2. The explicit form of Θ for spin $\frac{1}{2}$ particle is given by

$$\Theta = K e^{-i\pi S_y/\hbar} = K e^{-i\pi \frac{\hbar}{2}\sigma_y/\hbar} = K e^{-i\frac{\pi}{2}\sigma_y}$$

$$= K\left(\cos\left(\tfrac{\pi}{2}\sigma_y\right) - i\sin\left(\tfrac{\pi}{2}\sigma_y\right)\right)$$

$$= K \sum_{j=0}^{\infty} \frac{\left(-i\frac{\pi}{2}\sigma_y\right)^j}{j!}$$

$$= K\left(1 + \left(-i\frac{\pi}{2}\sigma_y\right) + \left(-i\frac{\pi}{2}\sigma_y\right)^2\frac{1}{2!} + \left(-i\frac{\pi}{2}\sigma_y\right)^3\frac{1}{3!} + \dots \right.$$
$$\left. + \left(-i\frac{\pi}{2}\sigma_y\right)^4\frac{1}{4!} + \left(-i\frac{\pi}{2}\sigma_y\right)^5\frac{1}{5!} + \dots\right)$$

repeated terms as $(\sigma_y)^2 = I$ & $(i)^5 = i$

$$= K\left(1 - i\sigma_y\frac{\pi}{2} - \frac{(\pi/2)^2}{2!} + i\sigma_y\frac{(\pi/2)^3}{3!} + \frac{(\pi/2)^4}{4!} + \left(-i\sigma_y\frac{(\pi/2)^5}{5!}\right) + \dots\right)$$

$$= K\left[\left(1 - \frac{(\pi/2)^2}{2!} + \frac{(\pi/2)^4}{4!} - \frac{(\pi/2)^6}{6!} + \dots\right) - i\sigma_y\left(\frac{\pi}{2} - \frac{(\pi/2)^3}{3!} + \frac{(\pi/2)^5}{5!} - \dots + \dots\right)\right]$$

$$= K\left[\cos(\pi/2) - i\sigma_y \sin(\pi/2)\right]$$

$$= K(-i\sigma_y)$$

$$= -Ki\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = -K\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = K\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Act with this on our ~~matrix~~ state : $\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

$$\Theta \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = K \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = K \begin{bmatrix} -\beta \\ \alpha \end{bmatrix} = \begin{bmatrix} -\beta^* \\ \alpha^* \end{bmatrix}$$

$$\Theta^2 \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \Theta \begin{bmatrix} -\beta^* \\ \alpha^* \end{bmatrix} = K \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -\beta^* \\ \alpha^* \end{bmatrix} = K \begin{bmatrix} -\alpha^* \\ -\beta^* \end{bmatrix} = - \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$\Theta^2 |\Psi\rangle = -|\Psi\rangle$, giving us what we expected for a fermion.