Notation : $(\_,\_)$ Upper index (Contravar) ; Lower index (Covar)

# P4. Index Notation

GradQM.BSS.

(a) i. $x_\mu x^\nu = T^{\mu\nu}$ . (X) as $(1,1) \neq (2,0)$

ii. $x^\mu y^\alpha = x^\nu T^\mu{}_\nu$ . (x) as $(2,0) \neq (1,0)$

→ $\nu$ is summed over & we have only one free index on RHS.

iii. $x^\beta = \Lambda^\beta{}_\alpha \Lambda^\rho{}_\sigma \gamma^\sigma y_\rho z^\alpha$ . (✓) as $(1,0) = (1,0)$

→ All indices on RHS except $\beta$ (the same free index on LHS) are summed over.

iv. $G^{\mu\nu} = 8\pi T^{\mu\nu}$ . (✓) as $(2,0) = (2,0)$

→ Also, this is the EFE.

v. $\Lambda^{\nu\nu} = \eta^{\nu\mu} \Lambda^\mu{}_\nu$ . (x) as LHS is violating Lorentz index rules by having the same index upstairs twice.

(b) i. $x^\mu x_\mu = \eta_{\mu\nu} x^\mu x^\nu = \eta_{00} x^0 x^0 + \eta_{ii}(x^i x^i)$

$= (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$

$= 4 - 25 - 4 - 1 \quad = -26$

ii. $y_\nu x^\mu y^\nu = \eta_{\nu\alpha} y^\alpha x^\mu y^\nu = (\eta_{\alpha\nu} y^\alpha y^\nu) x^\mu$

$= ((y^0)^2 - (y^1)^2 - (y^2)^2 - (y^3)^2) x^\mu$

$= (25 - 1 - 4 - 4) x^\mu \quad = 16 x^\mu$

|| same as this bracket.

iii. $y_\nu x_\mu y^\nu = \eta_{\mu\alpha} x^\alpha (y_\nu y^\nu) = 16\, \eta_{\mu\alpha} x^\alpha = 16(\eta_{\mu 0} x^0 + \eta_{\mu 1} x^1 + \eta_{\mu 2} x^2 + \eta_{\mu 3} x^3)$

$= 16(2\eta_{\mu 0} + 5\eta_{\mu 1} - 2\eta_{\mu 2} + \eta_{\mu 3})$