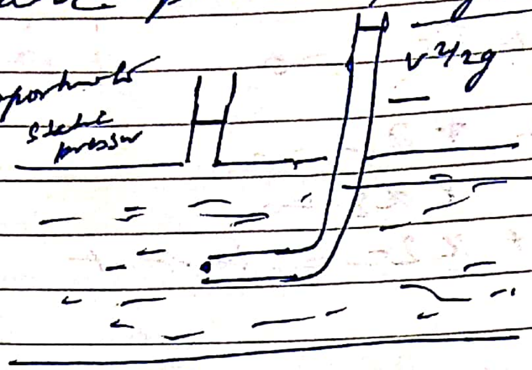
PITOT TUBE

Static pressure, Dynamic & Stagnation



$$P + \rho\frac{V^2}{2} + \rho g z = \text{const (along a stream line)}$$

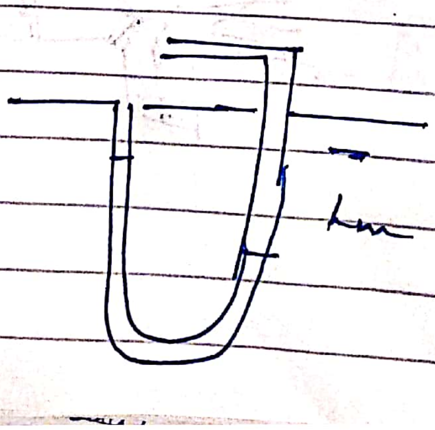
Static pr. — dynamic pr. — hydrostatic pressure

Sum of static & dy press

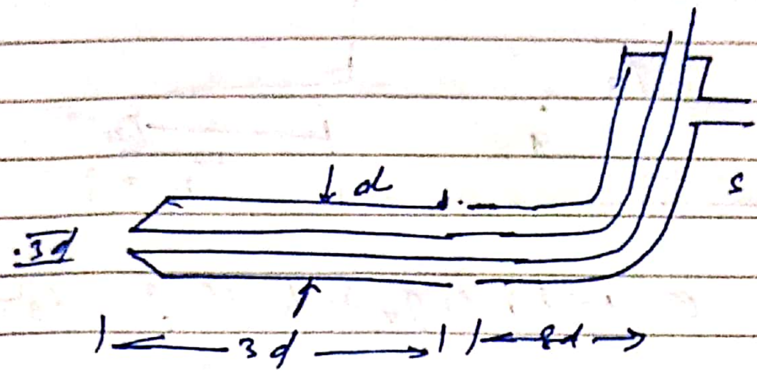
$$P_{stag} = P + \rho\frac{V^2}{2}$$

$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

The stagnation pressure represents the pr. at a point where fluid is brought to a complete stop isentropically.



$$p_{stag} - p = h_m (w_m - w_f)$$
$$= h_m (\gamma_m - \gamma_f)$$
$$= g h_m (\rho_m - \rho_f)$$



6/8 holes.

$$V = \sqrt{\frac{2g (p_{stag} - p)}{\gamma_f}}$$

$$= \sqrt{2g h_m \left(\frac{\gamma_m - \gamma_f}{\gamma_f}\right)}$$

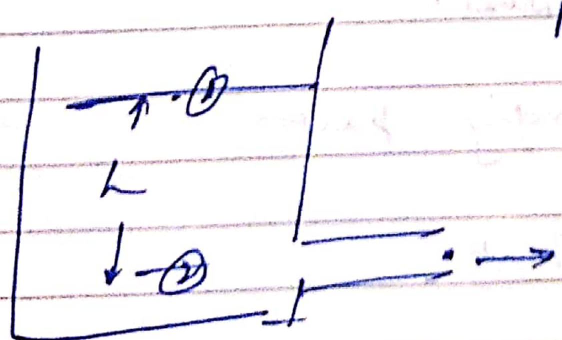
$$S_m = \frac{\text{sp. grav. of manometric fluid}}{\text{sp. gr. of flowing fluid}}$$

$$= \sqrt{2g h_m (S_m - 1)}$$

$$V = C_v \sqrt{2g h_m (S_m - 1)}$$

$C_v = 0.95 - 1$

C_v = coefficient of velocity.



$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$p_1 = p_2 \quad v_1 = 0$

$z_1 - z_2 = h$

$v_2 = \sqrt{2gh}$

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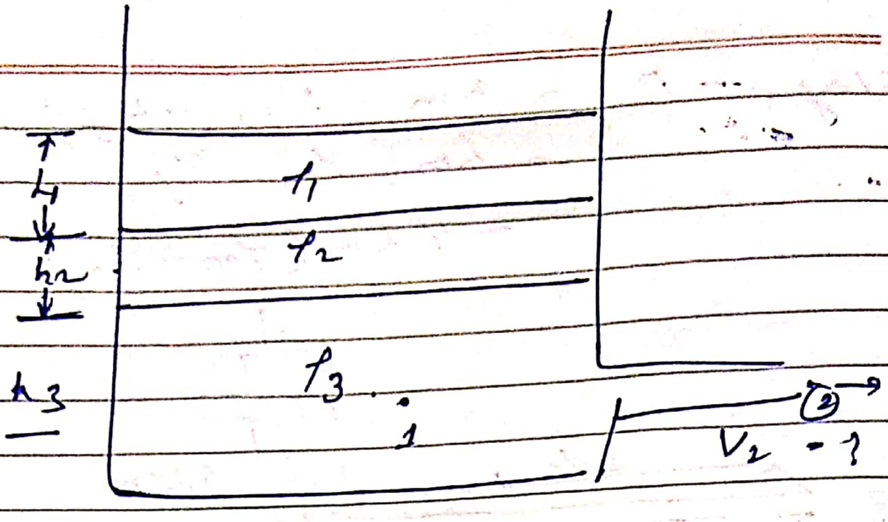


$$p_1 = 0 + h_1\rho_1 g + h_2\rho_2 g + h_3\rho_3 g$$

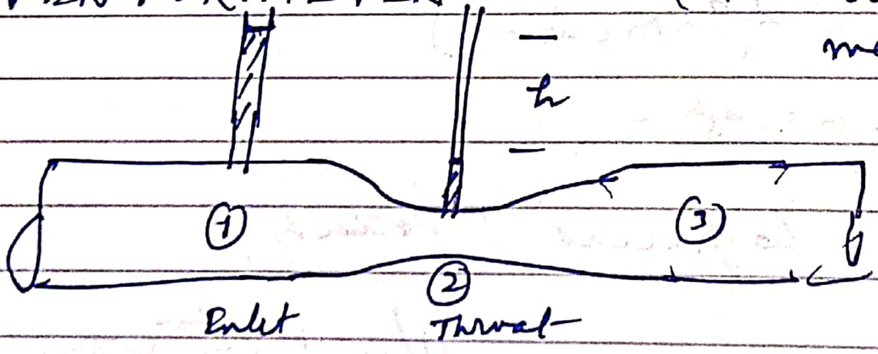
1/2

$$\frac{p_1}{\rho_3 g} + 0 + 0 = \frac{p_2}{\rho_3 g} + \frac{V_2^2}{2g}$$

$$V_2^2 = 2g\left(h_3 + \frac{h_1\rho_1}{\rho_3} + \frac{h_2\rho_2}{\rho_3}\right)$$

$$V_2 = \sqrt{2gh_3\left(1 + \frac{h_1}{h_3}\cdot\frac{\rho_1}{\rho_3} + \frac{h_2}{h_3}\frac{\rho_2}{\rho_3}\right)}$$

VENTURIMETER. (- for discharge measurement)



1. Short converging part
2. Throat
3. Diverging part

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2}$$

from continuity eq^n

$$A_1 V_1 = A_2 V_2 \Rightarrow V_2 = \frac{A_1}{A_2} V_1$$

$$\frac{p_1 - p_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$h = \frac{V_1 \left\{ \left(\frac{A_1}{A_2}\right)^2 - 1 \right\}}{2g}$$

$$V_1 = \frac{A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

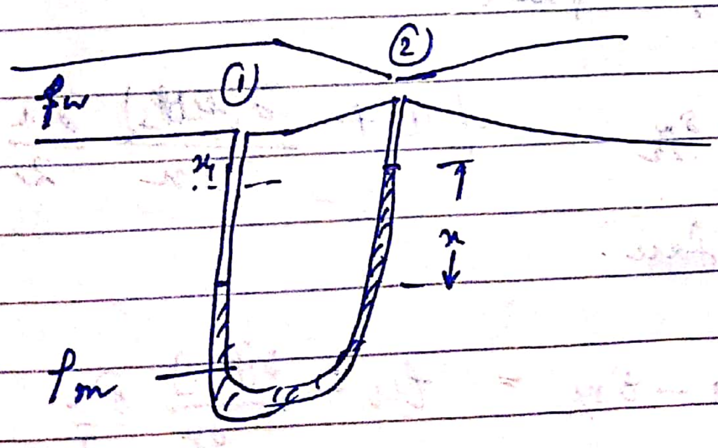
$$Q_{th} = A_1 V_1 = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$$

$Q_{Act} < Q_{th}$ due to friction.

$$\frac{Q_{Act}}{Q_{th}} = C_d \text{ (coefficient of discharge)}$$

$$Q_{Act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}.$$

$C_d \simeq 0.9$



$$+x_1 g\rho_w \qquad -\cancel{x_2 g \rho_w}$$

$$p_1 + \rho_w g x - \rho_m g x = p_2$$

$$p_1 - p_2 = x(\rho_m - \rho_w) g$$

$$\frac{p_1 - p_2}{\rho_w g} = x\left(\frac{\rho_m}{\rho_w} - 1\right) g$$

$$\boxed{h = x\left(\frac{S_m}{S_w} - 1\right) g}$$

If suppose $S_m < S_w$

$$h = x\left[1 - \frac{S_m}{S_w}\right] g$$

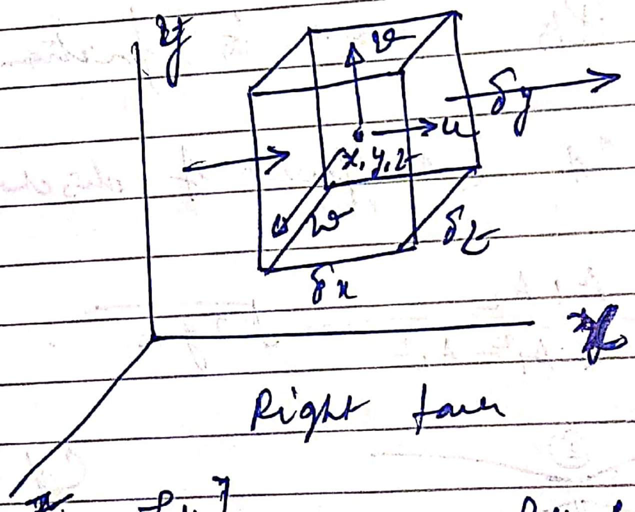
$m \rightarrow$ manometric fluid

$w \rightarrow$ working fluid

Orifice $\rightarrow$ discharge

meter / orifice plate

Continuity Equation.



Right face

$$\rho u\Big]_{x+\delta x/2} = \rho u + \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}$$

Left face

$$\rho u\Big]_{x-\delta x/2} = \rho u - \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}$$

Net rate of mass out flow in x dir

$$= \left[\rho u + \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}\right]\delta y\,\delta z$$

$$- \left[\rho u - \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}\right]\delta y\,\delta z$$

$$= + \frac{\partial(\rho u)}{\partial x}\delta x\,\delta y\,\delta z \quad \text{—— (I)}$$

in ~~Net~~ rate of mass out flow in y direc

$$= + \frac{\partial(\rho v)}{\partial y}\delta x\,\delta y\,\delta z \quad \text{—— (II)}$$

. . . . z direct

$$= + \frac{\partial(\rho w)}{\partial z}\delta x\,\delta y\,\delta z \quad \text{—— (III)}$$

accumulation change in unit time

Total rate of mass out flow = (I) + (II) + (III)

$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall = \frac{\partial\rho}{\partial t}\delta x\,\delta y\,\delta z \quad \text{—— (IV)}$$

(I) + (II) + (III) + (IV) = 0

$$\Rightarrow \frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\Rightarrow \frac{\partial\rho}{\partial t} + \nabla\cdot\rho\vec{V} = 0.$$

for steady flow

$$\nabla\cdot\rho\vec{V} = 0$$

Incompressible / Steady flow

$$\nabla \cdot \vec{V} = 0$$

or $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

Dimensional Anaysis

Dimensional Homogeneity and its application

M / L / T.

6 Eqns describing a physical phenomenon must be dimensionally homogeneous and the units therein must be consistent.

For example for simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

LHS

$$[T] = T$$

$$RHS = \frac{L^{1/2}}{(LT^{-2})^{1/2}} = T.$$

Some well known example of dimensionally homogeneous eqn are.

$$\phi = AV$$

$$V = \sqrt{2gh}$$

$$p_1 - p_2 = \frac{32\mu V l}{d^2}$$ Hagen Poiseuille Eqn

$$h_f = \frac{4fLV^2}{2gd}$$ Darcy Weisbach Eqn

Dimensional Analysis

Rayleigh Method.

* Gather all independent variables which are likely to influence the value of the dependent variables
* Write the function relationship

$$y = f(x_1, x_2, x_3 \ldots)$$

$$y = k\, x_1^a\, x_2^b\, x_3^c \ldots$$

$\underline{MLT} \longrightarrow a/b/c$

* If the number of exponents (a, b, c, d) involved is more than 3, then exponents of the properties D, V, & ρ are evaluated in term of other exponents.
* Substitute the value of exponents in the main eqn and form non dimensional parameters by grouping the variables with like exponents.

DSK Ex 6.7 — Show that the resistance F to the motion of a sphere of diameter D moving with a uniform velocity V through a real fluid of density ρ and viscosity μ is given by

$$F = \rho D^2 V^2 f\left(\frac{\mu}{VD\rho}\right).$$

Sln

$$F = f(D, V, \rho, \mu)$$

$$F = K\, D^a V^b \rho^c \mu^d$$

$$[MLT^{-2}]^{a} = (1)\,[L]^{a}[LT^{-2}]^{b}[ML^{-3}]^{c}\,[ML^{-1}T^{-1}]^{d}$$

$$1 = c + d$$
$$1 = a + b - 3c - d$$
$$-2 = -b - d$$

express the exponent of D, V, ρ in terms of μ

$$c = 1 - d$$
$$b = 2 - d$$
$$a = 1 - (2-d) + 3(1-d) + d$$
$$= 2 - d.$$

$$F = K\left[D^{2-d}\, V^{2-d}\, \rho^{1-d}\, \mu^{d}\right]$$

$$= K\rho D^2 V^2 \cdot \left[\frac{\mu}{DV\rho}\right]^{d}$$

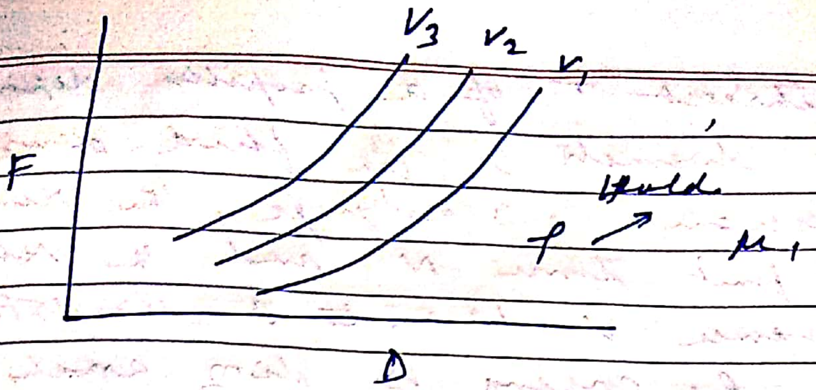
$$F = \rho D^2 V^2\, \phi\left[\frac{\mu}{DV\rho}\right]$$

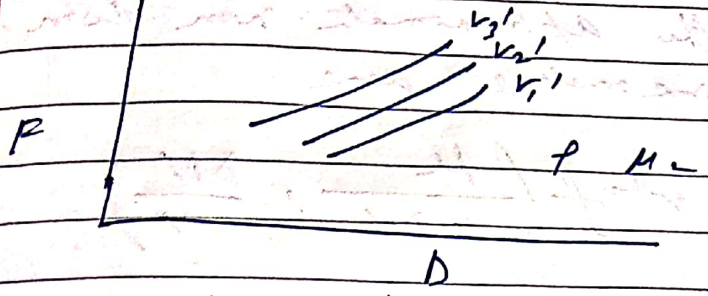
Method I

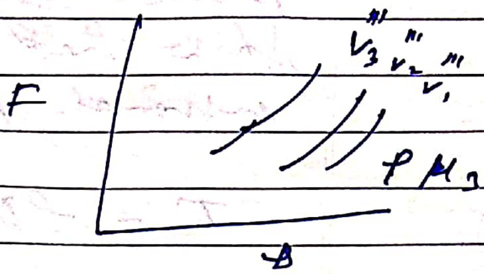
Solution without dimensional analysis

$$F = f(D, V, \rho, \mu)$$

Can be depicted graphically by varying only one parameter at a time and keeping all other const.







The drag force F has been plotted against D for various values of V and keeping ρ and μ constant. Evidently many more such plots will be required for an effective description of the flow situation.

Further

D is also ~~var~~ varied.

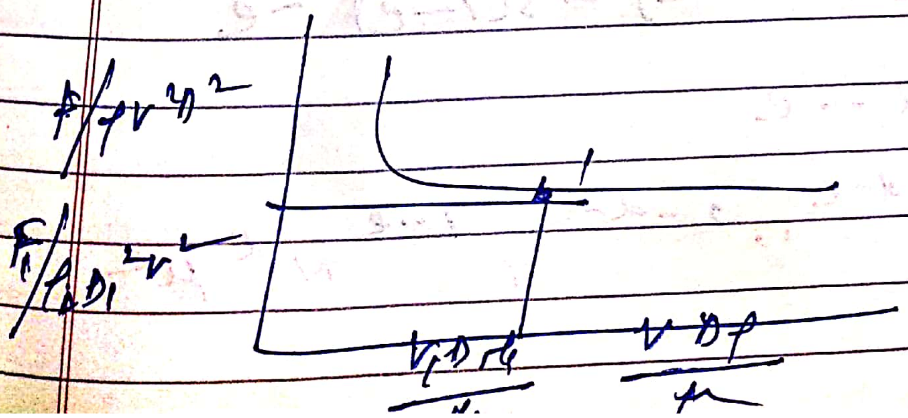
a number of fluid of varying μ and ρ.

Laborious and time consuming and expensive.

Method II

Solution with dimensionless parameter

$$\frac{F}{\rho V^2 D^2} = f\left(\frac{VD\rho}{\mu}\right)$$



Less expensive

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Ex 6.4 > The thrust T of a propeller depends on its diameter D, the fluid density ρ, dynamic viscosity μ, the revolution per unit time N and the velocity of advance V w.r.t. to the undisturbed fluid. By means of dim analysis show that the approximate non dimensional parameters are

$$T = \rho D^2 V^2 f\left(\frac{\mu}{VD\rho}, \frac{DN}{V}\right)$$

Soln

$$T = f(D, V, \rho, N, \mu)$$

$$T = K D^a V^b \rho^c N^d \mu^e$$

$$[MLT^{-2}] = (M)[L]^a [LT^{-1}]^b [ML^{-3}]^c [T^{-1}]^d [ML^{-1}T^{-1}]^e$$

M: $1 = c + e$

L: $1 = a + b - 3c - e$

T: $-2 = -b - d - e$

$c = 1 - e$

$b = 2 - d - e$

$1 = a + (2 - d - e) - 3(1 - e) - e$

$a = 2 + d - e$

$$T = K D^{2+d-e} V^{2-d-e} \rho^{1-e} N^d \mu^e$$

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$$= k \, D^2 V^2 \rho \left(\frac{DN}{V}\right)^d \left(\frac{\mu}{\rho V D}\right)^e$$

$$= \rho D^2 V^2 f\left[\left(\frac{\mu}{\rho V D}\right), \left(\frac{DN}{V}\right)\right]$$

Buckingham's Pi . Theorem :-

" If there are n variables in a homogeneous eqn and if these variables contain m primary variable, the variable can be group into (n − m) non dimensional parameters.

The non dimensional groups are called Pi terms.

$$f(x_1, x_2, x_3 \ldots \ldots x_m) = 0$$

$$\phi(\pi_1, \pi_2, \pi_3 \ldots \pi_{m-n}) = 0$$

~~MLT~~ ~~3 variable~~ These variable are not to form non dimn para ~~any~~ amongst themselves.

Repeating variable

1. A geometrical property such as length (D)
2. A fluid property (such as) ρ
3. A flow characteristics (such as) velocity V.

Ex 1.8 Show by the use of Buckingham π theorems, the velocity through an orifice is given by

$$V = \sqrt{2gh}\, f\left(\frac{D}{H}, \frac{\mu}{\rho V H}, \frac{\sigma}{V^2 D}\right)$$

$\sigma \rightarrow$ S.T.

Soln $f(V, D, H, g, \rho, \mu, \sigma) = 0$

$\left.\begin{array}{l}\rho \\ V \\ H\end{array}\right\} \rightarrow$ Repeated variables.

$7 - 3 = 4\ \pi$ terms

(i) $\pi_1 = \rho^{a_1} V^{b_1} H^{c_1} D$

$$[M^0 L^0 T^0] = [ML^{-3}]^{a_1} [LT^{-1}]^{b_1} [L]^{c_1} [L]^1$$

$0 = a_1$

$0 = -3a_1 - b_1 + c_1 + 1$

$0 = -b_1$

$c_1 = -1$

$$\pi_1 = \rho^0 V^0 H^{-1} D = \frac{D}{H}.$$

(ii) $\pi_2 = \rho^{a_2} V^{b_2} H^{c_3} g$

$[M^0 L^0 T^0] = [ML^{-3}]^{a_2} [LT^{-1}]^{b_2} [L]^{c_3} [LT^{-2}]$

$[M] \quad 0 = a_2$

$[L] \quad 0 = -3a_2 + b_2 + c_2 + 1$

$[T] \quad 0 = -b_2 - 2$

$b_2 = -2$

$0 = 0 - 2 + c_2 + 1$

$c_2 = 1$

$\pi_2 = \rho^0 V^{-2} H^1 g = \dfrac{gH}{V^2}$

(iii) $\pi_3 = \rho^{a_3} V^{b_3} H^{c_3} \mu$

$[M^0 L^0 T^0] = [ML^{-3}]^{a_3} [LT^{-1}]^{b_3} [L]^{c_3} [ML^{-1}T^{-1}]$

$[M] : 0 = a_3 + 1$

$[L] : 0 = -3a_3 + b_3 + c_3 - 1$

$[T] : 0 = -b_3 - 1$

$a_3 = -1$

$b_3 = -1$

$0 = +3 - 1 + c_3 - 1$

$c_3 = -1$

$\pi_3 = \rho^{-1} V^{-1} H^{-1} \mu$

$\pi_3 = \dfrac{\mu}{\rho V H}$.

(iv) $\pi_4 = \rho^{a_4} V^{b_4} H^{c_4} \sigma$

$[M^0 L^0 T^0] = [ML^{-3}]^{a_4} [LT^{-1}]^{b_4} [L]^{c_4} [MT^{-2}]$

$[M] : 0 = a_4 + 1$

$[L] : 0 = -3a_4 + b_4 + c_4$

$[T] : 0 = -b_4 - 2$

$a_4 = -1 \qquad c_4 = -1$

$b_4 = -2$

$+3 - 2 + c_4 = 0$

$c_4 = -1$

$\pi_4 = \rho^{-1} V^{-2} H^{-1} \sigma$

$= \dfrac{\sigma}{\rho V^2 H}$

$\phi(\pi_1, \pi_2, \pi_3, \pi_4) = 0$

$\phi\left[\dfrac{D}{H}, \dfrac{gH}{V^2}, \dfrac{\mu}{\rho V H}, \dfrac{\sigma}{\rho V^2 H}\right) = 0$

$\dfrac{V^2}{gH} = f\left(\dfrac{D}{H}, \dfrac{\mu}{\rho V H}, \dfrac{\sigma}{\rho V^2 H}\right)$

$V = \sqrt{2gH}\, \phi_1\left[\dfrac{D}{H}, \dfrac{\mu}{\rho V H}, \dfrac{\sigma}{\rho V^2 H}\right).$

SIMILITUDE :-

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Model Studies

Prototype: Prototype is the full size structure employed in the actual engg design. The prototype operates under actual working condition.

Model: Model is a system by whose operation the characteristics of other similar system can be ascertained. Experimental observations made on a model bears a definite relationship to the protype. Model, a mech analogy of the prototype is generally a small scale replica of the prototype. However in some cases, a model may be even larger or of the same size as prototype dims depending upon the need and purpose. For example, the performance of a carburettor and a wrist watch is made on a large scale model.

SIMILITUDE

Similitude refers to the theory and art of predicting prototype conditions for model observations. It prescribes the relationship between a full scale flow and flows involving smaller but geometrically similar boundaries.