CE 529a

FINITE ELEMENT ANALYSIS

Course Project

Analysis of shell intersection of pipes under loading

Under the guidance of

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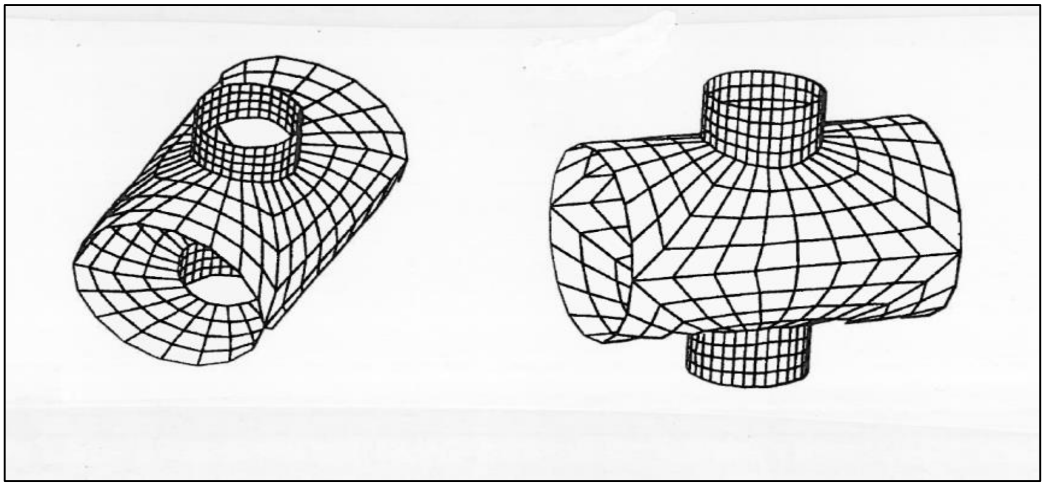
TABLE OF CONTENTS

1. INTRODUCTION-PROJECT OVERVIEW

- This project consists of modelling and analysis of a 3D shell- pipe element using ABAQUS software and comparing the results obtained (displacements) with the results obtained from MATLAB program created using the theory of finite element analysis.
- The project involves the use of 8 noded 3D shell element for analysis and modelling. The element is tested for various test cases as like bending, twisting to check its functionality and is then used to model the pipe element.
- Figure below shows the typical project model



PROJECT MODEL

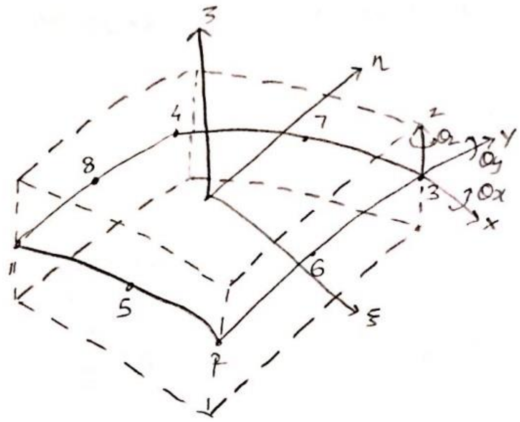
- Only one eight of the model is modeled because of the symmetry of the model about both of its axis.
- Hence this project is basically the computer implementation of finite element methods. The implementation is defined so at mimic the real world working environment in industry.

2.THEORY

Geometry of The element

For 3D shell element of This type, The displacements and rotations are calculated wrt midsurface plane.

For natuaral coordinates system, ξ & η are along midsurface plane & ζ is along Thickness.



Unit vectors along ξ & η directions are -

$$\hat{q}_1(\xi,\eta) = \hat{q}_{1j}(\xi,\eta)\,\hat{i}_j$$
$$\hat{q}_2(\xi,\eta) = \underbrace{\hat{q}_{2j}(\xi,\eta)\,\hat{i}_j}_{\text{summed index.}}$$

global position vector is -

$$R_0(\xi,\eta) = \sum_{j=1}^{3} x_j(\xi,\eta)\,\hat{i}_j = \tilde{x}(\xi,\eta)\,\hat{i}_1 + x_2(\xi,\eta)\,\hat{i}_2 + x_3(\xi,\eta)\,\hat{i}_3$$

Unit normal is -

$$\hat{q}_{3k} = \hat{q}_{3kj}\,\hat{i}_j$$

Unit vector tangent to ξ & η are –

$$q_1(\xi,\eta) = \frac{\partial R_0(\xi,\eta)}{\partial \xi} \qquad q_2(\xi,\eta) = \frac{\partial R_0(\xi,\eta)}{\partial \eta}$$

$$\therefore \hat{q_1} = \frac{1}{|q_1|} \cdot q_1 \qquad \hat{q_2} = \frac{1}{|q_2|} \cdot q_2$$

Unit normal – $\hat{q_3} = \frac{1}{|q_3|} \cdot q_3(\xi,\eta)$

For these, orthogonal set of vectors is obtained as –

$$\hat{V_1}k = \hat{q_1}k$$
$$\hat{V_2}k = \hat{q_4}k \qquad \text{where } \hat{q_4}k = \hat{q_3}k \times \hat{q_1}k$$
$$\hat{V_3}k = \hat{q_3}k$$

Displacement Approximation

$$U_j(\xi,\eta,\zeta) = \sum_{k=1}^{N} \psi_k(\xi,\eta) U_j^k + \sum_{k=1}^{N} \zeta \frac{t_k}{2} \psi_k(\xi,\eta)\left[-V_{2kj}\,\theta_1^k + V_{1kj}\,\theta_2^k\right]$$

JACOBIAN MATRIX.

$$[J]_{3\times3} = \begin{bmatrix} \sum_{k=1}^{8}\left[\frac{\partial \psi_k}{\partial \xi} x_{k1} + \frac{\zeta}{2}\frac{\partial \psi_k}{\partial \xi} t_k \hat{V_3}k_1\right] & \sum_{k=1}^{8}\left[\frac{\partial \psi_k}{\partial \xi} x_{k2} + \frac{\zeta}{2}\frac{\partial \psi_k}{\partial \xi} t_k \hat{V_3}k_2\right] & \sum_{k=1}^{8}\left[\frac{\partial \psi_k}{\partial \xi} x_{k3} + \frac{\zeta}{2}\frac{\partial \psi_k}{\partial \xi} t_k \hat{V_3}k_3\right] \\ \sum_{k=1}^{8}\left[\frac{\partial \psi_k}{\partial \eta} x_{k1} + \frac{\zeta}{2}\frac{\partial \psi_k}{\partial \eta} t_k \hat{V_3}k_1\right] & \sum_{k=1}^{8}\left[\frac{\partial \psi_k}{\partial \eta} x_{k2} + \frac{\zeta}{2}\frac{\partial \psi_k}{\partial \eta} t_k \hat{V_3}k_2\right] & \sum_{k=1}^{8}\left[\frac{\partial \psi_k}{\partial \eta} x_{k3} + \frac{\zeta}{2}\frac{\partial \psi_k}{\partial \eta} t_k \hat{V_3}k_3\right] \\ \sum_{k=1}^{8}\frac{1}{2}\psi_k t_k \hat{V_3}k_1 & \sum_{k=1}^{8}\frac{1}{2}\psi_k t_k \hat{V_3}k_2 & \sum_{k=1}^{8}\frac{1}{2}\psi_k t_k \hat{V_3}k_3 \end{bmatrix}$$

$$
\begin{Bmatrix}
\frac{\partial u_1}{\partial \xi} \\ \frac{\partial u_1}{\partial \eta} \\ \frac{\partial u_1}{\partial \zeta} \\ \frac{\partial u_2}{\partial \xi} \\ \frac{\partial u_2}{\partial \eta} \\ \frac{\partial u_2}{\partial \zeta} \\ \frac{\partial u_3}{\partial \xi} \\ \frac{\partial u_3}{\partial \eta} \\ \frac{\partial u_3}{\partial \zeta}
\end{Bmatrix}
=
\underbrace{\begin{bmatrix}
\frac{\partial \psi_1}{\partial \xi} & 0 & 0 & \zeta\left(\frac{\partial \psi_1}{\partial \xi}\right) C_{111} & \zeta\left(\frac{\partial \psi_1}{\partial \xi}\right) C_{211} & 0 & \cdots \\
\frac{\partial \psi_1}{\partial \eta} & 0 & 0 & \zeta\left(\frac{\partial \psi_1}{\partial \eta}\right) C_{111} & \zeta\left(\frac{\partial \psi_1}{\partial \eta}\right) C_{211} & 0 & \cdots \\
0 & 0 & 0 & \psi_1 C_{111} & \psi_1 C_{211} & 0 & \cdots \\
0 & \frac{\partial \psi_1}{\partial \xi} & 0 & \zeta\left(\frac{\partial \psi_1}{\partial \xi}\right) C_{112} & \zeta\left(\frac{\partial \psi_1}{\partial \xi}\right) C_{212} & 0 & \cdots \\
0 & \frac{\partial \psi_1}{\partial \eta} & 0 & \zeta\left(\frac{\partial \psi_1}{\partial \eta}\right) C_{112} & \zeta\left(\frac{\partial \psi_1}{\partial \eta}\right) C_{212} & 0 & \cdots \\
0 & 0 & 0 & \psi_1 C_{112} & \psi_1 C_{212} & 0 & \cdots \\
0 & 0 & \frac{\partial \psi_1}{\partial \xi} & \zeta\left(\frac{\partial \psi_1}{\partial \xi}\right) C_{113} & \zeta\left(\frac{\partial \psi_1}{\partial \xi}\right) C_{213} & 0 & \cdots \\
0 & 0 & \frac{\partial \psi_1}{\partial \eta} & \zeta\left(\frac{\partial \psi_1}{\partial \eta}\right) C_{113} & \zeta\left(\frac{\partial \psi_1}{\partial \eta}\right) C_{213} & 0 & \cdots \\
0 & 0 & 0 & \psi_1 C_{113} & \psi_1 C_{213} & 0 & \cdots
\end{bmatrix}}_{\substack{[C]_{9\times 6} \\ \text{for } k=1}}
\underbrace{\begin{Bmatrix}
v_1' \\ v_2' \\ v_3' \\ \theta_1' \\ \theta_2' \\ \theta_3'
\end{Bmatrix}}_{[d]_{48\times 1}}
$$

for $k=1$ 6×1

for 2 to 8

where

$C_{1kj} = -\frac{1}{2} t_k V_{2kj}$

$C_{2kj} = \frac{1}{2} t_k V_{1kj}$

$$
\begin{Bmatrix}
\varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23}
\end{Bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0
\end{bmatrix}
\begin{Bmatrix}
\partial u_1/\partial x_1 \\ \partial u_1/\partial x_2 \\ \partial u_1/\partial x_3 \\ \partial u_2/\partial x_1 \\ \partial u_2/\partial x_2 \\ \partial u_2/\partial x_3 \\ \partial u_3/\partial x_1 \\ \partial u_3/\partial x_2 \\ \partial u_3/\partial x_3
\end{Bmatrix}
$$

Stiffness matrix formulation -

$$K' = \int_V [C]^T [B]^T [A]^T [E] [A] [B] [C] \, dv \; [d']_{48 \times 1}$$

$$[B] = \begin{bmatrix} [J^{-1}]_{3\times3} & 0 & 0 \\ 0 & [J^{-1}]_{3\times3} & 0 \\ 0 & 0 & [J^{-1}]_{3\times3} \end{bmatrix}_{9\times9}$$

$$[K''] = [K'] + \begin{bmatrix} 0 & & & & & & \\ & 0 & & & & & \\ & & 0 & & & & \\ & & & 0 & & & \\ & & & & 0 & & \\ & & & & & \epsilon & \\ & & & & & & \ddots \end{bmatrix}_{48\times48}.$$

$$K = [L]^T [K''] [L]_{48\times48}$$

where,

$$[L_K] = \begin{bmatrix} [I]_{3\times3} & 0 \\ 0 & [\lambda^K]_{3\times3} \end{bmatrix}_{6\times6}$$

$[\lambda^K]$ - direction cosines

$$[\lambda^K] = \begin{bmatrix} \hat{V}_{11} & \hat{V}_{12} & \hat{V}_{13} \\ \hat{V}_{21} & \hat{V}_{22} & \hat{V}_{23} \\ \hat{V}_{31} & \hat{V}_{32} & \hat{V}_{33} \end{bmatrix}_{3\times3}$$

THERMAL FORCE CALCULATION

Using Gauss Point formula,

Thermal stress equation can be written as -

$$\sigma_t = \sum [D]^T [E] [D] [\varepsilon_s] \, |J| \, d\xi \, d\eta \, dz$$

where, ε_s is thermal strain

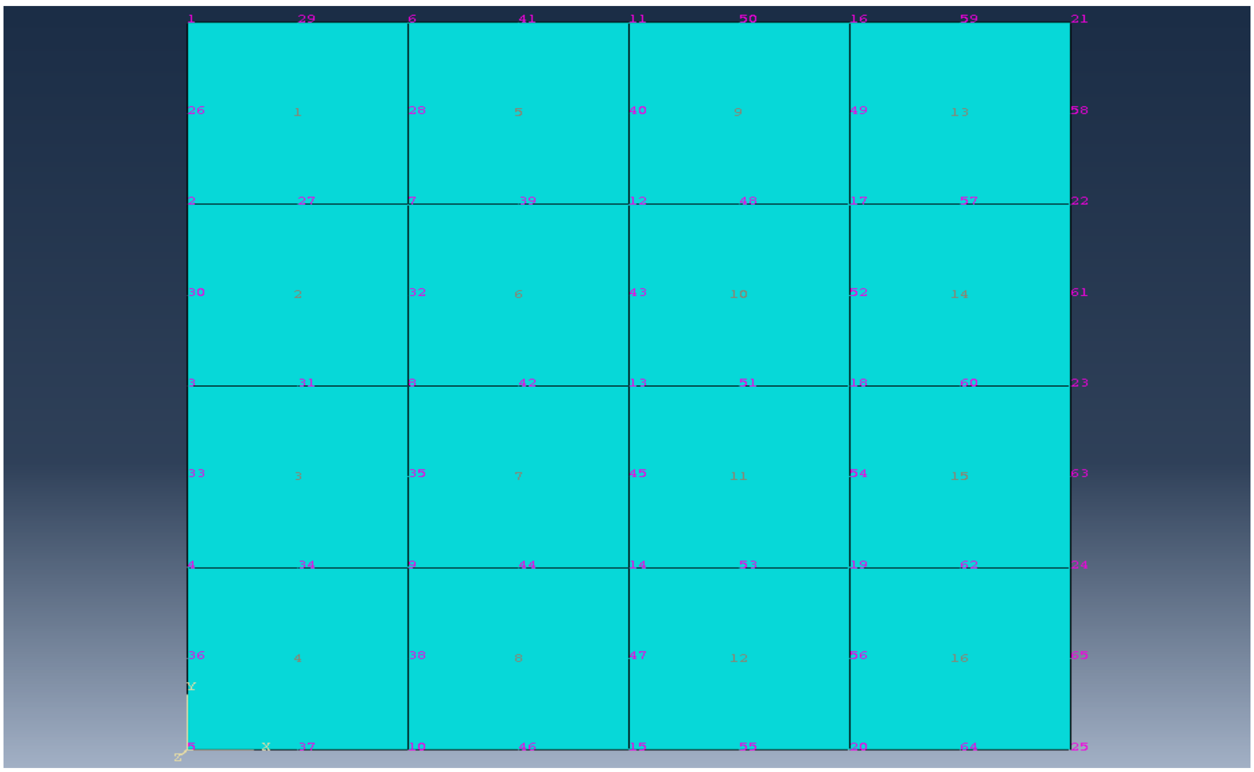
$$\varepsilon_s = \alpha \Delta T$$

3. TEST CASE NANLYSIS

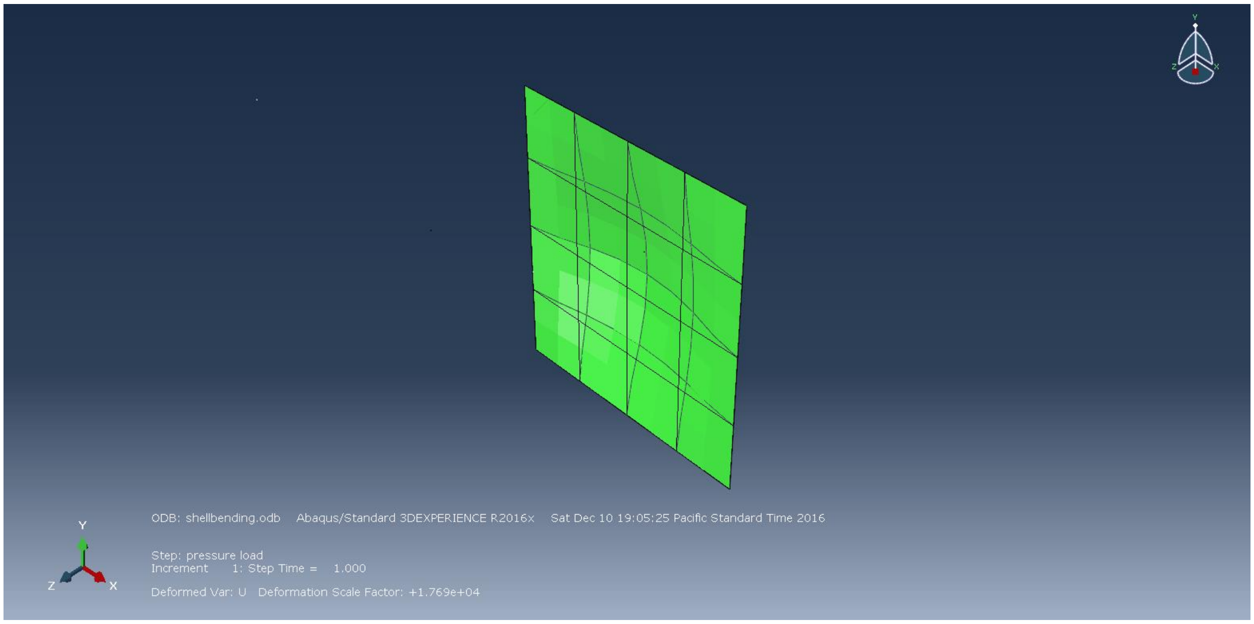
3.1 PLATE BENDING PROBLEM

INPUTS
E = 29E6 psi
Poisson ratio = 0.3
Thickness = 2 inch
Pressure = 10 psi
Mesh = 4 X 4 element grid
Dimensions 20 inch x 20 inch

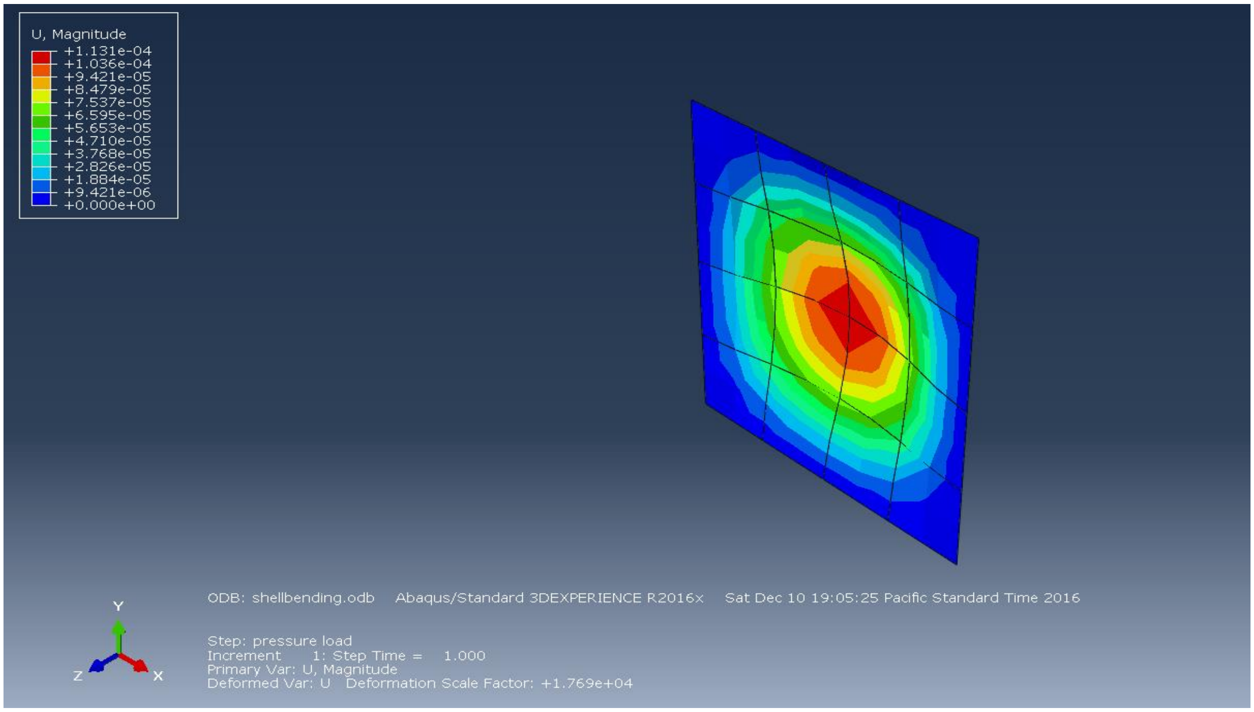
Element numbers and node numbers



Deformation Shape



Displacement



Comparison between ABAQUS and MATLAB displacements (U3 at all nodes in the Interior)

Sr No	NODE	ABAQUS U3 (inch)	FEA3D-MATLAB U3 (inch)	PERCENTAGE ERROR
1	7	-0.0000458	-0.00005	8.4
2	8	-0.0000717	-0.00008	10.37
3	9	-0.0000458	-0.00005	8.4
4	12	-0.0000717	-0.00008	10.37
5	13	-0.0001131	-0.00013	13
6	14	-0.0000717	-0.00008	10.37
7	17	-0.0000458	-0.00005	8.4
8	18	-0.0000717	-0.00008	10.37
9	19	-0.0000458	-0.00005	8.4

3.2 TWISTING PLATE MODEL

INPUTS

E = 29E6 psi

Poisson ratio = 0.3

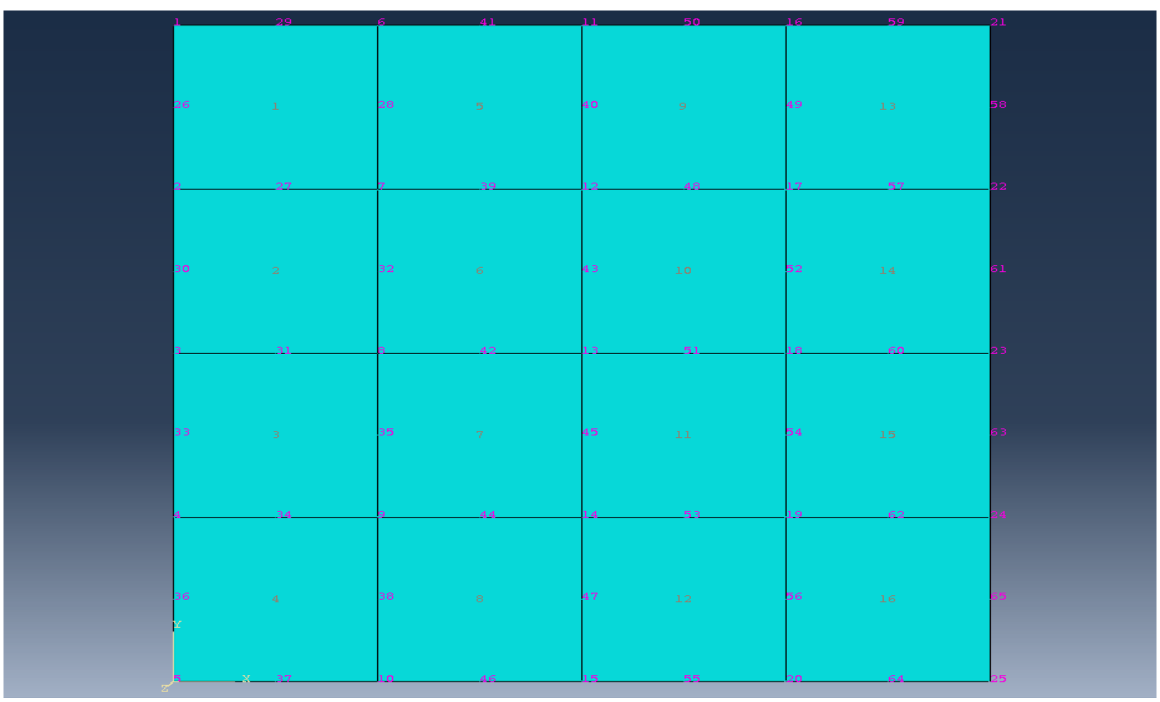
Thickness = 2 inch

Pressure = 200 lb

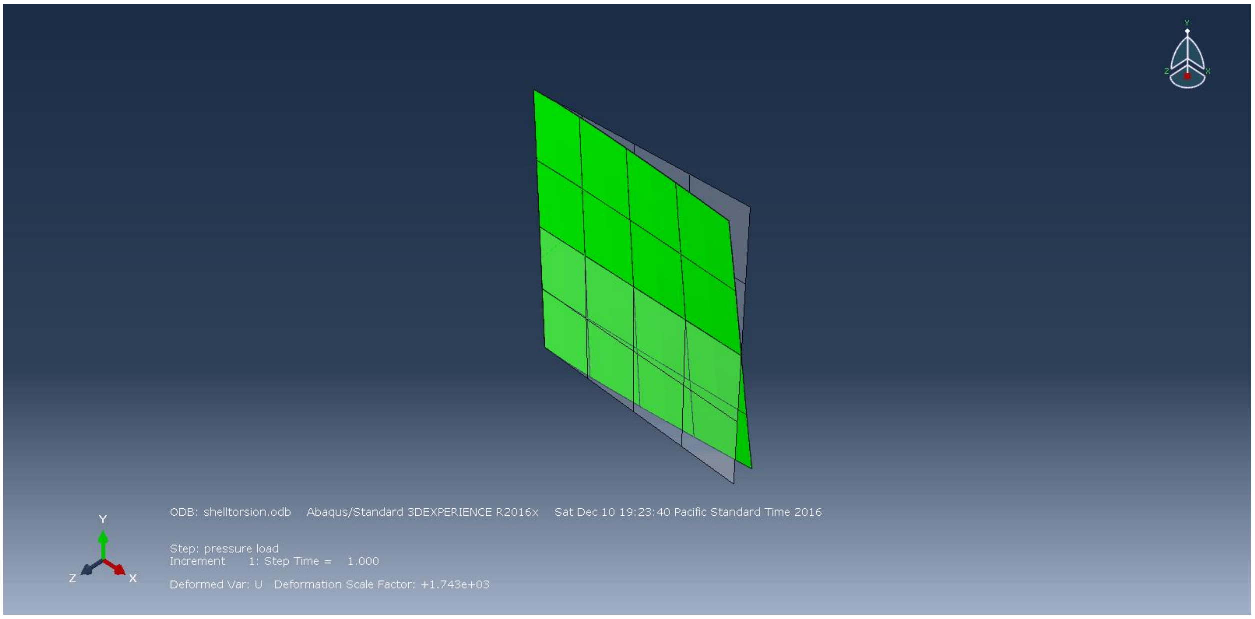
Mesh = 4 X 4 element grid

Dimensions 20 inch x 20 inch

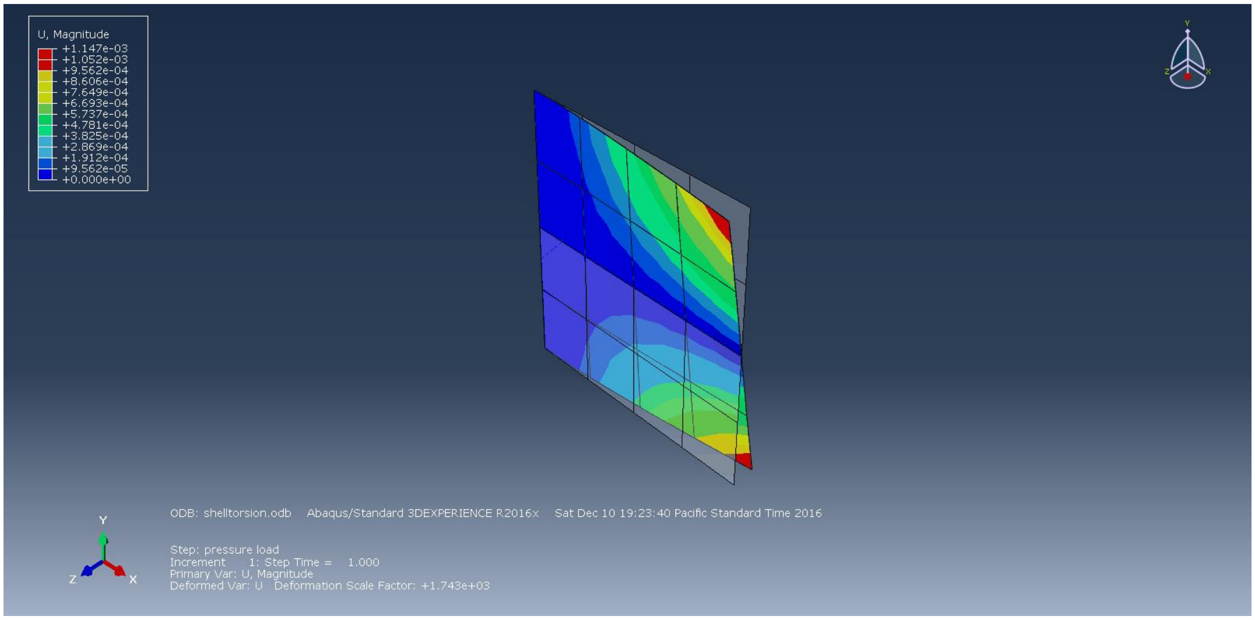
Element numbers and node numbers



Deformation Shape



Displacement



Comparison between ABAQUS and MATLAB displacements (U3)

Sr No	NODE	ABAQUS U3 (inch)	FEA3D-MATLAB U3 (inch)	PERCENTAGE ERROR
1	21	0.00114742	0.00107	7.23
2	25	-0.00114742	-0.00107	7.23

3.3 BATHE ARCH MODEL WITH CONCENTRATED FORCE MODEL

INPUTS

E = 3102.75E6 N/m^2

L = 0.508 m

R = 2.54 m

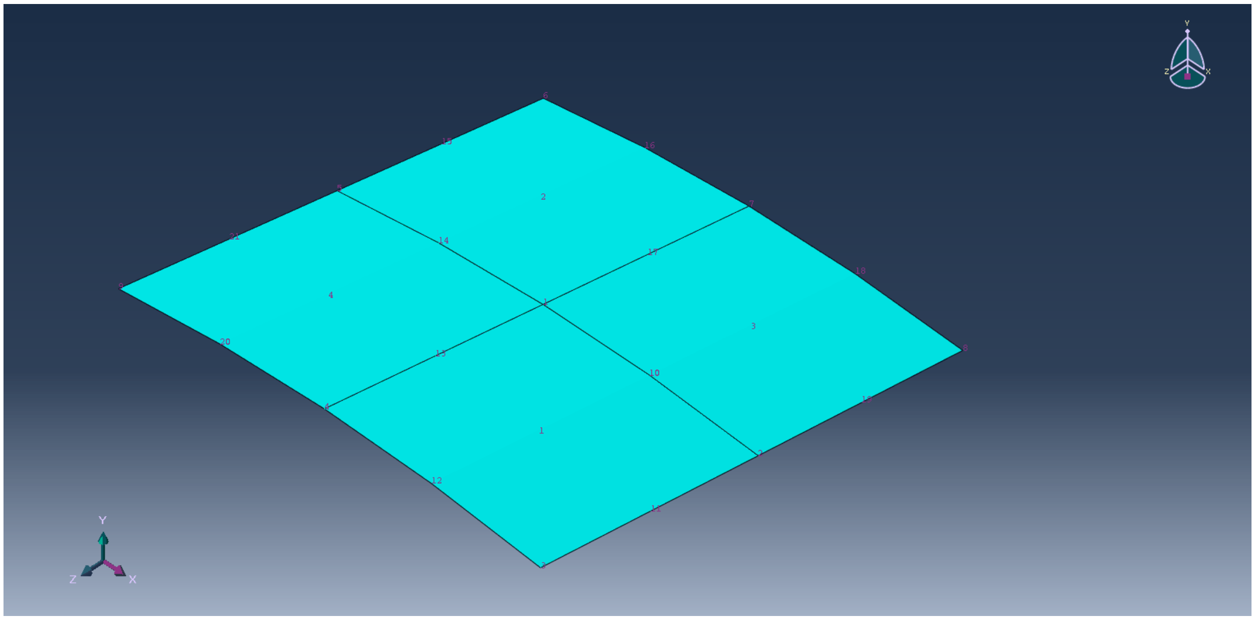
Θ = 5.73 ^{0}

Poisson ratio = 0.3

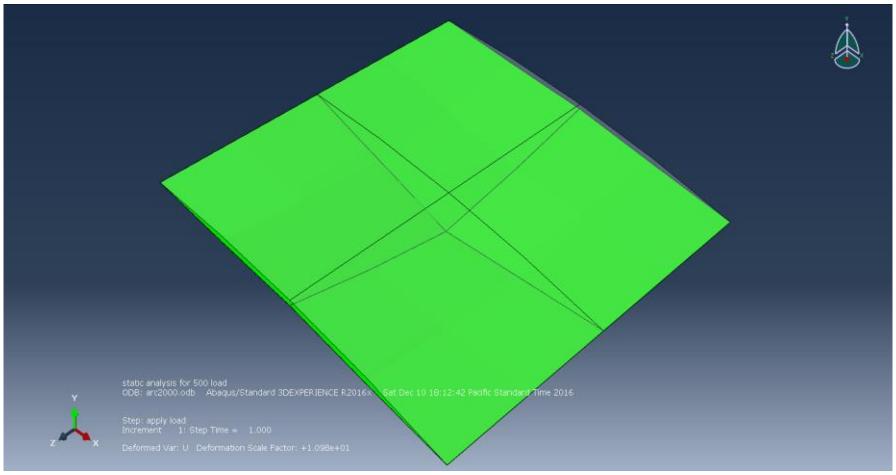
Thickness = 12.7E-3 m

Mesh = 2 X 2 element grid

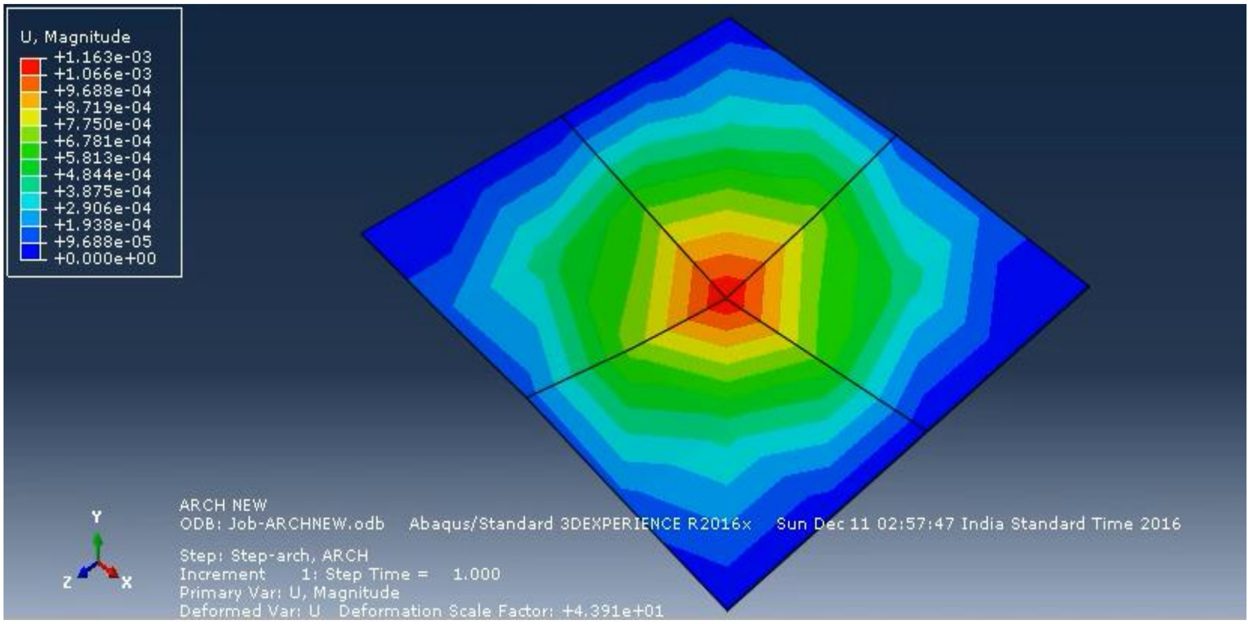
Node numbers and number for elements



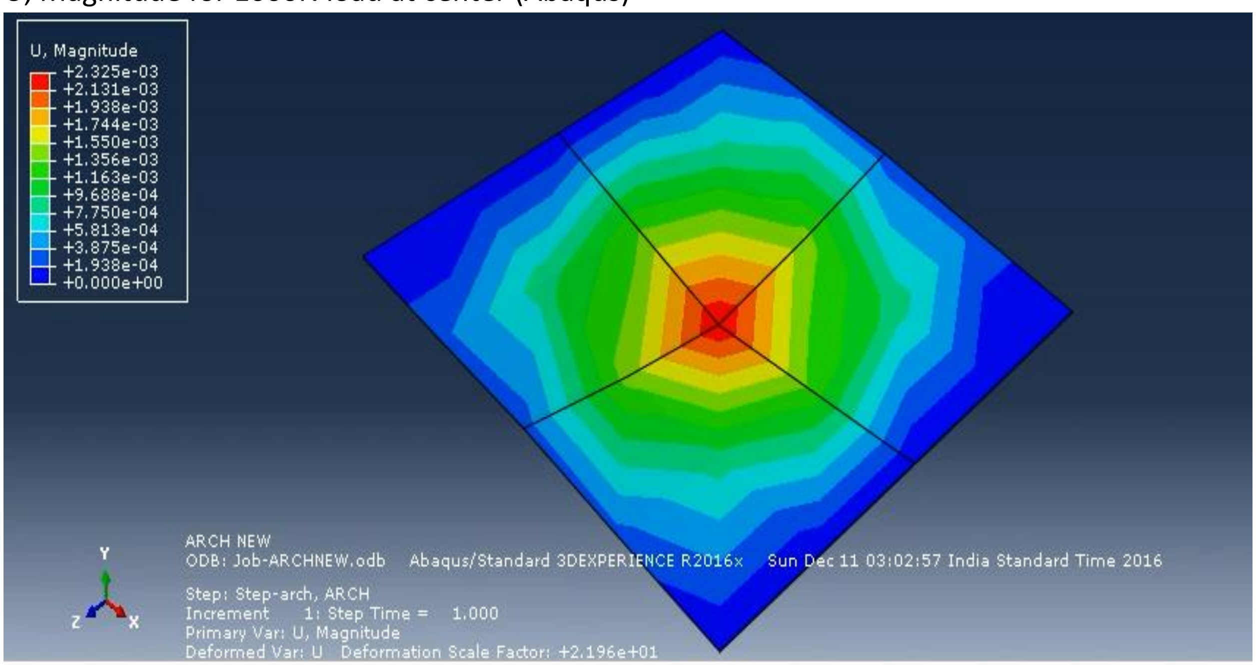
Deformation



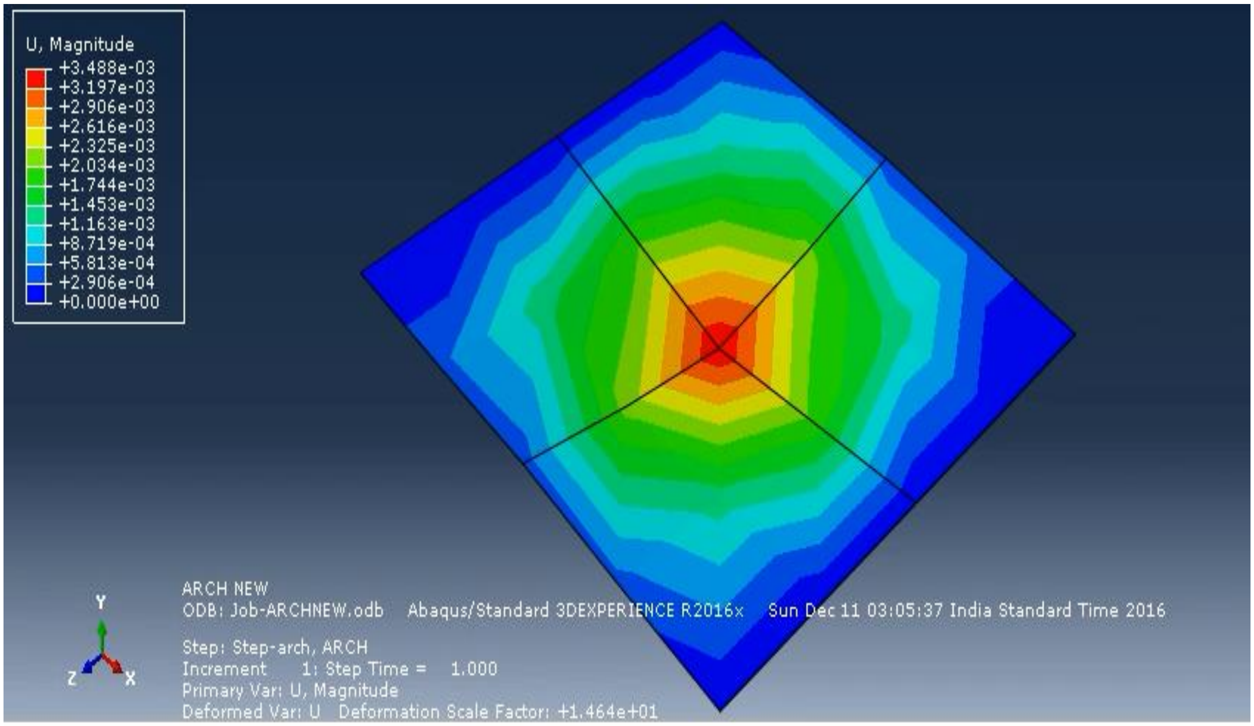
U, Magnitude for 500N load at center (Abaqus)



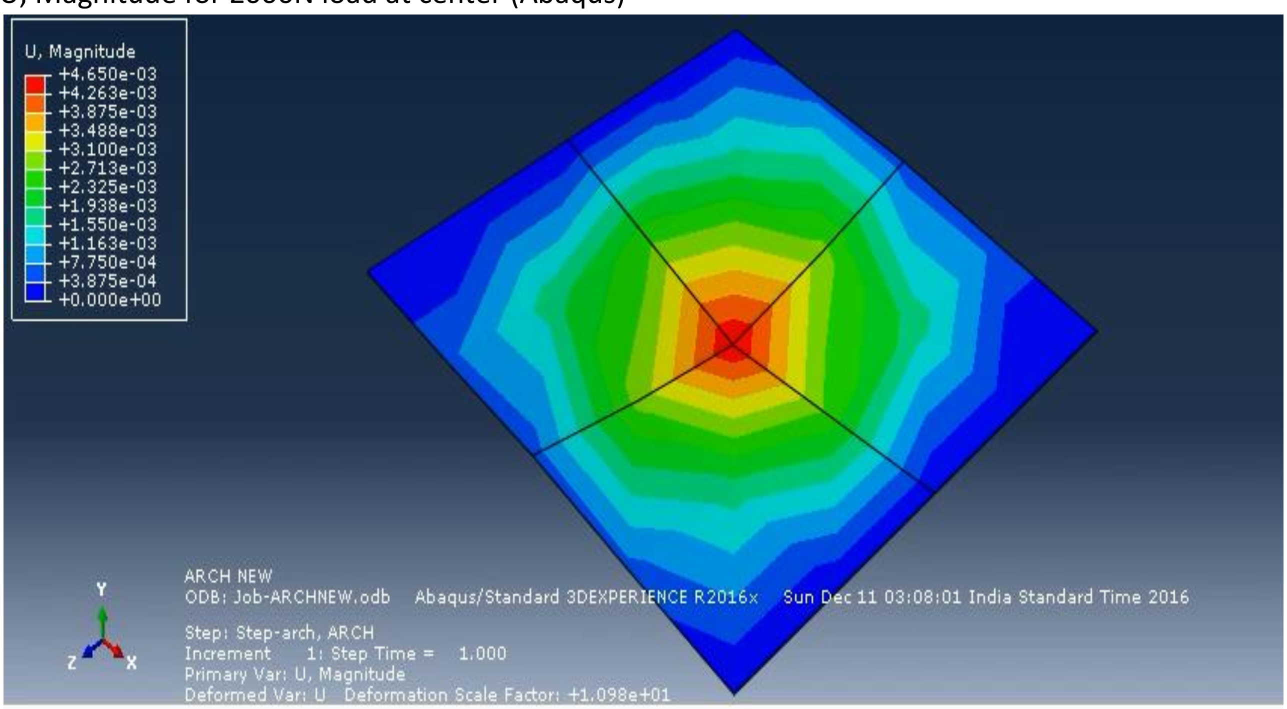
U, Magnitude for 1000N load at center (Abaqus)



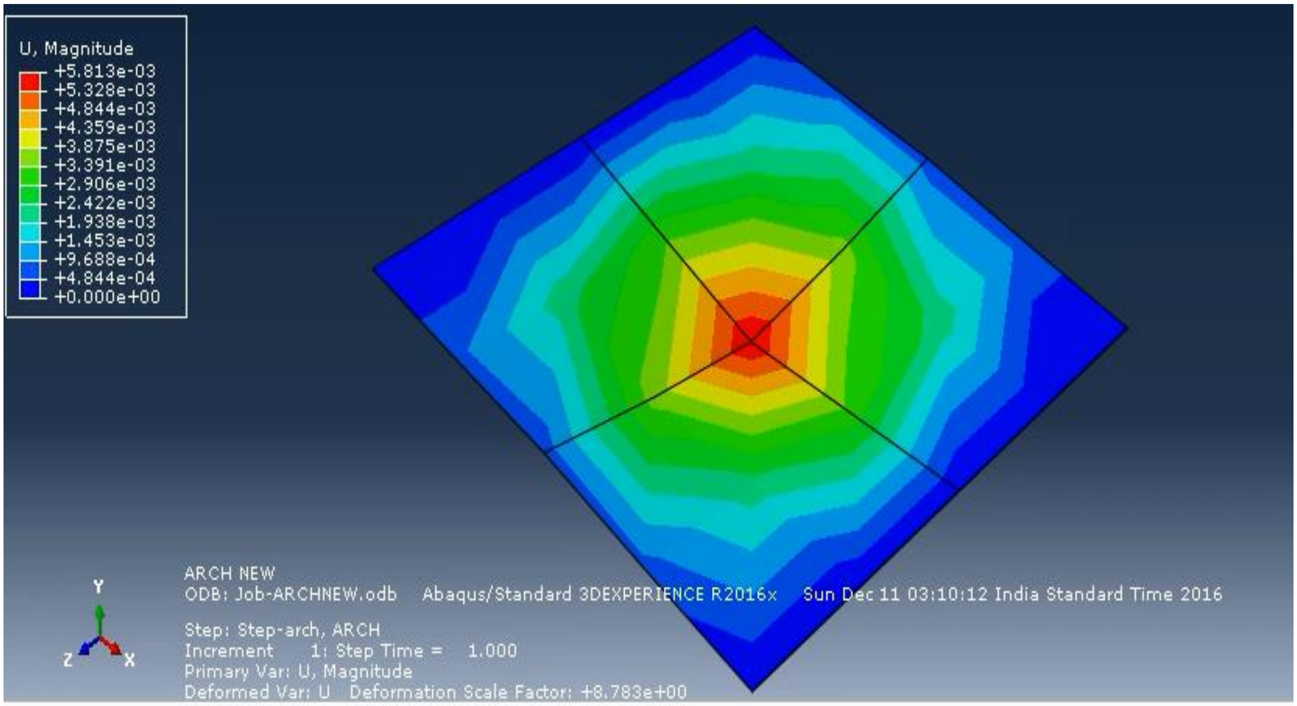
U, Magnitude for 1500N load at center (Abaqus)



U, Magnitude for 2000N load at center (Abaqus)



U, Magnitude for 2500N load at center (Abaqus)



Comparison between ABAQUS and MATLAB displacements (U2)

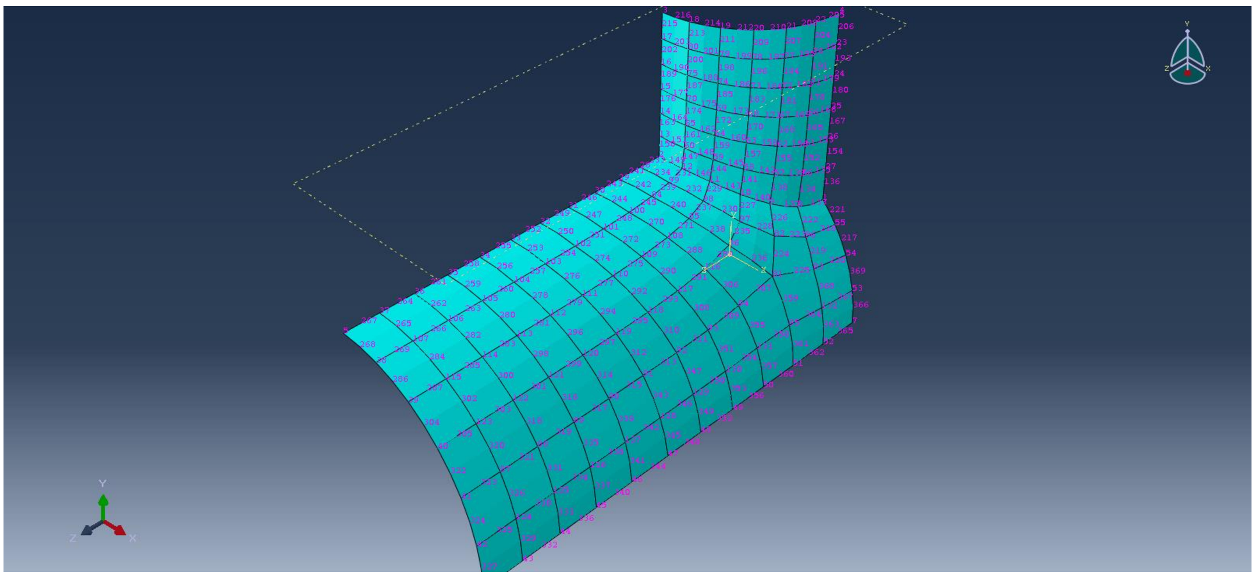
Sr No	**LOAD (N)**	**ABAQUS U3 (inch)**	**FEA3D-MATLAB U3 (inch)**	**PERCENTAGE ERROR (using absolute values)**
1	500	-0.00115	0.00113	1.7
2	1000	-0.00231	0.00170	3.5
3	1500	-0.00347	0.00227	5.2
4	2000	-0.00463	0.00284	6.3
5	2500	-0.00578	0.00340	7.0

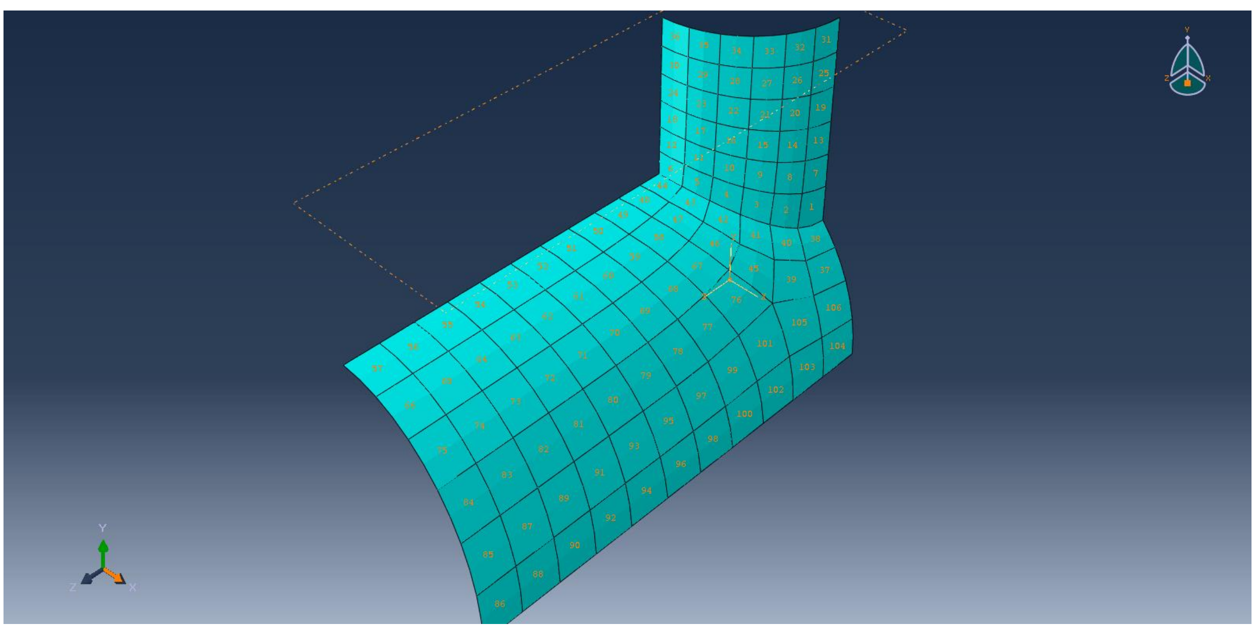
4. SHELL INTERSECTION MODEL

INPUT

E = 29E6 psi
Poisson ratio = 0.3
Shell edge load = 55000 lb/ft
Mass density = 0.0008 lbf-sec2 /inch^4
Coefficient of Expansion = 7E-6 in/in/deg F
Conductivity = 0.00023 lbs in/hr/deg F/in
Thickness = 2.5 in

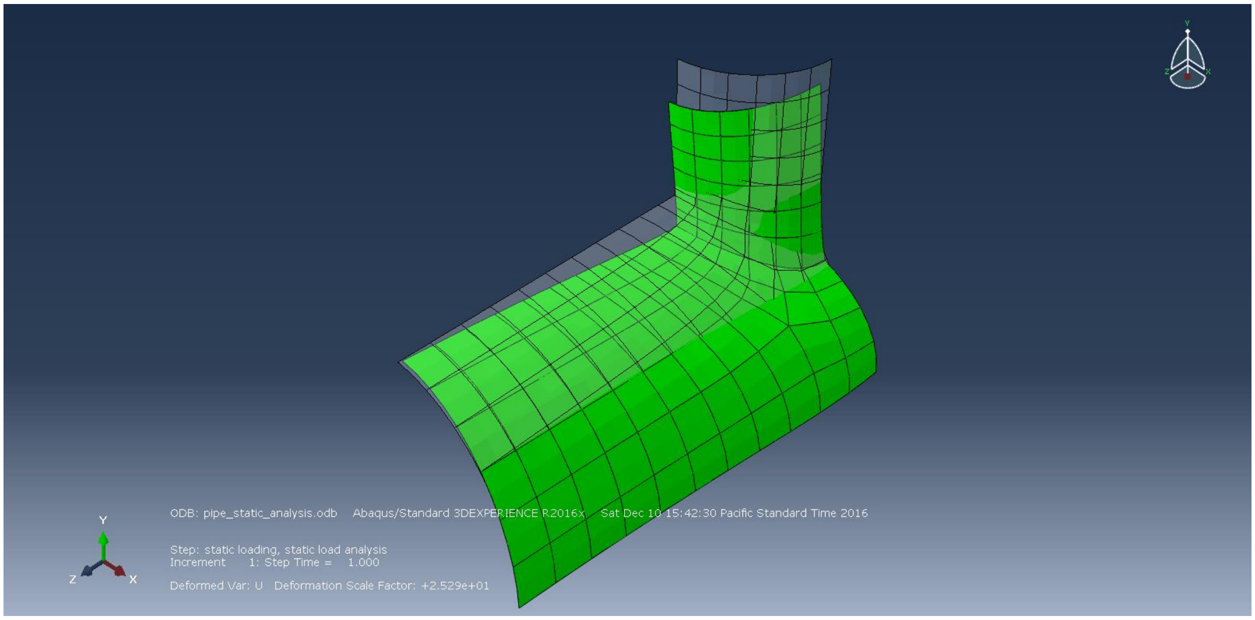
Element numbers and node numbers



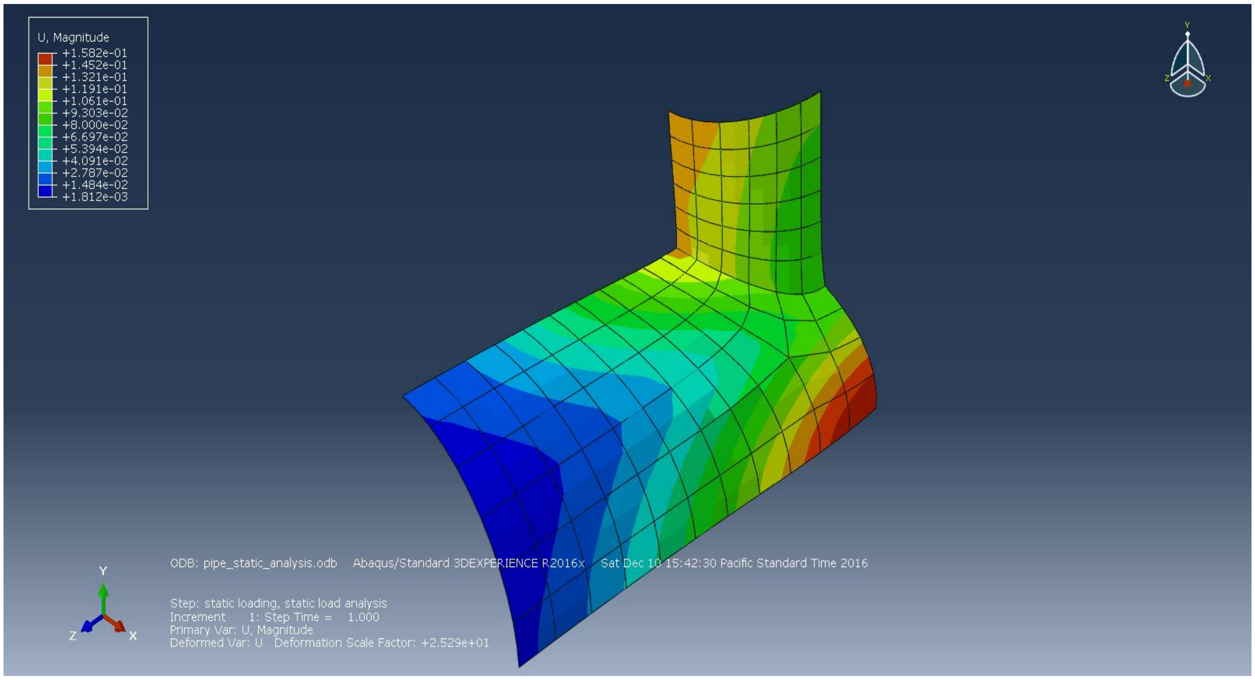


CASE1- STATIC ANALYSIS

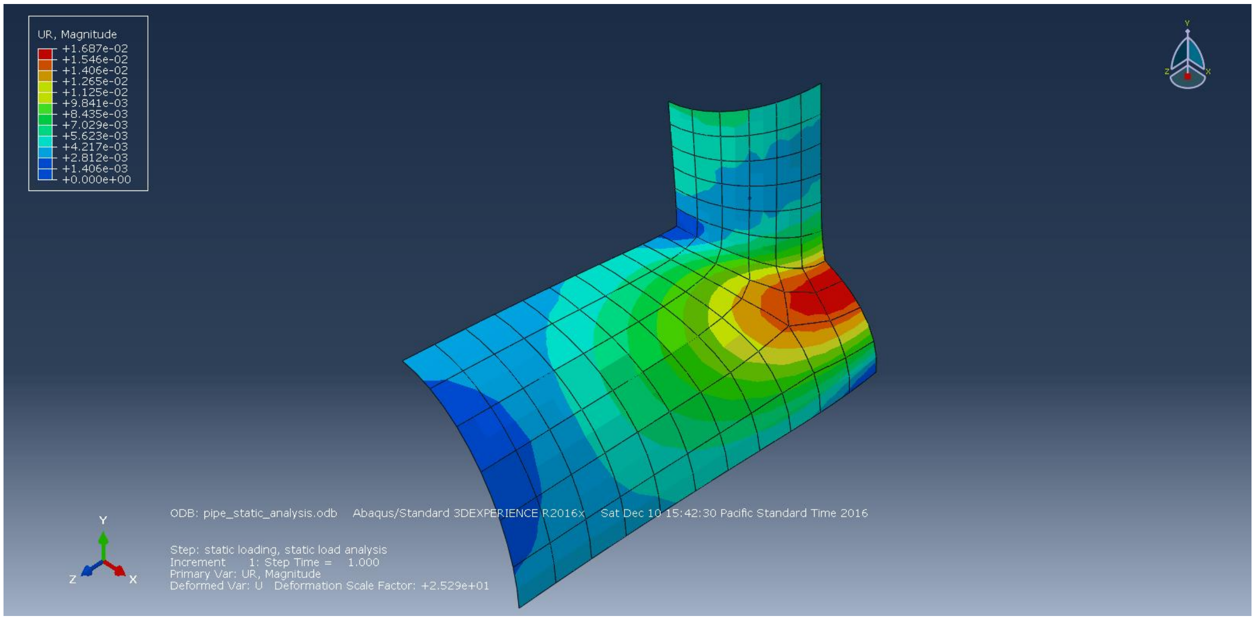
Deformation



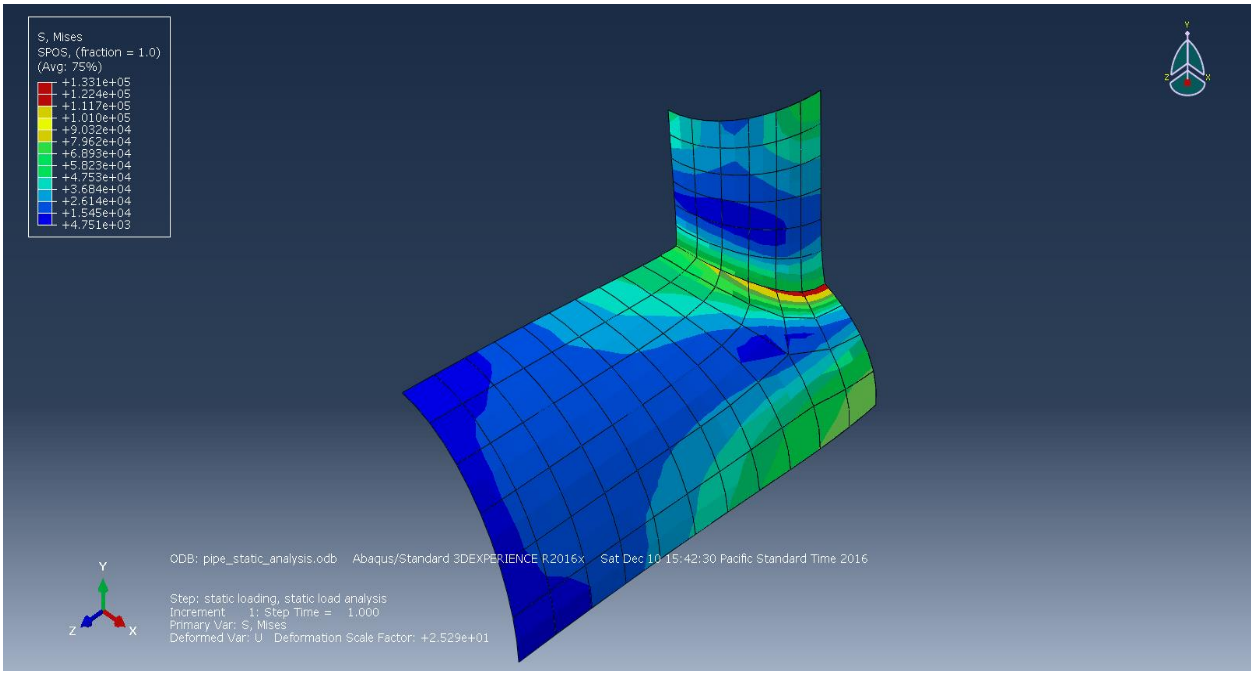
Umagnitude



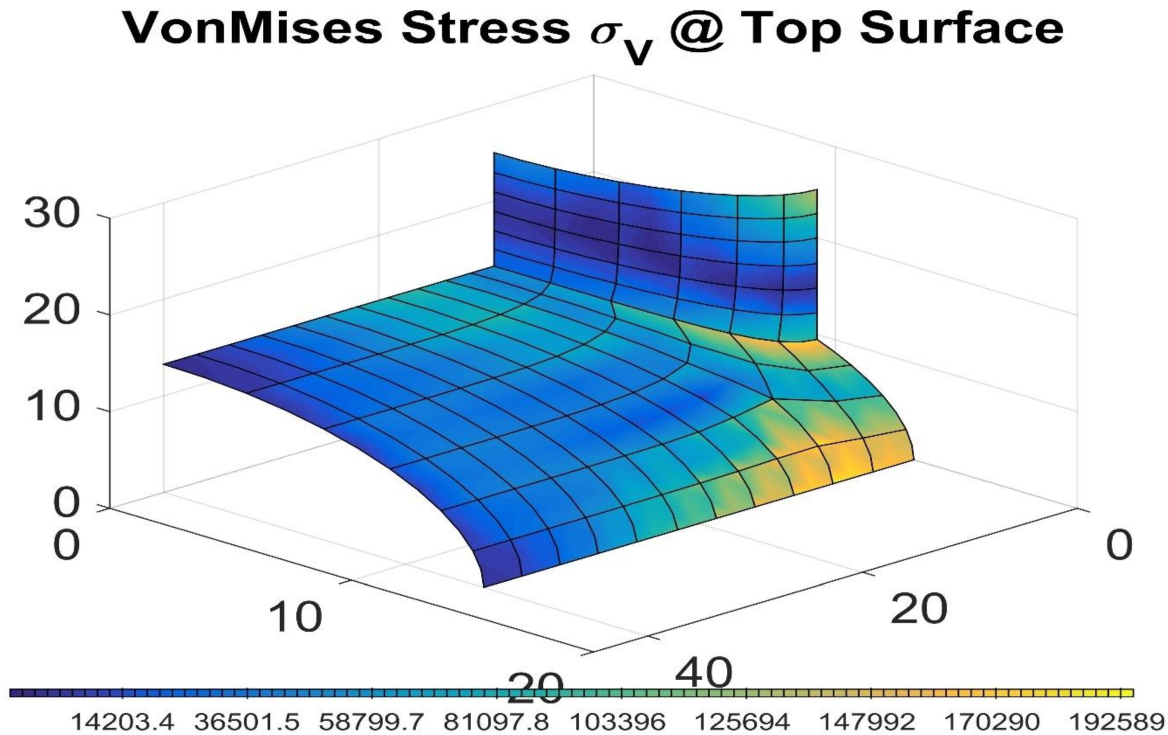
URmagnitude



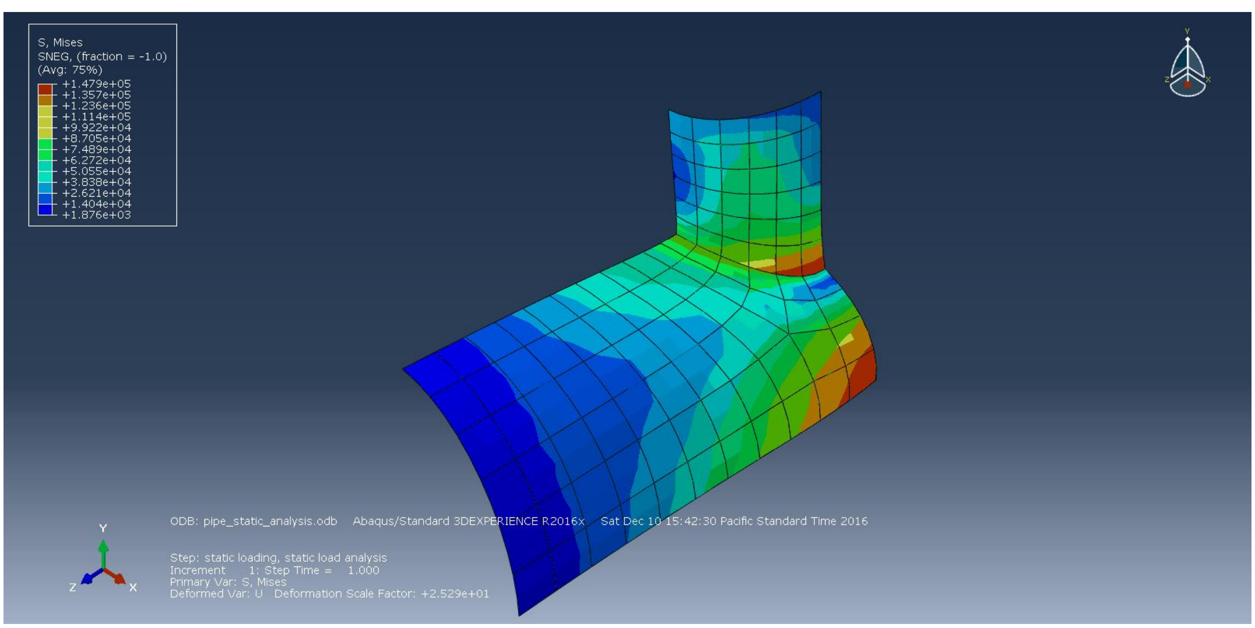
Von mises stress – Top surface(ABAQUS)



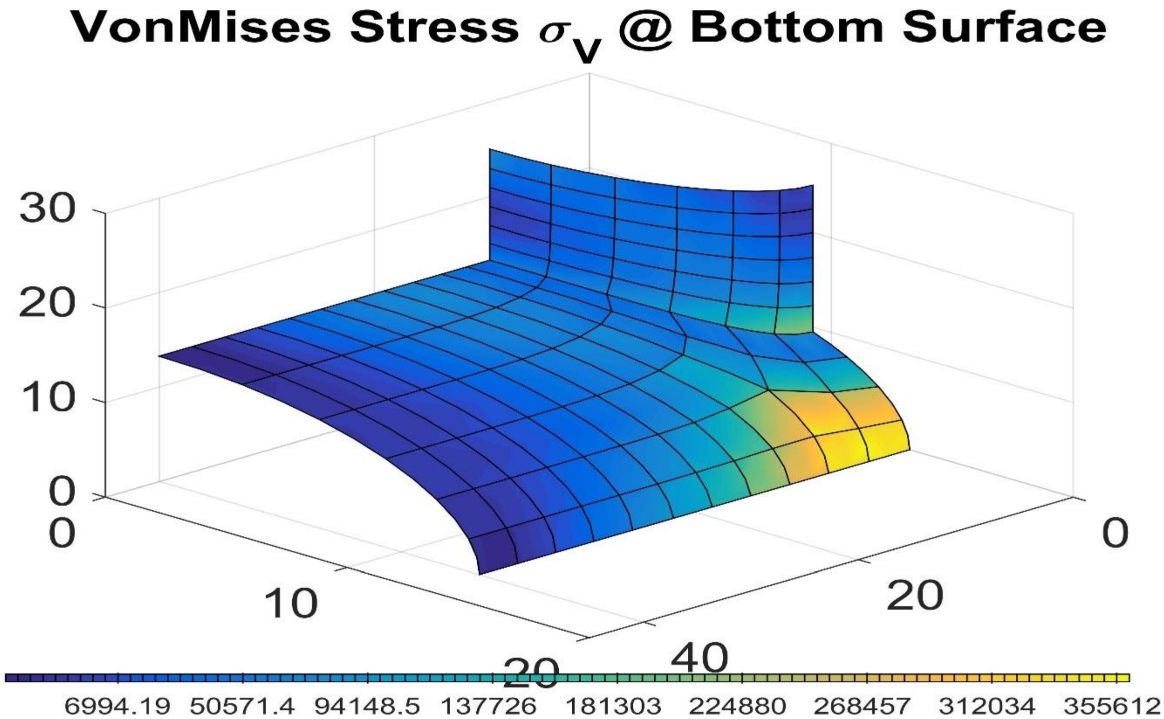
Von mises stress – Top surface(MATLAB)



Von mises stress- Bottom surface(ABAQUS)



Von mises stress- Bottom surface(MATLAB)

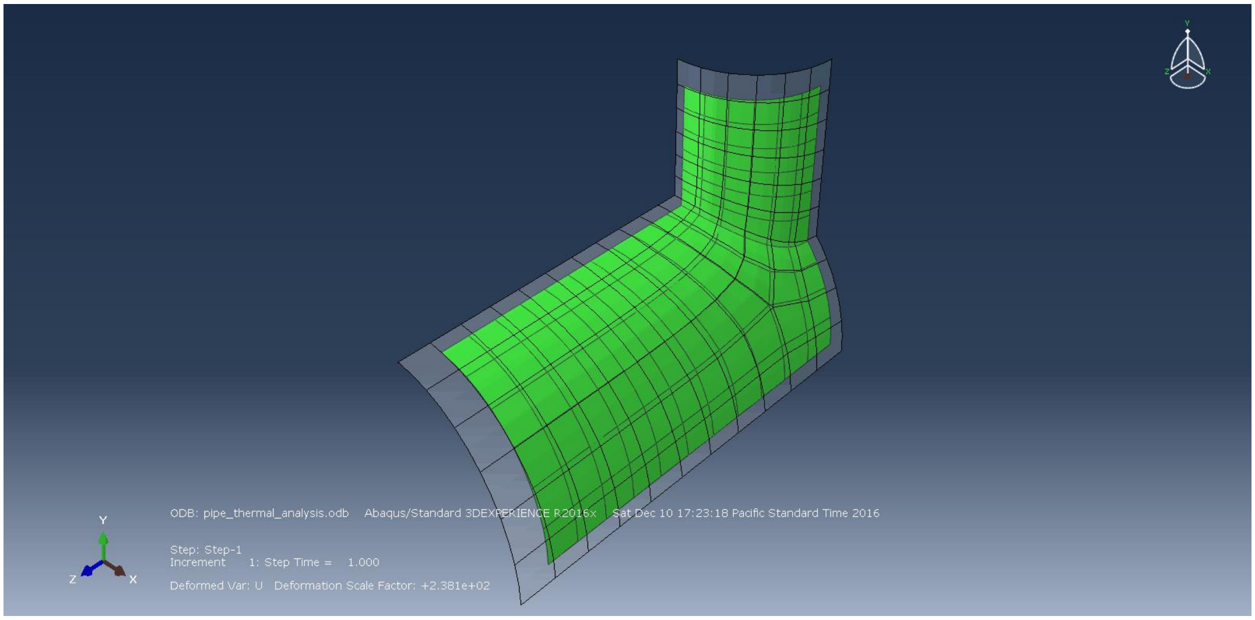


Comparison between ABAQUS and MATLAB displacements at point A,B,C

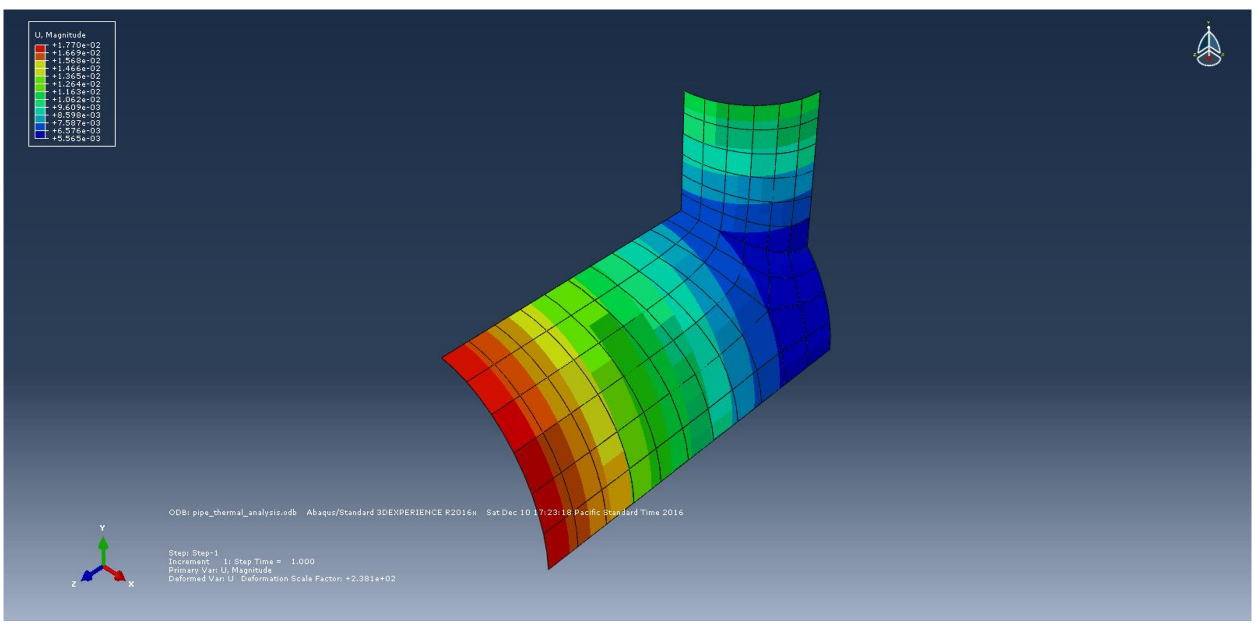
		POINT A	POINT B	POINT C
ABAQUS	U1	9.38e-33	0.1581	1.729e-32
	U2	-0.004	-3.97e-32	-0.123
	U3	-0.0152	3.63e-32	0.0423
	UR1	-0.0029	1.596e-34	0.0057
	UR2	-4.8e-35	-3.123e-32	1.318e-32
	UR3	2.5e-33	8.14e-32	8.15e-35
FEA3D-MATLAB	U1	0	0.1335	0
	U2	0.00236	0	-0.107
	U3	-0.01256	0	0.04438
	UR1	-0.00279	0	0.00638
	UR2	0	0	0
	UR3	0	0	0
PERCENTAGE ERROR (Taking percentage error using absolute values)	U1	0	18.42	0
	U2	69.4	0	14.95
	U3	21.01	0	4.68
	UR1	3.9	0	10.6
	UR2	0	0	0
	UR3	0	0	0

CASE2- THERMAL ANALYSIS

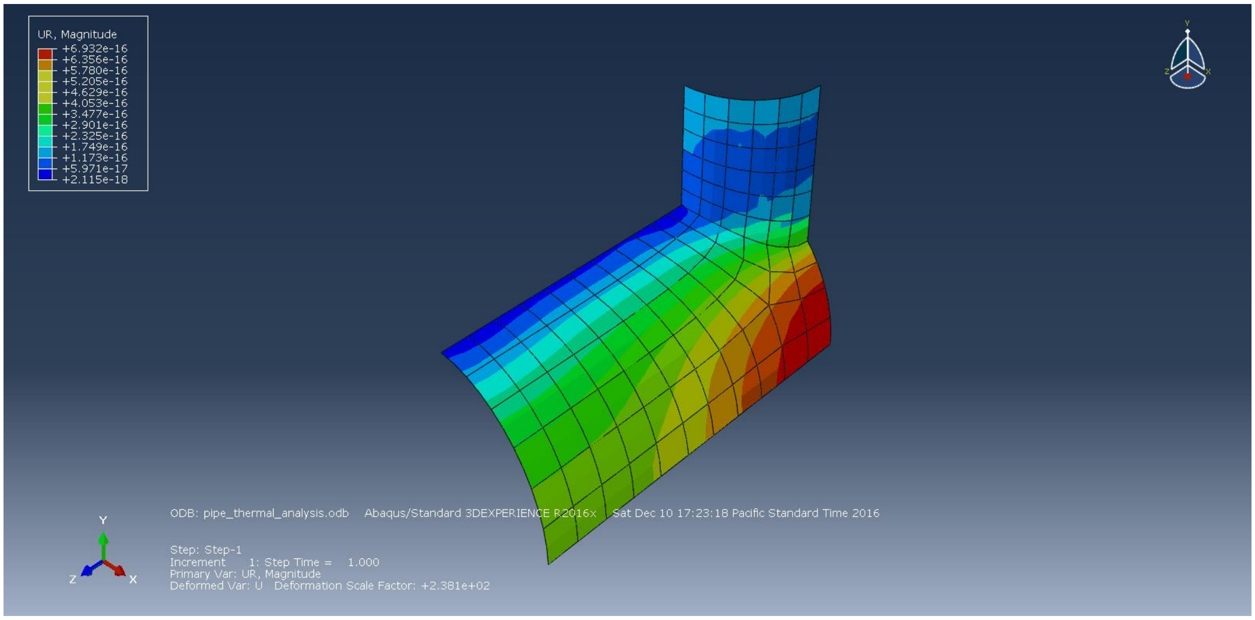
Deformation



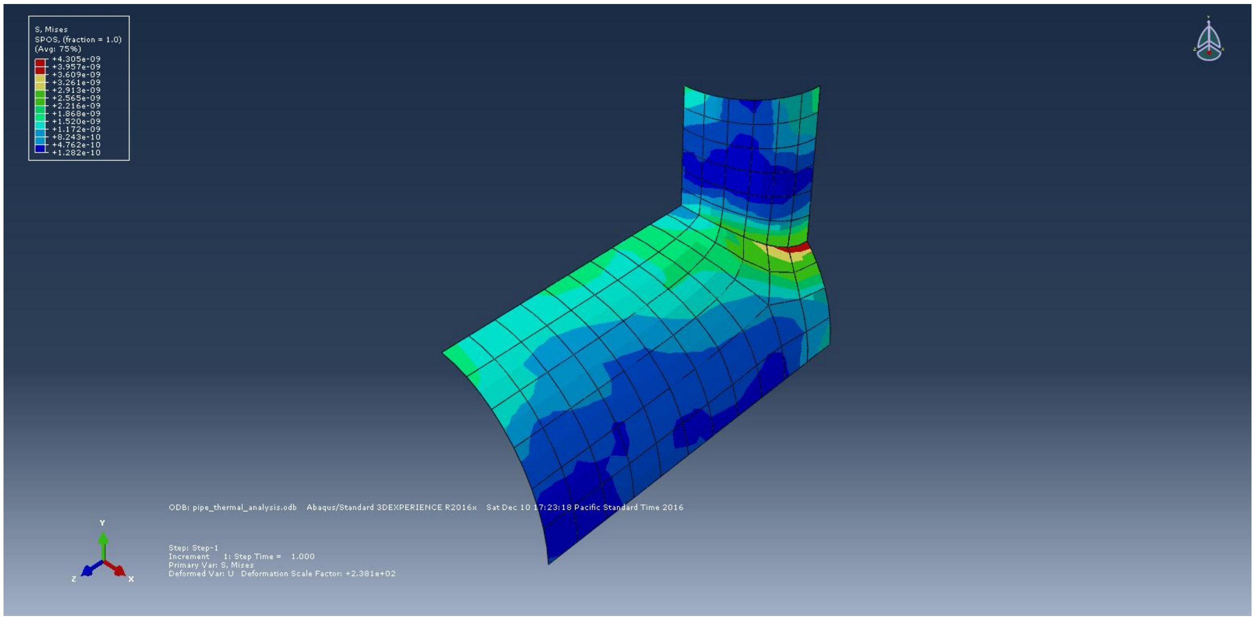
Umagnitude



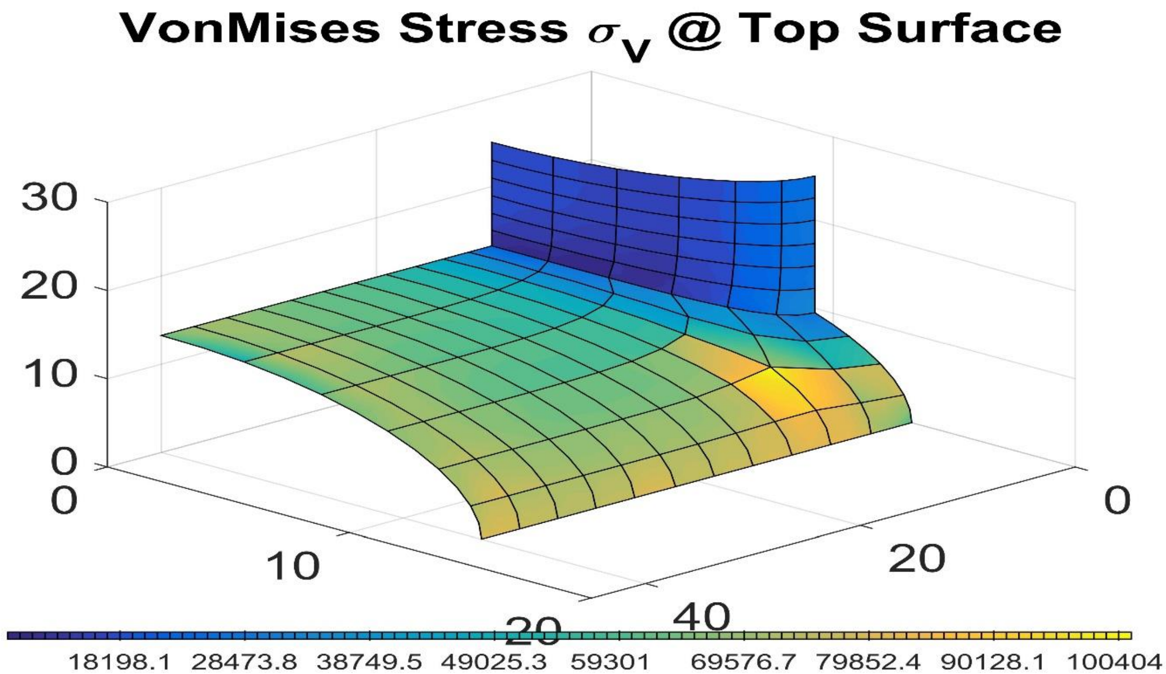
URmagnitude



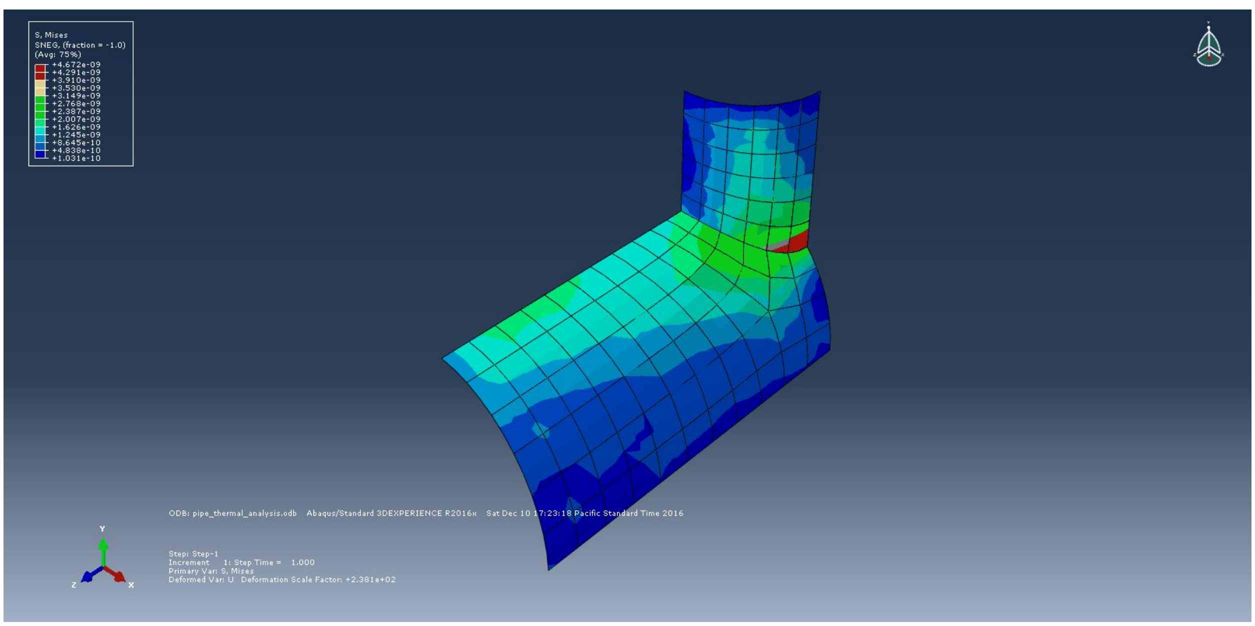
Von mises stress – Top surface(ABAQUS)



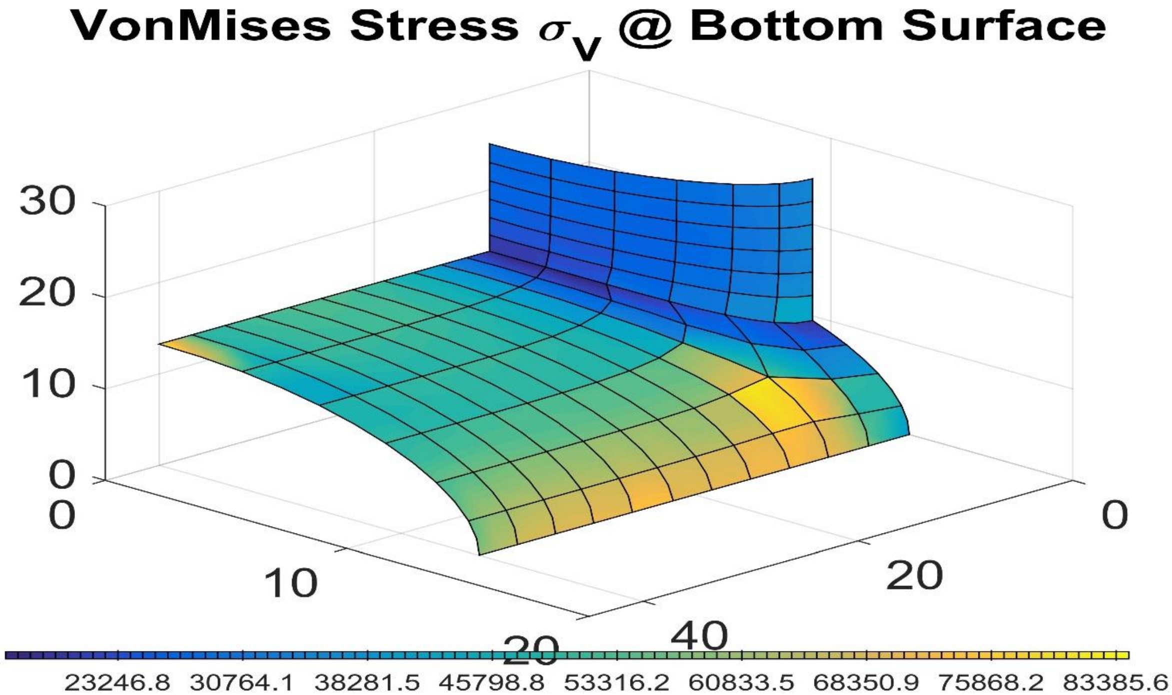
Von mises stress – Top surface(MATLAB)



Von mises stress- Bottom surface(ABAQUS)



Von mises stress- Bottom surface(MATLAB)

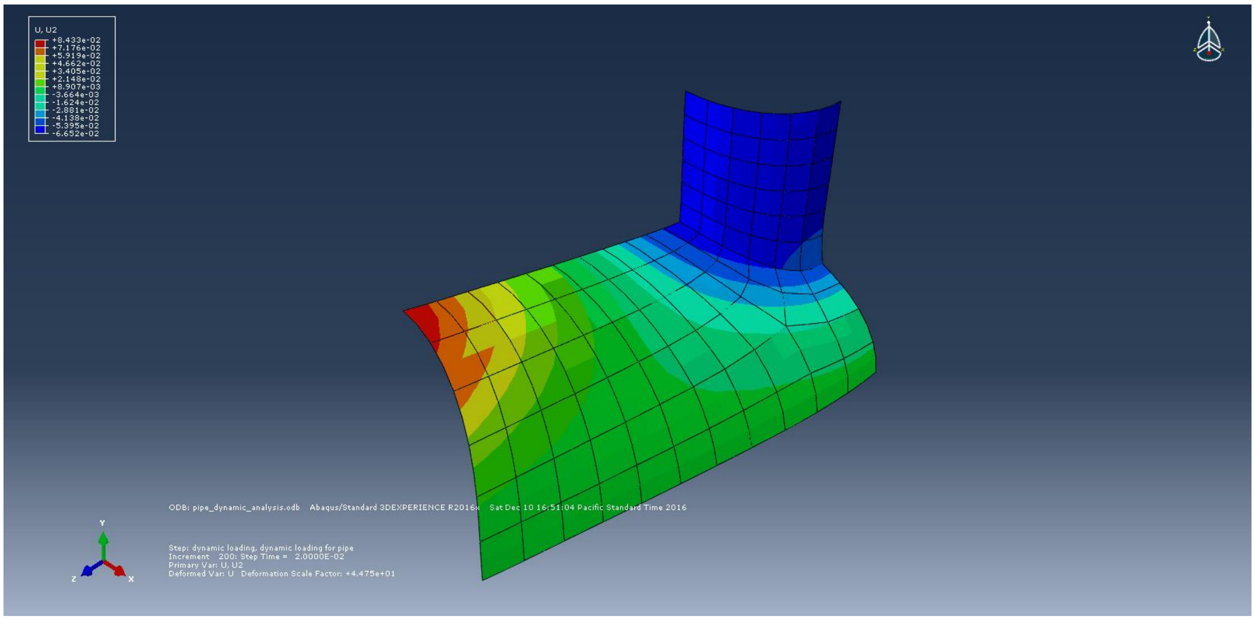


Comparison between ABAQUS and MATLAB displacements at point A, B, C

		POINT A	POINT B	POINT C
ABAQUS	U1	-2.279e-32	-0.0055	-1.521e-32
	U2	-0.0055	2.279e-32	-0.0150
	U3	-0.0168	2.126e-32	-0.0038
	UR1	3.79e-17	0	1.436e-16
	UR2	0	0	0
	UR3	0	6.932e-16	0
FEA3D-MATLAB	U1	0	-0.00612	0
	U2	-0.01080	0	-0.00981
	U3	-0.0199	0	-0.00434
	UR1	0.00004	0	0.00032
	UR2	0	0	0
	UR3	0	0	0
PERCENTAGE ERROR (Taking percentage error using absolute values)	U1	0	10.01	0
	U2	49.07	0	52.9
	U3	15	0	12.44
	UR1	100	0	100
	UR2	0	0	0
	UR3	0	0	0

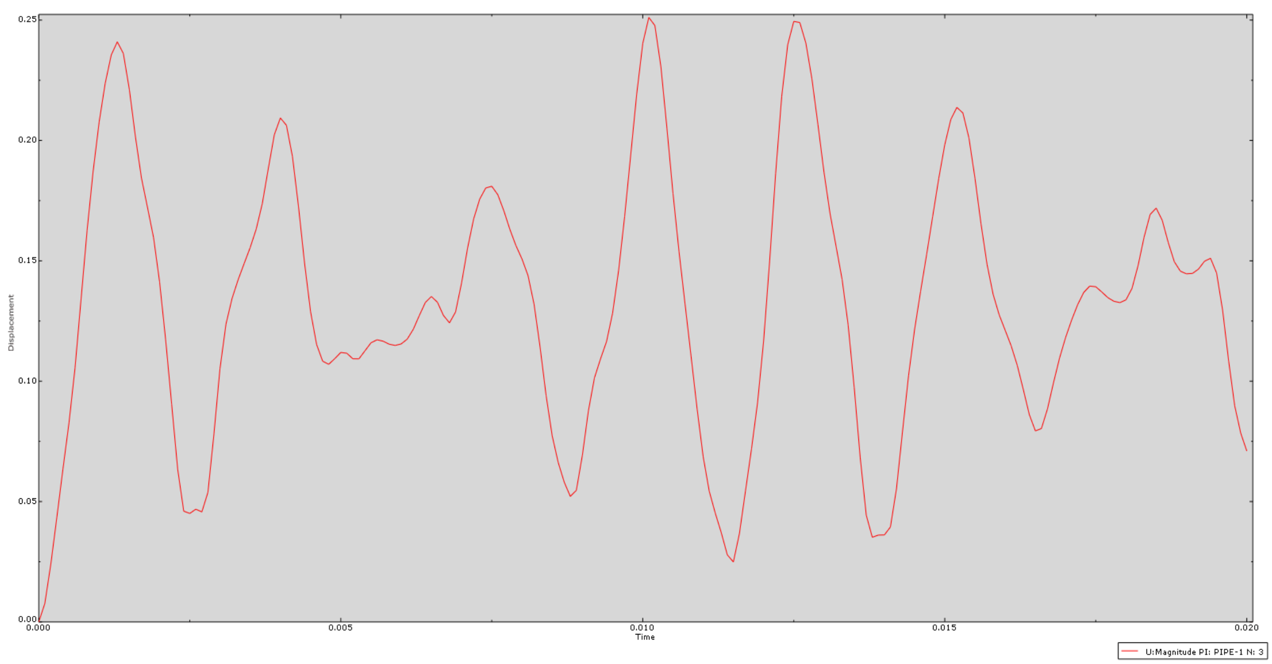
CASE3 DYNAMIC ANALYSIS

U2value

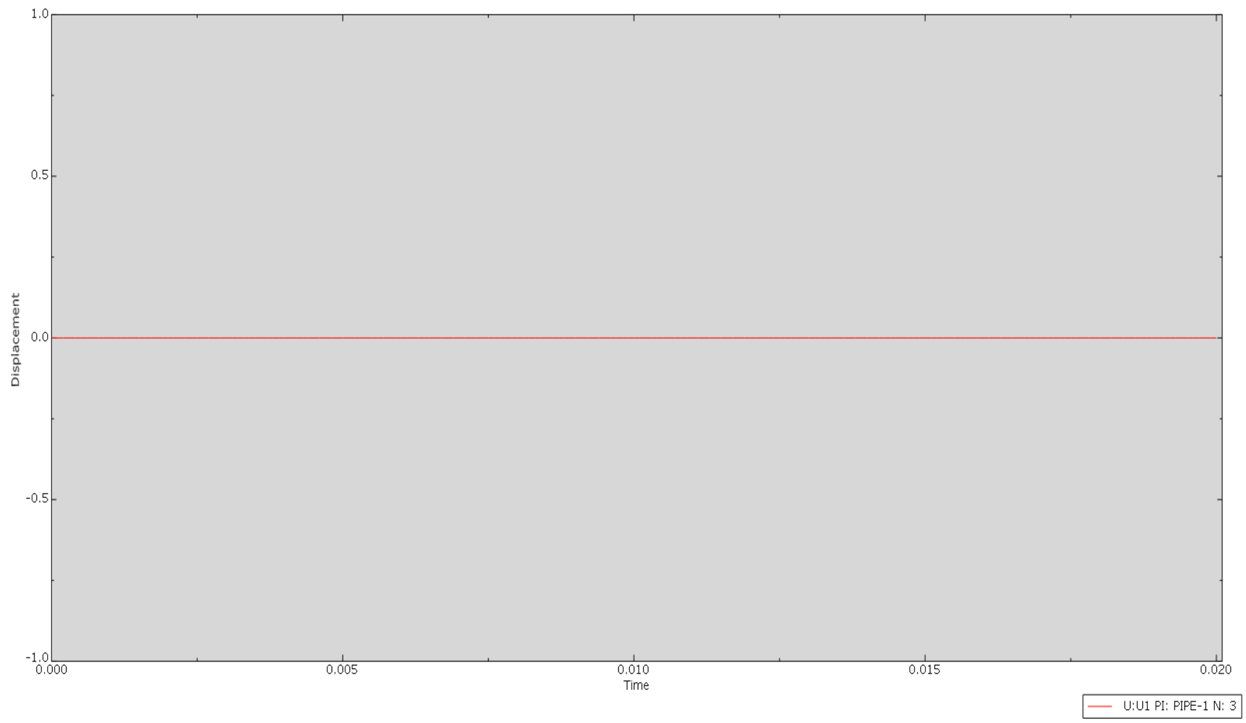


PLOTS

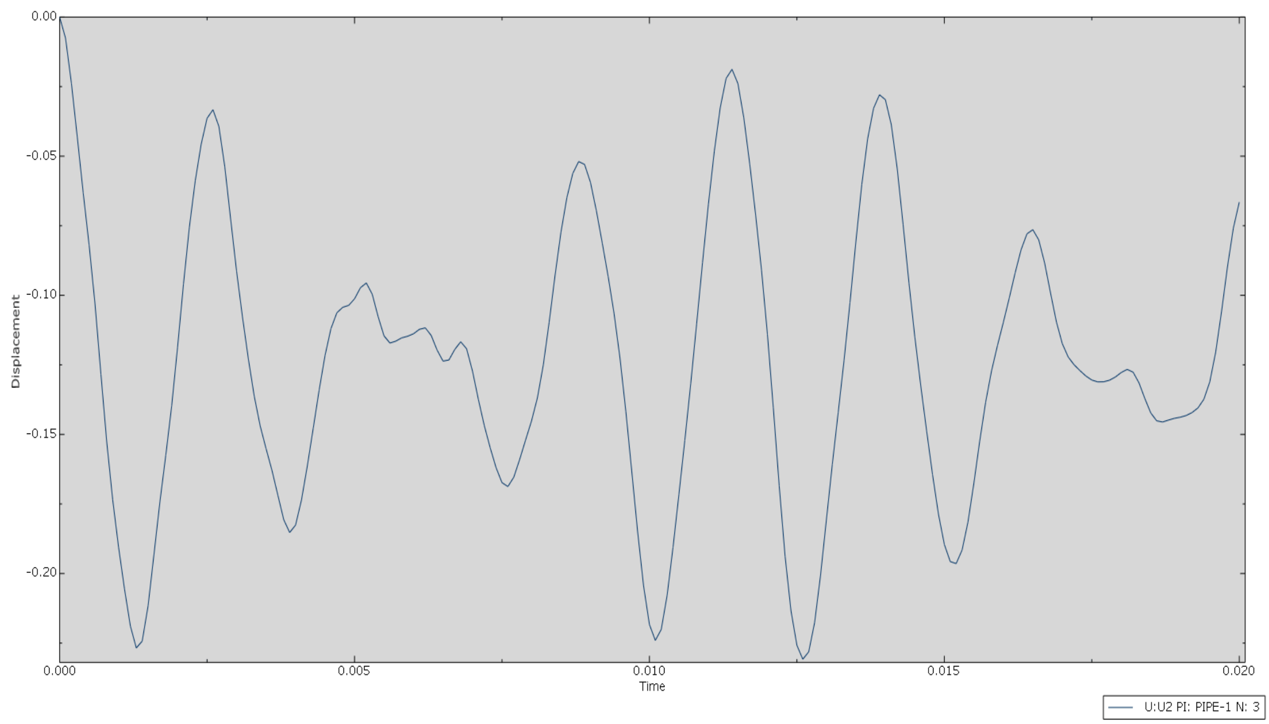
1. Umagnitude



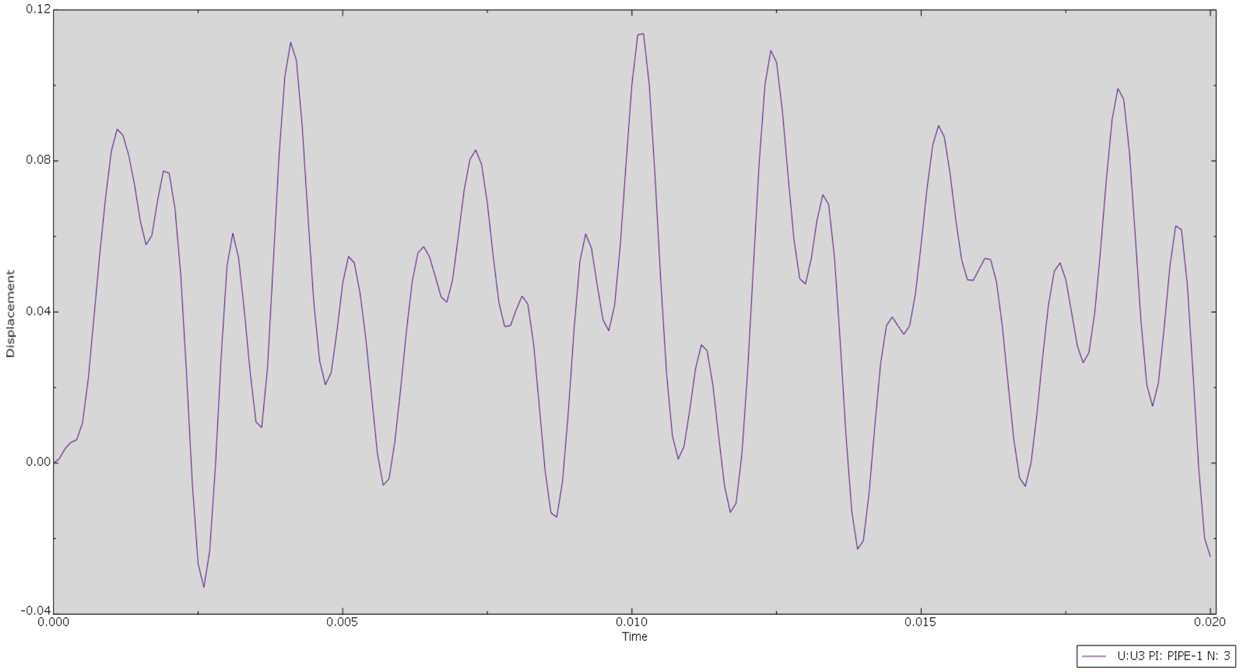
2. U1


1.0
0.5
0.0
-0.5
-1.0
Displacement
0.000
0.005
0.010
0.015
0.020
Time
U:U1 PI: PIPE-1 N: 3

3. U2


0.00
-0.05
-0.10
-0.15
-0.20
Displacement
0.000
0.005
0.010
0.015
0.020
Time
U:U2 PI: PIPE-1 N: 3

4. U4



0.12
0.08
0.04
0.00
-0.04
Displacement
0.000
0.005
0.010
0.015
0.020
Time
U:U3 PI: PIPE-1 N: 3