Surface Denoising Using Implicit Mean Curvature Flow

The diffusion equation $f_t = \mu\Delta f$ describes how a function f changes over time by a scalar coefficient μ times its spatial laplacian. In mesh processing, this equation is used to smooth out geometrical details on a mesh by replacing the regular Laplace operator by the Laplace Beltrami operator and the function f by vertex positions. Since the Laplace Beltrami operator of vertex positions corresponds to the mean curvature

$$\Delta \mathbf{v} = -2H(\mathbf{v})\,\mathbf{n}$$

geometrically the equation moves all vertices in the normal direction by a strength proportional to the mean curvature. The Laplace Beltrami operator can be computed using the cotangent formula while the change in vertex positions over time is approximated by

$$\mathbf{v}_t = (\mathbf{v}(t+h) - \mathbf{v}(t)) / h$$

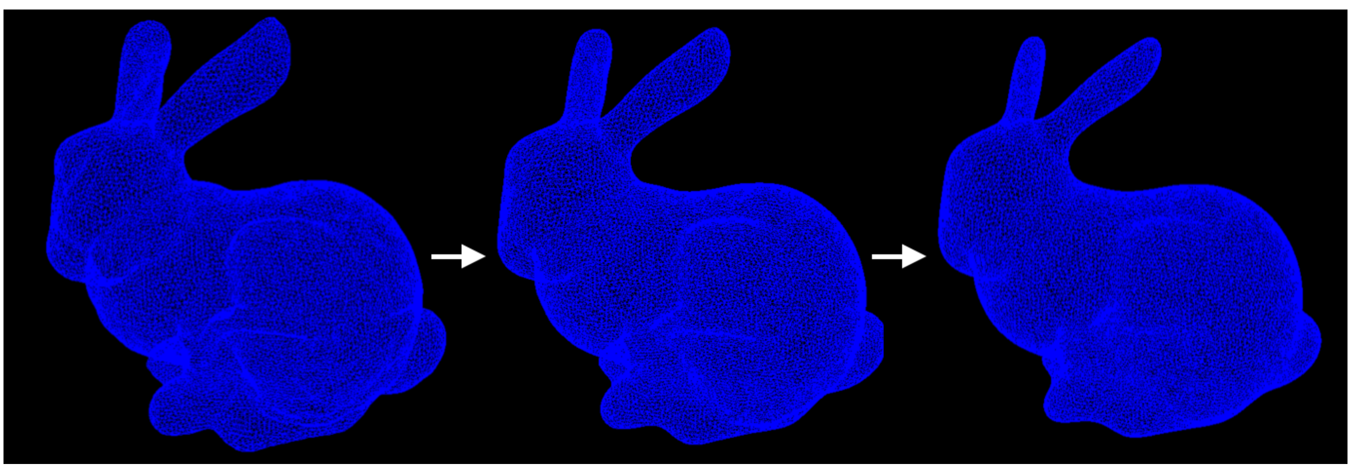
for some duration $h > 0$. Solving for $v(t+h)$ yields the explicit Euler integration scheme:

$$\mathbf{v}(t+h) = \mathbf{v}(t) + h\Delta\mathbf{v}(t)$$
$$\mu = 1$$

which is not numerically robust for large time steps. Implicit time integration should be used instead

$$\mathbf{v}(t+h) = \mathbf{v}(t) + h\Delta\mathbf{v}(t+h) \quad \Leftrightarrow \quad (\mathrm{Id} - h\Delta)\mathbf{v}(t+h) = \mathbf{v}(t)$$

The resulting linear system of equations is not too expensive to compute as the matrix $(\mathrm{Id} - h\Delta)$ is highly sparse.



The implicit mean curvature flow scheme has a few shortcomings: It blurs geometrical features such as sharp edges, leaves geometric discontinuities and is not volume preserving, all of which become apparent after a few time steps.

Implementation: https://github.com/rohan-sawhney/denoising