Civil Engineering
USC

CE 541a - Computer Assignment No. 1

Consider the forced SDOF equation of motion:

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x = f(t) \tag{1}$$

in the special case where $f(t)$ is piecewise linear between equally spaced time points t_1, t_2, t_3, etc.

In this Eq. (1) may be written as

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x = f_i + \frac{\Delta f_i}{\Delta t}(t - t_i) \tag{2}$$

where

$$\Delta t \equiv t_{i+1} - t_i = \text{constant} \tag{3}$$

and

$$\Delta f_i \equiv f_{i+1} - f_i \tag{4}$$

The exact solution of (2) may be expressed as

$$\left\{ \begin{array}{c} x_{i+1} \\ \dot{x}_{i+1} \end{array} \right\} = [A(\zeta,\ \omega,\ \Delta t)] \left\{ \begin{array}{c} x_i \\ \dot{x}_i \end{array} \right\} - [B(\zeta,\ \omega,\ \Delta t)] \left\{ \begin{array}{c} f_i \\ f_{i+1} \end{array} \right\} \tag{5}$$

where the 2×2 matrices A and B have the elements:

$$\left.\begin{array}{rcl} a_{11} & = & e^{-\zeta\omega\Delta t}\left(\frac{\zeta}{\sqrt{1-\zeta^2}} \sin\omega_d\Delta t + \cos\omega_d\Delta t\right) \\ a_{12} & = & \frac{e^{-\zeta\omega\Delta t}}{\omega_d}\sin\omega_d\Delta t \\ a_{21} & = & -\frac{\omega}{\sqrt{1-\zeta^2}} e^{-\zeta\omega\Delta t}\sin\omega_d\Delta t \\ a_{22} & = & e^{-\zeta\omega\Delta t}(\cos\omega_d\Delta t - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d\Delta t) \end{array}\right\} \tag{6}$$

and

$$\left.\begin{array}{rcl} b_{11} & = & e^{-\zeta\omega\Delta t}\left[\left(\frac{2\zeta^2-1}{\omega^2\Delta t} + \frac{\zeta}{\omega}\right)\frac{\sin\omega_d\Delta t}{\omega_d} + \left(\frac{2\zeta}{\omega^3\Delta t} + \frac{1}{\omega^2}\right)\cos\omega_d\Delta t\right] - \frac{2\zeta}{\omega^3\Delta t} \\ b_{12} & = & -e^{-\zeta\omega\Delta t}\left[\left(\frac{2\zeta^2-1}{\omega^2\Delta t}\right)\frac{\sin\omega_d\Delta t}{\omega_d} + \left(\frac{2\zeta}{\omega^3\Delta t}\right)\cos\omega_d\Delta t\right] - \frac{1}{\omega^2} + \frac{2\zeta}{\omega^3\Delta t} \\ b_{21} & = & e^{-\zeta\omega\Delta t}\left[\left(\frac{2\zeta^2-1}{\omega^2\Delta t} + \frac{\zeta}{\omega}\right)\left(\cos\omega_d\Delta t - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d\Delta t\right)\right. \\ & - & \left.\left(\frac{2\zeta}{\omega^3\Delta t} + \frac{1}{\omega^2}\right)(\omega_d\sin\omega_d\Delta t + \zeta\omega\cos\omega_d\Delta t)\right] + \frac{1}{\omega^2\Delta t} \\ b_{22} & = & -e^{-\zeta\omega\Delta t}\left[\frac{2\zeta^2-1}{\omega^2\Delta t}\left(\cos\omega_d\Delta t - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d\Delta t\right)\right. \\ & - & \left.\frac{2\zeta}{\omega^3\Delta t}(\omega_d\sin\omega_d\Delta t + \zeta\omega\cos\omega_d\Delta t)\right] - \frac{1}{\omega^2\Delta t} \end{array}\right\} \tag{7}$$

where

$$\omega_d = \omega\sqrt{1-\zeta^2} \tag{8}$$

Write a computer program (Fortran, C, or an equivalent Matlab or Mathematica module) to generate and print the step-by-step exact solution of (2) for an arbitrary excitation $f(t)$.

The subroutine DHMAT simply takes in ζ, ω, and Δt, and returns the 2×2 A and B matrices. Its name and argument list should be

$$\mathrm{DHMAT(W,\ Z,\ DT,\ A,\ B)}$$

Then use your program to generate the response of a SDOF system to a unit step input

$$f(t) = 1 \ ; \ 0 \leq t \tag{9}$$

to check your results. Choose as a test case the parameters

$$\omega = 2\pi,\ \zeta = 0,\ \Delta t = 0.1,\ TMAX = 3,\ T_n = 1.0,\ \ \underline{x}(0) = \underline{0}$$

Also find the exact solution analytically, and compare answers at several times t with the numerical results. Discuss the errors, if any.

Note that the program requires another subroutine EXCIT which takes in T and DT, and returns values of the excitation f at times $t_i = T$ times $t_{i+1} = T + DT$. The output is a 2×1 vector F in the form

$$F = \left\{ \begin{array}{c} f_i \\ f_{i+1} \end{array} \right\} \tag{10}$$