

An example:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

$$\boxed{AX = B}$$

Elimination

$$M = \left( \begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & b_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & b_3 \\ a_{41} & a_{42} & a_{43} & a_{44} & b_4 \end{array} \right)$$

$x_2 \rightarrow \text{row 2}$

$x_1 \rightarrow \text{row 1}$

To eliminate  $A_{21}$ , row 2 with  $x_2 - a_{21}/a_{11} * x_1$ , we replace  $M_{2,i}$  with  $M_{2,i} - a_{21}/a_{11} * M_{1,i}$  for  $1 \leq i \leq 5$  ( $i = \text{columns}$ ).

One division to calculate  $\frac{a_{21}}{a_{11}}$ . 1 multiplication element to calculate  $a_{21}/a_{11} * M_{1,i}$  and one subtraction element to calculate  $(M_{2,i} - \frac{a_{21}}{a_{11}} * M_{1,i})$ .

In total: 1 division, 5 multiplications and 5 subtractions (additions).

Since  $M_{2,1}$  will be 0 without knowing to calculate it i.e.  $(M_{2,1} - \frac{a_{21}}{a_{11}} * M_{1,1})$ .

Thus, 1 division, 4 multiplications and 4 additions.

Eliminating  $A_{31}$  and  $A_{41}$  also require same operations. So total operations to eliminate  $M_{j,1}$  are 3 divisions, 12 multiplications and 12 subtractions (additions).

Now to eliminate second element, we require

$$M' = \left( \begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ 0 & a_{22}' & a_{23}' & a_{24}' & b_2' \\ 0 & a_{32}' & a_{33}' & a_{34}' & b_3' \\ 0 & a_{42}' & a_{43}' & a_{44}' & b_4' \end{array} \right)$$

2 divisions, 6 multiplications and 6 additions.

$$\left( \begin{array}{cc|c} a_{33}'' & a_{34}'' & b_3'' \\ a_{43}'' & a_{44}'' & b_4'' \end{array} \right) \rightarrow 1 \text{ division, 2 multiplications, 2 additions.}$$

Total:  $\underbrace{(3+2+1)}_6 + \underbrace{(12+6+2)}_{20} + \underbrace{(12+6+2)}_{20}$  | Back substitution:  $a_{44}'' x_4 = b_4'' \rightarrow 1 \text{ division}$

Similarly  $a_{22}' x_2 + a_{23}' x_3 + a_{24}' x_4 = b_2'$

2 multiplication, 2 subtraction, 1 division

last row.

3 multiplication, 3 subtraction, 1 division

next row  $a_{33}'' x_3 + a_{34}'' x_4 = b_3''$   
 $\rightarrow 1 \text{ multiplication, 1 subtraction, 1 division}$