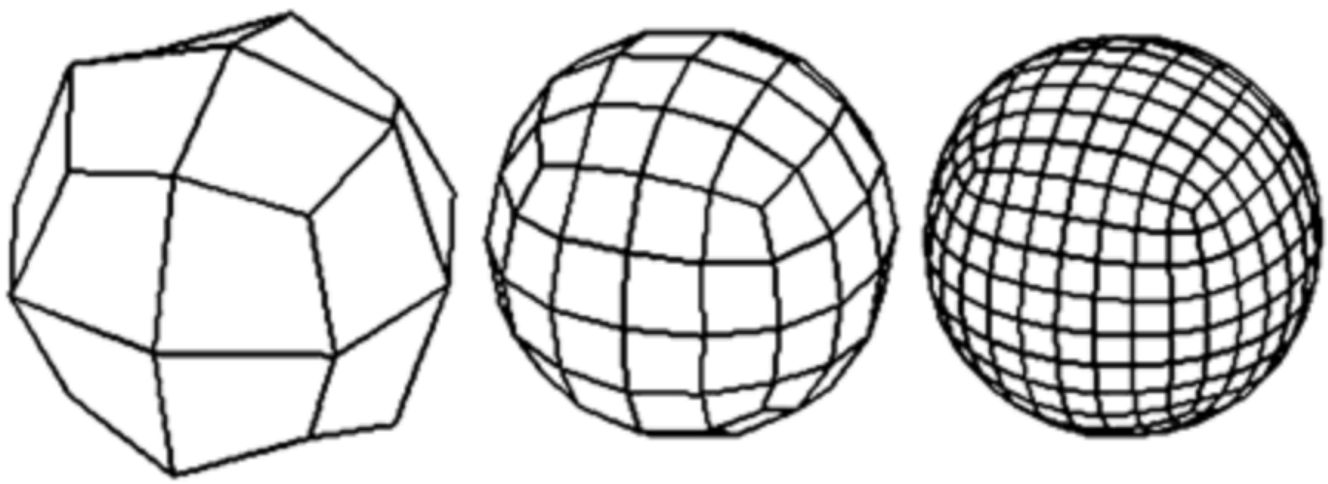
Approximating Real Objects with Polygonal Meshes

Digital surface representations of real objects can only be approximations in general. Very often, the representation is composed of small segments such as quads or triangles that approximate the given shape locally while its global error tolerance is controlled by the size and number of segments.

However, given that a polynomial function can approximate a smooth surface up to any desired precision, why are meshes generally represented by polynomials of degree one? In theory, the accuracy of an approximation with piecewise polynomials can be increased either by raising the degree of the polynomial or reducing the size of individual segments and using more segments for the approximation. It turns out a couple of practical and theoretical reasons exist to prefer the later approach. Processing a large number of simple objects on computer's today is generally more efficient than processing few complex objects. Additionally, for discretely sampled inputs such as point clouds, reasonable assumptions cannot be made about the boundedness of higher order derivations (needed for computing curvatures, normals etc). Smoothness conditions between segments are also difficult to satisfy for piecewise polynomials with with higher degree.



If a piecewise linear function is used to approximate a smooth surface, then from Taylor's theorem the approximation error is of the order $O(e^2)$, e being the maximum edge length (as opposed to $O(e^{p+1})$, p being the degree of a polynomial with degree greater than one). For a triangle mesh, the error is reduced to a fourth if the edge length is halved, that is, a triangle is split into four sub triangles, increasing the number of faces to four times the original. The approximation error is thus inversely proportional to the number of faces. However, instead of increasing the face count in all parts of the mesh to reduce the error, the face count can be adapted to the surface curvature such that flat areas have fewer faces (or are less sparsely sampled) while curved regions have more, thereby resulting in a mesh of moderate complexity.