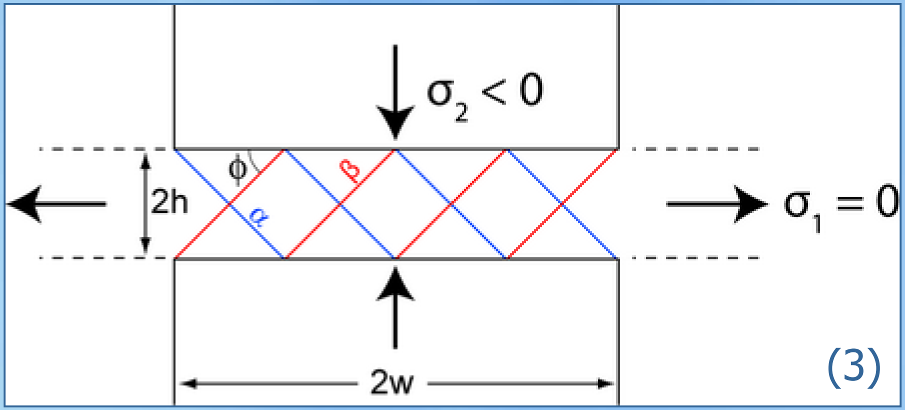
Slip-line field theory

MFT

Introduction

Slip-line field theory is used to model plastic deformation in plane strain only for a solid that can be represented as a rigid-plastic body. Elasticity is not included and the loading has to be quasi-static.

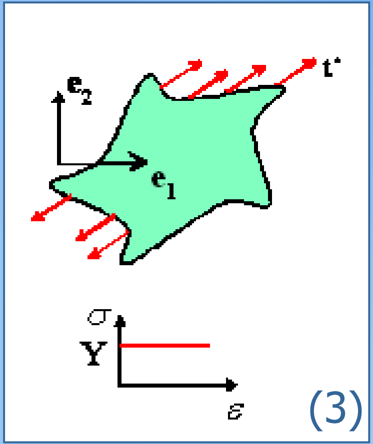
This method has been recently largely superseded by finite element method, but this theory can provide analytical solutions to a number of metal forming processes, and utilises plots showing the directions of maximum shear stress in a rigid-plastic body which is deforming plastically in plane strain.



Assumptions

Besides the usual assumptions that the metal is isotropic and homogeneous, the common approach to this subject usually involves the following:

- the metal is rigid-perfectly plastic; this implies the neglect of elastic strains and treats the flow stress as a constant,
- deformation is by plane strain,
- possible effects of temperature, strain rate, and time are not considered,
- there is a constant shear stress at the interfacial boundary. Usually, either a frictionless condition or sticking friction is assumed.



(3)

When the theory cannot be used

The principal ways in which slip-line field theory fails to take account of the behaviour of real materials are:

- it deals only with **non-strain-hardening materials**. Whilst strain-hardening can be allowed for in calculations concerned with loads in an approximate way, the manner in which strain distribution is altered because of it is not always clear
- there is no allowance for **creep** or **strain-rate effects**. The rate of deformation at each given point in space and in the deforming body is generally different, and any effect this may have on the yield stress is ignored.

When the theory cannot be used (cont.)

- all inertia forces are neglected and the problems treated as **quasi-static,**

- in the forming operations which impose **heavy deformations**, most of the work done is dissipated as heat; the temperatures attained may affect the material properties of the body or certain physical characteristics in the surroundings, e.g. lubrication

Despite these shortcomings, the theory is extremely useful; it is very important, however, to remember its limitations and not to expect too high a degree of correlation between experimental and theoretical work.

Plane plastic strain

Deformation which proceeds under conditions of plane strain is such that the flow or deformation is everywhere parallel to a given plane, say the (x, y) plane in a system of three mutually orthogonal planes and the flow is independent of z.

Since elastic strains are neglected, the plastic strain increments (or strain-rates) may be written in terms of the displacements (or velocities) $u_x(x, y)$, $v_y(x, y)$, $w_z = 0$, as below

$$\dot{\varepsilon}_x = \frac{\partial u_x}{\partial x} \qquad \dot{\gamma}_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

$$\dot{\varepsilon}_y = \frac{\partial v_y}{\partial y} \qquad \dot{\gamma}_{yz} = \frac{1}{2}\left(\frac{\partial v_y}{\partial z} + \frac{\partial w_z}{\partial y}\right) = 0 \qquad (1)$$

$$\dot{\varepsilon}_z = \frac{\partial w_z}{\partial z} = 0 \qquad \dot{\gamma}_{zx} = \frac{1}{2}\left(\frac{\partial w_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) = 0$$

State of stress

It follows from the Levy-Mises relation that τ_{xz} and τ_{yz} are zero and therefore that σ_z is a principal stress. Further, since $\acute{\varepsilon}_z = 0$, then $\sigma'_z = 0$ and hence $\sigma_z = (\sigma_x + \sigma_y)/2 = p$, say.

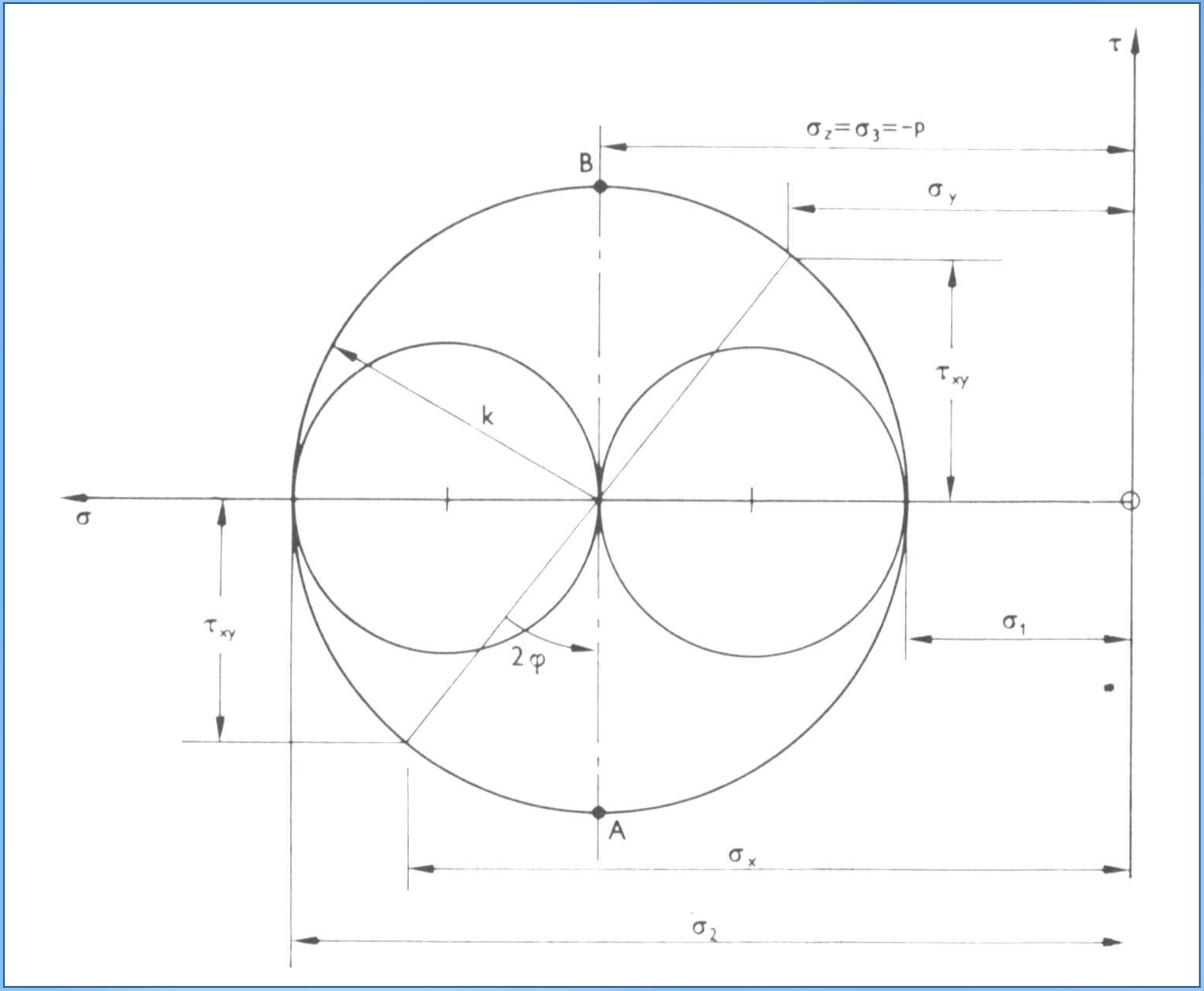
Because the material is incompressible $\acute{\varepsilon}_x = - \acute{\varepsilon}_y$ and each incremental distortion is thus a pure shear. The state of stress throughout the deforming material is represented by a constant yield shear stress **k**, and a hydrostatic stress **-p** which in general varies from point to point throughout the material. **k** is the yield shear stress in plane strain and the yield criterion for this condition is:

$$\tau_{xy}^2 + (\sigma_x - \sigma_y)^2 / 4 = k^2 \qquad (2)$$

where k = Y/2 for the Tesca criterion and k = Y/$\sqrt{3}$ for the Mises criterion.

Mohr's circle diagram for stress in plane plastic strain

The state of stress at any point in the deforming material may be represented in the Mohr circle diagram

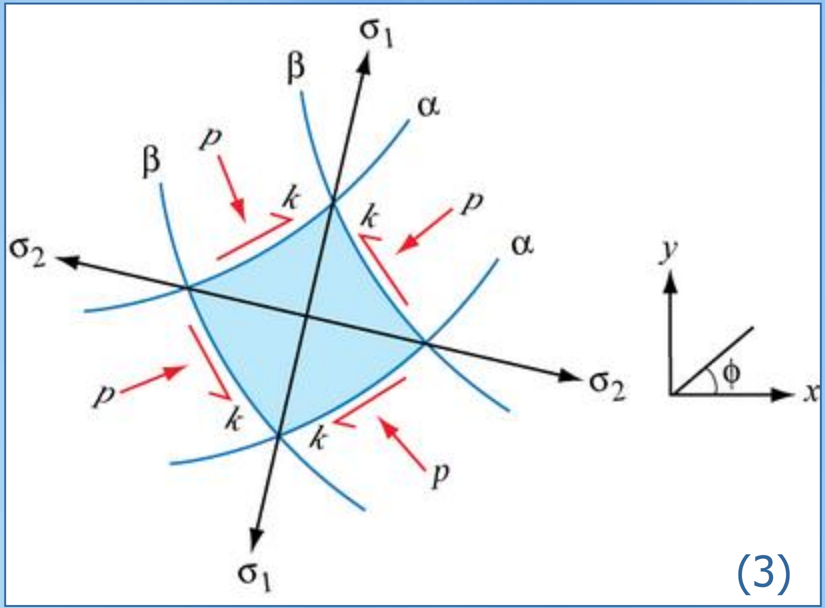


A and B represent the stress states (- p, ±k) at a point on planes parallel to the slip-lines through that point.

Directions of maximum shear strain-rate

For an isotropic material the directions of maximum shear strain-rate, represented by points *A* and *B* coincide with the directions of yield shear stress and that such directions are clearly directions of zero rate of extension or contraction. The loci of these directions of maximum shear stress and shear strain-rate form two orthogonal families of curves known as **slip-lines**.

The stresses on a small curvilinear element bounded by slip-lines are shown below:



Slip lines

The slip-lines are labelled α and β as indicated. It is essential to distinguish between the two families of slip-lines, and the usual convention is that when the α- and β- lines form a right handed co-ordinate system of axes, then the line of action of the algebraically greatest principal stress, σ_1 passes through the first and third quadrants. The anti-clockwise rotation, ø, of the α-line from the chosen x-direction is taken as positive.

Slip lines (cont.)

In order to determine the load necessary for a particular plastic forming operation, first of all the slip-line field patterns must be obtained. This means that equations for the variation of ***p*** along both α- and β-lines must be derived. Also, we must check that all velocity conditions along α- and β-lines are satisfied.

The Stress Equations

The equations of equilibrium for plane strain are, with neglect of body forces:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \qquad (3)$$

The above stress components σ_x, σ_y and τ_{xy} expressed in terms of *p* and *k* are:

$$\sigma_x = -p - k \sin 2\phi$$
$$\sigma_y = -p + k \sin 2\phi \qquad (4)$$
$$\tau_{xy} = k \cos 2\phi$$

p is the normal or hydrostatic pressure on the two planes of yield shear stress.

The Stress Equations (cont.)

Differentiating and substituting from equation (4) in equation (3) we have:

$$-\frac{\partial p}{\partial x} - 2k\cos 2\phi\,\frac{\partial \phi}{\partial x} - 2k\sin 2\phi\,\frac{\partial \phi}{\partial y} = 0 \qquad (5)$$

$$-2k\sin 2\phi\,\frac{\partial \phi}{\partial x} - \frac{\partial p}{\partial y} + 2k\cos 2\phi\,\frac{\partial \phi}{\partial y} = 0$$

If now the α- and β-lines are taken to coincide with 0_x and 0_y at 0, that we take $\phi = 0$, equations (5) become:

$$-\frac{\partial p}{\partial x} - 2k\,\frac{\partial \phi}{\partial x} = 0$$

$$-\frac{\partial p}{\partial y} + 2k\,\frac{\partial \phi}{\partial y} = 0 \qquad (6)$$

The Stress Equations (cont.)

Thus, integrating

$$p + 2k\phi = f_1(y) + C_1 \qquad (7)$$
$$p - 2k\phi = f_2(x) + C_2$$

If the hydrostatic stress ***p*** can be determined at any one point on a slip-line (for example at a boundary), it can be deduced everywhere else.

Thus

$$p + 2k\phi = const.\ along\ an\ \alpha - line \qquad (8)$$
$$p - 2k\phi = const.\ along\ an\ \beta - line$$

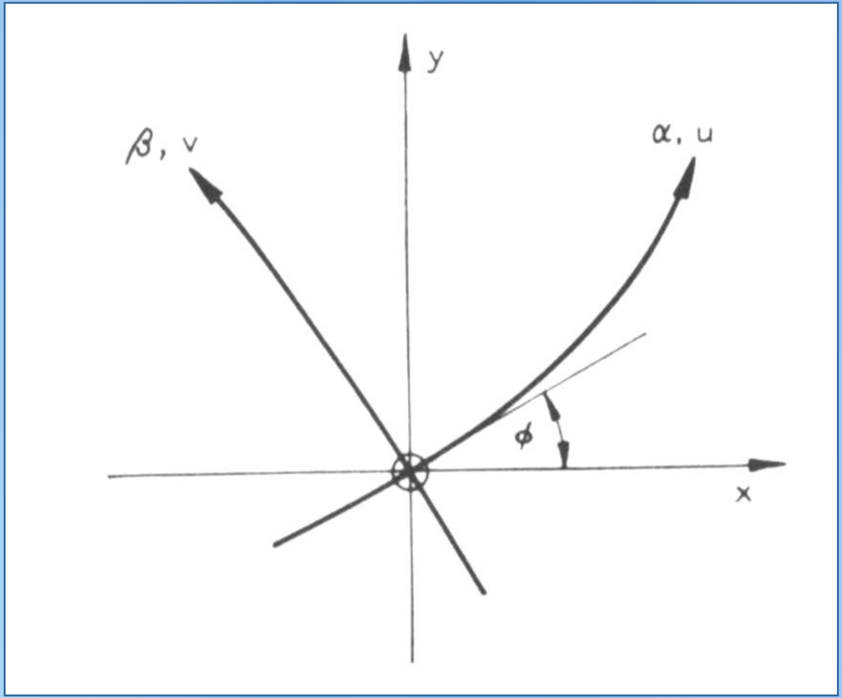
Relations governing hydrostatic stress along slip-lines (Hencky equations)

The equations (8) are known as the Hencky equations and are equivalent to the equilibrium equations for a fully plastic mass stressed in plane strain.

In general, the values of the constants C_1 and C_2 from equation (7) vary from one slip-line to another.

The velocity field (Geiringer equations)

In figure shown below ***u*** and ***v*** are the component velocities of a particle at a point *O* along a pair of α- and β-slip-lines the α-line being inclined at Ø to the *Ox* axis of a pair of orthogonal cartesian axes through *O.*



The velocity field (Geiringer equations) cont.

The components of the velocity of the particle u_x and v_y parallel to *Ox* and *Oy*, respectively, are then

$$u_x = u\cos\phi - v\sin\phi$$
$$v_y = u\sin\phi + v\cos\phi \qquad (9)$$

Taking the x-direction at point *0* tangential to the α-line, i.e.

Ø = 0.

$$\left(\frac{\partial u_x}{\partial x}\right)_{\phi=0} = \frac{\partial u}{\partial x} - v\frac{\partial \phi}{\partial x} \qquad (10)$$

The velocity field (Geiringer equations) cont.

Since $\varepsilon_x = \partial u_x / \partial x$ is zero along a slip-line

$$\frac{\partial u}{\partial x} - v\frac{\partial \phi}{\partial x} = 0 \ along \ an \ \alpha - line$$

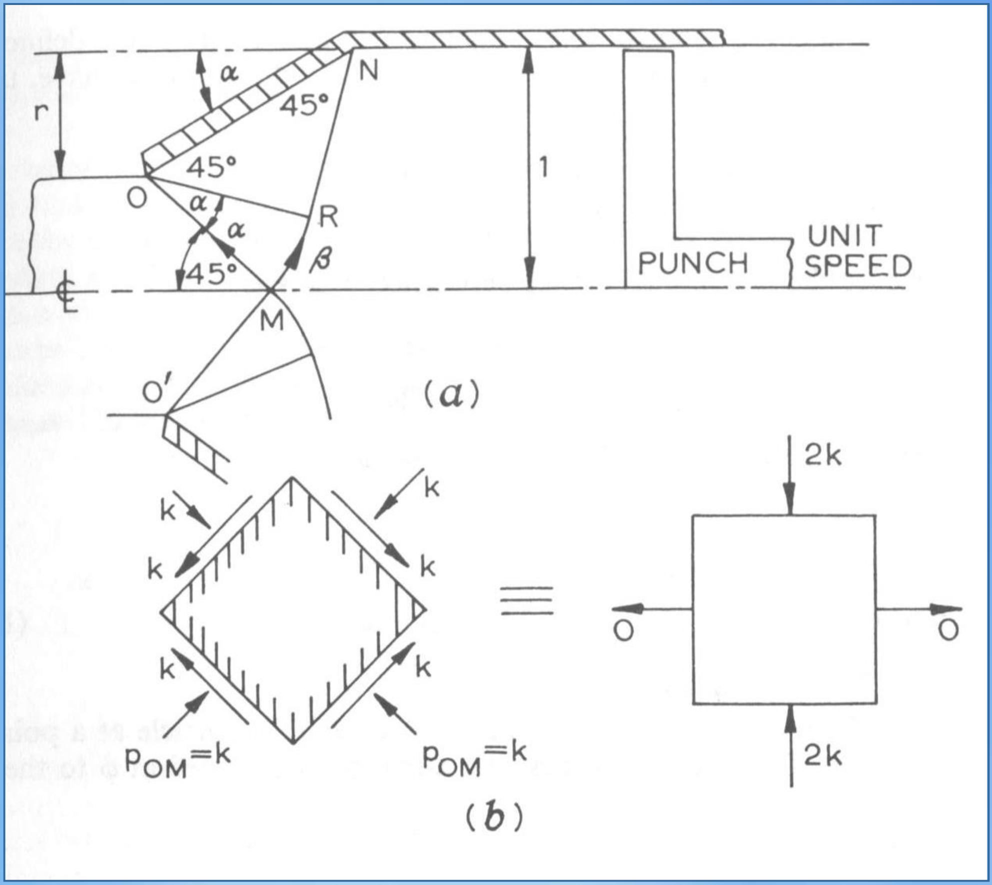
$$du - vd\phi = 0 \ along \ an \ \beta - line \qquad (11)$$

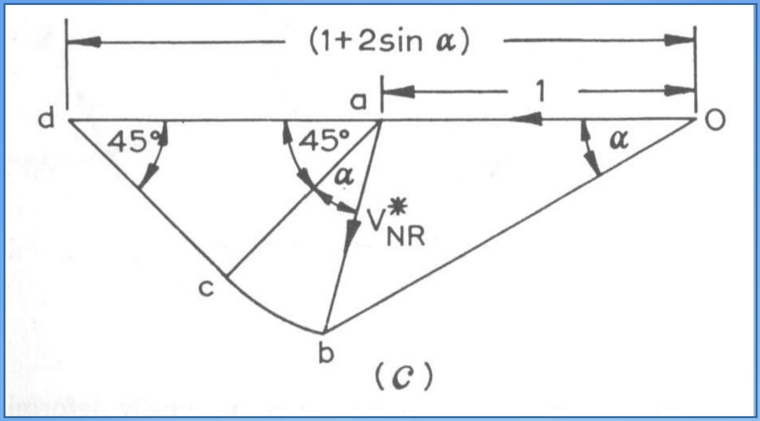
similarly it can be shown that

$$dv - ud\phi = 0 \ along \ an \ \beta - line \qquad (12)$$

Physically, it may be imagined that small rods lying on the slip-line directions at a point do not undergo extension or contraction.

Simple slip-line field solution for extrusion through a perfectly smooth wedge-shaped die of angle α



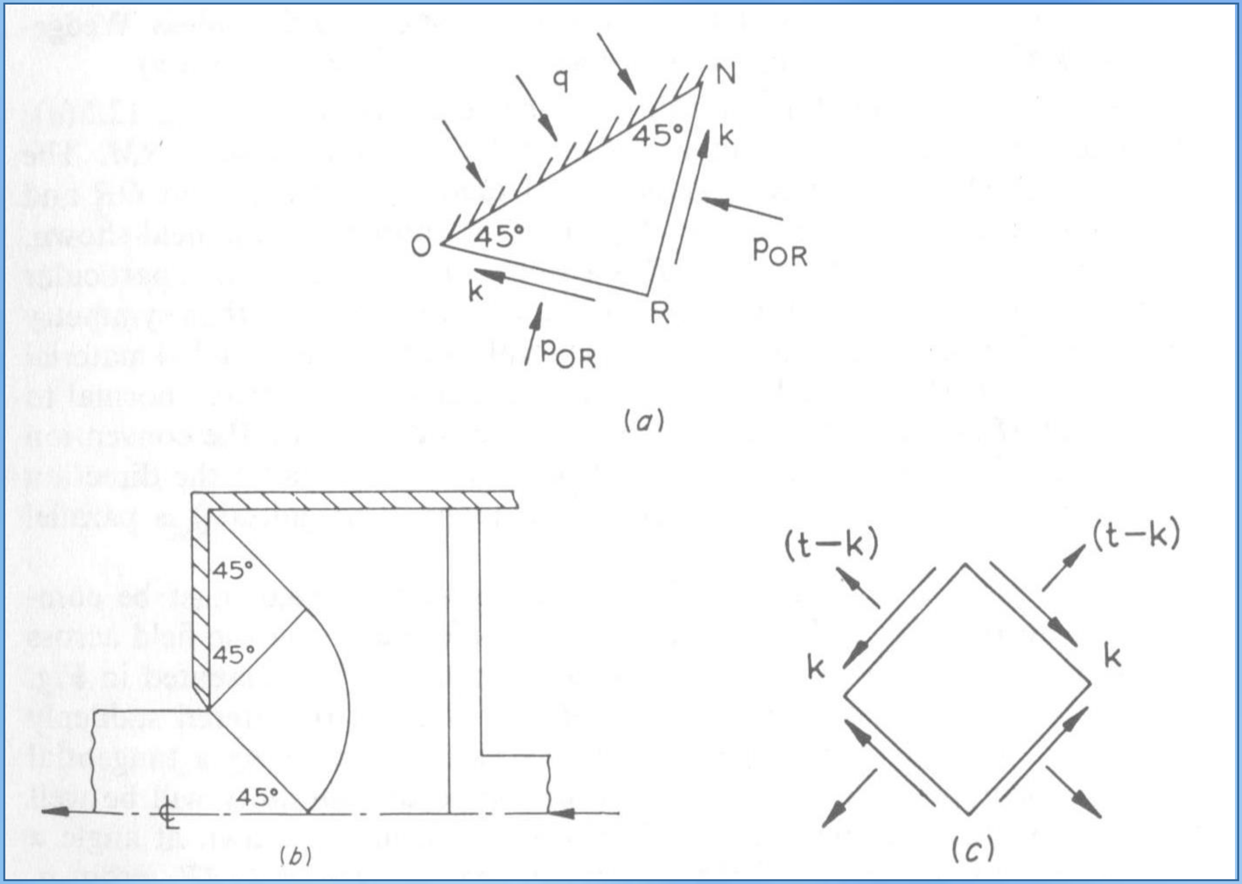


(a) Top half of extrusion only is shown symmetrical about centreline

(b) Stress systems at *M.*

(c) Hodograph to (a)

Simple slip-line field solution for extrusion through a perfectly smooth wedge-shaped die of angle α cont.



(a) To calculate stress on die face.

(b) A square die; container wall and die face both perfectly smooth; r = 2/3.

(c) Stress system at *M* of for drawing.

Refrences

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