Problem 3

Solution 3.(a)

We have placed ourselves in the rest frame of particle A, which has a spin $\frac{3}{2}$. It has disintegrated into two particles B and C, which have spin $\frac{1}{2}$ and 0 respectively. We use conservation of angular momentum here.

$$j_i = j_f \tag{86}$$

where the i, f subscripts correspond to initial and final.
It has been given that,

$$j_i = \frac{3}{2} \tag{87}$$

For the final angular momentum we have,

$$j_f = l_f + s_f = l_f + \left(\pm\frac{1}{2} + 0\right) = \frac{3}{2} \tag{88}$$

This gives us the relative final angular momentum,

$$l_f = \frac{3}{2} \pm \frac{1}{2} = \{1, 2\} \tag{89}$$

depending if particle B has a spin up or down.

Solution 3.(b)

All the possible states for $\left|l, m_l; \frac{1}{2}, m_s\right\rangle$,

- For $l = 1$ are,

$$\left|1, 1; \frac{1}{2}, \frac{1}{2}\right\rangle, \quad \left|1, 1; \frac{1}{2}, -\frac{1}{2}\right\rangle \tag{90}$$
$$\left|1, 0; \frac{1}{2}, \frac{1}{2}\right\rangle, \quad \left|1, 0; \frac{1}{2}, -\frac{1}{2}\right\rangle \tag{91}$$
$$\left|1, -1; \frac{1}{2}, \frac{1}{2}\right\rangle, \quad \left|1, -1; \frac{1}{2}, -\frac{1}{2}\right\rangle \tag{92}$$

- For $l = 2$ are,

$$\left|2, 2; \frac{1}{2}, \frac{1}{2}\right\rangle, \quad \left|2, 2; \frac{1}{2}, -\frac{1}{2}\right\rangle \tag{93}$$
$$\left|2, 1; \frac{1}{2}, \frac{1}{2}\right\rangle, \quad \left|2, 1; \frac{1}{2}, -\frac{1}{2}\right\rangle \tag{94}$$
$$\left|2, 0; \frac{1}{2}, \frac{1}{2}\right\rangle, \quad \left|2, 0; \frac{1}{2}, -\frac{1}{2}\right\rangle \tag{95}$$
$$\left|2, -1; \frac{1}{2}, \frac{1}{2}\right\rangle, \quad \left|1, -1; \frac{1}{2}, -\frac{1}{2}\right\rangle \tag{96}$$

Solution 3.(c)

From the previous parts of the problem we know that $l = 1, 2$. It is also given that $(-1)^l$ determines the parity of the state. For $l = 1$ we get parity to be -1 i.e. odd. Similarly, if we use $l = 2$ we get parity to be $(-1)^2 = 1$ i.e. even.

Solution 4.(d)

We have the particle A prepared in,

$$\left|s_A = \frac{3}{2}, m_A = \frac{1}{2}\right\rangle \tag{97}$$

We can change into a $|l, m_l; s, m_s\rangle$ basis by reading the right coefficients from the Clebsh-Gordon coefficients table,

$$\left|\frac{3}{2}, \frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|1, 1; \frac{1}{2}, -\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|1, 0; \frac{1}{2}, \frac{1}{2}\right\rangle \tag{98}$$

From here we can conclude the probability that the particle B is in spin state $\left|s_B = \frac{1}{2}, m_B = \frac{1}{2}\right\rangle$ is given by the squared coefficient in front of the second term in the RHS of the equation above, which is,

$$P\left(\left|s_B = \frac{1}{2}, m_B = \frac{1}{2}\right\rangle\right) = \frac{2}{3} \tag{99}$$