Fundamentals of optimization

Multivariate optimization with constraints

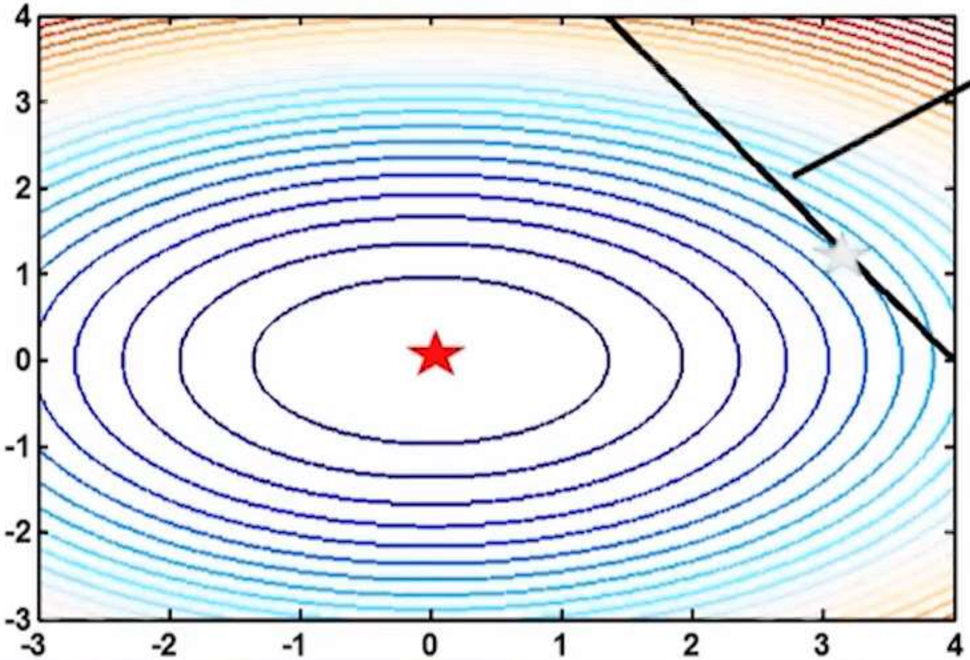
$$\min_{x_1, x_2} \; 2x_1^2 + 4x_2^2$$

$$st$$

$$3x_1 + 2x_2 = 12$$

Constrained minimum

Unconstrained minimum



$3x_1 + 2x_2 = 12$

All points on this line represent the feasible region

Unconstrained minimum is not the same as constrained minimum

Fundamentals of optimization

Multivariate optimization with equality constraints

At optimum (one equality constraint case)

$$-\nabla f(\overline{x^*}) = \lambda^* \nabla h(\overline{x^*})$$

In higher dimensions and when there are more than one equality constraint

$$-\nabla f(\overline{x^*}) = \sum_{i=1}^{l} [\nabla h_i(\overline{x^*})]\, \lambda_i^*$$

Gradient lies in the space spanned by the normal of the gradients

Fundamentals of optimization

Multivariate optimization

$$\min_{x_1, x_2} \; 2x_1^2 + 4x_2^2$$

$$st$$

$$3x_1 + 2x_2 - 12 = 0$$

First order condition

$$-4x_1 = 3\lambda$$
$$-8x_2 = 2\lambda$$
$$(3x_1 + 2x_2 - 12) = 0$$

solving →

$$\begin{bmatrix} x_1^* \\ x_2^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} 3.27 \\ 1.09 \\ -4.36 \end{bmatrix}$$