

Surface Denoising Using Implicit Mean Curvature Flow

The diffusion equation $f_t = \mu \Delta f$ describes how a function f changes over time by a scalar coefficient μ times its spatial laplacian. In mesh processing, this equation is used to smooth out geometrical details on a mesh by replacing the regular Laplace operator by the Laplace Beltrami operator and the function f by vertex positions. Since the Laplace Beltrami operator of vertex positions corresponds to the mean curvature

$$\Delta \mathbf{v} = -2H(\mathbf{v}) \mathbf{n}$$

geometrically the equation moves all vertices in the normal direction by a strength proportional to the mean curvature. The Laplace Beltrami operator can be computed using the cotangent formula while the change in vertex positions over time is approximated by

$$\mathbf{v}_t = (\mathbf{v}(t+h) - \mathbf{v}(t)) / h$$

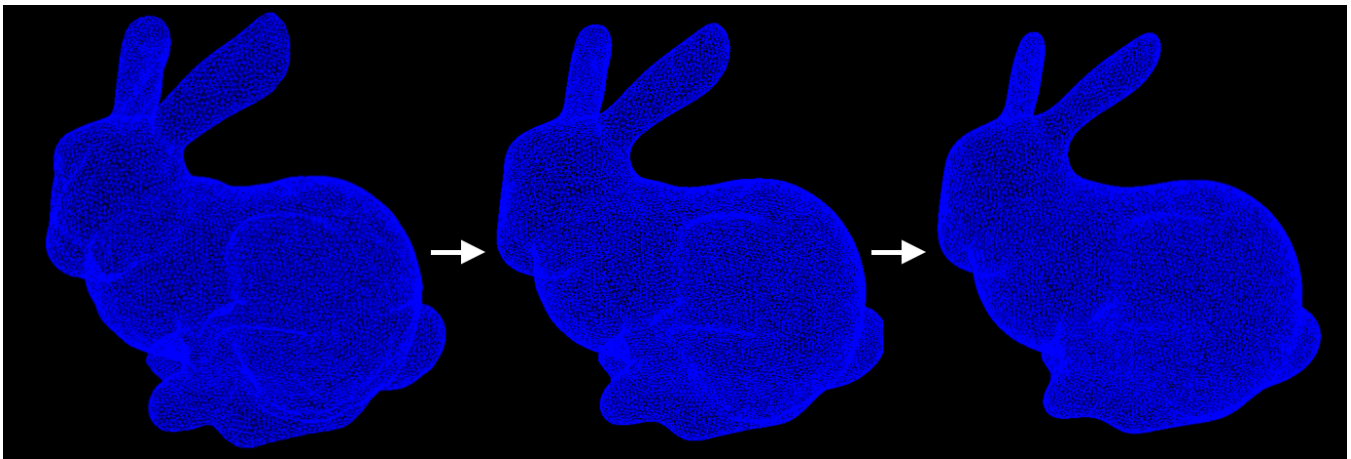
for some duration $h > 0$. Solving for $\mathbf{v}(t+h)$ yields the explicit Euler integration scheme:

$$\begin{aligned} \mathbf{v}(t+h) &= \mathbf{v}(t) + h\Delta \mathbf{v}(t) \\ \mu &= 1 \end{aligned}$$

which is not numerically robust for large time steps. Implicit time integration should be used instead

$$\mathbf{v}(t+h) = \mathbf{v}(t) + h\Delta \mathbf{v}(t+h) \quad \Leftrightarrow \quad (\text{Id} - h\Delta)\mathbf{v}(t+h) = \mathbf{v}(t)$$

The resulting linear system of equations is not too expensive to compute as the matrix $(\text{Id} - h\Delta)$ is highly sparse.



The implicit mean curvature flow scheme has a few potential shortcomings: It blurs geometrical features such as sharp edges and is not volume preserving, both of which become immediately apparent after a few small time steps.

Implementation: <https://github.com/rohan-sawhney/denoising>