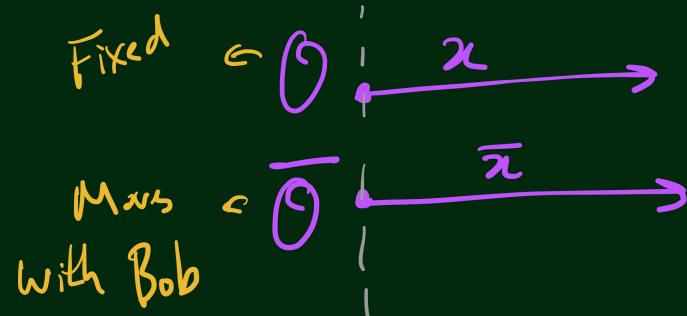
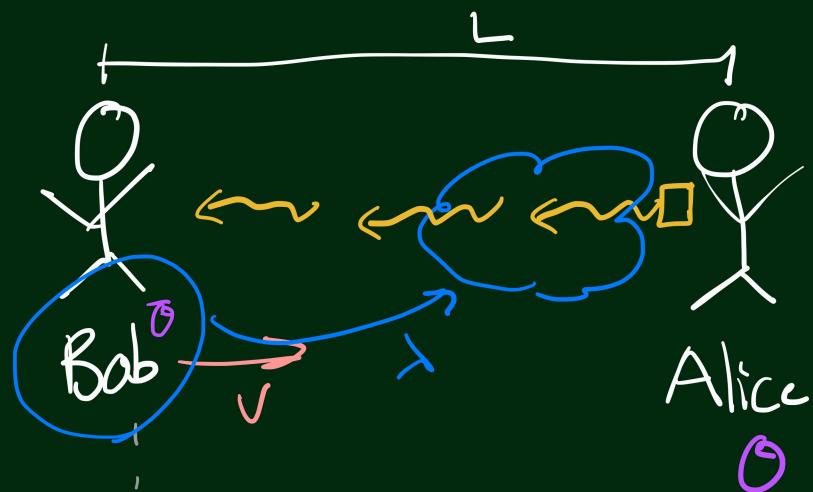


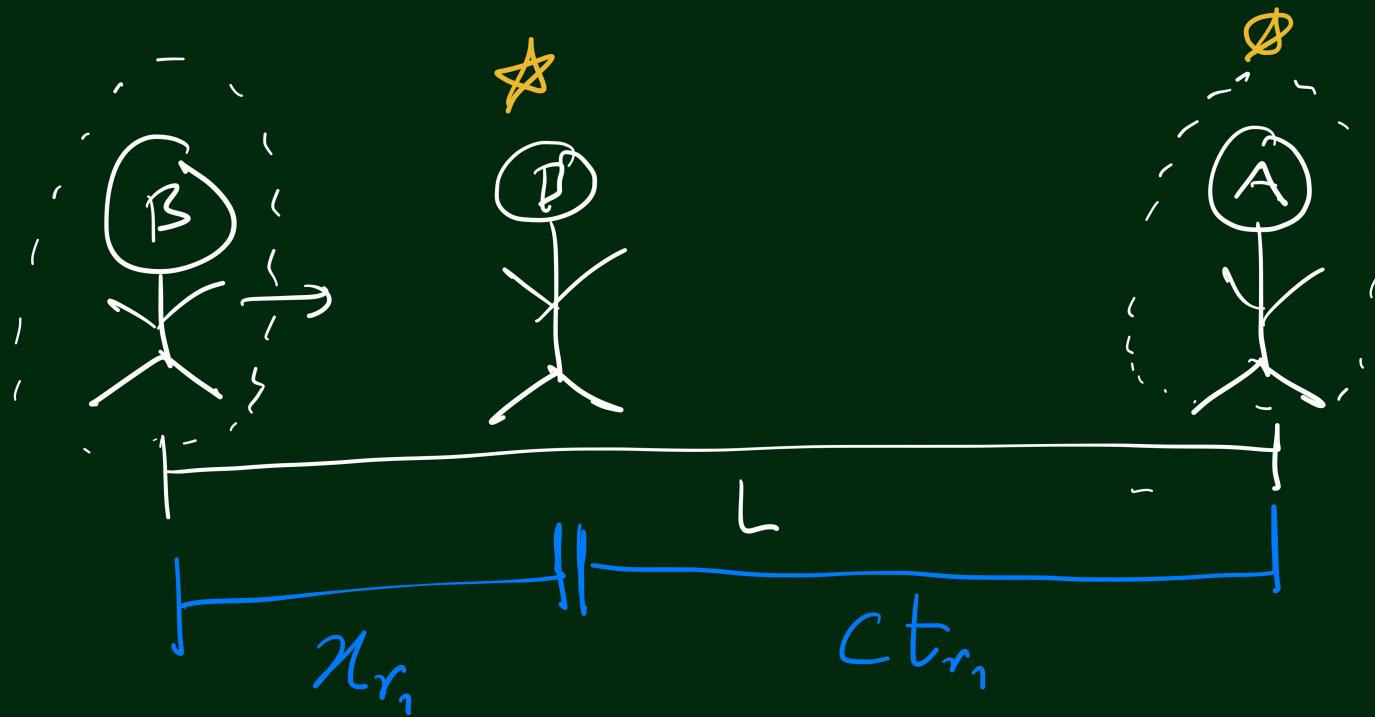
Exercise 1.1: Doppler effect in Special Relativity

(a) Alice is pointing a laser beam to Bob, who is moving towards Alice with relative velocity v . Obtain the formulae that relate the frequency, period and wavelength measured by Bob making use of the Lorentz transformations for the coordinates. Do not use the concept of energy and momentum of the photon (as in Sec. 1.12 of the lecture notes). Do this exercise applying only kinematic arguments. (1.5 pt)



- Emission
 - S₁ event of em signal 1 : $(t_{e_1}, x_{e_1}) = (0, L)$
 - S₁ event of em signal 2 : $(t_{e_2}, x_{e_2}) = (\tau, L)$

- Rec
 - S₁ ————— Signal 1 : (t_{r_1}, x_{r_1}) S₁
coordinates
at which
Bob will
receive the em
 - S₁ ————— 2 : (t_{r_2}, x_{r_2})



(From O's perspective)

$$x_{r_1} = \left(v t_{r_1} = L - c t_{r_1} \right)$$

$$v t_{r_1} = L - c t_{r_1} \Rightarrow v t_{r_1} + c t_{r_1} = L \Rightarrow$$

$$t_{r_1} = \frac{L}{v+c}$$

Coordinate: ST event
of the 1st
rec. signal,

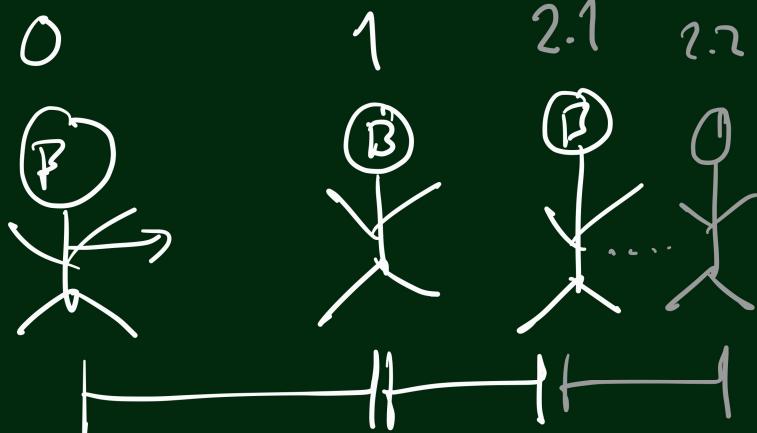
$$x_{r_1} = v t_{r_1} = \frac{v L}{v+c}$$

Next step : Calculate S7 event of 2nd rec. signal.

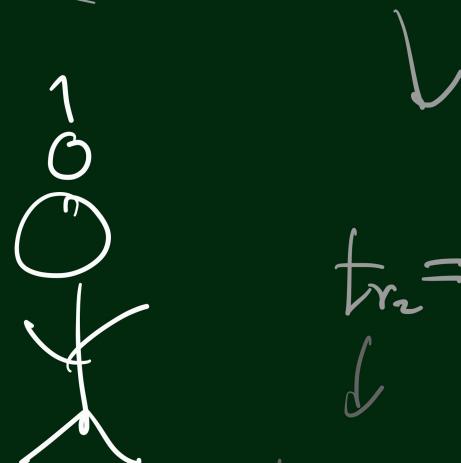
$$t_{r_2} = \frac{L + ct}{(v+c)}$$

$$x_{r_2} = \frac{v}{(v+c)} (L + ct)$$

$$x_{r_2} = vT + v\Delta t = v(T + \Delta t) = (L - c\Delta t)$$



$$x_{r_1} = vt_{r_1}$$



$$t_{r_2} = \frac{L + ct}{c + v}$$

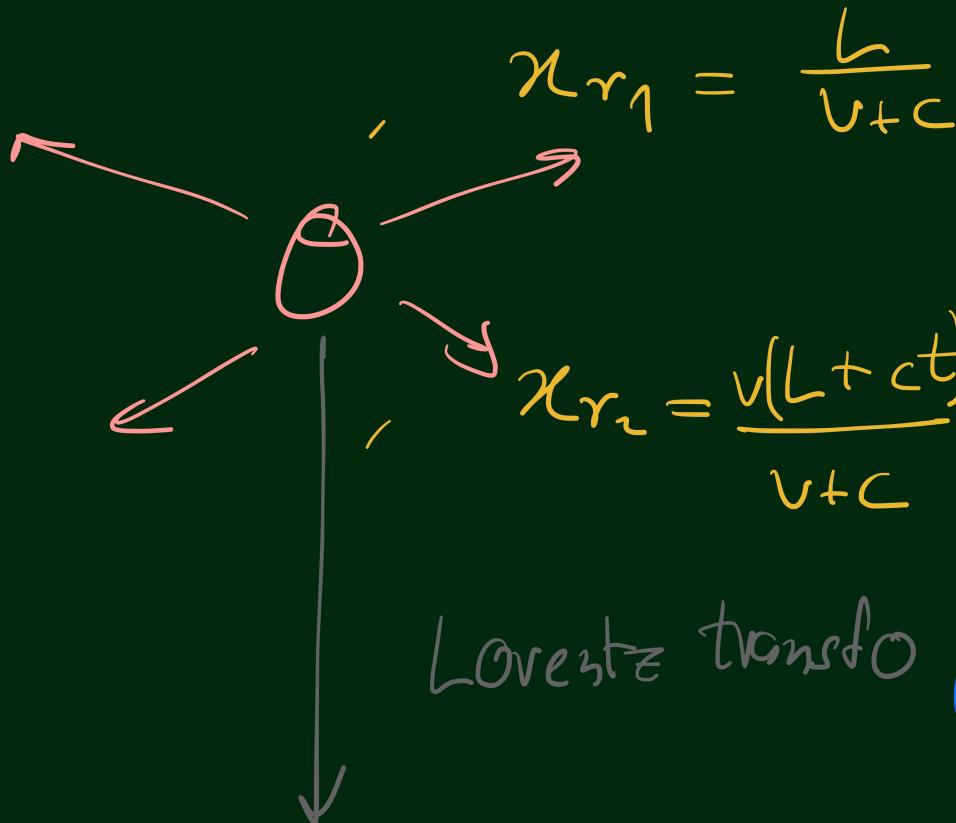
$$t_{r_2} = T + \Delta t$$

$$vT + v\Delta t = L - c\Delta t \Rightarrow v\Delta t + c\Delta t = L - vt \Rightarrow \Delta t(v + c) = L - vt$$

$$\Delta t = \frac{L - vt}{v + c}$$

$$t_{r_1} = \frac{L}{v+c}$$

$$t_{r_2} = \frac{L+ct}{v+c}$$



$$\bar{T} = \bar{t}_{r_2} - \bar{t}_{r_1} = \gamma \left(T - \frac{v}{c^2} X \right)$$

\bar{O}

$t_{r_2} - t_{r_1}$ $x_{r_2} - x_{r_1}$

$$= \gamma \left[(t_{r_2} - t_{r_1}) - \frac{v}{c^2} (x_{r_2} - x_{r_1}) \right]$$

$$= \gamma \left[t_{r_2} - t_{r_1} - \frac{v}{c^2} n_{r_2} + \frac{v}{c^2} n_{r_1} \right] \rightarrow n_{r_1} = v t_{r_1}$$

$\Rightarrow n_{r_2} = v t_{r_2}$

$$= \gamma \left[t_{r_2} - t_{r_1} - \frac{v}{c^2} v t_{r_2} + \frac{v}{c^2} v t_{r_1} \right]$$

$$= \gamma \left[(t_{r_2} - t_{r_1}) - \frac{v^2}{c^2} t_{r_2} + \frac{v^2}{c^2} t_{r_1} \right]$$

$$= \gamma \left[(t_{r_2} - t_{r_1}) - (t_{r_2} - t_{r_1}) \frac{v^2}{c^2} \right]$$

$$= \gamma (t_{r_2} - t_{r_1}) \left(1 - \frac{v^2}{c^2} \right) = \frac{1}{\left(1 - \frac{v^2}{c^2} \right)^{\gamma_2}} (t_{r_2} - t_{r_1}) \left(1 - \frac{v^2}{c^2} \right)^{\gamma}$$

$$= \left(1 - \frac{v^2}{c^2}\right)^{1/2} (t_{r_2} - t_{r_1})$$

~~$\frac{CT}{V+C} + \frac{L-K}{V+C}$~~

$$\bar{\tau} = \bar{t}_{r_2} - \bar{t}_{r_1} = \sqrt{1 - \frac{v^2}{c^2}} \overbrace{(t_{r_2} - t_{r_1})}^{\sim}$$

$$= \sqrt{1 - \frac{v^2}{c^2}} \frac{CT}{V+C} = \boxed{\bar{\tau} \sqrt{1 - \frac{v^2}{c^2}} \frac{C}{V+C}}$$

Chang. $C \rightarrow V$

$$= \bar{\tau} \sqrt{1 - \frac{v^2}{c^2}} \frac{1}{1 + \frac{V}{C}} = \bar{\tau} \sqrt{\frac{(1 - \frac{V}{C})(1 + \cancel{\frac{V}{C}})}{(1 + \frac{V}{C})^2}}$$

$$= \boxed{\bar{\tau} \sqrt{\frac{(1 - \frac{V}{C})}{(1 + \frac{V}{C})}}}$$

(b) Repeat the previous exercise, but considering that Alice sends a sound wave with velocity v_s in her reference frame \mathcal{O} , instead of an electromagnetic signal. (1.5 pt)

EM Signal

$$t_{r_1} = \frac{L}{v+c} ; \quad t_{r_2} = \frac{L+cT}{v+c}$$

Time dilation

$$\bar{\tau} = \tau \sqrt{1 - \frac{v^2}{c^2}} \quad \frac{c}{c+v}$$

Sound

$$t_{r_1} = \frac{L}{v+v_s} ; \quad t_{r_2} = \frac{L+v_s T}{v+v_s}$$

Time dilation

$$\bar{\tau} = \tau \sqrt{1 - \frac{v^2}{c^2}} \frac{v_s}{v_s + v}$$

$$\bar{v} = v \frac{1 + \frac{v}{v_s}}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow$$

• Goal: Compute $\overline{\lambda}$ (measured by Bob) $\overline{\theta} : \overline{v}_S$

$$\overline{v}_S = \frac{v + v_S}{1 + \frac{vv_S}{c^2}} = \frac{\overline{\lambda}}{\overline{\tau}} \rightarrow \boxed{\overline{\lambda} = \frac{\overline{\tau}(v + v_S)}{1 + \frac{vv_S}{c^2}}}$$

$$\overline{\lambda} = \lambda \sqrt{1 - \frac{v^2}{c^2}} \Rightarrow \overline{v} = v \frac{1 + \frac{v}{v_S}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- (c) Consider the results obtained in 1.1a and 1.1b. Do you recover the expected results in the non-relativistic limit? (1 pt)

Speed = freq wavelength

$$EM \text{ Signal} \Rightarrow \bar{\lambda} = \lambda(1)$$

$$\text{Sound: } \bar{\lambda} = \lambda, \quad \bar{v} = v \left(1 + \frac{v}{v_s} \right)$$