of loading.

In this case may be you know that when something suppose any specimen of suppose 50 mm length is you know its length is increase to 60 and then further coming down to 50. In that case if you look at the otherwise there is no change in it length. But in the case of plastic deformation basically where its going in increments, you will have to take into a account the strains in every increment. And then that way the total you know strain will be found by summing up the all the strain which have taken place.

So, this way you can take what very simple examples, suppose if you look at the a case where suppose a rod of 50 mm is there; so rod of 50 mm length.

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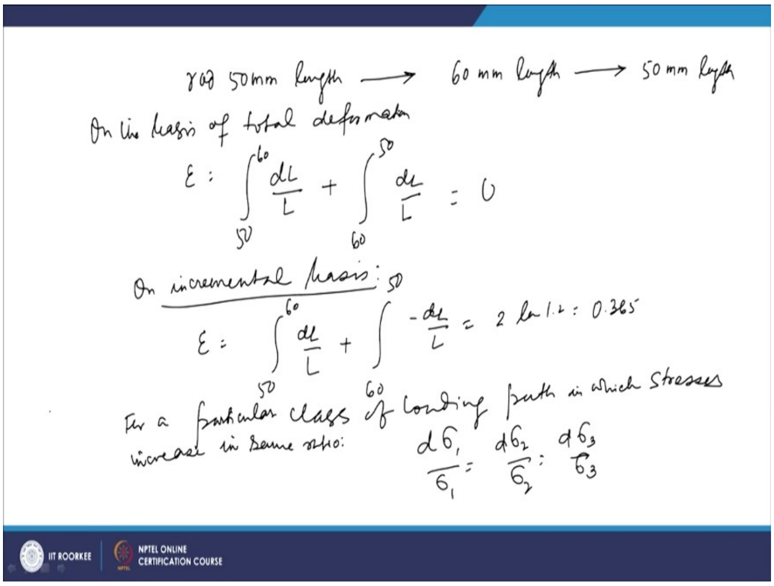

And suppose you are converting this rod to 60 mm length. So, if you look at this rod which is 50 mm length extended to 60 mm length and further this 60 mm length is compressed.

So, this is longer to 60 mm length, and then it is further compress to 50 mm length. Now you can see that if you talk about the total deformation, if you take the analysis based on the total deformation; in that case a on the basis of total deformation what you can find, a we will find like a $\int_{50}^{60} \frac{dL}{L}$. So, that is ln and then that that comes. And then further you have $\int_{60}^{50} \frac{dL}{L}$.

So, if you do in the based on total deformation case, you are ε or in the strain a that comes as 0. Whereas, if you talk about the strain which is occurring in the increment, in that case a on incremental basis. Non incremental basis (Refer Time: 04:17) see a such kind of you know find in the strain. So, what we see is, you have $\int_{50}^{60} \frac{dL}{L}$ and then you have $\int_{60}^{50} \frac{-dL}{L} = 2\ln 1.2 = 0.365$ which is basically the material has subjected to such is strain conditions.

So, what is say it is say it is no practical, and in the case of plastic deformation basically we are concern with the whole history of all you know increment of in which deformation take place and we are taking into a account all these increments. Again many a times what we see that, in certain types of loading what we see that the stresses are increases in the same ratio; so for a particular class of loading. So, you have many a times you deal with this loading path in which the stresses increase in the same ratio. So, in which stress increase stresses increase in same ratio.

So, this is also known as the proposed the loading sort of case, and in those cases what we see the $\frac{d\sigma_1}{\sigma_1}$ basically that usually becomes constant that is, $\frac{d\sigma_2}{\sigma_2}$ and same as $\frac{d\sigma_3}{\sigma_3}$. So, in this case say basically the plastic strains will be independent of the loading path. So, in such case is the plastic. So, final it will be depending upon only the final state of the stress. So, in such situations; otherwise what we will see that you have to go for the every increment and it will depend upon the all these you know incremental behaviors.

So, normally what we when we talk about the plastic stress strain equations, normally we deal with 2 types of or 2 categories of the relations. One is that the incremental or flow theories that is that will be basically relating the stress to the plastic strain increments. So, in that case normally we are neglecting the elastic strain components and in one case we are basically talking taking into account even the total plastic strain basically. So, so in that case we also take in to account the elastic part also.

So, based on that we have two theories, which analyze these stress strain; I mean relation between the stress and strain in the case of plastic deformation. So, we will deal with that. Now we will deal with the first one that is known as a Levy-Mises equation which is for the ideal plastic solid.

So, when we talk about this Levy-Mises you know equations that is for ideal plastic solid.

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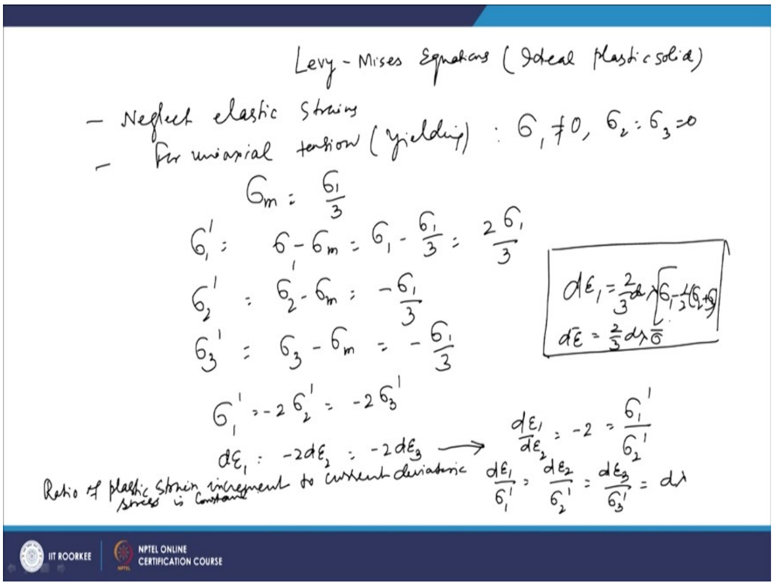

So, that equation that is known as Levy-Mises equations and it is for ideal plastic solid. Now in this case we normally neglect the elastic strain and so, we are neglecting neglect elastic strains.

So, this is for the ideal plastic solid where the elastic strain is neglected and they are known as the flow rules because we are dealing only with the plastic strain, and that is known as the Levy-Mises equation now in this case. So, what we do suppose we are assuming for a uni axial tension, where you have only σ_1 is there which is not equal to 0 which is non zero quantity, and σ_2 and σ_3 is a 0. So, if suppose for uni axial tension case, if you some specimen subjected to the uniaxial tension and in that there is yielding. So, uniaxial means you have σ_1 is non zero and σ_2 and σ_3 is 0.

So, for this have we know that you have σ_m is $(\sigma_1+\sigma_2+\sigma_3)/3$. So, that will be $\sigma_1/3$. Now if you see that then you have the deviatory history where stresses can be computed, and if you find the deviatory histories$\sigma_1^{'}$. So, that will be basically $\sigma_1-\sigma_m$, so $\sigma_1-\sigma_m$ that is $\sigma_1-\sigma_1/3$. So, it will be $2\sigma_1/3$. If you find the $\sigma_2^{'}$ that deviatory component for the σ_2 is the principle stress in the second you know direction. So, that will be again this will be $\sigma_2-\sigma_m$, and $\sigma_2$be in 0.

So, it will be $-\sigma_m$ and σ_m m is -. So, it will be minus $\sigma_1/3$ similarly that will be same as $\sigma_3^{'}$. So, it will be again $\sigma_3-\sigma_m$. So, that will be $-\sigma_1/3$. So, what we get in such case is in the such case of uniaxial tension, when specimen subject to yield in what do you see that, $\sigma_1^{'}=-2\sigma_2^{'}=-2\sigma_3^{'}$.

Now, in the case of plastic deformation we also know that there is constancy of volume, which is not the applicable thing in the case of elastic deformation. Now in the case of in the plastic deformation what you get is, you will get $d\varepsilon_1=-2d\varepsilon_2=-2d\varepsilon_3$. So, what you see from these 2 equations you get now; now from this equation you will further get $d\varepsilon_1/d\varepsilon_2=-2$.

So, So, this will be nothing, but if would see, it will same as $\sigma_1^{'}/\sigma_2^{'}$. So, so what we see? You can have this equation in a generalize way and that gives you the Levy-Mises equation and so, the equation comes you can have from here you can get that $d\varepsilon_l/\sigma_l^{'}$, it is seen to be same as $d\varepsilon_2/\sigma_2^{'}$ and that will be same as $d\varepsilon_3/\sigma_3^{'}$ and that we take as this dλ. So, what it shows this expression $d\varepsilon_1/\sigma_1^{'}=d\varepsilon_2/\sigma_2^{'}=d\varepsilon_3/\sigma_3^{'}$, now what we see that at any instant of deformation the ratio of this is the plastic strain increment ah.

So, ratio of the plastic strain increment at any instant and the this $\sigma_1^{'}$ is nothing, but the deviatory stress component. So, ratio of this plastic strain you know increment to the current value of the deviatory stress, that remains constant that is what we there is a stepped out from the this Levy-Mises equations.

So, ratio of plastic strain increment to current deviateric stress is constant. So, this is true for at any instant of deformation, and this is known as the Levy-Mises equation and you can further we have already seen the equation in the last chapter and from there, we can generalize this equations like you have $d\varepsilon_1=\frac{2}{3}d\lambda\left[\sigma_1-\frac{1}{2}(\sigma_2+\sigma_3)\right]$.

So, this is what. So, this similarly you will have $d\varepsilon_2=\frac{2}{3}d\lambda\left[\sigma_2-\frac{1}{2}(\sigma_1+\sigma_3)\right]$. Similarly $d\varepsilon_3=\frac{2}{3}d\lambda\left[\sigma_3-\frac{1}{2}(\sigma_2+\sigma_1)\right]$. So, what we see that we try to get this effective strain values and what we get is that if you talk in terms of effective strain $d\acute{\varepsilon}=\frac{2}{3}d\lambda\acute{\sigma}$ and effective same. We can also write in terms of these effective is strain and effective stress. So, $d\acute{\varepsilon}=\frac{2}{3}d\lambda$

$d\acute{\varepsilon}=\frac{2}{3}d\lambda\,\acute{\sigma}$. So, this way we get this expression for the plastic strain increment and the effective strain in such cases.

So, if you try to see the final Levy-Mises equation, the final Levy-Mises equation becomes.

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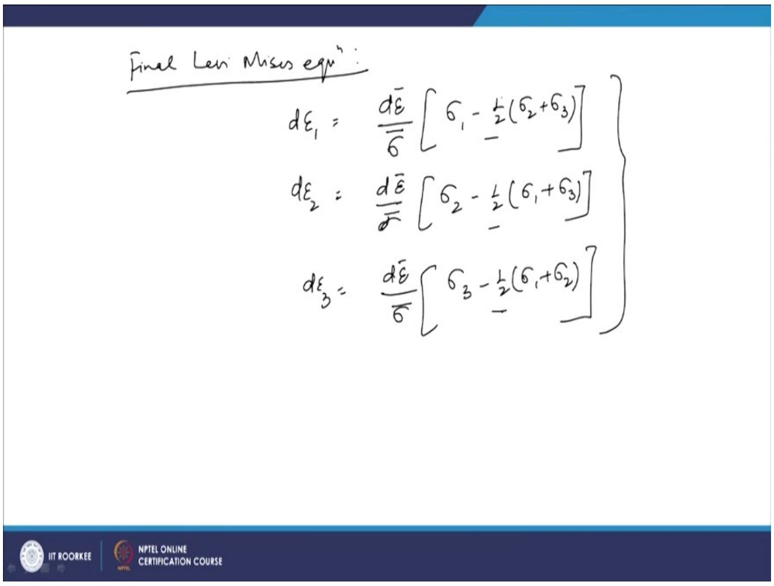

So, your final Levy-Mises equation will be coming if you look at, this will be $d\varepsilon_1=\frac{d\acute{\varepsilon}}{\sigma'}$ that is what if you displace this $\frac{2}{3}d\lambda$ term and then you get $\sigma_1-\frac{1}{2}(\sigma_2-\sigma_3)$. Similarly $d\varepsilon_2=\frac{d\varepsilon'}{\sigma'}$. And then this will be $\sigma_2-\frac{1}{2}(\sigma_1+\sigma_3)$ and $d\varepsilon_3$ will be again similarly the effective value of the strain divided by the effective stress, and then $\sigma_3-\frac{1}{2}(\sigma_1+\sigma_2)$.

So, if you try to see this equation with the equation which we got in the case of elastic strain, now you can see that in place of here in place of this $\frac{d\varepsilon'}{\sigma'}$, we got the 1/E. So, that this is this displaced by you know 1/E and then so, because as you know that stress by strain component was in that case the e values. So, it is strain by stress components it will be 1/E that was there in that case earlier which we discussed; and also here we are you were getting the ν term and you also found that in the case of plastic deformation, the ν was because of the constancy of volume and all other conditions involved, the ν was coming out to be 1/2.

So, that is ν is coming as they 1/2. So, this is the equations for a typical plastic you know stresses strain, which talks about these parameters, which are used in such cases. Now the next story which talks about these plastic stress strain equations, that is the Prandtl Reuss equation.

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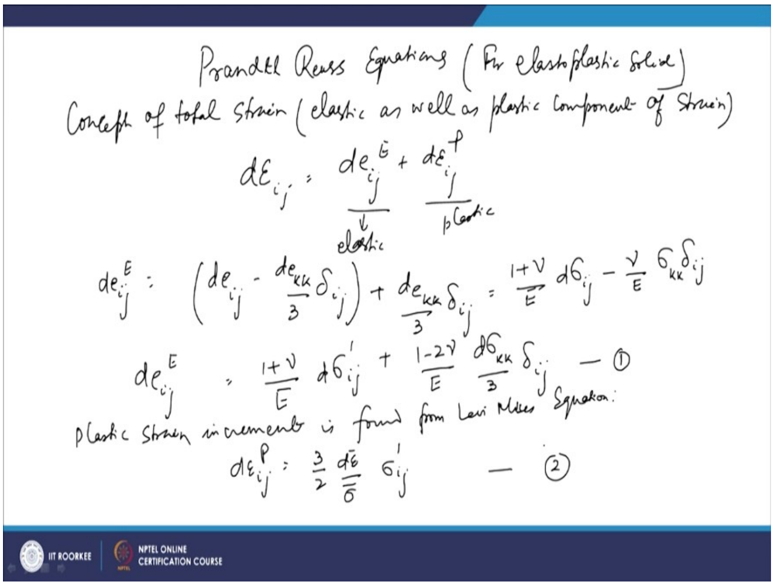

So, that is known as Prandtl Reuss equations, this is for normally elasto plastic solid.

Now, what we have seen in the earlier derivation was Levy-Mises equation, now that is basically for such case where the plastic strain component is very large, and basically we are neglecting the elastic strains. But when we are talking we are in the range of this elasto plastic reason, then it is you know more important not to neglect these deformations which occurred or strain which occurred in the case of the elastic deformation or in that rang of elastic deformation.

So, for that the Prandtl Reuss they have proposed the concept of total strain and so the concept of total strain was proposed. Total strain that is you will composing of elastic as well as plastic component of strain. So, what we doing that, in that basically we are dealing with it is elastic strain part also and the plastic strain part also, elastic part we have already discussed in our earlier lectures. So, they are from we get elastic strain part and the plastic strain parts we are getting from the Levy-Mises equation, which basically deals only for the plastic strain and which neglects the elastic strain part.

So, we can write we write $d\varepsilon_{ij} = de_{ij}^{E} + de_{ij}^{P}$. So, this is this part is the elastic part and this part is the plastic part. Now if you recall we can convert this elastic part, now if you see the equation for the elastic parts. So, we had got the equation for the elastic part and that can we further written in a in the form that is $de_{ij} = \left(de_{ij} - \frac{de_{kk}}{3}\delta_{ij} \right) + \frac{de_{kk}}{3}\delta_{ij}$. So, we can write certainly this because this part and this part will be cancelled, where we know that this is the chronical delta. So, you have that component and this is the main strain part. So, certainly this is the main part.

So, that is why we know that now this will be coming as the deviator. So, now, we can write this as $\frac{1+v}{E} d\sigma_{ij} - \frac{v}{E}\sigma_{kk}\delta_{ij}$. Now again you try to look at it, this part is suppose total strain. So, we are basically subtracting form there this part. So, this is the main part will strain part, and we get this $\frac{1+v}{E} d\sigma_{ij}$ and then this part has we know. So, this part will be as you know that this is $\frac{v}{E}\sigma_{kk}\delta_{ij}$. So, this way you define this.

So, you can write $de_{ij}^{E} = \frac{1+v}{E} d\sigma_{ij}^{'} + \frac{1+2v}{E}\frac{d\sigma_{kk}}{3}\delta_{ij}$. So, this can be return further which talks about this total elastic part of the strain. Now form the Levy-Mises equation you can find the expression for the plastic strain. Now for the plastic strain we have already what certain expression; so the plastic strain. So, increment is found from Levy-Mises equation.

So, we write these Levy-Mises equation as de_{ij}^{P} that is plastic, and it will be $\frac{3}{2}\frac{d\acute{\varepsilon}}{\acute{\sigma}}\sigma_{ij}^{'}$, that we had seem that expression and then income it as $\sigma_{ij}^{'}$. So, if you add the expression for the total is strain, which is the addition of this strain that is elastic strain and the plastic strain. So, this will be 1 plus 2 will give you in the total strain. So, if you look at the taking into account to both the components then for elasto plastic solid.

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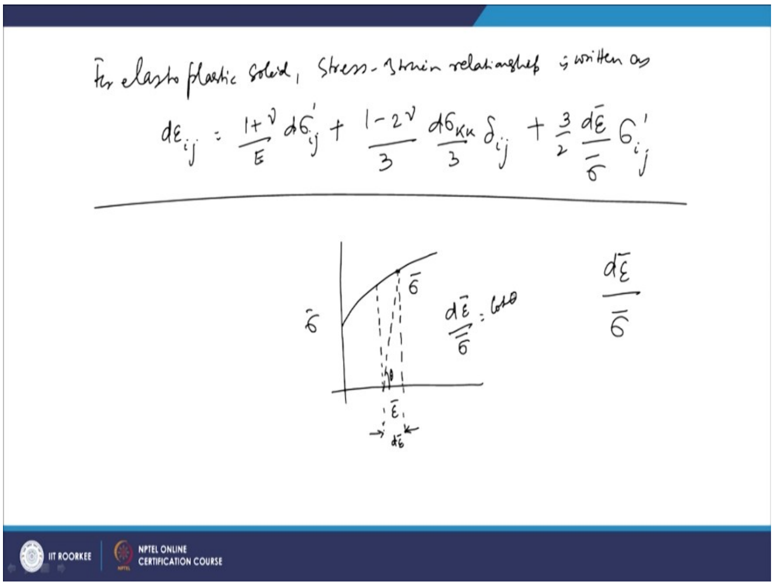

So, stress strain relationships can be written as now here you can write σ'_{ij}. So, this will be the sum 1 plus 2. So, you have. So, this 1 and this 2 we are adding, and that we getting the expression for the total strain. So, this will be get a $\frac{1+\nu}{E} d\sigma'_{ij}$, which is what we got $\frac{1+\nu}{E} d\sigma'_{ij}$, then will you have again you have one plus. So, this is coming $\frac{1-2\nu}{E}\frac{d\sigma_{kk}}{3}\delta_{ij}\frac{1+2\nu}{E}\frac{d\sigma_{kk}}{3}\delta_{ij}$.

So, So, that is what will write. So, this expression fin further be written there. So, it will be 1 -. So, we have this acquit $\frac{1-2\nu}{3}\frac{d\sigma_{kk}}{3}\delta_{ij}\frac{1-2\nu}{E}\frac{d\sigma_{kk}}{3}\delta_{ij}$. So, $\frac{1-2\nu}{3}\frac{d\sigma_{kk}}{3}\delta_{ij}$ and then, you have the summation of the plastic strain part and that comes as $\frac{3}{2}\frac{d\acute{\varepsilon}}{\acute{\sigma}}\sigma'_{ij}$.

So, this way, this is what we got in this and then from here we get the expression form the total a strain and that is the you know known as the Prandtl Reuss equation which basically involves the you know a calculates both these strains other be plastic or be the elastic, both are taken into account when we are dealing with. So, elasto plastic solid and we do not have the you know liberty to neglect these elastic you know component; because when that is very very large the plastic strain part is very very large as compared to elastic part then we can have, but otherwise they are cannot be neglect.

So, in that cases indorse cases we try to use this formula for finding the strain and then further you can have the calculation of the stresses using this. Now there is Levy-Mises and the Prandtl Reuss equations, they basically are providing the relationship between the increment of the plastic strain and the stresses. So, you have one side you have the equation for the elastic part that we have already earlier discussed, and in this case these 2 equations are basically given you the relationship between the increments of the plastic strain and the stresses. But the challenge which is their when we deal with certain situations such situation that, what will be the next increment of the plastic strain for a given you state of stress when the loads are increased basically incrementally.

So, that is how they are the basically essence is they are do to fine the you know increment in the you strain, because of the increment in the stress. And once you find the increment in the stress in every stage, in those case then what you can do is that, you can simply sum them and you can find the total plastic strain. That is what we have seen that in the case of plastic strain analysis, we are basically we cannot neglect or that that we have seen that we did for total stress analysis.

So, there in we saw that when we are between from 50 to 60 and coming back from 60 to 50, based on the total strain concept it was coming as 0. Whereas the actually when we do the analysis on the incremental basis, in that case it tells you that what is the strain which is subjected too. So, this way you use these plastic stress strain equations and also the yield criteria and other flow behavior of the material, we are like we use also the concept of the effective stress and effective strain you know concepts, to find basically the stresses and strains in the plastic flow analysis now.

So, this is about these 2 methods and normally what we have seen that in the Levy-Mises equation you get these. So, here you get these a $\frac{d\acute{\varepsilon}}{\acute{\sigma}}$ now if you look at this how to establish this. So, if you try to see in the in the equation for you suppose you have this σ' and this is the ε'. So, basically being the you know you are talking about pure plastic ideal plastic solid. So, in those cases what happens that you are if you look at this point, now if you see from here, now this suppose this becomes.

So, this is your $\sigma^{'}$. And now in this case you when a it is unloaded. So, that way now this part θ, now and this will be your part that is $d\varepsilon^{'}$ and that is why this $\frac{d\varepsilon^{'}}{\sigma^{'}}$ if you look, at this is from this curve if you look at this θ. So, that will be basically the $\cot\theta$. So, what happens that; we have seen that this we have the expression for the effective stress, effective strain curve for the increment of the strain that is a $d\acute{\varepsilon}$ which is there in the case of the plastic strain analysis or plastic stress strain analysis. And this way we try to stabilize this component that is $\frac{d\varepsilon^{'}}{\sigma^{'}}$.

So, depending upon so, we can deal with the different types of problems which we come across in the case of the plastic stress and plastic strain, and these two Levy-Mises equation as well as the Prandtl Reuss can be applied depending upon the type of the soli, whether it is real plastic or the elasto plastic solid.

Thank you very much.

Principles of Metal Forming Technology

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Lecture-16
Measures of yielding and ductility in tensile testing

Welcome to the lecture on Measures of yielding and ductility in tensile testing. So, in this lecture in this week mainly we will discuss about the different types of terminologies related to tensile testing, the failure of the material mostly related to the effect of the different parameters on the flow properties and all other associated you know topics.

So, in this lecture we are going to discuss about the terminology related to yielding and also ductility in the case of tensile testing. Now, when we earlier talked about the stress and strained, now as we know that we have the two types of stress and strain curve a nd normally when talk about stress and strain curve it refers to the engineering stress, engineering strain curve and in that normally the engineering the stress is defined as the load divided by the original area.

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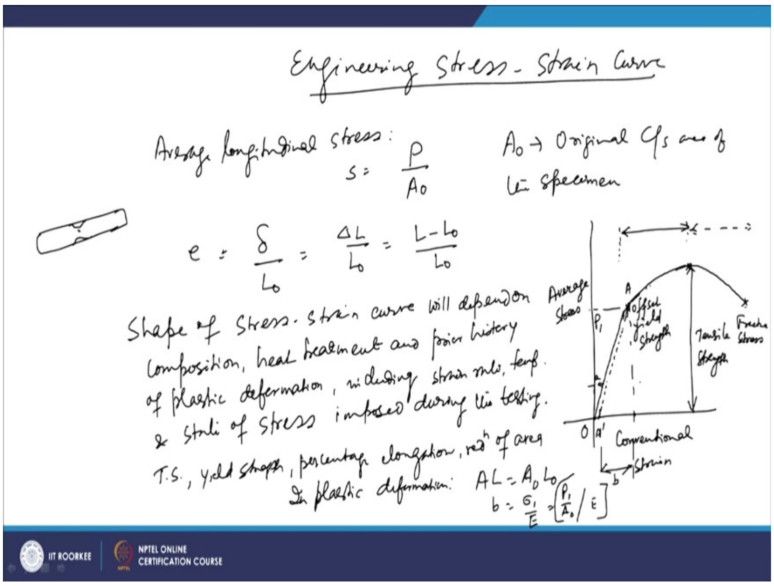

So, basically when we to talk about the engineering stress strains curve. Now, in this case we defined the engineering stress for that we have the load and the load is divided by the original cross-sectional area. So, we find the average longitudinal stress and for that so, what (Refer Time: 02:12) we get in the case of these engineering stress strain curve. Now, in this basically as you know that we try to subject the specimen to continuously increasing you know stress and then ultimately material fails. So, in this case what we do is we find the engineering stress and that basically is found by the load divided by the original cross-sectional area of the specimen.

So, this is basically original cross-sectional area of the specimen. Now, in this case the strain which we get that is basically the average linear strain and it is again divided by it is found by dividing the elongation of the gauge length of the specimen to the original length. So, we get e as $\frac{\delta}{L_o}$. So, that is this is the elongation in the gauge length that is your $\frac{\Delta L}{L_o}$ and that is nothing, but $\left(\frac{L-L_o}{L_o}\right)$.

So, this is how you get the engineering stress and engineering strain and you have a curve which talks about the this engineering stress strain curve, which initially increases and you have certain up to certain portion you have the linear region and then it deviates from linearity and then it goes on increasing to certain point and at that point there is a point where you know it has the maximum value of the these stress and then after that the stress value you goes on decreasing.

So, from there it will come and then finally, at some point of time it will be the fracture of the specimen. Now, the thing is that the stress and strain in this case the shape and magnitude of this stress strain curve. Now that depends upon many things like the composition of the material or the heat treatment of the material or the state of stress, which is imposed on the material; the shape of the stress strain curve will depend on composition, heat treatment and prior history of plastic deformation, including strain rate temperature and state of stress imposed during the testing.

So, what it means that the stress strain curve typically look at these stress strain curve. Now, what we see? We have already seen that it will go linearly up to certain point and from there it will we changing from the linearities vanishing and then it will come at one final point and then at this point; this is the point after that this average stress value is going on decreasing. So, this is your average stress and this is your conventional strain. So, what we see that now what we see that now this is the point what we call if you live it here.

Now, the actually add this point if you live there will be some amount of the plastic strain left and this point is so, normally this is known as the offset you were defining this value some percentage of some percentage or some value of the strain we defined. And that is why this point is the strength is an offset yield strength. Now, this point which is the maximum offset this value goes on decreasing, this point is known as the tensile strength of the material. And the stress which we get here that is known as the fracture stress. The thing is that what we see this is the case normally in this region you have the elastic deformation and then you go from here the plastic deformation starts and finally, the material is going under the fracture.

Now, the thing is that these terminology which are required for analyzing the stress strain behavior of the material. So, they are like so, as we see you have the tensile strength, then you have yield strength and then you have the percentage elongation. Similarly, you have the reduction of area. Now, what happens that since you if you are straining in the direction so, in the longitudinal direction. So, there will be certainly there will be increase in length. So, you will have percentage elongation again will be defined as the change in length divided by original length into 100. So, that will giving you the percentage elongation and as there is elongation as we know that we have the transfers direction, the transfer direction you have the compression area will go on decreasing. So, basically you have the reduction of area.

So, so, this way your engineering typical engineering stress strain curve looks like. Now, the thing is that as we discussed that in this elastic region the stress is proportional to the strain. Now, when this you know load will be exceeding a value corresponding to the yield strength, in that case the from here from this point where from the where the this loads increasing the yield, you know the stress value. Now, from here onwards you have the that is the plastic range of deformation and this is known as the plastic deformation. It means that if the material is unloaded even then also you know you will have some actually the deformation which is still there in the you know, permanent deformation will there of the material.

Now, if after these the stress required which is there in the plastic deformation range is basically increasing; that is what we see in this curve that stress requirement is going on increasing in this zone. Now, this is the zone where this you see that this is there is stress increasing, basically this is the zone of the where the material is getting hardened. So, as there is increase in the plastic strain the material is getting hardened and that is why this zone is known as the strain hardening zone. So, this is what the strain hardening topic is known as.

Now, the thing is that when we talk about the plastic deformation. So, as we already discussed that in the case of plastic deformation the volume constancy has hold good. So, volume constancy if it is there in that case you will have A ×L. So, in plastic deformation you have volume constancy will tell that $A_o L_o = ¿$.

So, it means that if the L is increasing, if L_o is the actual gage length, if the $L > L_o$ in that case A has to be smaller than A_o. It means that there will be change in the reduction of the cross sectional area. So, you will have the compensation to the change in length or increase in length by the reduction of the cross-sectional area. Now, what happens into these zones? In one zone what you see if you look at these zone there is a one zone where you see that is going on increasing and then in the second zone what you see is that it is decreasing. Now, what is the reason for increase or decrease?

So, what we see that in this zone there is basically the strain hardening being taking place and initially this strain hardening which is taking place. Now, this is more than compensate for the decrease in the area and that is why your stress value is continuing to rise with the strain. Whereas, after one point of time what happens that there will be the decrease in the cross sectional area, that basically is greater than the you know increase in the deformation load and that is why this basically in this zone it starts decreasing. So, so in this zone the load which there that dominates and in this zone basically the reduction, basically reduction becomes quite less load has lesser value

So, that is why what happens that in this zone this goes on decreasing and ultimately it will come at a point where it will fracture. So, as you know that once it comes here then the specimen starts basically necking. So, this is a point once you come to this state a point is reached and this point is in the specimen you will get a place which is weaker than the rest of the material or rest of the part of the specimen and from there basically the localized necking starts.

So, in this case the because during the plastic deformation the specimen starts thinning down or necking locally at that particular point; you may have any specimen and in that during the at any point it may start you know necking downs. So, it locally it starts thinning down and this area of cross sectional basically is decreasing locally quite, you know at a you know very rapid rate. So, with rapid rate it is decreasing the this is cross-sectional area and that is basically that is how these actual load requirement; basically if you look at the load which is because you are here it is the average stress which is nothing, but the load divided by the original area.

So, actually you have required lower load. So, the load becoming lesser and divided by original area actually that gives you the smaller value of these average stress values. But in actual case if you look at the load and since the area is decreasing a lot. So, if you find the actual stress value then that does not decrease

So, that can be seen from the graph of or the curve of the true stress true strain curve. In the true stress true strain curve it does not basically decrease in fact, the in that case although there is a decrease in the load you know which is required for further deformation, but at the same time there is rapid decrease in the cross sectional area. So, the stress value which is there generated that is still you know in the in the increasing side.

So, what happens after this also in the case of true stress true strain curve it does not go in the bottom side, but it will go in the upper side and then at one point of time it will fracture. So, that is there in the case of the true stress true strain. Now, if you look at these curve what you see if you can analyze this curve? If you look at this curve you see that this is suppose a point and suppose you are getting this point as A and it has come to $A^{'}$. So, this is total and then it has gone to this side.

So, we have recoverable strain elastic part; now this is the plastic strain. Now, the thing is that if you look in this zone which is which has a linear zone and we know that you have this is in the slope of this line this is nothing, but we find the models of elasticity of the material.

And you have the recoverable elastic strain and that will be basically the $\frac{\sigma_1}{E}$. So, that is what we get the recoverable elastic strain and if you define this recoverable elastic strength. So, if this recoverable elastic strain in defined as b so, this be will basically $\frac{\sigma_1}{E}$.

So, that is basically $\frac{P_1}{A_0}$ and then. So, if suppose this is P_1 in that case $\frac{\frac{P_1}{A_0}}{E}$ so, that is known as the b part. And you have then you have the permanent deformation that is there and it is the offset that is that is A. So, this A part which is shown this is the offset value which is taken and that is why this is known as the offset yield strength because that will be the permanent you know deformation which is retained in the material.

So, this material which is if suppose you are coming to the another point then from there it will; again if you go to any point it will come in this same fashion. You will have some of some as the elastic recoverable strain that will again divided by so, that stress value divided by the models of elasticity E and then other part will be your plastic strain. So, that way you find it so, and your path goes in that fashion. So, it will be parallel to this line you know up to which it was the linear one so, it will go and come in a in this direction. Now, we will deal with the different other terminologies which we come across in the case of plastic deformation.

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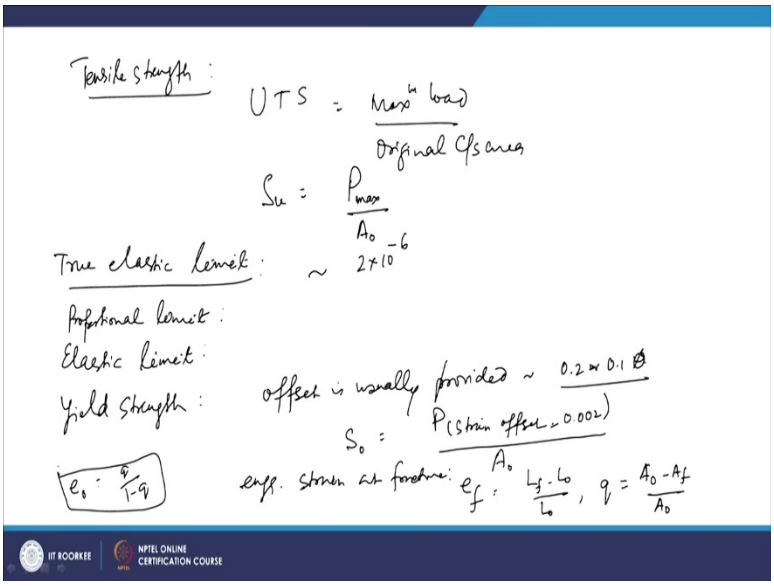

One of the term is tensile strength. Now, this tensile strength it is the so, if you we also call it as ultimate tensile strength or we also call it as tensile strength TS and this is basically the $\frac{maximum\,load}{original\,cross\,section\,area}$.

So, as we see that engineering stress strain curve now, this is where you have the maximum load and this divided by the original cross-sectional area that gives you the ultimate tensile strength. So, we define it has $S_u = \frac{P_{max}}{A_0}$. So, that is known as the ultimate tensile strength and it is very much important whenever we talk about the property of the materials.

So, for a tensile test we give a lot of significance to this parameter. And for the ductile materials it is measured as the maximum load basically which the material can with stand under the restrictive conditions of uniaxial loading. So, that is how we define these ultimate tensile strength. Now, we will discuss about the measures of yielding, what are you know what are the other you know terminologies which we discuss when there is yielding in those cases.

So, if you talk about the different you know criteria for the insistence of yielding. So, it will be depending upon many parameters and we will discuss one by one. Now, if you see if you we refer to the this graph; now in this graph we have seen that this point up to which the elastic there is linear region. Then we have some region where there is no linear, but still it is elastic it will come to back to its on position. But then after that you have the presence of the plastic zone.

Now, so, it means that the material gradually it is changing from the elastic to the plastic behavior and it is very sometimes very difficult where actually the plastic deformation in fact, starts. So there are various criteria, various parameters which basically define also that where how much sensitive that point will be. How you can say the different point I mean on this stress strain curve. So, if you try to see the different terminologies what we discuss one is true elastic limit.

So, true elastic limit is basically it is based on the macro strain measurement and it will be of the order of so, if you look at this strain value it will be order of 2×10^{-6}. So, it is basically because the movement of the few hundred dislocations only and elastic limit has very very low value in this zone. Then we come to the proportional limit. Now, the proportional limit it is the highest stress which is directly proportionally to the strains where we see that we are going to the zone at the end of this line where we reach.

And this point where this the maximum point which we reach where the stress is linearly proportional to strain and that point is known as the proportional limit. And this point can be understood by locating that place where the stress strain curve is basically deviating from the linearity. So, straight line proportion from where it is deviating from straight line proportion that point is the proportional limit; then you have the elastic limit. Now, the elastic limit it is the greatest stress which the material can which stand without any measurable permanent strain remaining in the material once you release the load. So, as you know that you had gone in a straight line and after that also you have a zone which is not following in the same path, which as some deviation from linearity.

But still if you are trying to unload the material you come to 0 stress 0 load, in that case the material will not have any you know plastic deformation. So, that is known as the the elastic limit of the material. And if the sensitivity of the strain measurement is increased; in those cases these elastic limit value is a decreased until this until this until you know at the limit where it will be equal in the true elastic limit determined from the micro strain measurement. So, truly a that that will be basically decreasing to as the sensitivity of the instrument will be increasing, you can come back come back to down to down values and ideally it should be the true elastic limit of the material.

Now, further you have to see that the other point is the yield strength value. Now, this is basically the stress which is required to produce the specified amount of plastic deformation. As we have seen that the yield strength yielding starts basically and we are specifying certain value of the plastic deformation and that basically normally is the offset yield strength.

So, you know we are basically deriving a line which is parallel to this straight part and where it is cutting. So, this part is the offset and this way we are reaching at a point so, if you leave the material at this point then you will reach at this point. So, if you take offset you know from this point and from here you draw parallel to this time then you will reach the point which is the offset yield strength point and normally we specify certain strain value.

So, for the offset yield strength the offset usually provide, offset is usually provided and this is normally of the order of 0.2 or 0.1 percent. So, this is how you are getting the yield strength. So, yield strength you can define as we define S_0 and that will be P that is load when the strain is basically strain offset is coming as suppose 0.002. So, strain is 0.002 or and then so, this is not percentage and then this by A_0. So, this way get the yield strength value and be define that.

So, we also call it as the proof stress value in many terminologies. Now, we will come to the measures of ductility as we know that in the case of metal forming your requirement is the material has to form, it has undergo the plastic deformation. So, it must have a adequate ductility and you know there is another appropriate term that is malleability, but they have the equal you know capability certainly in an difference sense. But now, you must know that the you must have the understanding of proper understanding of the ductility.

You know because it is very qualitative when we talk about the you know ductility term and for that what we do is that when we do the material undergoing these operational like rolling or extrusion. Now, in this case the it will be quantified in terms of the degree up which you can either roll them or extrude them till it fractures. So, that is how you basically indicate the extent to which this the material can be deformed or you also define in a very general way, you can plastically deform the material up to fracture. So, that indicates its ability to be ductile or ability to be worked upon.

Now, so this is about the you know the measures of the ductility of the material. We have discussed about the terminologies like you have engineering strain or you have also the so, you have change in length that is increase in length or reduce reduction in the area and that also is basically expressed as a percentage. So, if you look at these elongation so, if you look at the engineering strain at fracture.

So, when we do the tension test in those case engineering strain up fracture $e_f = \frac{L_f - L_0}{L_0}$.

Similarly, you have reduction in area so, that will $\frac{A_0 - A_f}{A_0}$. So, because the area is decreasing and length is increasing. So, that is how you find e_f and q. So, we find it and if you try to find the relationship between e naught and basically q so, basically $e_0$you can get as $\frac{q}{-q}$.

So, this can be found from the this expressions; you have other terms like modulus of elasticity which talks about the rigidity of the material, because that will be the that is found by finding the slope of these engineering stress strain curve where you see that it goes straight.

So, these are the terminologies then you have other terminologies like you have toughness, resilience. So, resilience will be there during that zone when the stress deformation is less and when we talk about the zone plastic deformation zone; they are the energy of absorbed or the energy inside these stress strain curve. The area under the stress strain curve that is indicative of the toughness of the material, when we talk about the plastic deformation.

And as we discuss that when we talk about the true stress true strain curve in those cases normally your graph is not basically decreasing but it will go and then it will be going and doing fracture like this. So, these are the different terminologies, which we come across in the case of these yielding of the materials.

Thank you.

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Lecture - 17
Instability in tension

Welcome to the lecture on Instability in tension. So, in the last lecture we discussed about the flow terminologies, in the stress strain diagram and in this lecture we are going to discuss few more about it and then also we are going to discuss about the stability criteria's; not criteria's basically that making which occurs in tension and mostly we will talk about the true stress true strain curve and its related terms.

So, in the last lecture we discussed about you know the terminologies in the stress strain diagram. You we saw the elastic limit. We had seen the proof stress, yield stress and then the specimen goes to a maximum load, then is further decreases and then finally, you have a fracture point.

So, in the initial reason the line which is there, the linear reason the, which is there its slope is nothing, but the elastic modulus of the material and as far as the modulus of elasticity is there, it is intrinsic property of the material. And actually this is because of the binding forces between the atoms. So, you can change it by only doing undergoing those treatments which basically changes the force of the attraction between the atoms of molecules.

And so, there are certainly certain methods by which you can somewhat change, but otherwise it is a structure insensitive property. So, that way you can do some changes by the heat treatment or alloying or so. So then apart form that, we discussed also about certain terminologies like resilience and toughness.

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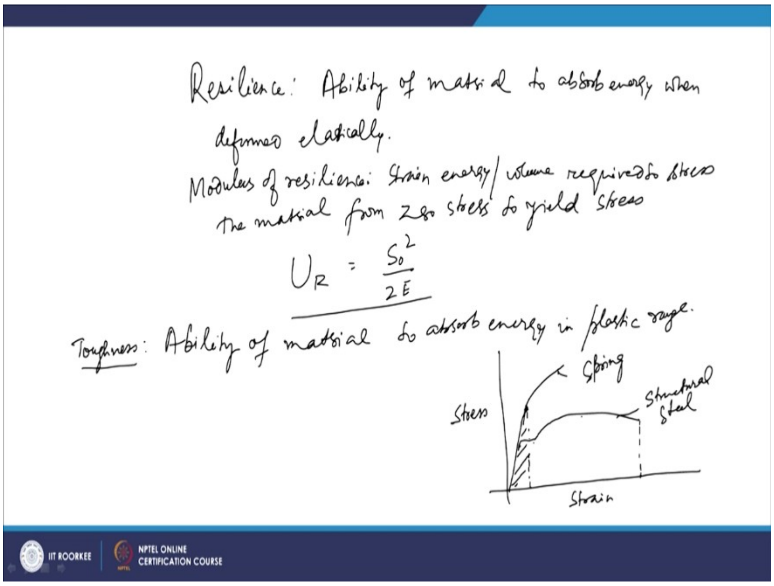

So, as we discussed that the resilience basically, this resilience which we talk about; now this resilience is the ability to absorb energy when deformed elastically. So, ability of material to absorb energy when deformed elastically; normally we measure it in the terms of a modulus of resilience which is nothing, but the strain energy per unit volume that is required for the material to stress the material from zero stress to yield stress. So, that is known as the modulus of resilience. So, if you try to define that this is basically the strain energy per unit volume required to stress the material from zero stress to yield stress. So, this property is known as the modulus of resilience and then this is basically defined as you know $\frac{S_0^2}{2E}$.

So, basically what we mean by this equation? This equation what we see you are that is your modulus of resilience. And this is basically the you know resisting energy load in application when the material must not undergo permanent distortion. So, that is basically we are talking about the material being deformed in the elastic region. So, mostly it is used typically we when we try to define the property of the springs where the deformation has to be elastic range, the when springs has to deform and then it has first further to gain to come to its own you know position. You cannot to afford to have the permanent deformation in the case of springs.

So, in those cases we define this property and you known for the springs, the springs which will have the higher yield stress and low modulus of elasticity; certainly they will have the values of the higher value of the modulus of resilience. So, that way this resilience is defined, but then another property which is further having the importance is toughness

So, we often come across this term toughness. Now, this also the same ability of material to absorb energy; but this is defined when you are in the plastic range. So, this is ability of material to absorb energy in plastic range. So, when the material goes beyond the elastic range and it reaches in the plastic range is there, its ability to absorb energy that is known as the toughness. And many a times you know it has the material has to undergo the stresses value which are more than the yield stress and that time the it is ability to absorb the that energy without fracture that is this property.

And normally many a times this is encountered in many applications like mostly for the gears or chains so, or the crane hooks. So, in those cases you required this property that is a toughness. So, you require this toughness of the material. Another way you can also think of the toughness is by finding the area under the curve that is the stress strain curve. So, basically you have two areas one is; the area under the zone which is the elastic zone, another will be the area under that zone which is the plastic zone.

So, if you typically look at a stress strain diagram if you see, now you may have a stress strain diagram which may go like this and then it may go here. And you know have another material which may go and which may go like this and then that way it will be it will be showing the, you know behavior like this. You may have two kind of material. Now if you look at this material, here you have the zone which is the you know elastic zone.

Now, if you look at these two curves what you see is that in this is the area under this curve. So, basically if you look at the work done per unit volume in this cases now, the total area under this stress strain curve; this is the amount of work that is per unit volume which can be done on the material without causing into fractures. So, that is this is how this area basically they are indicative of. So, this area will talk about how much energy this material, I mean the how much work per unit volume can be done on this material so, that before fracture. So, that is what the area is.

Now if you look at this curve, now in this you have two this is for two materials. Suppose this is for material which is you know for a spring. This is for the material this is for this normally the high carbon steels are used for making this steel and what you see is that it has this is indicative of the resilience. So, they have the higher resilience whereas, if you have the structural material structural steels are there. So, this is for that material. Now if you can see for this is spring material, it has basically the high yield strength because here your yielding occurs here from and in this case the yielding occurs here.

Now if they have the higher yield strength and tensile strength, then this structural steel which is having lower carbon you know material that is medium carbon steel. But what you see is that it has more resilience which it is more resilience. It has what you see is that this structural steel is having more ductility. It goes up to this point whereas; this does not go here itself. After that it does not go, so it fractures; but it is going to the larger value of the strain.

So, basically when it is comprising; so here you have the larger ductility as well as you have strength also, but it has a larger ductility and that is why since in the plastic zone, so this area whole area will be basically indicative of the energy or work which can be worked on this before fracture and that is why this material is said to be the tougher material. Whereas the spring steel this steel is having a more resilient is said to be more resilient. It is you know modulus of resilience is higher for this spring steel because of its higher yield strength.

So, this is something which is required when we try to deform the material. When we try to use the material for different applications in those cases, this is very much required.

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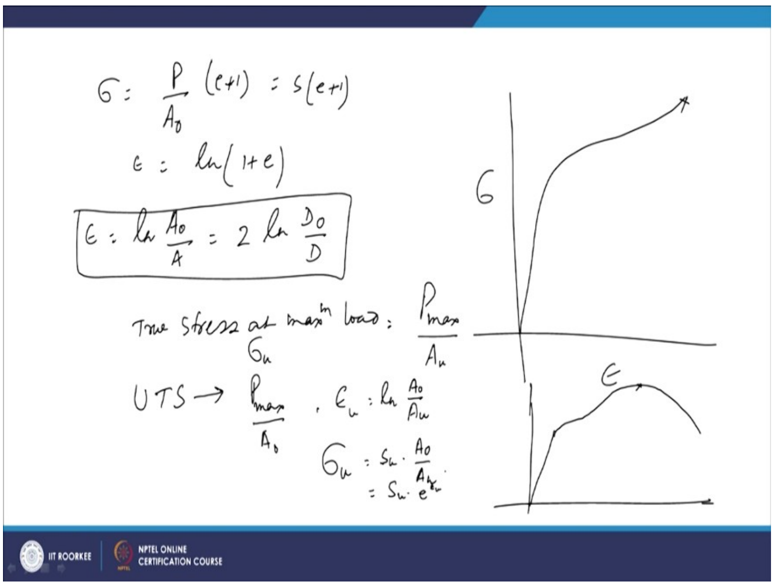

Now you can further try to refer to the, we can recall our we can recall our studies on the true stress true strain curve. So, as you see that when you have the true stress true strain curve so, you go to that and then this way it goes. So, this your true stress and true strain curve and here it fractures

Now, in this case the stress which is required I mean to be for the material to flow plastically that is known as the flow curve and here the difference as already discussed between them is that. In this case you calculate the stress based on the instantaneous area whereas, in the case of engineering stress calculation we find the load divided by the original area. So, that is how it is a found and what we see that in this case stress will be this will be $\frac{P}{A} A_0 (A+1)$. So, that is how we find. So, that will be E + 1.

So, also we get the true strain as $\ln(1+e)$. So, this way we get the value of the true stress as well as the true strain, also when you have because there is constancy of volume in the case of plastic deformation. So, we you also call this as the ratio of the area original to origin of area final and that is why we call it as $2\frac{\ln D_0}{D}$.

So, basically when you have the change in length, at the same time you have the decrease in the you know this is the diameter of the specimen. So, so that way you also define this true strain as this also. So, this is known about the true stress true strain curve. And if you see the true stress at maximum load, so true stress at maximum load also can be calculated and true stress at maximum loads anyway; in all these cases what you do is so, maximum load will be if someone maximum load is P_{max} and then that will be you have to derived by the area at that point.

So, basically you haveσ_u. Now before that you have to see the ultimate tensile strength of the material. When we decide about the ultimate tensile strength of the material, U T S at that time when you have the engineering stress strain curve that time we so, we normally go to this point. So, at this at this point you have the maximum load and this maximum load divided by the original area that is your ultimate tensile strength. But the true stress at maximum load that is σ_u that will be your $\frac{P_{max}}{A_u}$ and your ϵ_u that is ultimate strain at that point that will be again $\frac{\ln A_0}{A_u}$.

So, if you try to find the expression, you will find that the ultimate true stress. The true stress at the maximum load σ_u it will be basically $S_u \frac{A_0}{A_u}$. So, S_u can write that it will be $S_u e^{\epsilon_u}$. So, this way you find the ultimate you know a true stress at the maximum load and this will be required when we analyze this curves in the further you know analysis.

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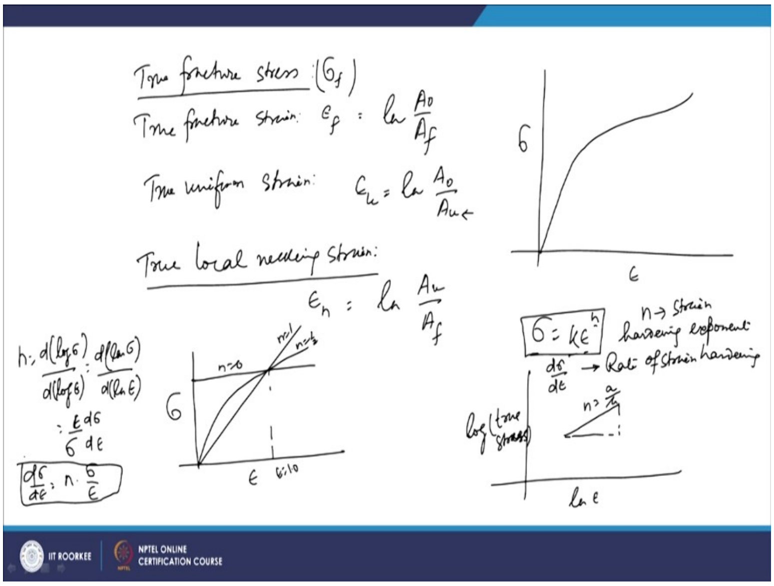

You must also know about the certain terms like you have the true fracture stress so, if you talk about those terminologies like true fracture stress. So, you go to the fracture point and at that time the true stress value which is calculated that is your true fracture stress and again that will be basically glowed at load which is there at the facture divided by the area of cross section which is at the fracture. So, that is true fracture stress. Similarly you have true fracture strain.

So, again true fracture strain will be so, that will be denoted by ϵ_f, this will be σ_f. So, true fracture is true stress this will be true fracture strain. So, that will be again $\ln \frac{A_0}{A_f}$. So, that is area which is there I mean that is reduced area at the fracture point. So, that will be $\ln \frac{A_0}{A_f}$. So, that is your true fracture strain. So, you can further converted in terms of the reduction of area so, that you can further do it, in terms of $\ln\frac{1}{1-q}$. So, that way true fracture stress can be found. So, this is $\frac{A_0}{A_f}$ or you can also write it as $\ln\frac{1}{1-q}$.

Similarly, you have a the other conditions like you have the true uniform strain. So, if you talk about this terminology, it is basically based on the strain up to the maximum load. So, it is basically that and it means it may be calculated based on the cross sectional area A_u and and that is why this ϵ_u is defined as $\ln\frac{A_0}{A_u}$. So, it is basically based on this A_u value what you get is that is known as true uniform strain and you have also the true local necking the strain.

So, true local necking strain can be calculated and in this case what we feel is that this is a for the strain which is required to deform the specimen from maximum load to the fracture point. So, so in the case of engineering strain, you have this from there actually the value of stress normally dips down whereas, it that does not happen in the case of the true stress true strain curve.

So, in this case the local necking strain it will be defined as $\ln\frac{A_u}{A_f}$ So, A_uu will be the area and for when this necking starts and then it will be going up to so at the maximum point and then from so, in the in the case of engineering stress curve and then from at the fracture where you have the final area which is occurring and that will be the ϵ_f. So, this way you try to find the true fracture strain.

We already discussed about the true stress true strain curve and what we saw in that is that you have that can be represented by the equation that is$\sigma = K.\epsilon_n$. So, you have 1, stress 1 curve which goes like this and we know that this is the strength coefficient and this is the you know n is the strain hardening exponent and this is the true strain and this is the true stress value.

So, you can find this n by finding ah; so that will be nothing, but if you find the graph between the $\log\sigma$ and $\log\epsilon$, then slope of that curve which can that will be a linear curve and the slope of that curve basically will give you the n. So, basically if it is the log of true stress and this is by log of true strain, then what you see you get one line and basically the slope of this line, basically this n; that is the known as the strain hardening exponent in those cases.

So, basically depending upon the value of this n, you will have different type of the curves you have $\sigma = K . \epsilon_n$. So, n may vary n may different values; n may be 0, n may be half or so. So n may be 1. So, if the n is 1, in that case you have a linear curve. So, you may have different kind of curves. So, if you have σand ϵ. So, if you look at n = 0 so if n = 0, so you will have $\sigma = K$. So, you will have this value for n = 0.

Now if you there is n = 1; so again n = 1 means this will be $\sigma = K . \epsilon$. So, you will have a liner equation. So, it will go like this. So, this will n = 1. You may have n as half and in those cases, if your n becomes half. So, n will go like this and this will be n =0.5. So, this way this is $\epsilon =$¿1. So, this way this power like equation that is this $\sigma = K . \epsilon_n$. This is known as power law equation and that basically changes in shape for the different materials.

There is another terminology which is required to be known to you and that is basically the rate of strain hardening. So, the rate of strain hardening and the strain hardening exponent, this n is known as strain hardening exponent. And there is a terms known as rate of a strain hardening. The rate of a strain hardening is basically different, then this so this is basically $\frac{d\sigma}{d\epsilon}$. So, the curve which is there so, $\frac{d\sigma}{d\epsilon}$is known as the rate of strain hardening.

So, you can use this curve analyze further to we find the rate of strain hardening and if you look at the value of n, if you try to find. So, value of n is nothing, but is d ¿¿ that is what we know. This is the slope of the log log curve $\frac{\log true\ stress}{\log true\ strain}$. Now this can be written as $\frac{d . \ln \sigma}{d . \ln \epsilon}$ and that can be written as; so this will be. So, this will $\frac{\epsilon . d\sigma}{\sigma . d\epsilon}$and this will be ϵ. So, it will be if basically $\frac{\epsilon . d\sigma}{\sigma . d\epsilon}$. So, what you see is that $\frac{d\sigma}{d\epsilon}$. So, which is nothing, but the rate of strain hardening that will be basically, if you look at this will be .n.σ / ϵ

So, this is if certainly a function there is a correlation, which exists between the rate of strain hardening and the strain hardening exponent. Further there has been many kind of these you known relationship between the true stress and true strain proposed by the different types of researchers.

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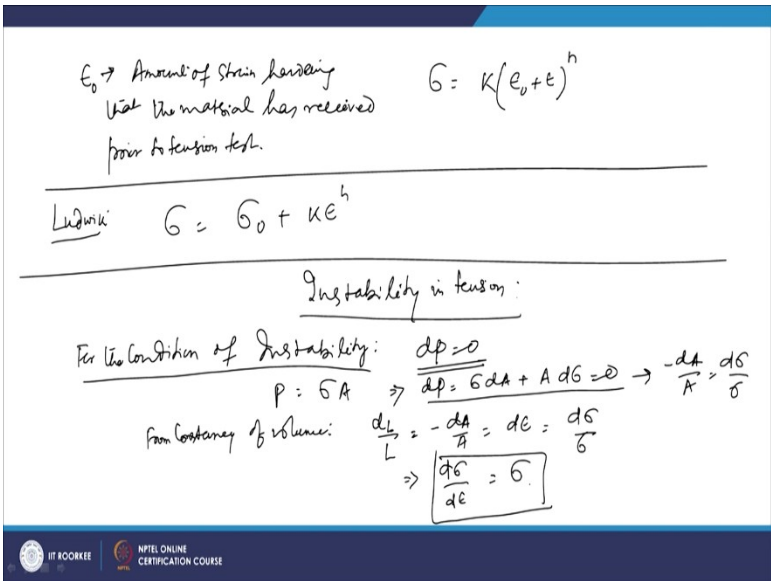

So, the another kind of relationship which has been suggested by (Refer Time: 24:09) and he tells that the sigma is basically can be expressed as $\epsilon_0+\epsilon^n$

Now, in this case this is also the value of true strain and here this epsilon naught. So, this is basically the amount of strain hardening that the material has received a prior to the tension test. So, material might have got certain strain hardening prior to the tension test which is, it is subjected to now. So, that the material has received prior to tension test.

So, in this case that is represented by$\sigma=K.(\epsilon_0+\epsilon)^n$. There is further one variation in this and that is proposed by Ludwik and Ludwik equation tells that $\sigma=\sigma_0+K.\epsilon^n$. Now in this case this $\sigma_0$is the yield stress and K and n are the you know strength coefficient. This is and then this is n is the same. They are same as what we saw earlier in those cases. So, this way they have given different types of expressions and from there you can have the value of you know you can find this slopes you can find the value of n or so.

Now, we will try to discus about the instability in tension. Now what happens that so, if we talk about the instability in tension. So, by this time we know that in the true stress true strain curve, it will come to a point and then from the there the plastic deformation starts and then ultimately the material starts you know necking down and finally, it fractures. So, if you look at the engineering stress strain curve at the point of necking, what you see is that the curve starts coming down.

Basically so, what you see that at this is the point maximum point and from there it starts coming down. So, that point can be found out by letting the derivative b = 0. So, basically the necking will be obtained necking. Necking will start at the maximum load during the tensile deformation of the ductile material. And the ideal plastic material is one where the no strain hardening occurs in those cases which would become unstable in the tension. So, once you apply it will become unstable and it will be start necking as soon as the yield you know starts.

So, as soon as the yielding will start, there basically the you know it will there will be no strain hardening and then there will be necking started. Whereas, in the normal cases what we saw that after yielding you have a reason of strain hardening and then once you reach at is as a different point, there the necking starts. So, basically in ideal plastic material from there actually necking will start. Now so, this is normally for the case of the real metal where the strain hardening goes on. Now necking or the localized this you know deformation, we think about this start at the maximum load and where the increase in the stress so, you as we have already discussed that in one case when it is increasing.

So, there one of the parameter is dominant whereas, the another parameters you have increase in the strain hardening, then you have the decrease in the cross section. So, increase in the stress due to decrease in the cross section area of the specimen is larger. So, that is that is why there will be you know the increasing strain hardening that is observed. And so, what we can say is that the condition of instability.

Now in this case the condition is that dp = 0. So, the maximum load is there. So, at that point when the dp = zero that time the necking will start. Now what we see is $P=\sigma A$. So, once you have $P=\sigma A$, so, that from here you can find $dp=\sigma.dA+A.d\sigma=0$.. So, this is the condition for the instability.

Now what you see that from since in the plastic deformation? You have constancy of volume.

So, in the constancy of volume, you we see that $\frac{d_L}{L}=\frac{-dA}{A}$ and which is nothing, but$d\epsilon$. So, what we see that if you use this $\frac{-dA}{A}$, it will be$\frac{d\sigma}{\sigma}$. So, that is why so, from here you get

$\frac{-dA}{A} = \frac{d\sigma}{\sigma}$. So, this $\frac{-dA}{A} = \frac{d\sigma}{\sigma}$. So, that will be equal to this. So, what you see is $\frac{d\sigma}{d\epsilon} = \sigma$. So, this is the condition of instability.

So, what we see that in the case of the necking point where the necking starts, you see that the in the true stress true strain curve. It is a slope becomes equal to the true stress value. And at that point where the slope becomes equal to the true stress value that is you know this point that will be at that point there will be necking started. So, that is how you try to find the point of necking. You can further have alternate expressions for this.

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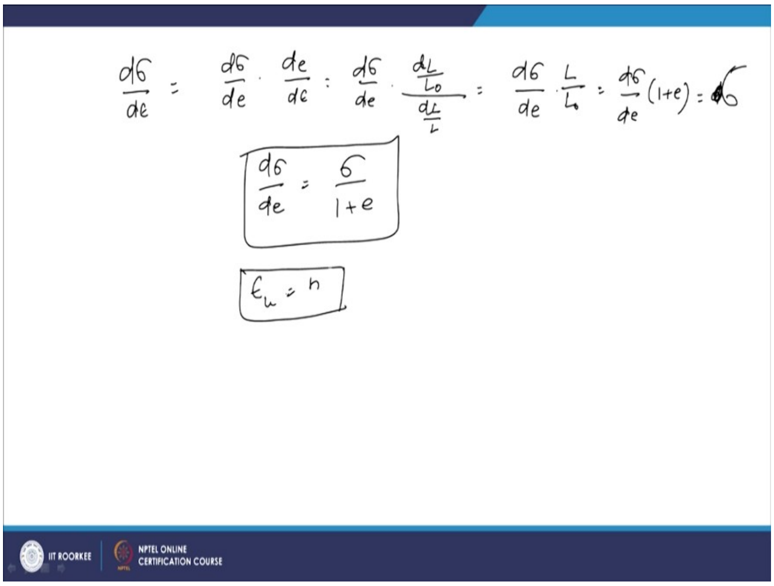

So, what we see that you get $\frac{d\sigma}{d\epsilon}$ and you can write further that as $\frac{d\sigma}{de} . \frac{de}{d\epsilon}$. Now you can write this $\frac{d\sigma}{de}$ that is your engineering strain and then the $\frac{de}{d\epsilon}$. So, *de* we known that $de = \frac{\frac{dL}{L_0}/dL}{L}$

So, what we can write is that $\frac{d\sigma}{de} . \frac{L}{L_0} = \frac{d\sigma}{de}(a+e)$.. So, it will be $\frac{d\sigma}{de} = \frac{\sigma}{1+e}$. So, that can be written as d. So, that is that we have already seen that this σ/de has to be equal to σ for the condition of instability. So, what we see is that $\frac{d\sigma}{de}$ will be $\frac{\sigma}{1+e}$ because *de* we know that *dσ* by. So, e is ϵ is 1 + e. So, we can write $\frac{d\sigma}{de}$as $\frac{\sigma}{1+e}$. So, these are the few conditions which we get to know where about the condition of instability in the case of the tension testing when the necking starts.

We also get some other you know the relationship values and from there what we see is that we get these two uniform strain that becomes equal to n also in the case of the instability in tension. So, what we have seen in this lecture that you have you must be quite conversion all these terminologies and then you have the condition instability where we try to see that how with under the different conditions. How we can have the different expressions, which express those conditions of the necking and those when the stability occurs and there is necking and then you have fracture. So, these expressions will be of importance when we discussed about the failure criteria or not failure criteria; in fact, when we discussed about finding the stress and strain.

Thank you.

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Lecture - 18
Strain rate effects on flow properties

Welcome to the lecture on Strain Rate Effects on Flow Properties. So, we have discussed about the true stress true strain curve. And we have seen that how these you know how the there is behavior of the true stress true strain curve. You have the point of maximum load you have the point of fracture. Now this is for we are not we have not talked about the strain rate, because you have many parameters which affect these flow characteristics of the material. And in that basically one of the important parameters which effects the flow properties of the material or the flow stress of the material is the strain rate.

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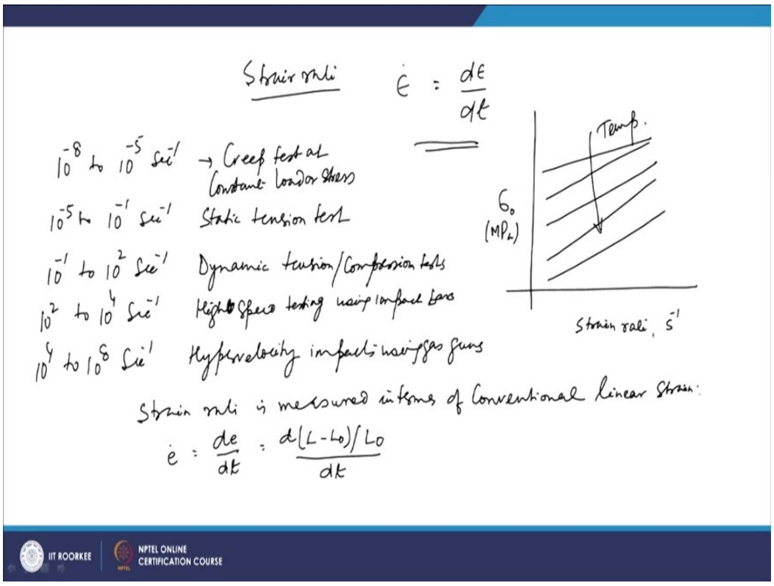

So, what we define the strain rate so, strain rate basically we defined it as this ϵ_0, $\dot{\epsilon}$and that is basically the way you the rate of straining, so, that will be $\frac{d\epsilon}{dt}$. And it has effect on the flow stress value is the point where the yielding will occur or the point at reach the fracture will start. They are affected because of the strain rates, and basic specially it is effect is there on the yield strength of the material. So, it is unit as you see so, it is unit will be per second. So, because the strain has no unit so, ultimately the strain rate has unit of per second here as it looks like.

Now, the in general, it you can understand that when you are increasing the strain rate, then it increases the flow stress value, and that can be found out by a normal, you know, a curve which talks about these strain rate dependence for the yield value. Now what we see in that this these values move like this. And they are with increase in the strain rate values, you know, they the yield strength will go on increasing certainly as this is for different temperatures.

So, you have this way I actually temperature is increasing. So, as we know that as the temperature is increasing, the flow stress requirement is smaller and, but the as the strain rate is increasing, this σ_0 that is the yield strength of the material in MPa that basically increases. So, yield stress and the flow stress at lower plastic strains are more dependent on this strain rate, rather than the tensile strength. So, when we are in that load plastic strain zone, in those cases the strain rate is very much important.

So, now if you try to see what is the implication of what is the basically importance of these strain rate how they affect. So, just have to have a feel, if you if you see the range of strain rate normally if you see the 10^{-8} to 10^{-5} sec^{-1}.

So, this is such small strain rate is what it is normally for the creep's test at constant load or stress so, this is for the creep test. Similarly, if the value is from 10^{-5} to 10^{-1} sec^{-1}. Now this is normally for the static tension test with a hydraulic or screw driven machines so, this is for static tension test, where we have the hydraulic or screw type of driven machines which do the testing. You have also the tension test or compression test the dynamic tests, and for them it goes from 10^{-1} to 10^{-2} sec^{-1} and this is for the dynamic tension or compression tests.

So, this is basically increasing, you have these values go like this. Then if you go further, 10^{-2} to 10^{-4} sec^{-1}if you see there is further increase of this strain rate. And that is basically for the high speed testing high speed testing using impact parts.

So, basically you have if you if you considered the wave propagation effects, when you do the testing with, you know, impact bars in those cases the strain rate values further increase and they become 10^{-2} to 10^{-4} sec^{-1}. And further if you go to further high value 10^{-4} to 10^{-8} sec^{-1}. This is a further higher value and this is known as hypervelocity. So, normally they these are encountered in the case of hypervelocity impact using gas guns so, this is hyper velocity impacts. Using gas guns, or you may have the expressively driven projectiles so, these are the examples of the strain rate different strain rates which are applied when we are trying to deform the material.

So, normally is static tension test we go for the strain rate values in the range from 10^{-5} to 10^{-1}, and otherwise once you go dynamic tension you have further and then high speed testing using impact parts and then hypervelocity impacts using gas guns or the expressively driven projectiles where you have the propagation of shock waves. So, in those cases your strain rates are even quite higher. Now when you apply the higher strain rates, in those cases are the materials which ordinary do not show the yield point, they also show the yield point like, you have if you do it on the low carbon steel which in ordinary case.

They do not show any yield point under the ordinary state of loading, and if you use these high strain rate then they show the yield point. So, that is what the difference of these strain rates are on the behavior or shape of the stress strain diagram. Now you must understand what is this strain rate.

So, if you look at the strain rate, what we see is that normally you have when we do the testing of the cylindrical specimen, you have one and fixed another from another side we are straining it. So, another side is basically attached to the this is a movable jaw one is fix jaw another is movable jaw, now that is the movable cross head of the machine. Now this cross head which is straining it, it is going at certain velocity, so, you have this strain rate. So, this strain rate it is basically measured in terms of the conventional linear strain. So, strain rate is measured in terms of conventional linear strain. So, what we see is if you look at the engineering strain rate $\dot{e}$, now this $\dot{e}$ it can be written as de/dt. Now de we can write as $\frac{d(L-L_0)/L_0}{dt}$,

So, $L_0$is the original length and L is the final length. So, de will be $\frac{d(L-L_0)/L_0}{dt}$. So, L_0 being the constant. So, it will go as $\frac{1}{L_0}$, and then this will be dL. So, because $dL_0$is fixed so, it will be$\frac{dL}{dt}$. Now $\frac{dL}{dt}$ is nothing but; so, that will be rate of change of this length, and this is actually expressed as $\frac{V}{L_0}$ now. So, what we see this V is nothing but the cross head velocity.

So, what we see that this is conventional strain rate so, we found the conventional strain rate.

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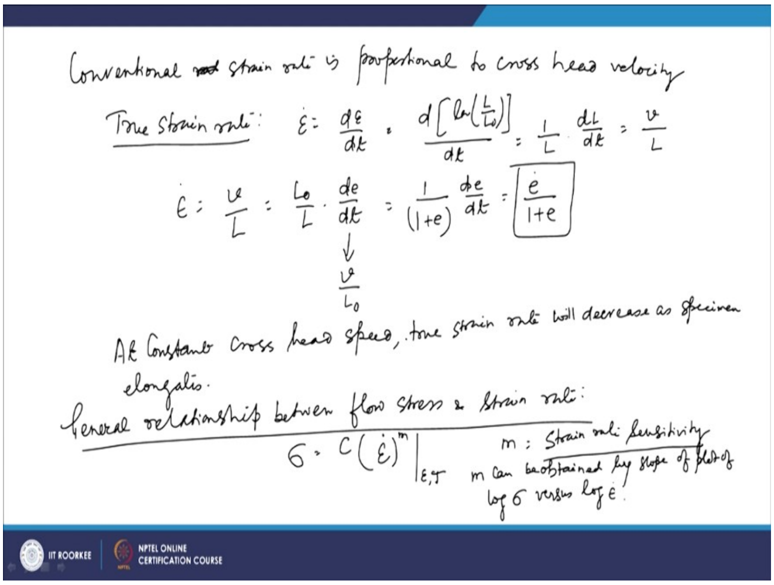

And conventional strain rate we found as V that is cross head velocity upon the gaze length; that is, the original gaze length that is L_0 so, it is proportional to the cross head velocity. Now this is how we see you measure, this conventional is strain rate, now in the modern testing machines these cross head velocity can be controlled. And this way, you can control the value you can do test or you can perform this testing at a specified strain rate. Now if you try to find the true strain rate.

So, true strain rate will that will be talking about the instantaneous length, rather than the L_0 so, it will be talking about L. Now in this case this is defined as $\frac{d\epsilon}{dt}$. So, that is in case of engineering conventional strain linear strain; that is, $\dot{e}$and it is $\dot{\epsilon}$that is $\frac{d\epsilon}{dt}$. And if you try to see the by definition this value of the you can write $d\left[\ln\left[\frac{L}{L_0}\right]\right]$ that is what the value of the $d\epsilon$ is you get the $\frac{d\left[\ln\left[\frac{L}{L_0}\right]\right]}{dt}$. So, so, we get $\frac{1}{L}$, and then we get $\frac{dL}{dt}$.

So, this way you try to find the value of the true strain rate, and that will be basically equal to $\frac{v}{L}$. So, there you got $\frac{v}{L_0}$, here you got the expression $\frac{v}{L}$. So, this is very clear that here it will be $\frac{1}{L_0}$and then $\frac{1}{L_0}$will be further multiplied. So, one L_0 will be, you know, anyway it will be cancelled.

So, you will get $\frac{1}{L}\frac{dL}{dt}$ that is $\frac{v}{L}$. Now what we see that if you can further try to, you know, have the correlation between the true strain rate and the conventional strain rate. So, you can say that true strain rate will be $\frac{v}{L}$. So, you can further write them as $\frac{L_0}{L}\frac{de}{dt}$. So, so that is how you can write this expression as the further what because we have already found the de. So, $e=\frac{de}{dt}$is basically $\frac{v}{L}$ is there $\frac{v}{L_0}$. So, L_0 the again here it comes over in the picture. So, this L_0will be changing because this is nothing but $\frac{de}{dt}=\frac{v}{L_0}$. So, that way L_0will comes $\frac{b}{L}$ will come. Now that can be further be written as $\frac{1}{1+e}$.

So, we know that there is expression relationship between the true strain and the engineering strain so, that way it will be $\frac{de}{dt}$. So,$\frac{L_0}{L}$ will be so that will be $\frac{1}{e}$. So, so, we can get $\frac{1}{1+e}\frac{de}{dt}=\frac{\dot{e}}{1+e}$ so, that way. So, once we know the, you know, engineering strain rate or conventional strain rate, and we know the conventional strain, then you can find the true strain rate. So, what we see that at constant crosshead speed so, at constant crosshead speed, now as you as you see that when you have a constant crosshead speed. In that case the true strain rate if you look at with the e will be more, if e will be higher than this will be going on decreasing.

So, the true strain rate true strain rate will decrease as specimen elongates. So, that is what, you know, as the specimen will elongate the true strain rate, to is the material is or the specimen is subjected to that will further where will be decreasing. Now you have to maintain that if you have to maintain this true strain rate, then certainly you may have you have to go for the, you know, open loop control in which your deformation velocity has to be increased. So, that way you can increase the deformation velocity from here.

So, $\frac{de}{dt}$is basically $\frac{v}{L_0}$ so, the V has to be increased. So, otherwise you have to also see that how you have to in close loop control methods ultimately in the later part you have to adjust so that your true strain rate becomes the same. So, you for maintaining this true strain rate to be uniform, you have to do the adjustments you have to change the cross head speeds. Now in this case you have a general relationship between, you have a general relationship between flow stress and strain rate.

So, as we discussed that to have the constant true strain rate, you will have to either go for the open, you know, loop control or the close loop control, and in those cases the V that is set will be a function of the ϵ_0 that is your true strain rate multiplied by the, you know, length of the original, you know, original gaze length that is L_0. And then exponential of theϵ_0, t. So, so, that way you try to have the constant, you know, velocity that value of the true strain rate. Similarly, when the plastic flow becomes localize. So, you have the non-uniform flow of I mean along the gaze length.

So, in those cases the open loop control is no longer satisfactory, because at that time the specimen elongates there is non uniform, you know, deformation that way. And in those cases you have the close loop control and they are again you have to adjust the; and the A, based on the A how you see that how the A is changing, so, based on that your true strain rate is to be controlled. Now the gender relationship between the flow stress and flow with the strain rate, that also has been suggested for A that is $\sigma . C(\dot{\epsilon})^m ¿_{\epsilon, T}$, and that is at a constant strain and the temperature. So, for a constant strain and the at a constant strain and at the constant temperature, this relationship holds good $\sigma . C(\dot{\epsilon})^m$. Now in this case this m that is exponent, this m is known as the strain rate sensitivity.

So, that will affect the flow stress of the material, and again similar to the power law curve. You can find this value of m by having the log plot of the true stress and true strain rate, and from their by binding the slope of the curve you can find this m. So, this m can be obtained by slope of plot of logσ vs log$\dot{\epsilon}$ that is the $\log \dot{\epsilon}_0$ that is your true strain rate. And this way you can have the value of the strain rate.

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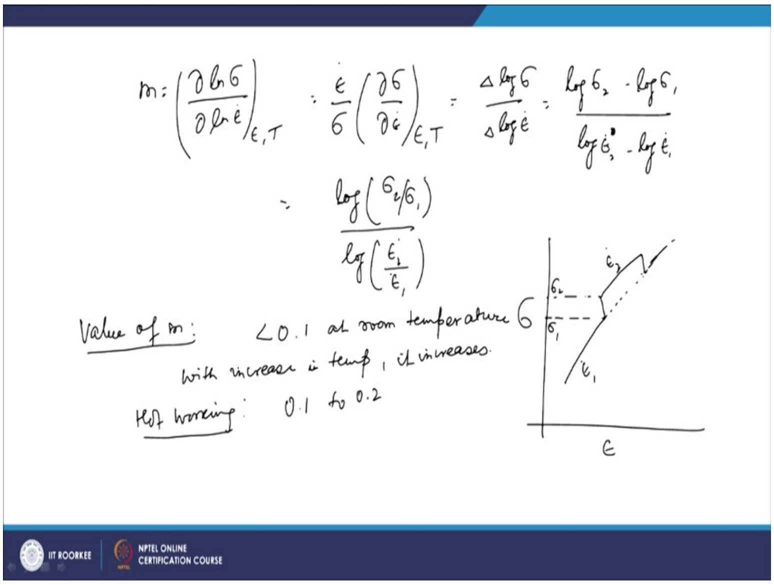

So, you can further see that this m is nothing but you have $\left(\frac{\partial \ln \sigma}{\partial \ln e}\right)_{\epsilon, T}$. That is what not this is the value of the strain rate sensitivity m, and this is at constant strain and temperature. So, again logσ will be $\frac{1}{\sigma}$ and $d\sigma$. So, so, it will be $\frac{1}{\sigma}$ will come and this way $\dot{\epsilon}$ will come so, it will be $\frac{\dot{\epsilon}}{\sigma}$. And then you will have $\frac{\partial \sigma}{\partial \dot{\epsilon}}$. That is true strain rate and this is at a particular or fixed strain as well as at the temperature.

So, this is nothing but you have this is nothing but your $\frac{\Delta \log \sigma}{\Delta \log \dot{\epsilon}}$. So, so, that way you can find if you have the graph you can find $\frac{\log \sigma_2 - \log \sigma_1}{\log \dot{\epsilon}_2 - \log \dot{\epsilon}_1}$. So, that way you get log¿¿¿¿.

So, this way you find the strain rate sensitivity of the material. Now it is value is normally quiet low at room temperature so, if you find the value of this m strain rate sensitivity. Now it is quite low at room temperature so, it is value is less than 0.1 at room temperature. But it is basically increasing with the increase in the temperature.

So, with increase in temperature it increases, so, specially the when the temperature is about above of half the absolute, you know, melting point at that time this value of m will be increasing. And if you go for the hot working conditions hot working is done about the reutilizing temperature and reutilizing temperature as we know, it something point 4 to point so, 6 times the absolute melting temperature. So, in those ranges it is value is comes 0.1 to 0.2.

So, this is something which talks about the strain rate, you know, the effect of the strain rate on the flow properties. So, that can be understood by so, if you have the stress strain curve. So, true stress and true strain curve, and if suppose you have put the strain rate and you have varied it along the testing. So, if suppose it goes initially like this and so, you have one strain rate, and then further you have changed it.

So, and then further it is going like this, now in those cases if you look at this is the $\dot{\epsilon}_1$, and another value is $\dot{\epsilon}_2$. So, what we saw, this is what we have done for this expression, in this expression and we found the value of these m. Now for this basically you have this is the σ_1, and this is the basically σ_2.

So, that way if you have the σ_1 and sσ_2 non and $\dot{\epsilon}_1$ and $\dot{\epsilon}_2$. In those cases, you can find the value of the strain rate sensitivity by referring to this expression $\log$¿¿¿¿. So, that way you find this value of m, now for many materials such, you know, equations do not hold good $\sigma = C(\dot{\epsilon})^m$, at constant ϵ and on the temperature. Now so, many times these strain rate dependence of the materials.

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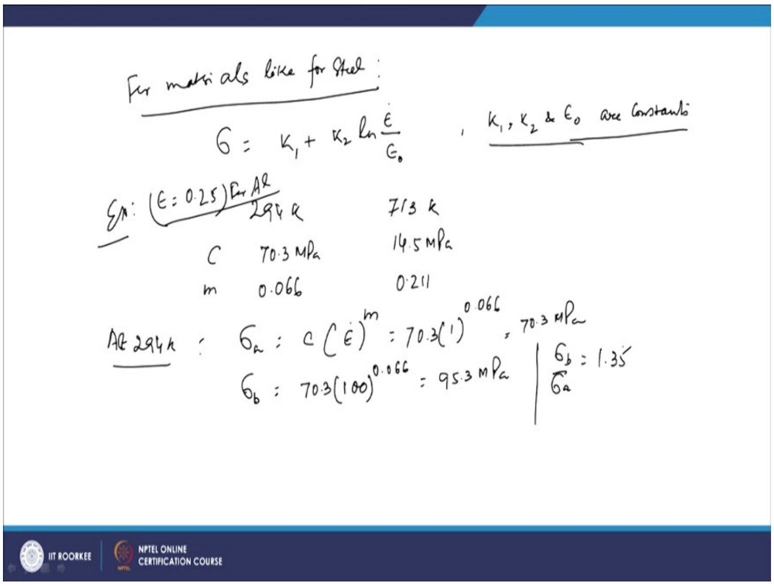

So, for many materials for materials like for steel; suppose, when we see that expression is not very, you know, complete that is not so completely in accordance with the with the flow stress behavior. And here you have a semi logarithmic relationship that a holds good, for the flow stress and strain rate. So, there it comes like $\sigma = K_1 + K_2 . \ln \frac{\epsilon}{\epsilon_0}$. So, this type of expression is seen to fit good. For some materials as far as the strain rate dependence is conserved. And here this K_1, K_2 and ϵ_0 they are the constants values.

So, this way we try to find the we try to understand the importance of these strain rate sensitivity in the case of the flow stress determination, and that way we can have the value of, you know, these parameters, they because when we do the, you know, deformation. Many deformations are done at very, very high strain rate and what will be it is effect on the on the flow stress values, they are required to be understood.

Also when there will be there maybe you may be dealing with certain cases, like there are suppose you have some parameters given. Suppose you are given with you are given with some true strain values for some material. And also you are given with the constants like C and m, m is the strain rate sensitivity. So, if you are given with the C and m; so, in those cases you can find you know the stress values, you know, the stress ratio of the stress values in those cases and, you know, what will be the change in the flow stress value at when the temperature is suppose increased or decreased for the temperature.

So, that can be, you know, calculated for example, suppose you have you are given with an one example; like, you are given at different temperatures suppose there are temperature of 294 k and 713 k for aluminium for aluminium whose at true strain rate of true strain of 0.25 suppose, and for aluminium you have some conditions given and suppose your C and m values are given, like this is 70.3 MPa C and at higher temperature it becomes less. So, this will be 14.5 MPa. Similarly, at the m value is 0.066 and the strain rate sensitivity value will be more at higher temperature. So, it will be dealing with 0.211. Now if you try to calculate the; you are told that what will be the change in the flow stress value for this change in the strain rate.

So, so, you can find that and for that you can see that if you calculate if you calculate this at 294 Kelvin, now this is I will be see of and $\dot{\epsilon}^m$. So, your C is given that is 70.3, and the $\dot{\epsilon}$ that is. So, for a 2 order change of magnitude so, for a 2 order change of a magnitude means signified you have to test for first the value of one and then it becomes 100. So, you can go for 1^m, m is given that is 0.066. So, that comes as the 70.3 MPa. And but if you go for the 2 order of magnitude higher values of $\dot{\epsilon}$. So, σ_b you can get as $17.3 \times 100^{0.066}$.

So, that way it comes out to be 95.3 MPa. So, what we see that you can find the flow stress ratio value and $\frac{\sigma_b}{\sigma_a}$ you get it as 1.35. So, that can when we computed for the higher temperature side, and this way you can say that what will be the changes in the this ratio of the these flow stresses change in the flow stresses as the, you know, strain rate value is increased. So, similar problems can be tackled, and as the time progress is we will try to understand it is significance for when we deal with the metal forming processes, that how this strain rate is sensitive how this strain rates are, you know, having the meaning towards the deformation characteristics of the material. So, even that can be done at even all this temperature also.

Thank you very much.

Principles of Metal Forming Technology

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Lecture - 19
Temperature effects on flow properties

Welcome to the lecture on Temperature effects on flow properties. So, we will discuss the effect of temperature and how it effects the flow properties of the material. Because many a times what we see that we do the test and we draw this stress strain diagram, and we are also interested to find the flow properties finding the flow stress values or the fracture (Refer Time: 00:56) fracture stress and all that.

So, basically we do it at the room temperature, but that testing is also done at the different temperatures may be in the temperature higher than the room temperature at that or even the temperature at which I mean it is below the room temperature. So, basically the selection of temperature at which the test is conducted it has its effect on the stress strain curve as well as the flow and the fracture properties.

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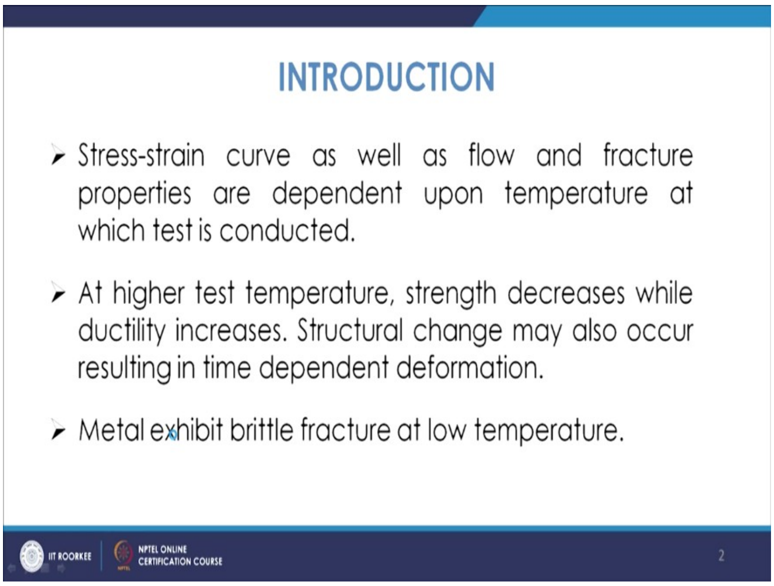

So, normally what we see that when there is higher temperature then the strength will be decreasing and the ductility will increasing. So, normally in the normal strain when we go to the higher temperature then there are many kind of a structural changes also occur. Like you have the strain edging which take place, then you have precipitation taking place, recrystallization taking place and that also has the you know effect on these properties.

So, we will try to see that how this temperature has the effect on the flow properties. If you look at if you try to see the you know properties of the material at higher temperatures, then in those cases if you try to draw the you know stress strain diagram for typical materials like this is the stress and this is the strain. Then what we see that if you do it for mild steel, now for mild steel it will go and it will go and behave like this; if you are doing the stress strain diagram at very low temperatures.

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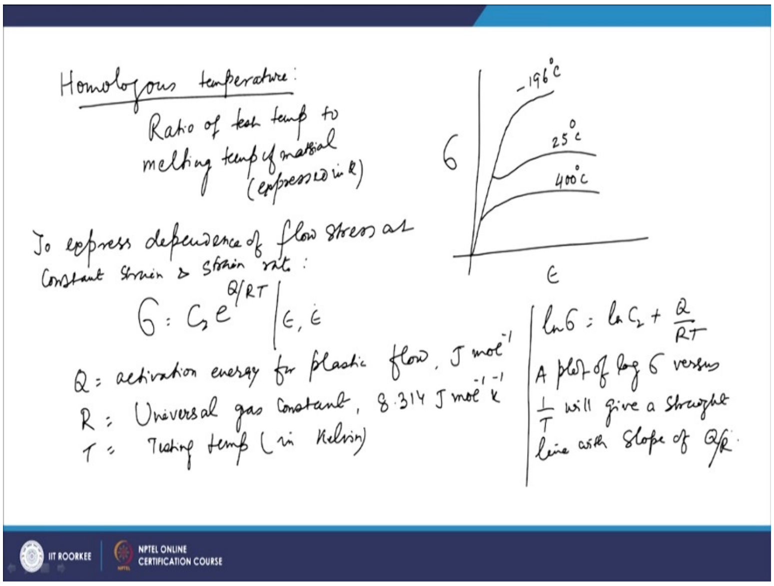

So, suppose you do it at minus of 196 degree centigrade; if the temperature is too low below the 0 rate centigrade temperature. If you go to minus of 196 degree centigrade you see that this is the stress strain diagram. Whereas, if you try to draw it at the room temperature something close to 25 degree centigrade then it looks like this so, it goes at about 25 degree centigrade. Whereas, if you further increase the temperature in those cases what you see is that this curve goes like this.

So, this will be about 400 degree centigrade. So, what we see that normally there will be the variation in the yield strength of the material you know with the temperature. Now, if you try to see the variation of the properties like if we try to see these curves.

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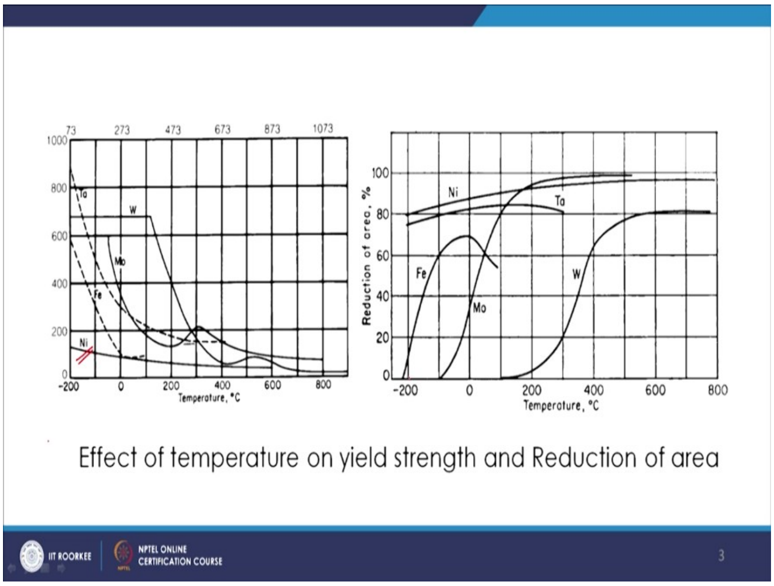

What we see in these curves? Now, this is for different materials the yield strength is shown. Now, if you look at this curve this is the curve for the tungsten. Now this tungsten, this is tungsten, this is tantalum, this is molybdenum iron and nickel.

Now, what we see that these materials tungsten, molybdenum or the iron or the tantalum they are the body centered materials. And what you see that as you increase the temperature there is drastic increase there is drastic decrease in the yield strength of the material; this is the yield strength in mega pascal. So, what you see is that once you increase the temperature at higher temperatures the yield strength decreases drastically. While, if you look at the material like you have nickel, nickel is the face centered cubic structure and here you do not see that many that much of appreciable change.

The certainly there is decrease in the yield strength value, but then that is not that much prominently decreasing as is happening in the case of the body centered cubic materials as its clear from this graph. Now, if you see the reduction of area also in these cases what you see is that again similarly we see the nickel which is you know (Refer Time: 05:57). So, here also, but the it is increasing, but if you look at these materials like iron, molybdenum or tungsten you see that as the temperature is increased the reduction in area is quiet appreciable.

So, basically that effects the you know ductility of the material. So, basically what happens that when you are this is a normal finding that the material, materials you know property I mean changes. Its strength will decrease as the temperature is increased or ductility is you know increased when the temperature is increased. So, strength decrease is and ductility is increasing in the case of the increase in the temperature. Now, the thing is that so, we have already seen that the fcc materials they are not showing that much of the dependence on the temperature. But the strain hardening exponent that basically will be decreasing with the increase in the temperature.

So, what happens that if you look at their you know stress strain diagram that will be flattering towards the later part. So, that is what to the normal trend is. Now, there is another parameter which is normally another terminology which is coming into the picture and this is regarding the temperature at which these test are carried out. And basically it is a I mean ratio is basically represented that temperature at which you carry the test and the melting temperature of the material and so, ratio of that is known as the homologous temperature.

So, you have the homologous temperature. So, this is basically ratio of test temperature to melting temperature of the material. So, basically this temperatures are expressed in Kelvin and this ratio is known as the homologous temperature. So, normally when we compare the flow stress of two materials at equivalent homologous temperature then we have it is advisable basically to correct for the effect of the you know temperature on elastic modulus by comparing the ratios $\frac{\sigma}{e}$. So, normally that is the you know so, homologous temperature is normally defined and then that way we compare the flow stress value for the two different materials.

Now, if you we try to analyze about that the dependence of this you know flow stress on temperature then normally when we try to talk about. So, to express dependence of flow stress at constant strain and strain rates. So, as we know that normally the flow stress will be depending upon the temperature strain and strain rate.

So, when we try to analyze about their dependence of the flow stress on the temperature in that case we assume that the strain and the strain rate is constant. And that can be expressed as $\sigma = C_2 . e^{\frac{Q}{RT}} \vee ¿_{\epsilon.\dot{\epsilon}} ¿$. So, this is at constant strain and strain rate. Now, in this case Q is the activation energy of the plastic flow. So, activation energy for plastic flow and its unit is J.mol^{-1}. Then R is the universal gas constant and as we know this is standard value that is

8.314 J.mol^{-1}K^{-1}, and T is the testing temperature that is in Kelvin. So, what we see that if you look at this curve this is $\sigma = C_2 . e^{\frac{Q}{RT}} \vee ¿_{\epsilon . \dot{\epsilon}} ¿ \cdot$

So, if you take the log function log on both the sides then $\log \sigma = \log C_2 + \frac{Q}{RT}$. So, that will be so, you will be taking the ln as exponential of function. So, you can have so, from there basically you can have the expression for Q. So, if you take you can you can see the if you take the log it will be $\ln \sigma = \ln C_2 + \frac{Q}{RT}$. So, so that way we get so, you have what you see is that $\frac{Q}{R}$again. So, you can have the expression so, you have one Q. So, what you do is now you have 1/T here and then you have $\log \sigma$. So, you can have plot of $\log \sigma$and then you have can have a plot versus 1/T.

So, it will have the slope of $\frac{Q}{R}$. So, what is there? So, in this case plot of $\log \sigma$versus 1/T. So, $y = mx + c$ so, you have x as 1/T. So, m is slopes this is $\frac{Q}{R}$. So, that will give a straight line with slope of $\frac{Q}{R}$. So, this way you can have the expression and you can find you can see the co-relation between the $\log \sigma$and the 1/T. Now, you can further find this value of Q which because if you have a temperature T_1 the stress flow stress value is σ_1 and if you have temperature T_2 the flow stress value is σ_2.

So, from there you can find the value of these you know Q. So, how can find that Q?

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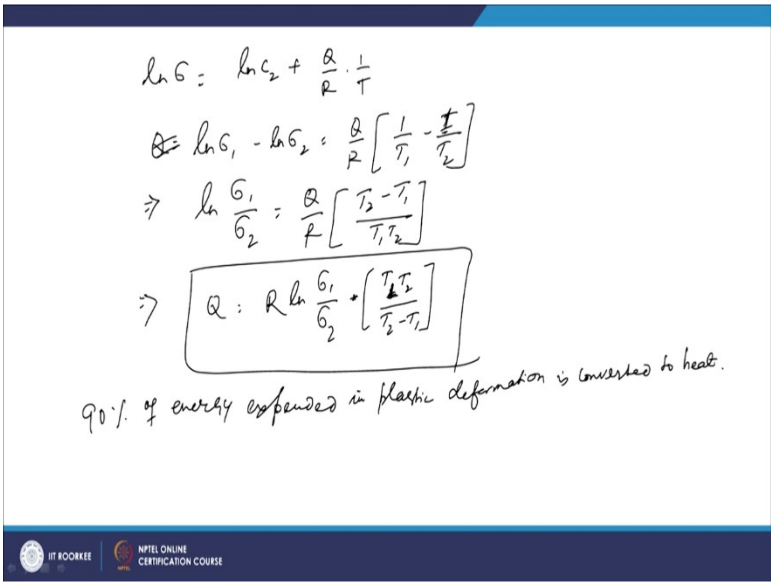

So, what we found is that $\ln \sigma = \ln C_2 + \frac{Q}{RT}$. So, suppose you want to find the value of Q so, now, Q can be found. So, if you have σ_1 at σ_1 flow stress I mean if the flow stresses is σ_1 at temperature T_1. So, $\ln \sigma_1$will be $\ln C_2 + \frac{Q}{RT_1}$ and similarly $\sigma_1$will be $\ln C_2 + \frac{Q}{RT_2}$.

So, if you subtract from this equation to the first equation to second equation there in that case this $\ln C_2$ term will cancel. So, $\ln \sigma_1 - \ln \sigma_2 = \frac{Q}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$. So, what we say we can find is that $\ln \sigma_1 - \ln \sigma_2 = \frac{Q}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$ ok. So, what you get from here is l n of sigma 1 by sigma 2 it will be $\frac{Q}{R}\left[\frac{T_2 - T_1}{T_1 T_2}\right]$.

So, you can find Q. So, $Q = R \ln \frac{\sigma_1}{\sigma_2} . \left[\frac{T_1 T_2}{T_2 - T_1}\right]$ yes. So, this way you can find this activation and at the so, you can have the test and you can if you measure the flow stress at the two different temperatures. Then in that case you can find the Q and once you know the Q so, that Q is determined from here that well.

Now, further what we see normally that when we do the plastic deformation; now 90 percent of the energy which is expended which is spend into the deforming the material that is basically converted to heat. So, basically 90 percent of energy expended in plastic deformation is converted to heat. Now, thing is that normally when you go for the plastic deformation of the material so, in most of the plastic deformation you have the inhomogeneous flow.

So, what happens that the deformation will be localized and the temperature rise will also be localized. So, what happens that only in the local region the temperature will increase. Now, since because of the inhomogeneous flow so, since they are the deformation takes place so, you will have increase in the temperature. Now, further what happens that if the temperature has increased into that region so, because of that the flow stress value will be chased. So, flow stress value will decrease because in that particular location where the there has been inhomogeneous flow there has been more plastic deformation.

So, there the temperature will increase and further with the increase the flow stress value will decrease. So, what happens that that process basically continuous and it continuous and ultimately fracture occurs there. So, that is why you have a localized place where this fracture occurs in the case of this plastic deformation. Now, there are some other you know conditions like when you have very high rate of deformation there you have very less time basically and in those case the there will be less times so, for the heat flow to occur.

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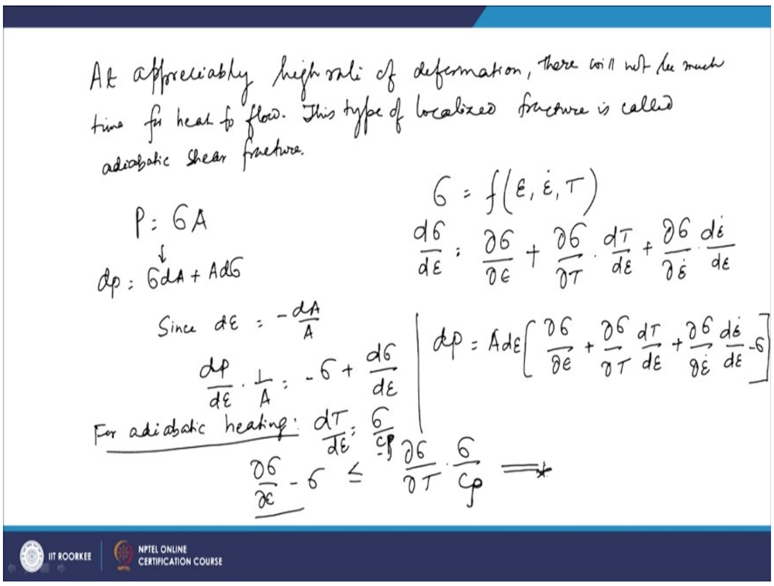

So, at appreciably high rate of deformation. So, those cases there will not be very much time there will not be much time for so, for the flow of for the heat basically. So, in those cases you know the heat flow to occur there will be not much time for heat to flow. So, in those cases you have a basically adiabatic condition and that is why the fracture which occurs under the those conditions. So, these are known as adiabatic shear fracture. So, this type of localized fracture so, normally is called adiabatic shear fracture. So, we have such cases occur whenever you apply the very high strain rate. So, that have occurs in those cases also normally when you do the temperature at low test.

So, such conditions occurs and there is a drop in the flow curve and also you will see that you will have the (Refer Time: 19:59) striations in the flow curve that is found in certain cases. Now, there has be certain finding by some of the researchers and Backofen has basically suggested for basically uniaxial loaded specimen. Now, we see is we can see that you we know that $P=\sigma A$ and also we know that the flow stress is a $f¿$.

So, what can be written? Now, if you try to see from this expression we can write $dP=\sigma dA+Ad\sigma$. So, we can write that now, this also if you try to get its derivative with respect to ϵ. So, $\frac{d\sigma}{d\epsilon}=\frac{d\sigma}{d\epsilon}+\frac{d\sigma}{dT}.\frac{dT}{d\epsilon}+\frac{d\sigma}{d\dot{\epsilon}}.\frac{d\dot{\epsilon}}{d\epsilon}$ if you try to find now, there will be $\frac{\partial\sigma}{\partial\epsilon}$ and then it will be you can have all these factors one by one.

So, it will be $\frac{\partial\sigma}{\partial T}.\frac{dT}{d\epsilon}$. Similarly, you this time you can take for $\dot{\epsilon}$that is strain rate. So, $\frac{d\sigma}{d\dot{\epsilon}}.$

$\frac{d\dot{\epsilon}}{d\epsilon}$. So, this way you can write these expressions.

Further now, what we get is because we know that d $\epsilon$is basically $-dA/A$. So, that we know because either dl/l or it is $-dA/A$. So, what we get is we can write the expressions $\frac{dP}{d\epsilon}$ we can write because 1/A so, this 1/A will come this side.

So, $\frac{dP}{d\epsilon}.\frac{1}{A}$can be written as $-\sigma+\frac{d\sigma}{d\epsilon}$. So, this is the expression which we get for from by substituting this value into the both the sides. Similarly, you can have the another value under expression we get $dP=Ad\epsilon\left[\frac{\partial\sigma}{\partial\epsilon}+\frac{\partial\sigma}{\partial T}.\frac{dT}{d\epsilon}+\frac{\partial\sigma}{\partial\dot{\epsilon}}.\frac{d\dot{\epsilon}}{d\dot{\epsilon}}-\sigma\right]$.

So, this way this σ value will come here from so, that will be minus sigma and the $\frac{d\sigma}{d}$.

So, this has this term and then an that way you have minus of the sigma value. So, a $\frac{\sigma . dP}{dA . \sigma}$ is there already. So, you can write these expressions. Now, from here what we see that if you look at for the case of adiabatic heating. So, for adiabatic heating now we can write the expression like you have d $\frac{T}{d\epsilon}$that will be $\frac{\sigma}{CP}$.

So, this is not P this is ρ. Now, in this case as we know the $C . dT$ and this will be $\sigma . d\epsilon$ that will be the you know energy which is a spent for that. So, which is going into it now that will be basically $C . dT$ temperature raise a because of the plastic deformation. So, plastic deformation that will be the energy which is you know spent for that will be $\sigma . d\epsilon$. And similarly this is the specific heat of the material and this is dT is a change in the temperature of the material because of this plastic deformation and this is your density of the material.

So, so this way what we see that for the you know for the low temperature deformation the strain rate dependence of the flow curve can be neglected and the flow curve basically the instability will occur when what we see is. So, we can tell that this $\frac{\partial \sigma}{\partial E}and-\sigma$. So, if you look at these curves and if you are basically for the low temperature case the, if you neglect these strain rate you know dependence and so, in those cases this $\frac{\partial \sigma}{\partial E}-\sigma$ this comes here.

Now, this has to be less than equal to you have these $\frac{\partial \sigma}{\partial T} . \frac{\sigma}{CP}$. So, this way this is expression which is valid in the case of these adiabatic shear fracture. So, what is clear from this curve this expression is that the load will drop due to this adiabatic heating and this is more pronounced at the low temperature. So, normally the specific heat will decrease at low temperature what we see is so, this is the C. So, the normally this as specific heat value that will decrease at the lower temperature values and you have the strong dependence of the you know temperature on the flow stress.

So, normally when the temperature becomes on the lower side you will have the dependence of the flow stress on the temperature is becoming more prominent. Whereas, when you have the you know in other conditions now, the this can be further seen in a way that when we talk about these temperature effects. So, normally you know that you have different working conditions in the metal forming, you have the selection of temperature.

Like if you take the temperature on the higher side you have the definition as the hot working or the cold working or the warm working where the you have the temperature in between the hot working and cold working. And in those cases these flow stresses can be predicted based on this the temperature and also you have other parameters like strain or strain rate.

So, certainly we have neglected certain parameter in this case, but then dependence can be checked by analyzing these equations. So, I mean you I hope that you are able to understand that how we write these equations. You must be able to understand all the terminologies. Here what we do is that since we have got the $\frac{dT}{d\epsilon}$ is $\frac{\sigma}{C\rho}$ and then for the load to you know for the instability or the load drop in those cases you have one term is $\frac{\partial\sigma}{\partial\epsilon}-\sigma$.

So, that is here and another term this term we are neglecting. So, you have these term $\frac{\partial\sigma}{\partial T}.\frac{dT}{d\epsilon}$. So, $\frac{dT}{d\epsilon}$ is again that is your $\frac{\sigma}{C\rho}$ that is why this comes here. And in those cases your this term $\frac{\partial\sigma}{\partial\epsilon}-\sigma$, if that is lesser than this $\frac{\partial\sigma}{\partial T}.\frac{\sigma}{C\rho}$.

So, this condition comes here and based on that you have this condition. So, that is why we can say that this expression tells this is the expression which is responsible for telling you that why the load is decreasing. Load is dropping due to adiabatic heating and it is more pronounced at the lower temperature site.

So, that is what we will discuss about the effect of these temperature on the deformation behavior, then that time this concept may be utilized.

Thank you very much.

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Lecture - 20
Influence of various parameters on flow properties

Keywords: Flow Properties, Deformation, Newtonian Viscous Flow

Welcome to the lecture on Influence of various parameters on flow properties. So, in this lecture we are going to discuss about certain other parameters other than the temperature which affect the flow stresses values or the stress strain behavior of the material. So, in that first of all we will try to study about the influence of a testing machine.

So, the machine on which we are testing you have the cross head velocity is there which is given certain velocity to you know to do the tension, you know test of the specimen. Now, if you see that even the testing machines also they are deflecting under the load. So, the testing machines normally of are of two types you have the load control machines.

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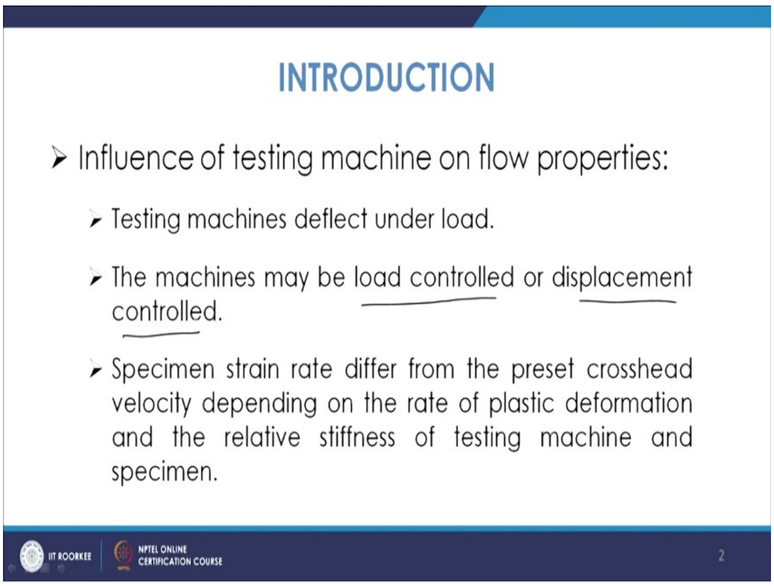

So, you have the either the load control machines or the displacement control machines. Now, in this load control machines you know the load and you do not have much control on the displacement so, you will have to live with whatever displacement comes. Now, in the another case when you have the displacement controlled machines so, you know the you have to have certain displacement for that it will adjust the load.

So, you have different type of mechanisms for controlling that and that way you have the earlier which were the hydraulic driven machines which were working they were the load control machines. Whereas, the screw driven machines are there which are the displacement control machines; nowadays you can control more accurately in the modern era where you have the computer control, you can control the things more accurately. So, basically the specimen strain rate will be differing from the preset cross head velocity depending on the rate of plastic deformation and the relative stiffness of testing machine and specimen. So, we will discuss how this comes to be true in such cases.

So, if you try to analyze the situation what we see that most of the testing specimen testing machines they will be deflecting under the load and that is to be you know taken into account, because you cannot directly convert them you this crosshead velocity into the deformation of the specimen. So, you will have to do the appropriate corrections in the system.

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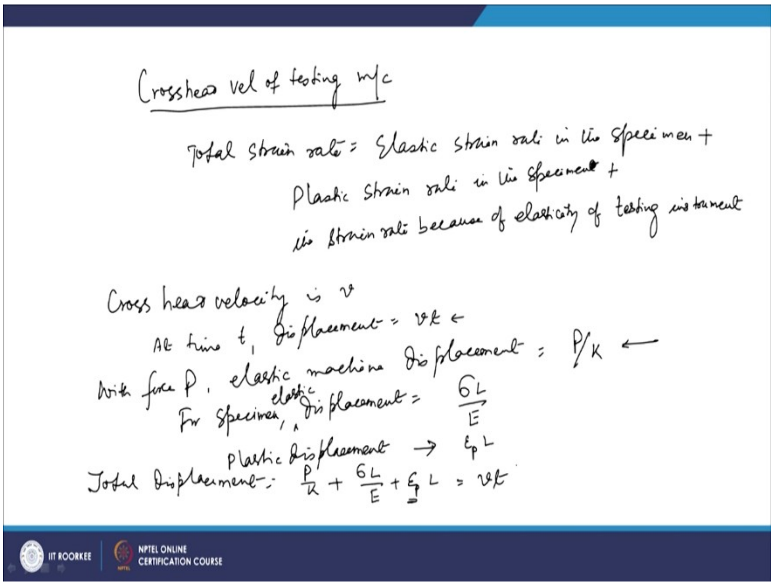

Now, if you look at the cross head you know velocity. So, if you look at these crosshead you know velocity of the testing machine. Now, these are applying the strain rate basically so, the total strain rate which is applied. So, total strain rate applied it will be sum of so, it will be sum of basically the elastic strain rate in the specimen plus the plastic strain rate in the specimen and the other part is that the strain rate which is resulting from basically the elasticity of the testing machine. So, the strain rate because of elasticity of testing instrument or machine.

So, you have two things one is the elastic strain rate in the specimen that in the elastic zone similarly, you have the plastic strain rate that is in the so, this is specimen and then the then again you have the strain rate because of the elasticity. So, there will be that deflection with there because of the load and because of the property of the instrument itself it subjected to certain load. So, that way it will have the effect on the strain rate and that is to be basically taken into account while we deal with that. Now, if you take the crosshead velocity as v if the crosshead velocity is so, we have already seen that crosshead velocity is taken as v. Now, so, at any particular instant I mean at particular time t the total displacement will be $v.t$.

So, at time t is displacement will be v.t. So, now, actually when we are applying the load P, now this force P will be creating the on the specimen it will be causing the you know you have the elastic machine displacement also; if the machine is there which is subjected to that load. So, with force P so, when we are applying so, that it basically acts on the machine also and in that case the elastic machine displacement it will be depending upon the machine stiffness. So, it will be the $\frac{P}{K}$. So, that is your because of the this machine properties then if you come to the specimen for specimen displacement will be $\frac{\sigma L}{E}$. So, that is basically for the specimen in the elastic region.

So, this is basically the elastic displacement. So, we can write these elastic displacement because we are talking about in the elastic region and in the elastic region you have the application of Hooke's law. So, you will have $\frac{\sigma L}{E}$. similarly, if you have we are talking about the plastic displacement. So, plastic displacement so, plastic displacement will be $\epsilon P.L$. So, so, you have three components one is for the machine that is $\frac{P}{K}$, you have for the elastic displacement of the specimen that is $\frac{\sigma L}{E}$. and then you have plastic displacement for the specimen and that will be $\epsilon P.L$.

So, what we see that the total displacement if you look at the total displacement will be the summation of the two these three components. So, total displacement it will be summation of this $\frac{P}{K}$. then you have $\frac{\sigma L}{E} + \epsilon P \, . \, L$. Now, what we see is that you have to take this ϵP. So, this ϵP you know you can it can be taken from the load time chart on the constant crosshead, you know velocity testing machine and you know that has to be corrected. So, you must have this as a constant value and for that all these are corrections required.

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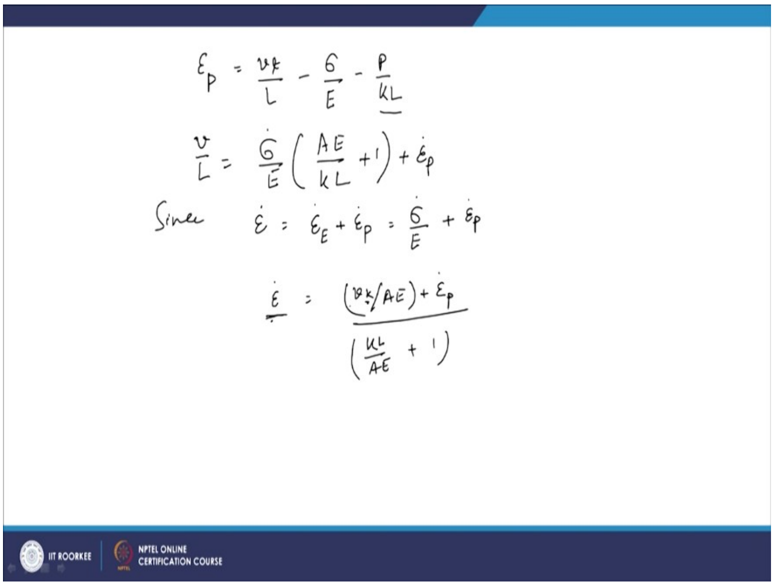

So, if you go to the expression what we get is you get the ϵP and that will be coming as $\frac{v \, . \, t}{L}$. So, if you look at this earlier expression now, this expression this is nothing, but the total displacement what we discussed the displacement $v \, . \, t$ so, this $v \, . \, t$ it will be similar to this. Now, if you go to further so, if you take ϵP on one side you will have $\frac{v \, . \, t}{L}$minus the two terms will go on other side. So, it will be $\frac{\sigma}{E}$ and then you will have $\frac{P}{KL}$. So, ϵP will be so, we have we have seen this expression $\epsilon P \, . \, L$. So, L will be divided on both the sides and that way you will have this expression$\frac{v \, . \, t}{L} - \frac{\sigma}{E} - \frac{P}{KL}$.

So, that is what $\frac{\sigma}{EL}$will go away and then you will have a $\frac{P}{KL}$. Now, what we see is that from here we can write so, what we see that there is a major influence you know of the machine and the specimen interactions. So, this is because of the machine interaction and you have a specimen interaction and they have effect basically on the strain rate. So, if you are treating that stress you know stress rate as $\dot{\sigma}$ and the strain rate as $\dot{\epsilon}$.

So, we can write so, we can write actually $\frac{v}{L}$as $\frac{\dot{\sigma}}{E}\left(\frac{AE}{KL}+1\right)+\dot{\epsilon}_P$. So, if we take the you know rate component now, if you look at this. So, this will be $\epsilon_P+\frac{\sigma}{E}+\frac{P}{KL}$ now, if you keep the $\frac{v}{L}$ on one side then in the in that case this will be $\frac{\sigma}{E}+\frac{P}{KL}$and this t will be you know because that way $\frac{\sigma}{t}$ will come. So, that will give you this $\dot{\sigma}$that is your stress rate and similarly you will have this term as the strain rate and then all the terms will come in between. So, that way you can have this $\frac{v}{L}$ as the this term $\frac{\dot{\sigma}}{E}\left(\frac{AE}{KL}+1\right)+\dot{\epsilon}_P$.

Now, as we know further since so, what we see that if you talk about the strain rate values now this is nothing, but you have in the elastic part plus you have the plastic part. And that can be written as the for the elastic part you have this by the elastic modulus and then you have the plastic part of the strain rate. So, if you talk about the strain rate finally, that that come can be written as $\frac{\left(\frac{vK}{AE}\right)+\dot{\epsilon}_P}{\left(\frac{KL}{AE}+1\right)}$.

So, this way you can have the value of these strain rate which can be controlled and this is to be control. So, once you have this K value know you must know that. So, what we say that for having the proper strain rate value these you know parameters have a say on non attending or the value which we try to achieve for calculating or for analyzing or for drawing the proper flow stress curve, flow curve or so.

So, now it is clear that this strain rate what are the specimen strain rate that will be basically differing from this v. So, this is the crosshead velocity and that is preset. So, if you have a given a preset crosshead velocity and that is not actually the actual strain rate basically, that is changing because of the stiffness of the machine. Now, the machine which has the you know more stiffness they are known as hard machines. Now, for the you know you have hard machines as well as the soft machines. So, hydraulic driven machines are known as the soft machines and screw driven machines are known as the hard machines because of the you know value of the K and K it will be ranging from 7 to 32 basically meganewton per meters.

So, based on that you will have the other you know this values that you know corrections which is to be there, it is to be incorporated and you will have the different you know impressions, different type of characteristics will be there in those stress strain curve when we do for on the hard machine as well as on the soft machine. Now, basically we are going to discuss about the different criteria that is the consideration of consideration of instability.

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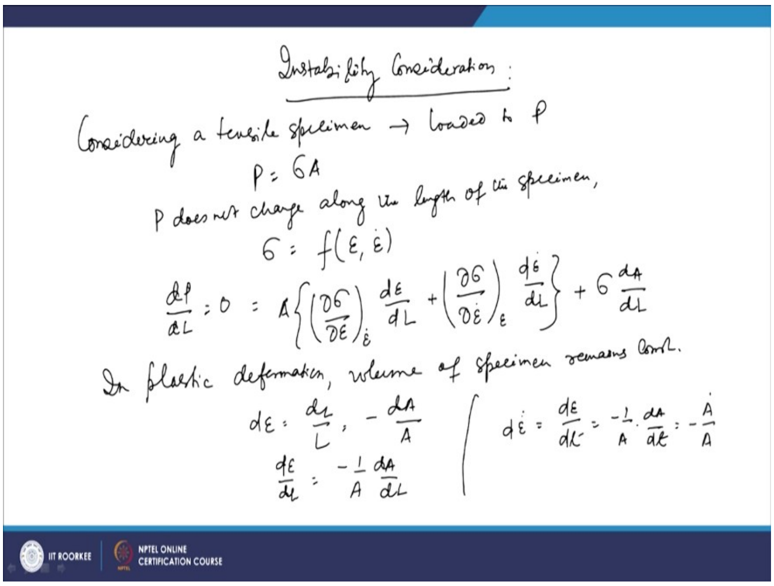

So, we are talking about the instability considerations. Now, we have already discussed about the stability in the criteria while, necking and normally we assume that the flow stress will be depending only on the strain. But we will see that how it is sensitive to the strain rate so, that is to be understood. Now, suppose you are considering a specimen so, considering a tensile specimen which where you apply the load value so, loaded to P.

So, we are loading them to a value of P now, any instant of time it is assume that the cross-sectional area is A so, P will be σA. Now, actually P is not changing while we are basically doing the tensile testing on the specimen. So, P does not change along the length of the specimens along the length it is not changing not during the specimen I mean testing, but along the length of the specimen and the flow stress value we are assuming that it is a function of strain and strain rate.

So, so, you can write that since it is not changing with the length of the specimen so, $\frac{dP}{dL}=0$ and that will be basically if you look at so, it will be $\frac{\partial\sigma}{\partial E}$ and that will be at constant strain rate. So, $\frac{dE}{dL}$ so, that is with respect to L and then you have also $\frac{\partial\sigma}{\partial\dot{\epsilon}}$ that is with respect to strain rate and then further you have at this is at constant strain and then $\frac{d\epsilon_0}{dL}$. So, that is what we are doing for the σso, σat this time A this is $d\sigma$and similarly you have σdA so, $\sigma\frac{dA}{dL}$.

So, that is what I mean you are getting $\frac{dP}{dL}$ so, dP will be $\sigma.dA+Ad\sigma$. So, $Ad\sigma$ if you find the $d\sigma$ it will be ∂(Refer Time: 17:57) first of all since σ is a function of it depends upon the (Refer Time: 18:03) the strain and the strain rate. So, we have a separately done this you know find the derivatives.

So, first with respect to ϵ at $\epsilon_0$at a constant value of strain rate then at a constant value of strain with respect to the strain rate, then $\frac{d\epsilon_0}{dL}+¿$ this so, this can be found. Now, when we are talking about the plastic deformation so, the there is no volume change. So, so, in plastic deformation volume of specimen remains constant. So, what we write? We write $d\epsilon_0$ as $\frac{dL}{L}$ and that is $\frac{-dA}{A}$.

So, we can write further that $\frac{d\epsilon}{dL}$ will be $\frac{1}{A}\frac{dA}{dL}$. So, so, this is to be further used when we go to that now, this equation that is $d\epsilon = \frac{dL}{L} = \frac{-dA}{A}$. So, that can be written further and we can write that $d\epsilon_0$so, we have to convert we have to express for the strain rate. So, $d\epsilon_0$ will be actually $\frac{d\epsilon}{dt}$that is what so, you have $d\epsilon$ you know we know it; now that can be $\frac{-dA}{A}$.. So, it will be $\frac{-1}{A}.\frac{dA}{dt}$.

So, we can write this as this is $\frac{dA}{dt}$is $\dot{A}$that is you know rate of change of the area. So, it will be minus of $\frac{\dot{A}}{A}$. So, this d epsilon naught can be taken as $\frac{-\dot{A}}{A}$. So, if you substitute this further so, in the expression for the $\frac{d\dot{\epsilon}}{dL}$; in those cases we can write that $\frac{d\dot{\epsilon}}{dL}$, it will be $\frac{-1}{A}$ and then $\frac{d\dot{A}}{dL}+\frac{\dot{A}}{A^2}.\frac{dA}{dL}$.

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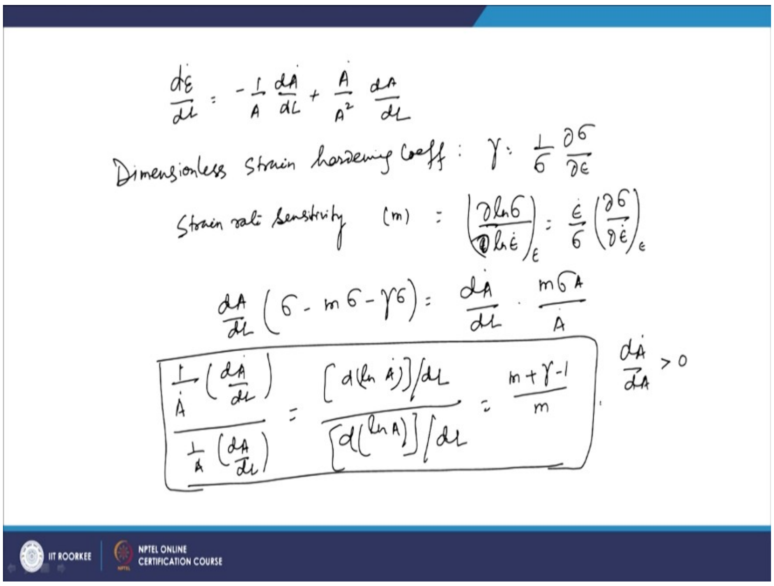

So, so this way you have the expression for the $\frac{d\epsilon_0}{dL}$. Now, what we do is we define some material quantities and the material quantities which we define is, one is dimensionless strain hardening coefficient. Now, this is basically we defined as $\gamma$and this will be $\frac{1}{\sigma}\frac{\partial \sigma}{\partial \epsilon}$. So, this is known as dimensionless strain hardening coefficient and similarly you we define this strain rate sensitivity.

Now, this strain rate sensitivity that is your m so, this we call it as m and this we know that this is $\frac{\sigma}{\epsilon}$. And then that so, $\frac{\partial \ln \sigma}{\partial \ln \dot{\epsilon}}$ and this is what the extended sensitivity is and this can be written as $\frac{\dot{\epsilon}}{\sigma}$because this will be $\frac{1}{\sigma}$. So, σ will come down and then this $\dot{\epsilon}$will come up and then you have the $\frac{\partial \sigma}{\partial \dot{\epsilon}}$. So, that is at a so, this is also at constant strain; so, this will be at constant strain. So, now, if you use these to you know equations from here to find.

Now, they can be further put in the earlier equations and what we get is we get these equations like $\frac{dA}{dL}(\sigma - m\sigma - \gamma\sigma) = \frac{d\dot{A}}{dL} . \frac{m\sigma A}{\dot{A}}$. So, this is what we get by substitute these values and from here if you do the final rearrangement you further do the rearrangement; what we get is we get $\frac{\frac{1}{\dot{A}}\left(\frac{d\dot{A}}{dL}\right)}{\frac{1}{A}\left(\frac{dA}{dL}\right)} = \frac{[d(\ln \dot{A})]/dL}{[d(\ln A)]/dL} = \frac{m+\gamma-1}{m}$.

Now, what we see is that these $\dot{A}$ and this can be expressed in terms of these you know dimensionless values, these strain rate sensitivity value m you have the dimensionless strain hardening exponent. So, that way you get this expression. So, this expression basically this expression what we get this expression actually that is describing the rate of change of area that is $d\dot{A}$you know and that is it will be correction for the you know onset of necking.

So, in those cases when the necking will be occurring what will be the correction required that can be you know seen from this expression. Now, for any specimen which is going to vary its length you know so, along its length actually you will have the variation in the across sectional area. And that may be because of many reasons you may have the taper, you may have the machining errors. So, where ever you go if you go along the length there may be the change in the area dA.

Now, in those cases there may be other reasons also like there maybe heterogeneities of the structure. Because of that also you may have at some point, you know you may have something like some impurities or some places where you had some shrink is cavities or source. Because of that at some localized position you may have the you know change in the cross-sectional area and because of that you will have the weaker section there and that will lead to the you know you know deformation starting there.

So, basically deformation becomes unstable there that place at those these you know smallest cross section area that will be shrinking faster than the rest of the things. So, that occurs so, for that to occur the condition is that this $\frac{d\dot{A}}{dA}>0$. So, for this condition what will happen your this specimen will shrink faster, now as long as this is ¿0.

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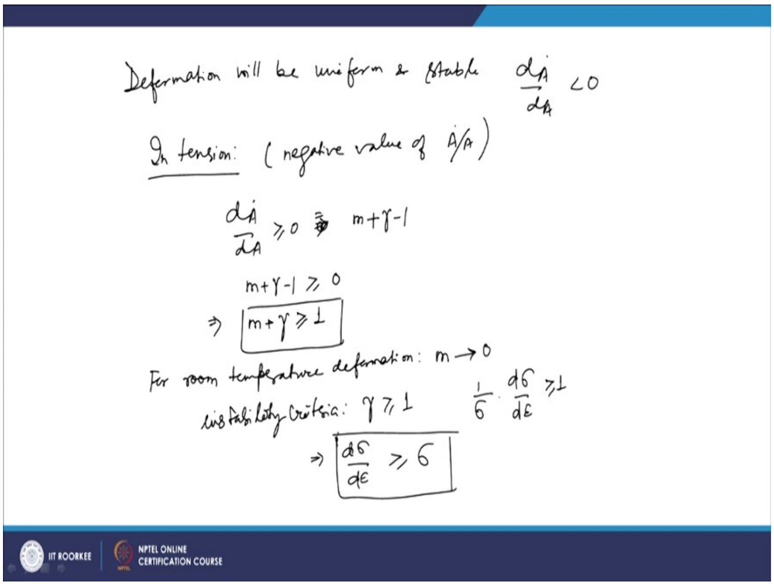

So, for the condition which can be interpreted is that the deformation will be uniform and stable. So, as long as this $\frac{d\dot{A}}{dA}<0$, but once you know if it is more in that case it will not be stable. So, now in when we talk in tension already what we know is that the $d\dot{A}$ will always be negative because the area is decreasing. So, the decrease from the final area to the so, the that difference between the final cross section to initial cross section will always be negative.

So, in the case of tension this value will be negative and so, it will be negative value negative value of $\frac{\dot{A}}{A}$. So, so, in those cases the stable deformation in tension will occur when this $\frac{d\dot{A}}{dA}>0$ and that is why so, $\frac{d\dot{A}}{dA}$; as we talk about because in the case of tension your $d\dot{A}$ is anyway negative. So, for that to be positive now, in those cases that has to be more than 0. So, that is why it will lead to you know in the case of $m+\gamma-1$ already we have seen these values.

So, this will be equal to $m+\gamma-1$. So, $m+\gamma-1$that has to be more than equal to 0 and that is why the condition for in for this tension case is that $m+\gamma\geq 1$. So, it shows that both these you know strain hardening strained hardening as well as these strain hardening both are suppressing the onset of necking. Now, this equation tells that both basically suppress the necking and when you are going for the room temperature deformation in that case so, for room temperature deformation. Now, for room temperature if deformation your m is basically turning towards 0.

So, that is why this instability criteria instability criteria will be you can that can be reduced to $\gamma\geq 1$. And we know that this $\gamma$is defined as so, so that is why this will be $\frac{1}{\sigma}\frac{d\sigma}{d\epsilon}\geq 1$. So, this is giving you the further necking criteria that we know that is basically $\frac{d\sigma}{d\epsilon}\geq\sigma$ will go that side so, this is the necking criteria. So, what we see is that you have if you talk about these necking criteria this is the criteria which is coming for by taking into these when parameters like strain rate or strain rate sensitivity or so.

So, so this can be further you know analyzed, you can further see its on other interpretation when we talk about the Newtonian viscous fluid. We which where without strain hardening case you have when the γ=0 and you have $m\geq 1$. So, that is for the Newtonian viscous flow that is opposite to its so, that is case for the instability in those cases.

So, that is how it is interpreted and we can use these concepts when we talk about the analysis of deforming processes in the coming lectures.

Thank you very much.

Principles of Metal Forming Technology
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Lecture - 22
Mechanics of metalworking and analysis methods

Welcome to the lecture on Mechanics of Metalworking and Analysis Methods. So, we will try to be aware about some of the terminology is in this what are the mechanics for the metal working processes; which and an some of the terminologies, we will try to be acquainted with which will be required to analyze the operations in metalworking. Now when we try to analyze that, we know that we have studied about the deformation theories. We have studied about the plasticity theories in that we have seen many points like we have discuss many points like, what are the conditions in the case of plastic deformation like the value of this Poisson ratio or also the other, you know, constancy of volume conditions and all that.

So, we will discuss about certain things because in most of the cases, you are concerned with the reduction of the, you know, or change in the dimension of the product or the cross section of the material. And in that case you need to you find the amount of stress which is required to do that, and what will be the associated things which will be occurring with that so, that will be discussed.

So, when we talk about the mechanics. In that case, the first thing which we come across in those cases is the constancy of volume approach.

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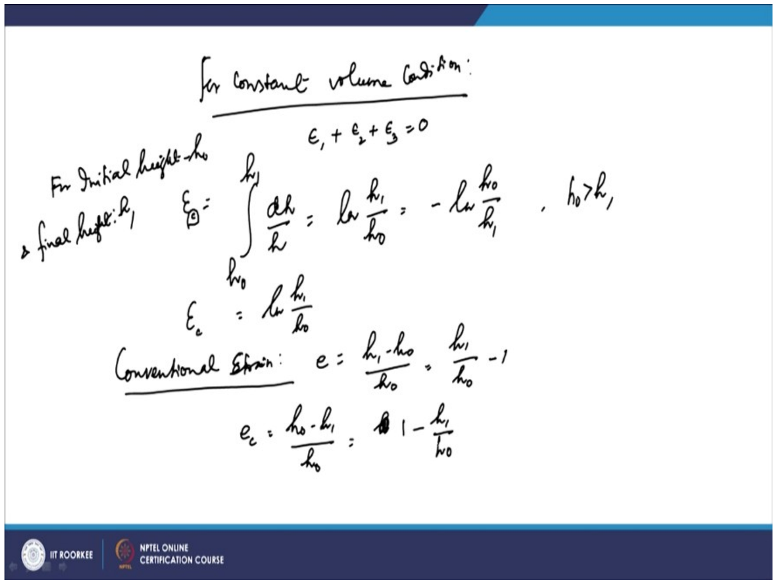

So, for constant volume condition so, in that case, what we get is that $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$; because we know that if the strain is there in one direction positive, then certainly you have in transfers direction you have the strains negative directions. So, that way this $\varepsilon_1 + \varepsilon_2 + \varepsilon_3$ is coming out to be 0.

Now, when we talk about the metal working processes, then what we see that normally when we talk about the stresses, and we know that the stresses are of primarily 2 natures either tensile or compressive. So, we take tensile stresses as the positive one and the compressive stresses as the negative one. So, but when we talk about the analyses of metalworking. So, normally we deal with the compressive force only because normally, the compressive forces are have applied.

So, in such cases in the cases of this metal working analysis, normally these compressive directions they are not taken as negative because every time you have to do it in the negative manner. So, so normally since they predominate in the analyses. We normally take it as the positive one, and for that we have the different convention. So, suppose we are compressing the material, in that case, if from the height h_0 to h_1 we are compressing. So, if suppose for initial height h_0 and final height h_1.

So, if you look at the compressive strain which is developed, so as we know that compressive strain will be so, if you write it like this a compressive strain or so. Now, in this case we will have the definition h_0 to h_1 and this will be $\frac{dh}{h}$ so, it will be $\frac{\ln h_1}{h_0}$. Now since the h_1 is smaller than h_0. So, we will write it like $\frac{-\ln h_0}{h_1}$; so because the h_0 is more than h_1.

So, normally what we do is, we we write these in this strain in the case of this plastic deformation as $\frac{-\ln h_0}{h_1}$. So, basically this is a this is the strain, but we write when we put this subscript c, that is for compressive, you know, strain when we do the metal working analysis. In those cases, we write we remove this negative sign, and we write $\frac{\ln h_1}{h_0}$. Similarly, if you try to find the conventional engineering strain; so for conventional strain, now in the case of conventional strain, what we see is it will be $\frac{h_{1-h_0}}{h_0}$.

So, again it will be $\frac{h_1}{h_0}-1$. So, what we see that normally you have the negative values, but when we talk about compressive, you know, values. So, we will write the engineering strain that is in compressive. So, it will be $\frac{h_{0-h_1}}{h_0}$. So, when we talk about the compressive value it will be like $1-\frac{h_1}{h_0}$. So, this is how we normally have the convention, when we talk about the deformation in the case of metalworking.

Now, further we know that when you are try to increase or decrease the length associated with that there will be the change in the reduction of the area or so.

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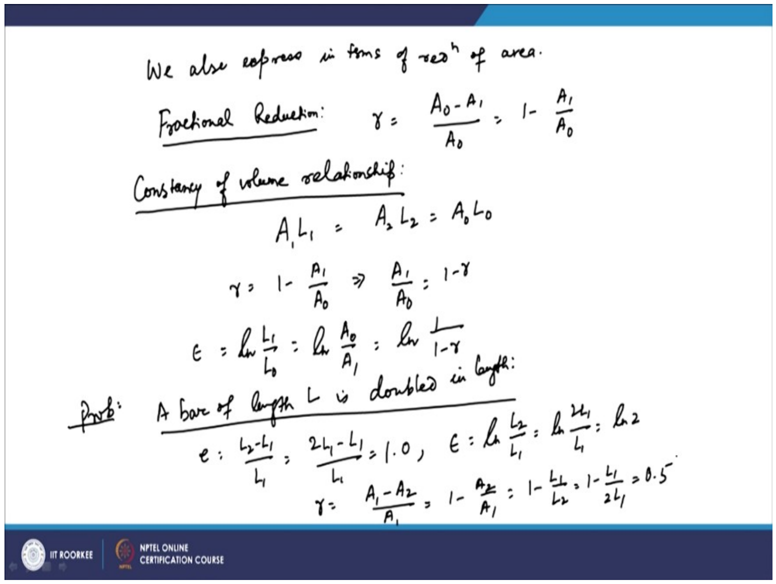

So normally we also express we also express in terms of reduction of area. So, in that the commonly used term is the fractional reduction.

So, the fractional reduction will be defined as r. And this will be $\frac{A_{0-A_1}}{A_0}$. So, this is the original cross sectional area which was there earlier, then what is the final area, and then divided by the original area and that is known as the fractional reduction of the area. So, if you take the constancy of volume relationship, now as we know that you have the change in length, and associated with that you have the change in the cross sectional area, and if the volume is constant then the, you know, area multiplied by length it will be always same.

So, you will have $A_1 L_1 = A_2 L_2 = A_0 L_0$. So, what we see is that r you can we are writing it as so here we write it as $1-\frac{A_1}{A_0}$. So, it will be r will be $1-\frac{A_1}{A_0}$, or $\frac{A_1}{A_0}$ is written as $1-$¿r. Now if you see the definition for the strain to strain value, it is $\ln\frac{L_1}{L_0}$, so, it will be $\ln\frac{A_0}{A_1}$. Because $\frac{L_1}{L_0}$ will be $\frac{A_0}{A_1}$ because $A_0 L_0$ will be equal to $A_1 L_1$.

So, you can write this as $\frac{1}{1-r}$, because it will be $\frac{1}{1-r}$; so $\ln\frac{1}{1-r}$. So, this way you can find so, this are these are the terminology is which are mostly use fractional reduction in area or the compressive strains or so. And they can be use for the analysis of the processes; you may have suppose, if you take the example certain example like if suppose you have a problem, and it is said that you have a bar which is a a bar of length L is basically doubled in length.

So, you may be told that you find these different, you know, value of the engineering strain or true strain or the, you know, reduction. So, suppose a bar of length L is doubled in length that is becomes L from L to 2 L. In those case if you try to find the e it will be $\frac{L_{2-L_1}}{L_1}$, so, L_2 is nothing but 2 L_1. So, it will be $\frac{2L_{1-L_1}}{L_1}$, so, it will be 1. Similarly, if you try to find the true strain there, in that case it will be $\ln\frac{L_2}{L_1}$. So, it will be $\ln\frac{2L_1}{L_1}$, and in that case it will be $\ln 2$. So, $\ln 2$ value is 0.693 so, that will be found out; similarly, if you try to find the r.

So, r will be suppose so, r you can get is as $\frac{A_{1-A_2}}{A_1}$. So, so that way you can find so, it will be $1-\frac{A_2}{A_1}$. Now $\frac{A_2}{A_1}$ is nothing but $\frac{L_1}{L_2}$so, it will be $1-\frac{L_1}{L_2}$, and L_2 is 2 L_1. So, $1-\frac{L_1}{2L_1}$; so it will $1-\frac{1}{2}$, so, 0.5. So, this way when L bar of length L is doubled in length, in that case it is basically you can calculate this true strain or engineering strain or the fractional reduction values in such fashion.

You can also calculate if it is halved in length, and that way you will have the values coming to the negative side in the case of engineering strain. And, further you can calculate other values like fractional reduction or true strain values and in compression whatever we have discussed. So, that way you can find those values.

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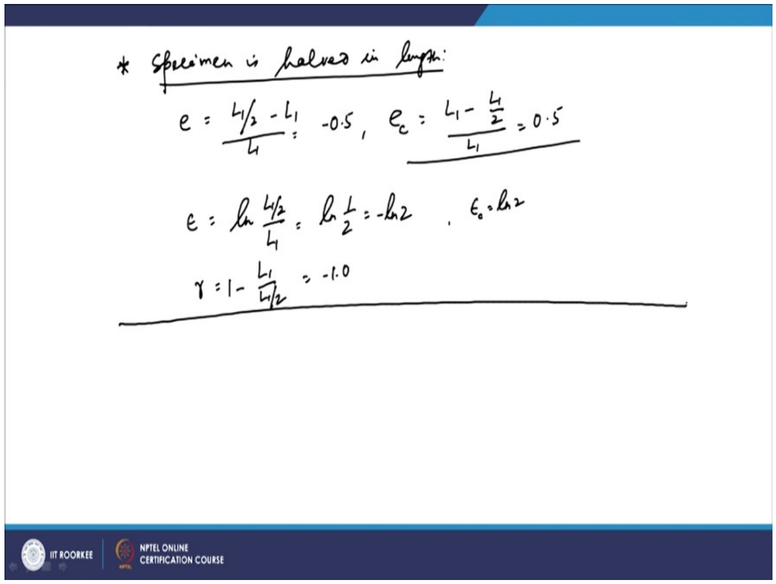

So, let us take for example, that if the specimen is halves in length. Now in this case, L_1 becomes $\frac{L_1}{2}$. So, we can find the engineering strain and engineering strain will be $\frac{\frac{L_1}{2} - L_1}{L_1}$. So, it will be minus of 0.5, but if you take the engineering strain in compressive if direction compressive strain engineering compressive strain; in that case we will do the reverse $\frac{L_1 - \frac{L_1}{2}}{L_1}$ so it will be basically 0.5.

So, this is what they convention is used in the case of the metal forming analysis where you largely you are trying with the you are dealing with the compressive stresses. Similarly, if you go for the true strain values so, true strain value also ln of final length by original length so, $\frac{\frac{L_1}{2}}{L_1}$. So, it is $\ln\frac{1}{2}$ so, ln 1 minus ln 2. So, it will be minus of ln 2, but if you talk about the true compressive stain it will be ln 2, because it will be $\frac{L_1}{\frac{L_1}{2}}$ so it will be so, that way it will be ln 2,.

And if you look at the reduction of area so, it will be $\frac{L_1}{\frac{L_1}{2}}$, and then 1 minus that so, it will be something like minus 1. So, that is how you tried to find these parameters whenever required in the, you know, metalworking analysis. Coming to the analysis part of the, you know, metal working operation we discuss about the zone in which we are going to confine and do the study about the metal working processes.

Now what is there in that basically you are applying the stresses, then you have many conditions you have the equations you are getting equations of equilibrium you have to adjust the forces which are being applied and you have to make them, you know, a balance in certain directions, and accordingly you try to get the values you try to find these stress and strain values and different points. So, that is what the aim is there in the case of metal forming in every point of the deformation zone. In the deformed reason you try to find the velocity you try to find the stresses or the strain.

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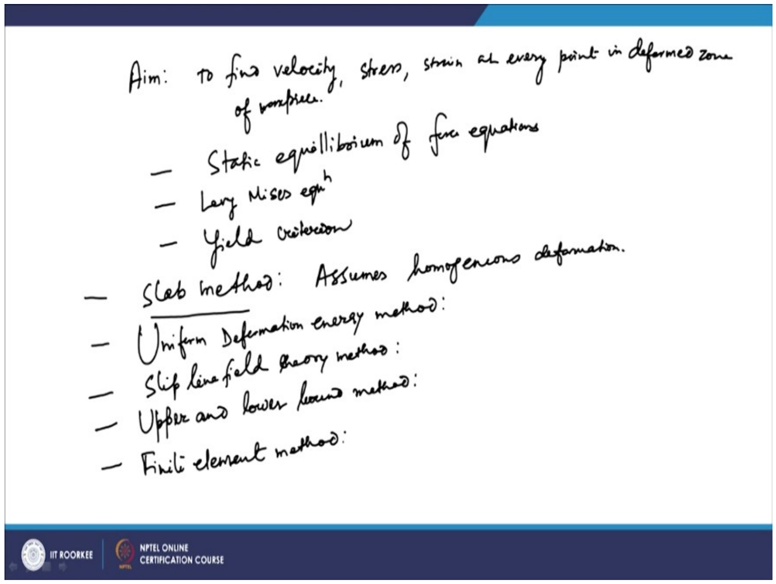

So, it is basically your aim is to find velocity stress strain at every point in deformation zone in deformed zone of work piece. Now you have many ways for approaching the problem in that case, because ultimately we what we are interested in that we want to know that with what velocity the material is deforming or the stresses or strain at every point in that. So, we are in the interested into it and you have many ways. And basically you have 3 sets of equation which will be coming up which has to be solved, which are to be solved to find all these values. Now these 3 sets of equations are that you have first the static equilibrium of force equations.

So, you have this static equilibrium of force equations which are to be, you know, solved. Then you have the levy-mises equation ; with so, this levy-mises equation will express the relation between the stress and the strain rates. So, based on that, you will have the equation and you will find you will use these equations for further finding, and then you have the yield criterion.

So, you have basically 9 independent equations, and they are to be solved and you have the 9 also the unknowns you have 6 stress components, and 3 the velocity component or the strain component, and these are to be solved and you have the 3, is this way you have the 9 equations and you have 9 unknowns and they are to be, you know, solved.

So, normally the solution is certainly tedious, you analytically equation I mean solution is not that way easy to solved them. And you have many methods which are used, and the different methods which are used are the, you know, slab method. So, this slab method it assumes homogeneous deformation. So, in this case when we are doing an analysis, and you have some element of, you know, certain shape; suppose, you have a square element is there. So, once you deform it then it will be converted into rectangular elements.

So, that way you have a homogeneous kind of deformation that is assumed in the case of slab method. Similarly, you have another approach which is used is uniform deformation energy method. Now in these cases what we is, then these that your applying you are giving the energy I mean for deforming the work piece. Now from these, you know, work of plastic deformation you are trying to calculate the average stress value. So, so that way that is why it is known as the uniform deformation energy method.

So, in this case is you must know that what is the work done on the machine, on the material and based on that you find you predict these average forming stresses. The second the third method is the slip line field theory method. So, it is basically calculating the point by point these stress values are, you know, calculated. And normally we assume the plane strain conditions in such case, and for every point by point, the values of the stresses are calculated. Then another method is upper and lower bound method. Now this is done based on the limit analysis, and basically it will be using, the so this upper and lower bound method.

You will have the limit, and you will use the reasonable, you know, stress value and the velocity field. So, that you calculate basically the bound with in which these actual forming load should be, you know, lying. So, based on that you calculate this parameter values, then the last method is the finite element method. Now this is basically also known as the matrix method.

And it will be allowing the large increment of deformation for the rigid plastic materials, and basically lot of computational time is reduced in this is the new approach is when latest approach which is used and in this case is the lot amount of these computational time is saved; however, the you need to have the proper understanding for, you know, making the matrix which is to be solved. So, basically all these methods are in the order of increasing complexity, and the slab method is the most easy one which is basically normally used for the analysis of the metal forming processes.

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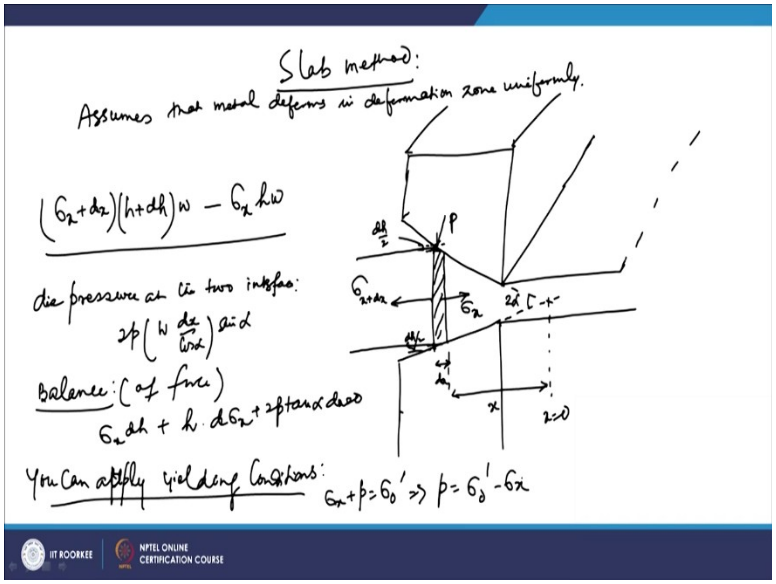

So, if you try to see that we try to analyze this slab method. So, this slab method it assumes that the metal is deforming; so, it assumes that metal deforms in deformation zone uniformly. The thing is the meaning is that if you have a square grid element, and if it is going under the application of stresses.

So, it will be converted it will be, you know, having a rectangular type of element it will be converted into rectangular type of element when it is, you know, uniformly basically, you know, deformed. So, that is what this approach is and this approach is used for. So, you have all these conditions we apply we apply these, you know, force balance equations you apply the levy mises equation also apply the yield criteria nd based on that we try to calculate the stress values.

So, suppose if we when we do for certain analysis, suppose we do for the strip drawing of a plate. So, suppose you have die which is going like this and you have this, die has certain width, and similarly you have this as another die. On the top as well as on the bottom, and then if suppose you have one strip is there the strip is basically, you know, we are this is strip basically we are applying this is a this middle point.

And this is suppose you one a strip is take an here. Now if you look at this is coming to suppose meeting at this point so, this angle is α that is 2α, this is included angle, now you want to a change the thickness of this is strip, and later on this strip will come out this thickness. Now in this case this is you are this is this way you have the width of this strips.

So, you are assuming the width to be constant, in that case you have constraint from both the sides, and this length is basically going in this particular direction. Now what is shown is that you have certain height on this side, and this is reduced to this small size, and how you will analyze this process now if you look at this. So, here you apply this pressure p. And similarly you have this is strip subjected to a stress of σ_x in this direction and if this is taken as the origin.

So, if suppose this is the point which is at x direction x distance, and if suppose this is the length dx. So, this is at $x + dx$ so, you will have stress value of σ_{x+dx} at this point. Now we take this height as h, and the reduction in this height is dh. So, if this height is h so, in that case you will have this is the half of the distance which will be ½ dh, similarly you will have ½ dh here. So, so h this will be h and this will be $h + dh$. So, this height is h and this side it is $h + dh$. So, this half will be $dh/2$, and here also you will have half will be $dh/2$, here and similarly you will have half will be $dh/2$.

So, that will be so, this if this is height h and this is $h + dh$, this is x equal to 0, other things are like when you are. So, this is strip which is there, you will have it is, you know, width along this. Now in this case we have to find, what will be the how you will find the stress which is at the, you know, when it is leaving the die; so in those case so, suppose you do the analyses of the forces balanced.

Now what you see is that you have σ_x at this point and this is σ_{x+dx}. At this points so, if you look at that way. So, you have σ_{x+dx}, at this point at this point, and your height is h and width is we are taking as constant so, this will at $h + dh$.

Similarly, so, this as this is one direction, and this will be the w that will be the, you know, if you try to find the, you know, force which is applied because of the so, you are the height with height you are multiplying with the width. So, you will have the area and that multiplied by the stress. Similarly, and on this surface you will have $\sigma_x h w$.

So, this way you have the, you know, force balance equation in the x direction. You may have the force balance equation in the y direction, and if you take the y direction so, basically this is 2α. So, you will have similarly you have α here. Now this case if you look at the die pressure at the 2 interface, now in this case you take because of this die pressure and if you take the, you know, force which is its component in the x direction. So, that will be basically $2p\left(w\frac{dx}{cos\alpha}\right)sin\alpha$

.

So, what we see is that if you look at this and this, the addition of them must be equal to 0. So, basically that should be in the equilibrium condition that should be equal to 0. So, σ_x, h and w so, and then σ_x h dx w, σ_x dx dh w and σ_x dx h w. Like that, so, if you dx and dw term is neglected, because they will be the small, you know, smaller values.

So, based on that if you take the balance of the forces of force equation if you see you will see, in the x direction you will get $\sigma_x dh+hd\sigma_x+2p\,tan\alpha\,dx=0$. Now similarly you get the equation also you have the force balance equation, may be in the y direction or z direction depending upon the other cases.

So, you will have such equation then, what you will do is you will also apply for the wider cases from there you will get certain condition. Then you apply the condition of, you know, yielding and you can apply you can apply yielding conditions. And from there you will apply suppose the tresca's criteria so, in that case you will have the $\sigma_1-\sigma_3$. So, maximum is σ_1 and minimum σ_3.

So, $\sigma_1-\sigma_3$ has to we $2k$. Now in this case when we do the balance in for the y direction (Refer Time: 30:41) that this is σ_y is basically coming as p. So, you have basically σ_x, and then that is coming out to be minus p. So, σ_x + p that will be coming out to be σ_0 so, p is coming as $\sigma_0'-\sigma_x$. So, this way what we get is now one we integrate that you will get certain equation and on integration, you will get $\frac{\sigma_x}{\sigma_0'}=\ln h+Const$.

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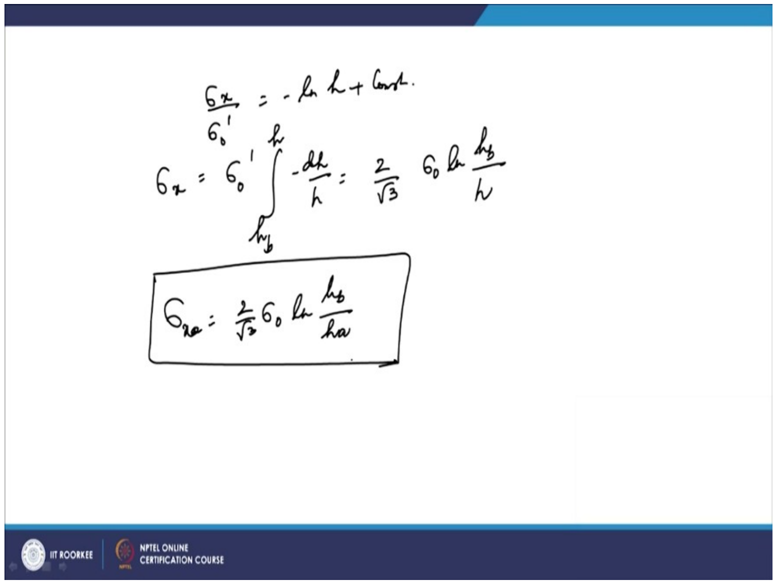

And then you can have the values of these, you know, a limits because the h will be varying from h to h_b.

So, you will have the on one side we have one height another side you have another height. So, if you do the analyses further so, that is why we do this analyses based on these methods, what we get is you get the equations. Finally, you can we can further see we will finally get the equation like h_b to h and then we get minus dh/h. So, what we get is you finally, get $2/\sqrt{3}$ and $\sigma_0 \ln \frac{h_b}{h}$. So, this way you can find, now in this case you find if you find at the exit where h is h_a. So, in that case, σ_{xa} can be found as $2/\sqrt{3}\ \sigma_0 \ln \frac{h_b}{h_a}$. So, this way you can also do it in terms of the reduction of area that terminology also.

So, what we see that this is a the simplest of the approach which is used in the case of the deformation analysis, and this is known as the slab method you have other methods also, and most of time we will deal with the calculation of the stresses using this slab method. So, that is about this slab method, you have other methods, also you can have more understanding about the other methods and we will see how this slab method is applied for finding, expressions for the stresses calculation and all that in the different forming processes.

Thank you very much.

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Lecture - 23
Determination of flow stresses in metal working

Welcome to the lecture on Determination of flow stresses in metal working. So, as we know that in the case of plastic deformation you need to know the load with which you are going to apply on the specimen so, that the, you know because every material will have some flow resistance. So, you required to have the stress value reached in all these circumstances so that there is plastic deformation. So, you are basically this is what is required then whenever you have any specimen any billet or slab or so and that will again depend upon its state. So, underwater state you are going to do the deformation. So, the deformation may be done at the temperature.

Deformation now, you know the amount of load which will be required may depend upon these value of stress, strain or you know I mean strain or strain rate or temperature or even sometimes it also depends upon the geometry of the work piece. So, if you try to see that what will be the, you know normally what will be the forming stress or pressure.

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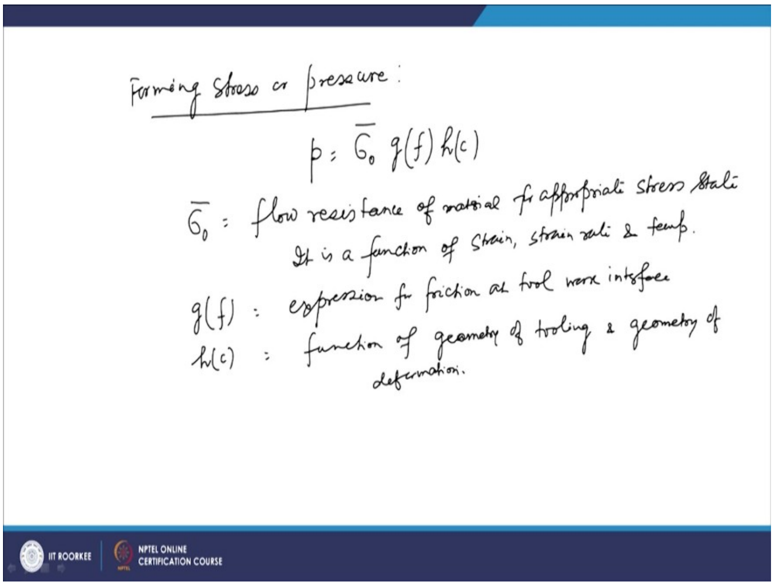

So, forming a stress or pressure which you have to apply that normally is said to be depending upon basically 3 terms and invariably you have 3 terminologies which define that what should be the forming load or pressure on that and that is basically defined as the term P equal to so this will be effective value of stress. So, that it is flow stress which is the average value of the flow resistance of the material and then you have a function g(f).

So, this is basically for the taking into account the friction which is acting between the tool and die interface and also another function which will be there it will be basically working for the tooling and the geometry of the work piece. So, basically this if you look at the $\acute{\sigma}_0$ this is known as the flow resistance of the material for appropriate stress state.

So, the material is subjected to you know different type of stress state, it maybe uni-axial bi-axial there may be no chase and all that. So, based on that under that condition the material has certain flow resistance and that is basically divided I mean presented by this term $\acute{\sigma}_0$ and it may be uni-axial or plane strain conditions or so. So it is basically a function of it is a function of strain; strain rate and temperature. So, as we know that this information has is being carried out at different conditions and these parameters are the ones which decide what will be the flow resistance of the material and that will be basically dependent upon the strain value or strain rate value or the temperature values, will discuss about how these parameters affect these value of the flow resistance of the material.

So, this function g(f) actually this is expression for friction at the you know tool work interface. So, this is expression for friction at tool work interface. So, there is the friction at that surface where the tool and die is meeting and that is to be counted into while we try to find the load you know the forging load or the forming load or the pressure. Then the, this term $h(c)$, so this is also one expression and this is the function of geometry of tooling and geometry of deformation.

So, we have certainly the tooling arrangement and we have also the deformation geometry how it is and where we have to deform in what shape. So, this function will be accounting in that factor and normally we try to avoid this becomes very redundant you know normally its contribution is very minimal. So, normally we try to you know neglect this part that is h(c).

So, this is how I mean now we have to see that how when we talk about the friction conditions, how they are going to affect how what is the effect of these friction conditions or friction between the tool and the work piece if it is, so how it is going to affect the you know load criteria.

We also must be we have the idea that when we go for the plastic deformation, in those cases your strain values are true strain values are quite in the higher side. So, we get the true strain value of about 2 to 4. So, basically normally when we draw these flow curves so, from the flow curves basically you I mean depending on the flow curve you will have the you know you are so, you will have the idea about finding these you know the forging or deforming loads.

And normally you have a larger value of typical value of strain from 2 to 4 which normally encountered in the case of the deformation analysis. Also you have you may feel to you may face even the large value of strain rates in many cases as it may go as high as 100 you know per second. So, so normally you cannot do with the ordinary test what will be the behavior of these you know flow curve, how can you draw the conclusion to find the you know flow stresses and all that it is very difficult.

Further as we discussed that your temperature is also going to be important parameter in determining that what will be the you know what will be the flow stress because at as you know increase we increase the temperature then the flow stress requirement becomes smaller. So, at what temperature we are doing all these you know analysis at what temperature the test is being carried out.

So, there itself you can have you have to have this value of the you know flow stress. So, so these things are you know important basically you must have the idea about the true strain rate then temperature and all that. Now the thing is that when we try to look at the compression test of certain you know specimen; so, you have that also gives you some indication about how the forming you know behaviors vary.

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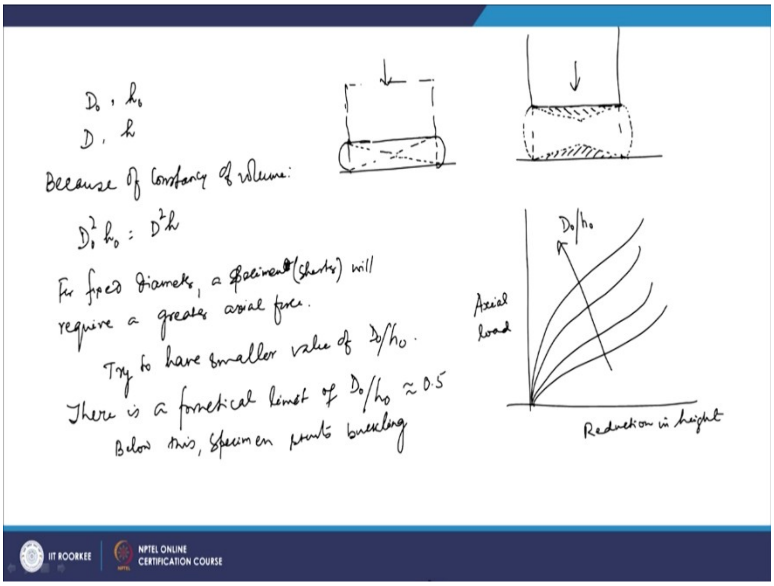

So, suppose you are going to do the you know compression of the problem in those cases, suppose what we see is that, suppose you take the example you will many a times you if you try to have the specimen like this and when you apply the you know pressure from here from the top you are applying the pressure. Now what happens that there is friction at this interface.

So, because of this friction of this interface you will have the resistance to flow at this place and you will have lesser resistance in these regions. So, what may happen is the you may have the you know presence of certain such kind of effect which is observed in many cases on these two sides and this is known as the barreling effect. Now what happens due to that basically because of this you will have a zone which is basically undeformed.

So, this will be the zone on both the sides because you have friction acting at both the places are the top as well as at the bottom and so, what we see is that you have a deformed zone at the you know interface, near that interface you have these as the you know undeformed zone basically and because of that normally what you see is you see the you know barreling type of effect if you look at the further the example.

So, that if suppose you have smaller specimen like this, so in those cases you will have again the barreling effect and if you will see so, you have the pressure being applied from here and in this case the barreling may see something it may come up to the central parts.

So, very small part is so, most of the part is now these are the undeformed regions basically and you will see the barreling and barreling is not very much desired in the you know metal working analysis.

So, what is seen if you try to see the you know in this case since you have So, the since we are doing the compression test. Now what happens that suppose you have the initial diameter as D_0 and then height as h_0 and it is converted to the dia D and the you know height converted to h.

Now what we see is that normally because of the constancy of volume principle. So, you will have $D_0^2 h_0 = D^2 h$. So, you know now this can be further used for the analysis for the finding of the forming load.

So, when the curve was plotted for the I mean load and deformation curve were plotted for the different value of the D_0 and h_0 then it was seen that your curve goes like this and it will move like that. So, when this where as we move in this directions so, the D_0/h_0 value is increasing and what we see that this is the axial load and this is basically the reduction in height. So, what we see that with this increasing value of this D_0/h_0.

The curve sees a trend which has which is so, it will be you know bending the in the upward direction. So, that is what is observed. Then it is also observed that the for fixed diameter a specimen, so, I mean a specimen that is shorter that is a shorter is specimen will require a greater axial force.

So, that is what it is seen in that if the there is h_0 is smaller in that case what you see that you require the more axial force which is there, which is evident from this curve.

So, normally you try to have the lower value of D_0/h_0. So, so try to have a smaller value of D_0/h_0; however, you cannot go this to this value of D_0/h_0 you know below certain limit because if you go to that value below certain limit in those cases you have more chances of the buckling. So, there is a practical limit of D_0/h_0 and that is normally taken as about 0.5.

So, basically you know instead of barreling the specimen will buckle. So, below this value the specimen starts buckling. Now so, we are discussing about the effect of this barreling and this is because of the you know friction which is occurring at this interface and in actual these friction which is there at the interface they are minimized. So there are ways to minimize this friction.

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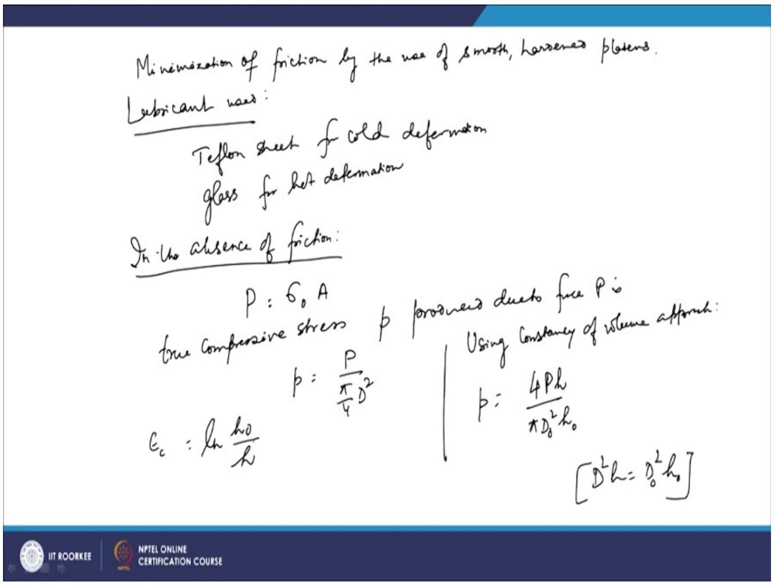

So, so, minimization of friction is done. So, that can be done by the use of the you know a platens or the dies. So, so by the use of smooth hardened platens. So, what we do is in those cases what we make we try to make the groups also towards the end so, that the lubricants are not lost.

So, so that way we try to retain the lubricant and further use it. So, that is how it is you know we try to neutralize you know you try to retain the lubricant and also we try to do the testing in the increment. So, we do it then further so, we do the you know the test in increment, so that you can also place the lubricant during the interval.

So, so the lubricant which is used so, lubricant which is used is different for solid and the you know for different materials. So lubricant used is normally we use Teflon sheet for cold deformation and we use the glass for hot deformation.

So, normally it is seen that you can reach the strain rate of strain value of 1 without you know with only very slight you know barreling a you try to observe these values in the in such cases. Now when you do not have the friction so, in the absence of friction, so when you do not have the friction, friction forces are very very small.

So, in those cases what we do is so, you have the P as $\sigma_0 A$. So, that will be the uni-axial compressive force you required to yield the material and the true compressive stress that is p which will be produced because of this force produced due to force P. So, you can find it by dividing with the area. So, p will be $\frac{P}{\frac{\pi}{4}D^2}$ because D is the diameter which has been achieved and since we are using the constancy of volume approach so, using constancy of volume approach you can have the value of p as $\frac{4\,Ph}{\pi\, D_0^2 h_0}$ because D^2h will be so, this will be $D_0^2 h_0$.

So, what we do is $\frac{D_0^2 h_0}{h}$ will be the D^2 so, h will go up. So, you can have the value of these you know p you know that compressive stress true compressive stress can be found using these values. Now if when basically so, we know that the D_0 and h_0 are the initial diameter and the height and D and h are the final you know diameter and the height, so you can find the compressive stress also.

A compressive strain true compressive strain and that will be given as $\ln\frac{h_0}{h}$. So, this way you can find these true compressive stresses and strains. Now the thing is that when we are doing these testing of these specimen, then in that case basically we need to have the true strain rate So, basically where the cross-head speed even though it is constant you cannot maintain the true strain rate as we are discussed in the past that when the cross-head speed beat the constant one, but the strain rates vary.

So, so the because you have to maintain a constant true strain rate and for that for maintaining the true strain rate you have some mechanism and normally you have the servo control testing machine so, upto the value of 10/sec you have the servo control testing machine which are used for controlling that true strain rate because as we discussed that as the a will be decreasing so, in that case the true strain rate basically will be increasing.

So, upto that it can be controlled using that servo control mechanism and if it goes to the higher values then you have a equipment known as cam plastometer that basically tries to maintain the constant true you know constant strain rate values because you know once you are doing the testing then your strain rate should be the same one.

Now when we work for the basically the you know thin sheets in those cases normally we apply the plain strain conditions and what we do is you have the you know we do the analysis by keeping the you know dimension in one of the dimension as the one which does not change and when we discuss about the thin you know when we do the plane strain compression test for the thin sheets in those cases if you look at, so, for plane strain compression test of thin sheets.

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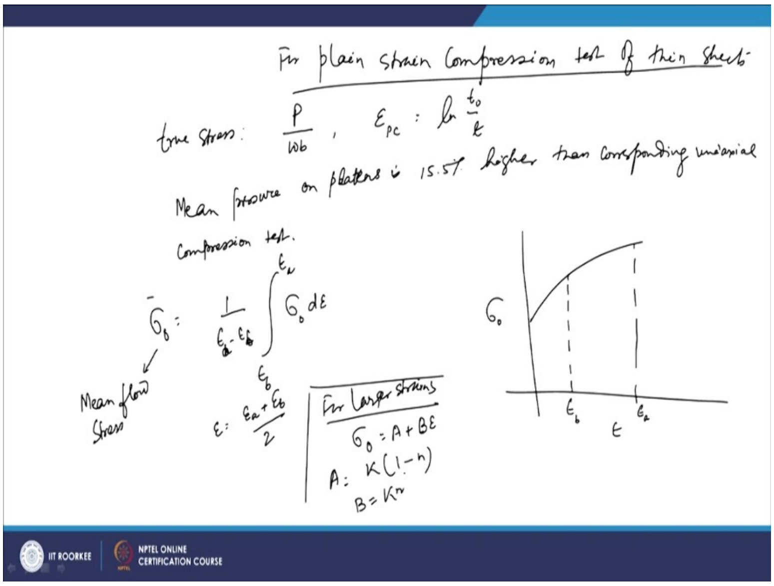

Now in those cases what we do is that you find the you know true stress and true strain. So, true stress will be found and that we found by find by the load divided by wb. So, so, this way we try to find this P and similarly if you try to find the true plastic strain, so, in the compression and that will be again $\ln\frac{t_0}{t}$.

So, you know this way you find the you know true stress and true strain in the case of the plane strain you know compression test for the thin sheets and normally the you know mean pressure on the platens is actually, so, so mean pressure on platens is about 15.5% higher than if you do that in the corresponding uniaxial you know compression test and corresponding uniaxial compression test. So, this way you try to find these true stress and true strain values in the case of the you know thin sheets.

Now normally when what is the happening is that when we have the you know variation of flow stress is there with you know strain. So, normally with strain the variation of flow stress is normally neglected in the you know analysis of true hot working, but when there is a strain hardening present, so as we discuss about we will discuss further about the you know effect of temperature and its effect on the flow stresses.

So, when we talk about the you know hot working in that case we normally you know neglect the effect of this strain on the flow stress, but when we talk about the cold working so, in those cases certainly we have some you know there will be effect on the flow stress values and in that case the concept of the mean flow stress comes into picture and so if you look at the flow stress; so, this is a typically you know strain hardening material because with strain hardening the you know the forming load is required is basically increasing.

So, so, in the case of such materials what we do is we have the concept of having the mean flow stress and that is basically suppose you have two regions; so, this is ε_b and this is the ε_a. So, and suppose in those cases concept of the mean flow stress and mean flow stress is computed like you have $\frac{1}{\varepsilon_b - \varepsilon_a}$ and so, this is $\varepsilon_a - \varepsilon_b$because $\varepsilon_a$is larger and ε_b is smaller and then you have ε_b to ε_a and then you have a σ_0 and $d\varepsilon$.

So, this is basically area under this you know curve. So, so, this is how you find the value of these you know flow stress requirement when the average mean flow stress which is required. So, this is your mean flow stress. So, from the value of a from the curve, which is you know get experimentally from there you can have it and also you can approximate these values by you know having the you know and the value ε as $\frac{\varepsilon_a + \varepsilon_b}{2}$.

So, so this way you can have the calculation of these flow stresses, also there we have studied that for many cases specially for the larger strains there has been another you know formula which is has been proposed for calculating the flow stress and that value is given as so, for largest strains. So, the proposed you know formula is $\sigma_0 = A + B\varepsilon$.

So in those cases we have one is we have seen the earlier flow curve equations earlier one there was σ was equal to $K\varepsilon^n$ that was the power law of and then we had other you know expressions of which was consisting of two terms.

So, in that on the similar line you have $\sigma_0 = A + B\varepsilon$ and it will be A is nothing, but you have $K(1-n)$. So, K is the strength constant which was found in those cases and $B = Kn$. So, so based on that the flow stress is approximated for such analysis.

So, so, what we see that normally you must have the idea about the flow stress calculations and this you know accurate prediction of the flow stress is required so that you can then you can predict the forming load or the you know load by which you are going that the load which is going to applied to create the plastic deformation of the material and we will see that how this flow stresses they are basically governed because of the change in the temperature conditions or other strain rate conditions so, that we can study later on.

Thank you very much.

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Lecture - 24
Hot working and cold working

Welcome to the lecture on hot working and cold working. So, as we know that when we do the deformation, deformation may be done at lower temperature or at high temperature. So, in this lecture we are going to discuss about the hot working and cold working, the trends of these processes what happens in these processes and then certainly when we are talking about the hot working and cold working. So, we are talking about basically the effect of temperature, how the temperature affects the processes.

Now, the thing is that when we talk about hot working; the hot working processes are the ones where you have the condition of temperature and strain rate are such that there is, you know, recovery taking place, where as in the case of cold working that recovery is not active. So, continuous with the deformation basically so, when we talk about the hot working if you talk about hot working.

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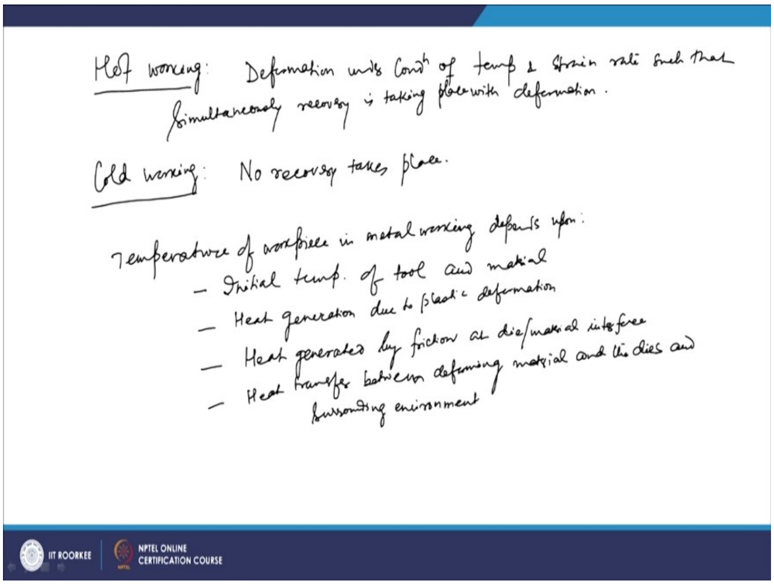

So, in that case as we know that we have discussed about the effect of temperature in strain rate. So, you have you have deformation under the action of under certain conditions of, you know, temperature and strain rate. And in this basically such that simultaneous recovery is taking place simultaneously recovery is taking place.

So, this is basically the difference between the hot working and cold working. In the case of hot working simultaneously the temperature is higher. So, the recovery is taking place you have 3, you know, basically, you know, things which happen recovery re-crystallization and grain growth. Now this recovery basically differentiates, this hot working and the cold working and so, in this case of hot working the recovery is actually simultaneously taking place with deformation so, that is with deformation. And in the case of cold working, you know, the difference is that the recovery is not taking place. So, the basically the material gets strain hardened as you go on deforming the material.

So, slowly and slowly the material gets harder and harder. And so, the load which is required the forming load or forming pressure which is required. Every time you deform the material in case of cold working, you see that the forming pressure requirement becomes higher. So, so that is the difference between the, you know, hot working and cold working, also when we talk about the hot working so, so in this case no recovery takes place.

Now in the case of hot working, basically you have the distorted grain structure also you have strain hardening that basically is rapidly eliminated because of the, you know, high temperature, so because of the re crystallization which is taking place at that high temperature. And that is why, you know, you can go for the larger degree of deformation in the case of hot working. Whereas, in the case of cold working as there is no recovery taking place there is strain hardening going on so, that is why the flow stress requirement or the stress required to remove to deform the material plastically that basically goes on increasing.

And basically you must that is why you have a degree of you have a limit up to which you can cold work the material. Because otherwise if you try to cold work the material beyond certain limit will be fracturing. So, there will be fracture so, that is how you try to also define the cold working that where you can up to what extent you can go so, that there is no fracture. You can increase the degree of cold work also when you try to give the annealing in between. So, you do the some cold working then you further anneal the process (Refer Time: 05:15) and then again do the cold working. So, like that you have, you know, hot working and cold working.

Now, this hot working and cold working as we know that since there is a temperature which defines, the either it is hot working in the hot working range or in the cold working range, and it is normally a function of the Melting temperature of the material and many a times for the material with larger melting point certainly it is higher.

But for many low melting point alloys (Refer Time: 05:48) even the room temperature working is considered as the hot working. So in fact, for tungsten like materials 1000-degree c is the cold working state, whereas, for tin the room temperature working is also the example of the hot working. Now coming to the generation of temperature, now what happens? That when we talk about the plastic deformation as we know, when the material is undergoing the plastic deformation there will be generation of temperature inside the work piece. So, so the temperature of the work piece so, temperature of work piece in metal working, it will be depending upon certain factors.

So, it will be depends it depends upon now first is that what is the initial temperature of the tool and the material. So, this is initial temperature of tool and material. Then the next is that because of the plastic deformation there will be heat generation, so heat generation due to plastic deformation. Apart from that as we know we have seen that when the die and the tool meet and at the interface you have large amount of friction. And because of this friction basically the heat will be generated and that heat goes into the work, work piece as well and that increases the, you know, temperature, so it is heat generated by friction at die or and material interface. And then you also have the heat transfer between the die material and the tool interface. So, you have heat transfer between deforming material and the dies and surrounding environment.

So, these are the, you know, factors which are you now the ones which are basically responsible for seeing any change in the temperature of the body. Now the thing is that when we talk about a frictionless deformation process then in a frictionless deformation process.

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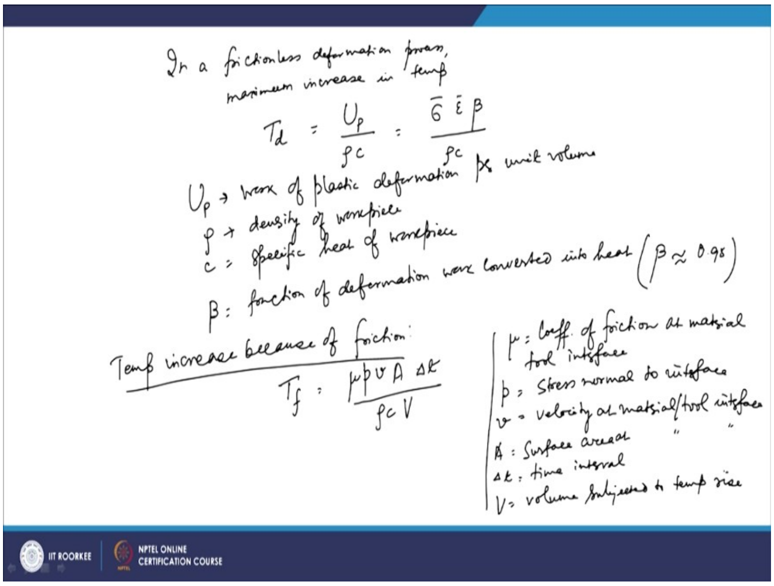

So, in those cases the maximum increase in the temperature. So, maximum increase in temperature so, that will be basically because of the work done on the material. So, that will be basically T_d so, we call it as the temperature because of the deformation of the plastic deformation of the, you know, work piece. And so, this increase in temperature, and that will be divided by the, you know, specific heat of the material, and then you have the density of the work piece.

So, that will give you the, you know, work of plastic deformation per unit volume. So, so here this way so, this U_p which is nothing but it is the work of plastic deformation per unit volume, and it can be, you know, seen as the average value or mean value of the stress, then mean value of the strain that is what we have understood how to calculate. And then you have a factor because this factor will be how much is converted into heat, and then that will by ρ_c. So, if you look at these terminologies you have U_p which is defined as the work of plastic deformation per unit volume. So, this is U_p which is that work which is done, on this ρ is the density of the work piece, and c is the specific heat of the work piece. So, ρ times the c and then ∇t we know that you had been for the, you know, mass take into consideration you have $mc\,\Delta t$ so, but it is per unit volume.

So, you have ρ and that is why you get this is the amount of plastic deformation, you know, work which is done for doing the plastic deformation. So, that is how and this is your mean stress mean strain and then beta is a conversion factor. So, β is actually the fraction of deformation work which is converted into heat. So, when we work on the plastically work on the material so, what fraction is converted into heat, and normally β is taken as high as normally closed to 0.95. So, rest is basically stored in the material as the defect structure that is stored in the material itself. Now we may have we may also find the change in the temperature, or increase in the temperature because of the friction. So, the temperature increase because of friction.

Now, see there is friction at the tool and die interface. So, this function will increase the temperature and this is basically denoted by T_f temperature increase because of the friction *f* and it will be μ times. So, μ is basically the coefficient of friction which is applied at the material and tool interface, then you have the *p*. So, that is the normal stress which is acting so, to the interface, then you have *v* that is velocity which is there at the, you know, material to interface. And then you have the surface area which is there in the picture and then you have Δt. So, that is for how much time you are doing this deformation and divided by *ρc* and you have *V*. So, in this case you have like μ is coefficient of friction and certainly it is at the, you know, material and tool interface.

So, at material tool interface, similarly you have *p* as the stress normal to interface. Then you have *v* as the velocity at material tool interface. And then you have term as *A*, *A* is the surface area at again material tool interface. Then you have term as Δt so, Δt is the time duration during which you have to consider the temperature rise. So, this is basically the time interval for which we are going to consider the temperature rise. And *V* is the volume subjected to temperature rise.

So, you can just see from so, *ρcv* into then you have Δt that is this one, and then that will be equated to this coefficient of friction and then that we have the reaction normal forces. And then accordingly you have the time also. So, that way this is equated and you can get the value of T_f from here and normally this temperature is highest at the tool and, you know, material interface where the friction will be generated. And then it will be falling off as you go inside the work piece, because it is maximum at this interface where the frictional force is generated, and many a times for simplicity we normally neglect this temperature gradient.

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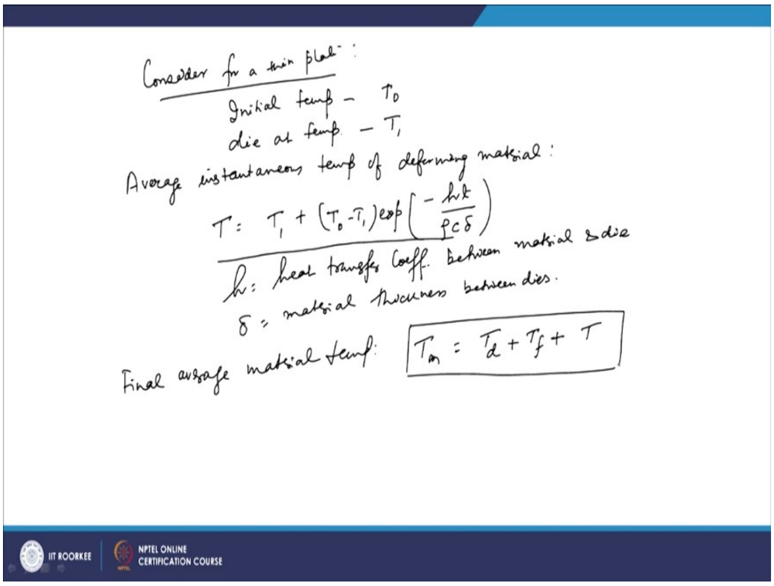

Now, if you consider for a thin plate. So, if you consider for a thin plate which is, you know, between, you know, two dies. And between so, and you will have it is initial temperature is T_0, and die is at temperature die at temperature is T_1.

So, in that case you have the average instantaneous temperature of the deforming material. So, average instantaneous temperature of the deforming material will be. So, that can be found there has been, you know, suggested by one researcher L. Tawn(Refer Time: 16:25). And we have so far that it is computed as basically T will be

$$T_1 + \left(T_0 - T_1\right)\exp\left(\frac{-ht}{\rho c \delta}\right)$$

So, in that case h is the heat transfer coefficient between material and the dies. So, heat transfer coefficient between material and die. And you have other things, you know, c is a specific heat, and δ is basically the material thickness between dies. So, what we see is that normally it will be discussing about. So, this material this equation basically this equation basically decides, you know, it talks about the average material temperature during the cooling of the material. So, normally for very thin, you know, thing plate which is cooled between two dies surfaces for that this expression is; and if you talk about the final average material temperature.

So, final if you consider all the temperature rises which are taking place because of the friction or because of the plastic work which is carried over them; so final average material temperature. So, it will be basically it will be material temperature will be T_d because of the deformation work which is increase then because of the frictional work which is friction with the increase in the temperature, and then this is the average temperature of the material. So, this way you can have the expression of the average material temperature at any instant. So, if you are given with certain conditions like if you are given any data for the for any material you are given the density or the specific heat, then certainly for a particular conditions you can find temperature rise, because of these conditions like may be because of the friction or because of the deformation work or so.

Now what we try to now see that when we talk about the temperature that is when we talk about the hot working temperature, now in these cases as we discussed that ; in these cases, your you have condition of temperature and strain rates that there is no recovery being taking place. So, essentially there is no hardenings so, in the hot working you do not have any strain hardening which is taking place.

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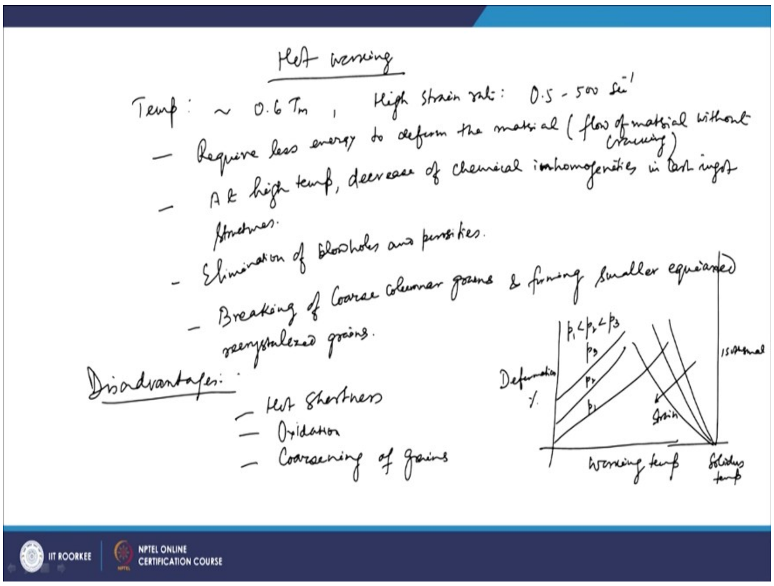

And, you know, in these cases the true stain values; which are achieved normally is quite high I mean it is there from 2 to 4 value is normally achieved. And normally hot forging or you have the hot extrusion or hot rolling is normally carried out, so that is what the advantage of the hot working is that you can go for the larger strain.

You know, large strains can be achieved with very much, you know, with not much of the increase in the flow stress requirement because once you go to the higher range. So, normally the temperature which demarcates these you know, hot working or cold working is normally 0.6 times the temperature of the melting temperature of the material in Kelvin. And you have also a very hot high strain rates which are achieved in the in these cases, and this is normally in the range of 0.5- 500 per second.

So, that type of, you know, conditions are there in the case of hot working. Now you also have other advantages of hot working not only that it is only giving less flow stress value for doing the deformation. Basically, so, certainly you the one advantage is that you require less energy to deform the material without cracking basically, to deform the material so, basically flow of material without cracking.

So, this is what ultimate requirement is that the materials must flow, and it should not crack. Also along with this process I mean along with this requirement being fulfilled that is the low value of the requirement of the flow stresses, there is another advantage that at that high temperature. you have the chances of decreasing the chemical inhomogeneities. So, at high temperature there will be decrease of chemical inhomogeneities, inhomogeneites in the casting that structures.

So, this is another example or another advantage of hot working. Further when we talk about the hot working, since we are doing at the high temperature, so, and we are deforming to a larger extent. So, the blow holes and the cavities they are basically welded they are eliminated. So, you have the elimination of blow holes and porosities. Then if you have the larger I mean columnar grain structure is there, now they are also broken when we deform the material at the high temperature, and then you get the equi-axed type of structures. So, basically we breaking of course columnar grains, and forming, you know, you get smaller equi-axed (Refer Time: 23:49), you know, re-crystallized grains. So, because of these changes in the structure of the material what happens that when you hot work the material, basically the ductility and the toughness normally improves there are certainly some disadvantage of the, you know, hot working.

So, if you talk about certain disadvantages of the hot working. So, the main disadvantage is that since you are doing at higher temperature. And the maximum higher temperature limit is it can go up to the solidus temperature of the melting temperature of the material. So, just if you take the difference of about 50 °C below that, you can go up to that temperature and you can do the hot, you know, working of the material. Or the thing is that when you have certain phases you have low temperature eutectics, below that then maybe sometimes burning occurs below that temperature in the presence of these eutectic phases which have the low melting points, you know, as compared to solidus temperature.

So, in those cases you have hot shortness is or burning is many a times, you know, encountered. Another advantages another disadvantage which is encountered is the example of, you know, a scaling. So, basically oxidation at the surface occurs so, if you are doing the formation forming process in air then at high temperature especially for ferrous materials it has a chances to form scales or oxides. And then that will require larger tolerances on the, you know, dimension of the material.

And also another disadvantage with these scales is, that this scales may go inside the deformation process itself and they can have the pits on the surface, you may have there may be scale pits on the surface. So, that may basically jeopardize the machined surface the finish of the surface. So, these are basically the, you know, the disadvantages of the hot working processes it has many advantages, but these are the disadvantage for which you have to take care of you have to have wider tolerances. And you have to have also see that it should not go to, and also when we do the hot working. And we do the stages then in that case you have to see that where you have to finish, because if you are in the higher range you can go give for more deformation, but if you are at the more higher side then you may have coarsening of the grains.

So, there are chances that you may have the coarsening of the grains, many a times when you are compressing the material you are giving the compressive stress at the material so, at the surface because being cooling fast. So, you have the final grains whereas, you will have the coarser grains towards the, you know, center part. And you will have because if the cooling is small rate is small then in that case you have the chances of the I mean formation of the coarse grains. So, what is seen is that when you are trying to see. So, there is a curve which can talk about the, you know, the working temperature if the working temperature is increased. So, with increase of the temperature and if this is a deformation percent; so you have the flow curves curve goes like this. So, you have a this goes like this. And in this case you have this is as p_3 this is p_2 and p_1 so, you will have p_1, p_2 and p_3. So, basically when you have temperature increasing then in those cases what we see is that with the increase of temperature deformation basically increases you can go from larger and larger deformation of the materials.

Similarly, you may have the strain rate also graphs are there, and you can have the strain rates graphs are shown like your strain rate is increasing in this fashion. So, this way this is your isothermal curve. So, in that case this is your solidus temperature so, you cannot go beyond this temperature. And in this case, this is how the curve behaves for the, you know, hot working cases. If you go for the cold working; now in the case of cold working what happens that the material undergoes the strain hardening.

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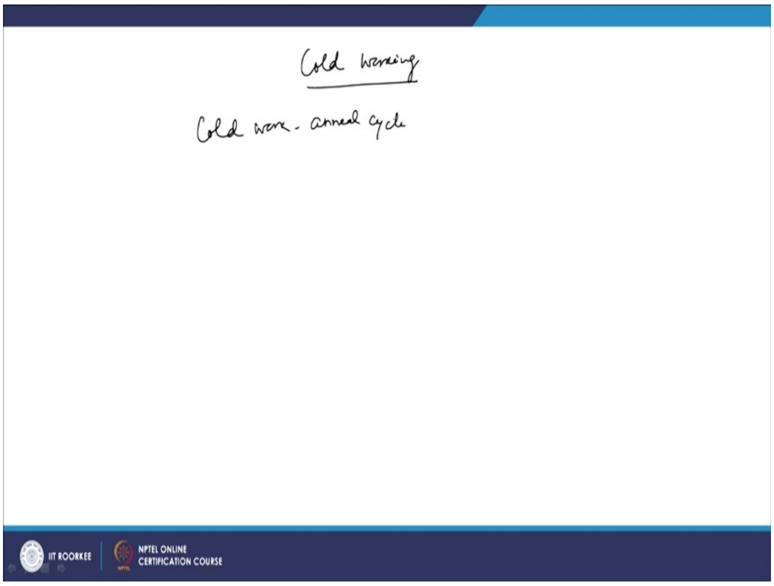

So, in the case of the strain hardening, basically material goes on getting harder and harder and your force as requirement as we have discussed that it goes on increasing. And in those cases you have the chances of cracking. So, what we normally do is, we are do the cold working stages you have a cold work anneal cycle.

Now, what we do is that depending upon the type of temper or type of hardness you require for the cord work of the materials, normally what we do you do the cold working then you go further annealing and then again you go for cold working. So, that way cold work anneal cycle is there, we know that when we do the cold working the dislocation density (Refer Time: 29:46) will be increasing and your stress requirement for further moving the dislocation or further the flow stress requirement will be more and more. So, this way there is a limit on the amount of cold work that can be done. Based on that based on the temperature also there is a warm working which is takes the difference advantage of both cold working and hot working. So, in the cold working you we know that there is no recovery taking place you have strain hardening that is there in the case of cold working. And then that way depending upon that you can analyze or the merits and demerits of the hot working and cold working processes.

Thank you very much.

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Lecture – 25
Metallurgical considerations in metal forming

Welcome to the lecture on Metallurgical considerations in metal forming. So, we discussed about the different type of forming processes. We discussed about hot forming, cold forming, others. Now, we discuss about some other parameters, which have the effect on the, forming you know methods or the criteria's while selecting the, you know particular forming processes. So, in that first of all, let us have the, you know effect of the strain rates. We have already discussed somewhat about the effect of the strain rate.

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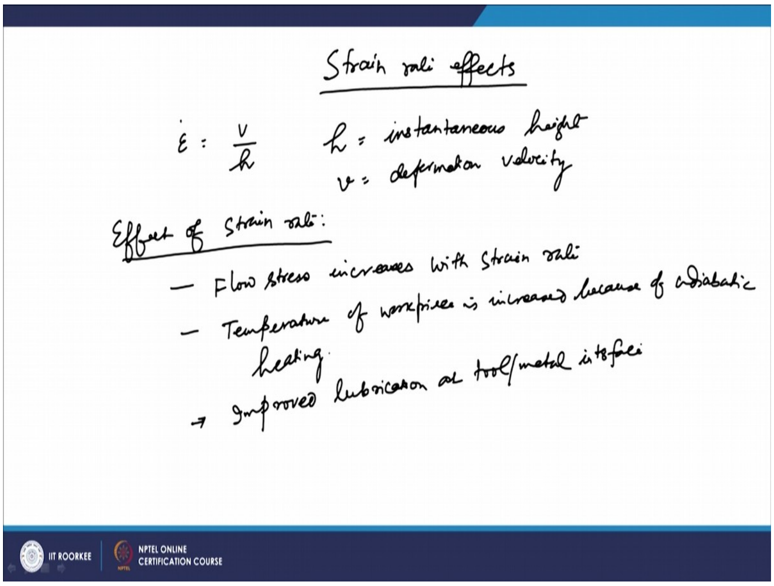

So, when we talk about the strain rate effects. Now, we know that how we define, this is strain rate and strain rate is nothing, but $\dot{\varepsilon}$ was there. So, $d\varepsilon/dt$ in that case, it was defined and normally, we define this is strain rate as v/h.

So, you have a, this wise our deformation velocity and this is, h is a instantaneous height. So, if you are trying to a compress a, cylinder or any specimen in that at any particular. So, this is basically, the true strain rate in that case. So, the true strain rate will be calculated using these methods. Now, depending upon the velocity the strain rate will be obtained and strain rate is normally expected to create certain effect, in that case that if you have the; so, in this case you have h as the instantaneous height and, v is the deformation velocity. So, this is basically the definition for the true strain rate for up setting, a cylinder in compression. Now, many a times we must know that how these strain rates are going to effect the deformation processes.

So, the effect of the, the effect of these, strain rate; so, effect of you know strain rate is that, one thing is that the flow stress will be increasing with the increase in strain rate. So, now, first effect is that flow stress increases with strain rate. This is typically true for metals, because when you increase the strain rate, the flow stress required for doing the, you know plastic deformation that will be increasing. The second effect is that the temperature will be increasing.

So, temperature of work piece is increased, because of adiabatic heating and it is, third effect is the improvement in the lubrication at the interface. So, so there is improved lubrication at tool metal interface. So, as long as this lubricating films are maintained, it will be improving the lubrication at the tool metal interface. This is the, you know effect of these, strain rate, when we talk about the deformation of the materials in any case, especially in the, plastic deformation region.

Now, many times we also try to, because as you know that we have seen earlier when we analyze the metal working, what we say is that as you move in certain direction. So, this h will be varying. So, when you have a converging die, the h will be varying as you move axially. So, in those cases, we try to define, these you know strain rate, as you know mean strain rate. So, that will be a function of the x. So, in, in those cases when you know, when we use the converging dies, where h varies with axial distance.

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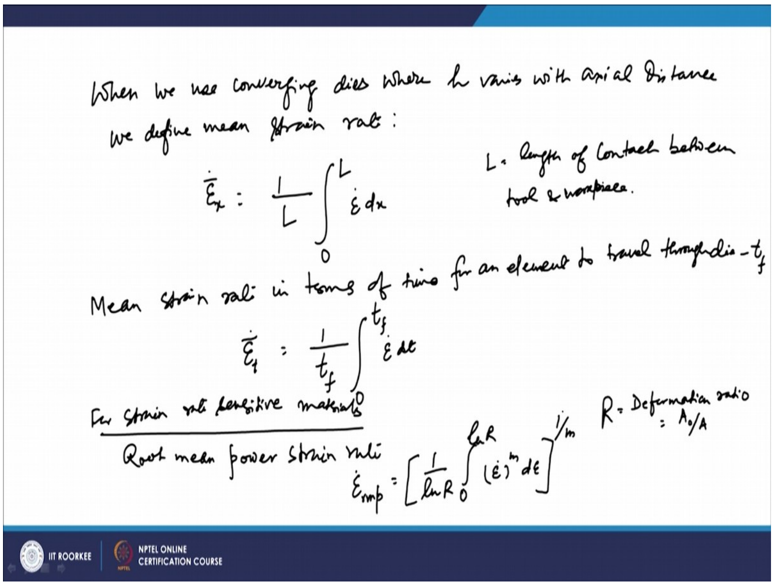

So, that is what we have. We had earlier seen in, in, in that case where we have the converging dies and as you move axially, that *h* will be varying. So, in those cases, we define, mean strain rate.

So, that will be basically in terms of, taking the parameter *x* into account. So, in that case we are going to define it as, this one $\dot{\varepsilon}$ and that will be basically; so, you have *L*. *L* is that, the length of contact. So, that will be contact between the tool and workpiece. So, we are dividing it and then we are getting the integrated value between 0 to L and that will be $\dot{\varepsilon}\,dx$. So, this way we get the mean stream rate and where *L* is the length of contact between tool and work piece.

Similarly, many a times, we also try to define these strain rate in terms of you know, the time, during which it is, you know going under the deformation process. So, in those cases, we are taking that *t* and we must know that for how much time that, you know it is going to travel through the die. So, again, you know when we can have the mean strain rate in terms of time, for element, for an element to travel through die; so, suppose it is t_f.

So, if t_f is that time see if, if you take that time into account, in that case we again define this, as compared to, for this parameter *t* and in that case we define it as again. We define it, divide it, by this total time, you know this time for which it is going to travel that element through that die and that will be again, you know multiplied by this integral and that will be, $\dot{\varepsilon}\,dt$ and that will be 0 to t_f.

So, this way you try to get the expression for the, you know mean stream rate in terms of time, further there has been, you know expression for this, you know, power strain rate in terms of root mean power strain rate. When we talk about the, you know strain rate, sensitive materials and for that; so, for strain rate sensitive materials. So, so we know that you have this parameter *m*. So, for that we will find again this root mean power strain rate.

So, there we defined as root mean power strain rate and this basically is defined by the expression and that expression is like

$$\dot{\varepsilon}_{rmp} = \left[\frac{1}{\ln R} \int_{0}^{\ln R} \dot{\varepsilon}^{m} \, d\varepsilon \right]^{\frac{1}{m}}$$

rmp root mean power, that is strain rate. So, that we know that *m* is the strain rate sensitivity and then you will have the $d\varepsilon$ and since we have, taken this power m.

So, we are going to have this power has *1/m*. So, that is how you know, you have taken the integration from 0 to *ln R*. So, R is basically the deformation ratios, in this case the R is deformation ratio. So, that is A_0/A. So, that is known as the deformation ratio and here. So, that is what you know, when we define the strain that, this *ln R* will be true strain in that way. So, and then once you are giving this power *m*, that is strain rate sensitivity, then ultimately you are, giving the exponent *1/m* to neutralize that effects.

So, you are getting this, root mean power strain rate. So, this way, these strain rates are, coming into picture, when we talk about, if the, the effect of these strain rates. Now, the thing is that, these strain rates or, or the velocity, you know have, you know the crosshead velocity. So, you have, then you have the deformation velocity based on, that you have the strain rates. Now, this strain rates basically, vary from very small value to very large value. So, when we talk about certain processes like you have normal tension test, in those cases the strain case values are very-very small and that is normally in the range of $6x(10)^{-7}$to $6x(10)^{-3}$.

So, that is the strain rate. So, that is velocity basically, you know, what we, encountered the velocity, in all the and based on that basically in what, you know, what is the zone, where the deformation is taking place, the strain rate will be determined so.

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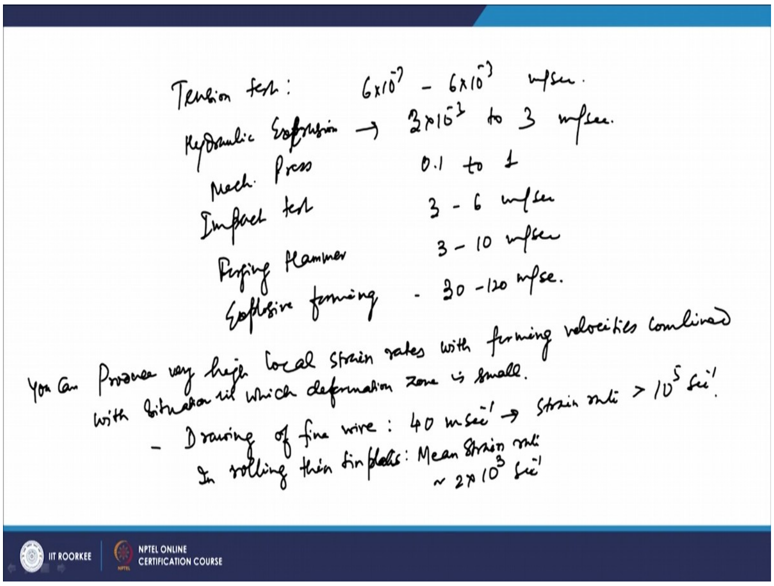

So, we have already seen that if you have normal tension test. The velocity which is encountered is $6x(10)^{-7}$- $6x(10)^{-3}$ m/sec. Similarly, if you have a hydraulic you know extrusion or that is through the press. So, in those cases you have $3x(10)^{-3}$ to 3. So, that is the, range of velocity. Similarly, you if you have mechanical press in those cases, it is coming from 0.1 to 1. So, then, then if you do, do the impact test like Charpy impact test, in those cases what we get the velocity, it will be 3 to 6 m/sec.

Similarly, if you have the forging hammer; now, forging hammer in those cases you get from 3 -10 m/sec and, the maximum what you get is normally for the explosive forming. Now, in, in those cases, in the case of explosive forming, you get the range from 30 -120 m/sec. So, what we say that this deformation velocity, this is the deformation velocity for the different testing operations and this is normally, you know more than the you know crosshead velocity in most of the, you know, for most of the equipments. So, it will be, larger than the cross header velocity in the standard, tension test.

Now, when these velocities are combined with the zone, you know, if they are working on a smaller zone, in those cases, you can get very high local strain rates. What we have seen that you have strain rate is v/h. So, now, the thing is that, one thing is that, when the, there is deformation going on, there also you see that as the h will be changing your strain rate will be changing, but another point is that if this deformation, you know it is, I mean defined or it is confined in a very smaller zone. In those cases, the locally, the strain rates are going to be, you know attained to a very high value. So, you can have. So, you can produce, you can produce very high local strain rates with forming velocities, combined with situation in which deformation zone is small.

So, basically, whenever you, if you combine this, you know forming velocity is which those situations where the, this deformation zone is the small, in those cases, you can have the very-very high, localized strain rate, values, for example, when you draw a fine wires.

So, when we are going for drawing of fine wire now, in this case, if you have a deformation velocity of 40 m/sec. Now, that can basically, result in a, you know strain rate in the axis of; so, strain may go in the axis of $(10)^5$ /sec.

So, So, that is how these you know, this velocity, you get this strain rate at the higher rate of, you know at the higher value of $(10)^5$ /sec. Similarly, you can get, you know in thin plates, in you know rolling thing plates, you get mean strain rate of around $2x(10)^3$ /sec. So, so. So, that is what we, get these, you know a larger, you know local mean strain rates. So, that is more prominent way of having the, large local, you know, strain rate values rather than increasing the velocity. So, you can increase the velocity, or you can decrease the size of these deformation zones and in that case you can have the large value of these, strain rates.

Now, based on these strain rate values, you have the classification of different type of, advanced forming processes and as compared to the conventional one, there are processes like high energy rate forming, you know processes. So, that is known as, that is in the category of HERF.

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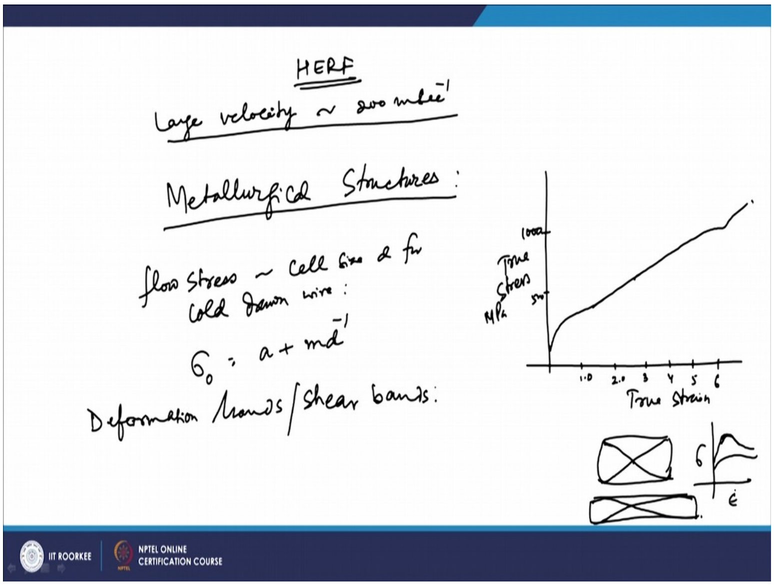

So,. So, in those cases you have, you know deformation velocity will be as high as about 200 m/sec and that gives to this. So, that will be imparting this energy at a very-very high rate. So, in those cases you have, large you know velocity and that will be of the order of about 200 m/sec. So, so there that will be used in normally in the case of forging or extrusion or; so, or sheet forming you can use and, they are normally coming under these category of HERF or high energy rate forming, because the energy of deformation will be delivered at a very-very fast rate as compared to the normal conventional, you know forming processes.

So, normally in these cases, you try to get, you know, what we do is, we use the explosives, where these deformation velocity very-very large velocities are achieved. We use the explosives or you know, explosive gases to create such high velocity and what happens that for many materials, you know, what happens that the elongation to fracture will be, you know, you know that will increase, with strain rate beyond that usual metalworking you know range. So, that again depends upon the critical strain rate value.

So, and after that it will be, falling off, so depending upon those cases in, in those cases. These explosive forming technologies are used and also that, that has also something you know, related to the, the, because when we do such kind of treatment that may also, you know affect the structure, you know structure of the material. So, that, that, that, that is also you know to be looked into and when we do that, in many cases that is not very much prominent, another extreme side will be with a very-very low strain rate values and as we know that when you have the, very high, you know strain rate sensitive materials and you, when you have very-very high, you know strain rate sensitive material.

So, they exhibit very much resistance to necking. So, so that defines another set of materials that is you know the super plastic forming. So, that is normally you know promoted or that is normally prominent when you have very-very, fine grained structures and the temperature also is more than 0.4 *Tm*, and strain rate is extremely small that is about 0.01, you know, per second. So, in those cases, you know there is little, you know chances of the, the necking. So, so there will be having the, they will have the higher resistance to the plastic instability and that is why they have the super plastic behavior in many cases.

So, that basically will be again a function of the strain rate and then also the temperature at which you are doing that you know, this deformation and so. So this way, you can, you know many difficult to work alloys, they are basically, you know, can be word using these concepts or many a times, when you have to go for the fine, you know details and impression to be obtained, in certain material like in embossed (Refer Time: 21:38) structures or so. In those cases, this concept is basically, utilized. Now, we will discuss about the metallurgical structures.

So, as we know that, when we do the, plastic deformation, specially, when we do the cold working of the material. In that case, as you go on working the materials, the, the stress values required. The flow stress requirement will go on increasing and it is also, because of the you know dislocations, which are you know dislocation density (Refer Time: 22:21), which is increasing and that has also to, to do with the structures of these dislocations, which is basically reported to have certain kind of you know, certain kind of structure, which is formed during these workings.

So, what has been seen, that when you do the, cold working of these wires and when you do the, drawing of cold drawing of the iron wire then if you draw the stress strain curve the, the stress strain curve looks like this. So, what happens that if you, this is the true stress and, and, and this is the true strain and you are doing the cold working of these iron wire. Now, what has been seen that it, it, it goes and then, it will be moving like this.

So, so it will be moving like this now ah. Now, in this case true strain will be having the values like it is 1, then you have 2. So, 3, then 4 and 5 and 6 and all that. So, this way what we see and this side you have MPa. So, so you have here as 500, then you have here as 1000

MPa or. So, now, what is seen. So, this is for the 0.007 % of carbon, steel ah. So, that iron is there.

Now, what we see that if you what we see here that even, if these you know, true strain value going above 6. These rate of strain hardening is not you know, decreasing, remarkably. So, they are not diminishing, the significantly now, that can be, what happens that when the deformation goes on, you have, you know some structure is formed. Some structure which will be forming and you will have the tangles of dislocation structure, which is, you know seen and what has been for seen that when we when we talk about two types of materials normally, if you talk about iron, you may have the BCC structure also you have the FCC structure and when we talk about the FCC structure they have, lower strain hardening as compared to the BCC structures and also the sub-cell (Refer Time: 25:07) size is also important. So, you have the development of stable cell size in the case of BCC iron alloys and that will be you know continuously decreasing as the deformation goes on.

So, as you see that when you talk about the cold deformed materials you have the elongated grains and basically the, the deformation band will be appearing. So, you have the flow stress. So, the, the flow stress, which is, found and if you try to find the cell size *d*, cell size *d* for a cold drawn, wire. Now, it has been found that the σ_0 is basically. So, this is the flow stress and it is basically $a+m\,d^{-1}$.

So, as the cell size, this will be decreasing, your flow stress requirement will go on increasing and, and in the case of these deep drawing, when you do or when you do the, you know why drawing operations in such cases. So, what has been seen that when you are going for cold working, you get equi-axed type of you know, cell structures, which leads to you know and, because of that basically, you have these, increase in these you know, flow stress values. So, further you, as we discussed about the formation of this shear bands or deformation bands.

So, you have also, you must be, you know conversant with these terms like deformation bands or shear bands. Now, they are defined as you know, they are the reason of distortion, where portion of grains are basically, there is a, being basically rotated, towards another orientation to accommodate the apply, the strains. That way you have these deformation band and shear bands. They are formed and if you try to, should look at these representation of these, bands.

So, what we see that when we increase the reduction. So, first of all you may have its appearance like you know. So, this bands are developed like this and if you increase the, the reduction. So, as you increase the reduction, these deformation bands move like this. So, they have also, you know you can see that in the middle portion, you know, at these places the shear stresses at and that way this shear bands are formed shear band will be found like this and these shear bands, you know they are basically, moving and that basically is, basically the, you know, you know principle, based on which these deformation occurs, plastic deformation occurs.

When we apply the, the forces also typically, when you work the hot deformation of the material. In those cases what happens that some of these strengthening mechanism become unstable and then you may also end up with sometimes the, the, the, usually if you have such kind of flow curve, after this deformation as the strengthening mechanism you know become unstable.

So, you get also such kind of you know change in this flow stress, you know curve. So. So, this way that is further, being seen in the hot deformed materials, which has been you know observed another thing which we must know is the, the deformation zone geometry.

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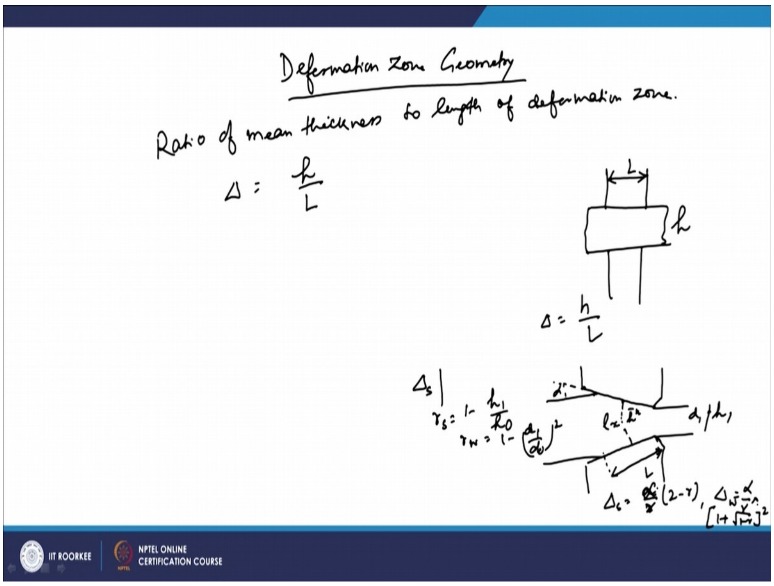

So, when we talk about. So, as we have discussed that deformation zone is very important and depending upon, you know it is, you know whether zone is small or larger, you can have the, you know presence of or the, you know production of, the larger value of strain rates or smaller value. So, when we talked about the deformation rates then there is you know, one particular parameter is important in those cases and here what we have, we have two things basically, you have the die and, in between the die you have a reason which is in contact with the die.

So, basically, you have a ratio of mean thickness to length of deformation zone. So, So, you have two things; one is the one is the material, which has certain thickness and then, you have, the die which is in, in touch with that and that is why this ratio is defined as delta (Δ) and this delta (Δ) is normally defined as *h/L*. So, for very simple situations like you have, a such kind of situation, where this is your, this is your *h* and if this is your *L*.

In those cases, you can have this deformation as *h/L*, but now again, when we talk about you know, based on the plane strain reduction, in those cases, you know sometimes we define it. So, suppose you have one, another you know type of, you know tool, that is, that is where you have such kind of, you know.

So, this is a converging type of, you know die and in this case, this is here. So, here the, the material comes and this is your angle. So, this is your, semi die angle α and in those cases again what we see, that here it will come like this. So, it will be having either d_1 or h_1 can be you know, seen and depending upon d_1 or h_1. You can define, this delta (Δ) and again here, you have this will be your L. So, this is the length. So, in this case these becomes the L and similarly, here you have, this will be your L.

So, this, this can be taken as. So, this is ah. So, here this you have h and here it will be varying. So, you have l or h' and in such cases. So, this will be average height. Now, in, in such cases what we do is, we can have the definition for the plane strain condition and this is

defined based on $r_s = a - \frac{h_1}{h_0}$ or if you are doing for the wires.

So, in those cases you have $r_w = 1 - \left(\frac{d_1}{d_0}\right)^2$ and that way, you have the definition of Δ_s or Δ_w

and $\Delta_s = \frac{a}{r}(2-r)$. So, this is α/r. So, this is $\frac{\alpha}{r}(2-r)$ and similarly, delta w is defined. So, that

will be again $\Delta_w = \frac{\alpha}{r}\left(1+\sqrt{1-r}\right)^2$.

So, this way, you define this parameter for the deformation zone, you may have different types of you know zones. Different type of geometries and based on that, because that will be affecting what will be the deformation pressure required and deformation pressure, that way depending upon the value of *h/L*, you will have the calculation of deformation. You know pressure or deformation loads, which will be calculated so that we can see when we talk about the calculation load, calculations in those situations.

So, what we discussed in this lecture, about the different aspects or different parameters, which are required to define the deformation zones or, or the metallurgical structures, which are important while we, do the deformation in the case of plastic deformation of materials.

Thank you very much.

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Lecture - 26
Introduction and Classification of Forging Processes

Welcome to the lecture on Introduction and Classification of Forging Processes. So, in this lecture we are going to now discuss about the type of forging processes and there classification and their introduction. So, among them we are going to start discussing about the process of forging.

So, first of all what is forging that we must know. So, forging is working of metal into a useful shape by hammering or pressing. So, what happens that many a times you require to get the different shape of the objects and for that normally you have the dies in the die you. So, in between the die the component is placed and then it is basically compressed with the in between the dies.

Now, that is what the forming processes are in the case of rolling; you have 2 rolls and in between the material will be compressed. Similarly in the case of forging you have 2 dies and in between the material is to be you know pressed. And in that there may be grove inside the die; so the thing is that whatever is the grove inside the die, when the material is pressed the metal will flow depending upon how where it has to flow where it is getting the way; so that it flows.

So, that way that is the processes of forging and during that process you know whatever happens in the case of forming methods like you have the development of fibrous structure, you have the disappearance of any kind of you know discontinuities or defects in the stock like you have the blow holes or shrinkage cavities, they are disappeared. You have the development of fibrous structure also when you we do that process in sequence in a proper manner.

So, that way this forging method is you know carried out and most of the auto mobile components they are made by this process. So, in this case basically you are applying the force from the top or you are applying may be on the side way is also you apply. So, that it will go and it will compress it or you may also draw it you make an increase its length you can decrease its length. So, it happens in that process that is the process of forging.

Now, let us discuss about the how we can classify these forging methods. So, as usual when we try to classify these different types of manufacturing processes then first of all we try to classified based on the temperature.

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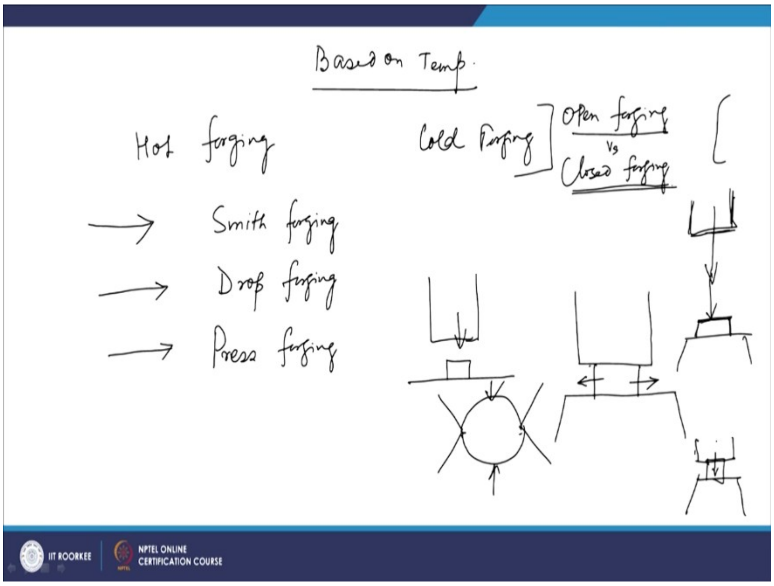

So, if we try to classify based on temperature then you have hot forging and cold forging.

So, as usual when we are going to talking about hot forging, when the stock is heated to a higher temperature higher than its recrystallization and temperature; then we call this process as hot forging temperature, hot forging process. And if you do it below that temperature then we call it as the cold forging you know processes. So, when you have the larger degree of deformation required. In that case you will go for the hot forging processes, because at higher temperatures you can go for higher degree of reduction because of flow stress requirement which is smaller.

And as the temperature will come down then the flow stress requirement will be larger and larger you require larger stress to deformed material plastically. So that will be in the difference between the hot forging and cold forging; hot forging will be done at higher temperature, higher than the recrystallization temperature. Then cold forging will be done at the lower temperature that is your lower than the recrystallization temperature like that. Then so that is why you can have the metals hot forged or the cold forged.

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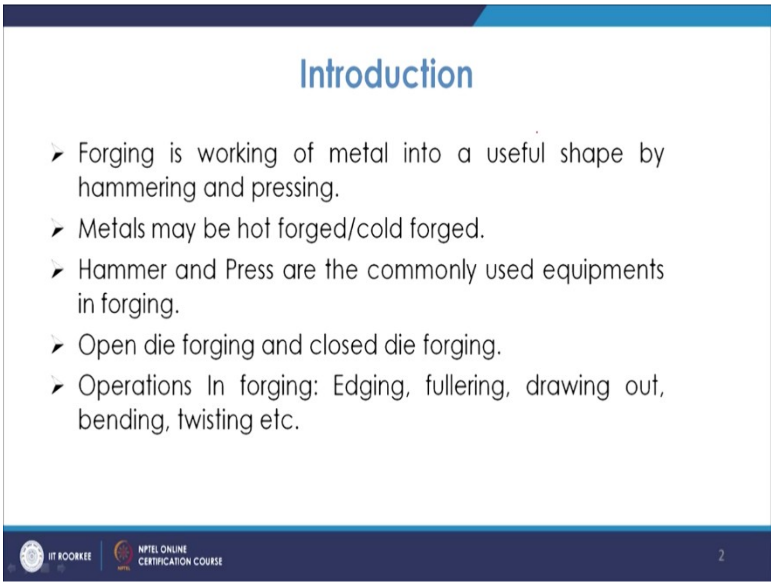

Similarly, you can you if you talk about the different types of forging operations what are the different you know what are the things which occur in the case of a forging processes or when we try to classify then in that case we also classify based on what way the forging process is carried out.

So, in that we have we have to come across certain terms like you have smith forging. So, is smith forging is something a type of open forging; so, before that let us talk about a forging process known as open forging versus closed forging. Open die forging versus close die forging we may see here that is open die forging and the close die forging. So, open die forging means again when we are; so in normal case forging as we have discussed that if you have a die and then you have this is your anvil; so, what we do is if you have an object then you tried to forge.

So, the moment will be in this directions; so if we say a flat die at this place and if you are try to forge it then in that case you have the moment of the metal in these lateral directions and wherever it does not have constraint the metal will move. So, and depending upon if you have the you know cavities or if you have the groves made in these places then metal will flow into it; so that way.

Now open die forging and close die forging; now open die forging means now when you have this is example of open die forging. So, you have this anvil and you have kept the metal and then you are you know allowing it to follow over it and then both are open there is no constraint from any side. So, normally that is known as open die forging and in that normally we you try to get to the simple sips, you when you have to increase its you know length or width on the cost of its thickness or so.

In those cases all you have to get the simple shapes you have to you know make the you know specimen flat or so; so those are the examples of open dies. So, dies are open basically you do not have any groves made in the die and it will be falling upon that. And then close die forging; in the close die forging the dies are closed basically when the die is finally, the both the die have to you know meet each other and they have to be closed.

So, ultimately they the dies have to you know like you know you may have die of one of this type and another may be of this type. So, both the dies; so this way you can come and ultimately if you have the; so ultimately you can have a you know this type of shape made in a close dies. So, ultimately initially they will not close because this process will be in a sequential manner and reduction will also be in succession and ultimately at the end the dies will be closed and the cavity which is their inside the die when the 2 dies are attached to each other, that will be the final shape of the material that you are going to you know get in the case of closed die forging.

Now coming to the different types of forging processes; so, again that is also a classification. So, based on what kind of dies are there; then depending upon type of processes how it is done. So, smith forging it is a type of open die forging and it is the name given based on the Smither's of blacksmiths in the village work.

So, normally you have an anvil and then you heat the material and you keep there and then you are hammering it. So, you are basically making the flat products or you are making the pointed once or. So, you may be doing some increasing in the length or you are sometimes making it flat or so, that is normally known as the smith forging.

Similarly you have you know drop forging and drop forging means you are dropping certain weight you know you have a die which as certain weight or the weight may be its own weight or you have the extra weight. Because of the velocity or acceleration with which it is moving so, and then it is dropping from certain distance.

So, you have a die here and you have a specimen here and something is there it is dropping through this distance; so that is known as drop forging. So, in this case basically you have this is the. So, normally you have use of hammers in these cases; so, basically it will come from certain height and its kinetic energy will be a stored into it will come with certain velocity.

And then it will be impacting on this job and this way it will try to deform it. So, that is the example of drop forging and normally we use the hammers in these cases. Now hammers also hammers are the one which used basically make the impact on. So, they basically have the impact type of force being applied on the specimen and then they try to deform the material.

So, so this is the type of forging process where we used this is hammers and that is drop forging. Then next will be you know the press forging; so, the press forging again you have here you have use of press. So, what we do is in hammer as we discussed that you have the material and with it is you know it is the specimen is subject to the impact force; from the hammer which is coming from the top at certain velocity, it is falling you know. And then that basically deforms the material where as in the case of this press forging what we do is that you do not apply the impact force rather; it is squeezed continuously.

So, in the case of press forging when you have the die then it will be in touch with this you know press and then it will slowly it will you know do the pressing or do the squeezing. So, in the continuous manner it will be pressing the material; so, that way that that is known as press forging.

So, here we use the press to do you know the forging processes. So, Smith forging any way is something you know normally for very you know in a general term which we which because of the Smither's we black smiths; we give this name otherwise you have drop forging and press forging are the main 2 apart from that you have also machine forging.

So, that is also there; so in that the material which be only upset to get the desired shape; so that is what is your the machine forging. Now coming to the open die forging as well as close die forging we have seen; that in the case of open die forging anyway the die is surfaces flat. So, it is you know free to you know expand in different directions.

And the material deforms over a flat surface, but when we talk about you know the close die forging. In the case of close die forging, what happens that you have certain shape in mind where the aim is that you have to get the whole you know final shape of that particular type.

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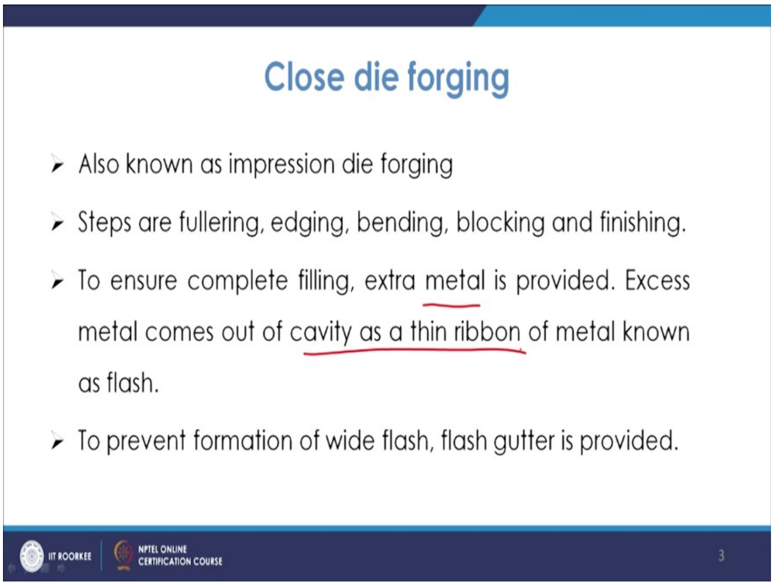

So, so that close die forging that is also known as the impression die forging because what happens in this case that you are ultimately trying to get the impression which is there into the dies. So, ultimate objective is to basically get that you know final shape of the material.

So, the in the case of this impression die forging; you have the steps like fullering, edging, bending, blocking and finishing. So, what happens that when we talk about the different methods; so as we can see that what is. So, first of all you will have the stock which is to be put into a final shape. So, they have to go in succession, you cannot think of you know converting the raw material which is there in the case of a billet (Refer Time: 15:17) or a slab or a stock or a rod. So, you it cannot be converted into the final shape.

So, what it has to do is that first of all you will have the fullering and in the case of fullering; normally what we do is you try to accumulate the metals wherever it is required. So, that is your fullering; so, what happens that suppose when we talk about. So, what we will do you have fullering dies.

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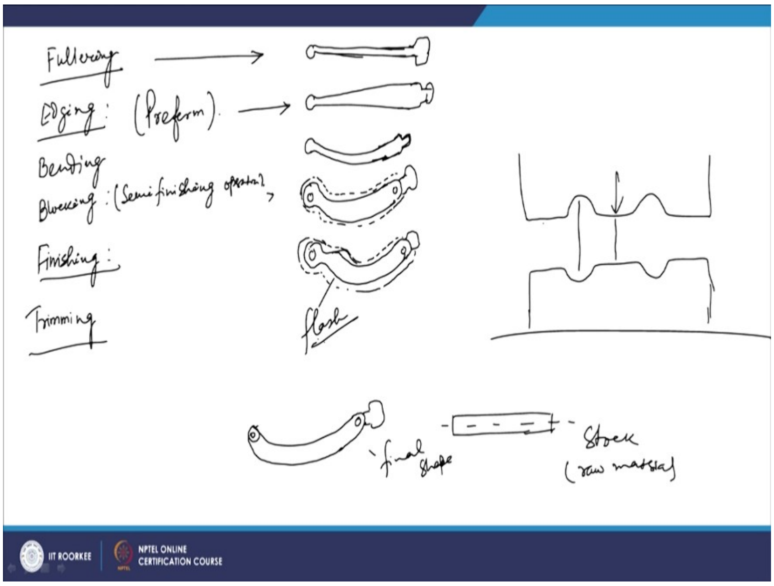

And suppose you have such kind of; so you have such kind of dies and in this case if suppose you are pressing these dies from this side. So, then what will happen the wherever it will get. So, suppose you have to make some component which is thicker here a thicker in these zone so and then you have thinner in these zones.

So, you will have those dies will which will try to push the material accordingly more materials in to this side and lesser one here. So, that way we try to you know you know even out the material. I mean depending upon where how much material has to be there in the that cross section; so, that way this process is known as fullering.

So, normally what way is done that one we talk about the fullering. So, in that case you know you have to reduce this stock to the desired size and in that you have to see that the this is basically; it is not upsetting basically, but it will be the reduction in the cross section at these different places; so that process is known as the fullering. Then the next is; so you have first is fullering, then you have the edging and edging is nothing but it is also name is the making the preformed.

So, what we do is in the case of edging we try to gather the you know suitable amount or exact amount at different cross section and also you know it is something like knowing the preform. So, it will the material the shape will be somewhat coming similar to what you have to achieve. So, you will have; so it will basically ensure that you have the defect free flow of material and you will have the complete die fill and minimum of the loss.

So, edging is nothing but you know it is all also something synonymous with making the preform of the material. And in this ensures the exact amount of material which is to be accumulated at the respective positions. Then if you required you can go for the bending operations like if you see the there are you know the material has the bent shape, then you have to go on the bending dies then you do the bending operations.

So, you have bending then after that you go for blocking. Now why bending is required because many a times if you can get that type of shape even by you know cutting the material and or machining the material and get that shape. But with that basically you have more chances of you know loss of the fibrous structure. So, that is why we go for the bending processes.

And then after that bending you have you blocking. So, basically it is also called this semi finishing operations. So, so you have the impression which is created which is semi finished. And because you know when we talk about the forging components; now you have the pockets you have basically when we talk about the intricate shaped components, you have many a places it has to go over the bent there are radios and curvature along which the material as to go and fill, you may have the deep pockets the sharp corners. So, you know you must have actually the blocking impressions.

So, those impressions are basically used so that you ensure that you know the material has to flow accurately I mean at those corner so or wherever the radio is there. So, basically you provide the larger radii and corner and fillets so that the material flows smoothly and there is no any kind of discontinuity at those places. So, in when we talk about the semi finishing impression that is blocking; in those cases you provide these larger radii and also you provide the fillets. So, that you ensure that material goes over that portion and completely fills the cavity.

But still we are not considering about the flash, we are concerned with the easy you know passage of the material or easy moment of the material on the on those corners and fillet positions. And so that the material moves uniformly without any kind of lap or any kind of defect structure in those cases; then after blocking you have the finishing operation. Now when we are finish the blocking then we are concerned with the larger you know fillet corner radii and so, but we are not concerned about the flash.

So in fact, when we talk about the final impression this is finishing is nothing but getting the final impression of the material which is going inside. So, ultimately it will be dealt I mean it will be dealing with the final shape of the material and basically that is ensured when you also you know come to a conclusion or that is characterized by some event like you are assured that you know; there are you have some extra material which have which was there you know in the stock and that extra material basically has come out.

So if you think about any casting process also in that if there is a riser (Refer Time: 23:12) which attached in a side. And if the metal goes into the casting and then through that is goes into you know the riser then filling of the riser will ensure that the casting is already filled. So, similarly in this case what we see is you have you basically supply some extra material and that extra material will form as the flash and that will be seen.

So, then once this finishing processes is over it means your material is ready with some extra material and that extra material is known as flash. Now the next process which is there in the case of this close die forging is trimming. So, in the case of casting we call it as fettling, where we remove all these you know extra attachments to the cast product.

So, by cutting or by flame cutting or by machining you could you remove them like gates or risers or so; in this case you have the formation of the flash and this flash needs to be removed and that is known as the trimming process. So, this is basically the component of this there are the different stage is in case of these you know close die forging so that is what it happens.

So, if you try to look at you know the different stage is in the case of the component which is made. Suppose if the component is made of made like that of a lever; suppose you which is used in the you know automobile. So, suppose the lever has the actual shape.

So, suppose you have this is the actual you know shape of the lever. Suppose that beings the one which is you know a lever is to be made and this is your final shape of that lever. Now the thing is you have a stock which is normally of you know any shape which is very plane shape or so, now, you have to convert these into a lever.

Now, for that first you know you will have a stocks; so, the stock is in this shape. So, this stock is there; so this is the stock it is raw material and this is your final shape. Now how to get it in this? So, this stock material will have to be passed, will have to pass under these 3 the successive you know stage is and finally, it will come to this shape.

So, first is you know fullering and as you know that you have in this side this is the size of which is here and this side you have here. So, this is somewhat smaller; so, first of all you what will do is you will make a component like this; you will make in the fullering die. So, so this way you get. So, so this way when you are trying to give under such die and it will you know increase its length it will you know draw out it and there it will come to its this shape that is known as fullering then coming to the edging.

So, for edging what is done now is now; now you will further you know bring and gather the metal to different regions and then you have this. So, so this way; so this way you are going to get, so that will be your edging. So, the here basically the amount which is required in those you know regions that will be accumulated. Then if you required the bending; so for that what you do is you put it under the bending dies and then it will be basically bent.

So, so it will be it will be bending like this and then so you have put under the die and then it has bent. So, that is your use of the bending dies to shape the component in the by bending it; then you have blocking and in the blocking what happens is that now you will put under the this is semi finishing operations that way. Now it is all on the bent specimen and then you will have the formation of these. So, but here you try to ensure that the material which is there which is flowing in the some extra material.

But any way here we are not concerned with defining that and when we go to the finishing; at that time again you have in that. So, here you ensured that the material has filled all the regions. So, that will be further the finishing stage and then so it will go like this and then you have this structure made and you have the extra regions which is the trim. So, here you ensure that this extra material which there the parting plane that is done and then ultimately you are removing this extra flash and that removal is known as trimming.

So, these are the different stage is in the case of the close die forging. So, this is about the you know impression die forging what we have discussed and there are certain you know considerations which are required to be seen, you provide these you know extra metal so, that there is complete filling of the cavity. Excess metal will come out of the cavity as thin even of metal known as flash and then to prevent the formation of wide flash; flash gutter is provided. So, that is because you know if the you know if the too wide then so that way it because that temperature goes less and then the resistance will be there. So, it will require larger and larger force to further press it downwards. So, that is why this gutter is provided in such cases.

So, these are the considerations when we talk about the close die forging.

Thank you very much.

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Lecture - 27
Equipments used in forging

Welcome to the lecture on equipments used in forging. So, in this lecture we will going to discuss about the different types of equipments which are to be used in the forging. So, as we discussed that normally depending upon the type of you know operations which we apply, in the case of forging you have one is drop forging So, where there we use the hammers.

So, these hammers basically they will be coming and they will be giving the impact force from certain height. And, they will transfer the energy on to the workpiece and then the material gets you know deformed. So, the energy of these rams, which are moving from top to bottom they are utilized to be formed the material.

So, that is how these hammers are used. Now, similarly as we discussed that you have the prsess, there we use the continuous squeezing action by the machine, and they apply the load on the you know component and that is why depending upon the load how much they apply the deformation will take place. Now, when we talk about the different types of you know hammers.

So, if you talk about hammers, you have the board hammer or power hammer or counter blow hammer.

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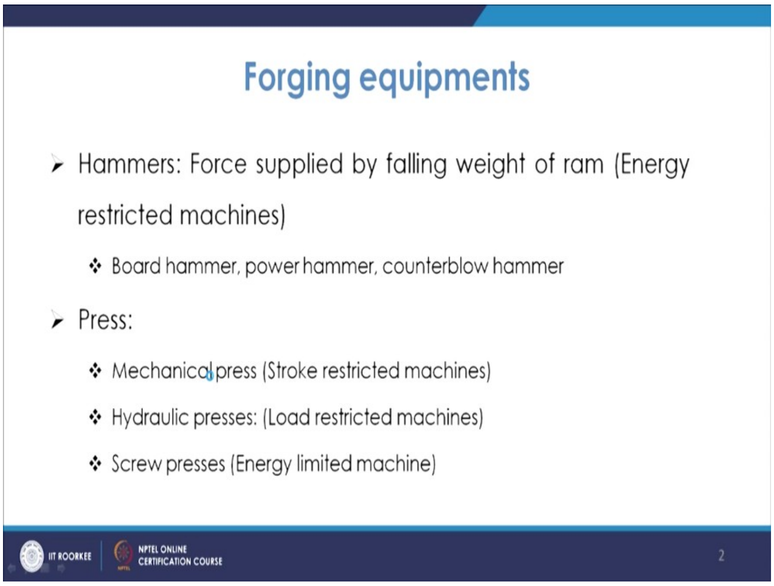

Now, in that board hammer or sometimes we also classify as the gravity drop hammer. And, then you have to drop hammer will be in fact, categorized as again for the gravity drop hammer and then you have the power drop hammer.

So, what happens that in those cases you require so you have; so when we talk about the drop hammers.

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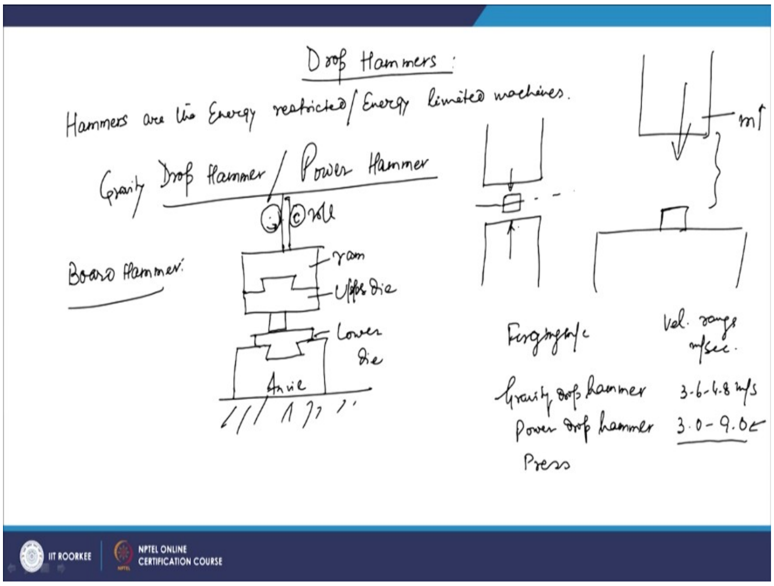

So, as we discussed that we have a ram (Refer Time: 02:49) and then you had it has to fall with it is weight. And, it has to fall from certain height and then it will come and then, it will do the impact force, it will be falling with impact on the specimen, which is here and it will transfer whatever energy it has it will transfer that energy due to that energy. So, this energy is utilized in deforming in this workpiece.

So, normally these hammers are known as the energy restricted machines, energy restricted or energy limited. So, because it is using its kinetic energy of this ram which is falling from the top. So, that is why it is known as energy restricted or energy limited machines. Now, in this you have drop or drop hammer or the power hammer. Now, in the case of drop hammer basically in both the cases they are dropping, but in the drop hammer gravity drop hammer.

So, is it is also known as gravity drop hammer. Now, in the case of gravity drop hammer, you know the ram falls of it is own weight. And, then that weight basically will be falling under the action of gravity, and that will be falling under the job and this way the material will get deformed. Whereas, you may have and certainly it will be further you know take a half and then further it is allowed to drop. So, that sequential blows take place in the case of these hammers normally, it is not that it will do the deformation process in one blow, it will be going on continuously.

So, actually it will be going on continuously and you it will be doing the deformation. So, normally the velocity also is there, when we talk about this you know hammers. They have a certain velocity with which they are basically falling over the machine.

Now, if you talk about this so, gravity drop hammers we have talked about now coming to power hammer. So, in those cases you accelerate with certain power source. So, and then that will give extra velocity and it will come and it will be doing that you know it will be doing that operation. So, that will be power hammer; now, this gravity drop hammer or the power drop hammer. So, this is basically your job is there on the anvil and anvil is attached to the foundation.

So, this foundation will be absorbing that energy impact energy and that is why in these cases your foundation must be strong enough. So, that because it will transmit that energy to the foundation, it will take some energy will be taken by this material and then that will be transmitted towards the foundation, so foundation need to be very very strong. In the case of the gravity drop hammer or the power drop hammer.

Now the difference between these two is also that, as you know that in the gravity drop hammer you have the top ram portion that will be falling over the work piece.

So, depending upon it is weight you know that it will have certain kinetic energy that energy it will be achieving. So, if you have to achieve. So, if your velocity is fixed. In that case and certainly when you are allowing it to free freely fall then you know that depending upon the height it will attain certain velocity from where it is released. So, depending upon the height from where it is released it will attain certain velocity.

So, that way you are going to get some energy that energy will be basically transferred to this material. So, depending upon this height how much it is? You will have the energy which is you know there into this ram and which will be transferred to the. So, that energy will be utilized for the deformation, but and if you have to have the larger value in that case you have nothing but you have only one option, that you increase the mass because velocity for that height will be you know fixed because of the height h. So, what you do is you make this heavier.

So, you have to increase it is m. So, that what we can only do, but once you increase the height it will be more and more bulky and then you have to have some power also while taking lifting it upward. So, what we do is that? Then, we have these machines like power hammer where the velocity is basically increased. So, you can increase the velocity ram being so that top die may be of smaller even with the same you know mass if you increase the velocity. In that case the energy which is you know achieved that will be larger. So, that will be for the power hammers.

Now, similarly you have also the presses now if you talk about. So, in the press as we discussed that when the in the press it will come and it will be pressing and squeezing continuous. So, if you talk about the speed ranges; so if you talk about the forging machine and if you talk about the velocity range in meter per second. So, it is seen that if it is a gravity drop hammer.

So, in that case the velocity range is 3.6 to 4.8 meter per second. So, that is the range of the velocity which we achieve and if you talk about the power drop hammer. Now, in these cases your velocity will be ranging from 3 to 9. So, you can see that you can achieve from smaller to the larger, you know this range is quite large and this high velocity you can achieve.

You have certainly some specialized you know machines and you have special methods, where you can achieve very large velocity ranges where we apply this forging pressure in very small area with large, you know velocity and they are used in HERF machines there it is going of the order of 6 to 24 meter per second. Now, if you talk about the mechanical presses.

So, if you talk in the case of press. Now, when we talk about the press so, before that we must know that you have a gravity drop and the power hammer. And, in those cases we use since we use the energy of the you know hammer that is ram which is coming down to deform the workpiece they are known as the you know energy restricted machines, and the power can be by any source you have you know the steam also is used as for increasing the powers then we normally call it as, the you know steam hammer we also call it as or are there are terms like you have board hammer.

So, they are you have the so, the ram will be there and then along the board the because of with the help of this you know pulleys or rolls. So, by the help of or use of the friction you take the board you know on the board you take the ram up and then further you allow the job to you know, allow this ram to fall down on the workpiece and do the job. So, that is your board hammer.

Now, so, in the board hammer. So, when we talk about this board hammer. So, you have upper die and the ram. So, you have you have an upper die suppose you have a die like this. And, this die will be attached with the ram and then so, this will be the ram. Now, in this case you have the; this is your die upper die and this is your ram. Now in this case they are raised by the friction rolls. So, griping the board. So, you will have a board and on this you have friction rolls.

So, on this they will be moving up and they will be so, this these roles will be moving in you know this direction. So, they will be taking this ram and the upper die together upward and then they will be coming and if suppose this being the job. So, you have a lower die and this basically will be attached to the lower you know die and that will be that will be at the bottom. So, this is your anvil and this is your lower die and this way this is your roll.

So, what happens since this board is used that is known as you know board hammer. Otherwise you know that this is the example of gravity drop hammer. So, while coming down, they will come and with this all it is weight it will be falling upon it. So, in this case this foundation is to be strong and these hammers are known as energy restricted machines, one of the other variety of this hammer is the you know counter blow hammer.

So, if you talk about these you know counter blow hammers. Now, we discussed that when in the case of these hammers they are applying this force, their energy is transferred that. So, the foundation needs to be stronger. Now, to have you know less requirement of the foundation strength, what we do if we do the hammering operation in between. So, so, what we do is the top as well as the bottom die both are allowed to move in opposite direction, and they will be coming and hitting the specimen at some position which is in between.

So, this way the force or the energy with which they are you know the energy, which is transferred to this workpiece know this die is moving in this direction and die is moving in this direction. So, there this amount of this energy which is transferred otherwise to the foundation and foundation needs to be stronger, that requirement becomes less in the case of these counter blow hammers.

Now so, depending upon you know what type of energy, what amount of you know energy, which is supplied that you can easily find out by finding the mass of this ram and the die its mass and with what velocity it is falling. So, that will be $\frac{1}{2}mv^2$. And, then if you are using the power drop hammer and if you are using the pressure and you know the area and also you know height.

So, that way you can also find that energy which is coming up and that way you can find the total energy, which you are able to achieve in the case of these hammer operations. Now, coming to the next type of equipment, that is your press.

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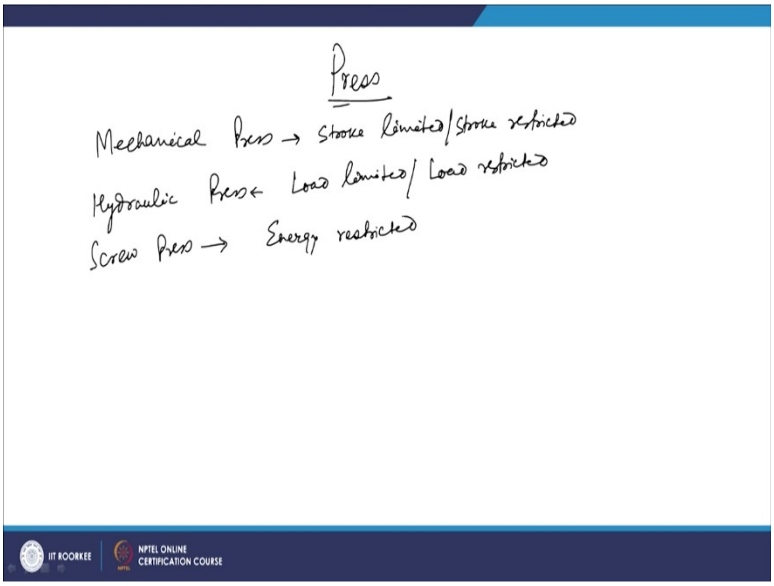

So, as we discussed that this presses also are of different type, you have mechanical press, you have hydraulic press and also you have screw press.

Now, as we discussed that the presses in the press you give a continuous squizzing type of action to deform the material. So, you apply this that load. So, so, hydraulic presses where you apply that load through the hydraulic load so, that is why they are known as the load limited or load restricted machines.

Now, again if you go to the mechanical type of press, now in this case you have the stroke. So, that stroke will be depending on, you have crank arrangement. And, depending upon the stroke position it will have you know the different way that you know load will be transmitted. So, depending upon the stroke position, that is why since that the deformation is limited upon the stroke positions. So, we call it as the stroke limited or stroke restituted stroke restricted machines. And, this hydraulic press will be load limited or load restricted machines. Similarly, you have the screw press.

Now in the screw press what happens you have a flywheel attached and what it does is normally it will be taking the extra energy absorbed, and that energy extra energy will be used for doing the deformation so basically this screw press is the energy restricted machine.

So, normally you have these are the 3 different types of the presses, which are used in the case of the presses and one being that load limited hydraulic press, mechanical press, is said to be the you know a stroke limited and the screw press is said to be the energy restricted.

So, now when we talked about the hammers now, we discussed that in the case of hammers, if we talk about the total energy which is supplied you know to the blow in a power drop hammer.

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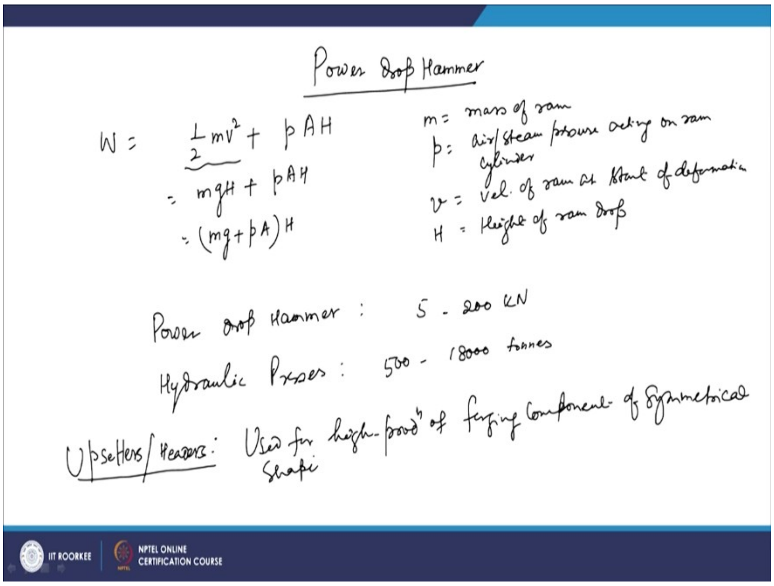

So, if you talk about a power drop hammer. So, in that case to the total energy supplied as we discussed that you have the mass of the ram, which is die also there and then that will be moving with certain velocity v.

So, you will have total energy (Refer Time: 19:48) so, that will be ½ mv^2 and then also you are you know using certain other source for increasing that you know energy. So, you have this p that is whichever pressure you are acting you are trying to act on the ram cylinder. So, that is your p and then you have the area over which this pressure will act and then you have the height of the ram drop.

So, based on that the total energy will be you know depending upon. So, you have m as the mass of ram. Similarly, you know you have p as the you know pressure or air or steam pressure acting on ram cylinder. So, during the down stroke (Refer Time: 20:54) that pressure which is acting on the ram that is your p, similarly v is the velocity of the ram at the start of deformation. So, at start of deformation then you have next remaining is H.

So, you will have this will be basically height of the ram drop. So, the ram is getting dropped from a certain height. And, it will be nothing but it will be *mg*. So, it will be falling over there. So, that will be nothing but the *mg*. So, it will be *mg* and then H and then you will have the *pAH*. So, that is your potential energy is that just says that, it is a kinetic energy and it is equivalent potential energy so, *mgH* and if you take $(mg+pA)H$. So, depending upon the pressure you apply, you can find its you know, you know total energy which is supplied on the workpiece.

Similarly, you can also calculate you know the total energy which is applied during the stroke of a press. And, that will be depending upon the moment of inertia of the flywheel and also the angular velocity. So, that that also has certain you know expressions by which you can calculate that energy which you apply. Now, talking about the you know machines configurations, when we talk about these forging hammers or the forging presses. So, normally these power drop hammers. So, they are they are ranging from 5 to 200 kN.

So, that is the range for these power you know drop hammers and here their size is from 5 to 200 kN. And, you know like a 13 kN something like in this power hammer. It can produce a forging road in the access of 600 tons. So, that is how the normally you calculate that what is the amount of energy, which is to be transferred so that they will give you certain idea about those things.

Similarly, when we talk about the presses so, the hydraulic presses when we talk about the hydraulic presses. Now, they are available in the ranges of suppose 500 to 18, 000 tons. So, even larger more than 50,000 tons hydraulic presses are also built and they are used the hummers normally are cheaper than the presses, because in the hammer basically a hammer you know we prefer these press machines, because here there is squeezing and continuous type of force being applied.

So, we prefer normally this presses normally for extrusion type of forging operation where you continuously squeeze and do the operations. So, there you go for this press machines, then you also have you know the forging machines that is known as upsetters or headers. So, among the other varieties of the machines you have upsetters or headers. Now, these are basically the kind of mechanical you know presses, which are horizontal acting and they will come and they will do that upsetting operations.

So, they will be decreasing it is length and increasing it is diameter. So, that way they do the job. So, in such cases you have these machines are known as the upsetters or headers and you have they are used for the high production so used for high production forging. So, mass production of forging component of symmetrical shape are basically you know produced using this upsetters or headers. And, normally you have rivets or you know gear blanks they are made using this process. So, that is the use of these upsetters or headers another which we discussed earlier.

So, we have discussed about different types of forging machines and different types of press machines. Then another classification of machines is HERF machines; that is high energy rate forming machines.

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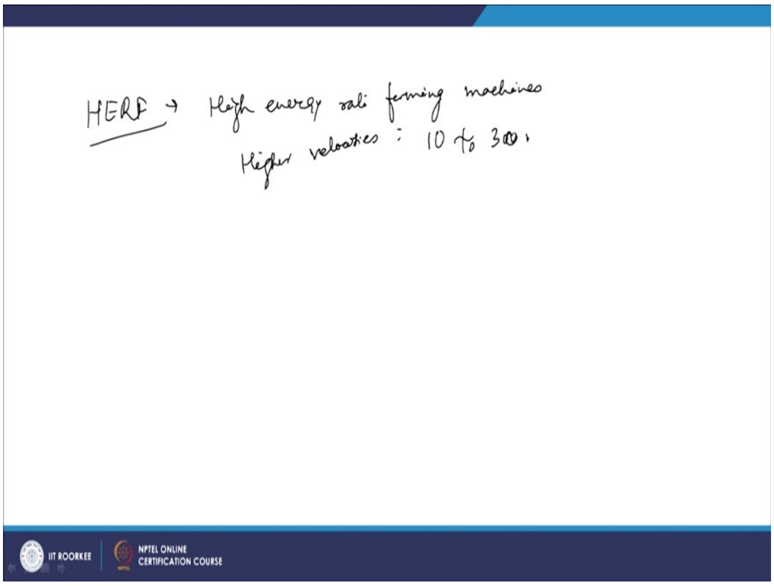

So, these HERF; HERF is high energy rate forming machines. Now, this is normally done in the case of incremental forging or such cases, where you have the velocity range quite high as compared to the normal velocity range, which is achieved in the case of either gravity drop hammer or the power drop hammer. So, in these cases the higher velocities are you know achieved and they are from 10 to 30 meter per second, about 10 to 30 meter per second of the velocity is achieved and, normally the processes that when you have to do deformation you know deformation in a localized position.

So, you will have you know sometimes example of orbital forging or incremental forging in that, the rate by which or velocity at which the forming takes place at the localization is quite fast and that way deformation takes place. So, such are the examples of so, that are done by these machines known as high energy rate forming machines. So, depending upon the application, you go for the type of machines. So, they are certainly the advanced kinds of machines.

So, normally in a nutshell we need to know the different kinds of machines, where they are to be used depending upon the type of application or depending upon the type of geometry, which are we are going to produce, you have different types of machines, you have the use of either you know hammers or you have use of presses, you have the use of counterblow hammers, you have the use of different types of presses like mechanical or you hydraulic or the screw presses, you may have the use of the counterblow hammers, whenever you have to reduce that vibration transmission to the you know foundation.

So, that way we have these different types of forging equipments which we must have the idea. So, that you whenever required you can design depending upon the load requirement or energy requirement you can design you can be quite versed with the specification of the machine, which has to be used for these typical applications.

Thank you very much.

Principles of Metal Forming Technology
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Lecture - 29
Introduction and Classification of rolling processes

Welcome to the lecture on Introduction and Classification of rolling processes. So, now, we will move towards the discussion about the rolling process you must have the idea about the rolling process which we discussed sometimes that you know in the rolling process, we use the rolls which are normally you know cylindrical in nature. So, they are very very hard; extremely hard rolls and these rolls are rotating in opposite direction and the work is fed in between the rolls.

Now when they are going to touch the roll at that time you know they are touching in a larger you know there thickness is more; but because of the friction basically and since they are moving in opposite direction. So, because of the friction which is there in between the work piece and the roll, they are forced in into the basically what happens that you have the rolls, you have normally this way of circular rolls.

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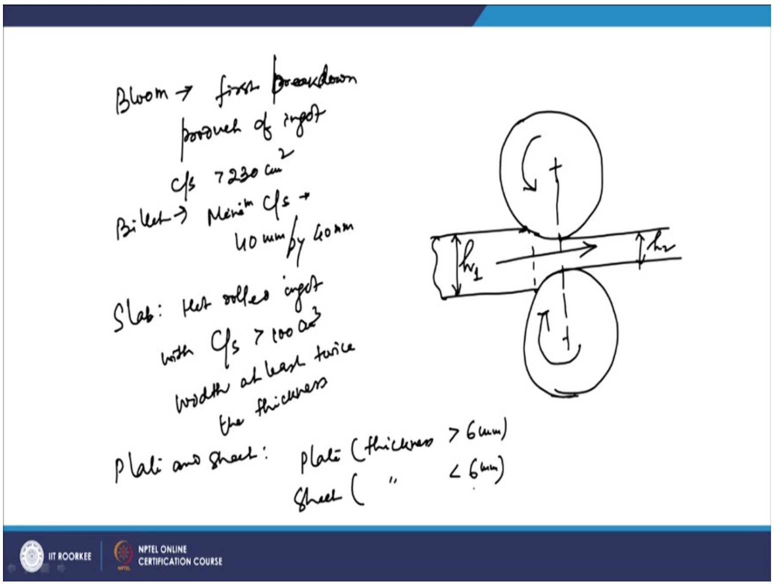

And what happens that these rolls; so, they will have this common center this is the center point and the. So, this will be rotating in the direction and this will be rotating in this direction.

So, this is basically in the opposite direction. This is moving in this direction; this is moving in this direction and the what piece basically fed from here and once it goes here also you we will have frictional force is acting at this place and because of this friction, the frictional force will be acting in that direction and this friction force basically will push this stock in to in between the rolls and ultimately the roll will come inside and then it will be going out.

So, while going out, it is thickness suppose here, it is this is h_1. So, this will be going as h_2. So, the height or thickness of the slab or this stock material is getting reduced and basically it is theatrically equal to the gap between these two rolls. So, this is roll gap.

So, this is normally a rolling process and these rolls are extremely, I mean hard they are made of hard materials and this is used for basically you know converting these ingots or to make this slab or slab, further to make billets or then and then finally, we are making the sheets also from these rolls.

So, based upon the temperature, we classify this rolling process as hot rolling or cold rolling depending upon the temperature at which we do this processing operation. So, if it is done at a temperature higher than its recrystallize and temperature, it will be hot rolling; otherwise it will be the cold rolling.

Now, in the industries you have if you talk about the large industries; even in earlier days or even now. So, now, they use to make the ingots, now these ingots basically are further processed and you make the different shapes of the products you know different cross section, difference are based upon the different sizes; you define those you know products.

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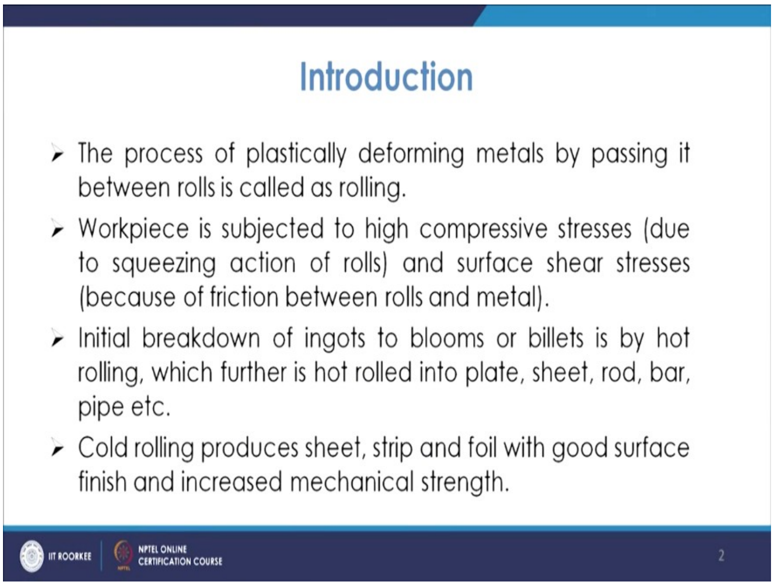

So, what we see is that the roll process of plastically deforming metals by passing it in between the rolls. So, that is known as rolling. Then Work piece is subjected to very high compressive stress due to the squeezing action of the rolls.

So, rolls are basically squeezing on the metal and because of that and the surface shear stress, there will be you know shear stress developed and because of the friction between the roll and the metal and that basically is you know useful or responsible for doing the plastic deformation you know in the material.

So, initial breakdown of ingots; ingot is basically it will be breakdown to the blooms or billets. So, in that basically the degree of reduction is quiet high. You have to deform you know appreciably. So, in that that is done at the high temperature and that is basically under the you know umbrella of hot rolling processes. Because there it has to deform you know to large extent.

And then, further when you are going to define I mean to roll to get the final shape and size or so. And when the dimensional accuracy is required finish is required, in those cases we go for the cold rolling. So, and when we can go for the hot rolling for so; first of all, you have to convert to the simpler shapes like blooms or billets. And then, you can go for you know making the plates sheets rods or bars or pipes.

So, they can be you know hot rolled and you have many a times you make many products by the rolling process by making the you know proper, you know type of roll; in that roll you have that type of rude geometry is there. So, based on that that kind of cross sectional product can be produced.

So, you have that is hot rolling and cold rolling will be producing the sheet strip and foil with good surface finish and you have the increased mechanical strengths. So, as you know that whenever we talk about the cold forming methods like cold rolling or any cold you know forming method, in those cases the you know it is done to finally, shape the material to very smaller, thicknesses.

And in that case basically because of cold forming the strength is increased. Because of the strain is hardening.

So, and also the finish is better because when you are at these lower temperature side; the chances of scale formation will be smaller, the chances of surface oxidation will be smaller and that way we go for these cold rolling of the materials. Now based upon you know we try to define these products. So, you have like Bloom. So, the bloom is basically it is the when we first break down this ingot. So, that is known as blooms.

So, this is the first break down product; break down product of ingot. So, normally its cross sectional area is greater than 230 cm^2 and also its width will be equal to its thickness. So, and its cross sectional area will be you know more than 230 cm^2.

So, that is what normally we specify the bloom as. Then, further when we do that reduction of dimension of these bloom, then we get the billet. So, we get this billet and we have the minimum cross section area of these billets and that is 40 mm by 40 mm.

So, this way you have the smaller products, that is known as a billet. Similarly, you have the slab and slab is basically the, you know hot roll ingot with cross sectional area greater than 100 cm^2 and width is at least twice the thickness. So, this way these bloom billets and slabs they are you know defined.

And then comes the sheet metals you know which are the very fine, you know less thick products and they are into the you know normally you will go for the finishing mill products and they are you have you know plate and sheet, and so there you will get this plate and sheet. Now plate has the thickness more than 6 mm and normally if it is less than 6 mm then you know it is sheet.

So, plate has thickness more than 6 mm and if it is sheet, it has thickness less than 6 mm. So, this is what the classification of the plate and sheet is and when we talk about the strip. So, normally the strip is also the roll product and its width is not more than 600 mm.

So, so that is what sheet is of you know larger width. So, that is known as the strip. So, the strip will be basically referred as the product with not greater than more 600 mm width; whereas, the sheet will be having the larger width then that.

So, this way you have basically the different types of products. There is also certain you know process known as powder rolling where the powder is in between the rolls and then, then they are basically you know they are in between these rolls and compacted into a green strip. And then, that is further sintered and then that way you make these powder rolling of the materials.

So, this way you classify the different types of you know products which are made from these rolling you know processes. Now thing is that where the shop, where there is these are these machines are there, where the rolling process is carried out, they are known as the rolling mills. So, you have the set ups where the rolling process is being carried out and it will be consisting of the rolls.

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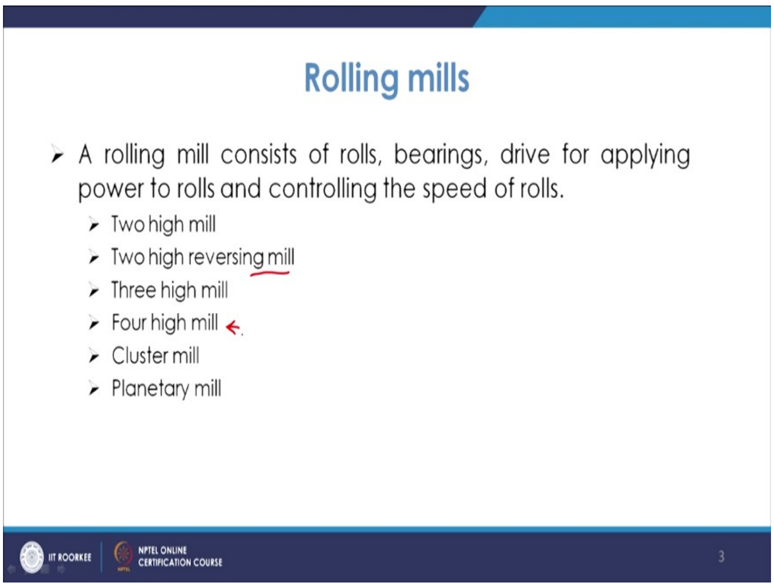

So, the rolling mill will become consisting of rolls, bearings, drives for applying power to rolls and controlling the speed of rolls. So, that is how a set up will be there. As we know that in the rolling mill you have you must have the means to rotate these rolls.

So, for that you have the, you know drive for the applying power to the rolls, then you may have to control this speed of these rolls. So, sometimes you have to control because at what speed the, you know material is coming or stock is coming from one side. So, based on that you have to control the speed of the rolls, you must have the controlling mechanism for the speed. So, this is what a rolling mill and that place where these machines are there; where the rolling processes is going on continuously. So, that is known as the, you know Rolling mills. Now coming to the different types of mills; as we know that in the case of rolling as we have seen if you take the simplest type of rolling mill.

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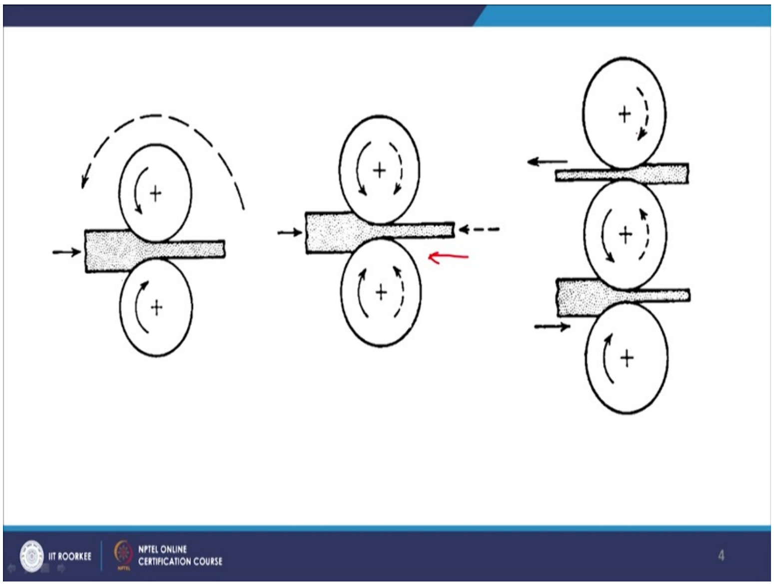

Now you see here.

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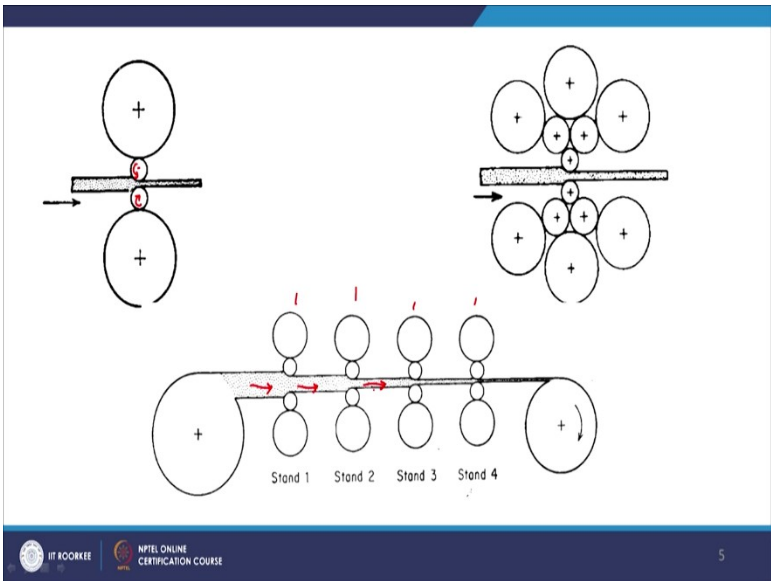

So, now in this case in now here. So, in this case these are the two rolls and they are as we see, they are rotating in opposite direction. This is rotating in the anti clock wise and this is rotating the clock wise direction. So, and then you as you see we have seen that your stock which is there.

So, it will be coming from this side and ultimately to it will be coming out. So, they are known as the two high rolling mill because you have 2 rolls which are used for you know deforming the work piece. So, that is why it is known as two high roll mill. Then there is two high reversing mill. Now the thing is that in this case there is only entry from these sides.

So, the material which is rolled in this direction; now it has to be further fed into you know at this place only. There is no feeding from this side. Because the speed you know is of the direction of the you know movement of this rolls is fixed.

So, thisis the moving in this direction. So, we have only to feed from this direction. So, it is known as two high whereass if we if we try to save the time. Because you have to if you have to further change it’s you know do the reduction you have to bring this side and then do it..

Now in this case what we do is you have two this in this you can see that you have feeding from both the sides. So, you feed from this side its reduction is done; you can adjust its gap between the rolls and then, further you can give the entry you know from this side.

So,. So, this way your in one go itself. So, from here it will go, then you have to reverse its you know direction of rotation. So, it will you start rotating now in this direction and this will be rotating in this direction. So, and the metal will be fed in this direction. So, depending upon the gap in between the roll which is adjusted after one stage, you will have the further you know reduction of its dimension and it will be reduced to the smaller size. So, this is known as two high reversing mill and that is what it is? This is two high reversing mill.

Then coming to the three high mill. So, now in the case of three high mill as we see that this in this there are rolling process going on at these two places and you are using the three rolls and if you look at this. So, if you see that this is rotating in one direction and this will be opposite to it and this will be opposite to this.

So, ultimately this is and this; so, this is moving like this and this is moving like this. So, they have similar you know movement direction. They are rotating in the same direction, but whereas, this is opposite to both because the metal will be coming suppose from this side and it will go here and from here it will move it will go here.

So, basically we have not to reverse the, you know direction of the rotation of the rolls. But you can see that they are fed from one side and they are fed from other side. So, this way the you can have two places the there is movement of the you know. So, this slabs or the billets which is getting reduce; this is known as three high you know rolling mill, where the three rolls are used for the deformation.

Then you have this is known as the Four high rolling mill. So, this is here this is four high rolling mill. Now if you look at the four high rolling mill again it has 4 rolls which are used for the deformation.

Now in this case what happens that if suppose you have these two rolls, they are the rolls which do the deformation; whereas, these are the rolls which are the backup rolls. They are basically giving the support; they are they give the support to rigidity to these rolls.

So,. So, that is why since you are using these 4 rolls, we call it as the four high rolling mills and this way you have because this will be certainly moving in in these direction and this will be moving in this directions.

So, that way, they will be taking the, you know stock from this side and then, then the finish product will be going in the you know other side. Another example of the rolling mill is the cluster mills. So, it is a cluster arrangement and in this case it is being supported by all these rolls and then, this you know you know this type of reduction takes place in such mills known as cluster mill.

So, what we see suppose in this cluster mill or if you look at the 4 high rolling mills, here actually the size of the roll which is doing the deformation is small and basically the smaller rolls, the power requirement will be you know smaller. But then, it is it has to be given it will have lesser strength; it will be given that strength and rigidity support by these you know backup rolls and they are supported by these large die metal backup rolls in such cases.

Coming to another kind of you know, now what this is a case of you know the here you see that this is a you know continuous continuously you see that this ingot is coming when it is of larger dimension ingot billet, or slab and they are ultimately you see at its converted into strip.

So, this is basically a continuous type of mill. Now see that you have many mills; this is one mill, this is second mill; this and you have further mill. So, you have continuous you know mill and you ultimately make the strip. So, this is basically a strip rolling mill and basically otherwise you have to take from here to next mill and then, further arrange further you know the next one or so.

But you can have such kind of arrangement which is known as the continuous you know mill; where if you look at this dimension is getting changed every after passing every you know mill and you have ultimately you see that you get these strip being produced of a very small thickness.

So, this is known as the you know you know continuous mill for finding a strip and here you must be having certain concept about it. Because you must be knowing that when it goes basically its speed will be larger than the speed of the rolls.

So, then this speed at which it will be moving this will be the incoming velocity, while entry. So, what happens in the rolls we did not discuss that way that when we talk about the rolls.

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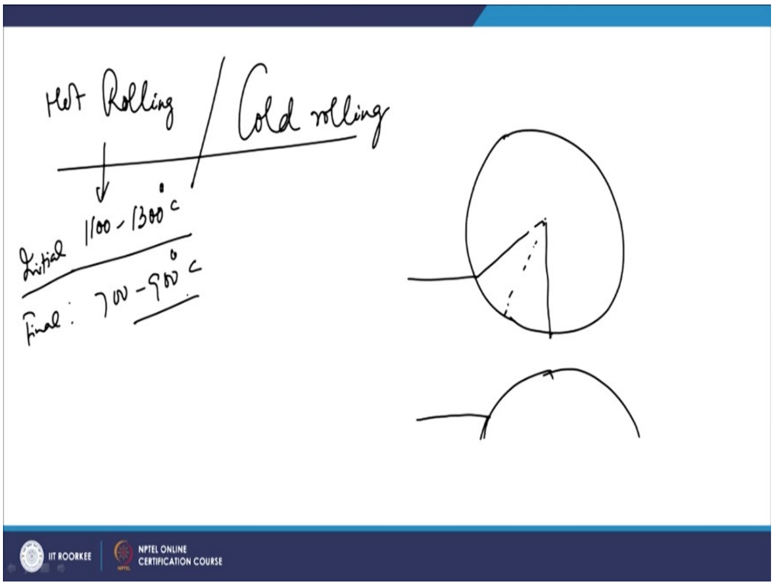

Now, in the case of a rolls when we see now in such cases suppose now, this is how the roll is. Now what happens that now if this is a center.

So, this is yours this angle which is there that angle is known as the angle of byte. The thing is that when it is coming inside; now it because of friction, it will be going you know towards this side and then while it is leaving and because of the you know you know constancy of volume, what happens that the velocity with which it is moving, it will be you know smaller I mean larger because if the width is you know same.

So, the cross sectional area and the velocity that will be your volumetric flow rate. So, the since the thickness is reducing. So, and width is the same.

So, in that case your velocity has to be maintained. So, velocity will be more while exit; while coming, it will have lesser velocity because the frictional force is acting. It means at one point you will have a plain at which the velocity will be same as the velocity of the roll and that is why that plain is known as the Neutral plain. Now this is known as the angle of byte.

So, basically this way what we mean to say that the velocity will be changing. Now in the case of this continuous rolling mill where we directly find get the strip, in that case you will have to have you know the, you know calculation to see that that what velocity is coming here and with what velocity it is going out. Now the velocity with which it is going out will be the entry point for this roll and then that velocity will be different here.

So, this way you will have to have that calculation and based on that you will have the rolling speed to be maintained and you can get these you know type of you know strips, in one go you are getting from this roll you know shape to strip production.

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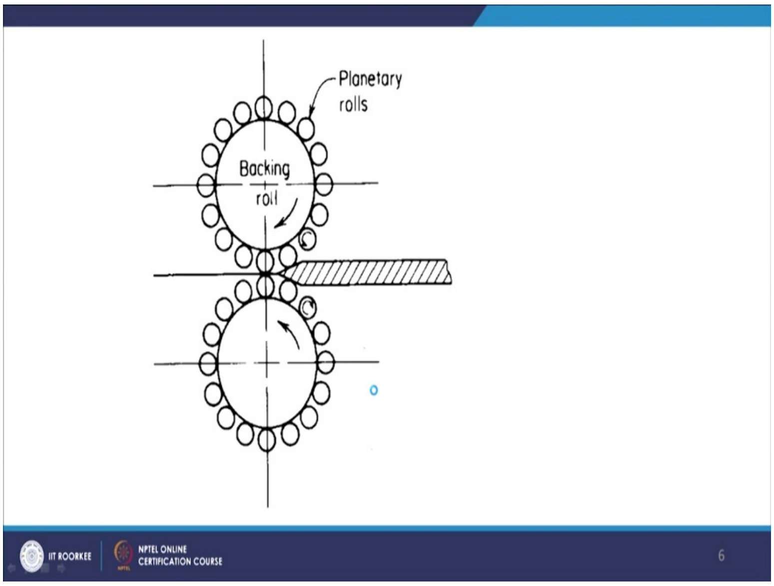

Then you have another kind of you know rolling, where we use these planetary rolls and this is known as the planetary rolling mill. So, this is planetary mill.

Now in this case as you see these rolls you see that this is the backing roll and these are the rolls which are doing the deformation. And basically as you see that there movement direction is also shown and this, what happens that, this is directly converted into a very thin sheet or strip.

Now what happens that slowly this will be coming up then after that another will come. So, they. So, this way you will have it is you know at this point the clearance between these 2 rolls is very very small. So, based on that basically you can have the production of very thin you know sheets possible by the use of these planetary rolling mills.

Because this like a planet it is these rolls are there on the backing rolls; that is why it is name is planetary roll. And this way, you can get these you know formation of very thin you know product in the rolling operation. So, basically it is also something like you know it is like a forging process because it will come, it will forge and then it will move.

So, that way now also it is like forging rather than rolling process you can say in such cases. Now as we discussed that depending upon the temperature, we try to defined this rolling process also as the either hot rolling or the cold rolling. So, now when to use the hot rolling and when to use the cold rolling now, as we know that in case of hot rolling, we will adopt these hot rolling normally you have the roughing mills are there.

So, when your degree of reduction has to be quiet high initially you ingot is to be converted to slab and the you know surface finish is not the criteria; basic criteria is to reduce the you know cross section, to reduce the thickness in that case you go for the hot rolling.

And normally what we do is normally we use that 2 high reversing mills for these hot rolling and the you know roll diameter is from 0.6 to 1.4 meter of the you know that roll and you have you know reversing you know you know planning mills. So, in those cases normally the purpose is that as we discussed that in the case of hot rolling, since we do at higher temperature.

So, in these cases the surface roughness will be there because there will be chances of surface oxidation; there will be chances of certain you know because we have discussed that in these process what happens that when we go for subsequent you know rolling, in that case because the scales are formed on the surface they may be trapped in between the rolls surface and so, there may be the trapment of these oxides in between the metal surface and the rolls.

So that, that may be you know in between. So, there may be indentation on the you know surfaces of the metal and so where we go for hot rolling, whenever you need to. So, that is why basically roughing operation basically. And the cold rolling basically is normally you have the finishing operation.

So, you have you know high speed four high tandem mills are normally used. So, with 3 to 5 strands are used for the cold rolling of a steel, aluminium or like copper alloys and in the plant you will see that these thinner sheets which are used as you know in place of the asbestos sheets you can see that these corrugated sheets are there.

Now they are made by these you know cold rolling mills; they are not made by the hot rolling. Whereas, when we are try to deform, where the degree of the deformation is larger; in those cases you go for the hot rolling mill and you can have different type of rolling mill arrangement which can do these hot rolling. Normally, you have two high and two high reversing which is used for the you know rolling mills.

And once you are further converting the ingot; ingot to bloom or then to slab or the billet and all that so, that is done in the succession. The temperature which is done in the case of hot rolling for the steel is normally 1100 to 1300 °C. So, as you know that at this high temperature, you will have more chances of formation of the scales.

So, we you have to see that where to finish the last finishing stand; what should be the temperature? Because you cannot do the finishing at a larger temperature range; larger temperature side because that will lead to the growth of the grains inside the structures.

So, normally we try to see that the last finishing stand, the temperature should be from 700 to 900 should be there. So, initial this will be initial temperature and in the final you know you know final finishing stages, you have 700 to 900 degree centigrade is a kept. So, that you have fine you know equi-axed ferrite (Refer Time: 29:59) grains which are to be obtained.

Now, in the you know degree of reduction will be you know achievable more in the case of hot rolling; whereas, in the case of cold rolling the degree of reduction is normally less may be from 50 to 90 percent is reported in the case of the cold rolling.

And also you have to see that you know you have to see that how much should be the reduction in every stage because during the cold rolling your strain hardening you know takes place.

So, how much you should do the cold rolling that is required to be you know analyzed; otherwise that may lead to basically the, you know brittleness of the sheet and that may you know develop the anisotropy in the material and that may lead to the failure of the material.

So, so that way you have to balance, you have to see that how much degree of reduction is required; you know in the last pass to permit the better control of flatness, gause or the surface finish in the case of cold rolling.

So, this is about introduction about the hot rolling and the cold rolling operations. We will discuss about the analysis part in our next subsequent lectures.

Thank you very much.

Principles of Metal Forming Technology

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Lecture – 30
Analysis of rolling load calculations

Welcome to the lecture on Analysis of rolling load calculations. So, we have discussed about the different types of rolling processes and, now will have the analysis about the load calculations in the rolling. So, let us first see that what happens in the case of rolling.

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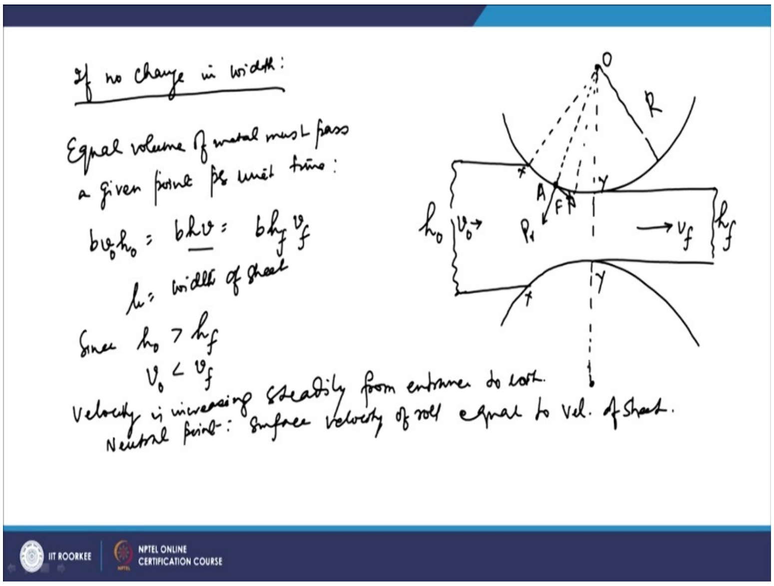

So, as we know that in the case of rolling you have two rolls and so, and then you have this as the centerline. So, centerline connecting the rolls centre; now, you have a slab which is coming and into the contact of the rolls and then they finally, you will have the exit of slab of this thickness. Now, what we see that we must know the deformation zone geometry in the case of rolling, now this is the these are the centre of these rolls.

Now, let the centre of the roll be ‘O’ and you have this radius as ‘R’. So, this is ‘R’ is the roll radius of both of them. Now, the thing is that when we try to calculate the rolling load. So, it will be basically nothing, but the pressure divided by the area of contact and that will be nothing, but these length of contact projected area and then that multiplied by the width. So, width we assume that it does not change so, *b L P*.

Now, we will find it anyway before that, now this angle which is formed from at the centre this angle is known as the angle of byte in the case of rolling. So, suppose this X-X is that section where, it is you know in touch with the roll this is your Y-Y from where it is exiting the path and, it is entering with the velocity v_0 and the initial height is h_0.

So, with h_0 height and with v_0 velocity, this you know slab or billet is moving in between into the roll. Now, it will be going inside these in between the rolls and, then ultimately it will be exiting and with exit while exit if you assume that h_f is the final height of the slab. And then it is going at a velocity of v_f that is your final velocity, this is v_0 is the initial velocity before it enters into the roll and this v_f is the final velocity of the roll.

So, what we see that if we are assuming; so, if no change in width results. So, in that case you know you are compressing the you know slab vertically and, then it is elongating in the axial direction and, what happens that there must be equal you know volume of metal so, equal volume of metal must pass given point per unit time.

So, this condition will tell you that $b v_0 h_0 = bhv = b v_f h_f$. So, this will be at any particular instant inside the zone at any point you have instantaneous value of 'h' and instantaneous value of 'v' and this should be equal so, 'b' is the width of the sheet.

And you know as we know that 'v' value will be in between the v_0 and v_f. Now, the thing is that what we see that when the sheet is trying to enter into the rolls, now what we see that if you try to look at now, as the h_0 is more than so, since h_0 will be so, if you are compressing, it is more than you know h_f.

So, v_0 will be you know less than v_f. So, v_f is basically increasing it means that, when you are going into the sheet is going out of the rolls, it has a higher velocity. Now, the thing is that this velocity which is increasing it must steadily increase from the entrance to the exit and at one point along this (Refer Time: 06:22) surface of the roll what will happen that this velocity will be basically equal so, the velocity with which it is moving it will be similar to the velocity of the roll and, that point is known as the neutral point or neutral you know we call it as the no slip point.

So, velocity is basically increasing steadily from entrance to exit. Now, it means that at one point, once it moves at that point this velocity you know velocity of this you know velocity which is there at any point, it is same as the velocity of the surface at the surface of the roll. So, sheet velocity is same as the roll velocity. So, so that point is known as neutral point.

So, this point where the sheet so, at this point you have a surface velocity of roll equal to velocity of sheet and, this point is indicated somewhere maybe at some point on that that is known as 'N' that is neutral point. Now, we will see that how this you know how you do the forces you know a calculation of the forces, which are acting at different points on this in that deformation zone.

So, what we see is that now you have two forces which are acting on the metal now, suppose this is the point A, now what happens that you have a radial force that is acting and this force is basically P_r. And similarly you have a tangential force also that is acting and this force is the 'f' so, that is basically you know frictional force which is acting.

Now, if you look at and analyze the forces which are acting, when it is this sheet has contacted these rolls at that point the frictional force basically is acting towards this neutral point and, when it leaves the neutral point region basically it is a plane. So, so till the neutral point this frictional force has the direction towards the neutral point.

So, basically this sheet is allowed to go in between these rolls, because of this value of these you know the frictional force. So, frictional forces is basically assisting this sheet to go into there, if no frictional force, it will not be going inside the in between the rolls. So, friction is required and if this no friction, if will not be you not going inside or in between the rolls.

Now, when it is going from the neutral point ahead, now from there the direction of this frictional force is changing and, it will be towards the neutral points so, basically the direction of these neutral, you know frictional force will be towards the neutral point and, this is reversing once it goes from the neutral point towards the delivery you know region.

Now, when we talk about the component of this P_r so, if you talk about this its component in the vertical direction that is basically the rolling load so, it is the load with which it is basically you know pressing against the metal. So, that is how you calculate this rolling load. And it is also that load by which because it will try to separate these two rolls together so, it is also known as the roll separating forces.

Because this is the horizontal component and that will try to so, that is why your rolls are basically you know rigid and, they are having support. So, but they have to have enough support because, otherwise they will be you know instant they will be pushed apart. So, that is why it is also known as the roll separating forces. Now, what we have you know a point that is specific roll pressure and, that is basically the roll pressure divided by the contact area and, we have to find this contact area between the between the rolls.

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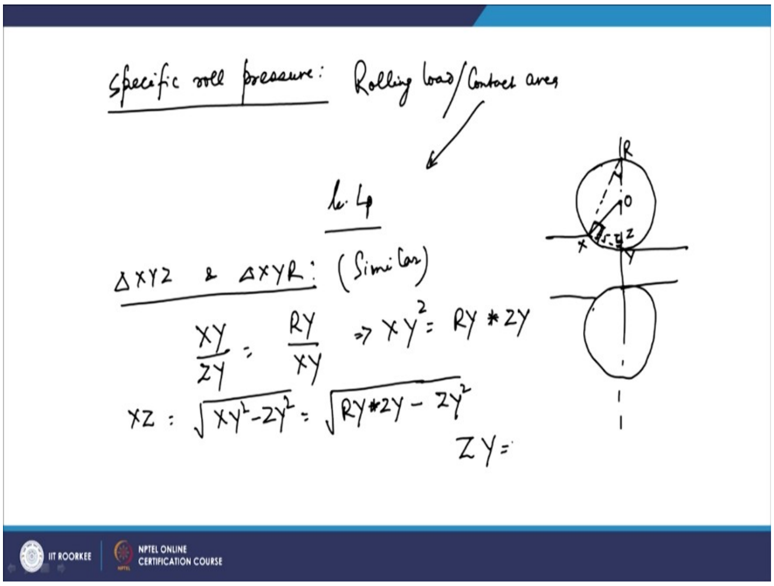

So, if you try to find the you know so, if you take the specific roll pressure. Now, what is this specific roll pressure? This roll pressure is rolling load divided by contact area. So, that is now what is the contact area? So, you have 'b' that is width which is there which is not changing. So, 'b' is there and then that will be multiplied with the projected arc of length so, arc of contact that is L_p.

So, this contact area will be nothing, but $b\ L_p$. So, that 'b' we know and how to calculate this L_p, now for finding that L_p, you can further see the previous figure and, in that figure you can see that if you look at this figure. Now, in this figure you can see that you have you just you know take from here; a line so, so that can further be seen maybe you can draw.

So, you have two rolls and this is the you know central line of the rolls and, you have this is coming and, then it is going like this. Now, in this case this is your O and if you have this as this one is X point and this is also X. And this way you have this point as Z. And this is the point which is known as Y so, this is we have already seen Y.

And this point is you know this was A basically so, we have nothing to do with that. Now, what we see that you if you take these two so, this is you know if you take this as R. Now, what we see that we are going to have the two triangles XYZ and another is XYR so, triangle XYZ and triangle XYR.

So, if you take these to you know triangles, what we see is that in XYZ this is the right angle and since being this is the diameter this is a right triangle. So, this angle is equal to this angle, similarly you know you have this angle as the common one and so, so this angle triangle YXZ will be same as triangle XRY. So, that way what we see that by using the AAA method all the angles are respective angles are equal so, the two triangles XYZ and the XYR they are similar.

So, what we can get if you take the XY which is you know XY which is in front of this you know right angle triangle. So, XY and XY is the in front of this angle and, this will be divided by this is equal to this angle so, this will be ZY so, XY will be divided by ZY. Now, again it will be you can take this RY, now RY is the in front of these you know 90° angle, angle RXY and this will be same as this angle is 90° so, in front of that you have XY side so, it will be XY.

So, you can from here you can write, that $XY^2 = RY \times ZY$. Now, what you can get XZ which is there. So, XZ will be nothing, but $XY^2 - ZY^2$. So, it will be $XY^2 - ZY^2$, now XY^2 is $RY \times ZY$ so, it will be $RY \times ZY - ZY^2$. So, what we get it will be ZY will be common, $RY - ZY$.

Now, what is ZY? So, if you take ZY, ZY is nothing but so, if you take this is your Z. So, this Z this is your Y. So, ZY is nothing but this is your h_0 and this is your h_f so, it is $(h_0 - ¿\ h_f)/2$. So, this is the total reduction this side also. So, so basically your ZY is $(h_0 - ¿\ h_f)/2$. So, now the thing is that we can have the expression for XZ so, you can write XZ.

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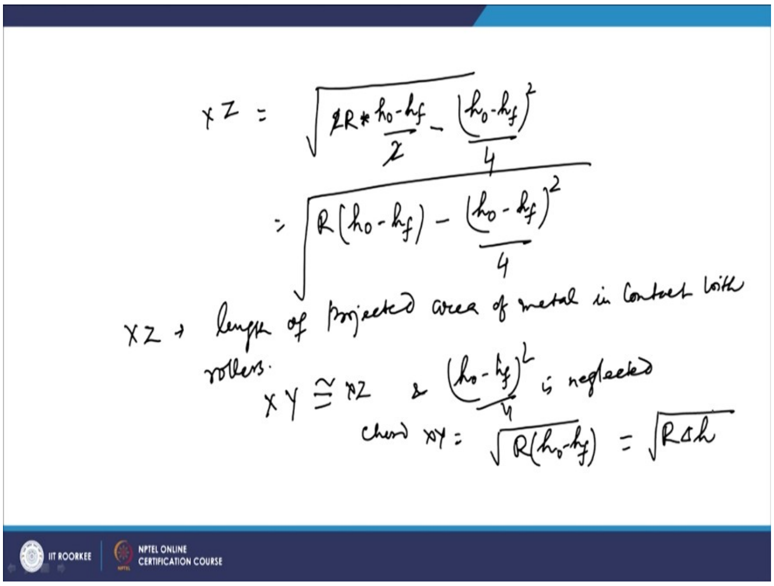

So, XZ can be calculated and exactly you have seen that $\sqrt{RY \times ZY - ZY^2}$. So, you have under root ZY is basically $(h_0 - h_f)/2$ and you have RY. So, you have if you look at this expression this is so, this was we had taken this R and this is Y this is nothing but 2R so, so RY is 2R. And then you have this ZY is $(h_0 - h_f)/2$.

So, this is your first term minus and then you have ZY^2. So, this is as $\left(\frac{h_0 - h_f}{2}\right)^2$. So, it will be $\frac{(h_0 - h_f)^2}{4}$ so, that comes out to be so, this 2 and 2 will cancel so you will have $\sqrt{R(h_0 - h_f) - \frac{(h_0 - h_f)^2}{4}}$. So, that is what you get the expression for the XZ.

Now, this XZ is basically known as the length of the projected area of the metal. So, XZ is length of projected area of metal in contact with rollers. So, once you know that then that basically will be multiplied with the 'b' and that will be give you total area. So, that will be you know multiplied again with specific roll pressure.

So, again what we see in this case we get this XY. So, if you look at this XY basically this XY will be normally equivalent to you know XZ because, this $\frac{h_0 - h_f}{2}$ is also and also that

$\frac{h_0-h_f}{2}$ is very very small and so, as XY so this is your this is your XY, XY is basically will be equal similar to the XZ so, that is why we write that XY is will be somewhat similar to XZ. And also this term $\frac{(h¿¿0-h_f)^2}{4}$¿, this is basically neglected as compare to because this is small and further it is square will be further is small.

So, $\frac{(h¿¿0-h_f)^2}{4}$¿ is neglected. So, we can find this chord XY, it will be basically $\sqrt{R(h_0-h_f)}$. So, so that is why this cord XY can be approximately said to be $R\Delta h$. So, so we get this $R\Delta h$ and, we get this specific roll pressures.

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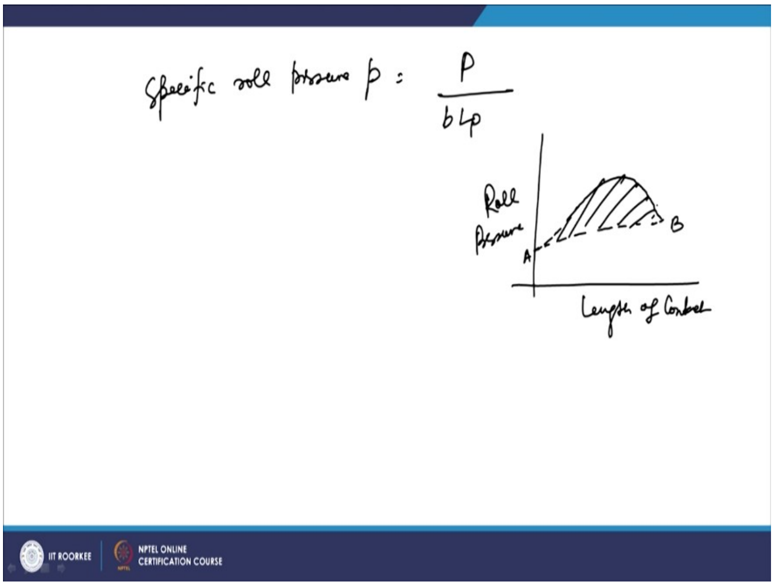

Once we get that we can get the specific roll pressure ‘p’ so, specific roll pressure ‘p’ it will be the rolling load divided by b times L_p. Now, if you try to find the distribution of this roll pressure along the you know arc of contact, then what is seen that the pressure will rise to a maximum, you know it will be maximum and the neutral and then it falls.

So, if the roll pressure is calculated and that is your length of contact. So, so this is how your actually this is for deformation and this is for overcoming this frictional forces. So, this is your A and this is B and this is the total you know forming load, which has can you calculated which is under the whole curve, but this curve which is shaded that will be for overcoming the frictional force, which is there and then this is for the deformation. So, that is how you calculate the total rolling load.

Now, further what we see that we have seen that when the roll is going inside, in that case because of this frictional force which is towards the neutral point the sheet is basically allowed to go in between the rolls. So, the there are two forces you have one is the force, which is the component of 'f', if you have the 'f' it is component 1 is there 1 is so, if you try to analyze the figure. So, if you have this as the roll.

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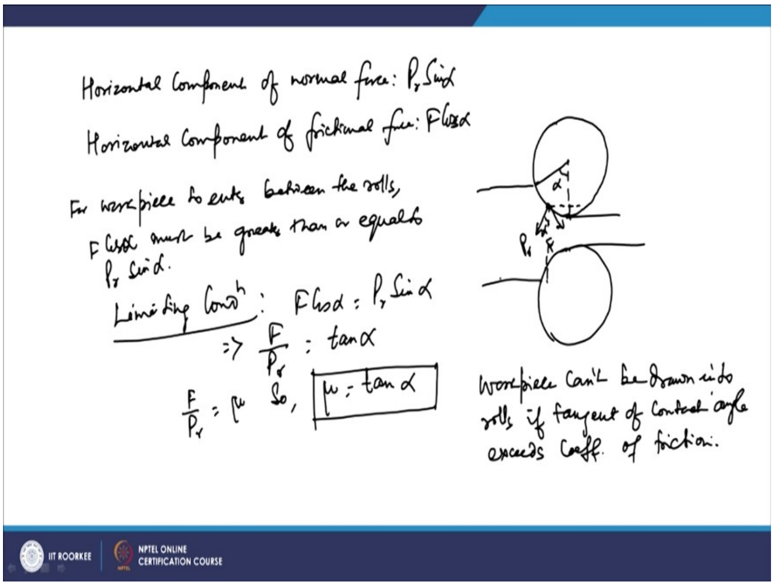

So, let us take next one you have the roll and at this point so, suppose this is how it goes and then finally, it will move from here. So, let us assume that it goes like this, now in this case what we see is that at this point we had seen that this is your P_r and this is how your 'f' goes.

Now, the thing is that this P_r its horizontal component so, this is your angle now the thing is that you have horizontal component of this normal force, now you can refer to this earlier picture here.

So, this is your if you take so, this is being the angle alpha, now angle α total angle is known as the angle of byte. Now this it is P_r and it is horizontal component if you try to find, now its horizontal component will be opposing the entry of the sheet in between the rolls and the so, you have this as R. So, this will be $P_r \cos\alpha$ you look at that way.

So, so your this angle so, is being this angle as α this will be also α. So, this horizontal component will be P_r. So, $P_r \cos\alpha$ will be the vertical component and $P_r \sin\alpha$ will be the horizontal component. So, that will be your $P_r \sin\alpha$. Similarly you have 'f' acting as that and this being α so, your $f\cos\alpha$ will be it is so, this angle being α its 'f' components will be $f \cos\alpha$ here.

So, you have the horizontal component of the frictional force that will be $f\cos\alpha$, now the thing is that $f\cos\alpha$ you know must be greater than or equal to $P_r \sin\alpha$. So, that the sheet (Refer Time: 26:43) enters into the rolls. So, the for work piece to enter between the rolls, now what we should be happen that this $P_r \cos\alpha$ should be less than or $f\cos\alpha$ must be greater than or equal to $P_r \sin\alpha$.

So, so that is why the limiting condition is that the should be equal that $f\cos\alpha = P_r \sin\alpha$. So, what we get is $\frac{f}{P_r} = \tan\alpha$.

Now, what we can see that $\frac{f}{P_r}$ basically that will be nothing, but μ, so $\frac{f}{P_r} = \mu$, so, from here you can get $\mu = \tan\alpha$, so in fact, what can be what we can conclude from here, that the work piece cannot be drawn into the rolls if the tangent of the contact angle is you know exceeding the coefficient of friction. So, in that case the work piece cannot be drawn into the rolls.

So, work piece cannot be drawn into rolls, if tangent of contact angle exceeds coefficient of friction. So, this is the you know condition for so, limiting value and based on that what should be the maximum you know you know how it has to enter in between that. So, that can be you know so, what should be the coefficient of friction because of which it has to must enter into it.

So, that you know condition can be derived by this formula, that further you can have the expression for you know further this μ.

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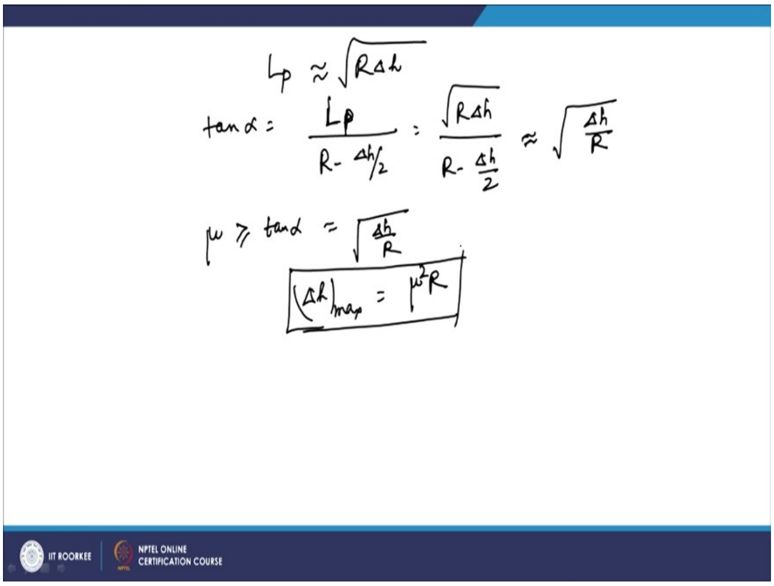

So, what we see that we have seen that L_p was taken out to be $L_p = \sqrt{R\Delta h}$. Now, the thing is that Δh is taken as the draft which is there in the rolling. Now, what we see that from the figure if you if you draw the inferences from the figure, now if you look at the $\tan\alpha$.

So, it will be nothing but this by this. So, it will be XZ/OZ in that case no so, if you try to find this $\tan\alpha$. So, $\tan\alpha$ will be the horizontal component that is L_p and divided by the vertical component and so, $\tan\alpha = \frac{L_p}{R - \Delta h/2}$ so, that is what $\tan\alpha$ is, L_p we have calculated $\sqrt{R\Delta h}$ divided by $R - \Delta h/2$.

So, you can write it as equal to somewhat $\sqrt{\frac{\Delta h}{R}}$, if the $\Delta h/2$ is neglected in that case you can take this $\Delta h/2$. Now, the thing is that depending upon the limiting condition for this coefficient of friction what we have achieved, we can write that μ has to be more than equal to $\tan\alpha$ and $\tan\alpha$ is nothing, but equal to $\tan\alpha = \sqrt{\frac{\Delta h}{R}}$.

So, $\frac{\Delta h}{R}$ maximum so, Δh maximum for that it is to be maximum value can be $\mu^2 R$. So, what could be the maximum you know redaction Δh maximum that can be found out using this formula $\mu^2 R$ so, based on that if you have a problem, where you have been given the radius of the rolls and the coefficient of friction the maximum reduction which is possible can be found out using this formula.

So, so that is how you can calculate. So, you can further do another analysis for the rolling load calculations and, when we talk about the effect of frictions and you have further taking the friction hill generated into account you can have the expression for the rolling load calculations and every time, you have to have this I mean average specific pressure required, then you have the width and also the L_p.

And based on that and depending upon the other conditions, you can have the calculation of rolling load in those situations. So, that is how for different situations, you can calculate the you know rolling load for the rolling applications.

Thank you very much.

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Lecture – 31
Defects in rolled and forged products

Welcome to the lecture on Defects in rolled and forged products. So, we have discussed about the processes of rolling and forging and, in this lecture we are going to discuss about the different type of defects, which are likely to come in the case of rolling and forging.

So, as you know we will start with the forging process and, in the forging process normally the type of defects which are common are like in complete forging penetration. So, this happens you know many a times because when the metal has to flow past the edges, or it has go into the cavities and, may be because of the improper temperature, or because of the improper design of the die, if you have not given proper fillet and corner radii. In those cases there may be in complete forging penetration.

So, that is one of the defect then you have cracking now cracking is also very prominent and, it may be surface cracking or flash cracking or flakes. So, surface cracking as we know that there may be cracks on the surface produced and, there may be many reasons for that, then you have the flash cracking.

So, the flash cracking is basically found at the flash. So, the flash is we know that this is the extra portion of the forged product just like in casting you have riser portions. So, here you have the flash portion. So, flash is the one which is outside the cavity of the you know of the forged product and so, flash goes out and there you may see the cavity in the flash.

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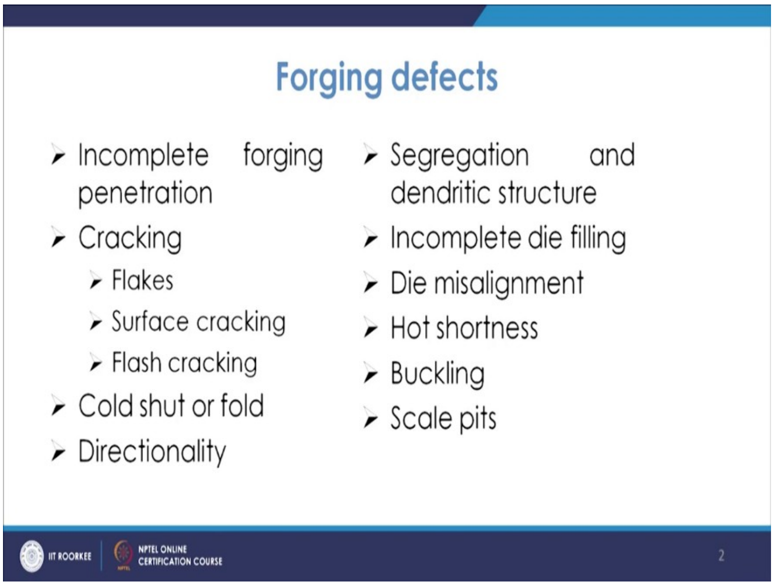

So, there may be reason for it may be thin may too thin flash or so, the severity is there in the case of flash cracking is that is a cracking if there in the flash and, if it has extended to the you know casting then the casting will be you know rejected.

So, that must not propagate inside the; you know forged product. So, that is there then you have flakes are basically the internal, you know cracks internal ruptures in the materials. So, it is because of the imbalancement of the cooling of the forged product.

Now, what happens that when there is a forging going on the top surface of the forged product is in contact with the die which is normally at lower temperature and the middle portion is at higher temperature. So, you have a differential you know temperature gradient that develops and that may lead to such uneven cooling (Refer Time: 03:40) may lead to the formation of flakes.

In the same way, you have other you know cold shut or fold that is because of the improper bounding between the two you know, from the two directions the forging is you know in that is progressing and, they are not able to properly overlap or joined, then that is cold shut or fold directionality is there that is related to the directional orientation of the grains, which certainly develop in the case of forging any and that is there so, (Refer Time: 04:20) this is a second time it has come.

Then you have segregation and dendritic structures so, that happens because what happens that, you may have the segregation of certain impurities are certain locations. Or many a times you try to see that dendritic structure should be removed maybe, but the in the inner internal part, because the compression force which you are applying that is limited to the top surface part.

So, that is you know not seen. So, this compression force is applied it is normally confined to the top you know surface layers. So, the dendritic structure which is there inside that is not you know broken. So, one of the purpose of doing these metal working processes is that the structure and needs to be improved, you have the dendritic structures so, they that also is broken when you heat them at high temperature in then, when you apply the you know compressive forces.

So, you are not able to control that you are not able to altere, or modify the structure. So, that is another defect which we are getting. incomplete die filling again it will be because of the improper die design, or improper temperature because of which the die proper die filling is not there you have also defect related to die misalignment or die shift.

So, many a times when you press the material, or when you are forging the material the top and bottom die, they may be misaligned because of many reasons. And if they are misaligned then that are the parting plane, or are the that plane where they are meeting there maybe you know defect that there may not be proper you know structure.

So, that may lead to the die I mean a shift at you know discontinues type of you know appearance at the plane, where the this two dies meet, hot shortness is another thing because of the sulphur environment many a times in the furnace atmosphere, we get such kind of defects that is hot shortness you have buckling also may be because of the you know improper, or incorrect you know length to width ratio of the specimen and, when you are applying the compressive force in that case there may be buckling effect.

So, that may be there and then you have a scale pits. Now, these scalp pits occur because of the scales which go in between the die and the work. So, what happens that in the case of hot working, or hot forging, when you take it from the furnace and the furnace is atmosphere is not controlled, then at high temperature there may be oxides are scales formed.

Now, what we do that we clean it in between and if you are not cleaning basically that scale is there. So, during the next operation these scales are embedded in between the die and the work you know work piece and, this is scales you know they are crashed and their impression is there on the work piece or work surface.

So, that creates a depression in the on the surface and that is known as scale pits. So, these are the different types of defects which normally occur. Now coming to the cracking; so, in the cracking normally you have the surface cracking cracking of flash cracked flash and flakes so, that is what we discussed earlier.

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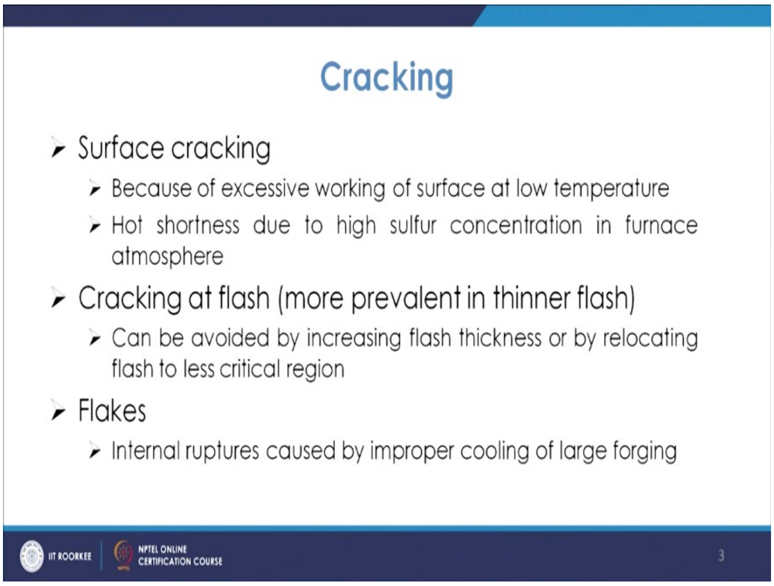

Now, the surface cracking is because of the excessive working of surface at low temperature and, also this hot shortness is due to the high sulphur concentration in the furnace atmosphere. So, that is what we see this these are the reasons, because of you know this the surface cracking takes place, cracking at the flash and this is more prevalent when the flash is thinner. So, normally you know the flash needs to be trimmed.

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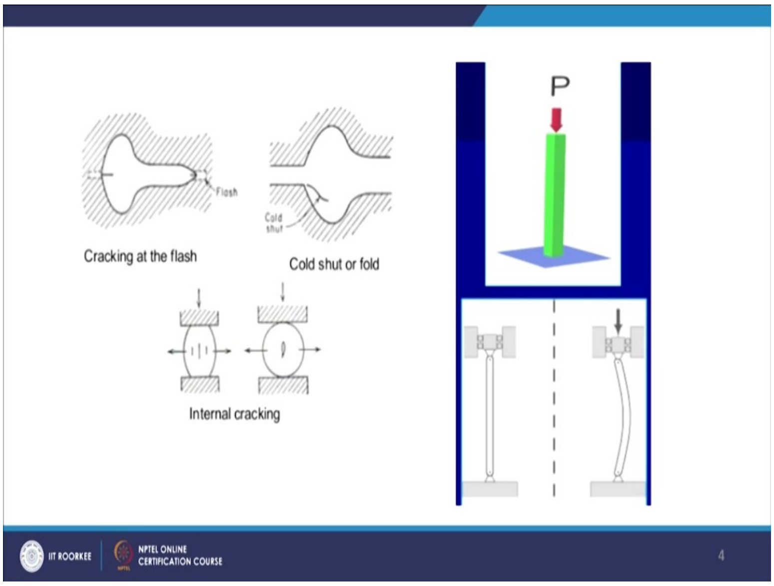

So, what we see here you can see that in this case, you have this is the flash and if this crack is there and this crack may be seen in the you know inside the casting itself.

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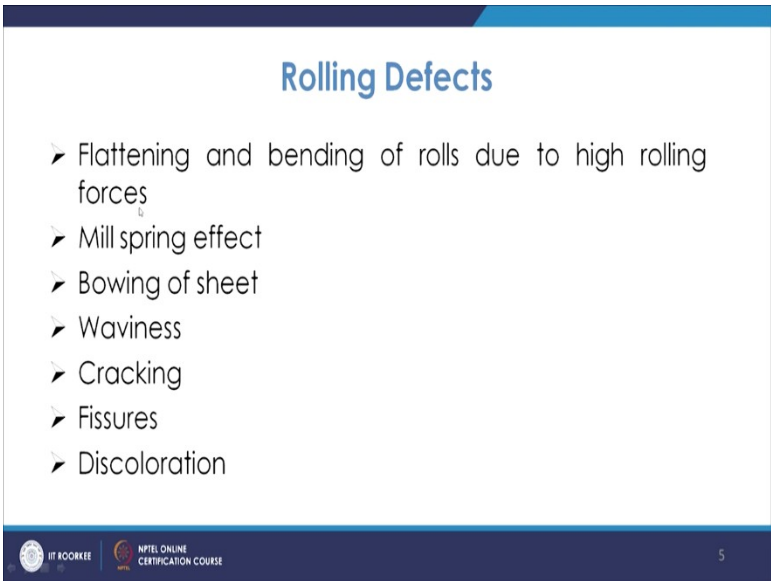

So, so, that is not desirable the flash as long as the crack is there till there, you know the end portion of the flash is fine, but if the chance is there then it goes into the body, then that may lead to the rejection of the you know forged part. So, normally what we do is that you have to increase the flash thickness. So, that the crack does not developed and, otherwise also if you there your flash is to a very critical region, then you must try to relocate this flash region to a less critical zone.

So, that the chances of even if there is a small crack, it does not you know effect the overall performance of the product. The only once we see so, that is why we have to have ensure, I mean you must ensure that the flash should not be attached, or it is not be provided near very critical region to avoid any such you know problem. Apart from that we discussed that we have a defect known as a cold shut, or we also know no know it as fold.

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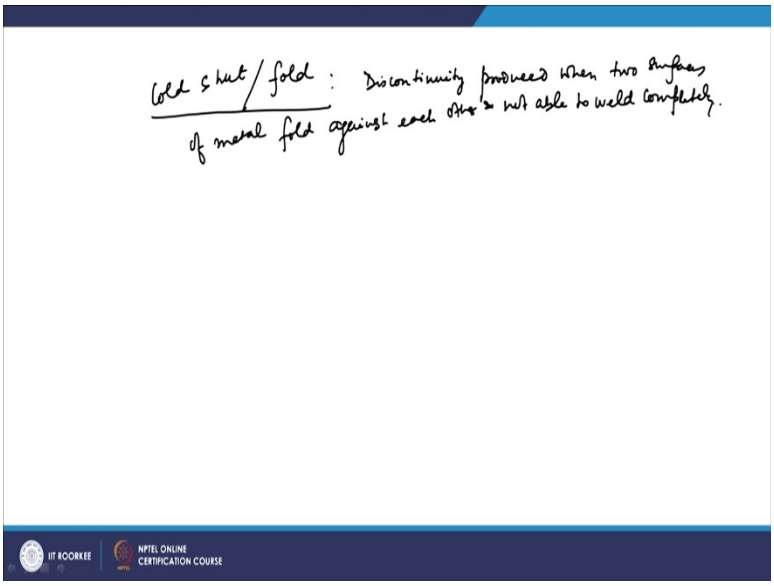

Now, this is the basically discontinuity which is produced, when the two surfaces of the metal are folding against each other and, they are not welding completely. So, this will be the discontinuity produced when two surfaces of metal fold against each other and, not able to weld completely.

So, that there is known as the you know cold shut or fold so, that this name is synonymous to what happens also in the casting process. So, the thing is that from the two sides the metals are coming. So, they have to they have to just completely weld each other and, because of the very sharp corner or because of the excessive chilling this may happen, because what of the stream from the one side the metal, when it is coming and, it has lost it is heat suppose in that case they may not be able to weld completely.

And it may also happen when there is a high friction. So, that also may be the reason so, this leads to the formation of cold shut and, also you have if the die radius is very small in that case also you may have the formation of these cold shuts. Now, if you see the flakes, now this flakes are nothing, but they are the internal ruptures as we discussed that you have basically you know the differential cooling at the surface and at the centre.

So, that differential cooling basically leads to the formation of stresses inside the forged part, then that leads to the cracking of the you know forged product. So, that leads. So, that is known as the internal raptures. Then you have we had discussed about other type of defects like improper fiber structure is formed many a times the fibrous structure, which you are getting that is not proper, that is improper that maybe is the that may not be basically a suitable product.

Because as you had discussed earlier that, if you are not able to get the fibers properly oriented, in that case you will not be able to get the proper you know you know property proper strength of the material.

So, these are normally the you know defects, which we on encounter in the case of this forging we discuss that we have hot shortness which is normally because of the presence of the Sulphur (Refer Time: 14:47) environment in specially, when we deal with steel or nickel. In those cases, these hot shortness is normally observed, when it is seen that you know that that may also we lead to the rejection of the forged products.

In proper flow lines as we discussed that is another you know defect that is that must be avoided, because flow line has to be you know proper. So, that you get the adequate property of the material. Next is the defect related to rolled products. So, as we know that in the case of rolling, you have two rolls and the material is going inside the roll the slab or the billet is going in between the rolls and, they are subjected to the compressive forces. The main thing is here that as we have already seen that there is a pressure back on the rolls.

So, the they will be roll separating forces and, also you have the deflection in the rolls. So, if the roll is not made of proper material, then you know in course of time the roll may lose the shape. And if the roll is loosing it is shape if the roll does not you know you know confirm to the actual shape, then in that case the same impression will be going on the product which is you know being made by these rolling process.

So, the thing is that you have the spring effect flattening and bending of rolls due to high rolling forces. So, the thing is that when there will be high rolling press forces and, since that reaction forces are there on the rolls. So, slowly they flatten and they also bend, there may be bending on the I mean you know of the rolls. And once you have the bending then that may lead to the improper you know shape of the product which is being formed.

So, so, that may lead to improper amount of stresses or the forces which you are generated on the sheet which is going under the rolls and, in that case the different part on the you know product different part on the sheet, will be subjected to different type of stress it at some place it may be subjected to tension whereas are some place, it may be subjected to compression.

And that may lead to the formation of cracks, you know wherever there is a tensile type of stress which is developed inside, then that may lead to the formation of cracks at some places, or in may fracture some fracture may also be seen. So, that is the you know backdrop of the flattening and bending of the rolls due to high rolling forces, then you have mill spring effect, but the thing is that when you have the rolling mill and since being it has certain elastic constant.

So, so what will happen now the material which is going it has some elastic constants. So, it will be under that compressive force from the rolls and, once it goes out of the rolls than that part is basically recovered and that is why the dimension which is there so, you expect that the final dimension will be the same as the roll gap or the distance between the two rolls, whereas when you will have so, what will happen that if you if you look at this.

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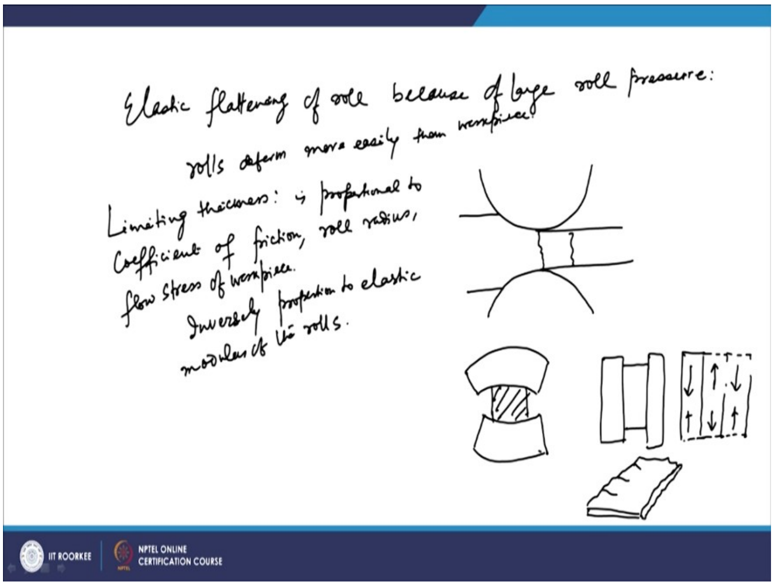

So, we know that in a case of rolling so, what we expect that to you have this sheet and then ultimately it will go like this. Now, the thing is that this we expect it to be the final thickness, whereas because of the mill spring effect, because spring back effect. Now, that will happen that this dimension will not the same as it should be.

So, in those cases the dimension will not be the actual and actual one and, you will have to know the elastic constant of these rolls and also, because of these you know mills, or the rolls and accordingly you will have to set the gap so, that you ultimately get the thickness or the height of the you know of actual dimension.

So, that may be the you know problem with this roll product, that may lead to the bowing of sheet waviness, cracking and, then you have a fissures and discolorations are the another you know problem in the case of rolling, where you have discoloration taking place on the sheet, or you have internal you know the fissures which are formed in the roll products so, that is also seen.

Now, what are the reasons for these you know flattening, or bending of the rolls. So, as we discussed that you have you know since the pressure roll pressure is quite high, now that leads to these you know rolls basically actually deforming.

So, you have elastic flattening of roll, because of large roll pressure. So, due to this what happens that rolls deform more than the you know material itself so rolls. Now, another thing is that when we try to deform and, specially the thinner sheets. In those cases normally we use the smaller rolls and, also you have the limiting thickness you cannot decrease the thickness beyond certain limits. So, that also is the limitation when you are doing the rolling process and, also that will be depending upon certain parameters like what is the coefficient of friction, then what will be the roll radius all that. So, you have what we see that you have the limiting thickness.

So, below that you cannot roll and, this limiting thickness it will be is proportional to so, you will have some parameters like coefficient of friction, then you have the roll radius and, then you have the flow stress of the work piece, but it is inversely proportional to the elastic modulus of the rolls.

And it is inversely proportional to elastic modulus of the rolls. So, so that is why you need to have a proper you know knowledge about the coefficient of friction, also you must know that what will be that limiting thickness, because that they depend upon all these parameters and based on that you will have to control these dimensions.

So, that you get the proper geometry or proper dimension of the roll product which you get finally, also many a times what we see is the because we assume that the roll gap which is there in between you know the rolls, they must be parallel ideally we assume that the roll gap, which is there in between the rolls they have to be you know parallel.

Otherwise what will be happening that you will have the edges of the sheet it will be decreased more, you know in thickness than the other so, at one point you will have more thickness and, another point you will have less thickness. So, that way you cannot get the sheet thickness to be uniform, if your gap between the rolls are not the same so, because you have the constancies of volume.

So, if at one place you have any you know you know decreasing the width that has to be accompanied by the strain in the other regions and, this leads to you know the basically improper dimension of the products. So, that is normally you have to control you have to see that the roll gap is same at all the places, the what we see in the case of rolling that if you look at these pictures.

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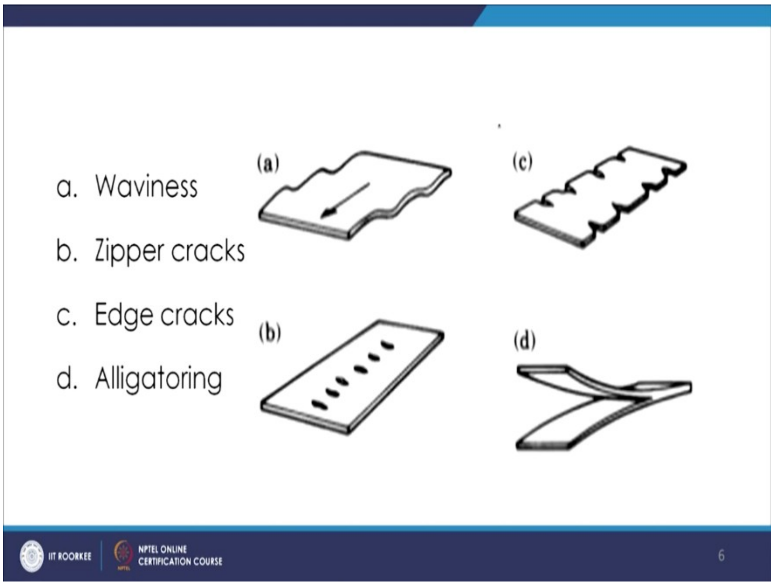

Many times we get some kind of a waviness also and this waviness is nothing, but the lack of flatness. So, you know sheet being remaining so, what we discussed that if you have you know if you have the improper roll gap at the different positions, then in that case you will have this formation of these waviness, or you will have because the strain has to readjust among themselves.

And this kind of you know appearance is seen in those cases. Now, this can better be understood by referring to certain figures like, if suppose many a times you have rolls like, they are coming like this portion. Now, in such cases if you rolls are like this, then what happens that if your body is which is to be compressed here.

Now, what you see that in this case the other side portion that side the middle portion will have will be basically compressed. So, that will be that will be leading to you have you know such kind of you know sheet will be developed.

So, you have this will lead to basically formation of the stresses and, if you see the different regions, what we see that in this portion this portion will be under the tension and this portion will be under compression. And in that case you may see that on the edges you will have the you know, you know waviness or you have the cracks which develop or they basically adjust.

So, on the edges you will see that you have such kind of appearance which is found on the edges. So, adjust the strain energy adjustment leads to the formation of such kind of also cracks, which is observed we have already seen that you have the cracks also coming at the central portion also. So, these are normally the you know type of rolling defects, which normally encounter we encounter in the case of rolling processes.

Thank you very much.

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Lecture - 32
Introduction and classification of extrusion processes

Welcome to the lecture on Introduction and classification of extrusion processes. So, we will discuss in this lecture about the important process such as extrusion in which what happens that you have a billet which is heated and you know, it is pressed from one side through a constricted region and then, the metal flows plastically deforms because of the you know being pressed at the die corner. And then, metal flows through that cavity.

So, we normally make the bars or even tubes also; the hollow you known parts are also made which are seamlessly made because if the billet is there so you can seamlessly make. So, if you press from one side you can get a you know seamless type of tubes or you can get a continuous rod which is extruded. And since, it is plastically deformed and it goes under that stress state. So, because of the forming you know attributes and forming properties the property of the material is better.

So, you have, actually in extrusion as we discussed that the block of metal is reduced in cross section by forcing it to flow through a die or a face under high pressure.

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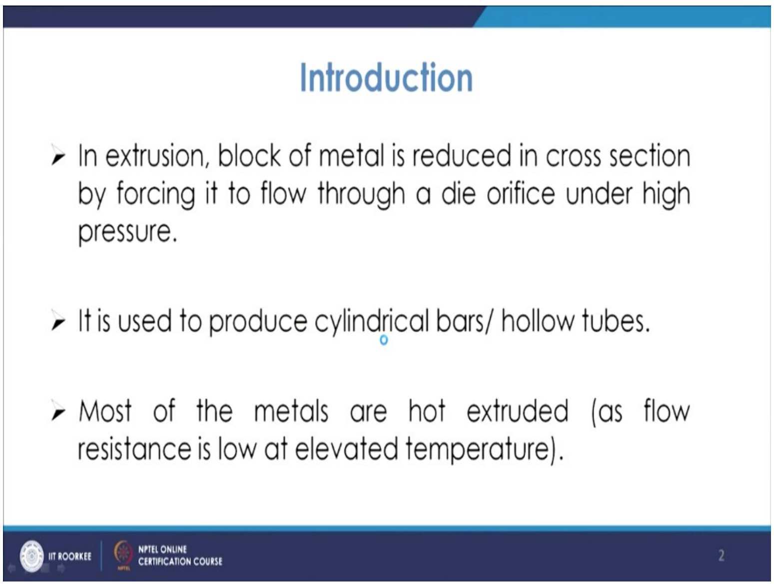

So, you are applying the pressure; you can apply the pressure from one side, and the metal will be extruded from the other side. So, for example, you have you know one container is there and you know this is something which is a billet, which is you know heated.

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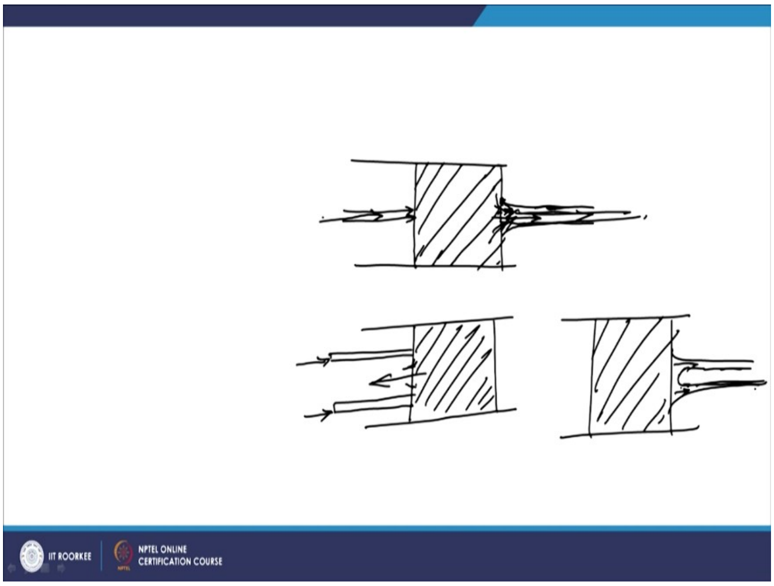

It is kept in this you know cavity and you will have lubrications on the walls so that you have the minimized the frictions or so.

Now, the thing is that once you are pressing it from this side then and so otherwise, this is a case of hydrostatic extrusion. If you are closing it because you will have the stresses from all the sides but then when you are giving a suppose constrained region to basically flow it.

So, what happens that at these places as we have discussed about the direct and indirect type of you know compressive you know forces which are developed in direct compression type, indirect compression type forming processes; it comes under that.

So, what happens that because of the reaction which is there at this point, then there will be the deformation of the failure of the material at this point and then material will try to go through this region. And then, you can you can go on steadily where you applying the pressure from here and this you will get it continuously.

So, that is this product is known as the Extruded product. Now the thing is that when you apply the pressure initially, since length is larger. So, in this case, when it becomes smaller;

now the thing is that the extrusion pressure will vary in certain process. Now that will be depending upon what type of extrusion process is there.

So, this is the example of extrusion, but the thing is that you are; so as we discussed that you are you know applying you were under high pressure. So, this is the, orifice and since you are applying the pressures from here by the RAM. So, under the pressure it will be metal will be coming out through this you know cavity and that is the process known as extrusion.

The thing is that when this the direction of the you know pressure which you apply of the Ram and also the direction of the extruded product which is coming out to through the orifice, they may be same or they may be different; they may be opposite to each other.

So, in another case what happen that you have the billet and what you do is you do some mechanism by which you apply this pressure from here there is a hollow Ram and then, what you do is when you apply this pressure from this side and if it will hollow and if you have a die opening here and then, metal will come out of this

So, what happens in this case? In this case basically the metal will be extruded or it will be coming out in the opposite direction. So, this is the example of indirect extrusion. So, normally you have the direct extrusion and indirect extrusion and depending upon the cross section which is you have here at die, you will be getting that type of cross sectional product; normally it is round or you can have a triangular cross section.

But you can have irregular type of also cross section which can be produced by doing this extrusion process. Now this is very important process and it is used for you know extruding the rods or bars or the tubes also. And in the when you have to make the tubes then what happens that when you are applying the pressure and the metal is about to go out of the you know you have to go through that cavity at that time you are placing a mandrel in middle portion of the cavity so that that is here.

So, if suppose it is coming here, but if you are putting a mandrel here. So, if you putting mandrel, you can just see that if you are you know slab or billet is here and it has to come through this and then, if you are putting a mandrel like this. So, if you are putting a mandrel, then the metal will only flow through these places.

So, you will get you know tube or a pipe of certain internal radius and external radius. This will be internal radius and that will be the external radius of the extruded product which is coming out. So, this way, you can get the product of different type may be it may be hollow; it may be solid and then, you will have the varieties that we will discussed.

So, it used to produce the cylindrical bars or hollow tubes and most of the metals are hot extruded because when you are doing the excursion at high temperature in that case the flow resistance will be low at that high temperature. So, flow stress as we know that once you increase the temperature at higher temperature, the flow stress requirement is a smaller. So, you required to put smaller pressure in those cases.

So, that is why we do the extrusion mostly the hot extrusion. Depending upon the again how we classify the forming process based on temperature you have; here also the classification is on the basis of temperature and you can tell it as hot extrusion or cold extrusion. So, most of the metals are hot extruded because of the you know low value of flow stress at higher temperatures. coming to the hot and the cold extrusion.

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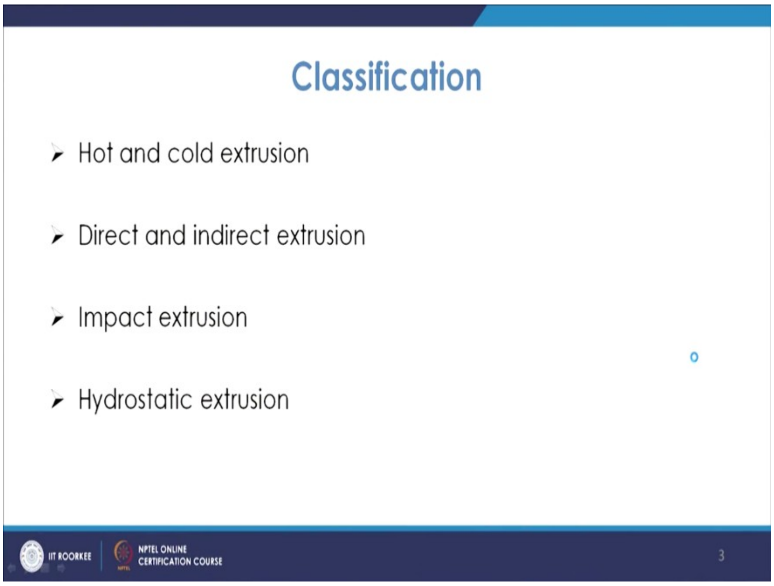

Coming to the hot and the cold extrusion. So, as we discussed that when we try to you know give a classification of the extrusion process, then you may you know classify as the at the basis of temperature, classify on the basis of temperature and if it is done at a temperature more than the (Refer Time: 09:06) recrystallizing temperature, we call it as the hot extrusion; otherwise we call it as cold extrusion.

So, that is the you know classification based on the temperature. Then as we discussed that you may have the classification based on how you are taking the extruded product out in what direction.

So, on that basis you have the direct extrusion and the indirect extrusion you know. As we discussed that in the direct extrusion you have the direction in which we have apply the pressure on the RAM and also the, you know the movement of the extruded product they are basically same.

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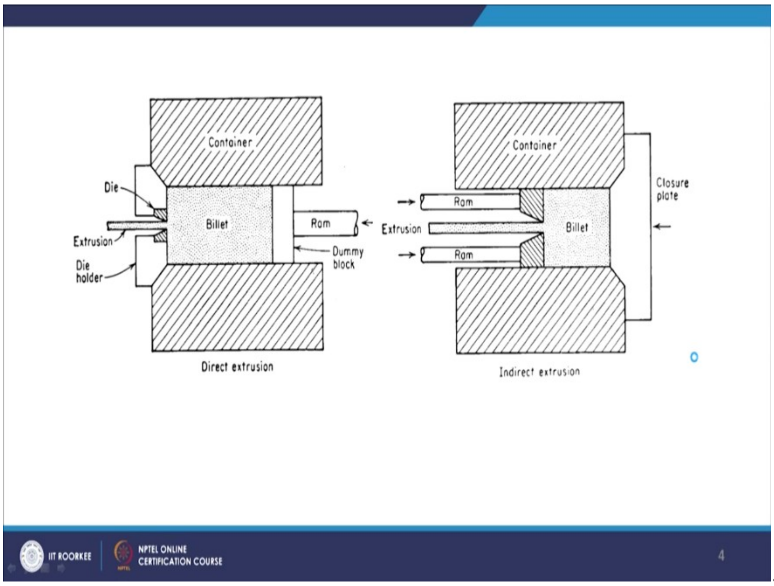

So, this is how we can understand the example of a direct extrusion and indirect extrusions; as we see that in this case you have the direct extrusion, you have a die holder here and you are having the RAM here; you have a dummy block placed here.

So, that there is no direct contact with this RAM otherwise RAM will be too hot and then you will have this container and this is the die holder. So, die holder is there to support the die to give. So, it holds the die into it and it give the rigidity to the die because there is lot of amount of pressure which is there. So, a large amount of force to which it will be subjected to.

So, then you apply this pressure on this RAM from this side and as we discussed that this billet which is heated one; since it is getting this constricted passage from here, it has only the way to exit. So, and also there has stresses reached at these corners as we discussed earlier that the kind of stresses which are you know generated at these points the metal flows through this point. And this is the example of the direct extrusion.

And if you come to the indirect extrusion at what we see is that we have the billet here and this is a hollow RAM and the you know this in between you have this portion from here the billet will be basically coming up; now there are differences between these direct extrusion and indirect extrusion processes.

So, now what we can see here? Now, in this case when the RAM is basically pressing this billet. So, this billet is getting pressed and there will be relative motion between the billet and the container wall. So, because of this friction which is there in between the billet and the container wall, you will have the requirement of pressure increased in such cases.

But, if you take the example of this indirect extrusion; in this case this RAM is kept stationary at its own position and the container along with the billet. This is basically moved. So, in that case there is no relative motion between this billet and the container wall in the case of indirect extrusion. So, the friction forces which are there in such cases are basically lower and so the power requirement will be smaller in the case of indirect extrusion.

So, that is the main difference between the, you know direct and indirect extrusion.

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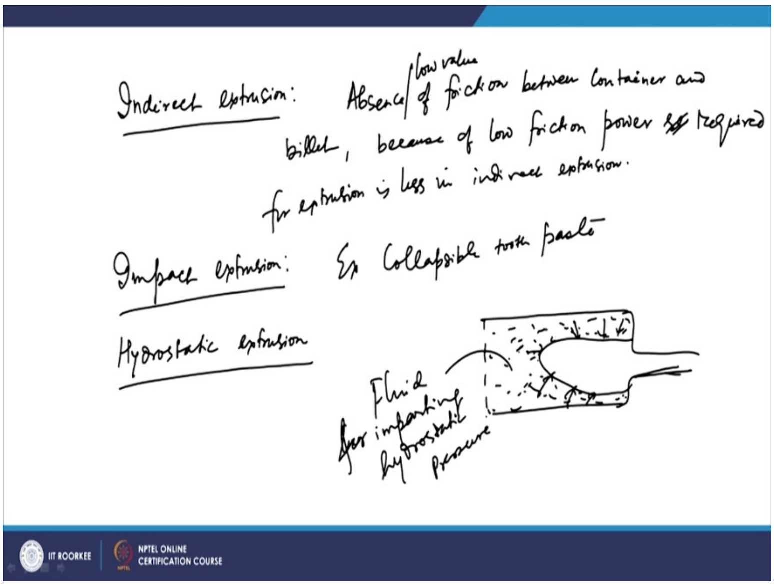

So, if you talk about indirect extrusion. So, the absence of friction between container and billet, because in this case the container along with billet that is basically pressed, so there will not be any you know friction which is there.

And so you are; so, because of low friction, now we can since, we can have or low value. So, because of low friction, the power required for extrusion is less in indirect extrusion. But, what we can see that there is a limit of these you know process in the case of indirect extrusion because the RAM is hollow.

So, certainly the pressure which you apply certainly there is limitation and that is why you will have that limitation, how you can how much load you can apply in the case of direct extrusion. As we discussed that if you apply the mandrel, you can if you apply mandrel in the case of.

So, you know at the end you know here, if you apply the mandrel; then in that case when the metal is coming out due to that mandrel, you can have the hollow shaped product which is formed in the case of you know this extrusion. So, you can get the hollow tubes and pipes by the use of mandrel. Now you have another process of extrusion that is impact extrusion.

Now, in the impact extrusion what happens that you know basically you have the extrusion which occurs because of the impact force which comes. So, the RAM will travel from certain height under impact and then, the metal will flow in the opposites. So, it is basically a type of indirect type of extrusion.

So, normally very short length type of hollow shapes, you know are produced using these you know impacts extrusion and impact extrusion basically is used for making these hollow, you know tubes especially the collapsible toothpastes.

So, normally it is a direct when direct extrusion and it is normally done with the help of a mechanical press which is very high speed and they have you have very high deformation rate which is achieved in this case and normally what you can see that the toothpaste, you know tube which is there. So, they are made of very soft materials which are there.

So, that goes on that RAM and then, the metal extrusion the opposite direction normally. You can go even in the direct extrusion type also and it takes the shape once it is subjected to the impact force.

So, that is the example of impact extrusion. Another extrusion is the hydrostatic extrusion. Now what happens that in the case of hydrostatic extrusion, the pressure is created by the fluid. So, what we can say that if you have you know a normally it is used for the materials which are very limited ductility even brittle materials are you know extruded by you know this process.

So, suppose you have a billet or so or any material which is now in this case you have this as the hydrostatic fluid which is present and once you apply the pressure, this hydrostatic fluid basically is applying the pressure, this fluid is applying. So, you have application of pressure from all the sides equally you know you have hydrostatic stresses which are developed.

So, that is why you have forces which are developed on this material and then it it flows through this constriction. So, basically you have the fluid for imparting hydrostatic pressure. So, that is how it is known as the hydrostatic extrusion. Next, we will discuss about the typical characteristics of these extrusion process. So, when we talk about the hot extrusion; so, if you talk about the hot extrusion.

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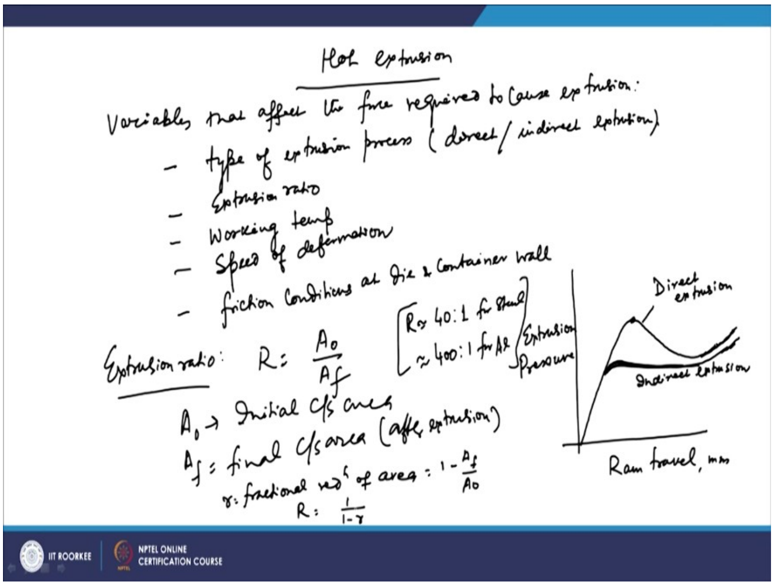

Now, there are many variables which have an influence on the force required to extrude the products. So, variables that affect the force required to cause extrusion. So, that will be first is type of extrusion process. So, you may have the direct extrusion or you may have the indirect extrusion. Then you have the Extrusion ratio.

Now, what is the final area of the billet and what is the final area I mean what is the initial area which of the billet which is to be extruded and what is a final area of the extruded product? So, the ratio of these two areas is known as the extrusion ratio. So, certain extrusion ratio if it will be more, in that case you your requirement for the pressure will certainly be more.

Then you have the working temperature. Working temperature means at what temperature you are doing that extrusion and certainly if the temperature will be more the pressure requirement will be smaller. Then after working temperature, you have the speed of deformation and then, you have the friction conditions at die and container wall.

So, these are the variables like you have the friction is there at the die and the container wall; then certainly you will have the requirement of a large extrusion pressure for doing these extrusion work. So, what we do many a times, we apply the lubrication; we apply solid lubricant also. So, we apply the lubricants.

So, that there is minimum friction at the die and container wall interface for that. Now if you apply, if you try to look at the variation or the pressure which is there in the case of extrusion process, what we see is that you have a rapid rise in the pressure requires if you try to see the graph between the RAM travel that is in mm.

And then, if you look at this as a function I mean this is with this how this extrusion pressure is going to vary. Then, it has been seen that in the case of direct extrusion the curve goes like this and then, if you have the indirect extrusion; then the curve goes like this. So, so that way your, this is for the direct extrusion and this is for indirect extrusion.

Now what happens that when you initially compress this die? So, initially when you are applying the pressure on the billet. So, what we see is that there is initial surge in that pressure, in this initial region and this is because of because with the pressure the whole container which basically is going I mean whole that extrusion is going to start and basically this rapid rise is because of the initial compression of this billet to fill that extrusion container, it will be completely filling that.

So, when you are pressing it will be the container will be completely filled because of that the pressure is initially increasing. Then it is coming to this highest point in the case of direct extrusion and at this point basically the extrusion starts.

So, this point is known as the Breakthrough pressure and after that the metal starts deforming and then, metal starts coming out of the you know outlet or the out of the die and you get the extruded (Refer Time: 24:53) product.

Now, after that you see that there is a decrease in the you know in the value of the extrusion pressure and in the case of direct extrusion because in the direction extrusion as you see that when it starts you know the product which starts going out of that die, then the you know the length to of the billet which is to be extruuded at that will go on decreasing.

So,. So, as it is decreasing the pressure requirement will be smaller because you require lower pressure to you know give the force on the material. So, that it goes through that outlet. So, it will go on decreasing and in the end basically again, it is there is a increase in this value which is going very high. At high rate it is increasing just to ensure that the RAM stops there and in the end, you have the, you know unwanted things which are stopped to this normally they are defects in the structures they are basically removed.

So, that is why at the end basically you have the stop being applied at this place. Now if you looked at this indirect extrusion curve, in this case it starts from here and you do not see much change in that extrusion pressure. Because there is no you know otherwise the friction between the billet and the container and it will be going on continuously at this point.

So, what we see is that you have the steady kind value of kind of extrusion pressure for that. But then, you have as we discussed that there is limitation for this because once you use the hollow RAM, in those cases you have the limitation of how much pressure you can apply by using that hollow RAM. Now, as we discussed that there will be small discard in the end and that will be defects.

So, being the unwanted product that for that there is pressure is you know end of the pressure will be building up and then, there that will be leaving up small discard in that. The terminologies which are you know otherwise important for this extrusion is one is the extrusion ratio. So, as we discussed extrusion ratio is basically the initial area upon the final area.

So, this is your A_0 is initial cross section; cross sectional area and A_f is the final cross sectional area that is after the extrusion. So, it takes value normally is 40:1. So, if we take the value of ‘r’. Now ‘r’ will be something like 40:1 for steel for hot extrusion and it may go as high as about 400: 1 for softer materials like aluminum.

So, if you depending upon the type of metal, you have the collection of these extrusion ratio; you have you can also define this extrusion in the form of you know reduction in area. So, if your ‘r’ is the fractional reduction of area. So, in that case, $r=1-\frac{A_f}{A_0}$.

So, you can have the expression for R as capital R as $R=\frac{1}{1-r}$. So, this way they are normally used for you know for the analysis in the case of extrusion process. Now, once you know this you know this $\frac{A_0}{A_f}$; then, what has been seen that the pressure which is required or the extrusion force which is required.

So, that is normally related to the natural logarithm of these $\frac{A_f}{A_0}$ that is your extrusion ratio. So, your Extrusion force extrusion force is normally seen to be.

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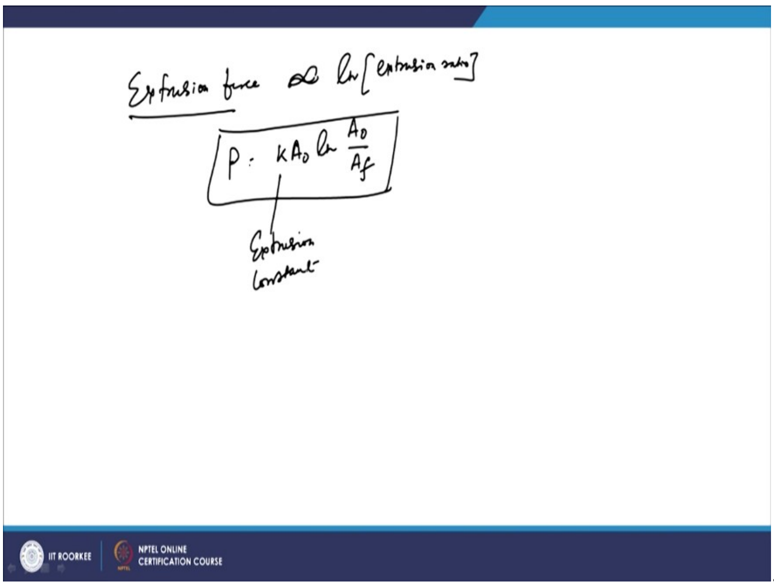

So, what happens that you know you have extrusion force; it is directly proportional to natural log of the extrusion ratio. So, what has been seen that the $P = k A_0 \ln \frac{A_0}{A_f}$. So, normally 'k' is known as the extrusion constant and we know other factors A_0 and A_f.

So, that way also you have many a times you must know the range at which the extrusion process is carried out; you may have hot extrusion or you may have the cold extrusion. Hot extrusion is normally for steel it is done in the range of 1100 to 1200 °C and the pressure which is required is 800 to 1200 MPa. If you go for cold temperature will be smaller; then in that case certainly the pressure requirement will be larger and that is what mostly you depending upon because the problem with the higher temperature side is sometimes the oxidation. So, normally you provide you know the lower temperature which suitable plasticity so that you can get the extrusion with you know with minimum of this kind of defects like oxidation or so.

Also a higher temperature is the maximum temperature will be the temperature at which the hot shortness occurs. So, that way you have to take the temperature range in to which you should do the extrusion process.

Thank you very much.

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Lecture - 33
Analysis of extrusion processes

Welcome to the lecture on Analysis of extrusion processes. So, in this lecture we are going to discuss about the expressions, which are required to find the extrusion load calculations, and all that by the different methods of analysis. So, if you try to see that you know analysis of this process now, if we use the so, by using uniform deformation approach deformation energy approach.

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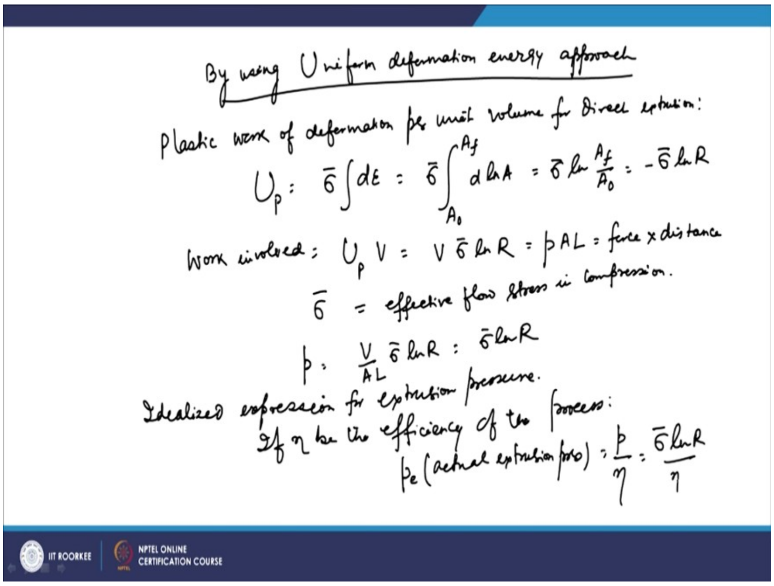

So, we have got some idea about uniform deformation energy approach, where we talk about the plastic work of deformation. So, the plastic work of deformation per unit volume if you look at for direct extrusion. Now that can be taken as U_p and it will be nothing but you take this average value of a stress, and then dε. So, that is the plastic work of deformation per unit volume, and this can be taken as further you have dε. So, it will be you are doing from A_0 to A_f and it will be $d\ln A$.

So, that will be coming out as, $\bar{\sigma}\ln\frac{A_f}{A_0}$, sigma bar that is average stress. So, it is nothing but $-\bar{\sigma}\ln R$. So, that is what if you do it through the uniform deformation energy approach you get this value U_p.

Now, if you take the work involved, now what is work involved? So, it will be if this is the plastic work of deformation per unit volume. So, this multiplied by volume, it will be the work involved. So, it will be $U_p V$ so, V is the volume of the work piece. Now it will be so, so you have V, and then you have $\bar{\sigma}\ln R$. So, that will be your work involved, and you can that will be basically the pAL.

So, this will be in a force into distance is nothing but the force into distance. So, pA will be the pressure. And then you have the distance so, it will be force into distance that is what the work done it is. So, as we know that you define this $\bar{\sigma}$, it is as you defined as effective flow stress in compression. So, from this expression $V\bar{\sigma}\ln R = pAL$, from here we get 'p' as the V/AL and then $\bar{\sigma}\ln R$. So, since AL is 'V' so, it will be $\bar{\sigma}\ln R$. So, what we see is that this is the extrusion pressure p is the extrusion pressure, and this is nothing but $\bar{\sigma}\ln R$. Now in this case this is basically the idealized expression for the extrusion pressure.

And so, this is idealized expression for extrusion pressure, because it is not considering either the frictional you know effects or the redundant deformation. So, that is why it is the expression for the idealized expression for the extrusion pressure. Now if we incorporate the efficiency of the process, so, if the if η is the be the efficiency of the process, now in this case the pressure which will be required will be nothing but p/η.

So, you can take actual extrusion pressure. So, p_e that is your actual extrusion pressure. That will be this 'p' divided by the efficiency η, and that will be your $\frac{\bar{\sigma}\ln R}{\eta}$.

So, this is how you try to find the you know expression for or calculate the value of actual you know extrusion pressure. Now the analysis by or by once you the measurement or the forces has been done by one of the you know scientist D. Pier. So, he has shown that he has shown that the force which is an extrusion it has many components. So, he has shown so the D. Pier has shown that the force in extrusion so, it consists of many you know components.

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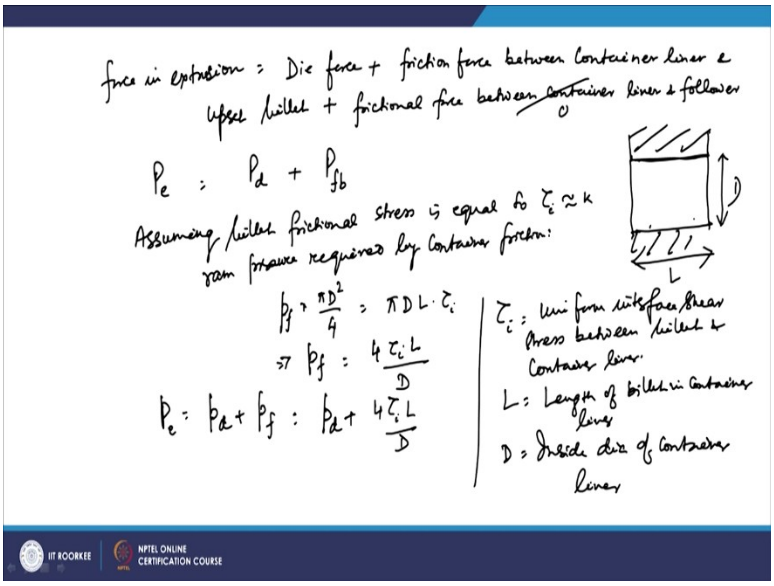

And it will be a sum of the die force, then you will have the friction between the billet and the container wall.

So, container liner and the upset billet. So, friction force between container liner and upset billet. Then there is another friction and that was between the container liner and the you know follower. So, the frictional force between container liner and follower; which is following from the back so, that basically is normally considered to be 0. So, you are taking basically the two types of you know pressure, force that is die force so, you are taking this force in extrusion, it will be the die force we call it as the P_d and the friction force between the you know container liner and the upset billet, and that we are basically representing by the f_b.

So, this is 'f' for the friction, and between the container liner and the billet. Now if you assume that this billet frictional you know stress, that is τ_i because and that is equal to 'k'. So, assuming billet frictional stress is equal to τ_i, that is similar to the 'k', in that case if you talk about the RAM pressure. So, if you look at the RAM pressure so, the RAM pressure required by container friction. So, in that case if you look at this RAM pressure which you calculate now if that is p_f so, that will be multiplied by $\pi \frac{D^2}{4}$.

So, this is been applied on that area $\pi\frac{D^2}{4}$. Now that will be basically you are if you talk about this friction which is acting on that you know in between that, cylinder that is liner of the container and the moving a billet. So, in that you have $\pi DL\tau_i$. So, that is what you can have from here, and if you try to find this p_f. So, this p_f will be 4 into. So, you have 4 go will go from here and D and D will cut on one side. So, it will be $4\tau_i$ and π and π will cut, D and D will cut so, $\frac{4\tau_i L}{D}$. So, what we see is that if you look at this expression, what is being found further. So, you will have if you to log of the pressure. So, this is *(p_d + pf)* and that is basically $p_d + \frac{4\tau_i L}{D}$. So, this terminology is τ_i, it is the uniform interfacial stress between billet and container liner. And similarly you have L which is they have which is the length of these container liner billet, that which is in contact with the container liner. So, length of billet and container liner, and D is the inside diameter of the container liner.

So, that is what I mean you know that you have these. So, so this becomes your L and this becomes D. So, this how this is how it because it has to move so, this is container. So, so that way this is your L and this is your D. So, once you measure this p_e and p_d, once you know that, then you can you know find the appropriate value of this fictional stresses. A simple approach by the slab method analysis has also been carried out.

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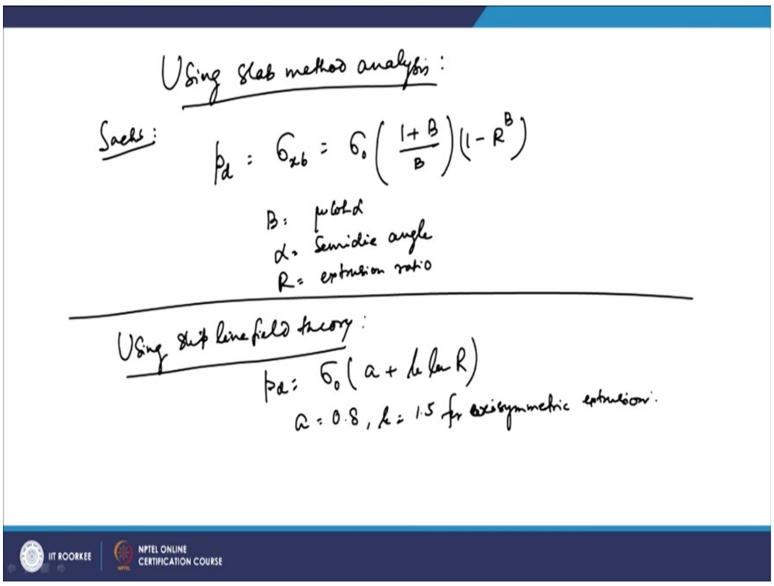

So, using that slab method analysis.

Now, in that basically that has been expression for this die pressure, and in that you know you have taken that friction into account, and you have the there is conical die, and the material is extruded through that conical die. On for that Sacks has performed you know the analysis. And they have done the analysis for using that coulomb friction. And coulomb sliding friction basically and this has been shown out. So, similar to that which is done for the wired drawing case also, now in this case p_d is said to be Pd as

$$p_d = \sigma_{xb} = \sigma_0 \left(\frac{1+B}{B}\right)\left(1 - R^B\right)$$

So, this is the expression for the die pressure which is required. In this case the 'B' parameter $B = \mu \cot \alpha$, α is the semi die angle in this case. So, α is semi die angle, and R is extrusion ratio. So, basically this work involve the friction, but does not involve taking into account the redundant force, which is there in the case of extrusion. There has also been analysis by using this slip line field theory, for the plain strain deformation cases and for these using (Refer Time: 14:26) slip line field theory. So, most of these expression which used to come will be of the form p_d will be actually equal to $\sigma_0 (a + b \ln R)$.

So, that has that work with this analysis in that which is coming to of this shape, 'a' and 'b' are said to be the constants, and 'a' normally is taken as 0.8 and 'b' as 1.5 for the axisymmetric type of you know extrusion. So, there has been reported by many researchers, like the hill has Robert hill R hill has given this expression, $p_d ¿ \sigma_0 (a + b \ln R)$, 'a' is taken as 0.8 and 'b' as 1.5 and this is for axisymmetric extrusion. Now even there has been analysis using the upper and lower bound you know upper bound analysis, and when the analysis has been done using the upper bound process.

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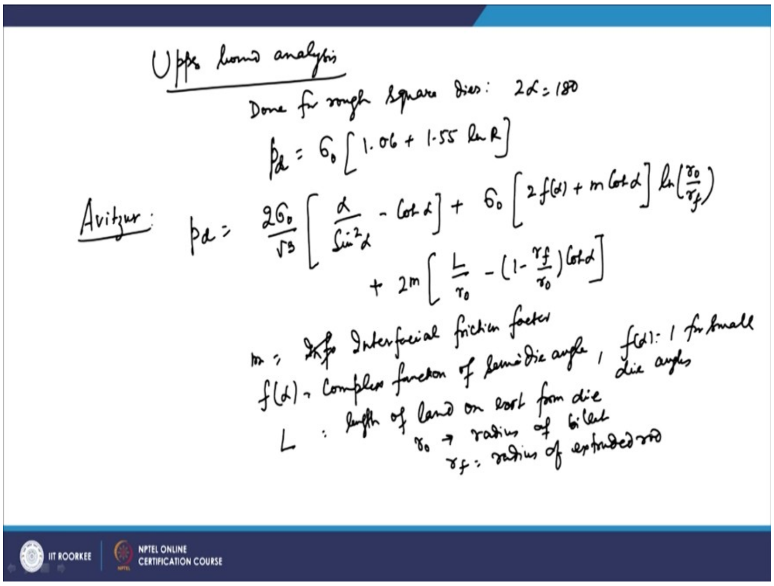

So, upper bound analysis has been performed by researchers and Kudo has done this analysis using this upper bound analysis. And they have done for the rough square dies so, they have done for rough square dies. So, in this case 2α is basically 180. So, in that case they have found Pd as sigma naught into 1.06 plus 1.5 ln R. So, in that case the A, which was there in the A in that axisymmetric type of case suggested by Hill. Now here Kudo has suggested, p_d¿$\sigma_0(1.06+1.55\ln R)$.

Now, there has been you know other expression which has been suggested by Avitzur. And Avitzur has given a more generalized expression using the spherical you now velocity field. So, Avitzur so, he has given more general type of generalized expression, using that spherical velocity field. And it will be applied for the lubricated extrusion through a die or semi die angle α. And we have shown the expression as

$$p_d=\frac{2\sigma_0}{\sqrt{3}}\left[\frac{\alpha}{\sin^2\alpha}-\cot\alpha\right]+\sigma_0\left[2f(\alpha)+m\cot\alpha\right]\ln\frac{r_0}{r_f}+2m\left[\frac{L}{r_0}-\left(1-\frac{r_0}{r_f}\right)\cot\alpha\right]$$

So, this is the generalized expression for the that is done by Avitzur. And in this case you have 'm' as interfacial friction factor. Then you have $f(\alpha)$ it is complex function of semi die angle. So, very for very small angle $f(\alpha)$will be normally one. So, $f(\alpha)$is one for a small die angles. Then 'L' is 'L' which we get in this case it is length of land on exit from die. And then you have this ‘L’ is coming here, than another parameter is r_0 so, $r_0$is the radius of billet and r_f is radius of the extruded rod.

So, if you know all these parameters you can find the expression for p_d which has been suggested by Avitzur. And further there has been other analysis using the upper bound analysis further, and there has been good agreement for the experimentations done with hydrostatic extrusion.

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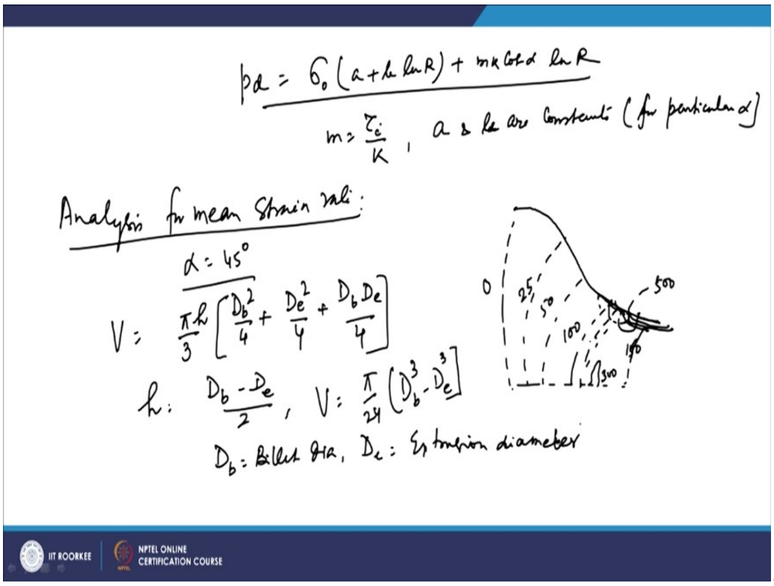

And D. Pier further shown that if you look at the expression by D. Pier, then he has shown that die pressure,

$$p_d = \sigma_0 (a + b \ln R) + mk \cot \alpha \ln R$$

So, in in this case, $m = \frac{\tau_i}{k}$, and ‘a’ and ‘b’ are the constants. So, depending upon the die angle semi die angle ‘a’ and ‘b’ are constants for particular α that is semi die angle, and for the die angle as it is increasing the value of ‘a’ and ‘b’ basically will be slightly increasing. And basically in the value of ‘a’ will be more increasing as compared to ‘b’. There has been further the analysis by studying the velocity field, in the case of this extrusion process.

And what has been seen that if you have a mean you know temperature of extrusion which is there inside the domain. Then strain rates are actually calculated, and they are seen to vary inside you know that domain. And it is seen that the strain rates are maximum at certain positions. And average rate is then computed for that particular you know truncated conical volume region.

So, if you are taking one semi die angle so, you have analysis for mean strain rate, and in that basically the flow field analysis has been carried out you have distribution of stress and strain rate and all that has been calculated, and you have the variation of temperature has also been recorded. And what has been seen that near the exit you have the maximum you know strain rate values. So, it was seen that it goes like this value so, what was seen that, this is the point of 0. Then as it goes it will you will have a you know strain rate value of 25, then you have 50, and what was seen that it was further increasing to 100, and then what was see in that here.

So, this is basically coming down so in this locality where there is further there is drop, now in this locality you have maximum, and that comes out to be around to about 500 maximum strain rate is recorded. Now here also it is recorded 300 something like. So, this way you have calculation of there further it is decreasing. So, this come comes out to be 150 or so. You have increase it to 200 or 250 or so. So basically if you do for these you know alpha, when it is taken as 45°, in that case, the how to calculate these mean you know strain rate or average mean strain rate.

So, for that what we do is we can define this 'V', 'V' as this should be having conical shape taking into account. So, you know that for conical shape you have $\frac{\pi}{3}$ you know. So, $\frac{\pi}{3}$is taken, and then so, you will your 'h' will be taken and then you have. So, D^2 term so, that is r^2 square term so, $D^{2/}4$. Now you have $\frac{D_b^2}{4}$, this is the first term, then you have $\frac{D_e^2}{4}$. And then you have $\frac{D_b D_e}{4}$. So, you know that the volume if you calculate it is $\frac{\pi r^2 h}{3}$.

$$V = \frac{\pi h}{3}\left[\frac{D_b^2}{4} + \frac{D_e^2}{4} + \frac{D_b D_e}{4}\right]$$

So, $\frac{\pi h}{3}$ is there and r^2 that is $\frac{D_b^2}{4}$. So, you have taken all these you know diameters into account $\left[\frac{D_b^2}{4} + \frac{D_e^2}{4} + \frac{D_b D_e}{4}\right]$. So, 'h' basically you can take average one and that you can take as $h = \frac{D_b - D_e}{2}$. So, that you can take it as 'h'. So, your V becomes, $V = \frac{\pi}{24}\left[D_b^3 - D_e^3\right]$. so, D_b is the below diameter and D_e is the extrusion diameters. So, that D_b is the billet diameter, and D_e is the extrusion diameter. Now if you look at the RAM velocity which is 'v' so, volume extruded per unit time so, for RAM velocity 'v', the volume extruded per unit time if you try to calculate so, that will be $v\frac{\pi D_b^2}{4}$.

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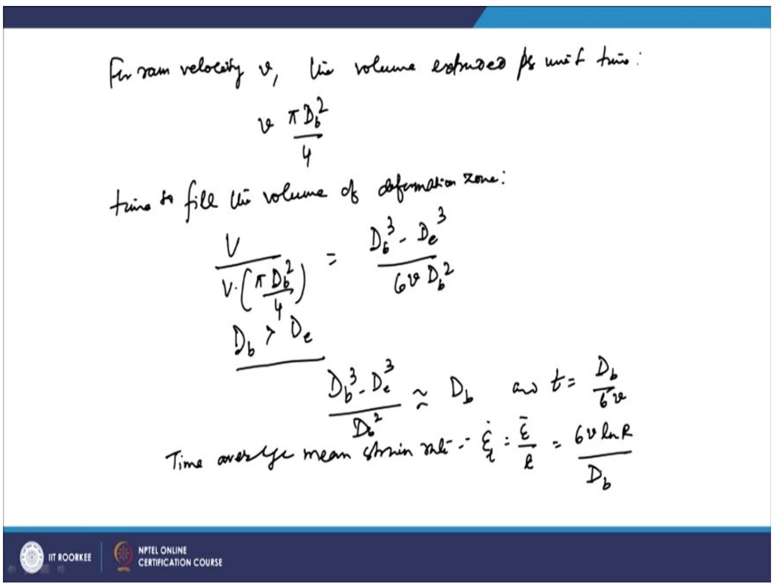

So, that will be the volume extruded per unit time. And if you see that time to fill, the volume of the deformation zone so, it is nothing but volume divided by this area multiplied by this. So, you have you know so, area multiplied velocity so, this is a same thing. So, volume by volumetric you know expansion per unit time so, that will be the time. So, it will be

$$\frac{V}{V\frac{\pi D_b^2}{4}}=\frac{D_b^3-D_e^3}{6\,v\,D_b^2}$$

So, if this since D_b is quite larger than D_e so, D_b being quite larger than D_e, in that case you can write that $\frac{D_b^3-D_e^3}{D_b^2}=D_b$, because this D_e term can be neglected, and you also you get $t=D_b/6\,v$. So, from this expression, you can have the expression for the mean strain rate. So, time average mean strain rate if you calculate. So, that will be basically

$$\dot{\varepsilon}_t=\frac{\acute{\varepsilon}}{t}=\frac{6\,v\ln R}{D_b}$$

So, this is basically the computed value of the time average mean strain rate.

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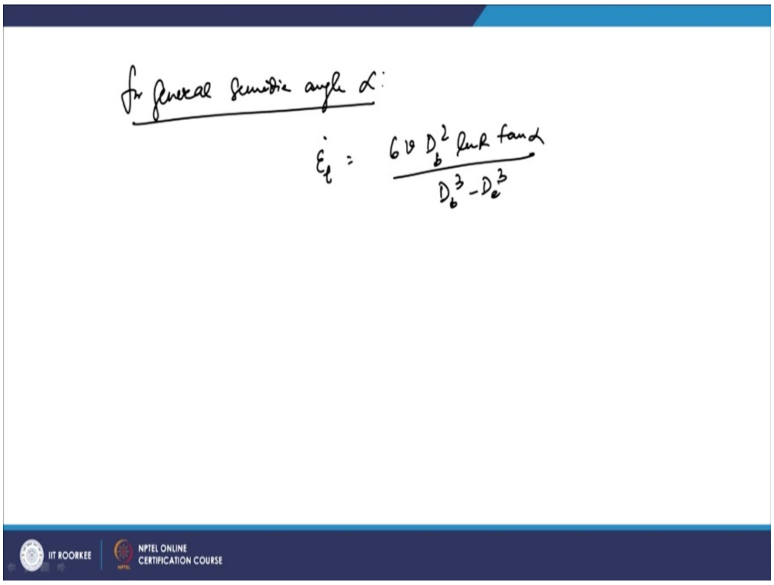

And if you lock c for the general semi die angle α, so, for general semi die angle so, for general semi die angle α, if you try to calculate so, this

$$\dot{\varepsilon}_t=\frac{6\,v\,D_b^2\ln R\tan\alpha}{D_b^3-D_e^3}$$

So, this basically expression can be used to calculate this strain rate, these mean average means strain rate, and that can be used further for calculating the flow stress values once you are given any problem in such type of you know analysis. So, it all can be seen from here how we use that you know flow stress related these when the analysis has been carried out. And using that mean strain rate you can calculate these average means strain rate values. And for that that can be used for solving the different type of problems.

Thank you very much.

Principles of Metal Forming Technology
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Lecture – 34
Extrusion of tubes and pipes, extrusion defect

Welcome to the lecture on Extrusion of tubes and pipes and extrusion defect. So, we discussed about the extrusion process and normally what we have so far studied that depending upon the cross section of the you know cross section at the die outlet. You can have that cross sectional product and you are basically extruding rod or so more conveniently.

However, if you have to make the tubes and pipes that can be made by the extrusion process also; in those cases we have to use the mandrel and with the help of that so what is done is that so when you are pushing from one side, in that case the there will be piercing first with the mandrel and that piercing will be going from one side till the die opening. And then, from there if when you are pressing, then the tube type of structure is produced.

So, if you look at some you know this process what we do is suppose you have a billet.

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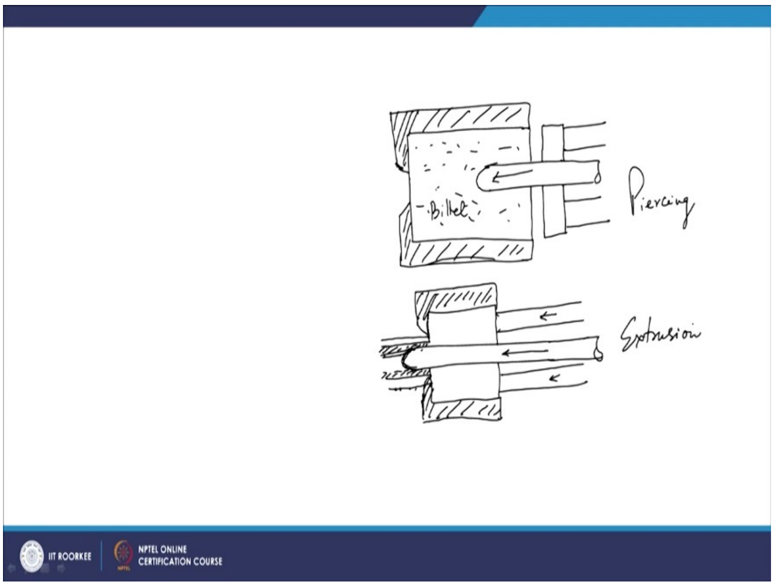

So, this billet is to be propose extruded and in that case you will have a mandrel here.

So, this mandrel actually this will be given the movement and this mandrel basically will be you know pushed from this side and what will happen that now this is your billet. So, this is your billet.

So, you can write that this is your billet and there will be you know piercing mandrel. So, this will be your piercing mandrel and (Refer Time: 02:34) that it will be basically pressurized from that side.

So, you will have here the force which will be working upon them and we see on this way on this side basically you have the die. So, you will have a die opening from here. So, that will be basically the container. So, this container will be there and on both the sides, you will have further on this side you have the container.

So, now, the thing is that this is your mandrel and this will be moving from this side. So, this will be for piercing and this piercing will be going on and also we can have this piercing once it goes. So, when it will be coming at the outlet, it will it will look like you will have the mandrel here coming all together up to the full length and this is your mandrel.

And now, you have this side that the clearance which is there in between that die wall. So, that will be the thickness of this tube which is produced and this way. So, from here you have the.

So, you have billet suppose coming to this part. So, similarly you have a billet on this part also. So, once the mandrel will be here; now once the extrusion process starts. So, what we can see is that you will have the you know die.

So, die will be appearing like this and die opening is there and that way you have the, you know again the pressure mechanism from this side. So, again the movement of this, it has to go till the end part. So, so this way you will have this internal diameter is maintained and this part is the solid pipe, the thickness of this pipe.

So, this way you are getting. So, you will be getting this is not there. So, it will be coming like this. So, this way you will have the production of these kind of seamless tubes which is formed in that. So, what we see is that initially you will have to pierce.

So this will be your piercing process and then, from here once it comes then it will be extruding. Now the thing that we also take we also take the hollow you know billet in in those cases so that you can pierce it; you can without much need of the piercing you can have, but certainly you will have to have because you have to pierce first of all through the billet.

So, you can have the, you know provision of the hollow kind of billet. So, in that case it will be coming here; but the things that when you have the hollow billets in that case the inside you know surface that may get you know oxidized because of the high temperature.

So, in such cases there may be difficulty in such cases. This is your piercing and then finally, you get the extrusion of the, these two type of structures.

Now, there are also many methods of making the seamless pipe and tubing and what has been done in those cases that there has been used of rollers basically and certainly the use of mandrel is there and in that basically because of the you know movement of the rollers of which is there on the top and bottom and many cases there has been 3 rollers also used.

There is a continuous you know production of the seamless you know pipes and tubes. So, in that what has happened there is one you know setup that is known Mannesmann mill.

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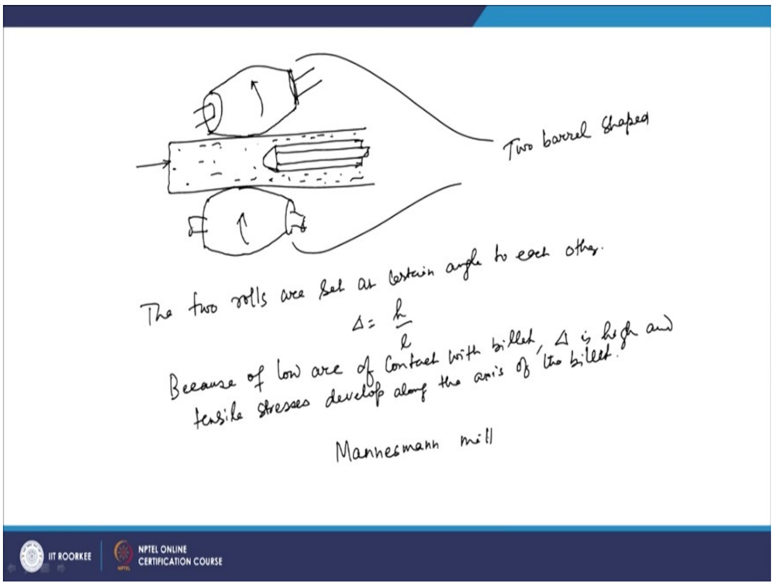

Now, in that case that have been done is suppose you have a billet which is there of suppose this thickness. Now in this case it has to be you know that tube to be extruded; now seamless pipe is to be extruded in the that case. Now this is used for normally the rotary piercing of the, you know steel and copper billets.

Now, in this case there is a mandrel here again. Now this mandrel will be having like this and now, in this case your mandrel has got the, this is attached and getting the support from this side.

So, that way you have this mandrel. Now what happens that you have basically on these 2 sides; what happens that you have the use of such kind of roller. So, this roller is used and this will be further having. So, you have such kind of you know.

So, this roller similarly there is another roller which is on this side. So, this roller will be like this and then, it will be having this shape. So, you will have this way it will be rotating in fact and now what is happening? In this case you have 2 barrel shaped rolls are there.

So, these are the two rolls which are rotating and these are the two barrel shape rolls. So, these are two barrel shaped. So, they are driven rolls and they are set at certain angle to each other. So, they are. So, that the two rolls are set at certain angle to each other.

Now, what is happening that the axial thrust will be developed because of their rotation? The axial thrust as well as the rotation is you know because of the rotation of these barrel shaped rolls, the axial thrust is developed on this and as you have the low arc contact with the billet as you see that there is very low arc of contact here on the billet.

So, the delta (Δ) is basically that is h/l that will be basically very very high and there will be tensile stress developed on these you know along the axis of the billet.

So, what happens that because of that because of this? So, the metal flows at this place and it will be pierce at this point and the metal will be moving. So, so there will be movement of the metals. So, once it is you know flow because of the low you know delta (Δ), there will be high value of delta (Δ) that is h/l.

So, because of, as we know 'l' as we know that $\Delta = h/l$. So, 'l' this is arc contact is very very small. So, because of that there will be you know large amount of tensile stresses which are developed along the axis and this because of this.

So, so, you have because of low arc of contact with billet. So, your delta (Δ) is high and tensile stress is developed along the axis of the billet. Now because of this what happens that you know the opening at the centre.

So, now, here since there will be a mandrel which is you know there; now because of the flow the metal will flow and the cavity will be generated. So, such kind of you know this is piercing is done at this point and you get seamless kind of you know structure.

So, once you have this metal now that that will be only going into this region. So, this is basically Mannesmann mill it is known as. So, this is basically Mannesmann Mill.

Now, the limitation with this Mannesmann mill is that it does not provide very large wall reduction. So, there are basically other types of the plug rolling mills and in those you know plug rolling mills what happens that you have the rolls.

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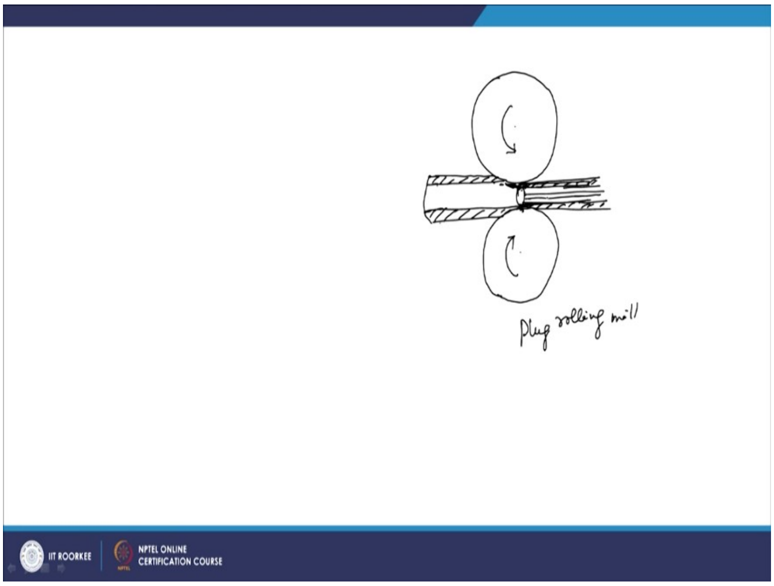

So, you have two rolls. Now in such rolling mill. So, you have basically coming up. So, this way you have the, you know you have the placement of mandrel here at the centre and because of.

So, you certainly there is certain gap between this. So from here basically this tube goes away with certain thickness. Now this is the thickness of the you know the tube which is produced and this is your plug mill. So, this is your mandrel which is there.

Now, here in this case you have a long you know you have tube which is over the long mandrel and you have a plug; this plug this is known as the plug, you have the use of the plug. So, that is why this in this case it is known as the Plug rolling mill. So, you have this is known as Plug rolling mill.

Now in this case this this role is rotating in this direction and this is rotating this direction; anyway this is basically bringing this into it and because of the presence of this plug basically now here the metal moves and it is basically you know deformed at the at this point. And then because of this clearance between this plug and in between and between these two rolls.

So, this is your gap between the rolls your metal comes. So, since we are using this plug we call it as the plug rolling mill. There is another setup which is using that 3 rolls and that is you know the they are the you know are known as the Three roll piercing.

So, you have the three rolls basically under the, that action further the tubing processes are carried out. One of the other variety is the reeling mill which will be burnishing the outside and inside surfaces.

So, these are the four methods which are used for making this seamless pipes and tubes in the extrusion industries. Now further we will discuss about the different types of, different way, the different way how the defects occur in the case of extrusion.

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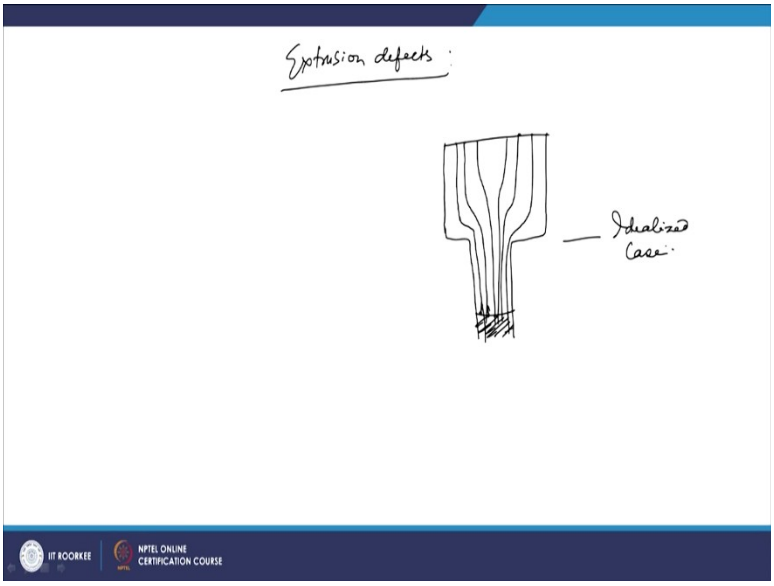

We will talk about extrusion defects. Now when we talk about extrusion as we know that when the metal is you know pressed from one side, the metal has to flow and there will be friction at the container, liner and the billet that interface there will be friction and for you know decreasing that friction we apply the lubricants.

But, again that lubricant is how much lubricant is used how is the you know what is the thickness of that lubricant and how the lubricant flow is maintained. So, it may so happen that the lubricant has properly been along on the surface as the extraction process goes on or it is not.

So, depending upon that basically the friction conditions which is there on the surface that will try to alter the you know the flow of the grains which occurs inside the extrusion and that leads to certain kind of defects in the extrusion.

More importantly when you have the flow of the material and if there is no proper lubricant, in those cases and also there are certain cases when suppose you have more chilling at the container; you know container side I mean on the surface of the billet and if the inside is somewhat cooler, then the surface will chill.

And when you are trying to apply the force, when the metal at the you know surface will resist the any kind of flow and there will be flow only from the middle portion.

So, the result becomes that you have may have the formation of dead zones at certain locations because there will be flow from the centre initially that will go through the cavity and then, later on once you are coming towards later part then your only outer periphery part is left and what you see many a times is that that may lead to certain kind of defects known as extrusion defects.

So, we discuss how you know how this flow of this you know flow of metal which takes place inside the extrusion die.

So, how that is affected. So, suppose what we see is normally what we do is we use the lubricants in the extrusion process and when we talk about the hydrostatic extrusion in that in that we know that in that case when the brittle material extruded. So, you have basically the presence of fluid from all this side. So, that increases the lubrication and that way you can extrude more effectively.

Now, what there has been some if you know efforts to show that how this container wall friction if it is increased how that will be you know visualized how it is it can be seen. So, suppose in normal case, when we talk about the extrusions; suppose you have this way you are doing the extrusion.

Now, in such cases what happens that typically when you have very low friction you know that there is low fiction, low container friction and you have well lubricated film. In that case if you look at your you know flow lines will be appearing like this.

So, they will be coming and then you have smooth flow. So, this is the indicative of the you know flow lines in such cases. So, like that you will have the you know flow which is occurring you know if you can take this as the end part. So, the flow lines appeared like this.

In the case of idealized condition, when you do not have you know very very low you know friction and there is well lubrication in such cases.

Now, if there is you know if there is increased container wall friction and. So, this is for idealized case. Now if you have the increase in the you know wall fiction.

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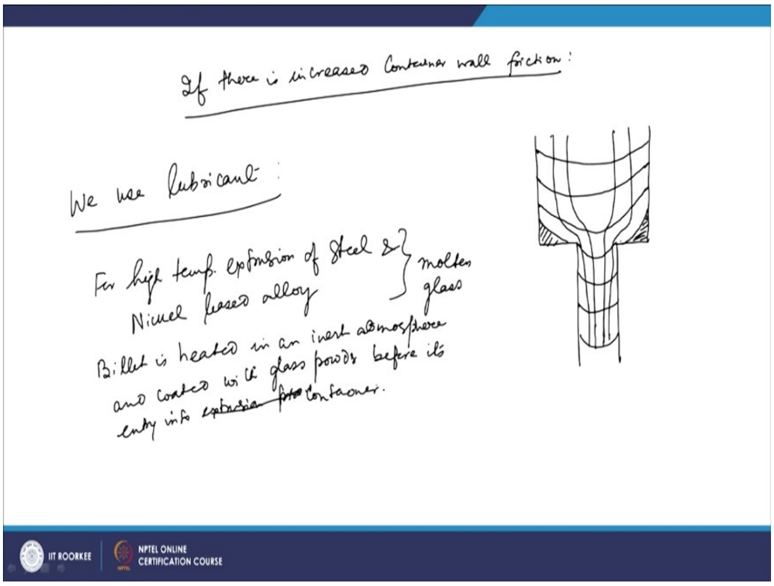

So, if there is increased container wall friction; now, what we see in the in such cases basically your you have distortion of this grid pattern takes place. So, in such cases, now in such cases what happens that when you have increased this friction between the container of wall and the billet.

So, what will happen the, this there will be now, at this places, you will have certain regions which are formed and these regions basically become the dead zones. So, and then you will have this flow going like this.

So, which was earlier you know straight. Now, in this case you will have because there is friction. So, you will have now you know more stress requirement at this sides and the metal will more importantly try to follow in order to flow from the you know middle sides. So, your flow lines will be going like this, after that it will be usual.

Now, in in such cases what happens that basically basic the basic characteristics is that there will be formation of these dead zones. Now main characteristics is that when you have increased wall friction; now what this zone which you see this becomes dead zone and dead zone means they are not taking part in the flow during the initial flow of the metal through that die.

And ultimately what happens that when you are towards the end of the process, then the you are trying to you know push it towards that that and they have basically also in between them there might be temperature loss.

So, now this part which is coming towards the end, there maybe you know extrusion defects and because of very high friction conditions that we led to those cases where you get the ring type of structure, circular ring type of structures if you look in a very transverse direction. Then, you look at the circular ring type of structures in such cases and because of the presence of the dead regions this type of defect known as extrusion defect.

So, also the effect is can be you know interpreted in the sense that you will have the shear zone development because of that and that is not basically productive. So, normally that is you know that you can classified as the redundant work because you are doing the work that is not of use.

So, that is redundant work and even if there is further increase in that you know wall friction on the container, then there will be further stepper zones will be you know formed shear planes will be formed on these sides.

So, this kind of defect basically you will have shear zone formation on the sides and that will be leaving a thin you know skin ultimately in the end which will be left in that. So, normally you must see and it will be very difficult.

So, further you have to clean that material portion if that zone becomes the share zone is further steeper, then a zone of a certain thickness is basically left over on the container part which is to be cleaned every time. So, that is another you know defect which is attached with such you know process.

So, what happens that for avoiding this, we use the lubricants and normally we use we use lubricant. So, what we do in the case of hot extrusion normally where we use the lubricant which has very low shear strength and which also should be stable enough to prevent any breakdown at high temperature.

Now, for high temperature you know steel for high temperature is extrusion when we are doing of steel and nickel based alloy; if you look at these alloys normally we use the lubricant as the molten glass. So, what is happening that you know billet is heated.

So, billet will be heated in an inert atmosphere and coated with glass powder before its entry into extrusion press. So, you know into the container; so, into the container. Now the thing is that once you apply this you know layer of lubricant and then it is placed there.

So, basically it all works first as the lubricant to reduce the friction also once you have applied this. So, that will also serve to reduce any kind of heat transfer which will be taking place from the surface and there has to be a lubricating film which is which has to be maintained and normally that is about 25 μm thick.

So, that you know thickness of the lubricant layer is to be maintained and also you have to have take into account the viscosity of this lubricant and this coating thickness which will be you know that will be depending upon.

So, that depends upon the rate at which the lubricant will be melting and is able to be melting or to be softening. So, based on that this lubricant and you know that coating thickness will be developed and also what is how your expression speed is there.

So, you may have high extrusion speed or you may have low extrusion speed that also you know decides that what should be the thickness; that also decide that how long this you know lubricant has to be you know.

It may so happened that in the initial part a large you know if the speed is basically a large part of the coating has been applied and later on the coating or the lubricant has become less which as increase the you know friction and that may lead to further the formation of the defects.

So, basically lubricant viscosity has a strong effect on the you know extrusion pressure in that case because the ones that lubrication is over that will try to increase the pressure on in the in. So, increase the pressure and then formation of this shear zone and that may lead to the extrusion defects. You may have basically the formation of pipes in between.

So, that is because of the inhomogeneous deformation in the case of direct extrusion because of the movement of the you know faster movement of the central portion of the billet as compared to the you know outer periphery as it happens whenever you know you have the lower temperature may be on the sides as we discussed.

So, in those cases that pipe, pipe may be formed towards the end and that pipe. So, that is why some part of it is basically discarded also. So, there is loss of productivity in those cases. So, these are typically the aspects which are related to the lubrication of this lubrication in extrusion and this is to be you know kept into account while designing this process.

Thank you very much.

Principles of Metal Forming Technology
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Lecture – 36
Analysis of wire drawing and tube drawing processes

Welcome to the lecture of Analysis on wire drawing and tube drawing processes. So in this lecture we are going to discuss about the; you know finding that you know expressions, how to analyze this process when we draw what are the stresses which are caused and how can be find drawing stresses values and so.

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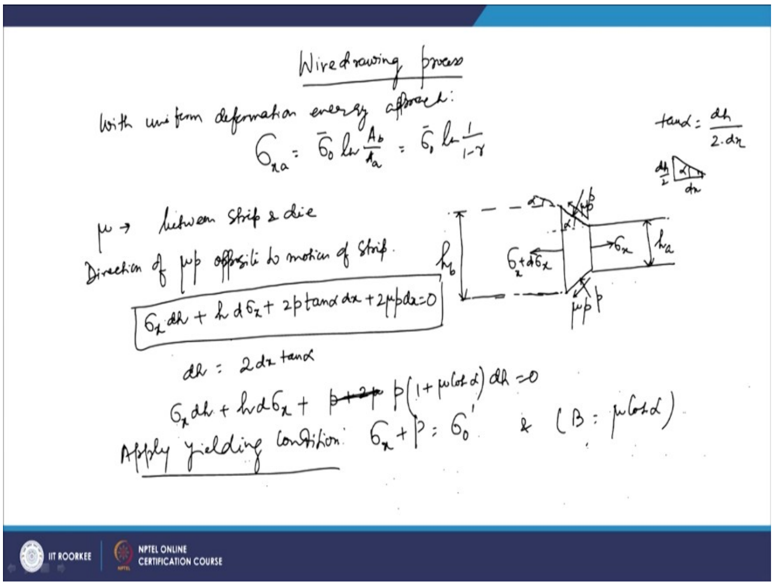

So, if you talk about the wire drawing operations. So, we will start with wire drawing process, now what we see is that if you use this uniform deformation energy approach; then normally with uniform deformation energy approach, we can get the you know draw stress value σ_{xa} and that will be basically average stress value and then $\ln\frac{A_b}{A_a}$.

$$\sigma_{ax} = \acute{\sigma}_0 \ln\frac{A_b}{A_a}$$

So that is what we normally come across and this we also write it as so this is

$$\sigma_{ax} = \acute{\sigma}_0 \ln \frac{A_b}{A_a} = \acute{\sigma}_0 \ln \frac{1}{1-r}$$

So, $\frac{A_b}{A_a} = \frac{1}{1-r}$, So that is how we do using that uniform deformation energy approach and in this case we are not taking into a count the friction. So we are basically neglecting friction which is not very realistic type of consideration and also we are neglecting the effect of transverse stresses which are you know generated and also the redundant you know deformation or shearing deformation. So, that is the neglected in this analysis.

Now if we take this friction into account then as per the line of the strip drawing what we have done earlier, while doing the analysis for the you know strip drawing process initially. We can do similar thing on the wire drawing process also and in that we will see that how we come; so suppose this also in that case also you have.

You have a conical die and so and then so what happens that this your so you have you know a rod which is of height or thickness h_b and then you are drawing with h_a so this is your h_a. So, that is how you know draw and in this case you have this angle as we take it as α. So this is your angle that is α, now what we do so this is your α now the thing is that you apply this pressure ‘p’ on on both the sides.

And what happens that because of the you know movement of this in this zone, deformation zone, movement of the you know metal in the deformation zone you have the you know application of the frictional stress, frictional force will be acting and its direction will be opposite to the movement so the μp will be acting in this direction.

So, your μp is acting in this direction, so this μp and this is also μp so μp is the frictional you know stress which is acting in that case. Now apart from that we can take this as the at this point we can have the value of stress as σ_x and at this point we will have stress being of dx you know this being dx, so, this will be $\sigma_x + d\sigma_x$.

So we can further do the you know you know force balance equations, you can put in and then again what we have done earlier we will use the equation of equilibrium or the you know criteria of failure of material and from there you will get the expression. So you will have basically the μ is existing that the coulomb friction is there, so this is coulomb coefficient of friction it is between strip and die, so this is your coulomb friction coefficient and we know that $\mu\, p$ its direction will be opposite to the you know motion of the strip.

So direction of $\mu\, p$ opposite to motion of strip, now the thing is that if you try to take into account the balance of the forces in the x direction what we see is that the

$\sigma_x dh + h\, d\,\sigma_x + 2\,p \tan\alpha\, dx + 2\,\mu\, p\, dx = 0$

So, this is this equation you will be getting from this you know by if you assume if you take the balancing of the forces because, you have this is your $\mu\, p$ so $\mu\, p\, dx$ so that will be your its component in that direction. You will have $h\, d\,\sigma_x$ so because σ_x will be removed and then this way you will have this expression coming up.

Then now the if you take that *dh* if you take the *dh, dh* is basically $dh = 2\,dx \tan\alpha$ because $\tan\alpha$ will be this by this, so this is your *dh/2* and this is your *dx*. So as you see that this *dh/2* and this is your *dx* and this is your α, so $\tan\alpha = \frac{dh}{2\,dx}$, so that is why you got $dh = 2\,dx \tan\alpha$.

So, if you put that what you get is

$\sigma_x dh + h\, d\,\sigma_x + p dh + \frac{\mu\, p\, dh}{\tan\alpha} = 0$

So it will be here since $h = 2\,dx \tan\alpha$, because this is $dx = \frac{dh}{2\tan\alpha}$, $\tan\alpha$ will be in the denominators that will be $\cot\alpha$. So that way we got

$\sigma_x dh + h\, d\,\sigma_x + p(1 + \mu\cot\alpha) dh = 0$

So this is the term which you got. Now if you apply the you know the yielding criteria so if you apply the yielding condition. Now, if you apply the yielding condition what you say we have already seen that you will have $\sigma_x + p = \sigma_0'$ and also we take the $\mu\cot\alpha$ as B. So we take $B = \mu\cot\alpha$, so it will be *1+B*, so p will be $\sigma_x - \sigma_0'$.

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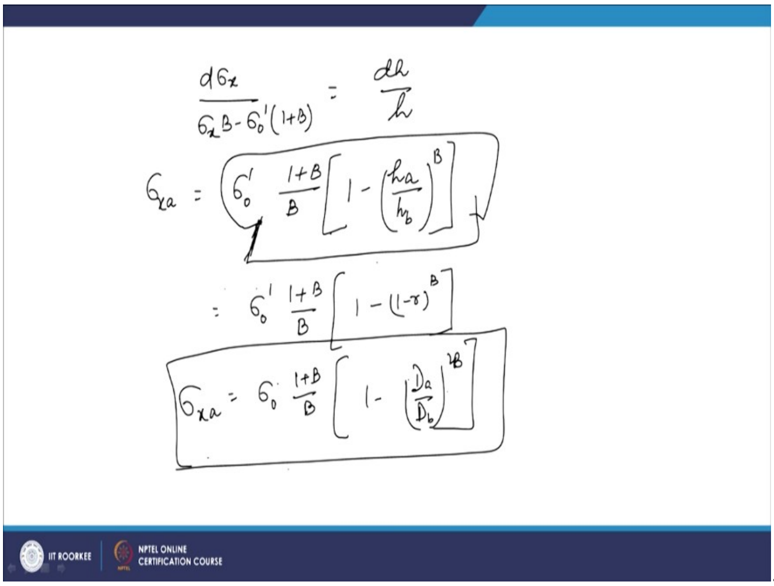

So, if you rearrange this terms then what we get we get that

$$\frac{d\sigma_x}{\sigma_x B - \sigma_0^{'}(1+B)} = \frac{dh}{h}$$

So this is by rearranging this term so in this expression in place of p we will be putting, so $\sigma_0^{'} - \sigma_x$ then into $1+\cot\alpha$ and $\mu\cot\alpha$ will be B so $(1+B)\,dh$.

So, that way you will be having dh term at one place and $d\sigma_x$ at one place, so that way you can get this expression $\frac{d\sigma_x}{\sigma_x B - \sigma_0^{'}(1+B)} = \frac{dh}{h}$

Now we have to integrate it and if we are taking B and $\sigma_0^{'}$ as the constant values in that case we can directly integrate them to find the drawing stress and in that case what we get that expression as sigma x a, because again you have to put these boundary conditions in that x equals to B you will have sigma x values will be 0. So that can be put in and then accordingly once the integration constant will come then you can further get the expression, so this expression what you get is

$$\sigma_{xa} = \sigma_0^{'} \frac{1+B}{B}\left[1 - \left(\frac{h_a}{h_b}\right)^B\right]$$

So that is what we get the value of the you know; so this value we are getting in the case of the σ_x, and this can further be written as $\sigma_0' \frac{1+B}{B}\left[1-(1-r)^B\right]$, and $\frac{h_a}{h_b}$ will be $(1-r)$.

So what we see here is that we can find the value of this σ_{xa} in that situation. We can also we can do that by using that another terminology, so if you take that circumference into picture in that case since you have the conical dies so taking the circumference of the die into you know picture you may have the expression like:

$$\sigma_{xa}=\sigma_0 \frac{1+B}{B}\left[1-\left(\frac{D_a}{D_b}\right)^{2B}\right]$$

So that way you have the alternate expressions which are coming in that that case now coming to the different analysis by another researcher that is your Johnson and Roweigh (Refer Time: 13:22) has given certain analysis and they have talked about the surface area of contact between wire and die.

So if based on this Johnson and Roweigh (Refer Time: 13:34) analysis when the surface area between the die and the you know wire is taken into account, if you look at that so suppose you take the you know 1 such case.

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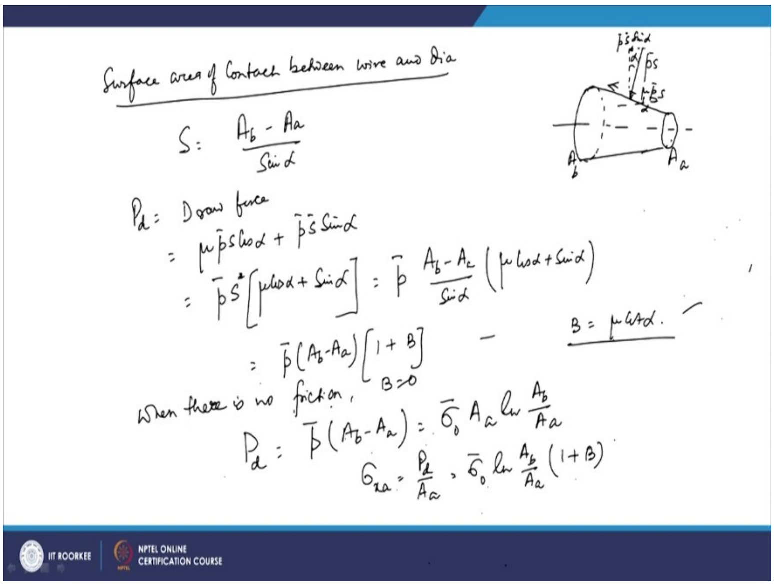

So, now this is the you know wire which is drawn and you have, so here you have this is your here this is this area is A_b and this is area is A_a. So in that case and your 'p' is being acted in this fashion, so what we see is that you have this as α. So what you see is that this one this will be $\acute{p} S\ \alpha$, so that is we will see how this you know s will be looking at.

Now this surface area of contact so if you talk about the surface area of contact between wire and die, so this surface area basically will be $S=\frac{A_b-A_a}{\sin\alpha}$, this is the you know surface area. Now the mean pressure so this will be now this you know which is acting here so this is basically this into as this force which is acting here the this multiplied by'S'.

Now, the draw force which is calculated if you try to find these draw force is P_d that is your draw force, so it is basically to be balanced by the you know frictional force and the horizontal component of the you know pressure this one this normal pressure, so it will be basically $P_d=\mu\acute{p} S\cos\alpha+\acute{p} S\sin\alpha$.

So basically that is your you know draw force you have this frictional force as well as the normal pressure which is there on this it has its component and based on that that will be so you have frictional force acting that way this also is acting in that direction, so $\acute{p} S$ its alpha part being normal part and its horizon its horizontal part it will be $\acute{p} S\sin\alpha$.

Similarly, you will have $\mu\acute{p}$ will be acting in this direction $\mu\acute{p} S$ will be acting in this you know direction and its horizontal component will be $\mu\acute{p} S\cos\alpha$. So $\mu\acute{p} S$ is since it is acting in this direction this being α and this is being $\mu\acute{p} s$ so it will be $\mu\acute{p} S\cos\alpha$ and $\acute{p} S\sin\alpha$. So what we can write is that we can write as $\acute{p} S[\mu\cos\alpha+\sin\alpha]$. So what we can write 'S' we have already found $S=\frac{A_b-A_a}{\sin\alpha}$, so it will be $\acute{p}\frac{A_b-A_a}{\sin\alpha}[\mu\cos\alpha+\sin\alpha]$.

So it can further be written as so $\frac{\cos\alpha}{\sin\alpha}=\cot\alpha$. So, we can further write$\acute{p}(A_b-A_a)[\mu\cot\alpha+1]$. So $\mu\cot\alpha$ is we have defined as B so it will be $\acute{p}(A_b-A_a)[1+B]$. So this is how we can find these draw force.

Now, what we can come we can also find we can also draw the conclusion from this expression that if B is 0. Now B will be 0 when α will be 0 or μ will be 0, so when the frictional you know there is no friction in the case of absence of friction so when there is no friction in that case your B becomes 0.

So, your P_d draw force that becomes $P_d = \acute{p}(A_b - A_a)$, so that is what it is coming and then you can write as $\acute{\sigma}_0 A_a \ln \frac{A_b}{A_a}$. So this is the same one which is similar to the one which we had earlier done during the uniform deformation energy approach.

So, there we had got the so from in those cases if you, so this is for the without the case of friction so if you apply the friction you will have this expression. Now in that case if you so we draw if you apply try to have the expression with draw stress with friction, so ultimately $\sigma_{xa} = \frac{P_d}{A_a}$. So if you are trying to draw that I mean find the value of draw stress and if taking friction into account so friction is here, so,

$$\sigma_{xa} = \frac{P_d}{A_a} = \acute{\sigma}_0 \ln \frac{A_b}{A_a}(1+B)$$

So, that is expression when we try to if you try to see the approach by the Johnson and Roweigh (Refer Time: 20:15) they have done the analysis we get the similar you know you know methods for similar expressions for finding the draw stress values. So you may be told to sometimes solve the problems based on such you know wire drawing operations.

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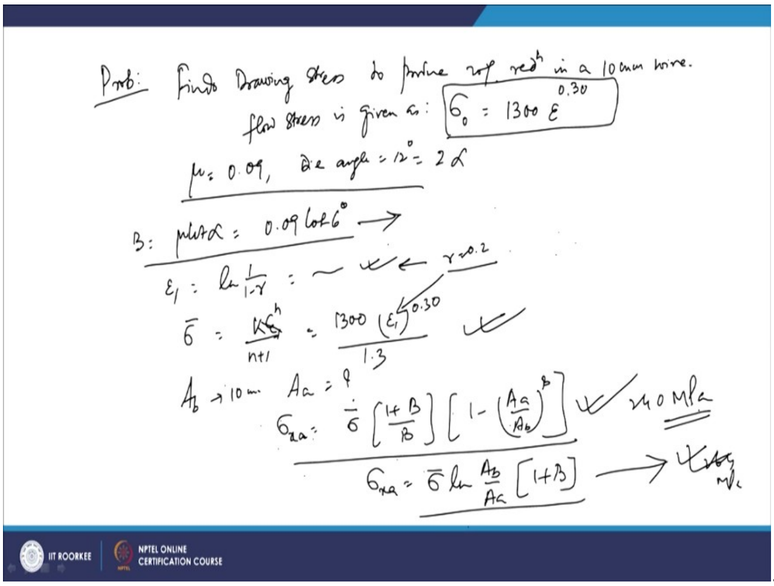

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So you may have certain problem suppose you may face with this problem suppose you have a problem where you have to find the drawing stress where you have to produce 20% reduction and in a 10 mm wire, so the flow stress may be given and and flow stress is given as; so the flow stress is given as suppose $\sigma_0 = 1300\,\varepsilon^{0.3}$. Suppose this is how the flow stress value given in terms of MPa this is the flow stress given you have to find the drawing stress.

So, you can find the drawing stress with the help of the different you know approaches and so you have one by using that strip drawing approach another may be by the method which is given by the Roweigh (Refer Time: 21:39). So suppose in that you know that you have been given this you know the so die angle is given as the 12° so the die angle will so μ is given as 0.09 and die angle is given as suppose bar 12°. Now the thing is that based on this you can have so first of all you will find the B and we know that the B is $\mu \cot\alpha$, so μ is 0.09 and then $\cot\alpha$ this is basically 2 α, so cot 6 degree this way you can get the value of B.

Similarly, you can find the so now, this $\varepsilon_1 = \ln\frac{1}{1-r}$. So we know that there is a 20% reduction so it will be $\ln\frac{1}{1-0.2}$, so it will be *1/0.8* and that that value may be calculated. So 'r' is basically your 0.2 so this way you can get this value also then now you can get the value of every stress value and that will be that is given by this value.

So this will be normally $\frac{k\varepsilon_1^n}{n+1}$, so once you know this 'k' is given 1300 and then this ε_1 is taken from here and then that raised to the power you know 'n', so 'n' is given as 0.3 and then you have *1+0.3,* so 1.3.

So, this way you can get the you know average stress value and then you have the value of A_b and A_a is as given so based on that A_b = 10 mm, so similarly A_a = 8 mm and based on that you can have the expression: $\sigma_{xa} = \acute{\sigma}_0 \frac{1+B}{B}\left[1-\left(\frac{A_a}{A_b}\right)^B\right]$

So, you know every term and you can get this value, similarly you can use the Roweigh analysis and from there you can get the expression: $\sigma_{xa} = \acute{\sigma}_0 \ln\frac{A_b}{A_a}(1+B)$

So from here also you can get the values and also you can see that how much they are approaching each other.

So, you can do the work you can do the analysis and in this case it comes 240 and this comes as 264 MPa. So you can check this by solving and you will get the confidence that how to you know get these values in such cases.

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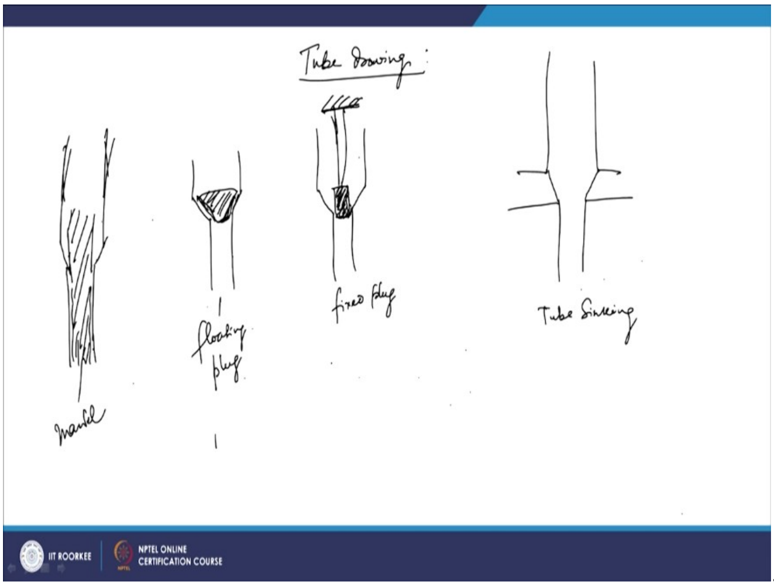

Now, we will come to analyze the tube drawing process. So, as we discuss that we do the tube drawing process either without using the plug or mandrel or we can use the plug or mandrel, now when we do not use these you know plug and mandrel and then it is known as tube sinking and other wise when we use the plug you have the fixed plug or the floating plug or you have the mandrel. So what we do is normally when you have you do not use when we do not use the you know this any kind of plug or mandrel then it is known as tube sinking. Then further when we use the we use the plug which is fixed or may we use the plug which is floating, so in that case if suppose you have this as the die.

Now, in that case if you are using a plug, so this plug will be like is and this plug will come it will be coming here and then this may be fixed. So if it is fixed at these point and if you are doing that you know tube drawing known as fixed plug process of you know this is a case of fixed plug.

You may have a floating plug also, so the floating plug comes like this you have this situation and then you have the floating plug which will be the in between there will be clearance and then you will have this as the floating plug. So this is a case of floating plug otherwise you may have the use of the mandrel, so in that case you have see the use of the whole mandrel is there.

So this mandrel will go like this and based on that so this is your whole mandrel. So, you can use the mandrel itself directly and that is the use of mandrel to get, so these are the different methods of using for making the tubes in the you know in case of the tube drawing operations now when we talk about the analysis of this tube drawing processes, now what happens that in the in the case of this tube drawing you have either use of the plug or the mandrel.

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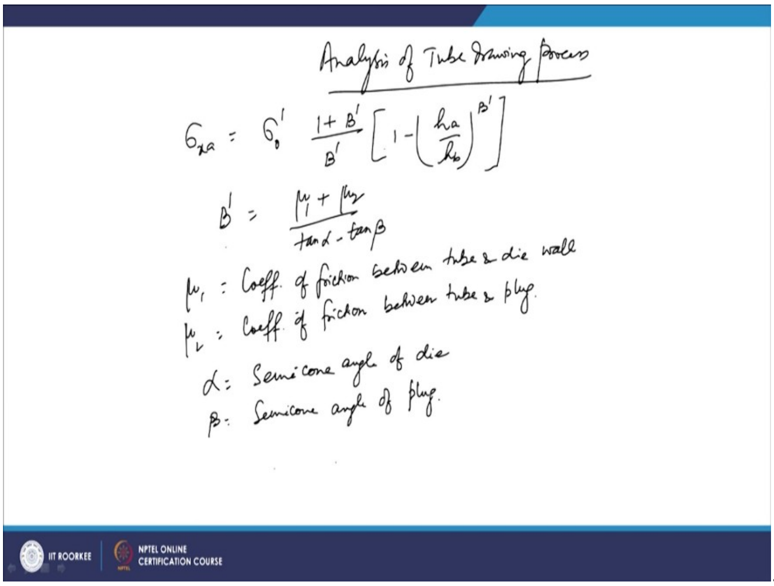

And when we are drawing with a plug in that case similar to the equation which we have done earlier, we can have the σ_{xa}. So here you have at the tube so you have:

$$\sigma_{xa} = \sigma_0' \frac{1+B'}{B'}\left[1-\left(\frac{h_a}{h_b}\right)^{B'}\right]$$

So, what happens that now in this case you have μ_1 and μ_2 both are you know working. Now in this case what happens that your B' another parameter is defined and this parameter is

$B' = \frac{\mu_1 + \mu_2}{\tan\alpha - \tan\beta}$. Now in this case the μ_1 is the coefficient of friction between tube and die wall and that the and μ_2 will be your coefficient of friction between tube and plug.

Now, what we see you can see from here you would have two places where you have coefficient of friction between and between this and the you know tube and then again you have the coefficient of friction between the tube as well as the plug, you have two interfaces which is there and you will have the friction acting there and that is why you will have you know two friction terms in such cases. So you will have and that is why the B' is defined by taking into this and μ_1 and μ_2 otherwise you had only one interface or the friction was working.

When we use this plug or mandrel in those cases you have this $B' = \frac{\mu_1 + \mu_2}{\tan\alpha - \tan\beta}$ and α is the semi cone angle of die and similarly you have β as semi cone angle of plug. So if you have a cylindrical plug then that case that β will be 0 and based on that you will have the you know value of this σ_{xa} can be calculated.

So, that happens normally in the case of tube drawing and finding all these parameters you can find the value of σ_{xa}. Now this B' that may take the different value may be you may have depending upon the mandrel or the which is moving many times then in that case it will be $\frac{\mu_1 - \mu_2}{\tan\alpha - \tan\beta}$ and in that case you will find the B' to be different and then accordingly you will find the value of σ_{xa}, so that is how so the thing is that you must know that why this term is coming this term is coming.

Because there is friction at two places one is at these you know there are two places where this pipe is facing the friction one is on the left side and another is toward the right from the plug or mandrel side. So because of that this is coming, so there are many analysis by different authors.

And basically the concept is the same depending upon that either uniform deformation energy approach or by is tube drawing approach process approach you can or using that slab method you can have the calculation of the σ_{xa} and so this is how this analysis should be carried out you can solve problems based on that you have more clarity on these topics.