* Algebra done in Mathematica.

We have,

$$T^{(2)}_{\pm 2} = \frac{1}{2}(x \pm iy)^2 = \frac{1}{2}(x^2 - y^2 \pm 2ixy)$$

$$T^{(2)}_{\pm 1} = \sqrt{2}\, r_{\pm} z = \mp z(x \pm iy) = \mp xz - iyz$$

$$T^{(2)}_0 = \frac{1}{\sqrt{6}}(2r_0 r_0 + 2r_+ r_-) = \frac{1}{\sqrt{6}}(2z^2 - x^2 - y^2)$$

Using these we have,

$$(x^2 - y^2) = T_2 + T_{-2}$$

$$xy = \frac{-i}{2}(T_2 - T_{-2})$$

$$xz = -\frac{1}{2}(T_1 - T_{-1})$$

$$\therefore\ 2z^2 - x^2 - y^2 = 3z^2 - r^2 = \sqrt{6}\, T_0^2$$

$$Q = \sqrt{6}\, e \langle \alpha, j, m=j | T_0^2 | \alpha, j, m=j \rangle$$

$$= \sqrt{6}\, e \langle \alpha, j \| T^2 \| \alpha, j \rangle \times \langle j, 2; j\, 0 | j, 2; j, j \rangle$$

$$e \langle \alpha, j, m' | x^2 - y^2 | \alpha, j, m=j \rangle = e \langle \alpha, j, m' | T_2^2 - T_{-2}^2 | \alpha, j, m=j \rangle$$

But, $m' \leq j$, so matrix element of T_2^2 gives 0.

$$\Leftrightarrow e \langle \alpha, j, m' | T_{-2}^2 | \alpha, j, m=j \rangle = -e \langle \alpha, j \| T^2 \| \alpha, j \rangle \times \langle j, 2; j, m' | j, 2; j, -2 \rangle$$

$$= -\frac{Q}{\sqrt{6}} \frac{\langle j, 2; j, m | j, 2; j, -2 \rangle}{\langle j, 2; j, 0 | j, 2; j, j \rangle}$$

How'd we get T_2^2, T_1^2, T_0^2?

$$T_2^2 = \langle 1,1;1,1|2,2\rangle T_1^1 T_1^1$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 (X+iY)^2 = \frac{1}{2}\left(X^2 - Y^2 + 2iXY\right)$$

$$T_1^2 = \langle 1,0;1,1|2,1\rangle T_0^1 T_1^1$$

$$+ \langle 1,1;1,0|2,1\rangle T_1^1 T_0^1$$

$$= \frac{2}{\sqrt{2}} Z\left(\frac{-1}{\sqrt{2}}(X+iY)\right) = -Z(X+iY)$$

~~$T_0^2 \Rightarrow$~~ and so on & so forth.