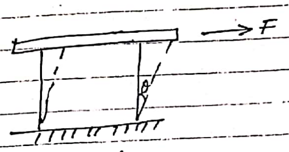
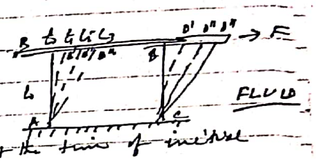
Fluid Mechanics → Fluid + Mechanics.

FLUID — A fluid is a substance which deforms continuously under the application of shear (Tangential) stress no matter how small the shear stress may be.

OR

A fluid is a substance that cannot sustain a shear stress when at rest.





Solid

After same time it will attain equilibrium (become stable).

t_0 = at the time of initial

$t_0 < t_1 < t_2 < t_3$

The fluid keeps on deforming continuously.

Fluid Mechanics is that branch of science which deals with the behaviour of fluid (liquid & gases) at rest as well as in motion.

It deals with static, kinematics and kinetic aspect of fluids.

REST ⟶ Fluid Statics

Motion

(i) Force (& pressure) not consider → fluid Kinematics

(ii) Pressure force + motion } consider ⟶ fluid Dynamics.

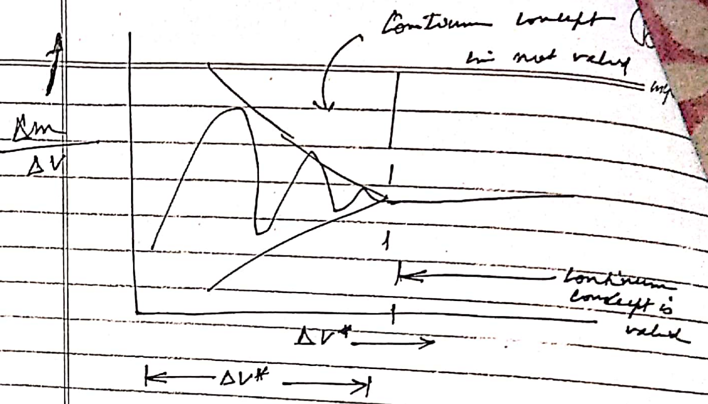
CONCEPT OF CONTINUUM.

Althrough the properties of a fluid arise from its molecular structure, eng'g problems are usually concerned with bulk behaviour of fluids. The members of molecules involved is immense and seperation between them is normally negligible by comparison with the distances involved in the practical situation being studied. Under these conditions, it is usual to consider fluid as continuum or a hypothetical continuous substance and the conditions at a point as the avarage of every large number of molecules surrounding that point within a distance ~~(Althou~~ which is large compared with the mean inter molecular distance. This is although very small in absolute terms)

Quantities such as velocity and pressure can be considered to be constant at any point and constant and changes due to molecular motion may be ignored. Variation in such quantities can be also assumed to take place smooth from point to point.

For Rarified gas these assumption is not valid (continuum)

$$R = \frac{\text{Mean free path of molecules}}{\text{Dimension of the problem}} >>$$



$$\rho = \lim_{\Delta V \to \Delta V'} \frac{\Delta m}{\Delta V}$$

ΔV^{*} is about $10^{-9}\ mm^3$ for all liquid and gas at atmospheric pressure. For air this volume contains 3×10^7 molecules.
At low pressure for gases molecular spacing and mean free path are comparable to or larger than physical size of the system. In such case continuum concept is invalid.

Fundamental Difference between Solid Mechanics and Fluid Mechanics.

	SOLID MECHANICS	FLUID MECH.
(1)	In SM. Strain = f (Applied stress)	In FM Strain = f (Rate of strain)
(2)	Deformation disappear where force is removed (If elastic point is NOT exceeded)	A fluid continuous to flow as long as the force is applied will NOT regain its original form when force is removed.

Difference between LIQUID and GAS.

	Liquid	Gas
(1)	Difficult to compress IN-COMPRESSIBLE	Easy to compress COMPRESSIBLE
(2)	Free surface forms if volume of liquid is less than container	No free surface occupy, gas occupy whole container

CHARACTERISTIC Properties of fluid

(1) Density or mass density (ρ)

$$\rho = \lim_{\delta V \to \delta V^*} \frac{\delta m}{\delta V} = \frac{\text{mass}}{\text{density}}$$

Unit — $kg\ m^{-3}$ Dimension = ML^{-3}

at $P_{air} = 1.013 \times 10^5\ N\ m^{-2}$, $T = 288.15$ } $\rho_{air} = 1.23\ kg\ m^{-3}$

$\rho_{water} = 1000\ kg\ m^{-3}$. at $4°C$.

(2) Specific weight γ

= weight per unit volume

$\gamma = \rho g$ unit $N\ m^{-3}$

Dim $ML^{-2}T^{-2}$

(3) Relative density (Sp. ~~density~~ gravity).

It is defined as the ratio of mass density of a substance to some standard mass density.

Solid / liquid $\rightarrow$ water

gas $\longrightarrow$ H_2

$$S_{sub})_{SOLID/LIQ} = \frac{\rho_{sub}}{\rho_{H_2O}}$$

$$S)_{gas} = \frac{\rho_{sub}}{\rho_{H_2}}$$

(4) Specific volume

— Ratio of volume of a fluid occupy by mass of fluid.

$$v = \frac{\text{volume of fluid}}{\text{mass of fluid}} = \frac{1}{\rho}.$$

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Bulk Modulus (E_v) of Elasticity

$$E_v = \frac{dp}{-\frac{d\forall}{\forall}}$$

Where dp = differential change in pressure to create differential change in volume $d\forall$ of a volume $\forall$

Since

$$m = \rho\forall \Rightarrow 0 = \rho\, d\forall + \forall\, d\rho$$

$$\Rightarrow \frac{d\forall}{\forall} = -\frac{d\rho}{\rho}$$

$$\Rightarrow E_v = \frac{dp}{d\rho/\rho}$$

Compressibility is the reciprocal of Bulk modulus of elasticity.

for gas

Isothermal process

$$\frac{p}{\rho} = \text{const}$$

$$\Rightarrow E_v = p$$

for adiabatic or Isentropic process

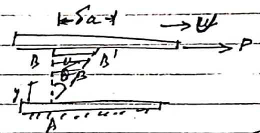
$$\frac{p}{\rho^k} = \text{const} \qquad k = \frac{c_p}{c_v}$$

mono atomic = 1.66
dia — =
air = 1.40

$$\Rightarrow E_v = kp$$

Drop of honey and ...

VISCOSITY — Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another layer of fluid.



b

$$P = \tau A$$

$A \rightarrow$ effective area of upper plate area

No slip condition

$U \rightarrow$ velocity of

$u = u(y)$ if this is linear relationship

$$= \frac{Uy}{b} \qquad \frac{u}{y} = \frac{U}{b} = \frac{du}{dy}.$$

δt
$\delta\beta$

$$\tan\delta\beta \simeq \delta\beta = \frac{\delta a}{b}$$

$$\delta\beta = \frac{U\,\delta t}{b}$$

rate of shear strain

$$\dot{\gamma} = \lim_{\delta t \to 0} \frac{\delta\beta}{\delta t} = \frac{U}{b} = \frac{du}{dy}.$$

In some fluid (Newtonian fluid)

$$\tau \propto \dot{\gamma}$$

$$\tau \propto \frac{du}{dy}$$

$$= \mu \frac{du}{dy}$$

$\mu \rightarrow$ viscosity, or absolute viscosity or dynamic viscosity.

$$[\mu] = \frac{[\tau]}{[\frac{du}{dy}]} = \frac{FL^{-2}}{LT^{-1}} \cdot L$$

$$= \frac{M\cancel{L}T^{-2}L^{-2}}{\cancel{LT^{-1}}} L$$

$$= ML^{-1}T^{-1}$$

unit (SI)

$$= \frac{N/m^2}{\cancel{m}/s} \cdot \cancel{m}$$

$$= Ns/m^2 = Pa\,s$$

CGS unit = dyne s / cm² = dyne s cm⁻² → poise

$$1\,Ns/m^2 = \frac{10^5\,dyne\,s}{10^4\,cm^2}$$

$$= 10\text{ poise}$$ $\boxed{\text{Water} = 1\,cP}$

1 centipoise = 1/100 poise

~~Water = 1 cP~~

1 P = 100 cP

KINEMATIC VISCOSITY (ν)

It is the ratio of dynamic viscosity and the density of the fluid.

$$\nu = \frac{\mu}{\rho}$$

$$\text{Unit (SI)} = \frac{Ns/m^2}{kg/m^3} = \frac{\cancel{kg}\,m\,s^{-2}\,s\,m^3 m^{-2}}{\cancel{kg}}$$

$$= m^2/s$$

$$[\nu] = M^0L^2T^{-1}$$

CGS = cm²/s = stoke

100 centistoke = 1 stoke.

NEWTON'S LAW OF VISCOSITY

There are some class of fluids in which it is found that:

" The shear stress (τ) on a fluid element is directly proportional to the rate of shear strain.

$$\tau \propto \frac{du}{dy} = \mu \frac{du}{dy}$$

OBEY → Newtonian fluid

DO NOT OBEY → Non Newtonian fluid.

VARIATION OF VISCOSITY WITH TEMPERATURE

(I) LIQUID

As temperature increases, the viscosity of liquid decreases.

$$\mu = D e^{B/T}$$ empirical rel^n Andrade's eq^n

D and B are constant, T is the absolute temperature.

Reason for decrease

(i) Decrease in cohension force. due to increase in intermolecular spacing.

(II) Gas.

As temperature increases, the viscosity of gas increases.

$$\mu = \frac{C T^{3/2}}{T+S}$$

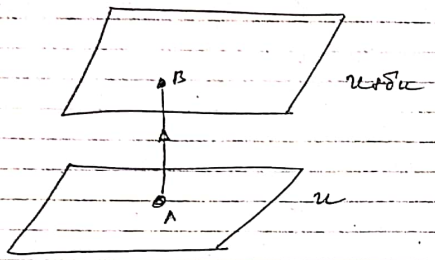
C and S, are emperical constant

Sutherland eq^n

T is absolute temperature

Reason

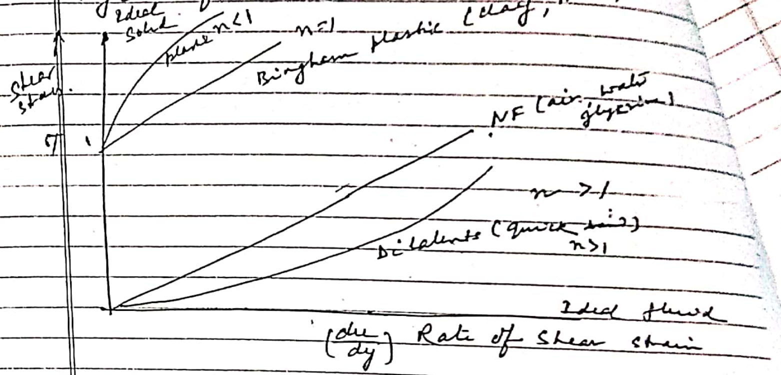
① ~~Molecular~~ Molecular momentum transfer



momentum of A is less than B If A goes to B Layer. It will have the tendency to decrease the velocity of the molecules of B Layer so this phenomena* will be increase in viscosity. If temp increases, due to molecular vibration increase, this momentum transfer will be more, thus ~~reduce~~ increase in viscosity. Therefore in gases as temp. increases, viscosity decreases.

* Molecular momentum transfer.

Regimes of Fluid

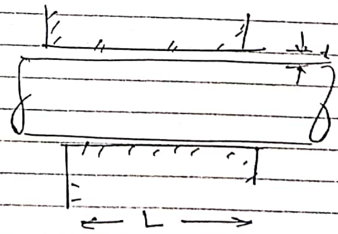


$$\tau = A + B\left(\frac{du}{dy}\right)^n$$

$\mu_a = f(\text{time})$

as time ↑ μ_a ↑ Thixoplastic

JOURNAL BEARING



200 rpm N rpm
L = 20 cm
t = 1 mm
d μ = 1.2 poise
D = 10 cm

$\tau = \mu \dfrac{du}{dy}$ $\mu = 0.12\ Ns/m^2 = 0.12\ Pa\,s$

$= 0.12 \times \dfrac{1.047}{0.001}$ $du = \dfrac{\pi D N}{60}$

$= \cancel{209\ N/m^2}$ $= \dfrac{\pi \times 0.2 \times 200}{60}$

$= 125.6\ N/m^2$ $= \cancel{2.09\ m/s}$

$= 1.047\ m/s$

$F = \tau A$

$= \tau \times \pi D L$

$= \cancel{2.09 \times \pi \times 0.2 \times 0.}$

$= 125.6 \times \pi \times 0.1 \times 0.2$

$= 7.89\ N$

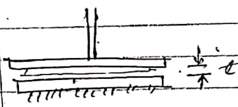
$T = F \times \dfrac{d}{2}$

$= 7.89 \times \dfrac{0.1}{2} = 0.39\ Nm$

Power lost $= \dfrac{2\pi N T}{60} = \dfrac{2\pi (200)\ 0.39}{60}$

$= 8.26\ W$ Ans

Prob.

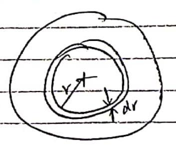


$N \rightarrow$ rpm

$D \rightarrow$ Dia

t — thickness gap

$\mu \rightarrow$ viscosity



$\tau = \mu \dfrac{du}{dy} = \dfrac{\mu}{t} u$

$du = \dfrac{2\pi r N}{60} = u$

$dF = \mu \dfrac{u}{t} \cdot dA$

$= \dfrac{\mu}{t} \cdot \dfrac{2\pi r N}{60} \cdot 2\pi r\, dr$

$= \dfrac{\mu \pi^2 N r^2\, dr}{30t} = \dfrac{\mu}{15t} \pi^2 N r^2\, dr$

$dT = r\, dF$

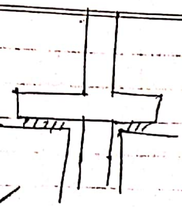
$= \dfrac{\mu}{15t} \pi^2 N r^3\, dr$

$T = \displaystyle\int_0^{D/2} \dfrac{\mu}{15t} \pi^2 N r^3\, dr$

$\dfrac{\mu \pi^2 N}{15t} \left[\dfrac{r^4}{4}\right]_0^{D/2} = \dfrac{\mu}{15 \times 4 \times 16} \pi^2 N D^4$

Power lost $= \dfrac{2\pi N T}{60}$

Collar bearing

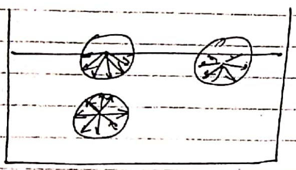


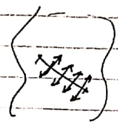
Lecture 3 #

Surface Tension. (ST)

— Surface tension is defined as the tensile force acting on the surface of a liquid in contact with gas or other surface between two immiscible liquid such that the contact surface behaves like a membrane under tension

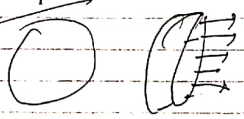
σ. N/m.





The free surface of a liquid act like a very thin film under tension of the surface of the liquid act as through it is an elastic membrane under tension.

ST on liquid droplet

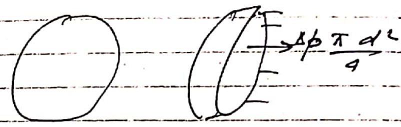


$\sigma \rightarrow$ ST
Δp — pressure intensity inside the droplet [in excess of outside pressure]

$$\sigma \pi d = \Delta p \frac{\pi d^2}{4}$$

$$\Delta p = \frac{4\sigma}{d}$$

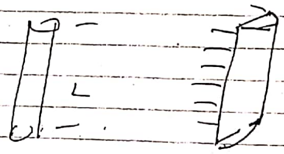
(2) ST Hollow Bubble



$$2\sigma \pi d = \Delta p \frac{\pi d^2}{4}$$

$$\Delta p = \frac{8\sigma}{d}$$

(3) ST liquid jet



$$\sigma\, 2 \times L = \Delta p\, L d$$

$$\Delta p = \frac{2\sigma}{d}$$

CAPILLARITY

— is defined as a phenomenon of rise and fall of a liquid surface in a small tube relative to adjacent general level of liquid when tube is held vertically in liquid

The rise of liquid surface is known as capillary rise and the fall of liquid surface is known as capillary depression.

$h = f(w, d, \sigma)$

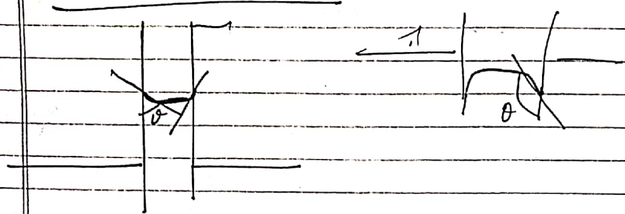
$h \rightarrow$

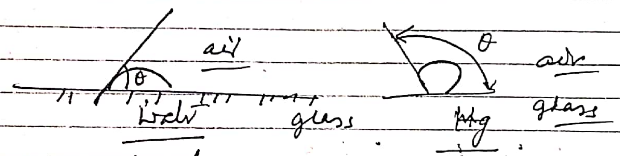
$w \rightarrow$ sp. weight

$d \rightarrow$ dia of tube

$\sigma \rightarrow$ ST

Contact angle θ



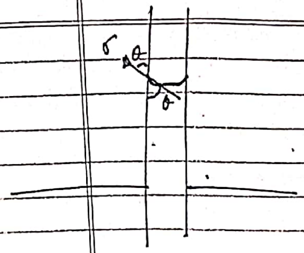


Adhesion / cohesion force

F_A F_C

$F_A > F_C \longrightarrow \theta < 90°$ wet

$F_A < F_C \longrightarrow \theta > 90° \rightarrow$ doesnot wet the surface



Weight of column of liquid

$= \rho g \left(\frac{\pi}{4} d^2\right) h$ — (I)

vertical component of ST ~~ten~~ force

$= \sigma \pi d \cos\theta$ — (II)

(I) = (II)

$\sigma \pi d \cos\theta = \rho g \frac{\pi}{4} d^2 h$

$\Rightarrow h = \frac{4\sigma\cos\theta}{\rho g d} = \frac{4\sigma\cos\theta}{wd}$

for water + clean glass

$\theta \simeq 0°$

$\cos\theta = 1$

$h = \frac{4\sigma}{\rho g d}$

Manometer tube (dia) > 9 mm.

$\theta \simeq 128°$ for Hg + glass.

vapour pressure (v.p)

Cavitation – is the phenomenon of formation of vapour bubbles in a region where pressure of liquid falling below vapour pressure and suddenly collapsing of these vapour bubbles in a region of high pressure. When vapour bubbles collapse a high pressure is reached. causing pitting of metal surface.

Equation of State (gas)

$$pv = RT$$

$$p = \rho RT$$

$$pV = mRT$$

m – kg, R – gas constant

$$\underbrace{R_0}_{\text{Universal gas}} = \underbrace{M}_{\text{molar mass}}R \qquad R_{air} = 288\ J/kgK$$

unit $R_0 = 8.314\ kJ\ K\ mol^{-1}\ K^{-1}$

$M_{air} = 28.97\ kg/kmol$

$$R_{air} = \frac{8.314 \times 10^3}{28.97} = 287\ J/kg\ K.$$

Lecture 4 (Fluid Statics)

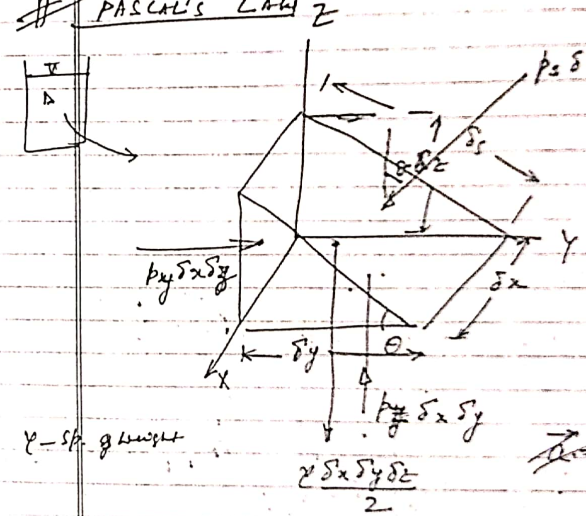
Pressure. $p = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$

$$= \frac{dF}{dA}$$

unit – Pa (N/m^2).   Bar

column of water   column of Hg.

PASCAL'S LAW



Triangular wedge. x direction force is not shown

$\delta y = \delta s \cos\theta$

$\delta z = \delta s \sin\theta$

γ – Sp. g weight

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$$\sum F_z = 0$$

$$\boxed{p_z\,\delta x\delta y - p_s\,\delta x\delta s\cos\theta - \rho\frac{\delta x\delta y\delta z}{2}a_z - \gamma\frac{\delta x\delta y\delta z}{2} = 0}$$

$$\sum F_y = 0$$

$$\boxed{p_y\,\delta x\delta z - p_s\,\delta x\delta s\sin\theta - \rho\frac{\delta x\delta y\delta z}{2}a_y = 0}$$

$$\Rightarrow p_z - p_s = (\rho a_z + \gamma)\frac{\delta z}{2}$$

$$p_y - p_s = \rho a_y \frac{\delta y}{2}$$

We are interested in a point

$\delta x \to 0 \quad \delta y \to 0 \quad \delta z \to 0$

$\Rightarrow p_y = p_s \qquad p_z = p_s$

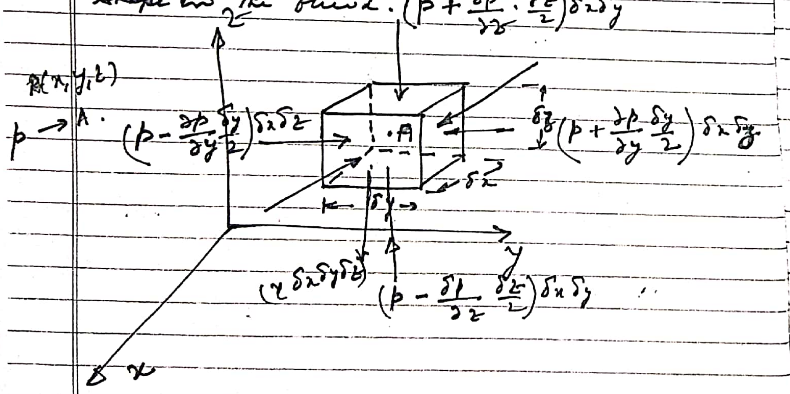
Since θ is arbitrarily chosen

$$p_s = p_x = p_y = p_z .$$

"The pressure at a point in a fluid at rest or in motion is independent of direction as long as there is no shearing stress present."

How pressure vary from point to point

Consider a fluid element of cuboid shape in the fluid.



Body force + Surface force = Resultant force.

Ps· SURFACE FORCE.

Resultant surface force in y direction

$$\delta F_y = \left(p - \frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x\,\delta z - \left(p + \frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x\,\delta z$$

$$= -\frac{\partial p}{\partial y}\,\delta x\,\delta y\,\delta z$$

Similarly

$$\delta F_x = -\frac{\partial p}{\partial x}\,\delta x\,\delta y\,\delta z$$

$$\delta F_z = -\frac{\partial p}{\partial z}\,\delta x\,\delta y\,\delta z$$

Resultant surface force acting on the element

$$\delta \vec{F_s} = \delta F_x\,\hat{i} + \delta F_y\,\hat{j} + \delta F_z\,\hat{k}$$

$$= -\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right)\delta x\,\delta y\,\delta z$$

$$\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k} = \nabla p$$

$$\nabla(\;) = \frac{\partial}{\partial x}(\;)\hat{i} + \frac{\partial}{\partial y}(\;)\hat{j} + \frac{\partial}{\partial z}(\;)\hat{k}$$

$\nabla \rightarrow$ gradient or del operator.

$$\Rightarrow \frac{\delta \vec{F_s}}{\delta x\,\delta y\,\delta z} = -\nabla p \qquad \text{——} (1)$$

Body force $\delta \vec{F_B}$.

Weight of element is

$$= -\delta W \hat{k}$$

$$= -\gamma \delta x \delta y \delta z \hat{k}$$

If $\vec{a}$ be the acceleration

Then, By Newton's second law

$$\sum \delta \vec{F} = \delta m \vec{a}$$

$$\rho \delta x \delta y \delta z \vec{a} = \delta \vec{F_S} + \delta \vec{F_B}$$

$$= -\nabla p \delta x \delta y \delta z - \gamma \delta x \delta y \delta z \hat{k}$$

$$\Rightarrow \boxed{-\nabla p - \gamma \hat{k} = \rho \vec{a}} \quad \text{—— (2)}$$

General Equation of motion of fluid

No. Shear stress is considered here

PRESSURE VARIATION IN A FLUID at REST

$\vec{a} = 0$

Equation (2) reduces to

$$\nabla p + \gamma \hat{k} = 0$$

$$\hat{i} \frac{\partial p}{\partial x} + \hat{j} \frac{\partial p}{\partial y} + \hat{k} \frac{\partial p}{\partial z} + \gamma \hat{k} = 0$$

$$\Rightarrow \frac{\partial p}{\partial x} = 0 \quad \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = -\gamma = -\rho g$$

Hydrostatic Law.

Rate of increase of pressure in a vertical direction is ~~equal~~ equal to its specific gravity of the fluid (Hydrostatic Law).

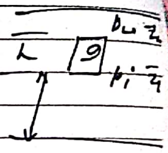
Here since

$$p = f(z).$$

$$\frac{dp}{dz} = -\gamma$$

For incompressible fluid.

γ (sp. weight) = c



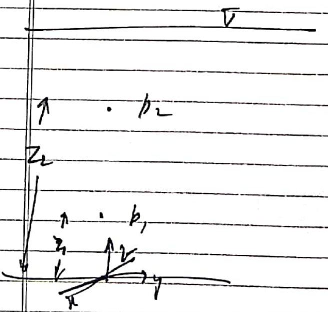
$$\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz$$

$$p_2 - p_1 = -\gamma (z_2 - z_1)$$

$$p_1 - p_2 = \gamma (z_2 - z_1).$$

$$p_1 > p_2$$

$$p_1 - p_2 = \rho g h = \gamma h$$



This type of pressure variation is called hydrostatic distribution, which is linear.

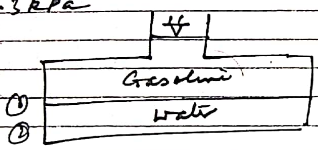
suppose pressure difference is 69 kPa

→ 7.04 m of water $\quad \gamma = 9810\ N/m^2$

0.518 m of Hg $\quad \gamma = 133\ kN/m^2$

pressure is not influenced by the size / shape of tank

$p_0 = 101.3\ kPa$



5.2 m

0.9 m

SG of gasoline = 0.68

pressure at (1) | absolute

(2)

Abs. pr.

gauge (gage) pr

vacuum pr.

Barometer

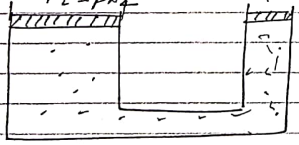
Abs. pr = atm pr + gauge pr.

or

Abs. pr = atm pr − vacuum pressure

Hydraulic jack

$F_2 = pA_2$ $\qquad F_1 = pA_1$ = Effort.



$$\frac{F_2}{A_2} = \frac{F_1}{A_1}$$

$$\Rightarrow F_2 = \left(\frac{A_2}{A_1}\right) F_1$$

Change of pr. w.r.t. height in atm

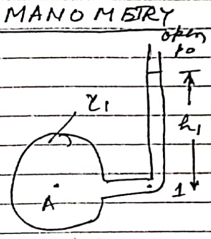
$pv = RT \Rightarrow \rho = \frac{p}{RT}$ or $\frac{dp}{dz} = -\frac{gp}{RT}$

$$p_2 = p_1 \exp\left[-\frac{g}{RT_0}(z_2 - z_1)\right]$$

Temp. Lapse Rate.

MANOMETRY

Manometers − (1) piezometer tube
(2) U Tube manometer



PIEZOMETER TUBE

$p_1 = p_0 + \gamma_1 h_1$

$= p_0 + \rho_1 g h_1$ → abs pr

$= \rho_1 g h_1$ → gage pr

Limitation

(1) for low pr. only

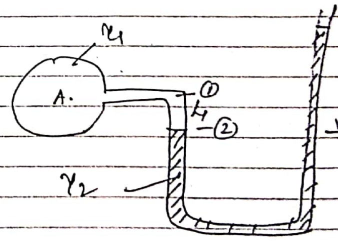
(2) Not meant for vacuum pressure

dia > 6 mm for avoiding Surface Tension effect. (capillary effect)

(3) only for liquid; not gas pressure can be measured

U Tube Manometer

p_A (gage pr)



$p_A + h_1\gamma_1 - h_2\gamma_2 = 0$

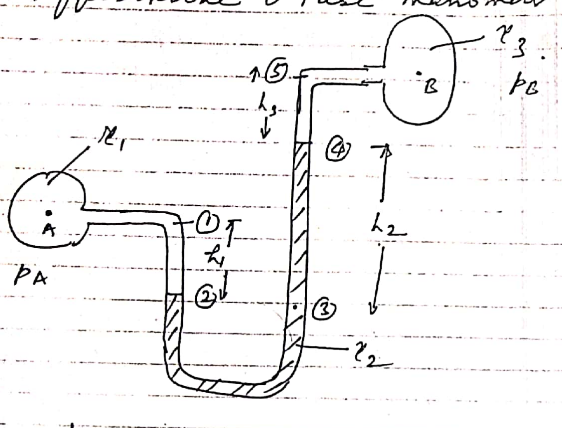
$p_A = \gamma_2 h_2 - \gamma_1 h_1$

If A is gas,

$\gamma_1 \approx 0$.

$\Rightarrow p_A = \gamma_2 h_2 = \rho_2 g h_2$

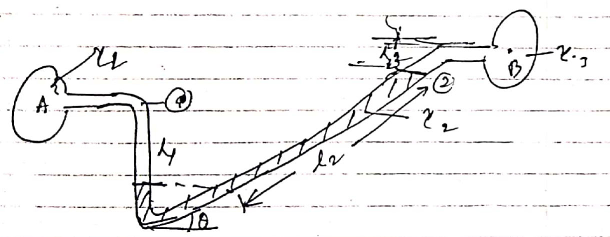
Differential U Tube manometer



$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$
$$= \rho_2 g h_2 + \rho_3 g h_3 - \rho_1 g h_1$$

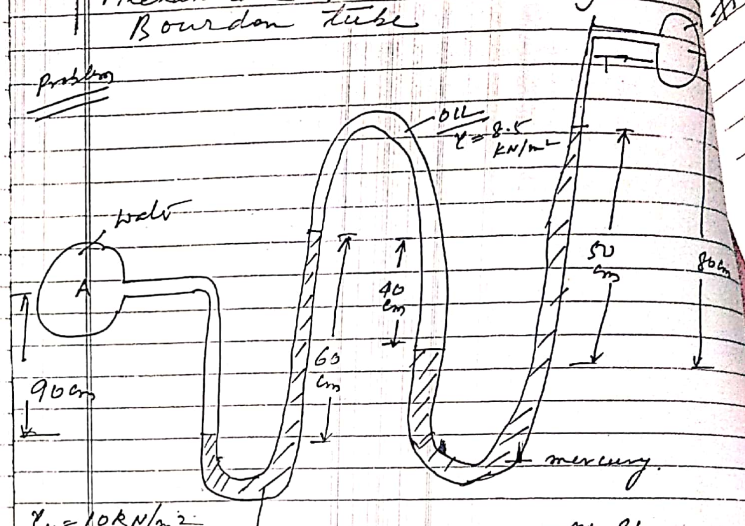
Inclined U Tube manometer



$$p_A + \gamma_1 h_1 - \gamma_2 l_2 \sin\theta - \gamma_3 h_3 = p_B.$$

Mechanical pressure measuring device
Bourdon Tube

problem



$\gamma_w = 10 kN/m^2$

~~oil~~ S.G. $(S.G)_{oil} = 0.85$

$$p_A + 10(0.9) - 136 \times 0.6 + 0.4 \times 8.5 - 136 \times 0.5 - 10 \times 0.3 = p_B.$$

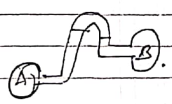
[Taking $g = 10 m/s^2$]

$$p_A - p_B = 140.2 kN/m^2 = 140.2 kPa$$
$$= 1.402 bar.$$

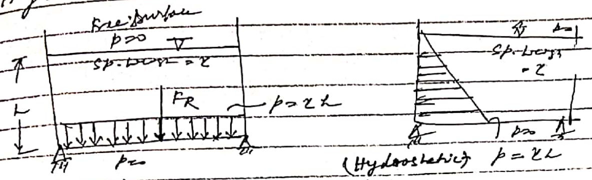
$$\frac{p_1 - p_2}{\gamma_{H_2O}} = \text{head of water (m)} \longrightarrow \text{pressure head.}$$

Inverted U Tube Manometer. — why.

for low pressure and low density manometric fluid.

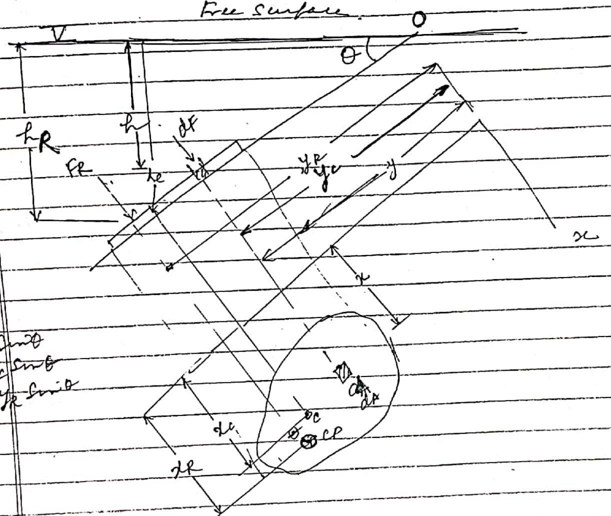


Hydrostatic Force on Plane Surface :-



(Hydrostatic) $p = \gamma h$

for inclined surface ~~f~~ (Force)



$h = y\sin\theta$
$h_c = y_c\sin\theta$
$h_R = y_R\sin\theta$

$dF = p\,dA$
$\quad = \gamma h\,dA$

$F_R = \int dF = \int_A \gamma h\,dA = \int_A \gamma y\sin\theta\,dA$

Taking γ = const

$F_R = \gamma\sin\theta\int_A y\,dA$

$\int_A y\,dA \rightarrow$ first moment of Area.

$y_c = \frac{\int_A y\,dA}{A} \Rightarrow y_c A = \int_A y\,dA$

$F_R = \gamma A y_c \sin\theta$
$\quad = \gamma h_c A$.

Force is independent of θ.

Resultant will not pass through centroid.
Taking summation of ~~ab~~ moment about x axis

$F_R y_R = \int_A y\,dF = \int_A \gamma\sin\theta\, y^2\,dA$

or $\gamma h_c A y_R = \gamma A y_c \cancel{\sin\theta}\, y_R = \int_A \gamma \cancel{\sin\theta}\, y^2 dA$

$y_R = \frac{\int y^2 dA}{y_c A}$

$\int y^2 dA = I_x \rightarrow$ Second moment of area (MOI) about x axis.

$y_R = \frac{I_x}{y_c A}$

parallel axis theorem says.

$I_x = I_{xc} + A y_c^2$

I_{xc} = MOI with respect to an axis passing through its centroid and ∥ to the x-axis

$$y_R = \frac{I_{xc} + A y_c^2}{y_c A}$$

$$\boxed{y_R = \frac{I_{xc}}{y_c A} + y_c}$$

Since $\frac{I_{xc}}{y_c A} > 0$ always for non horizontal surface. So resultant acts below centroid.

$$F_R x_R = \int_A \gamma \sin\theta \, xy \, dA$$

$$x_R = \frac{\int_A xy\, dA}{y_c A} = \frac{I_{xy}}{y_c A}$$

$$I_{xy} = I_{xyc} + A x_c y_c$$

$$x_R = \frac{I_{xyc}}{y_c A} + x_c .$$

$$h_R = y_R \sin\theta$$

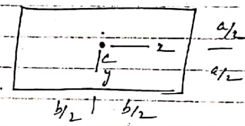
$$= \frac{I_{xc}}{y_c A} \sin\theta + y_c \sin\theta$$

$$\boxed{h_R = \frac{I_{xc}}{y_c A} \sin\theta + h_c}$$

$$\boxed{h_R = \frac{I_{xc}}{A h_c} \sin^2\theta + h_c}$$

I_{xyc} = Product of inertia with respect to an orthogonal coordinate system passing through centroid of an area and formed by translation of the x-y coordinate system.

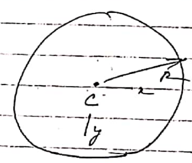
$I_{xyc} = 0$ If the area is either symmetrical to either x-y coordinate translated system.



$A = ba$

$I_{xc} = \frac{1}{12} b a^3$ $\quad I_{xyc} = 0.$

$I_{yc} = \frac{1}{12} a b^3$

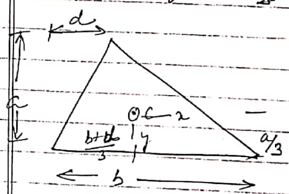


$A = \pi R^2$

$I_{xc} = I_{yc} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$

$I_{xyc} = 0.$

Centre of Pressure: Point at which Resultant Pressure force is acting on a surface.



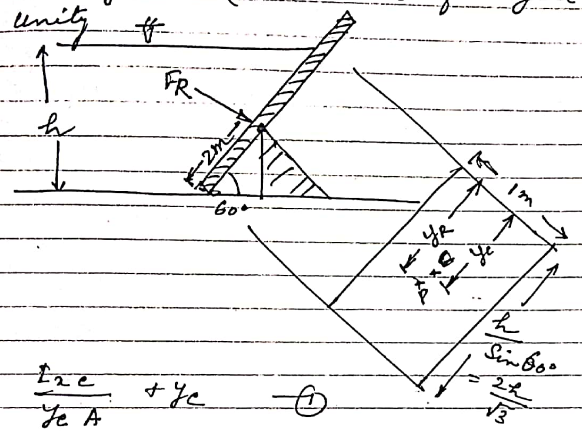
$A = \frac{ab}{2}$

$I_{xc} = \frac{b a^3}{36}$

$I_{xyc} = \frac{b a^2}{72}(b - 2d)$

Horizontal case
Vertical case }

A gate supporting water is shown in figure. Find out the height h of the water so that gate begins to tip about hinge. Take the width of the gate as unity.



$$y_R = \frac{I_{xc}}{y_c A} + y_c \quad \text{—(1)}$$

$$y_c = \frac{h}{2\sin 60°} \qquad y_R = \frac{h}{\sin 60°} - 2$$

$$I_{xc} = \frac{1}{12}(1)\left(\frac{h}{\sin 60°}\right)^3$$

$$y_c = \frac{h}{2\sin 60°} \qquad A = (1)\frac{h}{\sin 60°}$$

from (1)

$$\frac{h}{2\sin 60°} - 2 = \frac{\frac{1}{12}\frac{h^3}{\sin^3 60°}}{\left(\frac{h}{2\sin 60°}\right)\left(\frac{h}{\sin 60°}\right)} + \frac{h}{2\sin 60°}$$

or $\frac{h(2)}{\sqrt{3}} - 2$

$$= \frac{1}{6}\frac{h}{\sin 60°} + \frac{h}{2\sin 60°}$$

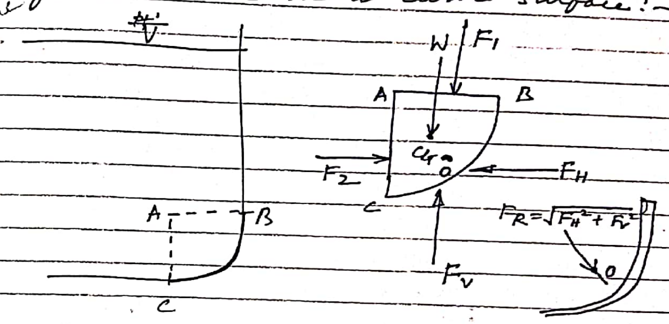
$$h\left[\frac{2}{\sqrt{3}} - \frac{1}{6}\cdot\frac{2}{\sqrt{3}} - \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\right] = 2$$

$$\frac{h}{\sqrt{3}}\left[2 - \frac{1}{3} - 1\right] = 2$$

$$\frac{h}{\sqrt{3}}\left[\frac{2}{3}\right] = 2 \Rightarrow h = 3\sqrt{3}\text{ m. } \underline{\text{Ans}}$$

Lecture 7.

Hydrostatic Force on a curved surface :-



F_1 = Weight of water column of volume AB (× width). ~~above~~ upto free surface

W = Weight of water in the curve area ABC

F_2 = hydrostatic force on AB surface

F_H = balancing horizontal force (Reaction)

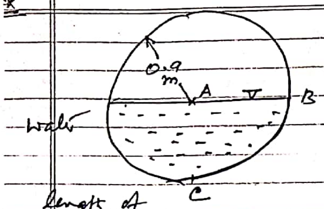
F_V = " vertical "

F_R = Resultant of acting force due to fluid on the curved surface

$F_H = F_2 \qquad F_H = F_2.$

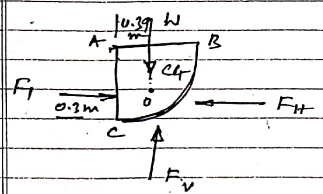
$F_V = F_1 + W$

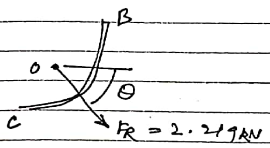
$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{F_H^2 + F_V^2}$



lengths of conduit = 0.3 m

Find the magnitude and line of action of resultant force that the water exerts on a 0.3 m length of the curved section BC of the conduit wall





$F_1 = \gamma h_c A = F_H$

$= 9810 \times 0.45 \times 0.3 \times 0.9$

$= 1192\ N = 1.192\ kN$

$W = \gamma V = 9810 \times \frac{\pi}{4}(0.9)^2 \times 0.3$

$= 1872\ N = 1.872 = F_V$

$F_R = \sqrt{F_H^2 + F_V^2}$

$= \sqrt{1.192^2 + 1.872^2}$

$= 2.219\ kN$

Buoyancy

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body, this force is known as buoyant force. A net upward vertical force results on an immersed body, because pressure increases with depth and pressure forces acting from the below are larger than the pressure forces acting from the above.

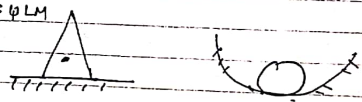
The buoyant force has a magnitude equal to weight of fluid displaced by the body and is directed vertically upward (ARCHIMEDES' PRINCIPLE).

STABILITY (~~IMMERSED~~ SUBMERGED & FLOATING BODY).

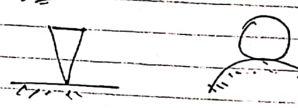
EQUILIBRIUM ⟶ STABILITY

Stable, Unstable, Neutral

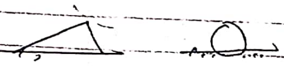
STABLE EQLM



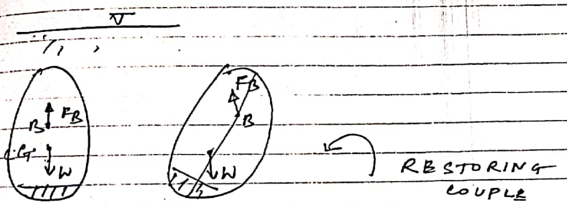
UNSTABLE Eqlm



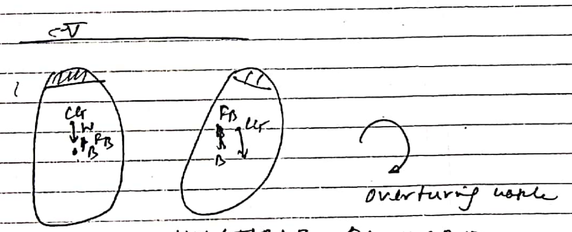
NEUTRAL Eqlm



SUBMERGED BODY



STABLE EQUILIBRIUM.



UNSTABLE EQUILIBRIUM

When G & centre of pressure B coincide
→ Neutral Equilibrium.

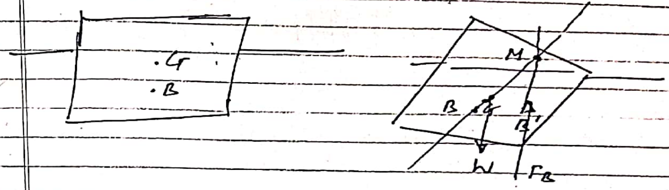
FLOATING BODY.

Metacentre and Metacentric height.

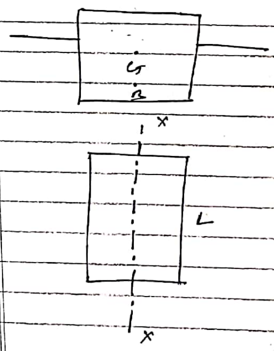
METACENTRE: It is defined as the point about which a body starts oscillating when the body is tilted by a small angle.

OR

It is also defined as the point at which the line of action of the force of buoyancy of displaced volume of fluid meet the normal axis of the body when the body is given a small angular displacement.



The distance MG i.e the distance between Metacentre of a floating body and the centre of gravity of the body is called Metacentric height.



$$\boxed{GM = \frac{I}{V} - BG}$$

I = Area moment of inertia about xx

V = volume of the body

BG = Distance from centre of buoyancy to CG.

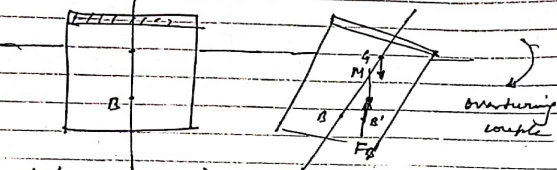
GM = +ve — stable eq/m

= −ve unstable eq/m.

Stability of floating body :-

a) When M is above G. → Stable

b) When M is below G $\rightarrow$ unstable eqlm



c) When M coincides G $\rightarrow$ Neutral equilibrium
No. couple will be produced by tilting the body.

$$T = 2\pi\sqrt{\frac{k^2}{(GM)\,g}}$$

$T \rightarrow$ time period of oscillation (s)
$k \rightarrow$ Radius of gyration of body (m)
$GM \rightarrow$ Metacentric height
$g \rightarrow$ acceleration due to gravity ($m s^{-2}$)

Lecture 8

Pressure variation in a fluid with Rigid body motion.

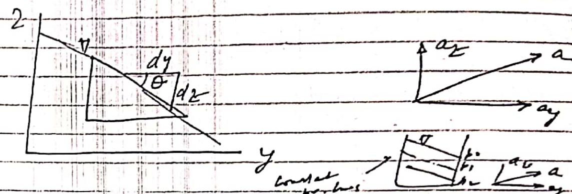
$$-\nabla p - \gamma\hat{k} = \rho\vec{a}$$

$$-\frac{\partial p}{\partial x} = \rho a_x ; \quad -\frac{\partial p}{\partial y} = +\rho a_y$$

and $$-\frac{\partial p}{\partial z} = \gamma + \rho a_z$$

With no shearing stress between layers of fluid

(A) Linear motion   taking



$$\frac{\partial p}{\partial y} = -\rho a_y \quad \text{and} \quad \frac{\partial p}{\partial z} = -\rho(g + a_z).$$

$$p = f(y, z)$$

$$dp = \frac{\partial p}{\partial y} dy + \frac{\partial p}{\partial z} dz$$

$$dp = -\rho a_y\, dy - \rho(g + a_z)\, dz$$

Along a line of constant pressure $dp = 0$ the slope of the line can be given by $dp = 0$

$$\Rightarrow \frac{dz}{dy} = -\frac{a_y}{g + a_z}$$

$$\tan\theta = \frac{a_y}{g + a_z}$$

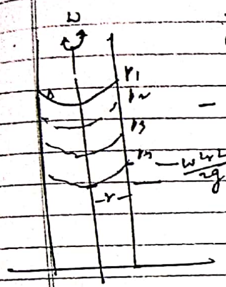
free fall + acceleration upward $a_y = 0$.

Rigid body Rotation   cylindrical coordinates sys.

In cylindrical coordinate system

$$\nabla p = \frac{\partial p}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial p}{\partial \theta}\hat{e}_\theta + \frac{\partial p}{\partial z}\hat{e}_z$$

$$\vec{a_r} = -r\omega^2 \hat{e}_r$$



$$\vec{a_\theta} = 0 \qquad \vec{a_z} = 0$$

$$-\nabla p - \gamma \hat{e}_z = \rho \vec{a}$$

$$\Rightarrow \frac{\partial p}{\partial r} = \rho r \omega^2$$

$$\frac{\partial p}{\partial \theta} = 0$$

$$\frac{\partial p}{\partial z} = -\gamma$$

Here $p = f(r, z)$.

$$\Rightarrow dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz$$

$$= \rho r\omega^2 dr - \gamma dz \quad \text{—(1)}$$

At surface or const pres. surface $\qquad \gamma = \rho g$.

$$dp = 0$$

$$\frac{dz}{dr} = \frac{r\omega^2}{g}$$

Integrating

$$z = \frac{\omega^2 r^2}{2g} + \text{const.}$$

free surface and constant pressure lines are parabolic in nature

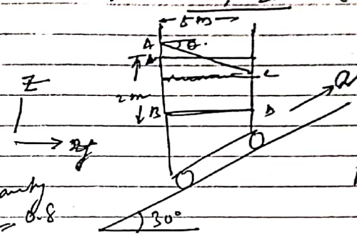
Integrating (1)

$$\int dp = \rho\omega^2\int r\,dr - \gamma\int dz$$

$$p = \frac{\rho\omega^2 r^2}{2} - \gamma z + \text{const.}$$

generally const for the r_0, z_0 (origin) are evaluated at centre.

Problem



sp gravity $= 0.8$

$\theta = ?$

pressure intensity at the bottom of the vessel at the front + rear end.

$p_B = ?$ $\quad p_D = ?$

$$a_x = a\cos 30° = 3.6 \times \cos 30°$$
$$= 3.12\ m/s^2$$

$$a_z = a\sin\theta = 3.6 \times \sin 30°$$
$$= 1.8\ m/s^2$$

$$\tan\theta = \frac{a_x}{a_z + g}$$

$$= \frac{3.12}{1.8 + 9.81}$$

$$\theta = 15°2'$$

$$AB = 2 + \frac{5}{2}\tan 15°2' \qquad p = \rho g h\left(1 + \frac{a_z}{g}\right)$$

$= 2.67\ m$ $= h_1$ $\quad \ldots$ ~~$p = \rho g h_1 (1 +$~~

$$p_B = \cancel{\gamma g h} = 0.8 \times 1000 \times 9.81 \times 2.67\left(1 + \frac{1.8}{9.81}\right) = 24.8\ kN/m^2$$

$eg = h_2 = 2 - \left(\frac{5}{2} \tan 15^\circ \cdot 2\right)$

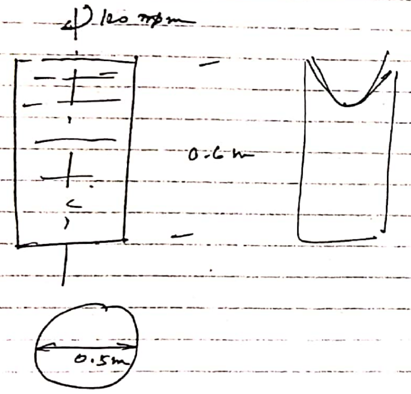
$= 1.33\ m$

$p_b = \rho g h_2 \left(1 + \frac{a_z}{g}\right)$

$= 0.8 \times 9810 \times 1.33 \left(1 + \frac{1.8}{9.81}\right)$

$= 12.353\ N/m^2$

Problem



$\omega = \frac{2\pi N}{60} = 12.57\ rad/s$

$Z = \frac{\omega^2 x^2}{2g}$

$x = 0.25\,m \quad \omega = 12.57$

$Z = 0.503\ m.$

volume of water spilled

$= \frac{1}{2}\left(\frac{\pi}{4} 0.5\right)^2 \times 0.503)$

$= 0.049\ m^3 = 49$ litres

Speed at which it just touches the centre base

$Z = \frac{\omega^2 x^2}{2g} \qquad Z = Z_1 =$

$x = r = 0.25$

$0.6 = \frac{\omega^2 (0.25)^2}{2 \times 9.8}$

$\omega = 13.72\ rad/s$

$N = 131.\ rpm.$

Lecture 9

Fluid Kinematics (FK)

FK is a division of engg. science that describes the geometry of fluid motion in terms of displacement, velocity and acceleration & any other quantities that can be derived from displacement and time.

Forces/Energies responsible for accelerating & decelerating the flow are not considered here.

LAGRANGIAN and EULERIAN Description.

Lagrangian Description requires us to ~~follow~~ track the position and velocity of each individual fluid parcel, which can be refer as fluid particle and can be taken as parcel of fixed identity. Similar to System analysis (Closed)

Problems of this technique.

1. Million of fluid particles
2. Identification is difficult
3. Fluid particle itself deform
4. Continuum concept hinders to distinguish between fluid particle

However, there are many practical application of this approach like Rarefied gas dynamics.

Eulerian Description (Approach)

— A finite volume called a flow domain / control volume is defined through which fluid flows in and out. Instead of tracking individual fluid particle, we define field variable which are the function of space and time ~~and~~ within the control volume.

field variable < scalar field variable (SFV) / vector field variable (VFV)

Pressure field

$$p = p(x, y, z, t) \rightarrow SFV$$

velocity field $\rightarrow$ VFV

$$\vec{V} = \vec{V}(x, y, z, t) \rightarrow$$

$$= u(x, y, z, t)\,\hat{i} + v(x, y, z, t)\,\hat{j} + w(x, y, z, t)\,\hat{k}$$

acceleration field $\rightarrow$ VFV

$$\vec{a} = \vec{a}(x, y, z, t).$$

Newton's law $\rightarrow$ Lagrangian Approach $\rightarrow$ for particle

LAG $\xrightarrow{RTT}$ EULER

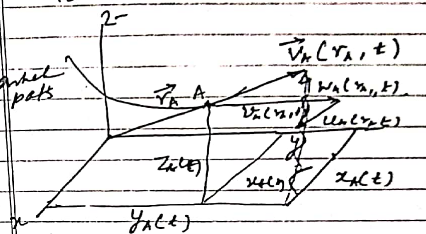
RTT $\rightarrow$ Reynold's Transport Theorem

The Acceleration Field

$\hookrightarrow$ In Eulerian method

$$\vec{a} = \vec{a}(t)$$

Material Derivative



$$\vec{V}_A = \vec{V}_A(r_A, t)$$

$$= \vec{V}_A[x_A(t), y_A(t), z_A(t), t]$$

$$x_A = x_A(t) \qquad y_A = y_A(t) \qquad z_A = z_A(t)$$

$$a_A(t) = \frac{d\vec{V}_A}{dt}$$

$$= \frac{\partial \vec{V}_A}{\partial t} + \frac{\partial \vec{V}_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial \vec{V}_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial \vec{V}_A}{\partial z}\frac{dz}{dt}$$

$$= \frac{\partial \vec{V}_A}{\partial t} + u_A\frac{\partial \vec{V}_A}{\partial x} + v_A\frac{\partial \vec{V}_A}{\partial y} + w_A\frac{\partial \vec{V}_A}{\partial z}$$

for All point in particular fn. particle value

for acceln field from velocity field:

$$\vec{a} = \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{local acceleration}} + \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\text{Advective acceleration / convective acceln}}$$

$$\Rightarrow a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}$$

$$\vec{\nabla} = \text{gradient / del operator}$$
$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$$

Local Acceleration is zero for steady flow

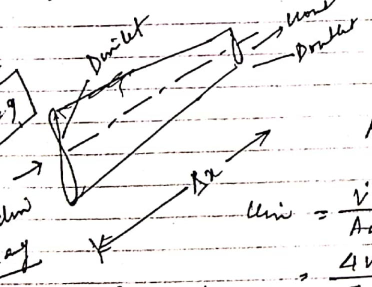
Steady / unsteady
Uniform / non uniform }

L / T , C / Inc.

Example

Steady

$\boxed{a \text{ along centreline} = ?}$



Subsitlay

$\Delta x = 9.91$ cm

$D_{in} = 1.07$ cm

$D_{out} = 0.46$ cm

$\dot{V} = 0.0530$ l/s

Accelrat at the centre line

$$u_{in} = \frac{\dot{V}}{A_{in}} = \frac{4\dot{V}}{\pi D_{in}^2}$$

$$= \frac{4 \times 0.053 \times 10^{-3}}{\pi \times (0.0107)^2} = 0.589 \text{ m/s}$$

$u_{out} = 3.19$ m/s

$$a_x = \cancel{\frac{\partial u}{\partial t}} + u\frac{\partial u}{\partial x} + \cancel{v\frac{\partial u}{\partial y}} + \cancel{w\frac{\partial u}{\partial z}}$$

steady $\qquad v=0 \qquad w=0$

$$= u\frac{\partial u}{\partial x} =$$

$$= \frac{u_{in} + u_{out}}{2} \cdot \frac{u_{out} - u_{in}}{\Delta x}$$

$$= \frac{u_{out}^2 - u_{in}^2}{2\Delta x}$$

$$= \frac{(3.19)^2 - (0.589)^2}{2(0.0991)} = 49.6 \text{ m/s}^2.$$

Alternate method

$$a_x \approx \frac{\Delta u}{\Delta t} = \frac{u_{out} - u_{in}}{\Delta x / u_{avg}}$$
$$= \frac{u_{out}^2 - u_{in}^2}{2\Delta x}.$$

$$\frac{D}{Dt} = \frac{d}{dt} + (\vec{V}\cdot\nabla)$$

TYPES OF FLOW :-

1. Steady and unsteady flow
→ Fluid characteristics like velocity pressure, density etc doesnot change with time at a point. → steady

$$\left(\frac{\partial V}{\partial t}\right)_{x_0 y_0 z_0} = 0 \quad \left(\frac{\partial p}{\partial t}\right)_{x_0 y_0 z_0} = 0$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{x_0 y_0 z_0} = 0 \quad \longrightarrow \text{Steady}$$

If changes → unsteady

$$\left(\frac{\partial V}{\partial t}\right)_{x_0 y_0 z_0} \neq 0, \left(\frac{\partial p}{\partial t}\right)_{x_0 y_0 z_0} \neq 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0 y_0 z_0} \neq 0.$$

$\vec{V} = \vec{V}(x, y, z, t) \rightarrow$ unsteady.

② Uniform and Non uniform flow

Velocity at any given time doesn't change w.r.t. to space

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{const}} = 0 \quad \text{uniform flow}$$

$$\neq 0 \quad \text{Non uniform flow}$$

$\partial s \rightarrow$ length of flow in the direction s

③ Laminar and Turbulent flow

— LF is defined as that type of flow in which the fluid particles move along with definite path and all streamlines are $\parallel$. The particles moves in laminas and layer smoothly over the adjacent layer

TF is that the type of flow in which fluid particles move in zig zag way. Eddies formation takes places and are responsible for high energy loss.

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho V D}{\mu} \quad \text{for pipe}$$

$Re < 2000$ LF

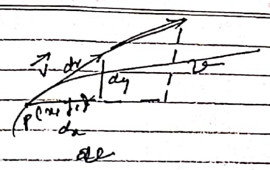
$Re > 4000$ TF

$2000 - 4000 \rightarrow$ may be LT/TF } generally for commercial pipe

FLOW PATTERN & FLOW VISUALISATION

Streamline

A Streamline is an imaginary line curve drawn through flowing fluid in such a way that ~~that~~ the tangent to it at any point gives the direction of velocity of flow at that point



$$\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

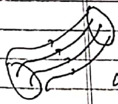
In 2D

$$\left(\frac{dy}{dx}\right)_{\text{along streamline}} = \frac{v}{u}$$

Solution contain constant c

Family of curves represents streamlines of flow field (for different c).

Stream tube consists of bundle of streamlines



Cannot ~~pass~~ cross each other (Streamlines)

Both Streamline & streamtube are instantaneous quantities) Can change with time.

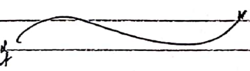
S. tube $dA \searrow$ $u \nearrow$

S. tube $dA \nearrow$ $u \searrow$

Pathlines

A path line is the actual path traveled by an individual fluid particle over some time period.

$t = t_{end}$.



at $t = t_{start}$

$$\vec{x} = \vec{x}_{start} + \int_{t_{start}}^{t} \vec{V}\, dt.$$

Streakline – is the locus of fluid particle that have passed sequentially through a prescribed point of in the flow

Lecture 10

INTEGRAL FLOW ANALYSIS:–

Reynold's Transport Theorem :–

Lagrangian Description → fixed pocket of mass → system

can change size and shape but always follow that pocket of mass.

Eulerian Description → fixed Region of interest → control volume can change size and shape and mass might cross the boundary

$$\sum \vec{F}_{sys} = m_{sys} \vec{a}_{sys}$$

But

$$\sum \vec{F}_{cv} \neq m_{cv} \vec{a}_{cv}$$

Suppose

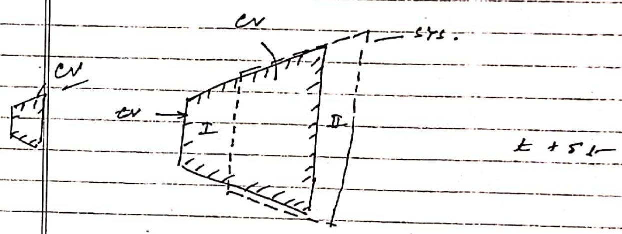
$B \rightarrow$ any extensive property (mass, momentum, volume, energy)

$b \rightarrow$ intensive property

$$b = \frac{B}{m}$$

$\forall \rightarrow$ volume.

$$B_{sys} = \int_{sys} \rho b \, d\forall$$



at t_0 → sys & cv coincides.

I → mass that enters cv to fill space sys on left

II → portion of sys no longer in cv

Initially

$$B_{sys}(t) = B_{cv}(t) \quad \text{—(1)}$$

$$B_{sys}(t+\delta t) = B_{cv}(t+\delta t) - B_{I}(t+\delta t) + B_{II}(t+\delta t)$$

$$\frac{\Delta B_{sys}}{\Delta t} = \frac{B_{sys}(t+\delta t) - B_{sys}(t)}{\Delta t} \quad \text{—(2)}$$

$$\frac{(\text{II}) - (\text{I})}{\Delta t}$$

Taking limit $\lim_{\Delta t \to 0}$

$$= \frac{DB_{sys}}{Dt}$$ [Material Derivative Lagrangian

$$\lim_{\delta t \to 0} \frac{B_{cv}(t+\delta t) - B_{cv}(t)}{\delta t} = \frac{\partial}{\partial t} B_{cv}$$

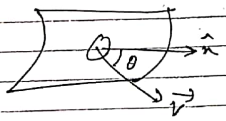
and for the things that cross a boundary

$$\lim_{\delta t \to 0} - \frac{B_I(t+\delta t)}{\delta t} = -\rho A V_1 b_1 = \dot{B}_{in}$$

$$\lim_{\delta t \to 0} + \frac{B_{II}(t+\delta t)}{\delta t} = +\rho A_1 V_1 b_1 = \dot{B}_{out}$$

$$\underbrace{\frac{DB_{sys}}{Dt}}_{\text{Change within sys}} = \underbrace{\frac{\partial B_{cv}}{\partial t}}_{\text{changes in cv}} + \underbrace{\dot{B}_{out} - \dot{B}_{in}}_{\text{stuff that crossed boundary}}$$

Generalization



$\hat{n} \to$ The surface outward normal with magnitude 1

$$\dot{B}_{out} = \int_{cs} \rho b \vec{V} \cdot \hat{n} \, dA$$

Along the surface $\vec{V} \cdot \hat{n} = 0$

$\dot{B}_{out} = -ve$

So, finally

$$\boxed{\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{cv} \rho b \, d\forall + \int_{cs} \rho b \vec{V} \cdot \hat{n} \, dA}$$

RTT

Suppose

$B = m$ (mass) $\qquad b = \frac{m}{m} = 1$

from RTT

$$\cancel{\left(\frac{Dm}{Dt}\right)_{sys}} = \frac{\partial}{\partial t} \int_{cv} \rho \, d\forall + \int_{cs} \rho (\vec{V} \cdot \hat{n}) \, dA$$

$=0$

$$\frac{\partial}{\partial t} \int_{cv} \rho \, d\forall + \int_{cs} \rho (\vec{V} \cdot \hat{n}) \, dA = 0$$

$\frac{d}{dt}$

$$\boxed{\frac{d}{dt}\left(\int_{cv} \rho \, d\forall\right) + \int_{cs} \rho (\vec{V} \cdot \hat{n}) \, dA = 0}$$

Continuity eqn in Integral form

Inward $-ve$
Outward $+ve$

for steady flow FIRST TERM $= 0$

$$\int_{cs} \rho (\vec{V} \cdot \hat{n}) \, dA = 0$$

$(\rho_i A_i V_i)_{in} = (\rho_o A_o V_o)_{out} \longrightarrow$ 1 D ...

outflow = inflow

$(\Sigma \dot{m})_{in} = (\Sigma \dot{m})_{out}$

If inlet and outlet are not one dimensional

$$\dot{m}_{cv} = \int_{cs} \rho (\vec{V} \cdot \hat{n})\, dA$$

for incompressible flow $\rho = c$

$$\int_{cs} \vec{V} \cdot \hat{n}\, dA = 0$$

$$\Sigma A_i V_i = \Sigma A_o V_o$$

$$\Sigma Q_{in} = \Sigma Q_{out}$$

Every flow field of possible flow must satisfy continuity equation.

Momentum equation
 & CV in inertial reference frame

$$\vec{V_r} = \vec{V} - \vec{V}_{cv}$$

$$B = m\vec{V} = \text{momentum (linear)}$$

$$b = \vec{V}$$

$$\frac{d(m\vec{V})_{sys}}{dt} = \frac{d}{dt}\int_{cv} \rho \vec{V}\, d\forall + \int_{cs} \rho V (V_r \cdot \hat{n})\, dA$$

$\Sigma \vec{F})_{\text{External on CV}}$ = (The time rate of change of linear momentum of contents of CV) + (The net flow rate of linear momentum out of the CS by mass flow rate)

Fixed CV

$$\Sigma \vec{F}\Big|_{ext} = \frac{d}{dt}\int \rho \vec{V}\, d\forall + \int_{cs} \rho \vec{V} (\vec{V} \cdot \hat{n})\, dA$$

$$\Sigma F\Big|_{\text{external CV}} = \vec{F_s} + \vec{F_B}$$

Steady

$$\Sigma F\Big|_{ext} = \int_{cs} \rho \vec{V} (\vec{V_r} \cdot \hat{n})\, dA$$

Momentum flux inlet & outlet

$$\int_{A_c} \rho \vec{V} (\vec{V} \cdot \hat{n})\, dA = \rho V_{av} A_c \vec{V}_{av}$$

$$= \dot{m} \vec{V}.$$

β = Momentum flux correction β

$$\Sigma \vec{F} = \frac{d}{dt}\int \rho \vec{V}\, d\forall + \sum_{out} \beta \dot{m} \vec{V}_{av} - \sum_{in} \beta \dot{m} \vec{V}_{av}$$

$$\int_{A_c} \rho \vec{V} (\vec{V} \cdot \hat{n})\, dA_c = \beta \dot{m} \vec{V}_{av}$$

$\beta =$

If C.S. slices normal to inlet & outlet

$$\beta = \frac{1}{A_c}\int_{A_c} \left(\frac{V}{V_{av}}\right)^2 dA_c$$