Supplementary Material

In this section, we show that our approach is incentive compatible, welfare maximizing, and can be computed in polynomial time.

1) Incentive compatibility:

Theorem .1. ***Incentive compatibility for dynamic intersections:*** *For each agent i for $i = 1, 2, \dots, n$ and $n = n_1 + n_2 + n_3 + n_4$ where n_j represents the number of vehicles on the j^{th} arm, bidding $\boldsymbol{b}_i = \boldsymbol{\zeta}_i$ is the dominant strategy.*

Proof:

1) Use SSA to compute an ordering over all agents. But this can generate false bids overbids and underbids.
2) For example, an agent may be assigned to go second on account of a higher priority vehicle.
3) But the latter vehicle may not even conflict with the first vehicle. Conversely, agent i might be asked to go first, risking a collision.
4) Therefore, it is important to identify conflicting and non-conflicting lane groups. Run the SSA in conflicting groups only.
5) At this point, we can say that since we have optimized conflicting groups and non-conflicting groups do not collide anyways, the resulting auction is incentive-compatible

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2) Welfare maximization: The next desired property in an optimal auction is welfare maximization [2], [3] which maximizes the total utility earned by every active agent.

Theorem .2. ***Welfare maximization for dynamic intersections:*** *Social welfare of an auction is defined as $\sum_i \boldsymbol{\zeta}_i \boldsymbol{\alpha}_i$ for each agent i for $i = 1, 2, \dots, n$ and $n = n_1 + n_2 + n_3 + n_4$ where n_j represents the number of vehicles on the j^{th} arm. Bidding $\boldsymbol{b}_i = \boldsymbol{\zeta}_i$ maximizes social welfare for every agent.*

Proof:

We can show that welfare is maximum for each set of conflicting lane group. We need to show that social welfare of the system is maximized when the welfare of each individual conflicting lane group is maximized. Our proof is based on induction. Base case ($n = 1$): for $n = 1$, this reduces to the static auction case [1]. Hypothesis: Suppose the current system consists of $n > 1$ conflicting groups and that the social welfare of this system is maximized. Then we want to show that the addition of $|n+1|$ agents (belonging to the $(n+1)^{\text{th}}$) conflicting group also maximizes social welfare of the system. Now, welfare of system with n conflicting groups is

$$\sum_{i=1}^{n} \sum_{j=1}^{|i|} \zeta_j^i \alpha_j^i$$

From the hypothesis, we know that the above is maximum. Now if the $(n+1)^{\text{th}}$ conflicting group is added, then we can maximize that using the static SSA auction [1]. The resulting sum is a sum of two optimized terms, so the resulting welfare of the system remains maximum. ■

Finally, we propose a novel strategy to address overflow by transferring a portion of the higher priority agent's bid to the lower priority agent i. We denote such a transfer by $(\hat{a}, \hat{b} = a \xrightarrow{c} b)$, where $\hat{a} = a - c, \hat{b} = b + c$. More formally,

Theorem .3. ***Overflow prevention:*** *In a current SSA, suppose there exists $(i, j) \in [n_k] \times [n_k], k = 1, 2, 3, 4$, such that $p_i > p_j$ and $s_i[t] > s_j[t]$. Let $\hat{\boldsymbol{\zeta}}_i, \hat{\boldsymbol{\zeta}}_j = (\boldsymbol{\zeta}_i \xrightarrow{q} \boldsymbol{\zeta}_j)$. If*

$$q < \boldsymbol{\zeta}_i \left(1 - \frac{\alpha_{i+m}}{\alpha_i}\right) - \sum_{s=i}^{i+m-1} \boldsymbol{\zeta}_{s+1} \left(\frac{\alpha_s - \alpha_{s+1}}{\alpha_i}\right),$$

the new SSA with $\hat{\boldsymbol{\zeta}}_i, \hat{\boldsymbol{\zeta}}_j$ as the new priority values for i, j is incentive compatible.

Proof:

The difference in utility for agent i by transferring some of her valuation to agent j is given by the following equation. Here, we assume that by executing this transfer, agent i gained a jump of m:

$$\begin{aligned}\Delta u = (v_k - q)\alpha_k - \big[v_{k+1}(\alpha_k - \alpha_{k+1} + v_{k+2}(\alpha_{k+1} - \alpha_{k+2} + \dots \\ + v_{k+m}(\alpha_{k+m-1} - \alpha_{k+m})\big] - v_k \alpha_{k+m}\end{aligned} \tag{1}$$

Rearranging the terms, $\Delta u > 0$ if,

$$\begin{aligned} v_k(\alpha_k - \alpha_{k+m}) - \sum_{j=k}^{k+m-1} v_{j+1}(\alpha_j - \alpha_{j+1}) > q\alpha_k \\ \implies q < v_k(1 - \frac{\alpha_{k+m}}{\alpha_k}) - \frac{1}{\alpha_k} \sum_{j=k}^{k+m-1} v_{j+1}(\alpha_j - \alpha_{j+1}) \end{aligned} \tag{2}$$

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References

[1] Rohan Chandra and Dinesh Manocha. Gameplan: Game-theoretic multi-agent planning with human drivers at intersections, roundabouts, and merging, 2021.

[2] Tim Roughgarden. *Twenty lectures on algorithmic game theory*. Cambridge University Press, 2016.

[3] Muhammed O Sayin, Chung-Wei Lin, Shinichi Shiraishi, Jiajun Shen, and Tamer Başar. Information-driven autonomous intersection control via incentive compatible mechanisms. *IEEE Transactions on Intelligent Transportation Systems*, 20(3):912–924, 2018.