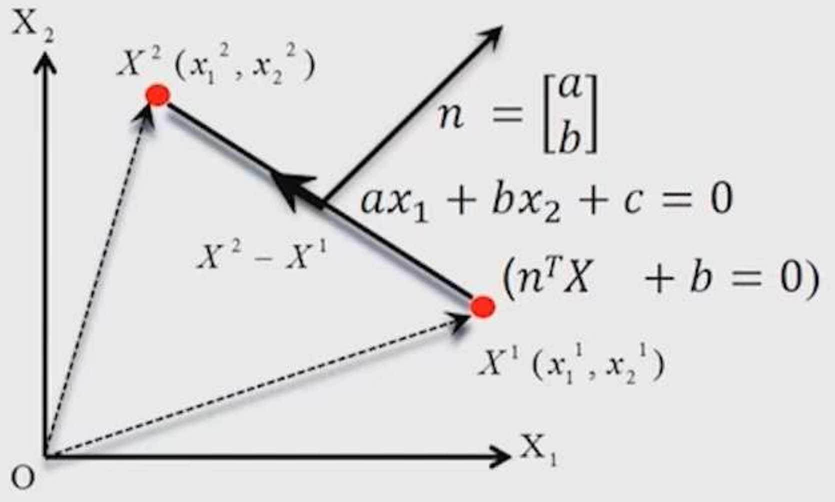
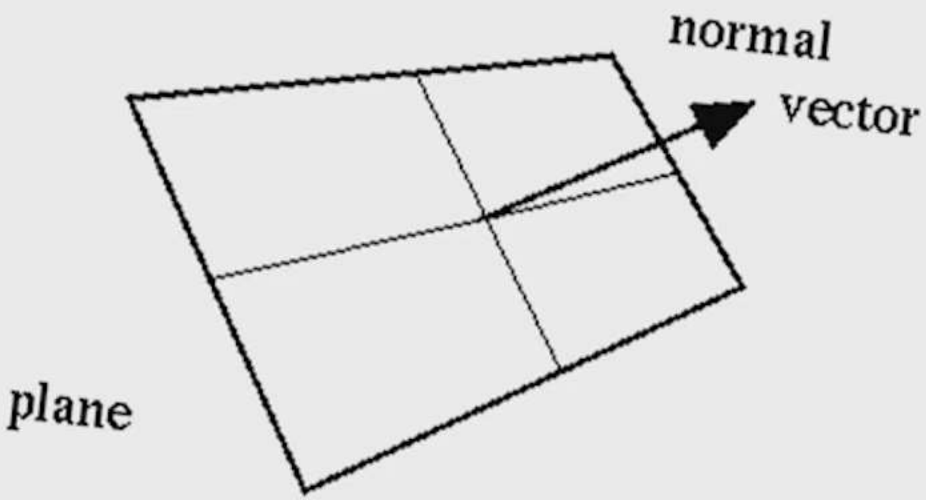
Representation of line and plane





Point X^1 lies on the line $\Rightarrow n^T X^1 + b = 0 - (1)$

Point X^2 lies on the line $\Rightarrow n^T X^2 + b = 0 - (2)$

Subtracting (2) from (1)

$n^T (X^2 - X^1) = 0.$

Thus n is perpendicular to $(X^2 - X^1)$

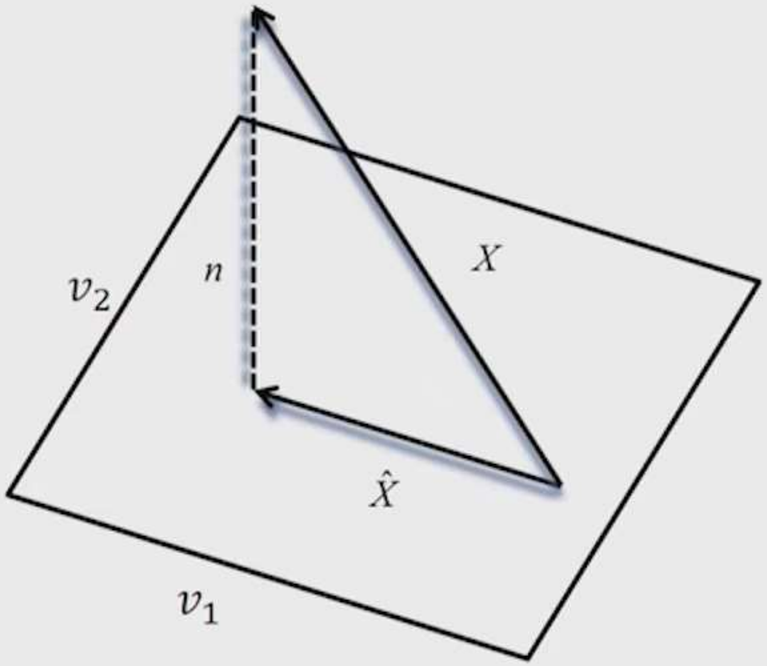
Projections

- We can define the projection ($\hat{X}$) of a vector (X) onto a lower dimension (two dimensions in the picture) mathematically as

$$\hat{X} = c_1 v_1 + c_2 v_2$$

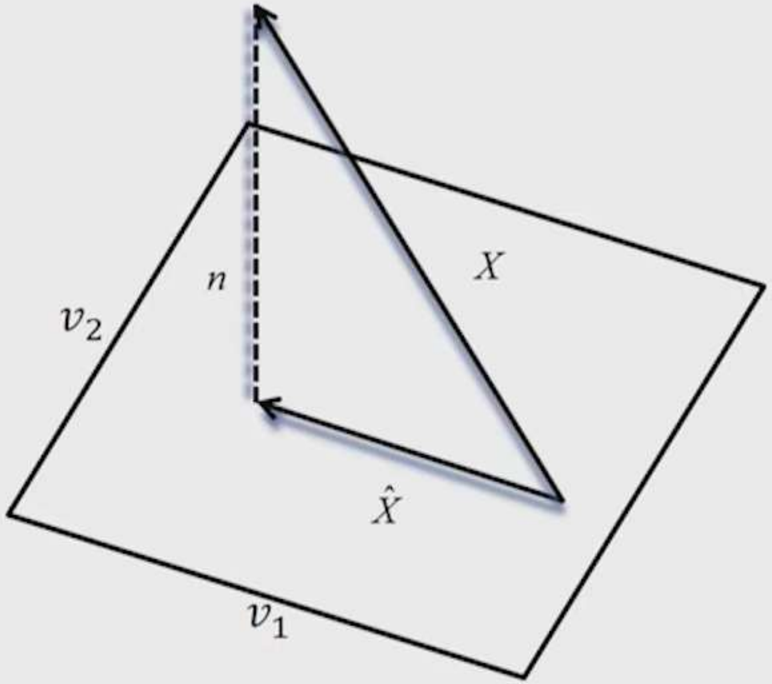
- Using vector addition

$$X = c_1 v_1 + c_2 v_2 + n$$



Projections

- Projections onto general orthogonal directions (two dimensions in this case)



$$v_1^T n = 0$$

$$v_1^T (X - c_1 v_1 + c_2 v_2) = 0$$

$$v_1^T X - c_1 v_1^T v_1 = 0$$

$$\hat{X} = \frac{v_1^T X}{{v_1}^T v_1} v_1 + \frac{v_2^T X}{{v_2}^T v_2} v_2$$

Projections: Example

$$X = [1 \;\; 2 \;\;\; 3]^T$$

- Projecting this vector onto the space spanned by the vectors $v_1 = [1 \; -1 \;\; -2]^T$ and $v_2 = [2 \;\; 0 \;\; 1]^T$
- Thus, finding the projection onto the plane defined by v_1 and v_2 is

$$\hat{X} = \frac{[1 \;\; 2 \;\; 3]\begin{bmatrix}1\\-1\\-2\end{bmatrix}}{6}\begin{bmatrix}1\\-1\\-2\end{bmatrix} + \frac{[1 \;\; 2 \;\; 3]\begin{bmatrix}2\\0\\1\end{bmatrix}}{5}\begin{bmatrix}2\\0\\1\end{bmatrix}$$

Projections: Example

$$\hat{X} = \frac{-7}{6}\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\hat{X} = \begin{bmatrix} 5/6 \\ 7/6 \\ 20/6 \end{bmatrix}$$

Projection -Generalization

- Projections onto general directions
- Consider the problem of projection of X onto a space spanned k linearly independent vectors

$$\hat{X} = \sum_{j=1}^{k} c_j v_j$$

$$\hat{X} = \begin{bmatrix} v_1 & \cdots & v_k \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_k \end{bmatrix}$$

$$\hat{X} = Vc$$

Projection -Generalization

- Using orthogonality idea

$$V^T(X - \widehat{X}) = V^T(X - Vc) = 0$$

$$V^T X - V^T V c = 0$$

$$c = (V^T V)^{-1} V^T X$$

$$\hat{X} = V(V^T V)^{-1} V^T X$$