**Geodesics**

In Euclidean space, the shortest distance between two points is a straight line. A geodesic is a generalization of straight lines extended to curved spaces such as surface meshes. Crane et al. introduced the heat method in [1] to compute geodesic distances in an efficient and robust manner on any geometric discretization such as regular grids, polygonal meshes and point clouds.

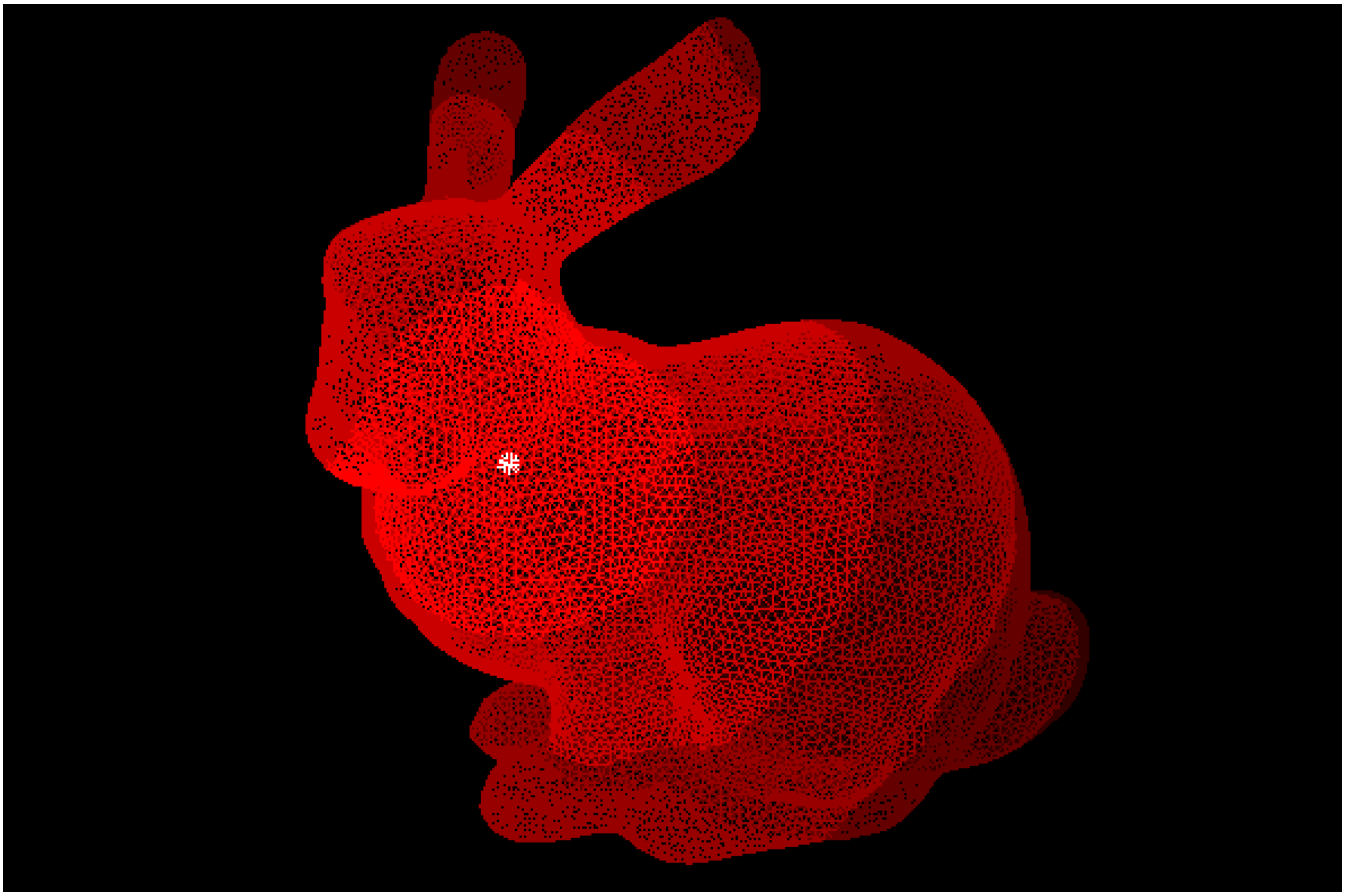

Figure 1. Geodesic distance from a single point on a surface

The method works in three steps:

1. Integrate the heat flow $\mathbf{v_t} = \mathbf{\Delta v}$ for a fixed time t. The heat equation can be discretized as:

$$\mathbf{(Id - t\Delta)v(t + h) = v(t)}$$

where Id is the identity matrix and $\Delta$ is the Laplace Beltrami operator. v(t) is initialized to zero except for the source vertex which is set to 1. **t** is chosen to be the square of the average edge length.

2. Compute the vector field $\mathbf{X = -\nabla v / |\nabla v|}$ where $\nabla$v is discretized as:

$$\mathbf{\nabla v = (\textstyle\sum_i v_i\ (n \times e_i)) / 2A_f}$$

Here $A_f$ is the area of the face, n is its unit normal, $e_i$ is the ith edge vector and $v_i$ is the value of v at the opposite vertex (Figure 2).

3. Solve the Poisson equation $\mathbf{\Delta\phi = \nabla \cdot X}$ where $\nabla \cdot$ X is computed as:

$$\mathbf{\nabla \cdot X = (\textstyle\sum_j \cot\theta_1\ (e_1 \cdot X_j) + \cot\theta_2\ (e_2 \cdot X_j)) / 2}$$

See Figure 3.

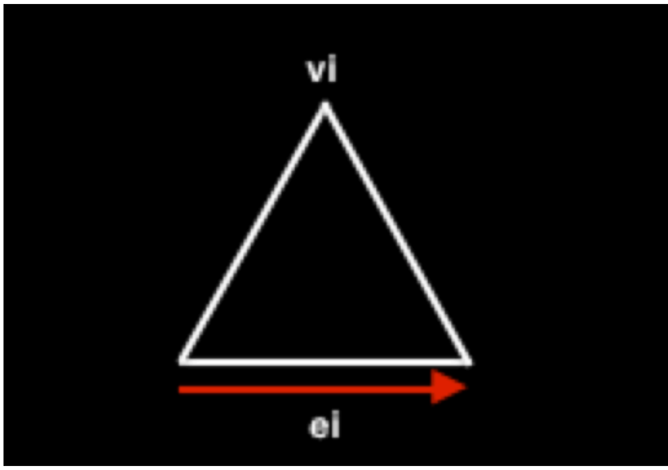


Figure 2

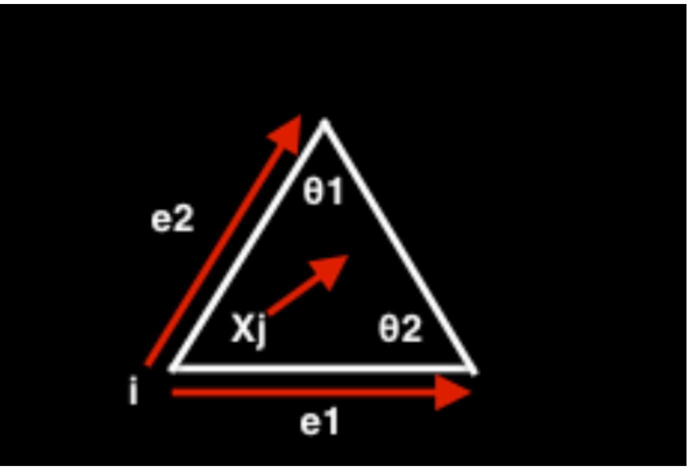


Figure 3

Implementation: https://github.com/rohan-sawhney/geodesics

[1] Crane et al. Geodesics in Heat: A New Approach to Computing Distance Based on Heat Flow