NBA Trade Sense: A Partially-Observable Markov Decision Process to Help General Managers Time Trades Better

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1 Introduction

Sports analytics is a rapidly growing field, attracting both fans and teams with its promise of deeper insights into the world of sports. For the average fan, the most visible aspect of sports analytics is in consumer-facing applications, for example, 538's NBA predictions. These tools often focus on answering the questions that most fans care about: Who will win the next game? How will this affect playoff chances? Though consumers invested in narratives and fantasy leagues usually love these analytics, there is not much publicly said about analytics used by the professional teams themselves.

Unlike the consumer-oriented models, which are more about predicting game outcomes, team-centric analytics delve deeper into strategic aspects that can directly impact a team's performance and future. Most NBA teams now have designated sports analytics departments that serve as support to the team's basketball operations, crunching numbers and using evidence to make more educated choices on player acquisition and management.

One significant area of focus is understanding trade patterns. By framing the player trading problem within a Partially-Observable Markov Decision Processes (POMDPs) framework, we aim to provide valuable insights into the dynamics of player trades, equipping GMs with a data-driven tool to navigate the complex landscape of trade decision-making in the NBA.

2 Data Collection and Aggregation

By-Game Player Data. Using the NBA API over the 10 years from 2013-2023, shortly after the NBA Lockout Season, which is often is considered a regime shift in the league, we gauge numerous stats across every active player in every NBA game.

Position and Salary Data. Using BeautifulSoup to scrape from ESPN, we garner the positions of each player in the league, as well as their average salary since 2013. Later on we will segmenting players by position, and, as explained later on in Model Architecture, by their relative performance amongst their position. Moreover, salary, or in other words, cost to a team, will be used as a part of our utility calculation within the POMDP.

3 Model Architecture

We focus on employing POMDPs to shed light on the optimal timing of NBA player trades, operating under the premise that general managers (GMs) may not always make trades optimally due to the challenge of dealing with unobservable information. The intricacies of player trades involve various uncertainties, such as past and future player performance, team dynamics, and external market conditions, and creating a complex decision-making landscape. Additionally, trading decisions often vary by the kind of player we observe and is therefore rather personalized. To take this into account, we build a separate POMDP model for each player position and enable states for varying levels of performance in each model. The positions are: *point guard (PG)*, *shooting guard (SG)*, *small forward (SF)*, *power forward (PF)*, and *center (C)*. Our modeling approach explicitly considers the limitations faced by GMs, accounting for the inherent uncertainty and incomplete information that permeate the decision to trade or retain a player.

3.1 States

Each state of a particular position's POMDP is a 2-item tuple: the first entry being the *tenure* t, representing the number of games a player has been on a team (which resets to 0 after a trade), and the second item being the *tier* S, a discretized measure of player performance categorized into tiers A, B, C, and D. The cutoffs of these tiers are based on the percentile quartiles of the distribution of performances for each position. The tenure refers to the cumulative number of games a player has played for a specific team. It serves as a key metric to quantify a player's historical association with any particular franchise (the tenure is team-agnostic). Notably, tenure is reset to zero following a player trade, acknowledging the player's transition to a new team. As for the tier determination, we chose to discretize the performance into these tiers to manage the complexity of the POMDP, preventing an excessive number of states and facilitating a more tractable modeling approach. The tier is determined by a custom metric that is calculated by player performance in a particular game. The composite metric, denoted as *METRIC*, is defined by an equally weighted combination of key NBA performance statistics obtained from the data:

$$METRIC = FG3\_PCT + FG\_PCT + FT\_PCT + PTS\_MIN + AST\_MIN + REB\_MIN + STL\_MIN + BLK\_MIN$$

$$- TOV\_MIN - PF\_MIN - PFD\_MIN$$

where each variable represents following specific player performance metrics: *FG3_PCT*: Three-point field goal percentage; *FG_PCT*: Field goal percentage. *FT_PCT*: Free throw percentage. *PTS_MIN*: Points per minute. *AST_MIN*: Assists per minute. *REB_MIN*: Rebounds per minute. *STL_MIN*: Steals per minute. *BLK_MIN*: Blocks per minute. *TOV_MIN*: Turnovers per minute (negatively weighted). *PF_MIN*: Personal fouls per minute (negatively weighted). *PFD_MIN*: Personal fouls drawn per minute (negatively weighted).

The metric is then aggregated for all games played by all players of a particular position, allowing for quartiling into 4 segments denoting A, B, C, and D level performance for each POMDP. In Figure 1 at the end of this section are some examples of specific players and their performance according to our custom metric, as well as how their performance compares to the quartile tiers we denote specific to their positions. From these plots, we can already get an understanding of how consistent or volatile a player's performance is and in what tier they mostly reside over cumulative games played.

Figure 1: Player Performance

3.2 Actions and Transition Matrices

The action is quite simple - at each state, the GM has the decision to either trade the player or retain the player. Given the action, our implementation builds a transition matrix to move from state i to state j. Each entry *(i,j)* of the transition matrix for a particular action represents this probability. For the matrix associated with keeping the player (NOT trading them), consider the 4 tiers assigned to player performance (A,B,C,D) and not assuming a player's tenure can stretch beyond 1000 games; this leads to a 4000 $\times$ 4000 transition matrix. The transition matrix thus describes the probability of the tier that a player will perform at in the next step of their tenure, which was calculated nonparametrically by averaging proportion of changing tiers across league-wide data, segmented specifically by tier and tenure.

For transition matrices for trading players, it is simply 4×4, as we know tenure will go to 0 after a trade, so we just need to predict which tier they jump to (in the code, we tile the 4×4 so that the dimensions are consistent with the non-trading, 4000×4000 transition matrix). In a similar fashion to the other transition matrix, we nonparametrically average the real world proportion of players of the each tier that shift to different tiers after being traded. Keep in mind, there is a pair of both of the above mentioned transition matrices for each position, given that we have chosen a different POMDP to model each position's trade dynamics.

3.3 Reward

We offer our own utility function that takes into account different factors to generate the reward of taking a particular action $a_i \in \{\text{retain, trade}\}$ at a given state $x_i = (t, S)$. Our utility function is aimed to take into account important considerations to generate the reward, namely, the player performance and the cost of having the player on the team (their salary). Our modeling assumption involves the cost of the salary because we presume that GMs would like to equip their team with better players but face the financial cost of paying better players higher salaries. Given a limited franchise budget, this cost should be factored into the decision-making of retaining the player on the team, and is therefore a negative reward in our function. The salary by player is aggregated by position and tier to generate an average salary by position and by tier. This salary, usually in tens of millions of dollars, is then scaled by a hyperparameter chosen to be either 10^{-7} or 10^{-8}, contrasting a 10x difference in how much the GM cares about financial cost of the player. We run our POMDP for each hyperparameter choice and observe differences in the value functions of the states.
Given the action is to retain the player on the team, the reward function is the sum of the scaled cost of the player and the average metric observed by a player of the particular position for the game at tenure t, as this represents the player's positive or negative performance for the team.

Given the action is to trade the player away, the reward function is negatively-weighted sum of the average player performance metric for all games following t, team-agnostic. This represents the cost or "regret" that the GM experiences by not having the player on the team anymore.

$$u(x_i, a_i) = \begin{cases} -(10^{-7} \times \text{average salary}) + \text{performance\_metric}(x_t), \text{ if } a_i = \text{retain on team} \\ -\text{average future metric, if } a_i = \text{trade} \end{cases}$$

4 Simulation

Given five independent models for each position, we simulate the POMDPs in action in order to draw insights regarding trade timing (how far into a player's tenure on a team is it a good decision to trade them) as well as compare our model's predictions to actual historical player trades in the past decade. The Bellman equation for POMDPs is:

$$V(x, \varepsilon) = \max_a (u(x, a, \varepsilon) + \beta EV(x, a))$$

As the value function is as function of an unobservable state, we aim to find the EV function for each state (t, S), that represents the expected value over all possible successor states. Notice here our partially-observable assumption, ε, is present in the calculation of the EV of a state, and we further make the additive utilities assumption that enables us to remove the ε from the utility function.

$$E_\varepsilon(f(\varepsilon(x', a')|x')) = E_\varepsilon\left(\max_{a'} (u(x', a') + \varepsilon(x', a') + \beta EV(x', a'))\right)$$

$$EV(x, a) = E_{x'} E_\varepsilon(f(\varepsilon(x', a')|x'))$$

Assuming that ε follows a Gumbel distribution, we can invoke the Gumbel trick and determine an expression for EV that is a function of our previously defined transition probabilities and utility function, allowing for simulation until convergence:

$$EV(x,a) = \sum_{x'} \left(\log \sum_{a'} \exp\left(u(x',a') + \beta EV(x',a')\right) \right) p(x'|x,a)$$

5 Results

Given our converged expected value, we can plot the EV for retaining the player (no trade) of a particular tier on the team over their tenure. An example is shown for power forwards (PF) in Figure 2 and 3. All plots (for df=0.96, and cost factor: 10^{-8}) are given in Appendix A.

To compare how the cost factor hyperparamater and the discount factor β may affect the EV function, we tested spanning the salary hyperparameter factor to either 10^{-7} or 10^{-8}, and a β value of $\beta \in \{0.7, 0.96, 0.99\}$, to represent different general managers who have varying degrees of cost aversity and forward-lookingness.

From the graphs below (df = 0.96, balanced outlook), we can also see that players of all different tiers start of with fairly consistent EVs for early in their tenure with a team, but the EV becomes increasingly more volatile as tenure continues. GMs with different risk tolerances may choose to time trades at different tenures. It is important to note that as the scaling factor decreases by a factor of 10x (from 10^{-7} to 10^{-8}), the magnitude of EV increases generally (notice the difference in scale of the y-axis) - this makes sense because the cost of retaining a player is de-emphasized, and the GM emphasizes performance more in making the decision. As expected, higher-tiered players give higher forward-looking value, though interestingly there is not a major difference in A and B tiered players especially in the direction of their volatility (similar number of instances of stellar performance). This may be attributed to the premium cost of retaining a tier A player creating a possibly acceptable tradeoff of taking a B-tiered player with for a lower price.

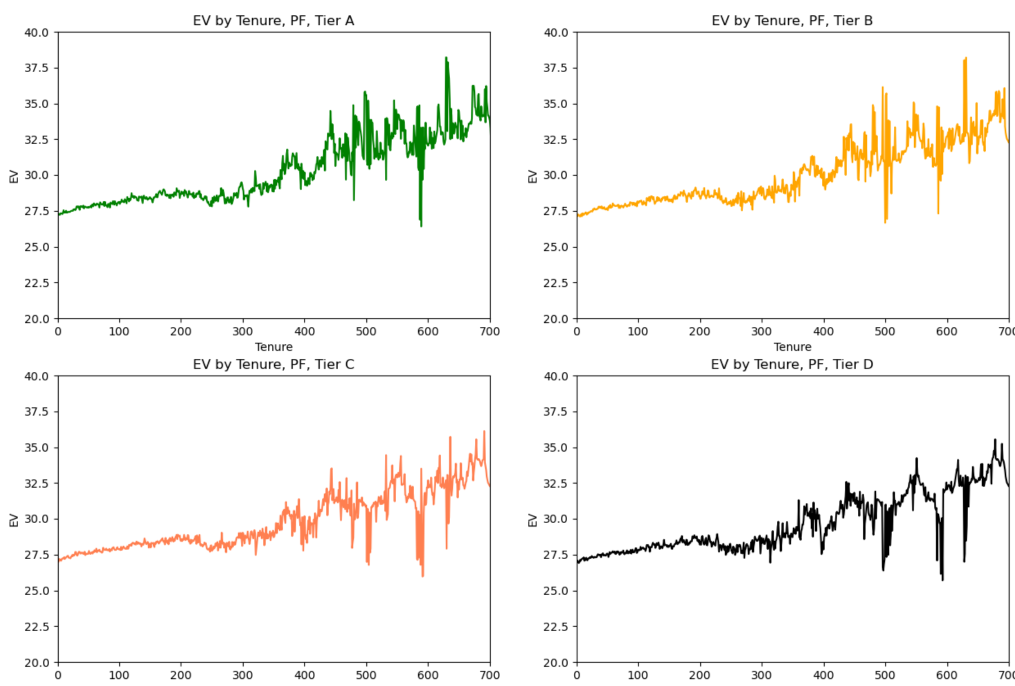


Figure 2: EV vs. Tenure (PF), Cost Scaling: 10^{-7}, df = 0.96

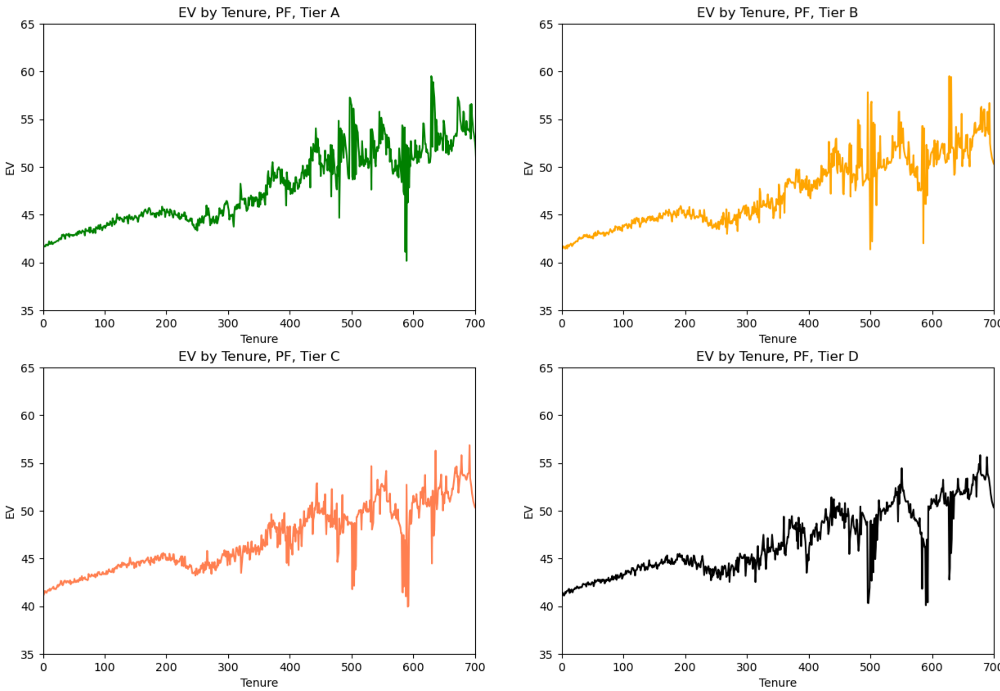


Figure 3: EV vs. Tenure (PF), Cost Scaling: 10^{-8}, df = 0.96

We do see a substantive difference in volatility if we compare A-tier players vs. D-tier players, as shown in Figure 4. Once overlaid, it's easier to see the difference in EV values, throughout all tenure ranges, as well as note how there is generally more volatility in the EV value of D-tier players compared to the best players. Sections of the tenure where there's high volatility usually see A-tier players have EVs that spike up, while D-tier players have EVs that spike down. This verifies a core performance difference between the lowest and highest tiered players.

Interestingly, the EVs when deciding to trade players are largely the same across different tiers. Figure 5 shows this for point guards (see Appendiex A for all positions). However, this makes sense in light of what 4 shows us. Recall that for traded players, we reset their tenure to 0. Notice how in figure 4, for all positions, A-tier and D-tier players generally start out at the same EV value at tenure 0. So, this result remains consistent with the rest of our EVs, although may shed light onto a limitation on the robustness of our methods to create the transition matrices.

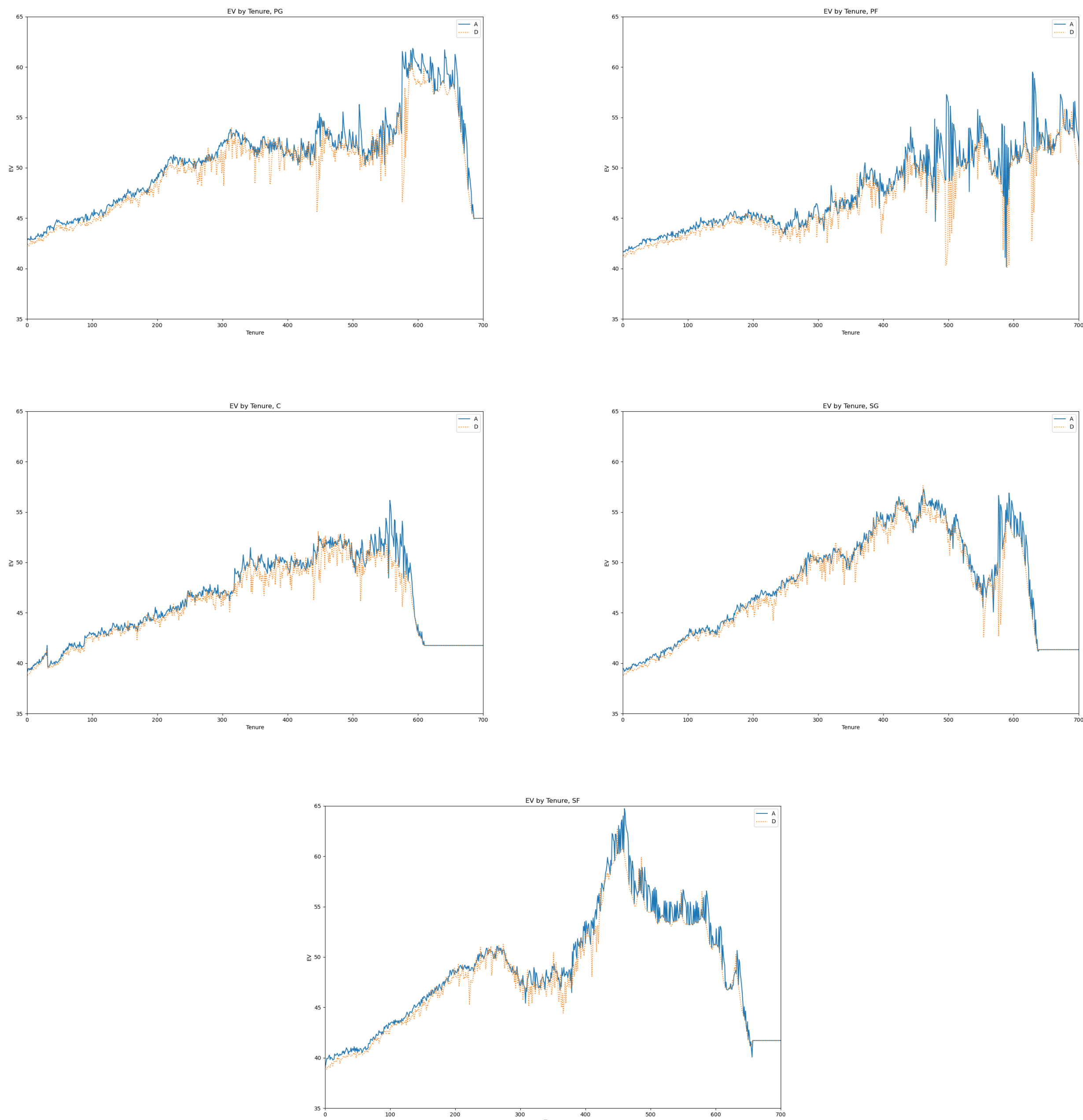


Figure 4: EV vs. Tenure, cost scaling: 10^{-8}, df = 0.96

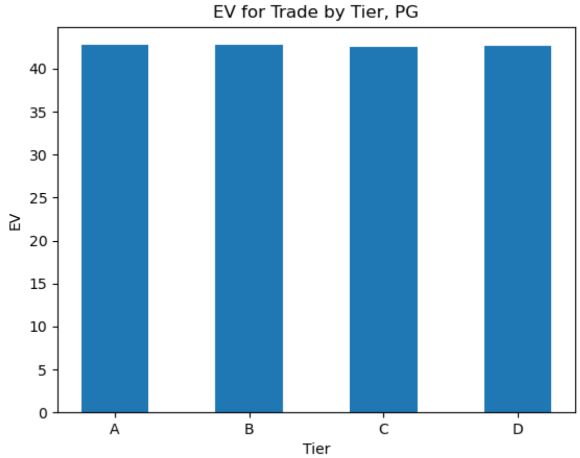


Figure 5: EV trading a PG. cost scaling: 10^{-8}, df = 0.96

We can also simulate our POMDP model for specific players and contrast when our model predicted to trade with high probability to when the player was actually traded. This involves calculating the choice probability for each POMDP at every decision point:

$$P(a_i = \text{trade}|x_i) = \frac{\exp\big(u(x_i, \text{trade}) + \beta EV(x_i, \text{trade})\big)}{\sum_{a'_i \in \{\text{retain,trade}\}} \exp(u(x_i, a'_i) + \beta EV(x_i, a'_i))}$$

We can extract from our data the exact historical times in a player's tenure (on a specific team) that they were traded. Figure 6 shows some choice probability results for recently-traded popular players. All plots for key players in the last decade are given in Appendix B. It's interesting to see that in general, historical trade times do line up with tenures of high probability, indicating that general managers are making near-optimal trades, or that our model reflects the attitudes of general managers in the league (or both). This of course is not always true. In Kevin Love's probability graph, you can see that he was traded at around 500 games, which was a period of low probability to trade, based on our model.

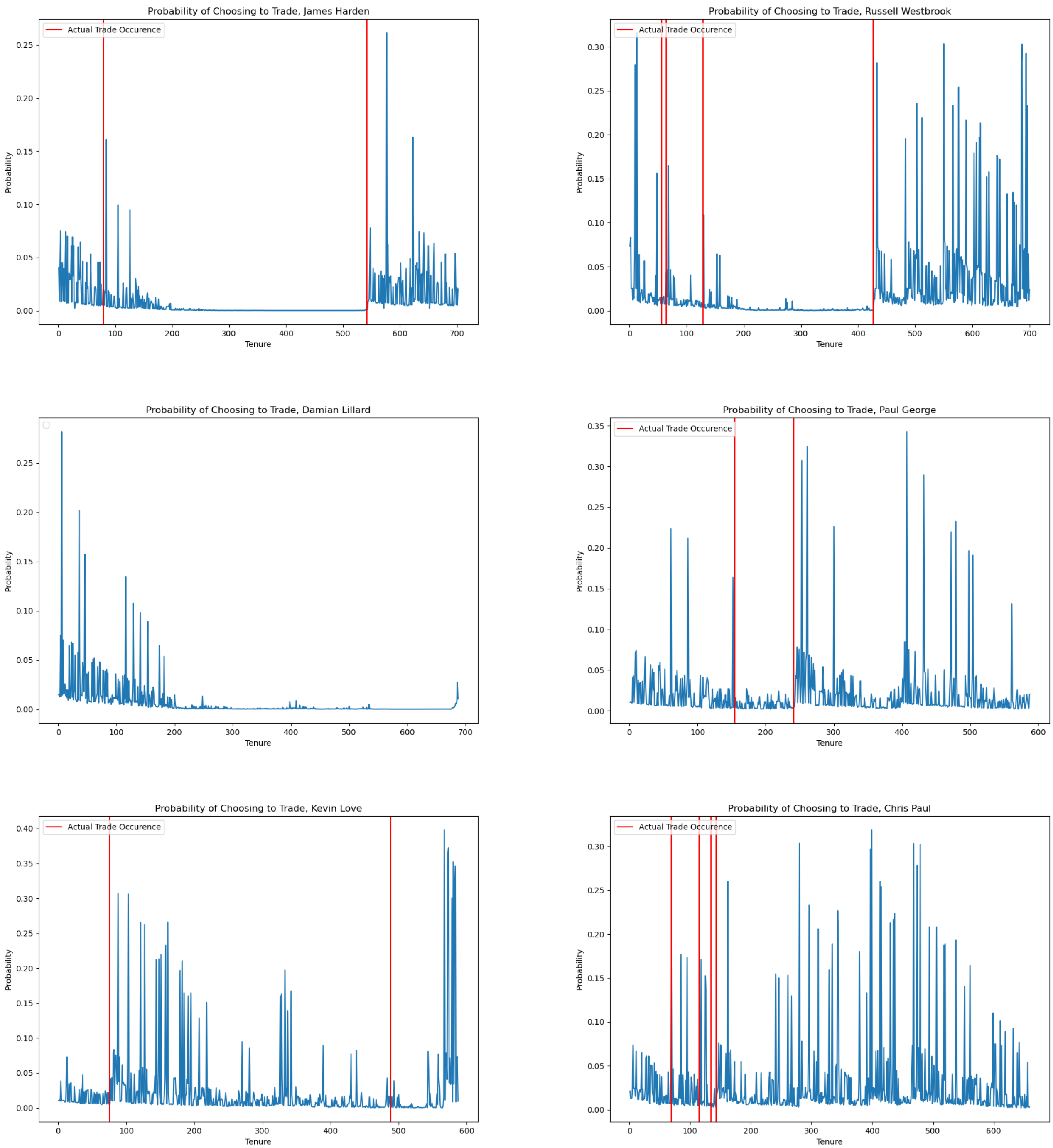


Figure 6: Popular Players: Choice Probability vs. Historical Trade, cost factor: 10^{-8}, df = 0.96

6 Implications on Decision-Making

From our results, one avenue of interpretation lies in the inherent value of different positions. Consider the graphs below (df = 0.99, forward looking, scaling factor = 10^{-8}), in Figure 7.

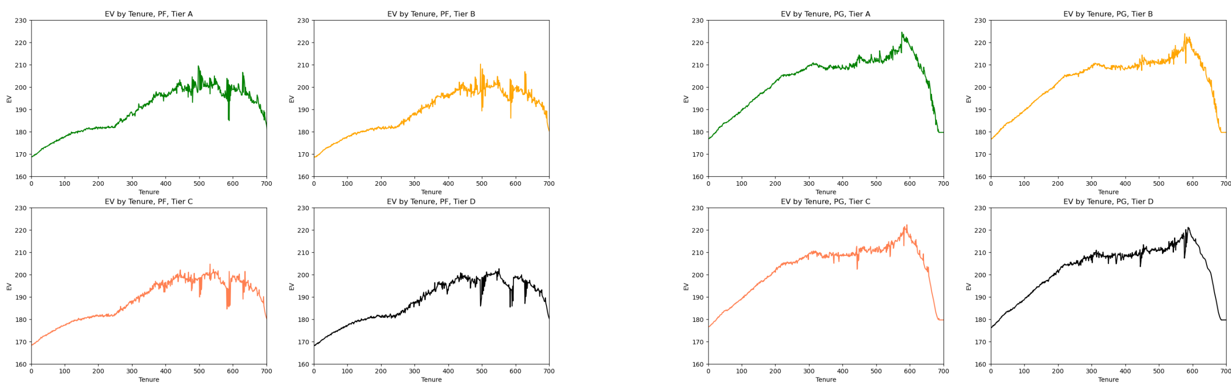


Figure 7: EV by Tenure, PF vs PG, df = 0.99, 10^{-8} scaling

We see that for both point guards and power forwards, EV seems to drop at and after between 500/600 games. This could be a sign to GMs to conciously look for trade options as players approach such a tenure. The key difference between these positions, however, lies in the magnitude of their EV. Power forward EVs peak at around 210, while point guards peak at above 220, with a consistent hover around 210 earlier in the tenure as well. This is perhaps an interesting indicator that point guards provide more value in the modern NBA than power forwards, and may sway GMs into putting more resources into PG scouting and development.

Now looking to a breakdown of the choice probabilities of key players such as LeBron James and Russell Westbrook (Figure 8), we see some trends down at the player level. First looking to cost, we see that for both players, when scaling factor is 10^{-8} instead of 10^{-7}, the probability of choosing to trade has a lower magnitude across the board — so much so, in fact, that James and Westbrook would be considered "untradable" for parts of their tenure. This outlines that teams more willing to take the cost burden of expensive superstars like these two will find their promise and hold onto them for longer, as key investments for their franchise.

Now looking to the other breakdown in the same figure below (9), we look at a balanced outlook GM (df=0.96) and a forward thinking GM (df=0.99). For LeBron, a player know to be very consistent, there are already relatively low trade probabilities under the balanced GM, but with the forward thinking GM, he is all but cemented in the future of the team, with near 0 trade probabilities in at the vast majority of tenure times. For Westbrook, on the other hand, a player known to be quite volitile at times, there are portions in his possible tenure where it is likely even a balanced GM would trade him, notably in the first 100 games or sometime after 400 games. However, with the forward thinking GM, they seem to be more willing to keep Westbrook, smoothing out volitile trade probabilities to near 0. In other words, Westbrook is a player that may be lucrative as a long-term investment for a forward thinking GM.

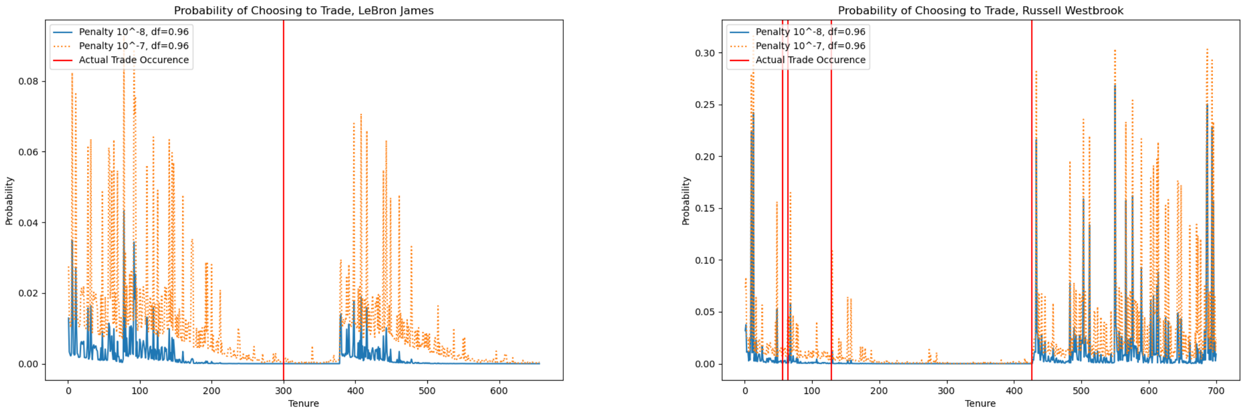


Figure 8: Popular Players: Choice Probability vs. Historical Trade, varying cost factors

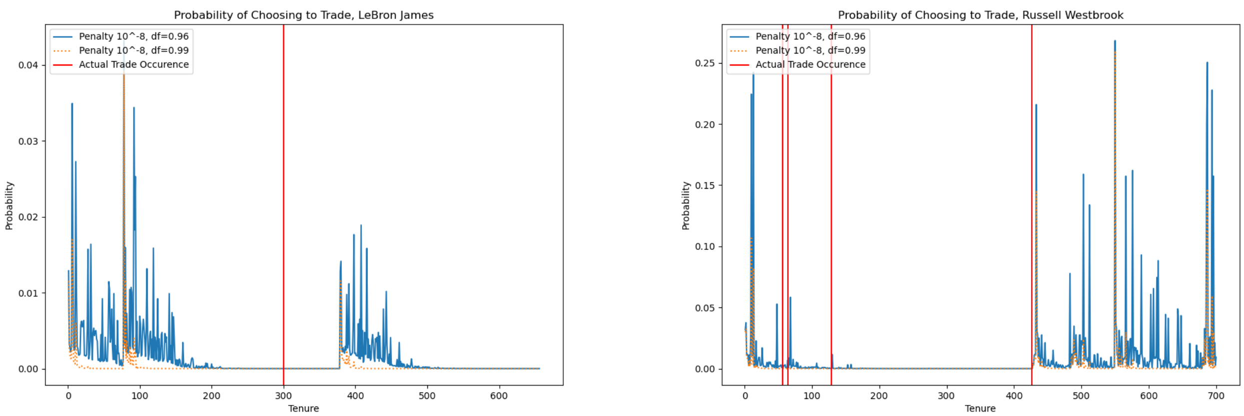


Figure 9: Popular Players: Choice Probability vs. Historical Trade, varying discount factors

7 Conclusion, Limitations, and Next Steps

At its core, our NBA TradeSense model is driven by POMDPs, due to their inherent ability to calculate metrics for decisions, with time in mind. This is highly applicable due to the fact that player performance is obviously dependant on their past performances. We utilized these models to collect insights into when a general manager may want to trade specific players, given how long they have been on their team and how well they have been playing recently. We can also simulate different general managers' attitudes, depending on if they are a forward-looking manager or a highly myopic manager, and also depending on how conscious they are of the costs of players. Finally, we compare when our models suggest trades vs. times when players were actually traded, which can give us insights into whether those general managers let go of those players at the "right" time.

A few clear limitations or potential improvements do come to mind. Our metric for player performance is quite generic (all stats balanced equally) as this was more of an exploratory exercise, however, a specific GM with a specific basketball philosophy may have some traits or attributes that they particularly lean to — perhaps a team oriented mindset centered around assists, or a "hustle" mindset centered around rebounds. Modifying *METRIC* to one's specific needs is a crucial optimization.

Another limitation is how we greatly simplify an actual trade. To simplify data analysis and collection, we assumed that all trades are one player for one player, and both players were performing at the same tier at the time of swap (i.e. assumes the price for each player is the same as well). This is clearly not always true - one common trade in the NBA is when one very strong player is traded for multiple weaker players. Our model is unable to account for these types of trades, which reduces its robustness when applying to real-world scenarios.

There are many possible future inferences that can be collected with the baseline model developed. One possibility is to analyze when a certain general manager in the league generally trades their players, and then attempt to find the cost-scaling factor and the discount factor that generate the best-fitting choice probabilities - in other words, generate choice-probability graphs that best align with that general manager's historic trade times. If fitted correctly, we would potentially see the general manager's historic trade times line up near perfectly with spikes in probability to trade their players.

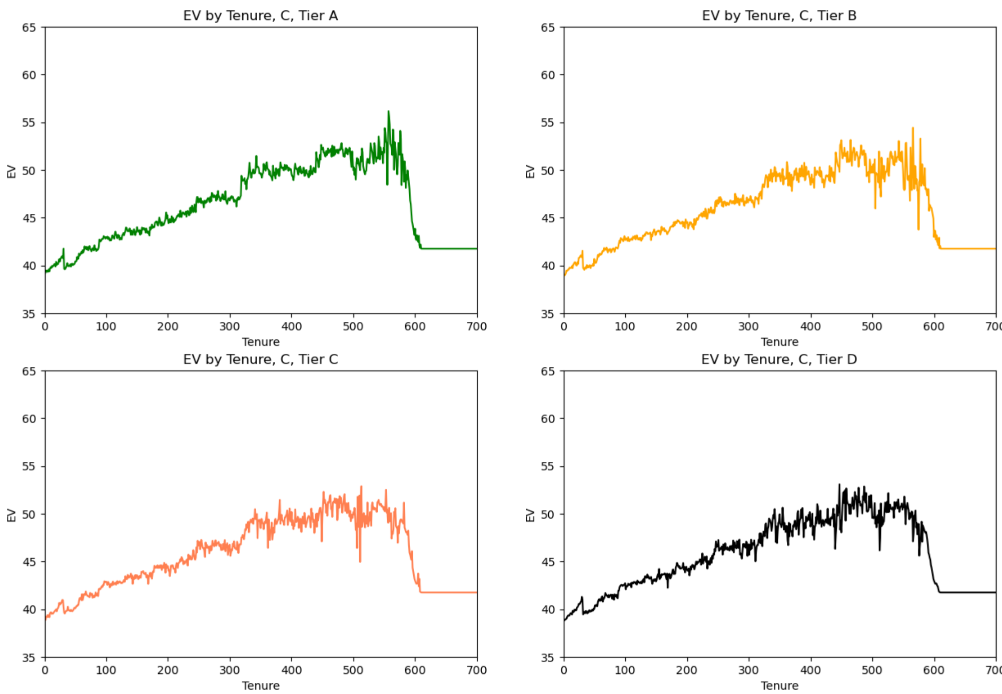
Another suggestion we have is to investigate thoroughly the performances of "all-star" players. These all-star players are usually a cut-above even your regular A-tier player in the league (i.e. they are outliers), and are crucial in forming championship level teams. Perhaps a tier above A-tiers (S-tier) should be created, to show players that perform at the 95+th percentile.

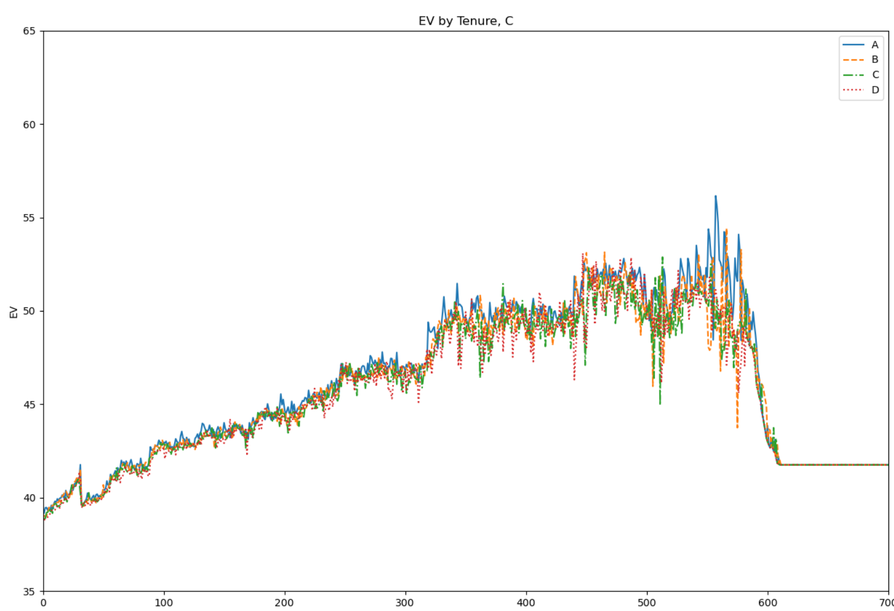
Ultimately, our TradeSense model is a baseline model that can be used by fans of NBA to better understand the decisions that general managers can make.

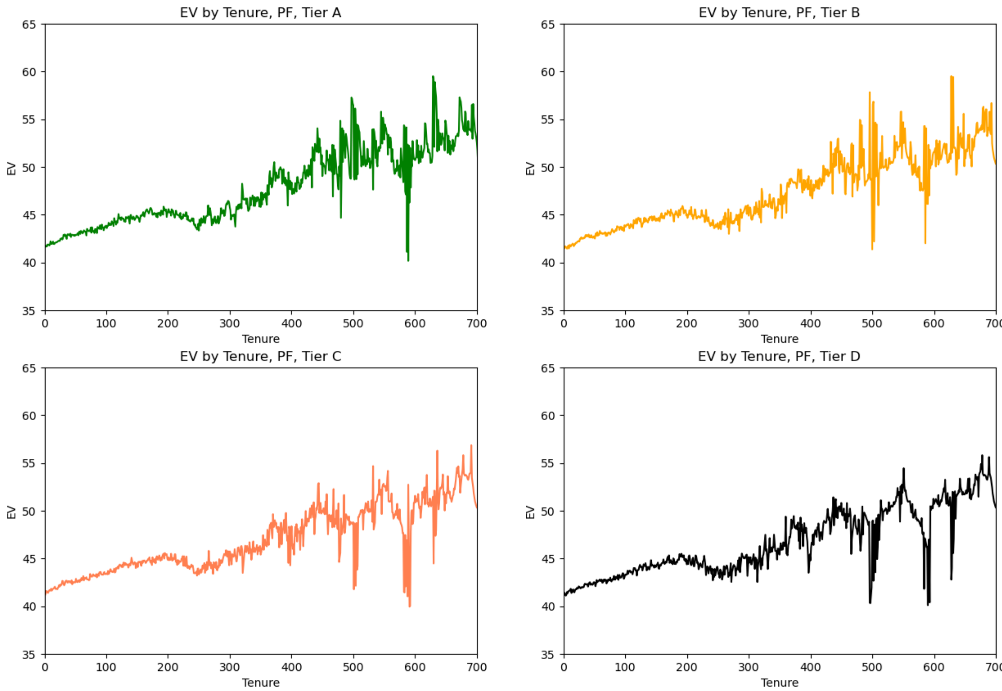
Appendix A

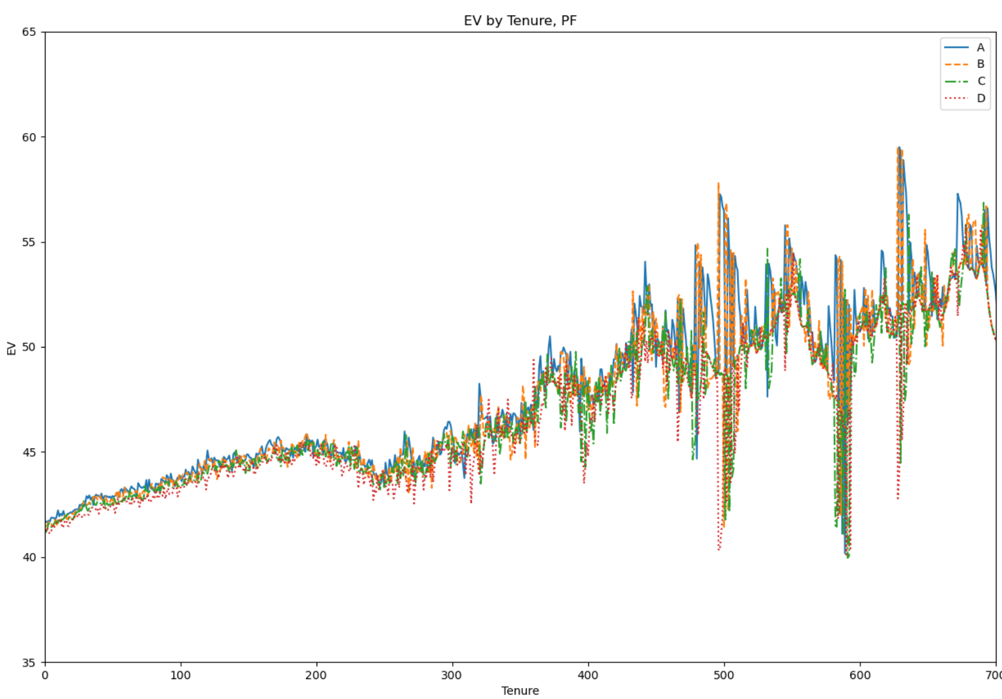
EV by Tenure and Tier for Various Positions.
All of the following graphs in Appendix A have:

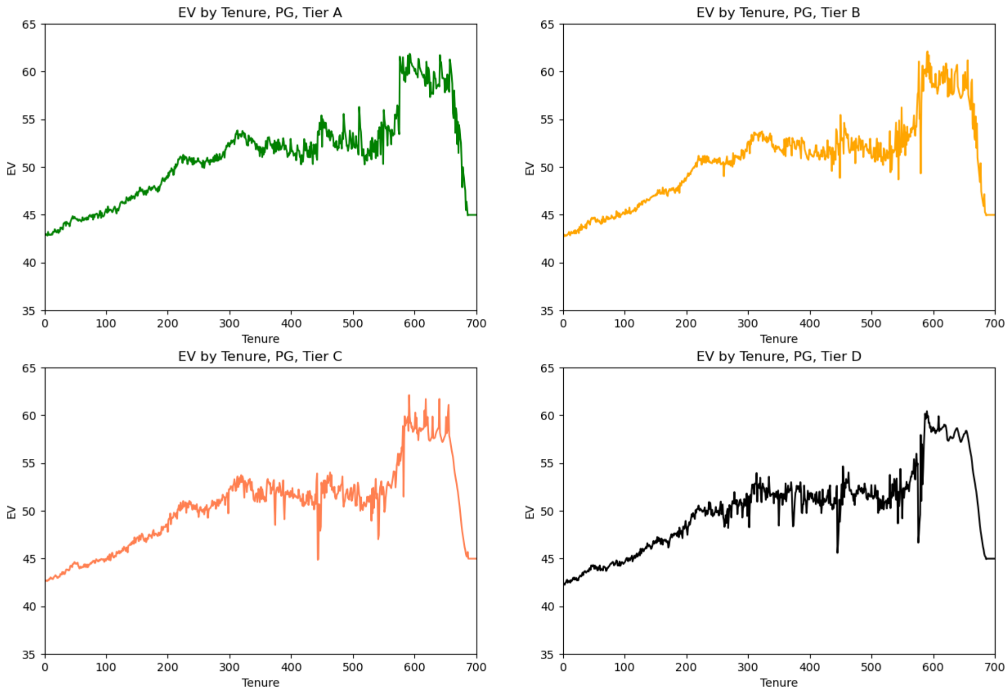
- Scaling factor to salary hyperparameter: 10^{-8}
- Discount factor (beta) set to 0.96.

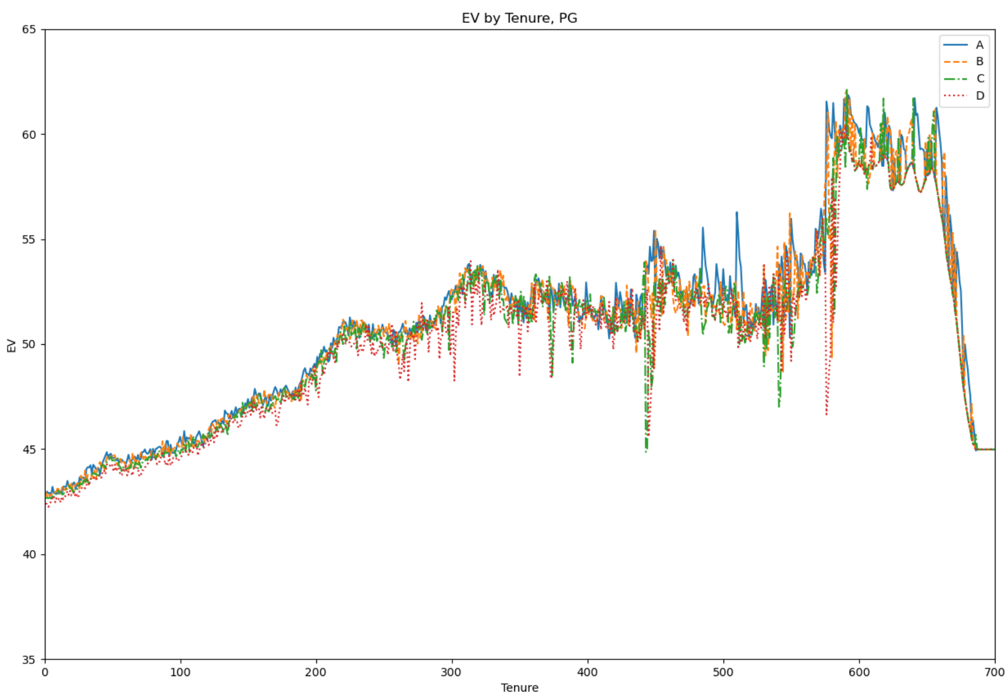


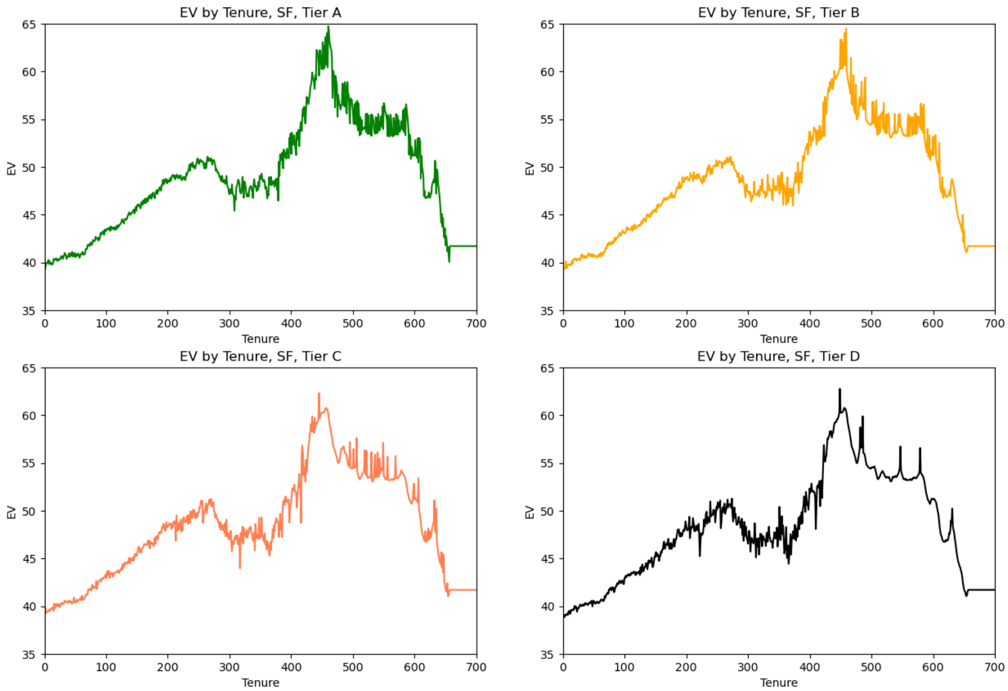


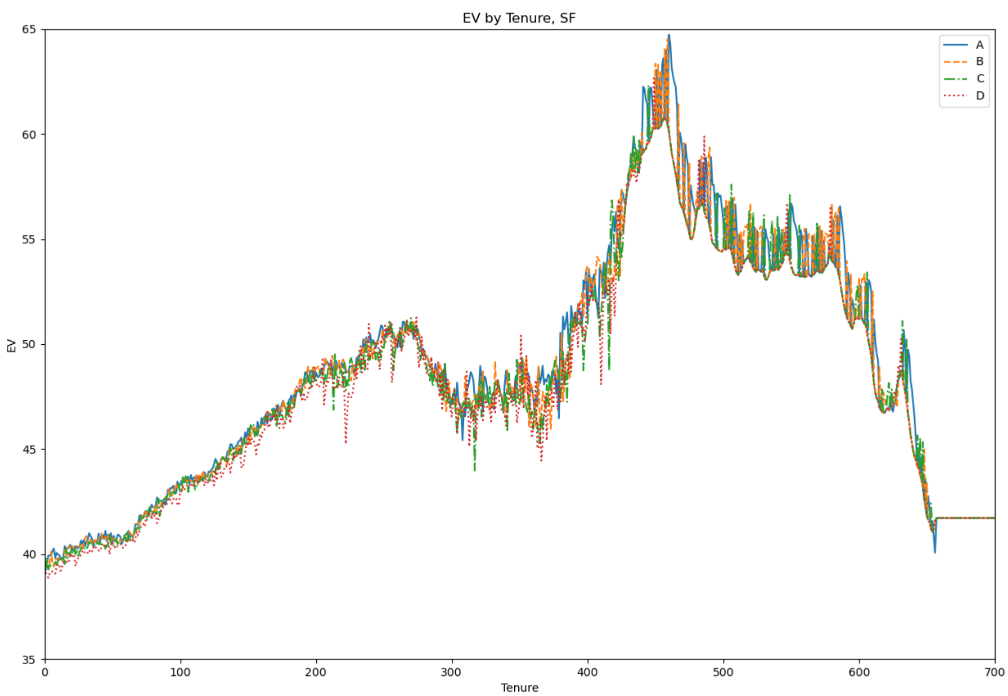

EV by Tenure, PF, Tier A
EV by Tenure, PF, Tier B
EV by Tenure, PF, Tier C
EV by Tenure, PF, Tier D
EV
Tenure

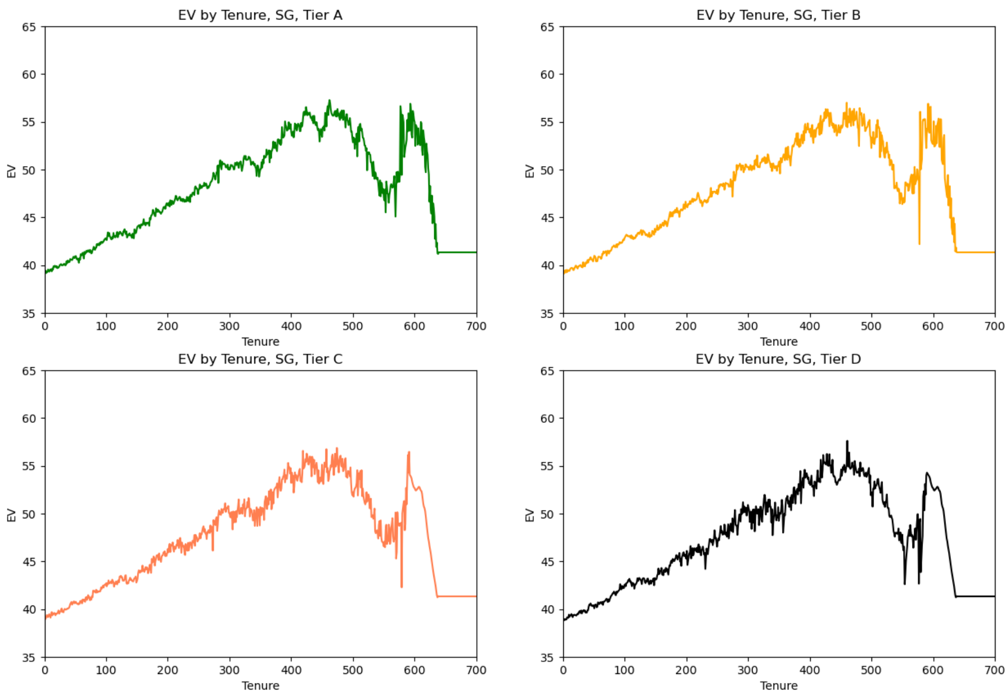

EV by Tenure, PF
A
B
C
D
EV
Tenure

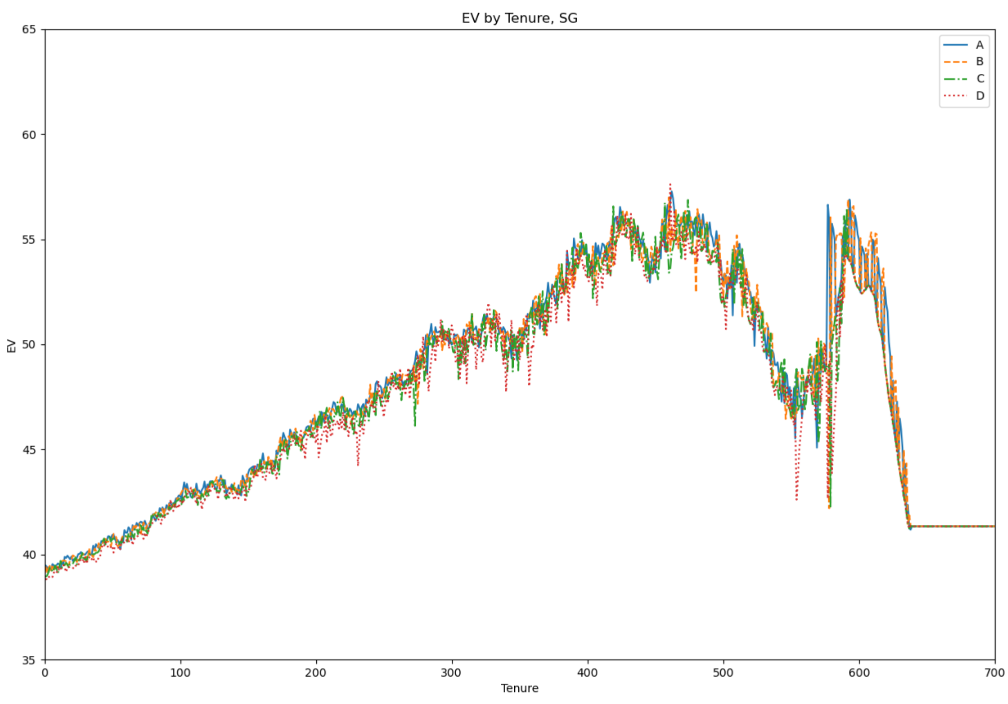

EV by Tenure, PG, Tier A
EV by Tenure, PG, Tier B
EV by Tenure, PG, Tier C
EV by Tenure, PG, Tier D
EV
Tenure


EV by Tenure, PG
A
B
C
D
EV
Tenure


EV by Tenure, SF, Tier A
EV by Tenure, SF, Tier B
EV by Tenure, SF, Tier C
EV by Tenure, SF, Tier D
EV
Tenure

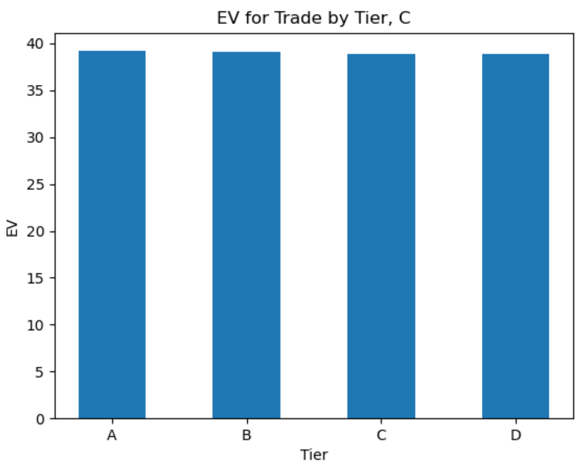

EV by Tenure, SF
EV
Tenure
A
B
C
D

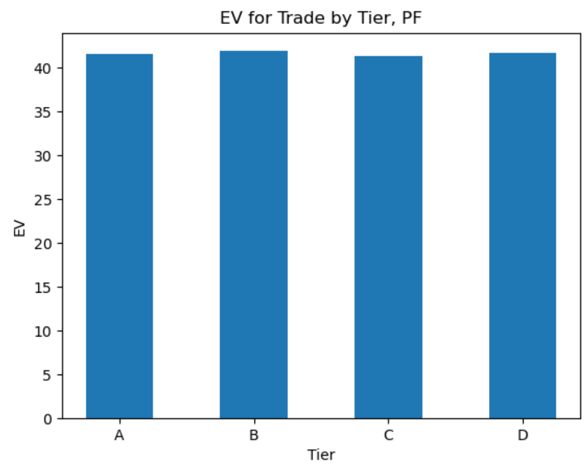

EV by Tenure, SG, Tier A
EV by Tenure, SG, Tier B
EV by Tenure, SG, Tier C
EV by Tenure, SG, Tier D
EV
Tenure

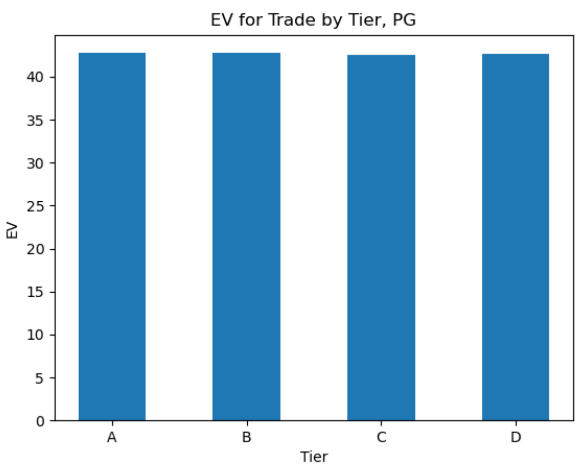

EV by Tenure, SG
A
B
C
D
EV
Tenure

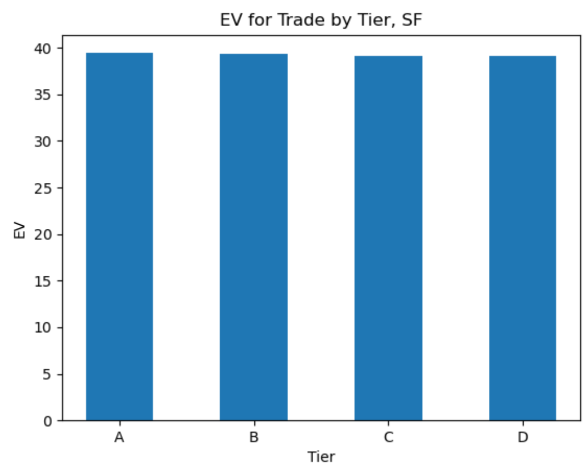
The following graphs show the EV when acting to trade players. All of the following graphs have:

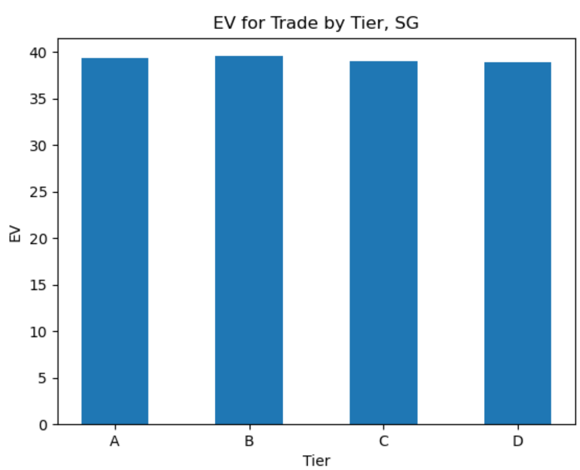
- Scaling factor to salary hyperparameter: 10^{-8}
- Discount factor (beta) set to 0.96.





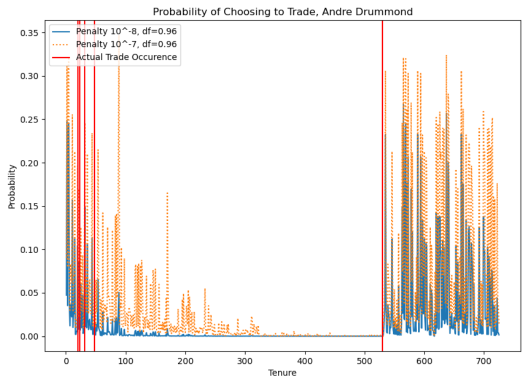


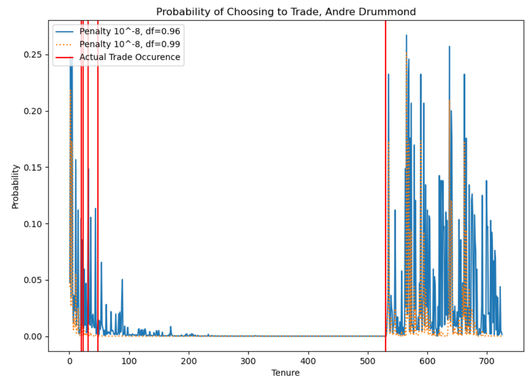


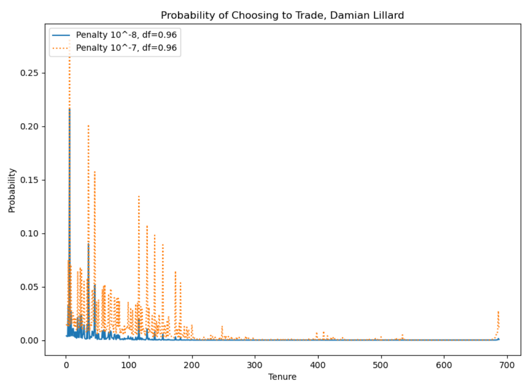


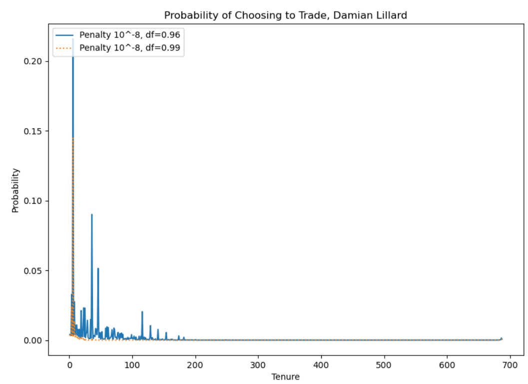
Appendix B

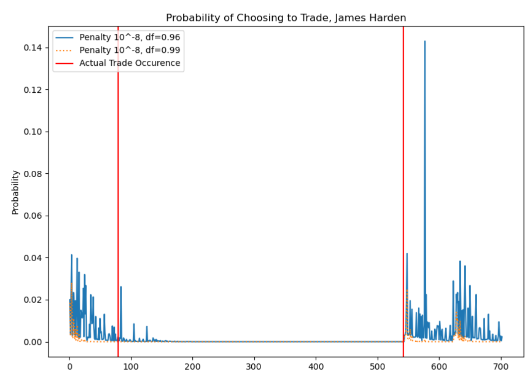
Probability of trading a specific player, given their average performance at specific tenures, modelled using collected EV values.



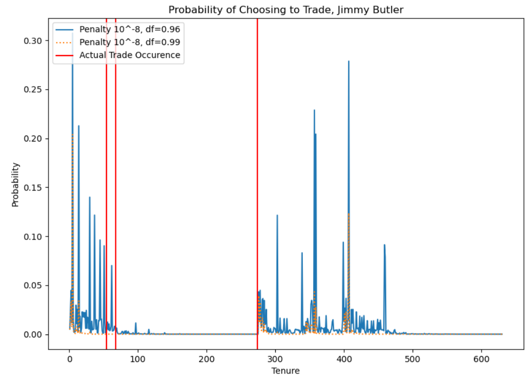


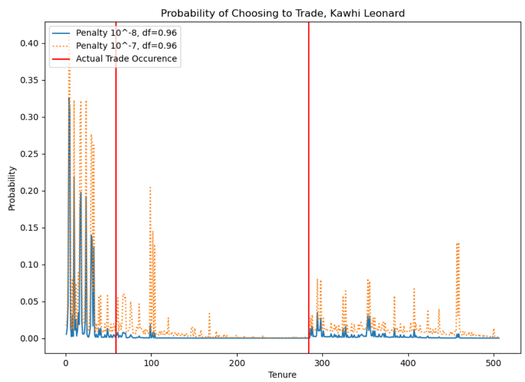


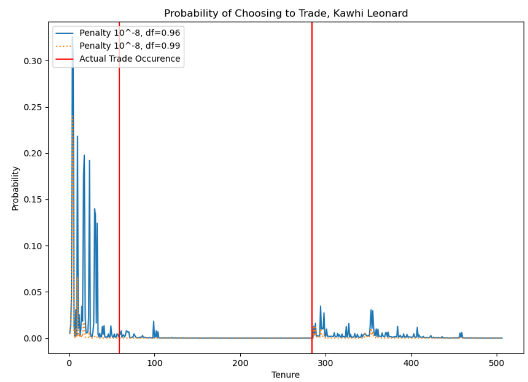


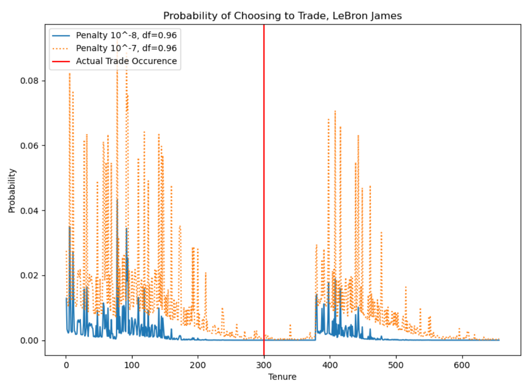


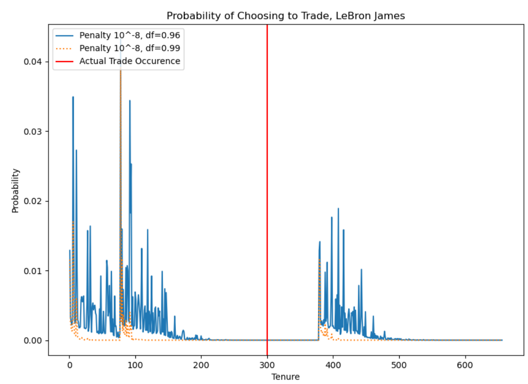

Probability of Choosing to Trade, Jimmy Butler
Penalty 10^-8, df=0.96
Penalty 10^-7, df=0.96
Actual Trade Occurence
Probability
Tenure

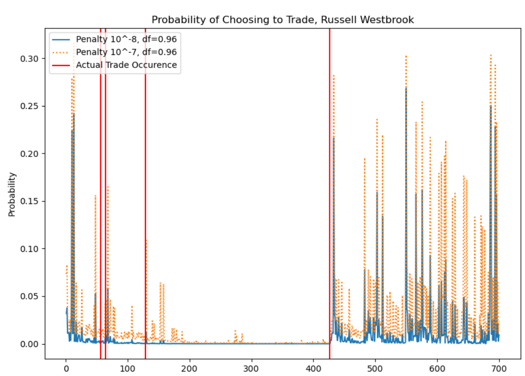

Probability of Choosing to Trade, Jimmy Butler
Penalty 10^-8, df=0.96
Penalty 10^-8, df=0.99
Actual Trade Occurence
Probability
Tenure

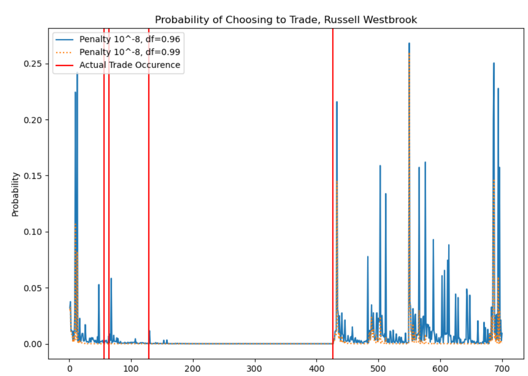

Probability of Choosing to Trade, Kawhi Leonard
Penalty 10^-8, df=0.96
Penalty 10^-7, df=0.96
Actual Trade Occurence
Probability
Tenure


Probability of Choosing to Trade, Kawhi Leonard
Penalty 10^-8, df=0.96
Penalty 10^-8, df=0.99
Actual Trade Occurence
Probability
Tenure


Probability of Choosing to Trade, LeBron James
Penalty 10^-8, df=0.96
Penalty 10^-7, df=0.96
Actual Trade Occurence
Probability
Tenure


Probability of Choosing to Trade, LeBron James
Penalty 10^-8, df=0.96
Penalty 10^-8, df=0.99
Actual Trade Occurence
Probability
Tenure


Probability of Choosing to Trade, Russell Westbrook
Penalty 10^-8, df=0.96
Penalty 10^-7, df=0.96
Actual Trade Occurence
Probability
Tenure


Probability of Choosing to Trade, Russell Westbrook
Penalty 10^-8, df=0.96
Penalty 10^-8, df=0.99
Actual Trade Occurence
Probability
Tenure