Efficient Allocation of Limited Resources.project

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Abstract

The aim is to look at models of allocation in order to distribute limited and scarce resources fairly and efficiently, considering all ethical values. This paper looks at the conclusions drawn by the model provided in the paper ”Efficient and Fair Healthcare Rationing” and compares its model to what Pathak et al. provided.

1 Pathak's Paper

1.1 Two Sided Matching Problem

The problem of healthcare rationing has recently been formally studied by market designers. Pathak, Sonmez, Unver, and Yenmez (2020, 2023) were among the first to frame the problem as a two-sided matching problem in which patients are on one side, and the resource units are on the other side.
Pathak et al. (2020, 2023) suggested dividing the units into different reserve categories, each with a subset of agents eligible for that category and its own priority ranking over the patients.

1.2 What do these priorities look like?

Agents are interested in a single unit of resource, and the resources are reserved for different categories. Category-specific Priorities represent the ethical principles and guidelines that a policymaker may wish to implement.For example, a category for senior people may prioritize the eldest citizens, or a category for healthcare workers may rank ICU personnel ahead of medical workers not directly exposed to the disease. category priorities are consistent with a baseline priority ordering. Agents are artificially endowed with Strict preferences over the categories.

1.3 What did they do in their paper?

They ran Deferred Acceptance Algorithm over the agents' preferences, which produces desirable outcomes for the Rationing problem. But, this approach may sometimes lead to matchings that are not pareto optimal.
Pathak et al. (2020, 2023) also consider a setting where given numbers of unreserved units must be allocated before or after allocating the units reserved for categories. They propose to use the Smart Reserves approach of Sonmez and Yenmez (2020).
Smart Reserves Algorithm computes a maximum size matching satisfying the basic axioms (eligibility compliance, respect of priorities, and non-wastefulness). Smart Reserve rules includes two extreme approaches with widespread appeal.
Minimum Guarantees Rule
Over-and-Above Rule

1.4 Problems in their Paper

His paper has not addressed the problem considering heterogenous priorities. Allowing heterogeneous priorities for categories is very much in the spirit of incorporating different ethical values. For example, it seems desirable that the priority ordering in the category of older people can be different from the priority ordering in the category of front-line workers. The former may rank individuals by age, and the latter may favor energetic medical professionals.

2 Contributions of Paper by Aziz and Brandl

- Respect improvements (do not penalize patients for rising in the priority ranking of a category)
- Allocation rules are strongly polynomial-time computable.
- Reverse Rejecting rules always allocate the largest feasible number of units and thus give a Pareto optimal allocation.
- Smart Reverse Rejecting rule thus obtained from Reverse Rejecting Rules , satisfies a new axiom we call order preservation, which is parameterized by how many unreserved units are processed first and last. Moreover, it generalizes two wellknown reserves rules—over-and-above and minimum-guarantees.
- In this paper, agents have dichotomous preferences(categories they are eligible for/ they are not eligible for)which allows us to obtain a polynomial-time algorithm for the problem.
- In this paper they have considered heterogenous priorities, that is the categories' priorities are not consistent with a baseline ordering.
- In this paper, they have allowed for weak priorities rather than strict ones.

3 Model discussed by Aziz and Brandl

- There are q identical and indivisible units of resources.
- A set N of agents with $|N|$ = n
- A finite set C of categories.
- Each category c has a quota $q_c \in N$ with $\sum_{c\in C} q_c$
- A priority ranking $\succsim_c$, a weak order on $N\cup \{\varnothing\}$
- An agent i is eligible for category c if $i \succ_c \varnothing$. We denote by N_c the set of agents who are eligible for c.
- $I = (N, C, (\succsim_c)_{c\in C}, (q_c)_{c\in C})$ is an instance of the rationing problem.

3.1 Matching

A matching $\mu : N \cup C \rightarrow \{\emptyset\}$ is a function that maps each agent to a category or $\emptyset$ and satisfies the capacity constraints: for each $c \in C$, $|\mu^{-1}(c)| \leq q_c$. For an agent $i \in N$, $\mu(i) = \emptyset$ means that it is unmatched (that is, does not receive any unit), and $\mu(i) = c$ means that it receives a unit reserved for category c.
Matching that follows the following axioms are desirable :-
1.Compliance with eligibility requirements A matching μ *complies with the eligibility requirements* (or is feasible) if for any $i \in N$ and $c \in C$, $\mu(i) = c$ implies $i \in e_c$.
2.Respect of priorities A matching μ *respects priorities* if for any $i, j \in N$ and $c \in C$, $\mu(i) = c$ and $\mu(j) = \emptyset$ implies $j \succ_c i$. If there exist $i, j \in N$ and $c \in C$ with $\mu(i) = c$, $\mu(j) = \emptyset$, and $j \succ_c i$, we say that j has justified envy towards i for category c.

Respect for priorities is equivalent to $envy - freeness$.
3.Non-wastefulness A matching μ is *non-wasteful* if for any $i \in N$ and $c \in C$, $i \in e_c$ and $\mu(i) = \emptyset$ implies $|\mu^{-1}(c)| = q_c$.
4.Maximum size matching A matching μ is a *maximum size matching* if it has maximum size among all matchings complying with the eligibility requirements.

3.2 Bipartite Graph

Instance I is associated with a graph B_I, called a *reservation graph.* $B_I = (N \cup C, E)$ is a bipartite graph with an edge from i to c if i is eligible for c. That is,

$$E = \{(i, c) : i \succ_c \emptyset\}.$$

we denote by $ms(B_I)$ the number of edges in a maximum size matching of B_I subject to given quotas (q_c).

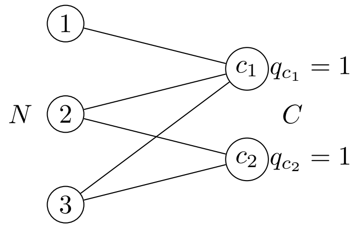

Figure 1: Bipartite Graph with following priority rankings

$$2 \succ_{c_1} 3 \succ_{c_1} \emptyset \succ_{c_1} 1 \quad 2 \succ_{c_2} \emptyset \succ_{c_2} 1 \succ_{c_2} 3$$

3.3 Allocation Rule

An allocation rule maps every instance I to a matching for I. An allocation rule f respects improvements if $f(I)(i) \neq \varnothing$ implies $f(I')(i) \neq \varnothing$ whenever i's priority increases from I to I'. Respecting improvements can be interpreted as a notion of strategyproofness

3.4 Algorithm to get desirable matching : Reverse Rejecting Rule

Whenever we apply Reverse Rejecting Rules, we consider some $\succ_\pi$ linear ordering over agents , which we call baseline ordering. Outcome of these rules depends heavily on $\succ_\pi$. Following are the steps to apply this algorithm to obtain a matching complying the above definitions.

- Let there be a set called set of Rejected Agents. and let it be empty at the start.
- consider the agents in ascending order of the $\succ_\pi$ and while considering each agent i, add it to this set, if and only if

$$ms\left(B_I^{-\mathcal{R} \cup \{i\}}\right) = ms\left(B_I\right)$$

- After the last agent has been considered , let R_I be the final set, then choose a maximum size matching out of reduced reservation graph $B_I^{-\mathcal{R}_I}$.If R is the set of rejected agents, $B_I^{-R} = ((N \setminus R) \cup C, E)$,

$$E = \{\{j, c\} : j \succ_c \emptyset \text{ and there is no } i \in R \text{ such that } i \succ_c j\}.$$

So idea is: Rev π Rules undertake an Instance, considers a linear ordering and returns a matching.
Following are the properties of these rules, these properties are satisfied by the matching that these rules output for any given instance.

- complies with eligibility requirements
- respects priorities

- returns a matching of maximum size among feasible matchings
- respects improvements
- can be computed in strongly polynomial time
- A matching complies with the eligibility requirements, respects priorities, and has maximum size among feasible matchings if and only if it is a possible outcome of REV π for some order $\succ_\pi$ By this, for every instance, there exists some matching that follows those axioms.
- A matching complies with the eligibility requirements, respects priorities, and is non-wasteful if and only if there is a profile of strict preferences for the agents over the categories so that the matching is the outcome of the Deferred Acceptance algorithm applied to the resulting two-sided matching instance. Note that not every matching with these properties has maximum size.

3.5 Order Preservation

Consider a matching μ of agents to categories in $C_p \cup \{c_u^1, c_u^2\}$. We say that μ is *order preserving* (with respect to c_u^1 and c_u^2) for the baseline ordering $\succ_\pi$ if for any two agents $i, j \in N$,

(i) $\mu(i) \in C_p \cup \{c_u^1\}$, $\mu(j) = c_u^1$, and j is eligible for category $\mu(i)$ implies $j \succ_\pi i$, and

(ii) $\mu(j) \in C_p \cup \{c_u^1\}$, $\mu(i) = c_u^2$, and i is eligible for category $\mu(j)$ implies $j \succsim_{\mu(j)} i$.

Maximum Beneficiary : A matching μ is a maximum beneficiary assignment if it maximizes the number of agents matched to a preferential category.

3.6 Smart Reverse Rejecting Rule

- Reverse Rejecting Rules are not equipped to handle unreserved categories.
- Pathak also proposed something called Smart Reserves Rules , but those doesn't handle hetergenous priorities. So, we discuss an extension of RR rules, called smart reverse rejecting rules as follows:
- step 1: Let the set of agents to be assigned unreserved units from c_u^1 be empty and call it, N1
- step 2 : We consider agents in the descending order of the baseline linear ordering.
- step 3 : When agent i is considered, add it to N1, if N1 contains fewer than qcu1 agents and agents in $N \backslash (N1 \cup i)$ can form a maximum beneficiary assignment.
- step 4: Once the set is complete, give each agent in N1 set, an unreserved unit from c_u^1.
- step 5 : Use Rev π rule to allocate reserved units from preferential categories C_p to remaining agents.
- step 6 : At last, give unreserved units from cu2 to remaining agents in descending order of baseline ordering.

Properties of Smart Reverse Rules:

1. complies with eligibility requirements
2. yields a maximum beneficiary assignment
3. respects priorities
4. respects improvements
5. satisfies order preservation
6. is polynomial time computable