$1 J = 6.24 \times 10^{18}$ eV
$= 6.24 \times 10^{9}$ GeV $\Rightarrow$ 1 GeV $= 1.602 \times 10^{-10}$ J

GradQM. PS4.

P3. (a) SI $\rightarrow$ Natural

1) $\hbar = 1.05 \times 10^{-34} [J\ s] = 1.05 \times 10^{-34} (6.24 \times 10^{9} [GeV]) [s]$

$= 6.55 \times 10^{-25} [GeV\ s]$ (✓)

2) $\hbar c = (1.05 \times 10^{-34}) [J \cancel{s}] \ (3 \times 10^{8}) [m \cancel{s}^{-1}]$

$= (1.05 \times 3 \times 10^{-26}) \ [J\ m]$

$= (1.05 \times 3 \times 10^{-26}) (6.24 \times 10^{9}) [GeV\ m]$

$= 1.96 \times 10^{-16} \ [GeV\ m]$ (✓)

3) $k_B = 1.38 \times 10^{-23} [J\ K^{-1}]$

$= (1.38 \times 10^{-23}) (6.24 \times 10^{9}) \ [GeV\ K^{-1}]$

$= 8.6112 \times 10^{-14} \ [GeV\ K^{-1}]$ (✓)

These three are pivotal relations in going from ~~to~~ SI $\leftrightarrow$ Natural.
As,

$\hbar = 1$ gives (From 1), • $1[s] = \left(\frac{1}{6.55 \times 10^{-25}}\right) [GeV^{-1}] = 1.53 \times 10^{24} [GeV^{-1}]$ (✓)

+

$c = 1$ gives (From 2) • $1[m] = \frac{1}{1.96 \times 10^{-16}} [GeV^{-1}] = 5.10 \times 10^{15} [GeV^{-1}]$ (✓)

$k_B = 1$ gives • $1[K] = \frac{1}{8.6112 \times 10^{-14}} [GeV] = 1.16 \times 10^{13} [GeV]$ (✓)

(b) Temp of solar core = $\rightarrow$ $T_{\odot c} = 15 \times 10^{6}\ [K]$

$= 1.5 \times 10^{7}\ [K]$

$$T_{\odot c} = (1.5 \times 10^{7}) \times (1.16 \times 10^{13}\ \mathrm{GeV})$$

$$= 1.74 \times 10^{20}\ [\mathrm{GeV}]$$

$$= 1.74 \times 10^{20} \times 10^{6}\ [\mathrm{GeV} \times 10^{-6}]$$

$$= 1.74 \times 10^{26}\ [\mathrm{keV}]$$

(c) $T = 40\ \mathrm{mK} = 40 \times 10^{-3}\ K = 4 \times 10^{-2}\ K$

$$= (4 \times 10^{-2})(1.16 \times 10^{13}\ \mathrm{GeV})$$

$$= 4.64 \times 10^{11}\ [\mathrm{GeV}]$$

$$= 4.64 \times 10^{11} \times 10^{9}\ [\mathrm{GeV} \times 10^{-9}]$$

$$= 4.64 \times 10^{20}\ [\mathrm{eV}]$$

(d) $t = 2.2 \times 10^{-6}\ s = 2.2 \times 10^{-6} \times (1.53 \times 10^{24})\ [\mathrm{GeV}]^{-1}$

$$= 3.36 \times 10^{18}$$

(e) $T = 2.7\ K$

$$E = aT^{4} = (4.72 \times 10^{-9}\ [\mathrm{MeV\ cm^{-3}}\ \cancel{K^{-4}}])\ (2.7\ [\cancel{K}])^{4}$$

$$= 4.72 \times 10^{-9}\ [\mathrm{MeV\ m^{-3}}] \times (100)^{3}$$

$$= (2.7)^4 (4.72 \times 10^{-9}) (10^6) (10^6) \quad [eV\ m^{-3}]$$

$$= 2.5 \times 10^5 \quad [eV^4] \left(\frac{1}{5.1 \times 10^{15}}\right)^3 [GeV]^3$$

$$= 2.5 \times 10^5 \times (10^9)^3 \left(\frac{1}{5.1 \times 10^{15}}\right)^3 \quad [eV^4]$$

$$= 1.884 \times \cancel{10^{5}}\ 10^{-15} \quad [eV^4]$$

(f) $\rho = \frac{3H_0^2}{(8\pi G_N)}$ in eV^4 & $\frac{m_p}{m^3}$, $H_0 = \frac{72\ km}{(s\ kpc)}$

$$= \left(\frac{3}{8\pi}\right) \frac{(72)^2}{(6.67 \times 10^{-11})} \frac{[km\ s^{-1}\ kpc^{-1}]^2}{[kg\ m\ s^{-2}\ m^2\ kg^{-2}]}$$

$$= 9.28 \times 10^{12} \left[\frac{km^2}{\cancel{s^2}} \frac{\cancel{s^2}\ m^{\cancel{2}}\ kg^{\cancel{2}}}{\cancel{kg}\ \cancel{m}} \frac{1}{(kpc)^2}\right]$$

$$= 9.28 \times 10^{12} \frac{km^2\ m\ kg}{(kpc)^2}$$

$$= \frac{9.28 \times 10^{12}}{(3.086 \times 10^{16})^2} \frac{\cancel{km^2}\ m\ kg}{\cancel{km^2}}$$

$=$

$1 m_p = 1.6 \times 10^{-27}\ kg$

(f) $\rho = \frac{3H_0^2}{8\pi G_N} = 9 \times 10^{-27} \left[\frac{kg}{m^3}\right]$

$$= 9 \times 10^{-27} \left(\frac{1}{1.78 \times 10^{-27}}\right) [GeV] \left(5.10 \times 10^{15}\right)^{-1} [GeV^3]$$

$$= \cancel{4.}\ 9.91 \times 10^{20} \quad [eV]^4$$

$$= 9 \times 10^{-27} \times \frac{1}{(1.6 \times 10^{-27})} \left[\frac{m_p}{m^3}\right] = 5.625 \left[\frac{m_p}{m^3}\right]$$