MFT-2

Topics

- Macroscopic Plasticity and Yield criteria
  - Tresca
  - Von Mises
- Effective Stress and Strain
- Flow Rules (In the plasticity regime)
- Theoretical strength of material
- Atomistic origin of elastic and plastic behavior

Quick Recap: Hydrostatic and Deviatoric components

- Deformation or strain can be caused by *dilation*, change in volume, or *distortion*, change in shape
- Mean strain or hydrostatic strain are involved in volume changes, while strain deviators cause distortion
- Similarly hydrostatic component of the stress tensor produces only elastic volume changes and does not cause plastic deformation
- Yield stress of metals is usually independent of hydrostatic stress (corollary: Plastic deformation does not result in volume change)
- Deviatoric stress involves shearing stress and hence is important in causing plastic deformation

Quick Recap: Constitutive Relations

- Hooke's law relates elastic stress and strain

$$e_x = (1/E)[\sigma_x - \nu(\sigma_y + \sigma_z)], \qquad \gamma_{yz} = (1/G)\tau_{yz},$$

$$e_y = (1/E)[\sigma_y - \nu(\sigma_z + \sigma_x)], \qquad \gamma_{zx} = (1/G)\tau_{zx},$$

$$e_z = (1/E)[\sigma_z - \nu(\sigma_x + \sigma_y)], \qquad \gamma_{xy} = (1/G)\tau_{xy},$$

- For an isotropic material out of the four elastic constants, E, G, B and ν, only two are independent

$$E = 2G(1 + \nu), \text{ or}$$

$$G = E/[2(1 + \nu)].$$

Plasticity

- In elastic regime, Hooke's law relates stress and strain, where any stress results in strain
- In plastic regime, a minimum stress (yield stress) must be reached before deformation can be attained
- The relation between plastic stress-strain is mere approximate, compared to elastic-stress-strain relations which are more "exact"
- In plastic regime, equations are obtained assuming certain models (Von-Mises, Tresca) by uniting experimental observations with mathematical expressions in a phenomenological manner

Yield Criteria

- Mathematical expression of the states of stress that will induce yielding or the onset of plastic deformation. In general: $f(\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy}) = C.$
- In terms of principal stresses: $f(\sigma_1, \sigma_2, \sigma_3) = C.$

Assumptions for isotropic materials:

- No Bauschinger effect (yield strength in tension is equivalent to yield strength in compression)
- Constancy of volume ("Poisson's ratio" = 0.5 for plastic regime)
- Magnitude of the mean normal stress does not influence yielding

- Constraints imply that the following criteria are not universally acceptable for all solids or for all conditions under which loads are applied
- Violations of these assumptions require new criteria

Von-Mises Criteria

- Reference: Section 3.4: Mechanical Metallurgy by Dieter, 3rd EDITION

$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2\right)\right]^{1/2}$$

$$\sigma_1 = k$$

$$k = \frac{1}{\sqrt{3}}\sigma_0 = 0.577\sigma_0$$

Tresca Criterion

Reference: Section 3.4: Mechanical Metallurgy by Dieter, 3rd EDITION

Combined stress test

Reference: Section 3.5: Mechanical Metallurgy by Dieter, 3rd EDITION

$$\sigma_1 = \frac{\sigma_x}{2} + \left(\frac{\sigma_x{}^2}{4} + \tau_{xy}^2\right)^{1/2}$$

$$\sigma_2 = 0$$

$$\sigma_3 = \frac{\sigma_x}{2} - \left(\frac{\sigma_x{}^2}{4} + \tau_{xy}^2\right)^{1/2}$$

Yield Locus

Under biaxial plane stress ($\sigma_2 = 0$)

Von Mises yield criteria:

$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_0^2$ ------ (1)

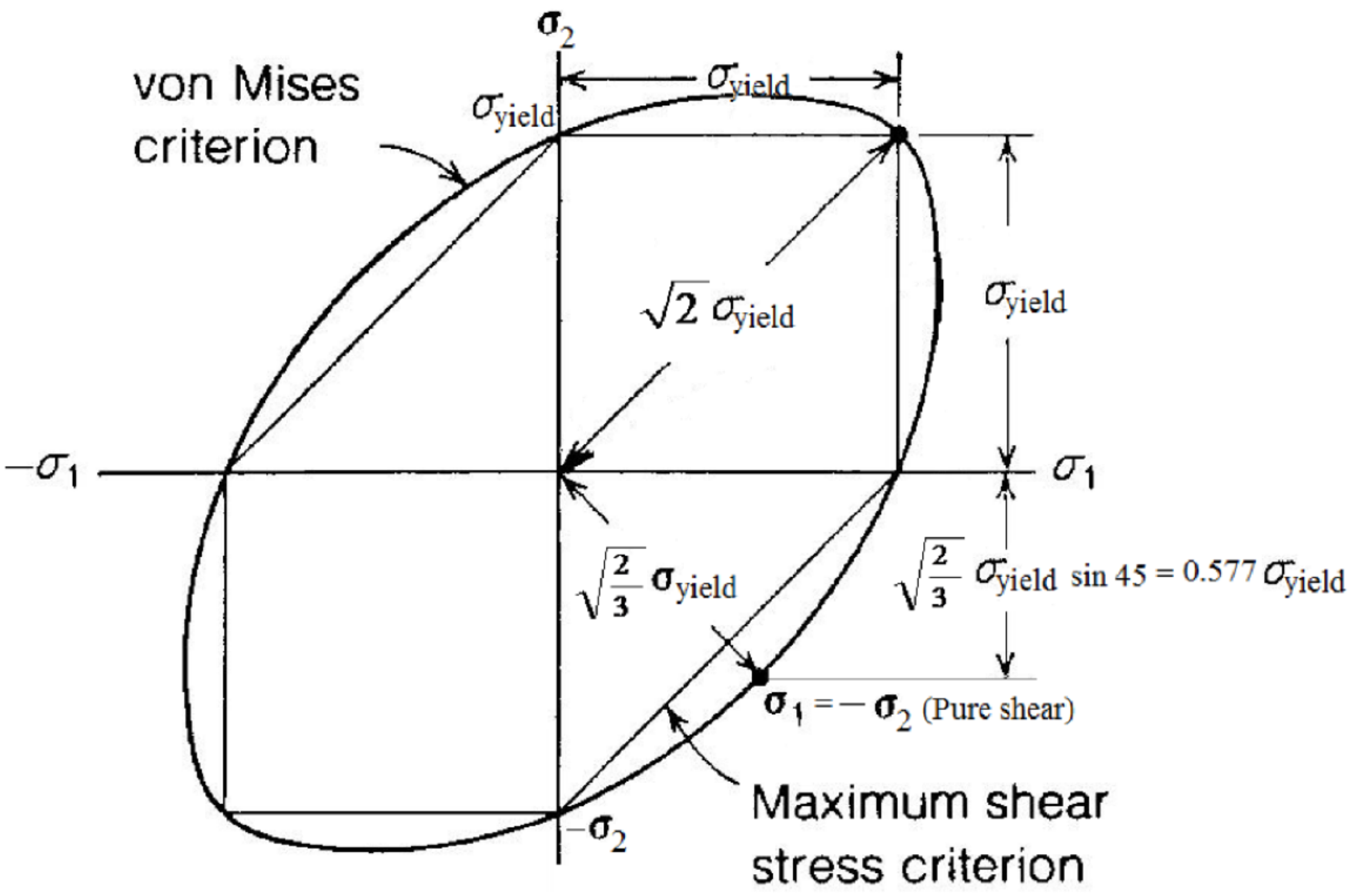
where $\sigma_0 - YS$

Eq. (1) ellipse

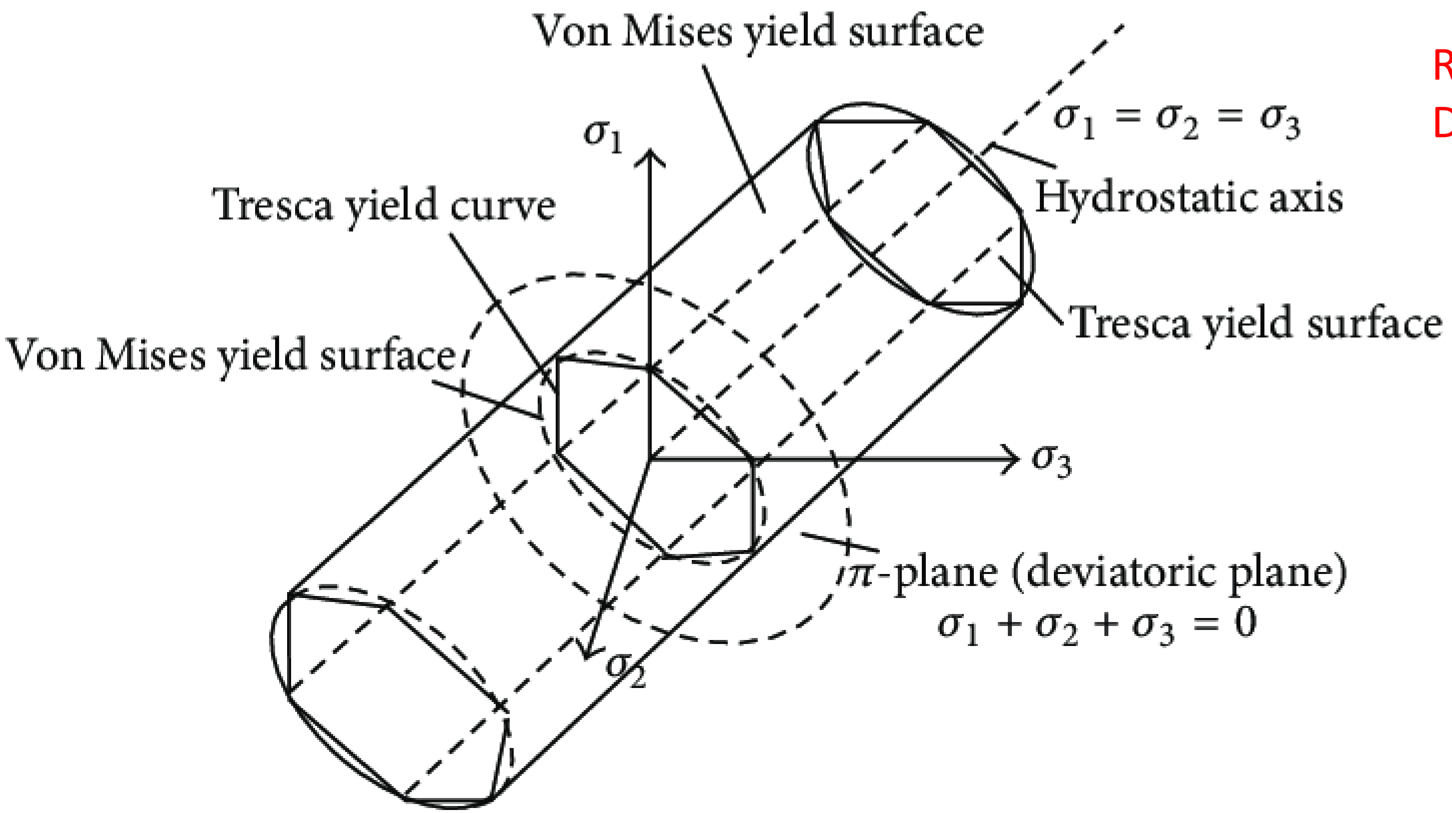
Major semi-axis $\sqrt{2}\sigma_0$

Minor semi axis $\sqrt{2/3}\,\sigma_0$

Plot of Eq.(1)→ Yield Locus

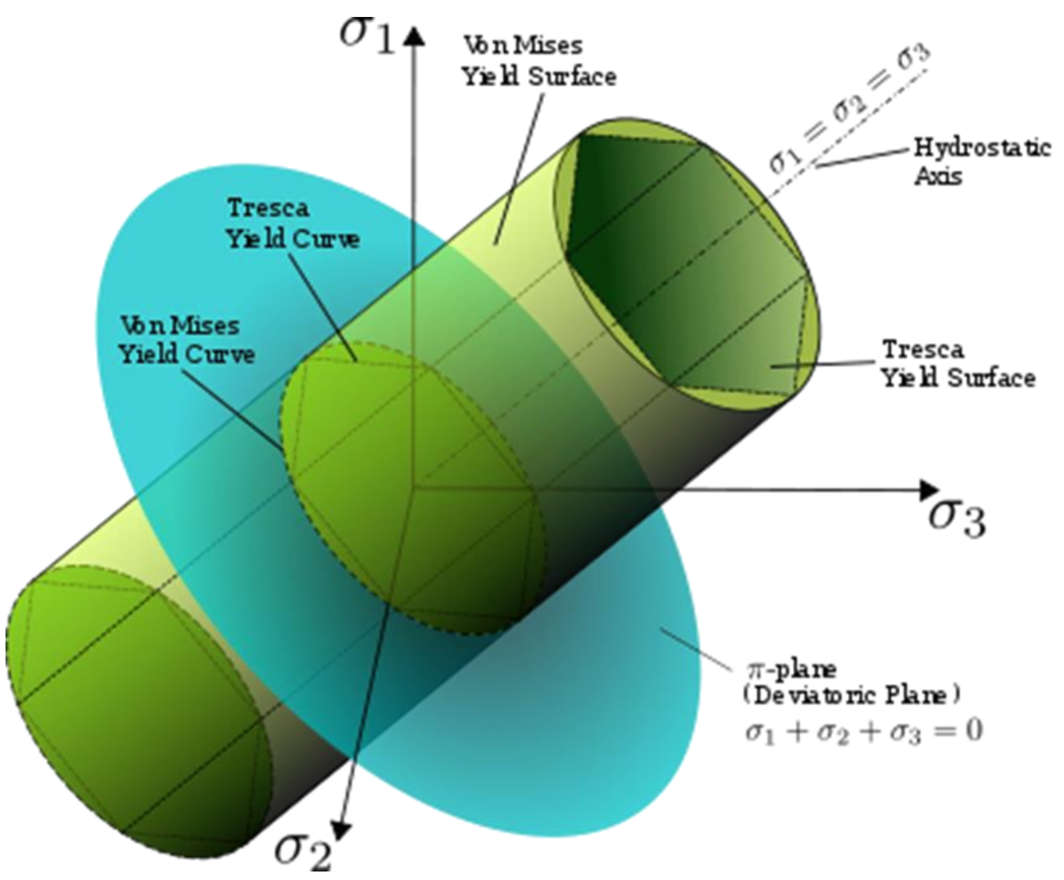


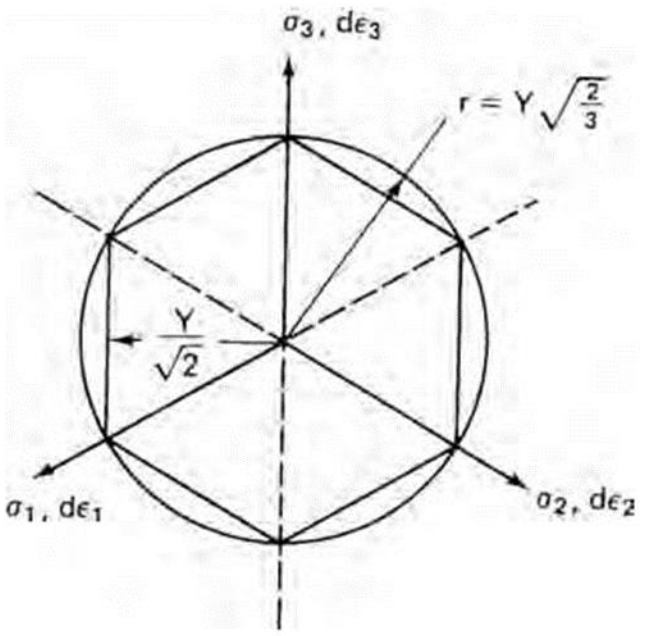
Yield Surface



Reference: Section 3.8: Mechanical Metallurgy by Dieter, 3rd EDITION

Yield Surface

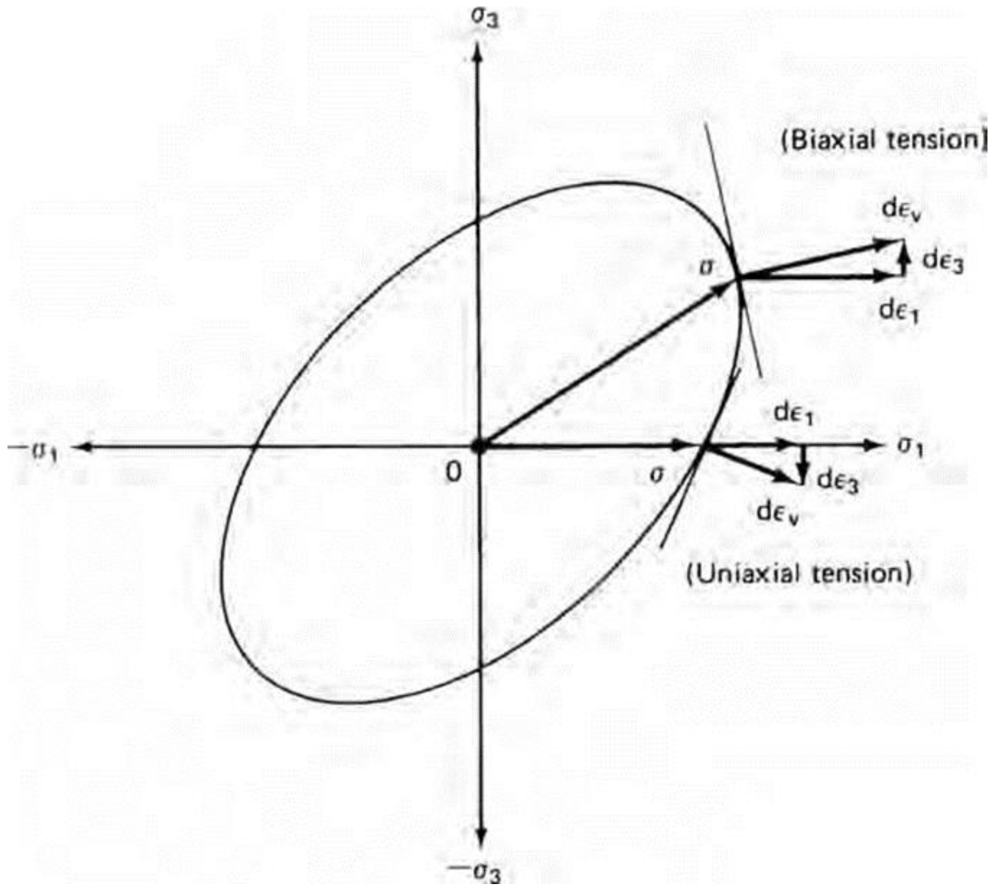




Cross-section for a constant hydrostatic stress:

$$\sigma_1 + \sigma_2 + \sigma_3 = c$$

Normality rule



Reference: Section 3.8: Mechanical Metallurgy by Dieter, 3rd EDITION

Elements of the theory of plasticity

Octahedral shear stress and shear strain

Reference: Section 3.9: Mechanical Metallurgy by Dieter, 3rd EDITION

Invariants of stress and strain

Reference: Section 3.10: Mechanical Metallurgy by Dieter, 3rd EDITION

Levy Mises equation

Reference: Mechanical Metallurgy by Dieter, 3rd EDITION