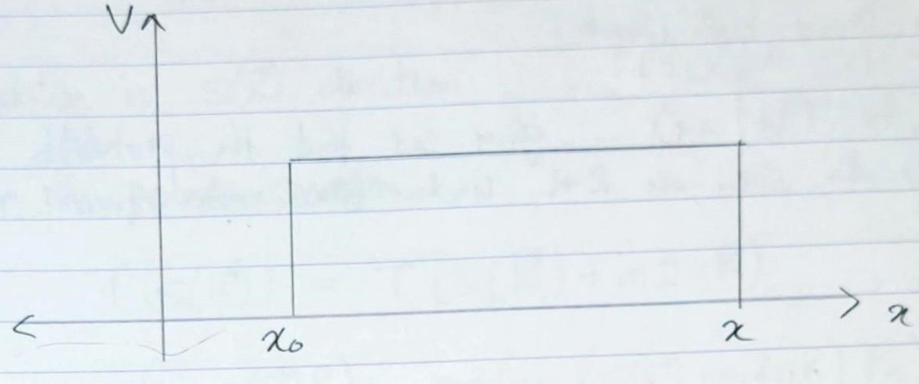
GRADQM : PS3

P3. Assume a symmetric potential like the following:



To show: $\langle x,t|x_0,t_0\rangle = \langle x_0,t_0|x,t\rangle$

$$\cancel{\langle x,t|x_0,t_0\rangle = \langle x|\mathcal{U}^\dagger(t,t_0)\,\mathcal{U}(t_0,t_0)|x_0\rangle}$$
$$\cancel{= \langle x|\mathcal{U}^\dagger(t,t_0)\,\mathcal{U}(t_0,0)\,T(x_0,x_0)|x\rangle}$$

Start w, $\langle x,t|x_0,t_0\rangle = \langle x,t_0|\mathcal{U}^\dagger|x_0,t_0\rangle$, $\tilde{\mathcal{U}} = \mathcal{U}(t_0,0)$

$$= \langle x|\tilde{\mathcal{U}}\,\mathcal{U}^\dagger\,\mathcal{U}|x_0,t_0\rangle$$
$$\mathbb{1}$$

$$= \langle x|\mathcal{U}^\dagger|x_0\rangle = \langle x_0|T^\dagger\mathcal{U}^\dagger|x_0\rangle$$

$$\mathcal{U}^\dagger = e^{\frac{-i\Delta t H}{\hbar}} \quad \& \quad T = e^{\frac{-i\Delta x\, p_x}{\hbar}}$$

Check the symmetry of this $T^\dagger\mathcal{U}^\dagger$ operator under Θ.

i.e $[\Theta^\dagger, T^\dagger\mathcal{U}^\dagger]$ or $[\Theta, T\mathcal{U}]$,

$$[\Theta, T\mathcal{U}] = T[\Theta,\mathcal{U}] + [\Theta,T]\mathcal{U}$$

$$\Theta\mathcal{U} = \Theta\exp\left(\frac{-i\Delta t H}{\hbar}\right) \underset{\substack{i\to -i \\ \Delta t\to-\Delta t,\ H\to H}}{=} \Theta\exp\left(\frac{i(-\Delta t)H}{\hbar}\right) = \exp\left(\frac{-i\Delta t H}{\hbar}\right) \leftarrow$$

$i \to -i$
$\Delta x \to \Delta x$
$p_x \to -p_x$

$$\Theta T = \Theta \exp\left(\frac{-i \Delta x\, p_x}{\hbar}\right) = \Theta \exp\left(\frac{i \Delta x\, (-p_x)}{\hbar}\right) = \exp\left(\frac{-i \Delta x\, p_x}{\hbar}\right)$$

which means both commute.

$\therefore$ $[\Theta, T\mathcal{U}] = 0$, gives us that the probability from $L \to R$ is the same as $R \to L$ under time ~~reve~~ reversal.