Deformation

Given a set of user defined anchor and handle vertices on a 3D model, surface deformation algorithms are designed to determine displacement vectors for the remaining unconstrained vertices such that the model is stretched or sheared as little as possible. This is generally achieved by minimizing the nonlinear shell energy:

$$\mathbf{E(S, S') = \int_\Omega k_s \,\| I' - II' \|^2 + k_b \,\| II' - II \|^2 \, du\, dv}$$

which measures the difference between the original shape S and its deformed version S'. Here, I and II are the fundamental forms of S and I' and II' are the fundamental forms of S'. k_s and k_b are stiffness parameters. In [1], Sorkine et al. observe that this energy is minimized when the local transformations that occur between S and S' are similar and as rigid as possible. They reformulate the shell energy as:

$$\mathbf{E(v) = \sum_{N(v)} w_{ij} \,\| (v_i' - v_j') - R_i (v_i - v_j) \|^2 = argmax_{R_i} Tr(R_i S_i)}$$

where w_{ij} are the Laplacian cotangent weights, $S_i = \sum_{N(v)} w_{ij} \, (v_i - v_j) \, (v_i' - v_j')^T$ and R_i are rotation matrices that best approximate rigid transformations. The singular value decomposition of $S_i = U_i \sum_i V_i^T$ yields a value for R_i $(= V_i U_i^T)$ that maximizes $Tr(R_i S_i)$ and thereby minimizes E(v).

E(v) can also be minimized by taking its partial derivative with respect to the vertex positions v_i' of S', which results in a sparse linear system of equations:

$$\mathbf{\sum_{N(v)} w_{ij} \, (v_i' - v_j') = \sum_{N(v)} w_{ij} \, (R_i + R_j) \, (v_i - v_j) \,/\, 2 \;\; \text{<->} \;\; Lv' = b}$$

The left hand side of the equation above is the discrete Laplace Beltrami operator applied to the vertex positions v_i'. Solving the above system for v_i' then results in the new shape S'. However, as the rotation matrices R_i are not known before hand, Sorkine et al. propose an iterative minimization scheme that is guaranteed to converge: given an initial guess for v_i', recompute the local rotations and use them to define a new right hand side for the linear system for a given number of iterations. The initial guess for v_i' is chosen to be the vertex positions from the previous frame.

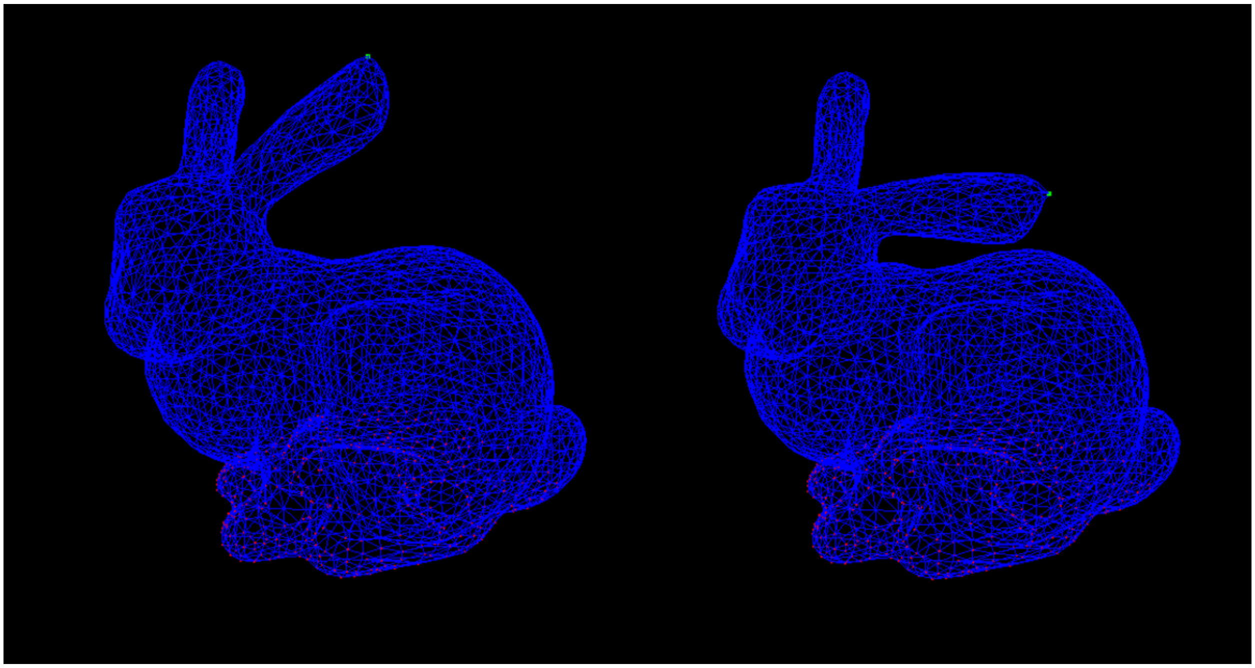

Figure 1: *Left*: Original Mesh. *Right:* Deformed Mesh

Implementation: https://github.com/rohan-sawhney/deformation

[1] Sorkine et al. As Rigid As Possible Surface Modeling