Another example for Gauss - Jordan method.

$$\begin{bmatrix} 0 & 2 & 0 & 1 & 0 \\ 2 & 2 & 3 & 2 & -2 \\ 4 & -3 & 0 & 1 & -7 \\ 6 & 1 & -6 & -5 & 6 \end{bmatrix}$$

Soln. Interchanging $R_1 \leftrightarrow R_4$ ( Pivot element being 0).

$$\begin{bmatrix} 6 & 1 & -6 & -5 & 6 \\ 2 & 2 & 3 & 2 & -2 \\ 4 & -3 & 0 & 1 & -7 \\ 0 & 2 & 0 & 1 & 0 \end{bmatrix}$$

Dividing column (1) by 6 and reducing all first column below

$$\begin{bmatrix} 1 & 0.1667 & -1 & -0.8333 & 1 \\ 0 & 1.6667 & 5 & 3.3667 & -4 \\ 0 & -3.6667 & 4 & 4.3334 & -11 \\ 0 & 2 & 0 & 1 & 0 \end{bmatrix}$$

Pivot

Interchanging $R_2 \leftrightarrow R_3$ and dividing row 2 by $-3.6667$ and reducing second column above and below

$$\begin{bmatrix} 1 & 0 & -0.8182 & 0.6364 & 0.5 \\ 0 & 1 & -1.0909 & -1.1818 & 3 \\ 0 & 0 & 6.8182 & 5.6364 & -9 \\ 0 & 0 & 2.1818 & 3.3636 & -6 \end{bmatrix}$$

No interchanging required as $6.8182 > 2.1818$, so divide row 3 by 6.8182 and reducing columns above and below

$$\begin{bmatrix} 1 & 0 & 0 & 0.04 & -0.58 \\ 0 & 1 & 0 & -0.280 & 1.56 \\ 0 & 0 & 1 & 0 & -1.32 \\ 0 & 0 & 0 & 1.5599 & -3.12 \end{bmatrix}$$

Now divide fourth row by 1.5599 and create zeros above.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 0 & 1.001 \\ 0 & 0 & 1 & 0 & 0.3333 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

Fifth column is solution.

Gauss elimination method to compute inverse

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

The augmented system is

$$\left[\begin{array}{ccc:ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} \to -3/2 \; ; \; -1/2 \\ \to \times + \end{array}$$

1st stage:

$$\left[\begin{array}{ccc:ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1/2 & 3/2 & -3/2 & 1 & 0 \\ 0 & 7/2 & 17/2 & -1/2 & 0 & 1 \end{array}\right]$$

II stage:

$$\left[\begin{array}{ccc:ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1/2 & 3/2 & -3/2 & 1 & 0 \\ 0 & 0 & -2 & 10 & -7 & 1 \end{array}\right]$$

This is equivalent

$$\left[\begin{array}{ccc:c} 2 & 1 & 1 & 1 \\ 0 & 1/2 & 3/2 & -3/2 \\ 0 & 0 & -2 & 10 \end{array}\right] ; \left[\begin{array}{ccc:c} 2 & 1 & 1 & 0 \\ 0 & 1/2 & 3/2 & 1 \\ 0 & 0 & -2 & -7 \end{array}\right]$$

Back substitution

$$\left[\begin{array}{ccc:c} 2 & 1 & 1 & 0 \\ 0 & 1/2 & 3/2 & 0 \\ 0 & 0 & -2 & 1 \end{array}\right] ; \begin{bmatrix} -3 & 5/2 & -1/2 \\ 12 & -17/2 & 3/2 \\ -5 & 7/2 & -1/2 \end{bmatrix} = A^{-1}$$

Here $|A| = -2$

If $|A| = 0$, no back substitution possible and matrix has no inverse.

For example:

$\frac{1}{2} y + \frac{3}{2} z = 0$

$y/2 - 3/4 = 0$

$z = -1/2$

$\boxed{y = 3/2}$

$2x + 3/2 + (-1/2) = 0$ Last column

$2x + 2/2 = 0$

$\boxed{x = -1/2}$

$\frac{1}{2} y + \frac{3}{2} z = 1$ middle column

$z = 7/2$

$\frac{1}{2} y + \frac{21}{4} = 1$

$y/2 = -17/4$

$\boxed{y = -17/2}$

$2x - \frac{17}{2} + 7/2$

$2x - \frac{10}{2} = 0$

$\boxed{x = 5/2}$

divide ...

Scanned with CamScanner

Given matrix $A$, find its inverse. First use the Gauss–Jordan method with exact arithmetic

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}.$$

Augment $A$ with the identity matrix and then reduce:

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 3 & -5 & -3 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{(1)} \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -5 & 0 & 1 & -3 \end{bmatrix} \xrightarrow{(2)} \begin{bmatrix} 1 & -1 & 0 & 1 & \frac{2}{5} & -\frac{6}{5} \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -\frac{1}{5} & \frac{3}{5} \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{2}{5} & -\frac{1}{5} \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -\frac{1}{5} & \frac{3}{5} \end{bmatrix}.$$

We confirm the fact that we have found the inverse by multiplication:

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & \frac{2}{5} & -\frac{1}{5} \\ -1 & 0 & 1 \\ 0 & -\frac{1}{5} & \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$