We obtained three tuples of integers in the cipher text of the form (a, password * g^a). Since, multiplication operation is from the group Z_p^* therefore it has to be done modulo p.

Let the password for the game be $x \in Z_p*$ and let the three tuples be $(a, n_1), (b, n_2)$ and (c, n_3). Therefore we will end up with three modular equations as shown below.

$$
\begin{aligned}
x * g^a &\equiv n_1 \ mod \ p \\
x * g^b &\equiv n_2 \ mod \ p \\
x * g^c &\equiv n_3 \ mod \ p
\end{aligned}
$$

Now, since $gcd(a, b) = 1$ therefore we can see that there exists a solution to the equation

$$\alpha \times a + \beta \times b + \gamma \times c = 0$$

Setting another constraint, on α, β and γ i.e.

$$\alpha + \beta + \gamma = 1$$

, and solving it simultaneously with the above equation we will get

$$\beta \times (b - a) + \gamma \times (c - a) = -a$$

. Now, the last equation also has a solution since, $gcd(b - a, c - a) = 1$ in our case. Hence, we came up with the values of $\beta \ and \ \gamma$ by solving the diophantine equation using Extended euclid's algorithm on this diophantine equation. On analysis using the C++ program, we got $\beta = -204768 \ and \ \gamma = 45036$. Now, using the fact that sum of $\alpha, \beta \ and \ \gamma$ is 1, we got $\alpha = 159733$.

Now, we did the following manipulations to obtain the final password or x.

$$
\begin{aligned}
x^\alpha \cdot g^{\alpha \cdot a} &\equiv n_1^\alpha \ mod \ p \\
x^\beta \cdot g^{\beta \cdot b} &\equiv n_2^\beta \ mod \ p \\
x^\gamma \cdot g^{\gamma \cdot c} &\equiv n_3^\gamma \ mod \ p
\end{aligned}
$$

Multiplying them together, we get

$$x^{\alpha+\beta+\gamma} \cdot g^{\alpha \cdot a + \beta \cdot b + \gamma \cdot c} \equiv n_1^\alpha \cdot n_2^\beta \cdot n_3^\gamma \ mod \ p$$

Plugging in the equations above for $\alpha, \beta \ and \ \gamma$, we get

$$x \equiv n_1^\alpha \cdot n_2^\beta \cdot n_3^\gamma \ mod \ p$$

Hence, we wrote a program to calculate the above expression modulo p and got the value of x to be $36085288503684007860367 25$.

$$\begin{aligned} x * g^a &\equiv n_1 \ mod \ p \\ x * g^b &\equiv n_2 \ mod \ p \\ x * g^c &\equiv n_3 \ mod \ p \end{aligned}$$

$$\begin{aligned} g^a &\equiv x^{-1} * n_1 \ mod \ p \\ g^b &\equiv x^{-1} * n_2 \ mod \ p \\ g^c &\equiv x^{-1} * n_3 \ mod \ p \end{aligned}$$

Again, as $gcd(a, b) = 1$, there exists a solution to the equation

$$\alpha \times a + \beta \times b + c = 1$$

Solving the diophantine equation using Extended euclid's algorithm on this diophantine equation in α and β,
$\alpha \ = \ 6953272 \ , \ \beta \ = \ -204768$

$$\begin{aligned} g^{\alpha \cdot a} &\equiv x^{-\alpha} * n_1^{\alpha} mod \ p \\ g^{\beta \cdot b} &\equiv x^{-\beta} * n_2^{\beta} \ mod \ p \\ g^c &\equiv x^{-1} * n_3 \ mod \ p \end{aligned}$$

Multiplying the above three modulo equations,

$$g^{\alpha \cdot a + \beta \cdot b + c} \equiv x^{-(\alpha + \beta + 1)} \cdot n_1^{\alpha} \cdot \ n_2^{\beta} \cdot n_3^1 \ mod \ p$$

Which simplifies to
$g \ \equiv \ x^{-(\alpha + \beta + 1)} \cdot n_1^{\alpha} \cdot \ n_2^{\beta} \cdot n_3^1 \ mod \ p$

Which gives us the value of g as 1928472839285002394817 29, which also fits into the template for g given in the message. Hence we conclude that our solution is correct.