

# FEATURE BASED ADAPTIVE MOTION MODEL

by

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## Abstract

We present a method to learn and adapt the motion model. The motion model can be influenced by environmental properties and is a crucial part of the navigation system. Examples are the drift that is accounted in the motion model can be different for carpets and tiles. The AUV can have a change in their motion model when they are moving from fresh water to sea water. Our algorithm is based on the Expectation Maximization Framework which help us to learn the right parameters for the model. The Expectation Step is completed by particle filtering and smoothing. The Maximization step involves finding the parameters for the model. We use side sonar images to extract landmarks which can gives us position estimates to help us evolve our motion model. This leads to a better position estimate of the robot. We found that the our learning motion model adapted well to the change in parameters and had low localization error compared to static motion model. This algorithm eliminates the need for laborious and hand-tuning calibration process. The main significance of learning the motion model is to have better navigation algorithms and also its a step towards robots being able to adapt to environment without human intervention. We validate our approach by recovering a good motion model when the density, temperature of the water is changed in our simulations.

## Acknowledgements

Thanks to all the little people who make me look tall.

# Chapter 1

## Introduction

### 1.1 Motivation

The core of human environment interaction is the ability of the person to know its position in the surrounding environment. The process of estimating the robot's position and orientation in the world is termed as Localization [15]. It is the answer to the question Where am I?. The current location can be determined by Global Positioning Systems, landmarks, maps etc. A common approach is to provide a prior map of the environment and a robot with help of sensors perceives the world and localizes itself in it. Another way to estimate the position of a robot is by knowing how a robot moves in the world . For example by knowing at what velocity a robot is moving we can predict its future location. There are various algorithms such as Particle Filter [6], Kalman Filter [9] etc. to estimate the state of a robot. The way the robot moves and senses the world are captured in motion and sensor models. These models are basic blocks to various algorithms such as path planning [11], Simultaneous Localization and Mapping (SLAM) [15] [7] etc. which require an accurate estimate of robot's position.

In order to account for the uncertainty in the environment Sebastian Thrun [15] represented the models probabilistically. Eliazar and Parr [5] pointed out that the essential input are the parameters to these models. The process of determining the parameters to a kinematic model is termed as calibration. Generally the robot is calibrated at the start of the experiment and the parameter values are not changed throughout the experiment. In practice wherever there is a significant change in the environment robots are manually re calibrated during the experiment. In most of the cases it is not possible to recalibrate while the robot is in operation. As our environments are dynamic our models need to be adapt to them. Eliazar and Parr [5] proposed an algorithm to learn parameters of a motion model for land robots. Teddy Yapp [16] took the work further and learned parameters for motion and sensor models

for land robots. Both of their algorithms used Expectation Maximization framework to learn right parameters for models.

For my thesis I specifically deal with water environments such as oceans, rivers etc. which are highly dynamic and will lead to changes in motion model for Autonomous Underwater Vehicle. Navigation in AUV relies on Inertial Navigation System(INS) which gives an estimate of velocity, position and orientation. All INS systems suffer from drift i.e. small errors in measurement of acceleration and angular velocity are integrated into progressively into larger error. Hegrenaes et al [?] pointed out that systems such as Doppler Velocity Log(DVL), surface GPS etc are used to compensate for the drift but they are situations where these systems fail or readings are discarded due to poor quality. To solve this particular problem they used velocity estimate from a static motion model to aid INS systems. This application shows the importance of accurate motion models for navigation. In my thesis I propose an online system for an AUV to adapt its motion model to the dynamic environment as well as automate the calibration process.

In Chapter 3 I demonstrate how the parameters of the motion model can be estimated during its normal operation. The algorithm used machine learning methods to learn the parameters as well as eliminate the necessity of identifying the parameters through a manual laborious calibration process. We use Expectation Maximization [2] framework to estimate the right parameters for the model. It is an unsupervised machine learning technique primarily used to estimate parameters. It alternates between the Expectation Step which creates an expectation of the log-likelihood using the current estimate of the parameters and the Maximization Step which computes parameters maximizing the expected log-likelihood found in the E step.

To calculate the likely trajectory of robot particle filtering [12] [1] and smoothing [3] [4] is performed using the sensor data collected during robot's normal operation. I choose particle filters because they can model non-linear transformations as well as they have no restrictions in model. Particle smoothing was performed because Russel and Norving [13] pointed out the state of the system is better estimated by smoothing as it incorporates more information than just filtering. I then calculate the maximum likelihood estimate of the parameters given the robot's trajectory and the motion data. For AUV the motion data is reported by the vehicle through the propellers



revolutions per minute. The landmarks are extracted from side sonar images and are collected as sensor data. I assume that the robot is equipped with a side sonar sensor and has access to its noisy motion data. Sound Navigation and Ranging (SONAR) is a technique based on sound propagation used for detecting objects underwater. The SONAR sensor is the only imaging tool which can work at high depth. Side scan sonar is a specific type of sonar used to image the topography of the sea floor.

To summarize I am proposing an algorithm to learn the right parameters of a motion model for an AUV. The parameters are learned on the fly without having a prior map or revisiting places. Results of the simulated experiment are presented in chapter 3.6 to show the effectiveness of the algorithm.

## 1.2 Contributions

The main contributions of the algorithm are:-

1) **Adaptive Motion Model**:- The motion model for AUVs adapt to changing environment. This automated process of calibration of the robots lead to no hand tuning of the models and gives us an online process which can be preformed during the robot's mission.

2) **Motion Estimation from Side Sonar Images**:- We present an approach to estimate the movement from side sonar images which can be coupled with existing motion model and can improve localization. It can be easily be performed on-board as limited amount of interest points are used which lead to lesser computation and memory usage.

## Chapter 2

### Background

In our introduction we talk about how we need to have an adaptive motion model. In this chapter we start by discussing how motion models are probabilistically represented as well as give an insight about motion models for AUV. This helps us in understanding on how motion model captures the probabilistic movements of robots. We then discuss a probabilistic state estimation algorithm i.e. Particle Filter [12] [1] which is at the heart of my algorithm as well as many other robotics systems. Lastly we discuss in detail about Particle smoothing [3] [4] which gives a distribution of the past states with taking into account all the evidence up to present.

#### 2.1 Motion Model

A motion model is responsible for capturing the relationship between the control input and the change in robot's configuration. Thrun [15] models the motion of a robot probabilistically because the same control inputs will never reproduce the same motion. A good motion model will capture the errors such as drift that are encountered during the motion of the robot. The motion model is an integral part of algorithms such as localization, mapping etc.

Let  $X = (x, y, \theta)$  be the initial pose of the robot in x-y space. Mathematically the motion model can be described as  $P(X'|X, u)$ , where  $X'$  is the pose after executing the motion command  $u$ . Based on the control input Thrun [15] divided the motion model in two classes 1) Odometry based motion model 2) Velocity based motion model.

The first class of motion models are used for robots equipped with wheel encoders. Odometry is generally obtained by integrating wheel encoders information and is more accurate than velocity. Velocity based models calculate the new position based on velocities and time elapsed. These models are implemented for Autonomous Underwater Vehicle(AUV) and Unmanned Aerial Vehicles(UAV). Both odometry as

Table 2.1: Notation used for marine vehicles

DOF		forces and moments	linear and angular vel.	positions and Euler an
1	motions in the x-direction (surge)	X	u	x
2	motions in the y-direction (sway)	Y	v	y
3	motions in the z-direction (heave)	Z	w	z
4	rotation about the x-axis (roll)	K	p	$\phi$
5	rotation about the y-axis (pitch)	M	q	$\theta$
6	rotation about the z-axis (heave)	N	r	$\psi$

well as velocity suffer from drift and slippage therefore the same control commands will not generally produce the same motion and the motion model  $P(X'|X, u)$  is represented as probability distribution.

The velocity motion model proposed by Thrun [15] assumes that robot can be controlled through two velocities a rotational and translational velocity. The translational velocity at time  $t$  is denoted by  $v_t$  and rotational velocity by  $w_t$ . Hence the control input  $u_t$  can be represented by

$$u_t = \begin{pmatrix} v_t \\ w_t \end{pmatrix}$$

The assumption is that positive rotational velocities  $w_t$  induce a counterclockwise rotation whereas positive translational velocities  $v_t$  correspond to forward motion. The set of equations to compute the next state of a robot for a velocity motion model are

$$\begin{aligned} x_t &= x_{t-1} + V_{t-1}/W_{t-1} \sin(\theta_{t-1}) + V_{t-1}/W_{t-1} \cos(\theta_{t-1} + W_{t-1}\delta t) \\ y_t &= y_{t-1} + V_{t-1}/W_{t-1} \cos(\theta_{t-1}) - V_{t-1}/W_{t-1} \sin(\theta_{t-1} + W_{t-1}\delta t) \\ \theta_t &= \theta_{t-1} + W_{t-1}\delta t \end{aligned}$$

To represent AUV's motion, 6 independent coordinates are necessary to determine the position and orientation of the rigid body.

The pose of AUV can be represented as  $s = (x, y, z, \theta, \phi, \psi)$ . The first three coordinates correspond to the position along the x,y,z axes while the last three coordinates describe the orientation. To estimate the position of an AUV we need to calculate the velocity at which the AUV is currently moving. The velocity can be computed in two ways: 1) Static Motion model 2) Dynamic Motion model

Hegrenæs [8] points that a way to implement a simple static motion model as table look-up based on experimental data.

$$u_r = f(n_s)$$

$u_r$ ,  $n_s$  are the water relative linear velocity in x direction and control system set point respectively. In a similar manner an expression can be established for  $v_r$ .

Another way to implement the motion model is through dynamics. The 6 Degrees of Freedom (DOF) rigid body equations of motion described by Fossen [14] are

$$\begin{aligned} X &= m[u - vr + wq - x_G(q^2 + r^2) + y_G(pq - r) + z_G(pr + q)] \\ Y &= m[v - wp + ur - y_G(r^2 + p^2) + z_G(qr - p) + x_G(qp + r)] \\ Z &= m[w - uq + vp - z_G(p^2 + q^2) + x_G(rp - q) + y_G(rq + p)] \\ K &= I_x p + (I_z - I_y)qr - (r + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - q)I_{xy} + m[y_G(w - uq + vp) - Z_g(v - wp + ur)] \\ M &= I_y q + (I_x - I_z)rp - (p + qr)I_{xy} + (p^2 - r^2)I_{zx} + (qp - r)I_{yz} + m[y_G(u - vr + wq) - z_g(w - uq + vp)] \\ N &= I_z r + (I_y - I_x)pq - (q + rp)I_{yz} + (q^2 - p^2)I_{xy} + (rq - p)I_{zx} + m[x_G(v - wp + ur) - y_g(u - vr + wq)] \end{aligned}$$

The equations described above can be expressed in a more compact form:

$$M_{RB}\mathcal{V} + C_{RB}(\mathcal{V})\mathcal{V} = \tau_{RB} \quad (2.1)$$

Here  $\mathcal{V} = [u, v, w, p, q, r]^T$  is the body fixed linear and angular velocity and  $\tau_{RB} = [X, Y, Z, K, M, N]$ ,  $M_{RB}$  is generalized vector of external forces and moments.  $M_{RB}$  is the rigid body inertia matrix and  $C_{RB}$  is Coriolis and centripetal matrix.

The right hand side of the vector 2.1 represents the external forces and moments acting on the vehicle. Fossen [14] classifies the forces into:

- Radiation-induced forces
  - added inertia
  - hydrodynamic damping
  - restoring forces
- Environmental Forces
  - Ocean currents

- Waves
- Propulsion Forces
  - Thruster/Propeller Forces
  - Control Surfaces/Rudder Forces

$$\tau_{RB} = \tau_H + \tau_E + \tau \quad (2.2)$$

Here  $\tau_H$  is the radiation induced forces and moments,  $\tau_E$  is used to describe the enviornmental forces and moments and  $\tau$  is the propulsion forces and moments. Equations 2.1 and equation 2.2 can be combined to yeild the following representation of the 6 DOF dynamic equations of motion:

$$M\mathcal{V} + C(\mathcal{V})\mathcal{V} + D(\mathcal{V})\mathcal{V} + g(\eta) = \tau_E + \tau \quad (2.3)$$

where

$$M \triangleq M_{RB} + M_A ; C(\mathcal{V}) \triangleq C_{RB}(\mathcal{V} + C_A(\mathcal{V}))$$

$M_A$  is the added inertia matrix  $C_A(\mathcal{V})$  is the matrix of hydrodynamic Coriolis and centripetal terms.  $g(\eta)$  is the restoring force.

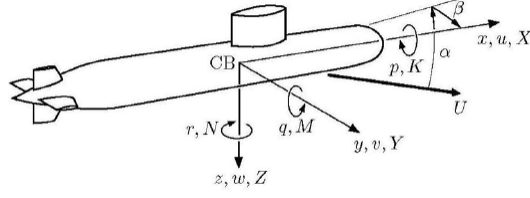
Lammas [10] points out that navigation equation of an underwater vehicle is:-

$$\mathcal{V} = M^{-1}(\tau - C(\mathcal{V})\mathcal{V} - D(\mathcal{V})\mathcal{V} - g(\eta)) \quad (2.4)$$

$\mathcal{V}$  can be integrated with time to get the velocity.

In the static model the velocity is calculated from a lookup table. In the other model we are computing forces and moments on the fly but the parameters to these forces are considered to be static. The paramters such as density, temprature etc of water can change with time and lead to an inaccurate estimate of velcoity in both the models. Hence the velocity needs to be adapted and Chapter 3.3 explains how it is done in my algorithm.

Figure 2.1: Body-fixed reference frames



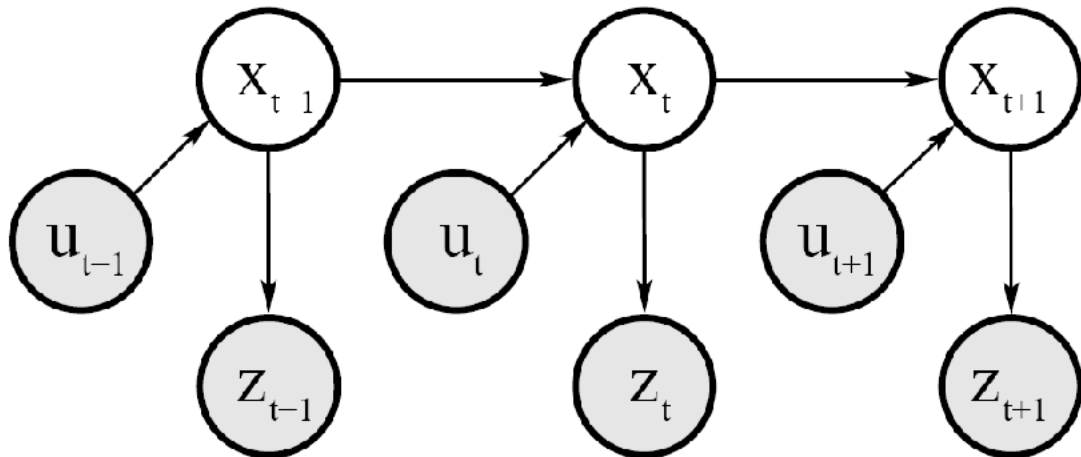
## 2.2 Particle Filter

Particle Filter is a state estimation algorithm based on a sampling method for approximating a distribution. It is an alternative non-parametric implementation of the Bayes filter. It also can be called as a sequential Monte Carlo algorithm. It was first proposed by MK. Pitt and N Shephard[refer the paper].

Particle Filters have no restrictions in model i.e. it can be applied to non-Gaussian models, they are super set of other filtering methods i.e. Kalman filter is Rao-Blackwellized particle filter with one particle. Additionally they can model non linear transformations of random variables therefore can be applied to any systems and measurement models. All the advantages make them a good alternative to Extended Kalman Filter(EKF) and Unscented Kalman Filter(UKF).

Particle Filters are used to calculate the approximate of a state in Markov chain. The key idea behind the particle filter is to represent the posterior  $bel(x_t)$  by a set

Figure 2.2: Markov Chain



of random state samples drawn from this posterior. The state  $x_t = p(x_t|x_{t-1}, u_t)$  is dependant upon  $x_{t-1}$  as well as the control input  $u_t$ . The observations are introduced in the system in the form of measurements. The  $p(z_t/x_t)$  are used to assign weights to the particles and give survival of the fittest mentality to the algorithm.

The algorithm for particle filters is described below:-

**Input:**  $X_{t-1}$ : particle set  $u_t$ : most recent control  $z_t$ : most recent measurement

**Output:**  $X_t$ : particle set

**begin**

**for**  $m=1$  to  $M$  **do** **do**

        sample  $x_t^m$   $p(x_t|u_t, x_{t-1}^m)$   $w_t^m = p(z_t|x_t^m)$   $X_t^- = X_{t-1}^- + (x_t^m, w_t^m)$

**end**

**for**  $m=1$  to  $M$  **do** **do**

        draw  $i$  with probability  $\propto w_t^{[i]}$

        add  $x_t^{[i]}$  to  $X_t$

**end**

    return  $X_t$

**end**

### **Algorithm 1:** Particle Filter Algorithm

The importance factor for each particle  $x_t^m$  is calculated and denoted by  $w_t^m$ . Importance is the probability of the measurement  $z_t$  under the particle  $x_t^m$ . Thus importance factor are used to incorporate the measurement into the particle set. In practice, the number of particles used is a large number(e.g.-:1000).

The key part of the algorithm is the re-sampling step in the particle filter algorithm. The algorithm draws  $M$  particles with replacement from a temporary particle set  $X_t^-$ . The probability of drawing the particles is given by the importance factor. The re-sampling step is a probabilistic implementation of the Darwinian idea of survival of the fittest. It refocuses the particle set to regions in state space with high posterior probability.

## **2.3 Particle smoothing**

The particle filter algorithm as described before is the first step in the Expectation process. The next algorithm that completes the Expectation Step is the particle smoothing. It is defined as the computing the posterior distribution of the past

states given all the evidence upto the present. Mathematically it can be represented as  $p(x_t|u_{1:T}, z_{1:T})$  for some  $t$  with  $0 \leq t \leq T$ . Russel and Norving showed that the state of the system is better estimated by smoothing as it incorporates more information than just filtering. We use particle smoothing algorithm proposed by Teddy N Yap and Christian R. Shelton which was based on the technique presented by Docuet et al. and Godsill et al. The algorithm is used to generate samples from the entire joint smoothing density  $p(x_{0:T}|u_{1:T}, z_{1:T})$ . This algorithm assumes that particle filtering is already done on the entire data set and resulting in a distribution of the pose.

**Input:**  $X_t, t = 0, 1, \dots, T$ : particle approximations to the posterior pdfs

$$p(x_t|c_{1:t}, s_{1:t}), t = 0, 1, \dots, T$$

$c_{1:T} = (c_1, c_2, \dots, c_T)$ : set of controls from time 1 to time T

**Output:**  $x'_{0:T} = (x'_0, x'_1, \dots, x'_T)$ : a sample from the entire joint smoothing density  $p(x_{0:T}|c_{1:T}, s_{1:T})$

**begin**

draw  $i$  with probability  $\propto w_T^{[i]}$   $x'_T \leftarrow x_T^{[i]}$

**for**  $t \leftarrow T - 1$  *down to*  $0$  **do** **do**

**for**  $i \leftarrow 1$  *to*  $N_s$  **do** **do**

$w_{t|t+1}^{[i]} \leftarrow w_t^{[i]} p(x'_{t+1}|u_{t+1})$

**end**

draw  $i$  with probability  $\propto w_{t|t+1}^{[i]}$

$x' \leftarrow x_t^{[i]}$

**end**

**end**

**Algorithm 2:** Sample the entire joint smoothing density  $p(x_{0:T}|c_{1:T}, s_{1:T})$

The particle filter process gives us a set of particles with their corresponding weights at the present time step. In the first step of smoothing we choose a particle with the probability proportional to the weight of the particles. Then we move a time step back till time=0 and calculate the new smoothed weights for every particle. They are calculated by the product of the forward probability and the weight of the particle.

$$p(x_t|x_{t+1:T}, u_{1:T}, z_{1:T}) = p(x_{t+1}|x_t, u_{t+1})p(x_t|u_{1:t}, z_{1:t})$$

To calculate the trajectory we draw particles according to the new smoothed weights. At every time step we pick a particle and add it our trajectory set till time=T. We can



repeat this process any number of time to get several trajectories. These trajectories are used in estimating the parameters of the motion model as they are treated as ground truth.

## **2.4 SURF**

It is a robust local feature detector, first proposed by Herbert Bay et al. in 2006. The sta

## Chapter 3

### Learning the motion model

#### 3.1 General Architecture

Figure 3.1: Block Diagram of the framework

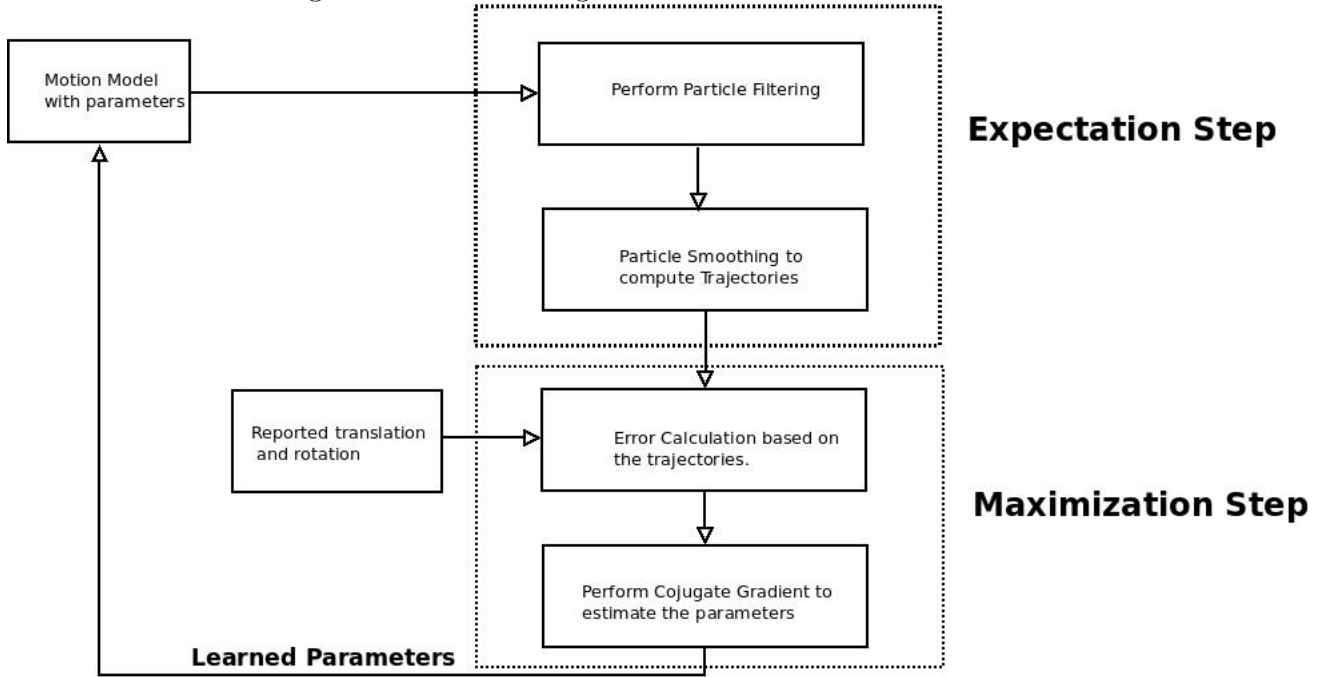


Figure 1.1 is a block diagram describing the whole system. The first step is to initialize the motion model with a set of parameters.

Given the prior distribution  $p(x_{t-1})$  of the robot's pose at time  $t-1$ , the motion model  $p(x_t|x_{t-1}, u_t)$ , the sensor model  $p(z_t|x_t)$ , the control commands  $u_{1:t}$  and the sensor measurements  $z_{1:T}$  we perform particle filtering using Algorithm 1 from time  $t=1$  to time  $t=T$ . After we perform filtering we have a set of particles with importance factors describing the distribution over the pose of the robot. To complete the Expectation Step particle smoothing is performed recursively backwards in time. A set of trajectories are generated by repeating Algorithm 2.

Based on the trajectories and the reported translational and rotational motion we

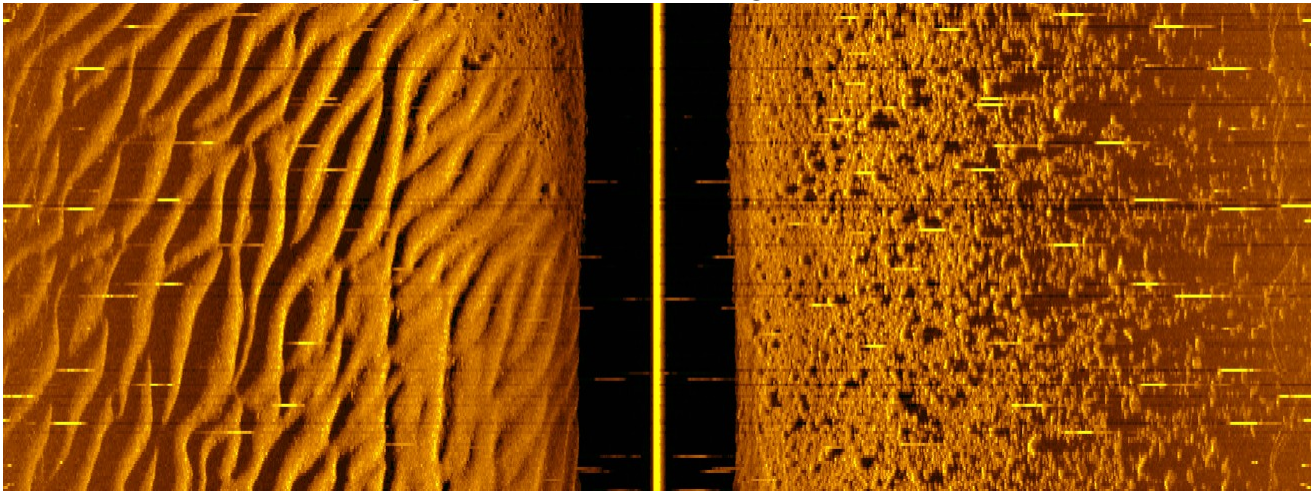
calculate errors at each time step. We perform newton conjugate gradient on the errors to give us the best estimate of the parameters. This completes the Maximization Step.

The learned parameters are re-assigned to the motion model which helps in adapting the model. The whole process is repeated at each time step so that we can dynamically learn the right parameters. Various algorithm such as SLAM, Kalman Filter are dependant upon the accuracy of the motion model and adapting our motion models is the most basic step towards it. We found that the adaptive motion model performs better than the static and the results are shown in the later chapters.

### 3.2 Dynamic Landmarks

In the particle filtering algorithm the sensor model is responsible to assign weights to the particles. The sensor model describes the process by which sensor measurements are generated in the physical world. It is generally done by using some sort of references in the world aka landmarks. Austin and Elizar in their papers used static maps of known environments to generate references for their sensor model. The probability of having static maps for underwater environments is pretty low and thus lead us to use side sonar images as references for the sensor model. As you can see in the image there

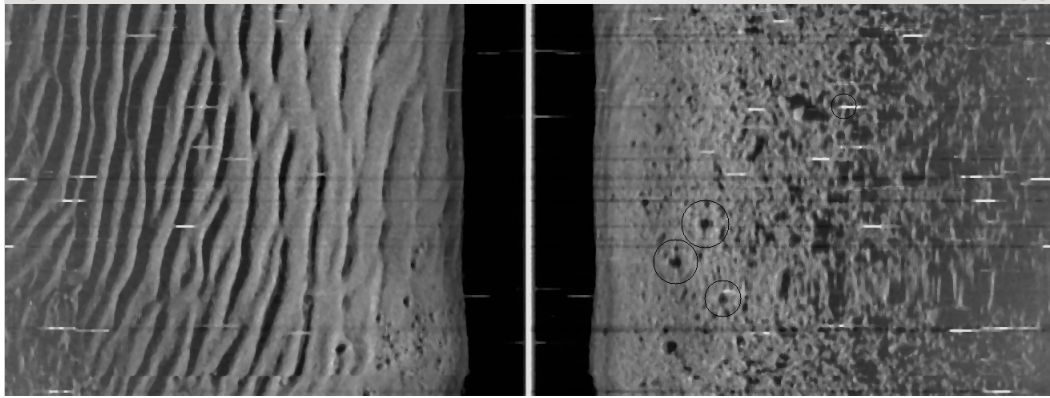
Figure 3.2: Side Sonar Image



are lot of horizontal lines which we treat as noise. For us to use the side sonar images we performed a pre-processing step in which we used a median filter on the image.

This helped us in getting rid of major noises in terms of horizontal lines but also blurred the image. In order to generate some landmarks we ran feature extractions techniques like SURF. This algorithm helped us in detecting interest points in the image and are shown below in circles. Andrew Vardy (refer the paper) ran multiple image registration techniques such as phase matching (you need to mention more) on side sonar images and showed that SURF performs better than the rest .

Figure 3.3: SURF Keypoints



These interest points can be treated as landmarks. We use a high Hessian threshold so that we have a maximum of 4 landmarks. The distance of the robot and the particles to these landmarks are used to assign weights to the particles.

The output of a side sonar is a ping of the surface and for an image to be created we need to combine pings over time. Andrew Vardy in his paper explains on how to combine pings to create an meaningful image. For our algorithm to run on the AUV we can use Andrew's algorithm as a black box and run our feature extraction on the output i.e. images of the seabed. This method allows us independence from a static map as well as helps in learning the motion model on the fly in a new environment.

### 3.3 Adapting the motion model by parameter estimation

Velocity as distribution

Expectation Maximization is an iterative process of finding maximum likelihood of parameters of a model which depend upon hidden variables. We use the EM algorithm as described by Christopher M. Bishop in his book. We have a joint distribution  $p(X, Z|\theta)$  where  $X$  are the observed variables and  $Z$  are the latent variables governed

by parameters  $\theta$ . As stated earlier the goal is to maximize the likelihood function  $p(X|\theta)$  with respect to  $\theta$ .

1. Have an initial estimate of the parameters  $\theta^{old}$
2. **E Step:** Evaluate  $p(Z|X, \theta^{old})$
3. **M Step:** Evaluate  $\theta^{new} = \text{argmax}_{\theta} L(\theta, \theta^{old})$

$$\text{where } L(\theta, \theta^{old}) = \sum_Z p(Z|X, \theta_{old}) \log p(X, Z|\theta)$$

4. Check for the convergence of either the log likelihood or the parameter values. If the convergence criterion is not satisfied then let

$$\theta \leftarrow \theta^{new}$$

and return to step 2

There are various convergence techniques that can be applied to step 4. In our algorithm we use Newton Conjugate Gradient to estimate the right parameters.

As stated in earlier chapters to learn the motion model we have a distribution around the translational and rotational velocity. We have assumed a Gaussian distribution and use the EM algorithm to learn the right parameters. In the first step i.e. Expectation Step we perform particle filtering and smoothing and the output is a set of robot trajectories which are treated as ground truth. At each time step we calculate the error between the distance given by trajectory and the actual distance moved.

$$\begin{aligned} \epsilon_{T_t}^{[j]} &= (\theta'_{t+1} - \theta'_t - r''_t) \text{mod} 2\pi \\ \epsilon_{D_t}^{[j]} &= (x'_{t+1} - x'_t) \cos(\theta'_t + \frac{r''_t + \epsilon_{T_t}^{[j]}}{2}) + (y'_{t+1} - y'_t) \sin(\theta'_t + \frac{r''_t + \epsilon_{T_t}^{[j]}}{2}) - d''_t \end{aligned}$$

$\epsilon_{T_t}^{[j]}$  and  $\epsilon_{D_t}^{[j]}$  are rotational and translational errors for  $t=0,1\dots T-1$  for  $j^{th}$  sampled trajectory.  $\epsilon_{W_t}^{[j]}$  and  $\epsilon_{V_t}^{[j]}$  are the rotational and translation errors in velocity and is given by  $\epsilon_{T_t}^{[j]}/\delta(t)$  and  $\epsilon_{D_t}^{[j]}/\delta(t)$ .  $r''_t$  and  $d''_t$  are the distance moved and are given by  $r''_t = w_t * \delta(t)$  and  $d''_t = v_t * \delta(t)$ .  $w_t$  and  $v_t$  are the reported velocity. Based on the motion model described

$$\begin{aligned} \epsilon_{V_t}^{[j]} &\sim \mathcal{N}(0, v_t^2 \sigma_{V_v}^2 + w_t^2 \sigma_{V_w}^2 + \sigma_{V_1}^2) \\ \epsilon_{W_t}^{[j]} &\sim \mathcal{N}(0, v_t^2 \sigma_{W_v}^2 + w_t^2 \sigma_{W_w}^2 + \sigma_{W_1}^2) \end{aligned}$$

The log likelihood functions are

$$\begin{aligned} \mathcal{L}(\sigma_{V_v}^2, \sigma_{V_w}^2, \sigma_{V_1}^2) &= -\frac{1}{2} \sum_j \sum_{t=0}^{T-1} [\log 2\pi + \log(v_t^2 \sigma_{V_v}^2 + w_t^2 \sigma_{V_w}^2 + \sigma_{V_1}^2) + \frac{(\epsilon_{V_t}^{[j]})^2}{v_t^2 \sigma_{V_v}^2 + w_t^2 \sigma_{V_w}^2 + \sigma_{V_1}^2}] \\ \mathcal{L}(\sigma_{W_v}^2, \sigma_{W_w}^2, \sigma_{W_1}^2) &= -\frac{1}{2} \sum_j \sum_{t=0}^{T-1} [\log 2\pi + \log(v_t^2 \sigma_{W_v}^2 + w_t^2 \sigma_{W_w}^2 + \sigma_{W_1}^2) + \frac{(\epsilon_{W_t}^{[j]})^2}{v_t^2 \sigma_{W_v}^2 + w_t^2 \sigma_{W_w}^2 + \sigma_{W_1}^2}] \end{aligned}$$

In the Maximization Step we minimize the likelihood function. In this case we maximize the log likelihood function because of the minus sign with respect to the motion parameters. We use newton conjugate gradient ascent for finding the local maximum of the function. The gradient of the function is taken as the first search direction while the next search direction are chosen in such a way that they are orthogonal to all previous search directions.

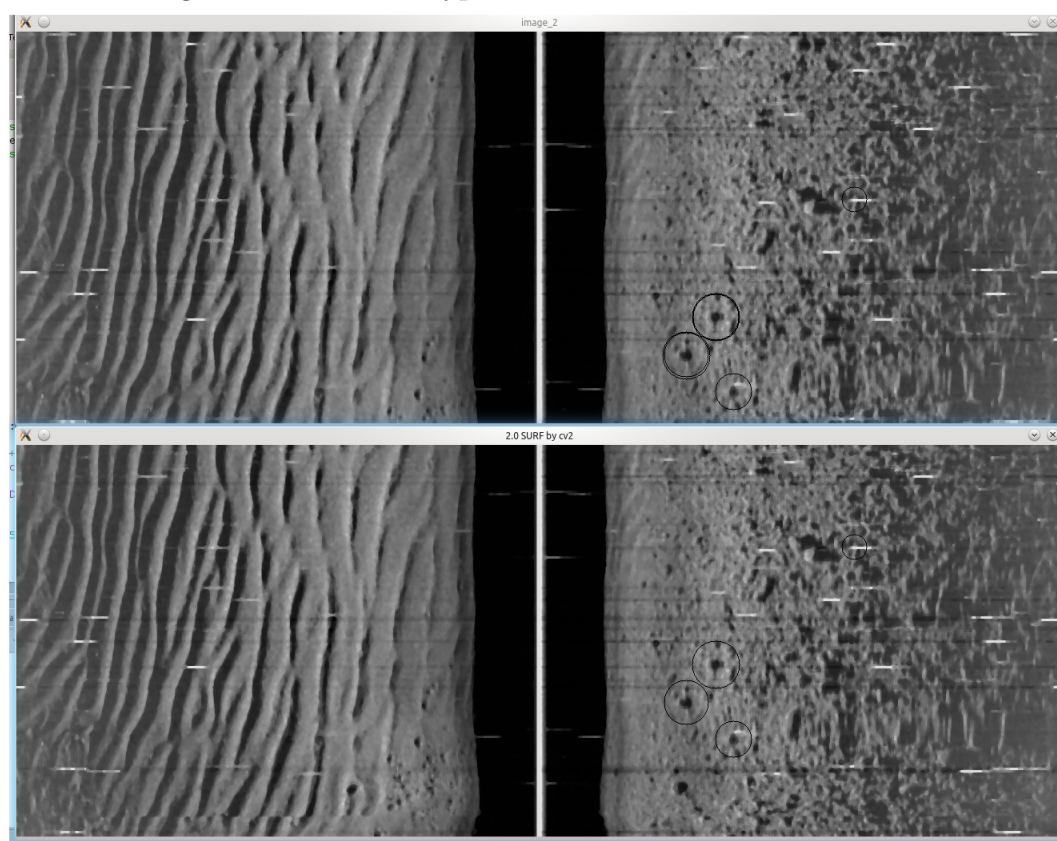
The output of the step described above is a set of parameters that maximize the function. We substitute these values in our motion model at every time step and the process is repeated throughout the robot's mission.

### 3.4 Visual Motion Estimates

In AUV the motion estimation is generally done through dead-reckoning. It is a process of calculating the current position based upon the previous position and the speed of vessel. The velocity of the AUV can be estimated by the acceleration measurement supplied by an Inertial Measurement Unit. Another way is to use a Doppler Velocity Log(DVL) that measures velocity in water by measuring the Doppler effect on scattered sound waves. Dead reckoning may have significant errors as the velocity and direction must be accurately known at every time step.

In this algorithm we propose to get motion estimates using side sonar images. As stated before we run SURF on the images and generate some key points. These key points are matched in the next image using a KNN based matcher. The matched key points gives us an estimate on the movement of the vehicle. In the figure we show two consecutive images and in the first image we have the key points marked in circle. In the next image we have the matched key points marked in green circles. This estimate can be coupled with the previous estimate to calculate the current position. The visual input to the dead reckoning algorithm has its pros and cons. The main advantage of using a visual estimate is that it doesn't suffer from drift which is prime concern for underwater vehicles. The disadvantage lies in the fact that we don't have side sonar images available every time. We can solve the problem by combining the visual input with the velocity estimates. We can pass the visual motion estimate and

Figure 3.4: SURF Keypoints



the DVL estimate to a Kalman Filter and use the output as an input to our dead-reckoning estimate. The second disadvantage is the computation power available on AUV. To specifically deal with the problems we use a high Hessian threshold to extract maximum of 4 landmarks so that feature matching is not computationally expensive.

We validate our algorithm on datasets consisting of side sonar images and the total distance the AUV moves.

### 3.5 Application

In this chapter we describe a specific application for learned motion model in underwater vehicles. A typical navigation sensor system would consist of Inertial Navigation Systems(INS) and some standard compass and pressure sensor. In some of the systems we could have position updates from long baseline(LBL),ultra short baseline(USBL) acoustics and surface GPS. Some high end systems would have Doppler Velocity Log(DVL) which estimates speed over ground or water. These systems are sometimes incorporated to limit the drift of Inertial Navigation System[refer the paper]. Even when a DVL is included there are situations in which DVL fails or the reading have to be discarded due to poor quality.

We need some sort of alternative velocity information that doesn't depend upon external sensors in order to compensate for the drift in INS. To get an estimate of velocity we can use the kinetic vehicle model to aid the INS.

In the traditional aided INS the readings from gyro and accelerometer measurements from the IMU are integrated over time to give an estimate of velocity,position and orientation. Due to the inherent errors in the gyros and accelerometers there is a drift in the INS system. There are aiding sensors such as DVL and surface GPS to compensate the drift for the IMU. The reading from the sensors are fused using a Bayes filters to get an enchained estimate of velocity ,position and orientation.

In the model aided INS the IMU suffers from the same drift but the output from the model is treated analogously to that of an external aiding sensor. In this system the DVL in traditional INS systems are replaced by vehicle model. The output from the model i.e. the velocity estimate is fused with the readings from the INS by a Bayes filter. The integrating of vehicle model with the IMU systems help in systems



Figure 3.5: Traditional aided INS

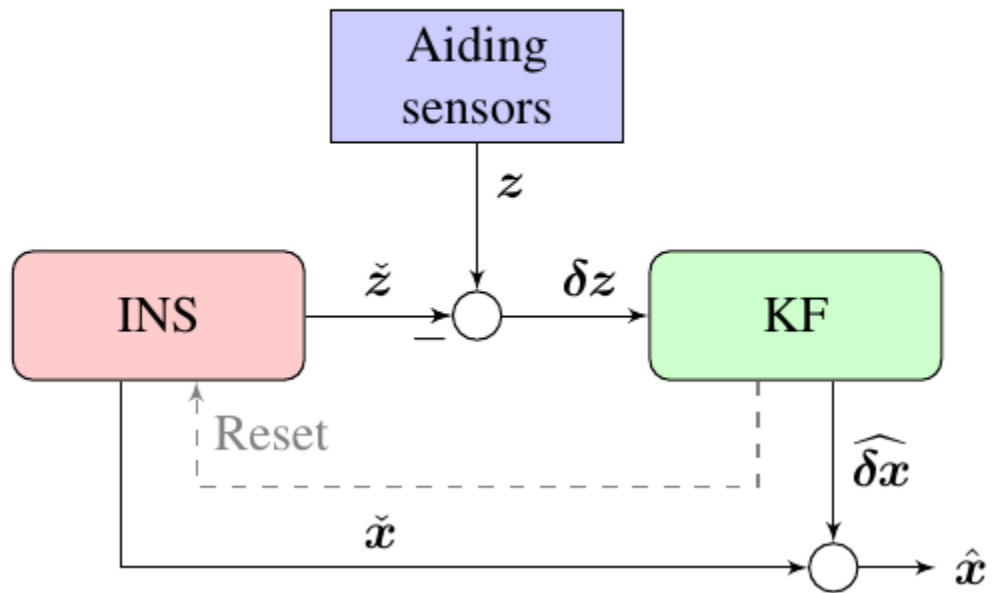
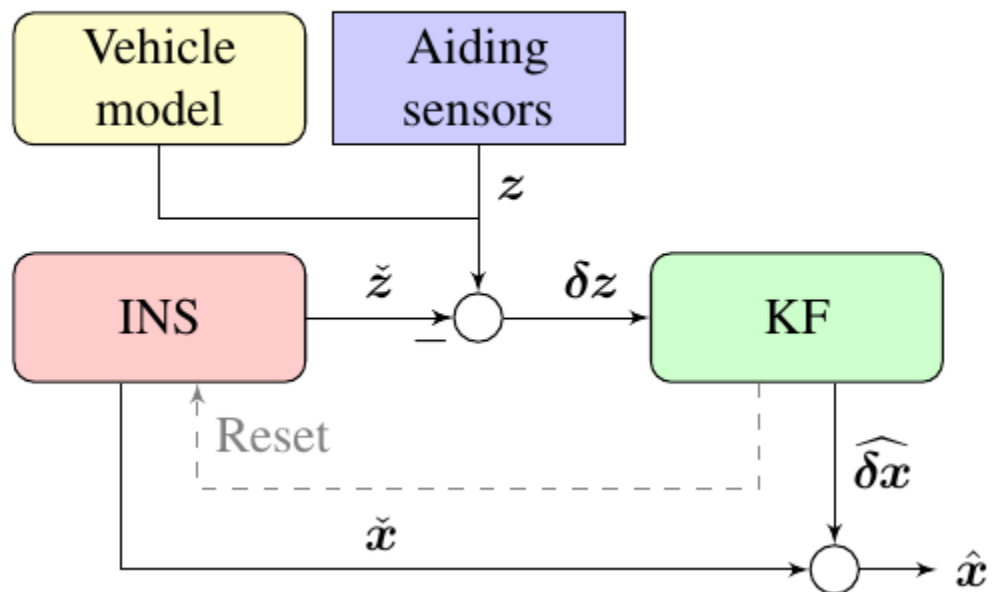


Figure 3.6: Traditional aided INS



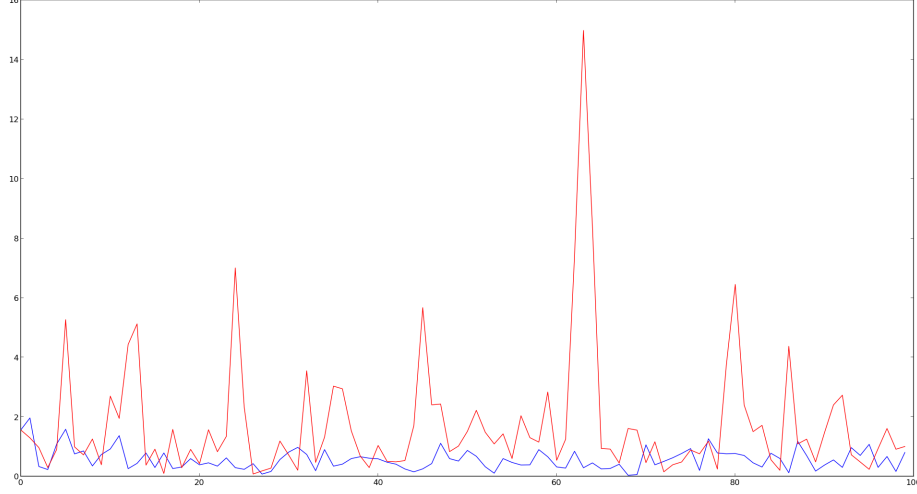
which lack external velocity measurements. The other implication is in systems where redundancy and integrity is important e.g. during sensor drop outs or sensor failures.

This specific application shows the importance of having a accurate motion model. As stated in earlier chapters the motion model can change due to various factors and would have a direct impact on the navigation systems of underwater vehicles. Wrong estimates from the motion model will not benefit INS systems and overall lead to wrong estimate about the position and orientation of the vehicle. Our algorithm shows on how to learn the motion model on the fly which can lead to better estimate of the velocity and therefore accurate navigation systems.

As you can see in the figure the input to the motion model is a control command and output is the velocity estimate. In HUGIN 4500 they had implemented a static motion model as table look up based on experimental data i.e.  $u_r = f(n_s)$  where  $n_s$  is the control system set point. The velocity estimate  $u_r$  is fused with the reading from IMU to get a better estimate of the position of the AUV. As shown before we try to learn the motion model to find the best estimate for the velocity so that it can aid the INS in this particular application.

### 3.6 Results

We use a simulation to demonstrate the effectiveness of learning the motion model. The simulation mainly consists of particle filter SLAM. We use a motion model described in the above chapters. The sensor model basically measures the distance from the four static landmarks defined at the start of the experiment. The experiment runs over 100 time steps and at every 5<sup>th</sup> time step we change the parameters of the motion model. As described in the above chapters the parameters that we intended to learn are  $\sigma_{D_d}^2, \sigma_{T_d}^2, \sigma_{D_1}^2, \sigma_{D_r}^2, \sigma_{T_r}^2, \sigma_{D_1}^2, \sigma_{T_1}^2$ . In the first stage of the experiment at every 5<sup>th</sup> time step we change  $\sigma_{D_d}^2$  or  $\sigma_{T_r}^2$  in our motion model. The following table and plots describes the five experiments that were conducted in the simulation.

Figure 3.7:  $\sigma_{D_d}=0.05$   $\sigma_{D_d}^* = 0.2$  Sensor Noise= 2.0

No.	$\sigma_{D_d}$	$\sigma_{T_r}$	$\sigma_{D_d}^*$	$\sigma_{T_r}^*$	Sensor Noise
1	0.05	0.05	0.2	0.05	2.0
2	0.05	0.05	0.2	0.05	5.0
4	0.05	0.05	0.5	0.05	2.0
5	0.05	0.05	0.5	0.05	5.0
6	0.05	0.05	0.05	0.2	5.0
7	0.05	0.05	0.05	0.5	5.0

$\sigma_{D_d}, \sigma_{T_r}$  are the parameters values that the motion model was initialized. These values are altered in order to simulate a change in the motion model and they are described by  $\sigma_{D_d}^*, \sigma_{T_r}^*$ . The sensor noise can be described as the confidence the robot has in its sensor model. The impact of the noise on the localization error can be seen in the following plots.

Figure 1 and Figure 2 are plots of the localization error with different sensor noises. We can see in both the cases the learned motion model performed better than the static motion model. Another important point is that the average error is less when the sensor noise is 2.0 as compared to the second case. This can be accounted for the fact that our localization algorithm is more confident on the sensor model as compared to the motion model.

Figure 3.8:  $\sigma_{D_d}=0.05$   $\sigma_{D_d}^* = 0.2$  Sensor Noise= 5.0

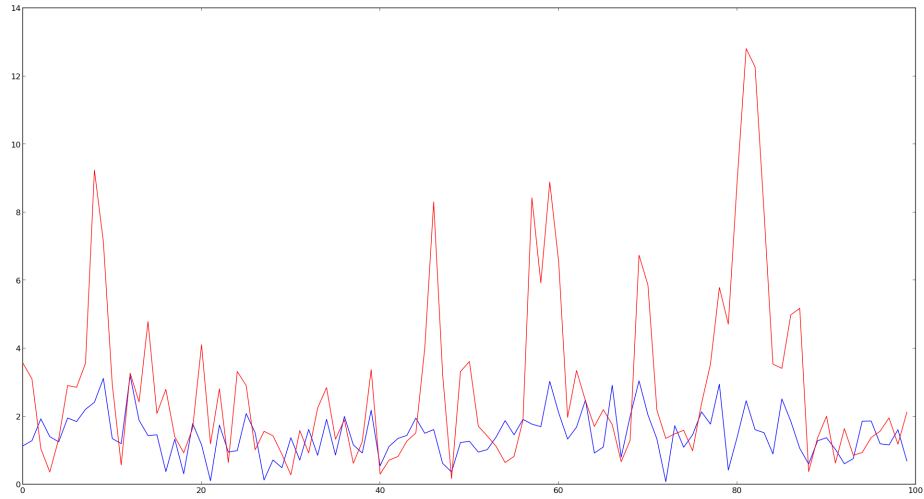


Figure 3.9:  $\sigma_{D_d}=0.05$   $\sigma_{D_d}^* = 0.5$  Sensor Noise= 2.0

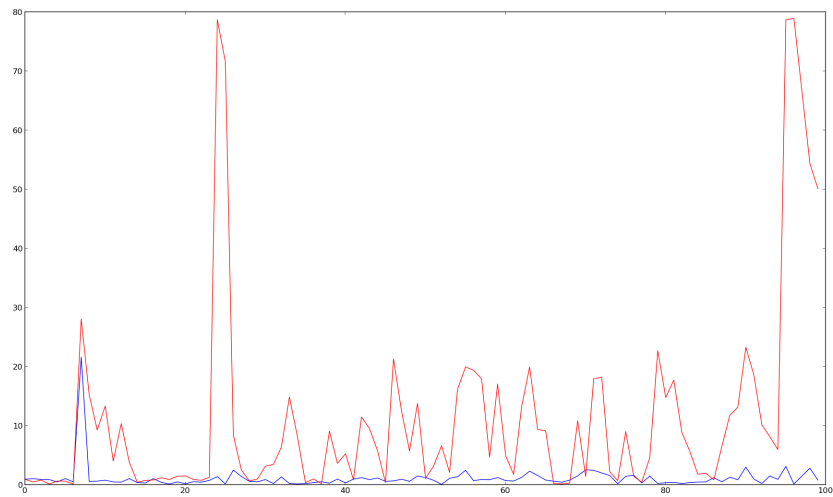
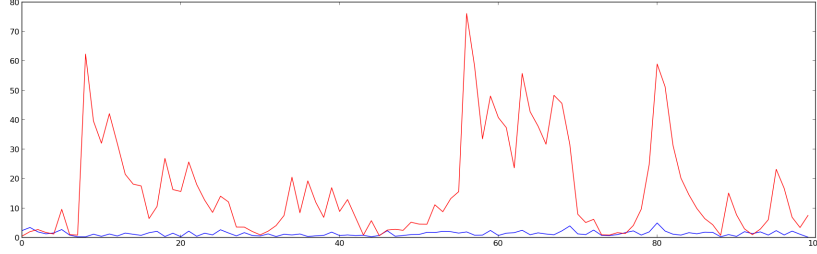


Figure 3.10:  $\sigma_{D_d}=0.05$   $\sigma_{D_d}^* = 0.5$  Sensor Noise= 5.0

As we can see in Figure 3 and Figure 4 at 5<sup>th</sup> time step the error shooting up but the learned motion model brings back the error whereas the static motion model takes time to recover back depending upon the sensor noise. In both the figures we can see that the error is pretty static in the learned motion model whereas in the static motion model there is a lot of fluctuation.

Figure 5 and Figure 6 describe the errors when the robot rotational motion is much more than the translational motion. We can clearly see that the learned motion model quickly adapts to the changes whereas the static motion model struggles to get the error down.

In all the cases it was very clear that we could see the adaptive motion model performing better than the static. The sensor noise had its impact on the overall error. Robot calibration is important to process in mobile robotics. The proposed algorithm is an automated process which can help us in better navigation of the robots and can be used for any motion model.

Figure 3.11:  $\sigma_{T_r}=0.05$   $\sigma_{T_r}^* = 0.2$  Sensor Noise= 5.0

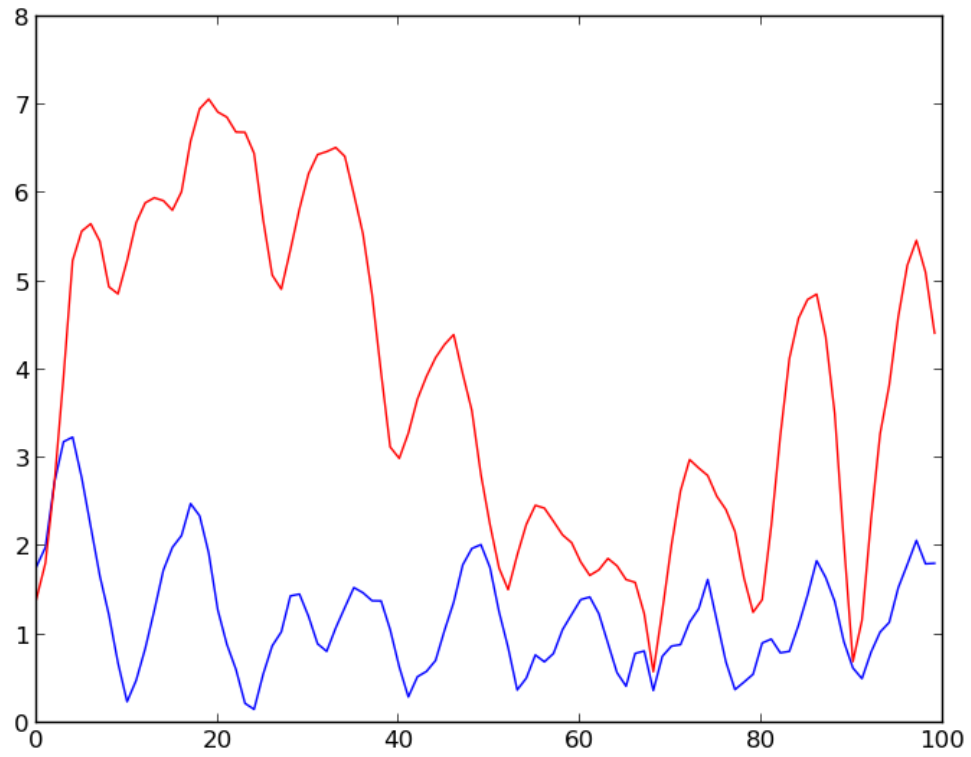
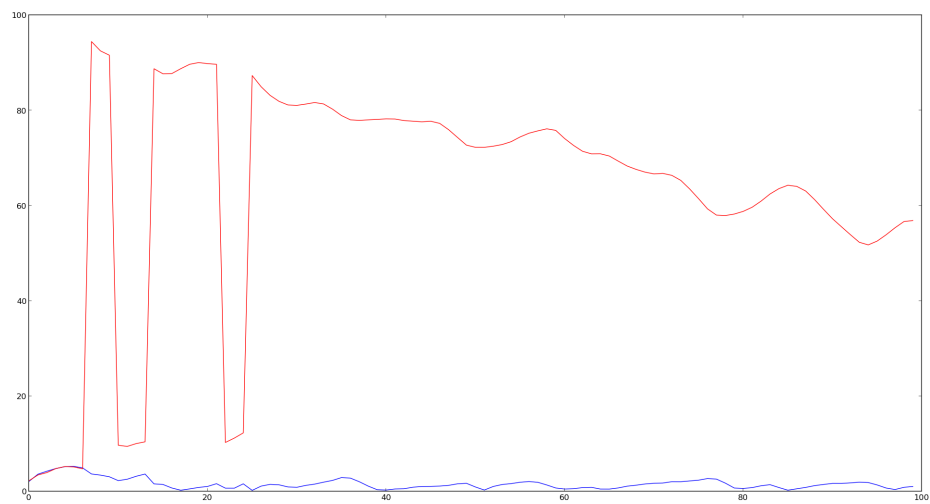


Figure 3.12:  $\sigma_{T_r}=0.05$   $\sigma_{T_r}^* = 0.5$  Sensor Noise= 5.0



## Chapter 4

## Conclusion

Did it!

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