FRM Part I Exam

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Study Notes - Financial Markets and Products

Last Updated: Jul 20, 2022

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Reading 25: Banks

After completing this reading, you should be able to:

- Identify the major risks faced by a bank and how these risks can arise.
- Distinguish between economic capital and regulatory capital.
- Summarize Basel Committee regulations for regulatory capital and their motivations.
- Explain how deposit insurance gives rise to a moral hazard problem.
- Describe investment banking financing arrangements, including private placement, public offering, best efforts, firm commitment, and Dutch auction approaches.
- Describe the potential conflicts of interest among commercial banking, securities services, and investment banking divisions of a bank and recommend solutions to the conflict of interest problems.
- Describe the distinctions between the “banking book” and the “trading book” of a bank.
- Explain the originate-to-distribute model of a bank and discuss its benefits and drawbacks.

Types of Banking

- **Commercial Banking**: involves the trading activities of receiving deposits and making loans. Commercial banking can either be retail (dealing with private individuals and small businesses) or wholesale (dealing with large corporations)
- **Investment Banking**: involves raising capital for companies, advising companies on mergers and acquisitions, and acting as broker deals to trade securities.

The Major Risks Faced by a Bank

Credit Risk

This is the risk that borrowers will fail to meet their obligations in line with agreed terms.

Credit risk arises when:

- A borrower defaults
- Some bank contracts that trade on derivatives may also give rise to credit risk. If the counterparty to a derivatives transaction defaults, the bank has a positive value, implying a negative value to the counterparty.

To hedge against credit risk, a bank builds expected losses into the interest charged on loans. Assuming a bank's cost of funds is 1.3%, the bank may charge 4.2% as the interest rate on loans. The difference (2.9%) is known as the net interest margin. If a bank predicts a loss of 1% of what it lends, the bank will remain with 1.9% to cover its other costs, for example, administration costs.

Market Risk

This is the risk of losses in a bank's trading book due to exposures to market variables. These market variables are also called risk factors, and they include changes in stock prices, interest rates, foreign exchange rates, commodity prices, and credit spreads.

Banks can use any of the three options below to hedge against market risk:

1. **Spot transactions**: A currency, say the USD, is bought or sold for immediate delivery.
2. **Forward contracts**: The price and amount of a commodity traded on a specified future date are agreed upon.
3. **Options**: The buyer/holder has a right but not an obligation to buy the underlying asset in the case of a call option, and a right but not an obligation to sell in the case of a put option.

Operational Risk

This is the possibility of loss resulting from failed internal processes, systems, people, or external events.

The following are some of the categories of operational risk identified by regulators:

- Internal fraud - e.g employees stealing from a bank.
- External fraud - which includes cyberattacks, bank robberies, etc.
- Clients, products, and business practices e.g. money laundering.
- Damage to physical assets such as computers and ATM machines
- System failures and business disruptions.
- Problems in execution, delivery, and management of business processes, e.g. data entry errors.

Liquidity Risk

This describes the risk resulting from the lack of a ready market for an asset, which in turn raises the specter of being unable to meet day-to-day funding needs.

Reputational Risk

Reputational risk refers to the potential for adverse public perception, negative publicity, or uncontrollable events to have a negative impact on a company's **reputation**, thereby affecting its revenue.

Economic Capital vs. Regulatory Capital

Banks need capital to fund their operations. Some of the capital is raised in the form of equity with the rest of it raised as debt.

Equity capital is sometimes referred to as going concern capital because it absorbs losses while the bank is a going concern. Providers of debt capital enjoy some protection because the bank has an obligation to meet contractual demands even when the bank is not doing well. Debt capital is therefore referred to as gone concern capital because holders only incur losses once the bank has been declared a failure or wound up.

To ensure that its activities proceed uninterrupted, a bank has to ensure that it maintains enough capital at all times. Although banks are allowed to use their internal models to estimate the amount of capital they need, regulators also have their capital requirements that must be met. This brings us to economic capital and regulatory capital.

Economic capital is a bank's own capital estimate of the amount needed to remain solvent and maintain its day-to-day operations. One of the main motivations for calculating economic capital is to obtain and maintain a high credit rating. This way, the bank will stir up confidence among both depositors and investors.

Regulatory capital is the minimum amount of capital a bank is required to hold by the bank regulators. Regulatory capital for credit risk is designed to sufficiently cover a loss expected to be exceeded only once every 1000 years. Capital requirements have evolved in recent years and are now enforced more strictly than they were in the past. If a bank's equity capital is USD 5 billion and there is a 1% chance that the bank will incur a loss higher than USD 5 billion over a year, both regulators and the bank itself will consider the equity capital insufficient.

Basel Committee Regulations for Regulatory Capital and Their Motivations

The Basel Committee was established in 1974. It is the primary global standard-setter for the prudential regulation of banks and provides a forum for banks and regulators worldwide to exchange ideas.

The motivations for Basel Committee regulations include:

Different capital requirements: Prior to 1988, capital calculation differed from country to country. This made cross-country comparisons and cross-country implementation of risk management tools impossible. In 1974, Basel regulations were introduced in an attempt to harmonize the calculation of capital across countries.

Evolution of bank activities: Initially, Basel regulations were designed to help banks cover losses arising from **credit risk**, particularly from defaults on loans and derivatives contracts. But with time, bank trading activities significantly increased, bringing about **market risk**. In response, the Basel committee in 1998 introduced modifications in the existing regulation, and banks were now required to keep capital for both credit and market risk. Later on, in 1999, **operational risk** was added into the portfolio of risks for which risk capital was required.

The 2007-2008 financial crisis: After several large banks went down during the crisis, the Basel committee pointed an accusing finger at the existing market risk capital requirements. The market risk capital framework was deemed insufficient. This led to yet another update of the regulations.

It's important to note that Basel Committee regulations are always being reviewed to reflect the changing needs of the banking sector. In fact, some of the latest proposals in Basel III are not expected to have been implemented in full before 2027.

In a nutshell, here's how different versions of the Basel Committee guidelines impacted the financial sector:

Basel I: All signatory countries pledged to calculate credit risk capital in the same manner.

Basel II: – Capital requirements for operational risk were introduced.

Basel 2.5– Capital requirements for calculating market risk were introduced.

Basel III – Equity capital was significantly increased.

The Link Between Deposit Insurance and Moral Hazard

When a bank becomes insolvent, depositors may end up losing a percentage of their money. However, in most developed countries, the government guarantees that if a bank fails, the bank's depositors will be in line for compensation. This is known as **deposit insurance**. Usually, depositors are able to receive a percentage of their deposit subject to a predetermined upper limit. For example, the U.K. government provides deposit insurance to most banks up to a limit of £85,000.

Moral hazard describes the fact that by being insured, customers will take little or no interest at all in the way a bank handles their money. After all, the depositors are assured of getting their money back even if the bank fails. In turn, the bank may relax its lending standards and its general policy on how it uses customer deposits.

In the absence of deposit insurance, depositors would maintain a keen eye on the bank's actions so that it does not engage in activities that may endanger their money. For instance, a depositor will be keen to scrutinize the loans being offered, the conditions required for credit, and the capital set aside to serve as a buffer against economic losses. However, in a system with deposit insurance, a lack of scrutiny means that banks are free to lend as much as they want to whomever they wish, besides investing in other income-generating assets of their choice.

Different Investment Banking Financing Arrangements

Investment banking mainly deals with the raising of debt and equity financing for corporations or governments. A typical arrangement starts with a corporation approaching an investment bank with a request for help in raising a specified amount of money. The two entities then agree on the form of finance desired – debt or equity – and the investment bank underwrites the issue. This means that the bank agrees to approach investors and ask them to subscribe to the issue. The bank sells the securities to investors. For example, an IPO would involve the sale of shares to investors.

The arrangement to sell the securities can take one of several forms:

I. **Private Placement:** The securities are sold to a small number of chosen investors. In other words, the sale is closed to the general public. Private placements are considered relatively cost-effective because they do not involve "going public" together with the associated costs, such as roadshows and ads. "Series A" Funding Rounds or "Series B" investments are examples of private placements.
   A price for the offering is determined by assessing the value of the issuer, then dividing this value by the number of securities to be offered. However, the offer price is usually less than the fair value of the issuer to increase the chances of a full subscription (all securities getting sold successfully).

II. **Public Offering:** A public offering involves the sale of equity shares or some other financial instruments to the public. In the U.S., this type of arrangement is subject to approval by the Securities and Exchange Commission. A public offering can take the form of a best-effort or a firm commitment. On a **best efforts** basis, the bank does as much as it can to place the securities with investors. The bank receives a fee that, in part, depends on the success of the placement. On a **firm commitment basis**, the investment bank buys the securities from the issuer and attempts to place them with investors. This type of arrangement is riskier for the bank because if it fails to resell all the securities, it will be forced to hold them itself or sell them at a lower price resulting in losses. The profit made on a firm commitment basis is the difference between the subscription price and the price paid to the issuer.

III. **Dutch Auction:** In a dutch auction, the price of the offering is set after taking into consideration all bids to determine the highest price at which the offering can be sold. In their bids, investors indicate the number of securities they are prepared to buy, and the price they are willing to pay for each. Securities are allotted to investors in order of bid prices, where the highest bid is considered first, then the next highest, until all the securities have been allotted. However, it's important to note that all investors pay the same price – the bid price. This is usually the lowest bid acceptable. A Dutch auction is meant to balance supply and demand in the market, therefore the price before and after an IPO (Initial Price Offer should be the same).

Apart from investment banking, banks engage in other income-generating activities. These include:

- **Advisory Services:** This entails giving advice to companies on mergers and acquisitions, restructuring, and divestments. The client could be the target or even the acquirer.

- **Securities Trading:** A majority of banks involve themselves in securities trading through the brokerage of both equity and debt instruments. Most investments are, however, short-term to ensure the bank has enough liquidity.

Initial Public Offering (IPO)

An IPO is the first time offering of a company's shares to the public. Before an IPO, a company's shares are held by its founders, venture capitalists, and those who provided early-stage funding.

Before the IPO, the shares of the company do not trade on any official exchange, and so it is difficult to estimate just what the share price will be after the IPO. To get a good estimate of the share price, the company must divide the estimated value of the company after the IPO (including the money raised) by the total number of shares it wishes to create. Most banks typically set the offering price slightly below their best estimate to increase the chances of success for the offering. After an IPO, the share price typically increases. The indication is that the issuer could probably have raised even more money by setting a slightly higher offer price. However, too high an offer price, and possibly flawed investor expectations, can result in a drastic stock price fall.

IPOs are a good investment but unaffordable to small investors.

Potential Conflicts of Interest in Banking

1. When giving investment advice, a bank might be tempted to recommend the securities being sold by its investment banking wing, even if such securities do not fit the profile of the customer.
2. The research division may mark a share as “buy” just to impress the management and create business for the investment banking division. This often happens when the research team is under pressure from management.
3. If a bank obtains confidential information that suggests one of its corporate borrowers may default in the near future, the bank may be tempted to push for floatation of a bond by the borrower, sometimes very aggressively. The bank would then use the proceeds to pay off the loan.
4. During the appraisal process for credit, banks oft obtain lots of information about the borrower. A bank may be tempted to pass that information to the investment banking division to help it provide advice to a potential acquirer.

In an attempt to avoid conflict of interest, some banks have Introduced **formal information barriers** where members of the investment banking division are barred from communicating directly with their research division counterparts. Some companies have gone as far as requiring any communication between the two divisions to happen only through the compliance department.

Distinctions Between the “Banking Book” and the “Trading Book” of a Bank

The **banking book** consists of assets on the bank's balance sheet that are expected to be held until maturity. In other words, the bank cannot sell them. Items in the banking book are subject to credit risk capital calculations. The VaR for assets in the banking book is measured at 99.9% confidence on a 1-year time horizon.

The **trading book** consists of assets available for sale, meaning that they are eligible for day-to-day trading. Under Basel II and III, the trading book has to be marked to market on a daily basis. In addition, the VaR for all assets making up the trading book has to be measured at 99% confidence on a 10-day time horizon. Items in the trading book are subject to market risk capital calculations.

If a bank has a desk for trading an instrument, that instrument falls under a trading book. Otherwise, it falls under a banking book.

The Originate-to-Distribute Model

Historically, banks used to originate loans and then keep them on their balance until maturity. That was the originate-to-hold model. With time, however, banks gradually and increasingly began to distribute the loans by selling them as securities to investors. By so doing, the banks were able to limit the growth of their balance sheet by creating a somewhat autonomous investment vehicle to distribute the loans they originated.

Advantages of the Model:

- It introduces specialization in the lending process. Functions initially designated for a single firm are now split among several firms.
- It reduces the banks' reliance on the traditional sources of capital, such as deposits and rights issues.
- It introduces flexibility into the banks' financial statements and helps them diversify some risks.

Disadvantages of the Model:

- Allowing banks to hive off part of their liabilities can result in the relaxation of lending standards and contribute to riskier lending. This implies that borrowers who previously would be turned away - possibly because of poor credit history - are now able to access credit.
- By splitting functions among multiple firms, the model can make it difficult for borrowers to renegotiate terms.
- The assets (loans) retained in the balance sheet become increasingly less representative of the role they play in the process of extending credit. In other words, the role and impact of banks as lenders in an economy are obscured.

Questions

Question 1

ABC Corp wishes to sell10 million shares using a Dutch auction. The underwriter starts the auction by offering a price of $50 per share. The following bids are received:

Price	Bids	Shares
$50	1	2,000,000
$48	2	1,000,000
$47	1	2,000,000
$45	2	2,000,000
$44	3	1,000,000
$42	5	3,000,000

Determine the price that will be paid by all the successful bidders.

A. $50

B. $45

C. $42

D. $44

The correct answer is B.

After the auction closes, the underwriter will calculate the highest price at which all shares could possibly be sold. Here, the auction wound up with bids for 28 million shares. However, the highest bids adding up to 10 million shares will be the winning bids. The price will be set equal to the lowest winning price bid on the 10 million shares.

At $50/share, 1 bid comes in for 2,000,000 shares. The underwriter will lower the price to $48/share, where 2 more bids come in for another 2,000,000 shares. The underwriter will yet again lower the price to $47/share, where 1 bid comes in for 2,000,000 shares. And after lowering the price to $45, 2 more bids come in for 4,000,000 shares. This makes a total of 10,000,000 shares. Thus, $45 is the lowest winning price bid, and all successful bidders will pay $45/share.

Question 2

Which of the following options best describes the link between deposit insurance and moral hazard?

A. The possibility of a surge in deposits at a bank due to increased trust and confidence among depositors

B. An increase in the deposit of funds with questionable sources, i.e., laundered cash

C. Relaxed lending standards at a bank in the knowledge that customers are well protected from incurring losses

D. Increased supervision and monitoring of banks resulting from pledges, by the government, to compensate depositors if the bank fails

The correct answer is C.

Moral hazard describes the fact that by being insured, customers will take little or no interest at all in the way a bank handles their money. After all, the depositors are assured of getting their money back even if the bank fails. In turn, the bank may relax its lending standards and its general policy on how it uses customer deposits.

Reading 26: Insurance Companies and Pension Plans

After completing this reading, you should be able to:

- Describe the key features of the various categories of insurance companies and identify the risks facing insurance companies.
- Describe the use of mortality tables and calculate the premium payment for a policyholder.
- Distinguish between mortality risk and longevity risk and describe how to hedge against these risks.
- Describe a defined benefit plan and a defined contribution plan for a pension fund and explain the differences between them.
- Compare the various types of life insurance policies.
- Calculate and interpret loss ratio, expense ratio, combined ratio, and operating ratio for a property-casualty insurance company.
- Describe moral hazard and adverse selection risks facing insurance companies, provide examples of each, and describe how to overcome the problems
- Evaluate the capital requirements for life insurance and property-casualty insurance companies.
- Compare the guaranty system and the regulatory requirements for insurance companies with those for banks.

Categories of Insurance

An insurance contract is an agreement between an insurer and a policyholder, where the latter receives protection against adverse events in exchange for premiums. Insurance takes two main forms: **Life assurance** and **nonlife (property) insurance**. (Since we cannot put a price/value tag on human life, the word "insurance" is replaced with assurance). Property insurance is renewed yearly, and the premiums payable may vary from year to year.

Types of Life Assurance

Term Life Assurance

A term life assurance contract is a contract to pay the beneficiary a predetermined amount of benefit, also called the sum assured, in case the policyholder dies within the term of the contract. For example, if the contract starts today and remains in force for the next 10 years, the sum assured will **only** be payable if the policyholder dies within the next 10-year period. If the policyholder survives to the end of the term, the contract comes to an end without any form of compensation.

Whole Life Assurance

Under a whole life contract, the sum assured is payable when the policyholder dies, regardless of when that happens. It provides protection for the life of the policyholder. Premiums are paid **throughout** the life of the policyholder. Unlike in a term life contract (where there's no certainty that the sum assured will be paid), the sum assured **is certain to be paid at some point in the future**, provided the policyholder continues to make the required premium payments up to the point of their death.

Variable Life Assurance

A variable life assurance policy is a type of whole life assurance with an investment component. A portion of the premium payable is invested in several sub-accounts available in the policy. For example, let's say John buys a variable life assurance policy where he pays an annual premium amounting to $10,000. The contract could be designed in such a way that $5,000 goes toward the sum assured (death benefit), say, $1 million, and the other $5,000 is invested in various instruments. Thus, the total benefit received on the death of the policyholder will be the sum assured plus a variable amount generated from the investment account.

Universal Life

Just like variable life assurance, a universal life contract is a type of whole life assurance with an investment component. However, a universal contract gives the policyholder a lot **more flexibility** in terms of the premium payable. The premium can even be reduced to a pre-specified minimum without the policy lapsing. Lapsing occurs when a policyholder quits paying premiums resulting in the withdrawal of the policy. Reducing premiums will, however, reduce the expected benefits.

Variable-Universal Life Assurance

This contract incorporates the benefits of both universal and variable life assurance.

Endowment Life Assurance

Under an endowment contract, the sum assured is payable either when the policyholder dies or at the end of the specified period, whichever comes first. There are many variants of endowment life contracts in the market today. Some may even have an investment component. Others may precondition payment of the sum assured on the survival of the policyholder to the end of the period.

Group Life Assurance

A group life assurance contract covers multiple persons, usually employees in a company. The policy could be contributory, in which case the premium payable is shared between the employer and the employee. In other cases, it could be non-contributory, meaning that the employee is obliged to pay the full premium amount.

Annuity Contract

An annuity is a contract that requires the policyholder to pay a lump sum. In return, the policyholder receives a regular series of payments at specified points in the future. This regular stream of payments is called an annuity. The annuity starts at a particular date and lasts for the rest of the policyholder's life. The annuity could start immediately after the lump sum has been paid. In other cases, it could start, say, 5 years after payment of the lump sum. Such a contract is called a deferred annuity. The insurance company funds the annuity by investing the lump sum in an investment vehicle of their choice, including secured bonds and mutual funds. An annuity helps the policyholder to defer the tax payable until they receive each scheduled annuity payment. The amount to which a policy holder's funds grow in an annuity contract is called the accumulation value.

Investments

Funds to be invested are provided by life insurance policies and annuity contracts. The funds are mostly invested in bonds. Sometimes the maturity of the bonds can be made to match the maturity of the liabilities. Companies will prefer government bonds to corporate bonds to avoid credit risk, but the return on corporate bonds is usually higher.

Mortality Tables

For each age, a mortality table gives the probability of a person at that age dying before their next birthday. In other words, it gives the survivorship of people from a given population. The mortality rate among men is different (and usually higher) from that of women. Thus, mortality tables are constructed separately for men and women.

Insurance companies use mortality tables to price insurance products, assess pension plan obligations, and project future insured events. Here is some important information on how to read a mortality table:

- **Probability of death within a year**: Provided that an individual is alive at year N, it is the probability that death will occur in the year following N.
- **Survival probability**: It is the cumulative probability of an individual living to year N. The survival probability at year 0 is therefore 1.
- The probability of survival to 50 years is the probability of survival to year 49 and the probability of death in year 50.
- P (survival until age N+1) = P (Survival until age N) × (1-Probability of death at age N+1)

The following is an extract from the mortality table developed by the U.S. Department of Social Security for 2015.

Period Life Table, 2015

		Male			Female	
Exact age	Probability of death within 1 year	Male number of lives	Life expectancy	Probability of death within 1 year	Female number of lives	Life expectancy
0	0.006383	100,000	76.15	0.005374	100,000	80.97
1	0.000453	99,362	75.63	0.000353	99,463	80.41
2	0.000282	99,317	74.67	0.000231	99,427	79.44
3	0.000230	99,289	73.69	0.000165	99,405	78.45
4	0.000169	99,266	72.71	0.000129	99,388	77.47
5	0.000155	99,249	71.72	0.000116	99,375	76.48
...	...	...	...	...	...	...
30	0.001626	97,393	47.75	0.000740	98,588	51.95
31	0.001669	97,235	46.82	0.000792	98,515	50.99
32	0.001712	97,072	45.90	0.000841	98,437	50.03
33	0.001755	96,906	44.98	0.000886	98,354	49.07
34	0.001800	96,736	44.06	0.000929	98,267	48.11
35	0.001855	96,562	43.14	0.000977	98,175	47.16

Source: https://www.ssa.gov/OACT/STATS/table4c6.html

Interpreting the table:

Consider the row corresponding to age 3. The second column gives the probability of a male aged exactly 3 dying within the next year (0.000230). The third column gives the number of male lives out of a cohort of 100,000 lives that attain age 3 (99,289). Finally, the fourth column shows that a male aged 3 has a remaining life expectancy of 73.69 years. When interpreted, that means on average, 3-year-old males will live to age 76.69 ($= 73.69 + 3$).

We can interpret the rest of the data in a similar manner.

Calculating the Premium Payable

Example: Calculating the Premium

Robert Myer, aged 30, buys a 2-year term assurance contract with a sum assured of $100,000.

Interest rates for all maturities are 6% per annum (with semiannual compounding), and premiums are paid annually in advance (at the beginning of the year). Calculate the break-even premium.

Solution

The present value of income (premiums payable) = present value of outgo (expected payouts) …………………. Equation 1

Working out the left side of equation 1,

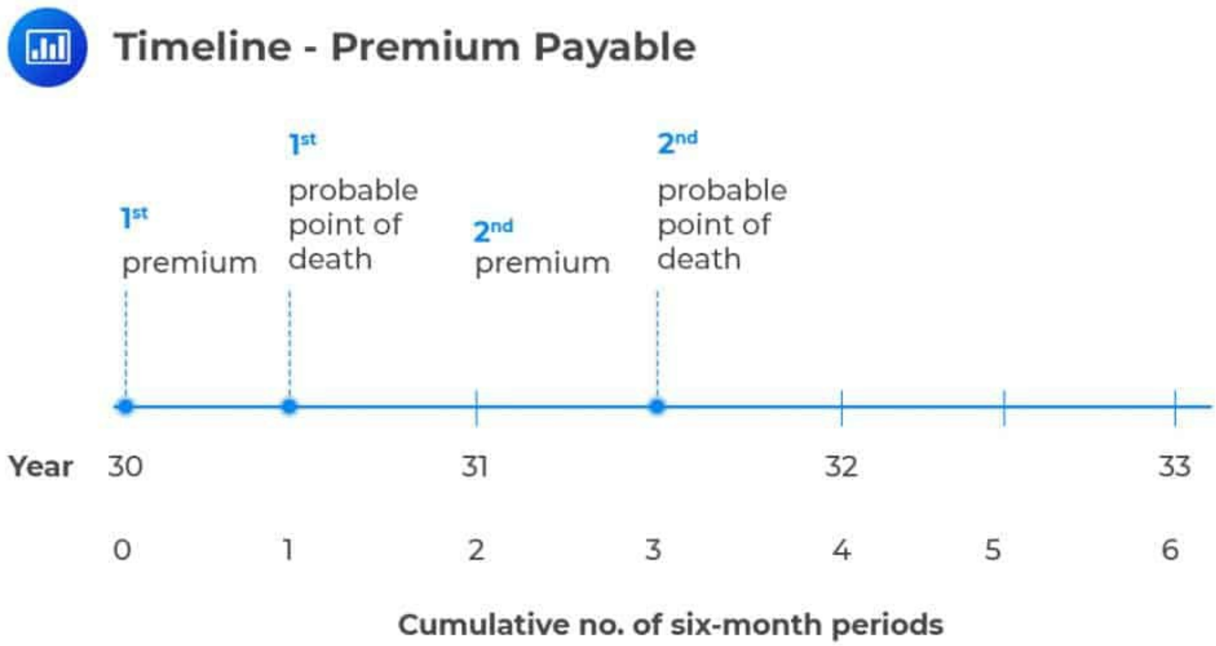
Let the annual premium payable be X. Since the first premium is paid immediately after the contract is signed, and its present value is still X. The probability of the second premium payment being made at the beginning of the second year is the probability that the man does not die during the first year (= 1–0.001626).

Hence,

$$\text{PV of income} = X + \frac{0.998374X}{1.03^2} = 1.94106X$$

Working out the right side of equation 1,

The expected payout if Robert dies in the first year is $\$162.60\,(= 0.001626 \times \$100,000)$. The expected payout if Robert dies in the second year is $\$166.63\,[= (1 - 0.001626) \times 0.001669 \times \$100,000]$. We must discount these to time 0. In most cases, the assumption is that death occurs midway through the year, and the benefit is payable immediately on death. Thus, we discount the first expected payout for 6 months and the expected payout in year 2 for 18 months.



Hence,

$$\text{PV of outgo} = \frac{162.6}{1.03} + \frac{166.63}{1.03^3} = \$310.35$$

Thus,

$$1.94106X = 310.35$$

$$X = \$159.9$$

Property and Casualty Insurance

Property insurance provides protection against damage to property, while casualty insurance provides coverage for liabilities arising from damages incurred by others arising from the actions of the insured party.

Property and casualty insurance allows an insurer to increase premiums if the risk of the insured event increases.

The risks faced by a property and casualty insurance company include:

1. **Catastrophic risks arising from many claims arising at once:**

   To insure against catastrophic risks, an insurance company may either reinsure itself or use CAT (Catastrophe) Bonds derivatives. CAT bonds are issued by insurance companies and pay a high interest rate.

2. **Risk of independently settling claims after using historical data to predict payouts:**

   Assume that two drivers, drivers A and B, have been placed in one risk category and given the same expected payout. It is highly unlikely that both drivers will make a claim arising from a road accident. Driver A's claim may be due to a road accident, while driver B's claim may be due to a different risk, say car burglary. These two drivers will receive different settlements despite being placed in the same risk category, with similar expected payouts.

Ratios in Property/Casualty Insurance

The **loss ratio** is the ratio of payouts made to premiums earned in a year. A ratio of 50 means that for every $1 of the premium received, a payout of $0.50 is given. The remaining $0.50 consists of profits and any other expenses. A high loss ratio indicates poor financial health. It implies that the insurer may not be collecting enough premiums to pay claims, expenses and still make a sizeable profit.

The **expense ratio** is the ratio of expenses to premiums earned in a year. It shows how efficient the insurer is in terms of cash management **before** factoring in claims and investment gains or losses. Expenses comprise ad fees, employee wages, commissions for sales agents, etc.

The **combined ratio** is the sum of the loss ratio and the expense ratio. For example, if the loss ratio is 75% and the expense ratio is 40%, the combined ratio is 115%. Suppose an insurer has a combined ratio of more than 100%. What does this imply? It means the insurer has had more losses plus expenses than earned premiums, i.e., it has lost money on its operations. But if the combined ratio is less than 100, it implies the insurer is profitable,i.e., it has more earned premiums than losses plus expenses.

A **gross ratio** consists of the combined ratio and any dividends paid to the shareholders. Assuming dividends of 2% are paid in the above scenario, then the gross ratio will be 117%.

The presence of interests reduces the gross ratio to form the **operating ratio**. Suppose an interest of 5% was earned in the above scenario; the operating ratio would then be 112% (117%-5%).

Major Expenses in an Insurance Company

1. Selling expenses
2. Loss adjustment expenses: expenses incurred by determining the validity of a claim

Health Insurance

In health insurance, premiums should only increase due to an increase in the cost of health and not due to increased risks to the policyholder that were absent at the onset of the policy.

Moral Hazard and Adverse Selection in Insurance

Moral hazard describes the risk that once individuals sign up for an insurance contract, they will behave differently than they would without the insurance. In other words, they will have an incentive to take risks they would otherwise not dare take. For example, in the presence of deposit insurance, banks will have an incentive to take more risks because the risk of losing customers is minimal.

The moral hazard increases the expected payout of the insurer. Some of the strategies adopted to mitigate moral hazard include:

- Use of **deductibles** such that the first part of a loss is covered by the insured
- Use of **co-insurance** provisions where the insurer can only cover a certain percentage

(less than 100%) of losses after taking deductibles into account.

- Use of **policy limits** so that there's a cap on the maximum payout

Adverse selection is the risk that a company may attract bad risks more than it attracts good risks. If the insurer sells a product at the same price to everyone, it might inadvertently attract more of the bad risks. For example, if smokers and non-smokers are charged the same premium for a whole life policy, the insurer will most likely attract more smokers.

Just like moral hazard, adverse selection increases the chances of claims overwhelming the insurer, something that can lead to insolvency.

The best way to mitigate the risk of adverse selection has much to do with making sure that **the insurer gathers as much information about prospective policyholders as possible**. That way, they will be more likely to unearth certain facts that will help them to price the policy appropriately. For example, insurers are normally very keen to dig into the health history of all prospective whole life policyholders. If the individual has a hereditary condition, for example, the premium payable will have to be adjusted upwards accordingly.

Mortality Risk and Longevity Risk

The mortality risk is the risk that people may die sooner than predicted. Mortality risk will negatively affect whole life policies as fewer premiums than anticipated will have been paid at the time of death. This has the potential to negatively impact insurance payouts because there will be more deaths than initially anticipated. On the other hand, increased mortality risk increases the profitability of annuity contracts because the policyholders end up receiving fewer scheduled payments than initially anticipated (most annuities lapse once the policyholder dies).

Longevity risk is the risk that a person will live longer than initially expected. Thanks to the rapid advancement in medical science and improved lifestyle among people. Increased longevity risk adversely affects annuity contracts because the sponsor ends up parting with more payments than initially anticipated. However, increased longevity will improve the profitability of life insurance contracts because the insureds will end up paying more and more premiums. (bear in mind that the benefit remains fixed all through).

Insurance companies hedge mortality risk through a combination of **careful pricing** and **reinsurance**, where they share the pooled risks with a third-party insurer. Longevity risk can be hedged using **longevity derivatives**. A typical derivative here is the **longevity bond**. Under this bond, a population is defined. The coupon payable as of a particular date is a function of the number of people still alive at that point.

Main Risks Facing Insurance Companies

In readiness for claims, insurance companies set aside funds generated from the premiums paid as well as from various investments. These funds are known as reserves.

The number one risk facing insurers is the risk that policy **reserves are not sufficient to cover the forwarded claims**. To mitigate this risk, actuaries tend to be **fairly conservative** when calculating the reserves needed.

Insurers also have to contend with the risk that their **investments will perform poorly**. Since most of such investments are in corporate bonds, an increase in default rates spells doom for insurers. This risk can be mitigated by diversifying investments over a range of industries or sectors in an economy.

Also, insurers are constantly faced with **liquidity risks**. Bonds can be quite illiquid, especially those offered via private placement. It may be impossible to quickly convert such bonds into cash in the face of an upsurge in claims.

Lastly, insurance companies face **credit risk** thanks to their transactions with banks and reinsurance companies.

Defined Benefit Plans vs. Defined Contribution Plans

In a **defined benefit plan**, the amount paid to the employee at retirement is specified by the plan. In addition, it's the employer who sponsors the plan in its entirety. Typically, the benefit payable is a function of the years the employee has worked and their salary.

For example, the benefit may be equal to the average earnings of the employee in the final three years of employment multiplied by the number of years worked. The employee has little control over the funds until they are received at retirement. The employer bears investment risk – they have to ensure that there are sufficient funds to pay the employee at retirement.

In a **defined contribution plan**, both the employee and the employer contribute toward the plan, and the total amount is invested in a range of stable, secure investments, usually mutual funds and money market funds. The amount paid to the employee at retirement depends on the performance of the investment.

The amount to which the invested funds have grown can even be converted into an annuity. Since the employer has little control over the funds' performance, defined contributions are considered low-risk to the employer.

In a defined benefit plan, all funds are pooled by the employer. Payments to retirees are made from the pool. In a defined contribution plan, however, each employee has their own account, and the pension payable is determined by the performance of that account.

Regulations

Unlike banks, insurance companies have no global regulatory requirements. However, the European Union implemented Solvency 11 as its regulatory framework. It specifies a minimum capital requirement (MCR) and a solvency capital requirement (SCR). Capital should not fall below SCR. If capital falls below MCR, the insurance company may not be allowed to take up a new business, and its policies may be transferred to a different Insurance Company. Capital is calculated using both standardized and internal models.

In the United States, every state has its own regulators. However, the National Association of Insurance Commissioners provides a forum for insurance regulators to exchange ideas.

In contrast to the deposit insurance system for banks, there are no guaranteed funds to protect policyholders. If an insurance company becomes insolvent, other companies contribute to the fund.

Questions

Question 1

The following is an extract from the mortality table developed by the U.S. Department of Social Security for 2015.

Period Life Table, 2015

	Male			Female		
Exact age	Probability of death within 1 year	Male number of lives	Life expectancy	Probability of death within 1 year	Female number of lives	Life expectancy
30	0.001626	97,393	47.75	0.000740	98,588	51.95
31	0.001669	97,235	46.82	0.000792	98,515	50.99
32	0.001712	97,072	45.90	0.000841	98,437	50.03
33	0.001755	96,906	44.98	0.000886	98,354	49.07
34	0.001800	96,736	44.06	0.000929	98,267	48.11
35	0.001855	96,562	43.14	0.000977	98,175	47.16

Chris Huckabee, aged 33, buys a 2-year term assurance contract with a sum assured of $1,000,000. Interest rates for all maturities are 4% per annum (with semiannual compounding), and premiums are paid annually in advance (at the beginning of the year). The break-even premium is closest to:

A. $1,742
B. $3,400
C. $1,650
D. $1,755

The correct answer is **A**

Present value of income = present value of outgo .... Equation 1

Working out the left side of equation 1,

Let the annual premium payable be X. Since the first premium is paid immediately the contract is signed, its present value is still X. The probability of the second premium payment being made at the beginning of the second year is the probability that the man does not die during the first year (= 1– 0.001755).

Hence,

$$\text{PV of income} = X + \frac{0.99825X}{1.02^2} = 1.95949X$$

Working out the right side of equation 1,

The expected payout if the policyholder dies in the first year is: $1,755 (= 0.001755 × $1,000,000). The expected payout if they die in the second year is $1,796.84 [= (1– 0.001755) × 0.0018 × $1,000,000]. We must discount these to time 0. Assuming that death occurs midway through the year, and the benefit is payable immediately on death, we should discount the first expected payout for 6 months and the expected payout in year 2 for 18 months.

Hence,

$$\text{PV of outgo} = \frac{1,755}{1.02} + \frac{1,796.84}{1.02^3} = \$3,413.80$$

Thus,

$$\begin{aligned} 1.95949X &= 3,413.80 \\ X &= \$1,742.19 \end{aligned}$$

Question 2

Smart Insurance, a U.K. based casualty insurance firm, posted the following summary of selected key ratios:

Loss ratio	65%
Expense ratio	30%
Combined ratio	C
Dividends	X
Combined ratio after dividends	Y
Investment income	Z
Operating ratio	O

C, **X**, **Y**, **Z**, and **O** are hidden values.

Which of the following statements is most likely INCORRECT?

A. The expense ratio of 30% excludes losses from the sale of assets such as shares and bonds

B. For each $1 in premium perceived, Smart Insurance pays out about $0.95 to its policyholders

C. The combined ratio after dividends is equivalent to (C + X)

D. The expense ratio of 30% includes advertising fees and commission paid to sales managers

The correct answer is **B**.

The loss ratio is the ratio of payouts made to premiums earned in a year. It represents the percentage of every dollar in premium that goes toward payment of claims. Since the ratio is 0.65, for every $1 in premium perceived, the insurer pays $0.65 to its policyholders.

Reading 27: Fund Management

After completing this reading, you should be able to:

- Differentiate among open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs).
- Calculate the net asset value (NAV) of an open-end mutual fund.
- Understand the various potential undesirable behaviors in trading at mutual funds.
- Explain the key differences between hedge funds and mutual funds.
- Calculate the return on a hedge fund investment and explain the incentive fee structure of a hedge fund, including the terms hurdle rate, high-water mark, and clawback.
- Describe various hedge fund strategies, including long/short equity, dedicated short, distressed securities, merger arbitrage, convertible arbitrage, fixed income arbitrage, emerging markets, global macro, and managed futures, and identify the risks faced by hedge funds.
- Describe characteristics of mutual fund and hedge fund performance and explain the effect of measurement biases on performance measurement.

Fund Managers

Fund managers are responsible for investing funds on behalf of their clients, based on clients' investment goals and risk appetites. The benefits of using fund managers include:

1. Fund managers have the necessary expertise needed to invest funds;
2. Large funds make it easier for diversification as opposed to smaller funds or when investors invest on their own;
3. Transacting larger trades is relatively cheaper than transacting smaller ones.

Mutual Funds

A mutual fund is made up of **a pool of money collected from many investors**. The money is then used to invest in securities such as stocks and bonds. The funds are operated by a fund manager whose mandate is to generate income or capital gains. However, mutual funds lack tax benefits, meaning that an investor's profits are subject to being taxed.

A mutual fund can be open-end or closed-end.

Open-End Mutual Fund

In an **open-end mutual fund**, shares **are traded at their net asset value** (NAV). The net asset value is the market value of all assets the fund owns at the end of each trading day minus liabilities divided by the number of shares outstanding., i.e.,

$$\text{NAV} = \frac{\text{Market value of assets at the close of the day} - \text{Liabilities}}{\text{Number of outstanding shares}}$$

The net asset value (NAV) changes on a daily basis, usually at 4 pm every day, to reflect changes in the underlying investments, which are usually stocks and bonds. All shares are also purchased or redeemed at the NAV. In an open-end fund, one deals with the **fund itself** when buying shares.

The open-end mutual funds can be further divided into:

- Money Market funds
- Bonds funds
- Equity funds

The funds that invest in more than one security are termed **hybrid funds** or **multi-asset funds.**

Money Market Funds

In money market funds, the fund manager invests in fixed income securities with a maturity period of less than a year. These act as an alternative to savings accounts in a bank since they usually offer higher interest rates than a bank savings account.

Bond Funds

These funds invest in fixed-income securities with a maturity period greater than a year.

Equity Funds

These funds can either be actively managed funds (where investors apply their expertise to achieve the objectives of the fund) or index funds (where the equities track a specific fund, for example, the FTSE 100).

To determine how well a fund has tracked its intended index, tracking error is applied. The most popular tracking area involves obtaining the mean square error, i.e., the squared root of the mean squared difference between the return of the funds and the index (hence the tracking error is also called root-mean-square error).

Example: Calculating the Tracking Error of Index Fund

The 5-year successive returns for FTSE100 are 3%, 10%, 11%, -7% and 4.0%. The respective returns on the fund is 3.0%. 9%, 10%, -5% and 4.0%. What is the tracking error of the fund?

Solution

$$\text{Tracking Error} = \sqrt{\frac{(3\% - 3\%)^2 + (10\% - 9\%)^2 + (11\% - 10\%)^2 + (-7\% + 5\%)^2 + (4.0\% - 4.0\%)}{5}}$$

$$= 1.10\%$$

The Expense Ratio

The cost ratio is the annual management fee expressed as a percentage of the assets' value under management. Other expenses charged on the mutual funds are the **front-end load,** which is the fee charged on mutual fund investors when they buy, and **back-load,** which is the fee charged on the investors when they sell.

Closed-End Mutual Funds

In closed-end funds, the number of shares is constant throughout the time. In other words, the number of shares in a closed-end fund does not change on a daily basis. Shares are **publicly traded on an exchange**, and therefore the price is a function of supply and demand. Shares are bought and sold through **brokers**. After the initial share sale, no more shares are issued.

Differences Between Open-End and Closed-End Mutual Funds

Basis for Comparison	Open-end Funds	Closed-end Funds
Subscription	Available for subscription throughout the year.	Available for subscription only during a few specified days.
Listing	Not listed on a stock exchange. Transactions occur directly through the fund.	Listed on an exchange for trading.
Transactions	Executed at the end of the day.	Executed in real time.
Maturity	No fixed maturity.	Fixed maturity period, say, 3-5 years.
Selling price	NAV.	Premium/discount to NAV.
Corpus	Variable.	Fixed.

Exchange-Traded Funds (ETF)

Exchange-traded funds combine the features of both open and closed-ended mutual funds. To create an ETF, an investor deposits money in exchange for shares in the ETF. The shares are then traded like shares of any other company.

Investors are allowed to give up their shares in exchange for the underlying assets. Investors can also obtain additional shares by adding assets with the same components as those already in the ETF.

Undesirable Trading Behaviour

Mutual funds and ETFs are significantly regulated in most jurisdictions to ensure that complete and accurate financial information is given to prospective investors. Despite the regulation, some undesirable trading behavior in mutual funds include:

- **Late trading**: This involves using market developments that occur after 4 pm to cancel or to carry out trade. An offense that is punishable by the law.

- **Market timing**: Market timing is brought about by the existence of stale prices (prices that do not reflect recent information or those that differ due to time zone differences). Market timing is not illegal; however, it requires large funds for it to be worthwhile.

- **Front running/Tailgating**: This is where traders use acquired information to trade for themselves before trading for their firm/clients. Front running is illegal in fund management.

- **Directed brokerage**: This Involves a contract between a mutual fund and a brokerage house. The mutual fund agrees to carry its trades through the brokerage house, which agrees to recommend the fund to its clients. It is legal but frowned upon by regulators.

Hedge Funds

Hedge funds are alternative investments that utilize pooled funds and use different investment strategies to earn active returns. For instance, hedge funds use derivatives and leverage to create high returns.

Hedge funds have fewer regulations than mutual funds, can follow a diverse approach of trading strategies, and are not required to disclose their holdings on a daily basis. They, however, have additional restrictions on how to solicit funds from investors.

Other differences between mutual and hedge funds are stated below:

Basis for Comparison	Mutual Funds	Hedge Funds
Flexibility	The manager has lots of constraints to deal with, e.g., limited use of leverage.	The manager has fewer constraints. Can use leverage, sell short, or even use derivatives.
Paperwork	Offered via a prospectus.	Offered via a private placement memorandum.
Liquidity	Investors can withdraw their money any day.	Investors can only get their money periodically.
Self-investment	The manager does not have to put some of their capital in the fund.	As a sign of good faith, the manager is expected to put some of their money in the fund.
Advertisement	May advertise freely.	Not free to advertise in the public.
Listing	Maybe listed, i.e., closed-end funds.	Cannot be listed on an exchange.

Hedge Fund Fees, Hurdle Rate, High-Water Mark Clause, and Clawback

Compared to mutual funds, hedge funds charge investors higher management/operational fees. These include:

- An annual management fee of 1%-3% of assets
- An incentive fee of 15%-30% of realized net profits

A typical hedge schedule that reads “2% plus 30%,” for example, indicates that the fund charges 2% per year of assets under management and 30% of net profit. These high charges are designed to attract the best hedge managers.

The management fee is computed on the assets at the beginning of the year, and the incentive fee is calculated after subtracting the management fee.

The incentive is equivalent to a call option on the net profit generated by the funds for an investor in a given year. For instance, consider the "2% plus 30%" fee structure of a fund. Then the incentive fee is calculated as:

$$\text{Incentive Fee} = 0.3 \times \max\left(R \times A - 0.02 \times A, 0\right)$$

Where

A = assets under management at the beginning of the year.

R = return on the assets during the year.

Note that the strike price of the above analogous call option is 2% of the assets under management.

Example: Calculating the Incentive Fee, Management Fee, and Return on a Hedge Fund

Century Capital is a hedge fund with a \$100 million initial investment. The fund charges a 2% management fee based on the beginning of year assets under management and a 20% incentive fee. In its first year, the capital earns a 25% return.

What are the management fee, incentive fee, and return on the capital?

Solution

$$\begin{aligned} \text{Management Fee} &= 2\% \times 100 = \$2\text{milion} \\ \text{Incentive fee} &= 0.2 \times \max(R \times A - 0.02 \times A, 0) = 0.2 \times \max(25 - 2, 0) = \$4.60 \text{ million} \\ \text{Total fee} &= \$2 \text{ milion} + \$4.60 \text{ million} = \$6 \text{ million} \\ \text{Return on the hedge fund} &= \frac{\$125 \text{ milion} - \$100 \text{ milion} - \$6.60 \text{ milion}}{\$100 \text{ milion}} - 1 = 18.40\% \end{aligned}$$

As a precondition for imposing high incentive fees, investors may be offered several guarantees. These include:

- **Hurdle rate:** This is the minimum return that should be earned before the incentive fees are imposed.

- **High-water mark clause:** This requires the fund to recoup any prior losses before the investment manager is allowed to impose an incentive fee. Prior losses may be comprised of performance losses, management fees, and administrative fees. A **proportional adjustment clause** may apply so that if the investor suffers a loss and simultaneously withdraws part of their capital, the amount of previous loss to be recouped is adjusted proportionally.

- **Clawback:** A clawback is an action where hedge investors take back the incentive fees previously awarded to the hedge fund manager so as to offset current losses. A portion of the incentive fees is held in a recovery account so that when the investor makes a loss, they receive some compensation from that account.

Example: Calculating the Return on a Hedge Fund Investment with a Hurdle Rate

Century Capital is a hedge fund with a $100 million initial investment. The fund charges a 2% management fee based on the beginning of year assets under management and a 20% incentive fee. Moreover, the fee structure specifies a hurdle rate of 5%, and the incentive fee is based on the excess of the hurdle rate.

In its first year, the capital earns a 25% return. What are the management fee, incentive fee, and return on the hedge fund investment?

Solution

$$\begin{aligned}
\text{Management Fee} &= 2\% \times 100 = \$2 \text{ milion} \\
\text{Incentive fee} &= 0.2 \times [125 - 100 - 5 - 2, 0] = \$3.60 \text{ million} \\
\text{Total fee} &= \$2 \text{ milion} + \$3.60 \text{ million} = \$5.60 \text{ million} \\
\text{Return on the hedge fund investment} &= \frac{\$125 \text{ milion} - \$100 \text{ milion} - \$5.60 \text{ milion}}{\$100 \text{ milion}} - 1 = 19.40\%
\end{aligned}$$

Prime Brokers

A prime broker handles the transactions of a hedge fund. These transactions include: lending them money, providing risk management services, providing hedging services, and carrying out stress tests on their portfolio. Common prime brokers are banks.

Common Hedge Fund Strategies

1. **Long/Short Equity:**

   As the name suggests, the long/short equity strategy involves maintaining long and short positions in equity and equity derivative securities. The fund manager buys the stocks they feel are undervalued while simultaneously selling those they feel are overvalued. Usually, the value of shares shorted is equal to the shares bought, and the sensitivity of both long and short positions should be the same as that of the market.

2. **Dedicated Short :**

The aim of dedicated short is to earn a return by maintaining a net short position in the market through a combination of long and short positions. This means that short positions take the lion's share of the fund's overall positions. In other words, a dedicated short fund focuses on identifying overvalued stocks.

3. **Distressed Securities:**

A distressed debt strategy is an event-driven strategy that tends to focus on companies in distress (financial trouble). Positions in bonds or stocks can be both long and short. Funds that employ this strategy imposes more stringent lock-up and withdrawal terms.

4. **Fixed-Income Arbitrage:**

Fixed income strategy seeks to profit from discrepancies in related fixed income instruments. The manager might buy-long a bond that they feel is undervalued and simultaneously sell-short a similar bond they think is overvalued.

5. **Convertible Arbitrage:**

Convertible arbitrage strategy seeks to profit from discrepancies in a company's convertible securities relative to its stock. It might involve taking a long position in a company's convertible securities and simultaneously taking a short position in its stock.

6. **Merger Arbitrage:**

Merger arbitrage strategy entails taking opposing positions in two firms that are about to merge. The goal is to exploit price inefficiencies that may occur before and after a merger. In most cases, a merger announcement is followed by a spike in the stock of the acquiring company and a dip in the stock of the target. The latter is especially associated with failed bids. For a cash bid, the price paid for each stock is usually **at a premium** to the market price. Offers can take the form of cash or stock of the bidding firm. For a cash bid, the risk arbitrage position consists of buying the target's stock and then waiting for it to move to the takeover price. For a stock exchange deal, the acquirer offers to exchange each target share for Δ shares of the acquirer. The arbitrage position, therefore, consists of a long position in the target offset by a short position of Δ in the acquirer's stock.

7. **Emerging Markets:**

   The emerging-market strategy involves debt/equity investing in emerging markets. It's a strategy that aims to identify emerging market shares that are overvalued or undervalued.

8. **Global Macro:**

   Global macro strategy is a general investment strategy that involves making investment decisions guided by the economic/political outlook of a country. In other words, the strategy reflects global macroeconomic trends. They look for countries where the market seems not to be in equilibrium and place bets that the market will adjust and attain equilibrium once again.

9. **Managed Futures:**

   The manager invests in financial and commodities futures markets. They make directional bets with long/short positions.

Research on Returns

Mutual Fund Research

Actively managed mutual funds have been unable to beat the market after expenses for the past decades. This has been attributed to the fact that the market return is the return to all investors before expenses.

Through a test called the persistence test, it has also been established that only half of the mutual funds could outperform a market in subsequent years after beating the market in the previous year. These findings have made investors prefer index funds to actively managed funds because index funds charge a lower fee and perform better.

Hedge Fund Research

Some hedge funds have been seen to report a good performance for a few years and then lose a large percentage of the funds under management.

The Barclay Hedge offers an index tracking of all hedge funds. Generally, Hedge funds perform better in bear markets and underperform in bull markets.

Risks Faced by Hedge Funds

- **Liquidity risk:** occurs when the fund invests in illiquid assets
  Liquidity is a function of (I) the size of the position and (II) intrinsic liquidity of the instrument.

- **Pricing risk:** some of the assets can be quite difficult to price, e.g., derivatives

- **Counterparty risk:** The manager gets into contracts with dealers, brokers, and clearing agents. There's always a risk that these parties will renege on their obligations, putting the fund on the path of unprecedented losses.

- **Short squeeze risk:** The fund manager may be forced to purchase security they had sold short sooner than anticipated when the investor from whom the security was borrowed comes calling early.

- **Settlement risk:** One or more parties in a transaction may fail to deliver securities as per the contract.

Hedge Fund Performance

There are no reliable data records that can help us to assess hedge fund performance over the years. Part of that has much to do with the discretion with which some hedge funds are managed. Studies have shown that most investors in hedge funds are drawn to the industry largely because of several biases. These include:

- Hedge funds tend to have a higher net return than bonds and stocks.
- Hedge funds exhibit lower volatility of returns compared to equities.
- Hedge fund Sharpe ratios tend to be higher than those of equities and bonds.

Question 1

Longeren Mutual fund had year-end assets of $40 million and liabilities amounting to $10 million. If the total number of outstanding shares is 1,0000,000, what is the fund's net asset value?

A. 30

B. 40

C. 50

D. 20

Solution

$$\begin{aligned}\text{NAV} &= \frac{\text{Market value of assets at the close of the day} - \text{Liabilities}}{\text{Number of outstanding shares}}\\ &= \frac{40,000,000 - 10,000,000}{1,000,000}\\ &= 30\end{aligned}$$

Reading 28: Introduction to Derivatives

After completing this reading, you should be able to:

- Define derivatives, describe the features and uses of derivatives, and compare linear and non-linear derivatives.
- Describe the specifics of exchange-traded and over-the-counter markets, and evaluate the advantages and disadvantages of each.
- Differentiate between options, forwards, and futures contracts.
- Identify and calculate option and forward contract payoffs.
- Differentiate among the broad categories of traders: hedgers, speculators, and arbitrageurs.
- Calculate and compare the payoffs from hedging strategies involving forward contracts and options.
- Calculate and compare the payoffs from speculative strategies involving futures and options.
- Describe arbitrageurs' strategy and calculate an arbitrage payoff.
- Describe some of the risks that can arise from the use of derivatives.

What Is a Derivative?

Derivatives are contracts that gain their value from the value of financial variables. Financial variables used to trade derivatives are also known as underlying. They include commodity prices, interest rates, oil prices, prices of metals, equity indices, real estate indices, Cryptocurrencies, temperature changes, etc.

Derivatives can either be linear or non-linear.

Linear vs. Nonlinear Derivatives

A linear derivative is one whose value is directly related to the market price of the underlying variable. What does that mean?

If the underlying makes a move, the value of the derivative moves with a nearly identical margin. In fact, there is a 1:1 relationship between the derivative and the underlying – explaining why linear derivatives are said to be "delta-one" products. However, the delta itself need not always be equal to 1. Examples of linear derivatives include futures and forwards.

A non-linear derivative is one whose value/payoff changes with time and space.

Space, in this case, refers to the location of the strike/exercise price with respect to the spot/current price. The payoff varies with the underlying value but also exhibits some non-linear relationship with other variables, including interest rates, dividends, or even volatility. Non-linear derivatives are generally referred to as options.

For non-linear derivatives, the delta is not constant. Rather, it keeps on changing with the change in the underlying asset. Examples include the Vanilla European option, Vanilla American option, Bermudan option, etc.

Uses of Derivatives

Derivatives are majorly used to hedge or to speculate. The following are specific examples of the uses of derivatives.

1. Companies use derivatives to manage various risks: interest rate risk, foreign exchange risk, and commodity price changes to risk.
2. Some mortgages have derivatives embedded in them to give the mortgagee the option of paying early, for example, when the interest rates are lower.
3. Employees are sometimes given the option of buying shares from the company at a future date at a predetermined price to compensate them.
4. Options have been embedded in capital Investment opportunities to give room for expanding or doing away with the project depending on the turn of events.
5. Some corporate bonds may have derivatives embedded in them. These derivatives will give

the bond issuers and holders the right to repay them or redeem them early/ convert them to shares respectively.

Over-the-Counter Trading vs. Exchange Trading

Over-the-Counter Markets

In the over-the-counter market, trading occurs between participants who contact each other directly or through a broker. Participants may be able to trade privately without the other party being aware of the terms, including the price. Stocks traded in an OTC market could belong to a small company that's yet to satisfy the conditions for listing on the exchange. The OTC market is also popular for large trades.

Since the 2007-2009 financial crisis, OTC markets are, however, increasingly being regulated. Some of the regulations include:

1. Standardized OTC derivatives must be traded on platforms called swap execution facilities.
2. The central registry must have all the records of any trade.
3. Dealers trading with other dealers should use a central counterparty

Exchange-Traded Markets

Investors trade in contracts that have been identified in the exchange. Traditionally trading was done using the outcry system (Investors met at the exchange floor and used signals to indicate their proposed trades.) Currently, trading is done electronically through a computer.

Advantages of OTC Markets over Exchanges

- There are fewer restrictions and regulations on trades.
- The participants have the freedom to negotiate deals.
- It's cost-effective for corporates as service costs lower.

- There's better information flow between a market maker and the customer, thanks to one-on-one contact.

Disadvantages of OTC Markets Compared to Exchanges

- There’s increased credit risk associated with each OTC trade.
- Less transparency.

Market Size

The years 1997-2017 saw the exchange-traded market and the OTC markets growing by a factor of 6 and 7.4, respectively.

Options, Futures, and Forwards

Options

An option contract is an agreement between two parties to transact on underlying security at a predetermined price called the strike price before some date called the expiration date. The option gives the holder a right but not the obligation to buy/sell the underlying at an agreed-upon date at the strike price.

Options not only hedge against risk but also provide additional protection against adverse price movements. In other words, they protect against negative risk while preserving upward payoffs.

All European options can only be exercised at maturity. On the other hand, American options may be exercised any time between the issue date and expiration. As such, the price of an option is directly proportional to its maturity date. For example, the premium paid for an out-of-the-money option on Apple expiring in one month will be less than the premium paid for an option with the same strike price expiring in one year.

A **call option** gives the holder the right but not the obligation to **buy** the underlying asset at the strike price before the expiration date. On the other hand, a put option gives the holder the right but not the obligation to **sell** the underlying asset at the strike price before the expiration date.

Forwards Contracts

A **forward contract** is a non-standardized contract – traded in an over-the-counter market –between two parties that specifies the price and the quantity of an asset to be delivered in the future. That it's non-standardized implies it cannot be traded on an exchange. Instead, they are traded in the OTC market. One party takes a long position and agrees to buy the underlying asset at a specified price on the specified date, while the other party takes a short position and agrees to sell the asset on that same date at that same price.

The agreed-upon price is called the forward price. The price at which the dealer wants to buy is called the bid price, while the price the dealer wants to sell is called the ask price.

Exam tips:

The bid price is the "quoted bid," or the highest price, which a dealer is willing to pay to purchase a security. The offer price is the price at which the security is offered for sale, also known as the "asking price." The bid-ask spread represents the difference between the offer price and the bid price.

Forward Contract Payoff

Consider a forward contract on a Stock. Let S_T be the stock price at the maturity of and K be the delivery (forward price at the initiation of forward contract). The payoff of the long and short positions is given below.

The payoff to the long position (the buyer of the forward contract) is given by:

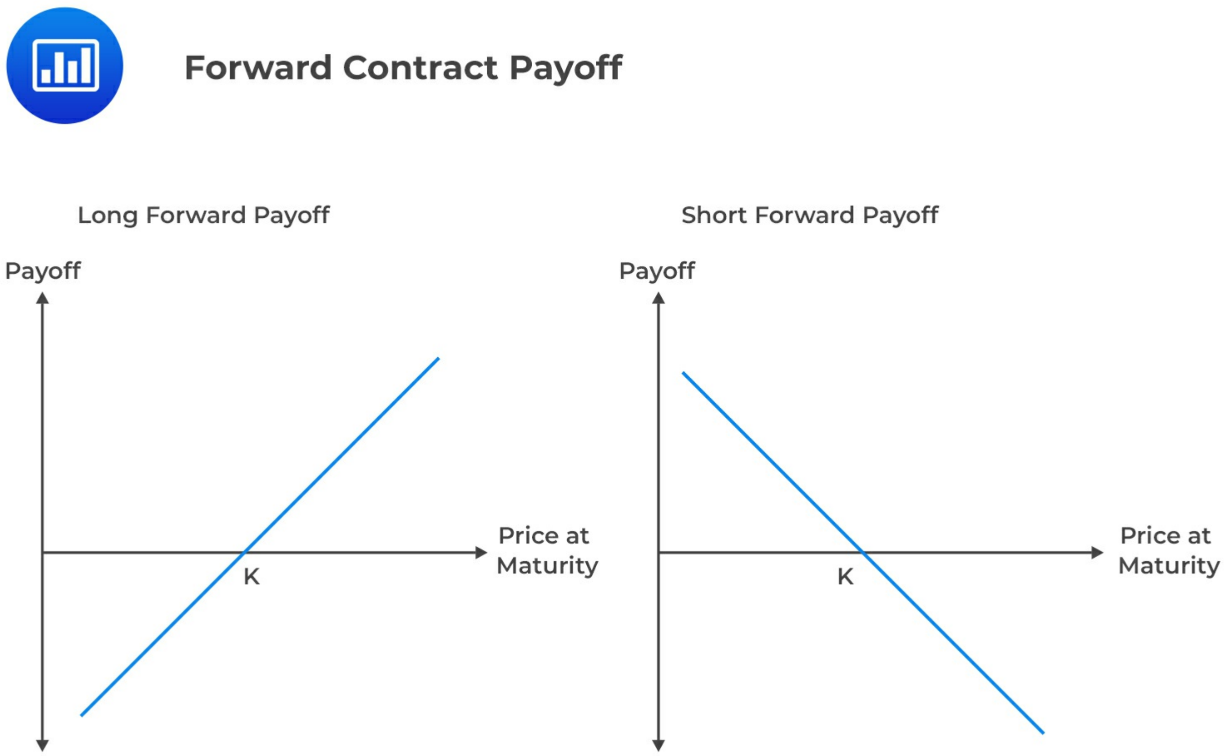
$\text{Payoff} = S_T - K$

Where:

S_T = spot stock price at maturity of the forward contract.

K = delivery price.

The payoff to the short position (seller of the) = $K - S_T$ Consider the following diagram:



The graphs above imply that for a trader in a long position, the payoff () will be beneficial when the underlying price (which is the stock price in our case) is high. On the other hand, for a trader in a short position, the payoff will be beneficial with a lower underlying price.

Example: Calculating the Payoff of a Forward Contract

Consider the forward contract on CAD- EUR exchange rate. The spot bid and ask prices per one euro are CAD 1.1080 and CAD 1.1083, respectively. The 6-month bid and ask prices are CAD 1.1120 and CAD 1.1125, respectively.

Suppose that company X enters into a long position to buy 10 million euros in six months. If the actual CAD- EUR exchange rate in six months is CAD 1.1200 per euro, calculate the profit to company X.

Solution

Based on the 6-month bid-ask exchange rates, company X buys 10 million euros for CAD 1.1125 (that is K = 1.1125). Consequently, the profit made by company X is

$$10,000,000 \times (1.120 - 1.1125) = \text{USD}75,000$$

Note: The short position of the above made a transaction made a loss of:

$$10,000,000 \times (1.1120 - 1.1200) = -\text{USD}80,000$$

Futures

A **futures contract** is a standardized, legally binding agreement – traded in on an exchange – between two parties that specifies the price to trade a given asset (commodity or financial instrument) at a specified future date.

Note that future contract offers similar payoffs as forward contracts. However, futures contracts trade on exchanges; that is, the underlying asset and possible maturity date are clearly stated in the contract.

Futures contracts differ from forwards in several other aspects:

- **Clearinghouse:** The clearinghouse is an interposed party between the buyer and the seller, which ensures the performance of the contract. In essence, futures contracts have no credit risk.

- **Marking to market:** Since the clearinghouse must monitor the credit risk between the buyer and the seller, it performs daily marking to market. This is the settlement of the gains and losses on the contract on a daily basis. It avoids the accumulation of large losses over time, leading to default by one of the parties.

- **Margins:** Daily settlements may not provide a buffer strong enough to avoid future losses. For this reason, each party is required to post collateral that can be seized in the event of default. The initial margin must be posted when initiating the contract. If the equity in the account falls below the **maintenance margin**, the relevant party must provide additional funds to cover the initial margin.

Options

Options are derivatives that offer the investor the right (but not the obligation) to buy or sell an asset in the future at a fixed price. Options can be found on exchanges and in the over-the-counter market. There are two types of options: call and put options.

In a call option, the holder has the right but not the obligation to buy the underlying asset (for example, stock) at a specified time within a specified period. In a put option, the holder has the right but not the obligation to sell the underlying asset at a specified price within a specified period.

An option contract involves two parties: the party with a long position and a short position in the option.

In the case of a call option, the party in a long position has the right but not the obligation to purchase an asset from a short position at a specified price called the strike price or exercise price within a given period.

For the put options, the party in a long position has the right but not the obligation to sell an asset from a short position at a specified price called the strike price or exercise price within a given period.

Option Payoffs

Call Option Payoff

Consider a European call option on a stock that will be exercised at time T. Let K be the strike price, and S_T be the stock price at time T. Consider the buyer of the call option (long position in the call option). By definition, the buyer will exercise the option if $S_T > K$ and thus the payoff of the buyer is $S_T - K$. Intuitively, if $S_T < K$ the option is not exercised, and thus the payoff to the buyer is 0.

The payoff to the buyer and seller is summarized below:

To the buyer (long position in the call option),

$$C_T = \max(0, S_T - K)$$

Where:

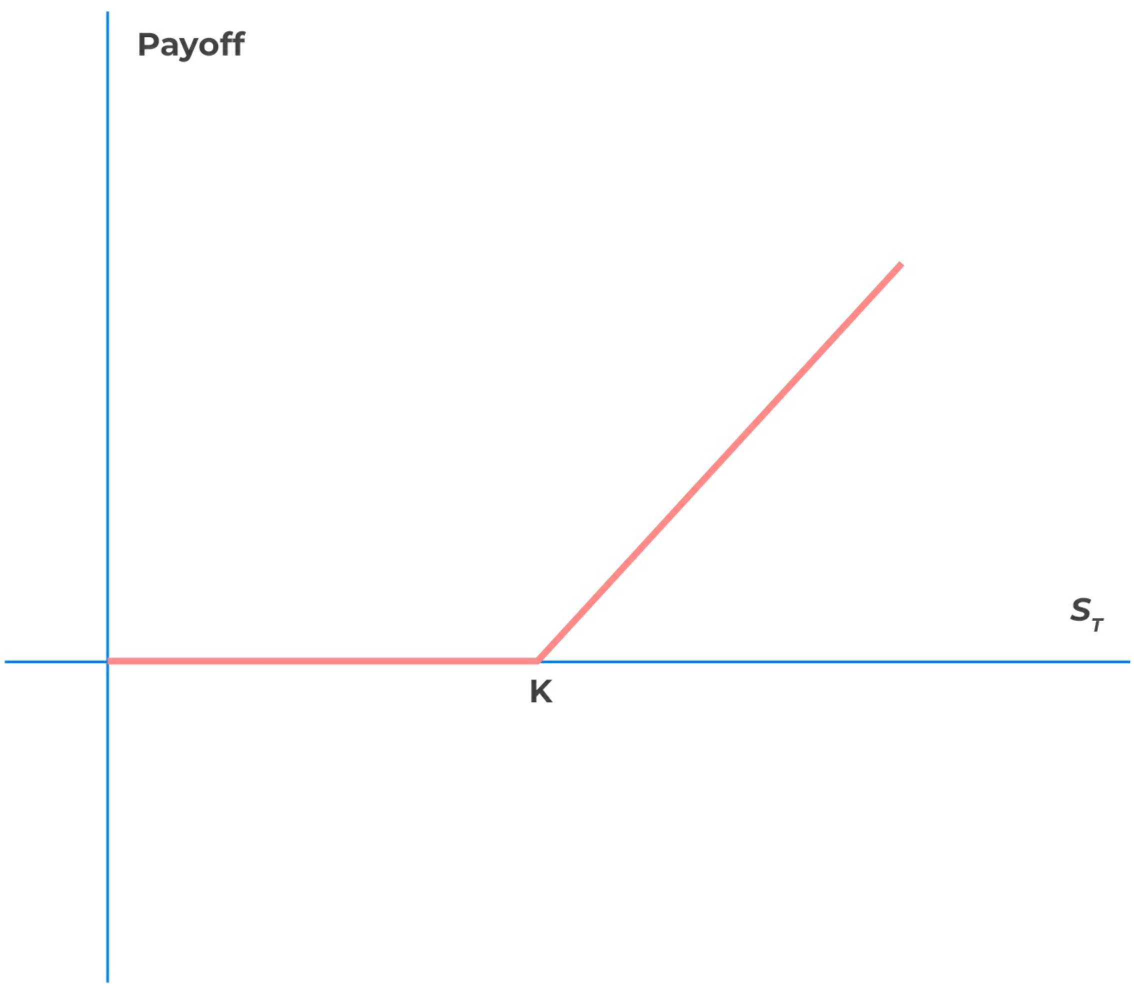
C_T = call option payoff at time T.

S_T = stock price at maturity.

K = strike price

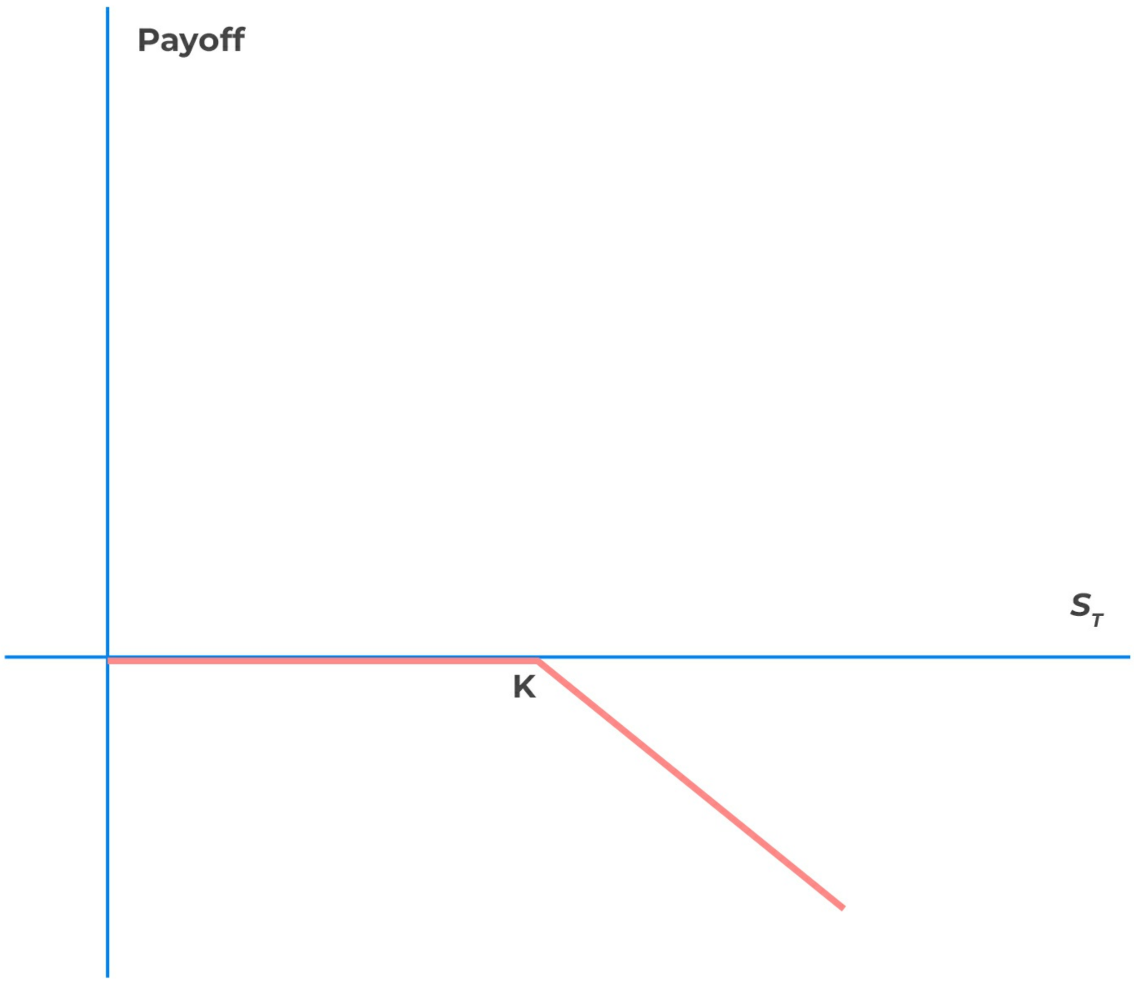
The graph of the payoff of a long position in the call option is shown below:

Call Option Payoff - Buyer



To the seller, payoff $= -C_T$. The payoff of the short position in the call option is shown below:

Call Option Payoff - Seller



The price paid for the call, C_0 is also called the call premium.

Thus,

Profit to the call option buyer $= C_T - C_0$

Profit to the call option seller $= C_0 - C_T$

Put Option Payoff

Using the same variables as a call option, and by definition of a put option, the European put option is exercised if $S_T < K$. In other words, the buyer of the option has the right to sell the stock for K. Therefore, if exercised, the payoff to the buyer (long position in the put option) is $K - S_T$. Intuitively, the put option is not exercised if $S_T > K$ and thus, the option is worthless to the buyer.

The payoff to the buyer and seller is summarized below:

To the buyer (long position in the put option),

$$P_T = \max(0, X - S_T)$$

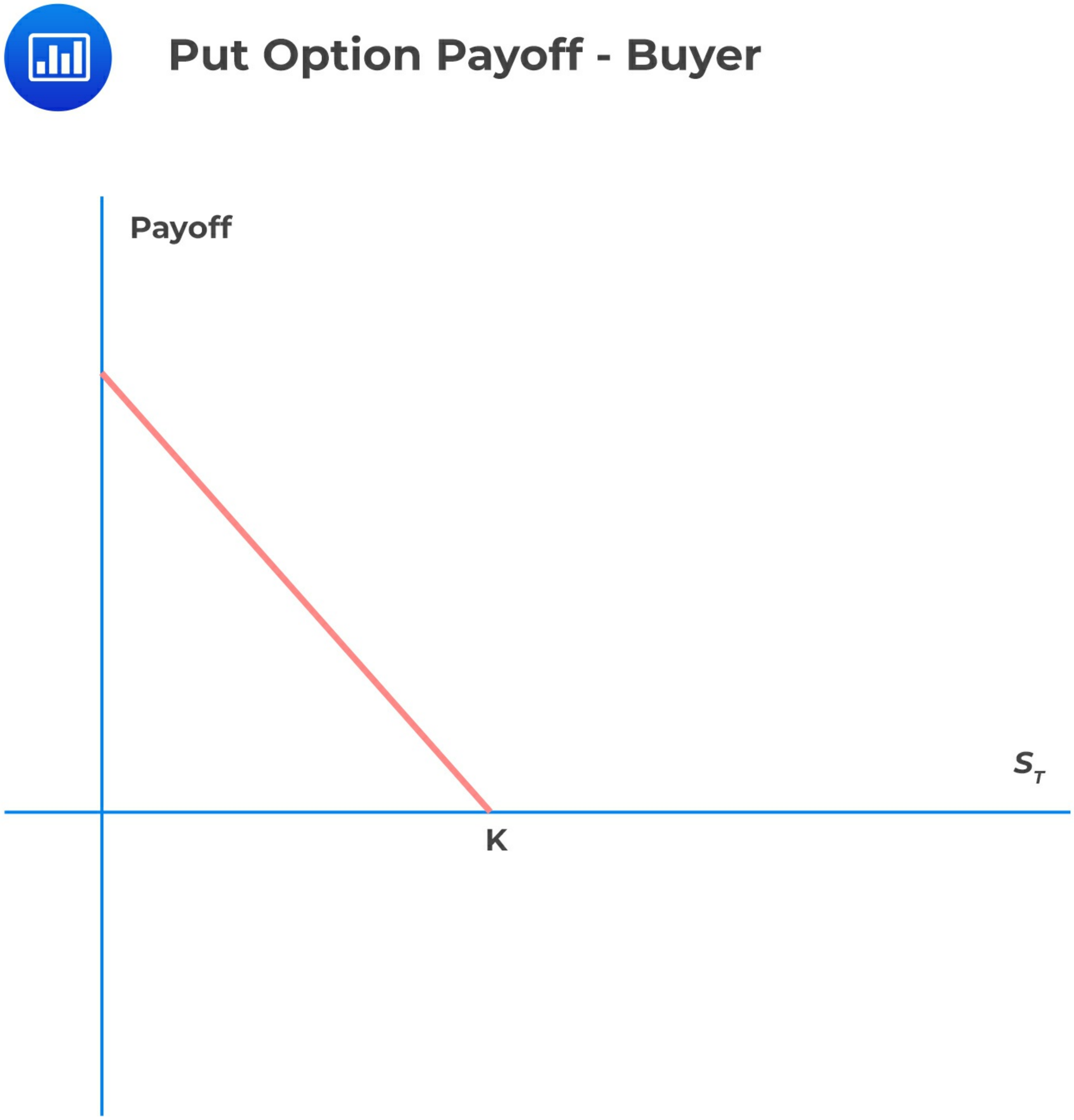
Where:

P_T =put option payoff.

S_T =stock price at maturity.

X =strike price.

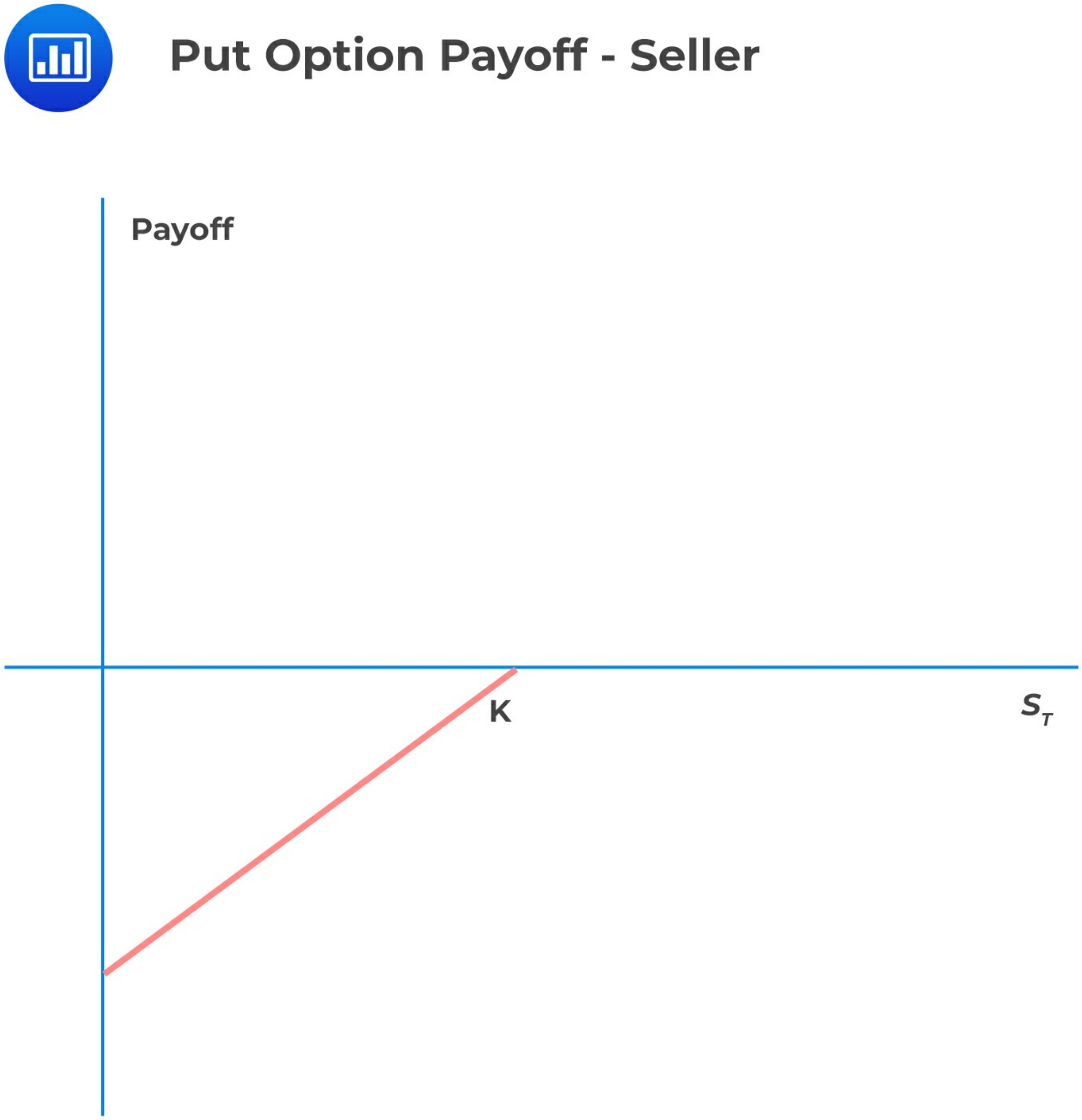
The payoff of a long put is shown in the following graph:



To the seller(short position in the put option),

payoff $= -P_T$.

The graph of the short put is shown below:



The price paid for the put, P_0 is also called the put premium.

Thus,

Profit to put option buyer $= P_T - P_0$

Profit to the put option seller $= P_0 - P_T$

Payoffs from Hedging Strategies Involving Options and

Forward Contracts

Hedging is the use of derivatives like futures and options to reduce or eliminate financial exposure. Before delving further into hedging, it is imperative to understand the following points:

- A long position/long exposure in security, e.g., a stock or bond, means the holder of the position owns the security and will make a profit if the price of the security goes up.

- Long exposure in a futures contract means the holder of the position is obliged to buy the underlying instrument at the contract price at expiry. The holder will make a profit if the price of the instrument goes up.

- A long position in an option implies buying a call option or a put option. A long position in a call option means the holder will make a profit if the price of the underlying soars above the strike price, provided the difference between the price at that time and the strike price is greater than the premium paid. A long position in a put means the holder will make a profit if the price of the underlying falls below the strike price. It helps to lock in the value of an asset whose price the holder believes will decline.

- A short position in security means the holder of the position borrows the security with the expectation of selling it at a profit and then repurchasing it at a lower price while returning it to the lender. The profit will be made in the face of an interim price decline. Conversely, the holder will make a loss if the price of the shorted security rises before repurchase.

- Short exposure in a futures contract means the holder of the position is obliged to sell the underlying instrument at the contract price at expiry. The holder will make a profit if the price of the instrument goes down. Conversely, they will make a loss if the price of the underlying rises dramatically.

- A short position in an option implies selling a call option or a put option. A short position in a call option means the holder makes a profit as long as the underlying price stays below the strike price. A short position in a put option means the holder makes a profit as long as the price of the underlying stays above the strike price, in which the put expires worthless, and the seller (writer) gets to keep the premium.

How Hedging Works:

An investor with a long position in an asset can hedge the exposure by entering into a short futures contract or buying a put option. An investor with a short position in an asset can hedge the exposure by entering into a long futures contract or buying a call option.

A forward contract helps the hedger to lock in the price of the underlying security. Forward contracts do not need any investment at the onset. However, the hedger gives up any movement that may have had positive results if they left the position unhedged. Let us look at an example:

Suppose a U.S.-based company is scheduled to receive £10 million in six months. The current exchange rate stands at 1.32$/£. The management is worried that the pound might depreciate against the dollar. It decides to hedge the exchange risk with a forward contract at 1.3$/£.

With the forward, the company will be guaranteed to receive $13million (=£10m×1.3$/£). Suppose the company does not hedge the position and the exchange rate in six months turns out to be 1.25$/£. The company will receive $12,500,000. Suppose further that the company does hedge the position at 1.3$/£, and the rate turns out to be 1.35$/£. In this case, the company will still receive $13 million but will be forced to give up the extra $500,000 it would have received if it did not hedge the position in the first place.

Market Participants

The most common categories of market participants are:

- **Hedgers.**
- **Speculators.**
- **Arbitrageurs.**

Hedgers

Hedgers use derivatives to reduce or remove risk exposure. We have already discussed how hedging works above. Consider the following example where foreign exchange risk is hedged using options.

Example: Hedging Exchange Rate Risk Using Options

A risk manager in company X (located in the U.S.) knows that his company is due to pay 10 million euros in 6 months, at the exchange rate of USD 1.1120 per euro. How can the risk manager hedge again foreign exchange risk using a call option?

Solution

The risk manager can hedge against the foreign exchange risk by buying the call option with a strike price of USD 1.1120. If in six months the exchange rate is more than USD 1.1120, the risk manager will exercise the option, getting the 10 million euros using the exchange rate of USD 1.1120.

If the exchange rate is less than USD 1.1120, the risk manager will not exercise the option and consequently acquiring the 10 million euro at a lower exchange rate.

If now, the risk manager's company is due to receive 10 million euros in six months, at a USD 1.112O exchange rate. How can the risk manager this position against the foreign exchange rate?

Alternatively, the risk manager could buy the European put option to sell 10 million euros at an exchange rate of USD 1.1120. If in six months the exchange is less than USD 1.1120, the risk manager exercises the option by selling the received for USD 1.1120. On the other hand, if the exchange is greater than USD 1.1120, the option is not exercised, and the risk manager acquires a favorable exchange rate.

Speculators

Speculative trading (regarding futures contracts) refers to the trading of futures contracts without the intention of obtaining the underlying commodity. Thus, speculators basically make bets on the market, unlike hedgers, whose priority is to eliminate exposures.

Speculators are motivated by the leverage that comes with futures contracts in which no initial investment is required. All that's needed is the initial margin required by the clearinghouse/exchange. The margin is no more than a percentage of the notional value of the underlying. The gains or losses associated with futures can be quite large, and payoffs are symmetrical.

Speculators trade in futures, intending to resell these contracts before maturity. They expect the futures price to move in their favor and make a profit when selling the contracts. However, there can be no guarantees that the price will move in their favor, and therefore this trading strategy is also laden with risks. If the price moves against a speculator's position, they could suffer substantial losses.

For options, speculators only need to part with the option's price at the onset, often just a few dollars for 100 shares worth of the underlying. However, options have asymmetrical payoffs. Going long on options can bring in significant gains, but losses are limited to the option's price paid.

In a nutshell, speculators buy assets for time and apply different strategies to benefit from price changes.

Example: Profit from Speculation

The current stock price is CAD50, and speculators believe that in one month, this price would have increased to at least CAD55. Moreover, the call option on the stock has a strike price of CAD52 with a price of CAD2. For convenience, assume that each call option represents 1 share of the underlying stock.

The speculator has an initial capital of CAD5000. If, at the end of one month, the stock price is CAD56, which strategy is more profitable?

I. Buying 100 (= 5000/50)shares and selling them after one month.
II. Taking a long position in 2,500 (= 5000/2) call options.

Solution

For strategy **I**, the profit made is given by:

$$(56-50)\times 100=\text{CAD}600$$

For strategy **II**, the profit made is given by:

$$[2,500\times(56-52)]-5000=\text{CAD}5,000$$

Hence it is more profitable to purchase call options.

Arbitrageurs

Arbitrage opportunities exist when prices of similar assets are set at different levels. Therefore, an **arbitrageur** attempts to make a risk-free profit by buying the asset in the cheaper market and simultaneously selling it in the overpriced market.

For example, suppose ABC stock is trading at $200 on exchange A and $198 on exchange B. Then, if you buy one ABC stock on exchange B and simultaneously sell it on exchange A, you can make a risk-free profit of $2 without any outlay of cash.

However, arbitrage opportunities are normally short-lived. The nature of efficient markets is that market forces will push up the asset's price in the underpriced market while simultaneously pushing down the asset's price in the overpriced market. At the end of the day, the asset will be priced equally in both markets.

Risks in Derivative Trading

Market Risk

There are no guarantees the market price will move in favor of the derivative trader. For example, an investor who is short in a put option has no guarantees that the underlying price will stay above the strike price, allowing them to keep the premium. The underlying's price could as well fall below the strike, in which case the option buyer exercises the option, forcing the seller to buy a stock at a price that's higher than its market price.

Counterparty Risk

There's always the risk that the buyer, seller, or dealer will default on the contract. This risk is particularly prevalent in OTC markets where regulations are not as strict as in exchange.

Liquidity Risk

Closing out a deal prior to maturity, e.g., in an American option that can be exercised before maturity, can at times be difficult. Even more likely, bid-ask spreads could be so large as to represent a substantial cost.

Operational Risk

There's always the risk that a trader with instructions to use derivatives as a hedging tool will be tempted to take speculative positions, possibly in the hope of making a "kill'. Such a move can be disastrous for the firm.

Question 1

A risk manager is worried that the price of gold will rise. He would like to hedge his position but is stuck between buying a futures contract on an exchange and buying a forward contract directly from a counterparty. The manager finds that the futures price is higher than the forward price. Both types of contracts would have the same maturity and delivery conditions. Assuming there's no arbitrage, which of the following factors would explain this price difference?

A. The forward contract party is more likely to default
B. The futures contract is more liquid and easier to trade
C. The futures contract has less transaction costs compared to the forward contract
D. All of the above

The correct answer is **D**.

All of these factors would make the futures contract **safer** for the investor. Hence, the futures contract would most likely be more expensive than the corresponding forward.

Reading 29: Exchanges and OTC Markets

After completing this reading, you should be able to:

- Describe how exchanges can be used to alleviate counterparty risk.
- Explain the developments in clearing that reduce risk.
- Describe netting and describe a netting process.
- Describe the implementation of a margining process and explain the determinants of and calculate initial and variation margin requirements.
- Compare exchange-traded and OTC markets and describe their uses.
- Identify the classes of derivative securities and explain the risk associated with them.
- Identify risks associated with OTC markets and explain how these risks can be mitigated.
- Describe the role of collateralization in the over-the-counter market and compare it to the margining system.
- Explain the use of special purpose vehicles (SPVs) in the OTC derivatives market.

How Exchanges Alleviate Counterparty Risk

An exchange is a central financial location where traders can trade (exchange) standardized financial instruments such as futures contracts.

A major risk when trading derivatives is counterparty risk, the other party's risk failing to honor their end of the contract. For example, if company A enters an interest rate swap to give fixed-rate payments in exchange for floating rate payments from company B pegged on the six-month LIBOR, A and B are said to be counterparties. If large moves in the floating rate occur, B could default, resulting in a financial loss for A.

Organized exchanges use several mechanisms to alleviate counterparty risk. These include:

Margins

These are assets that are transferred from one trader to another. They act as protection against the risk of defaulting of the other party. Consider two traders, trader X (seller) and Y (buyer), trading with a margin agreement in place. In case the price of the traded commodity goes below the agreed-upon price, the buyer (Y) can request a refund from the seller (X) for an amount equal to the difference between the agreed-upon price and the price of the commodity as of now.

This will reduce the chances of the buyer defaulting in a bid to buy the commodity at the current price (which is cheaper than the agreed-upon price). If the current prices are more than the agreed-upon price, trader Y (buyer) will be required to provide margin to trader X (seller).

Netting

"Netting" describes the procedure of combining long and short positions and calculating the CCP's net exposure to a member. Assume, for example, that Member X shorts five September oil contracts. At about the same time, Member X makes another trade in which it agrees to buy 10 September oil contracts. Even though the CCP member will now have two separate trades with the CCP, the two will be collapsed to a net long position of five September oil contracts (i.e., contracts to buy 5,000 barrels of oil in September).

Central Counterparties

Central counterparties are used in exchanges to clear transactions between members. Assume that traders A and B are in a contract to transact at a later date, with A as the seller and B as the buyer. The CCP will act as a central party to both A and B. A will sell to the CCP the agreed item at the agreed price. B will then buy the item from the CCP at the agreed price.

Benefits of Trading Using CCPs

1. Promotes trust between traders who will not even need to know the creditworthiness of each other.
2. CCPs help members to easily close out positions.

Ways in Which CCPs Handle Credit Risk

1. Netting
2. Variation Margin and Daily Settlement
3. Initial Margin
4. Default Fund Contributions

Netting

Netting involves long and short positions offsetting each other.

Variation Margin and Settlements

Future contracts are traded on a daily basis up-to-the the maturity period. A member who is trading with the CCP will have to pay the CCP in case the price of the traded commodity decreases. The payment made should correspond to the price decline of the commodity. Similarly, if the price of the commodity increases, the CCP will have to pay the member an amount that corresponds to the price increase.

Daily settlements have simplified the closing out of future contracts by making the contract's maturity date less useful. There are no interests associated with variation margin payments as they are settled on a daily basis and not on the maturity date.

Example

Suppose that trader A enters 4 June futures contract to buy 1000 barrels of oil. Suppose further that June futures price is 300 cents per barrel at the close of trading on Day 1 and 275 cents per barrel at the close of Day 2.

Trader A has lost

$$1000 \times (300 - 275) \quad \text{cents} = 25{,}000 \text{ cents or USD } 250$$

The trader is thus required to pay USD 250 to the CCP.

If June futures price is 310 cents per barrel at the close of Day 3, however,

Trader A has gained

$$1000 \times (310 - 275) \quad \text{cents} = 35{,}000 \text{ cents or USD } 350$$

The CCP is thus required to pay the trader, USD 350.

Initial Margin

In addition to the variation margin, a trader is required to deposit an initial margin with the CCP. Initial margins save CCPs from losses in case a trader is unable to pay the variation margin.

The initial margin amount is set by the CCP and is dependent on the changes in the future market prices. The CCP is therefore allowed to alter the initial margin at any point depending on market changes.

CCPs do not pay interest on variation margins because futures contracts are settled daily. It is true, however, that CCPs pay interest on initial margin because it is owned by the member who contributed it.
In the event that the interest rate paid by the CCP is deemed unsatisfactory, securities such as Treasury bills may be posted by members instead of cash. As a result, the CCP would reduce the value of the security by a certain percentage in order to determine their cash margin equivalence. The reduction is known as a haircut. When the price volatility of an asset increases, a haircut is usually increased

Default Fund Contributions

Default fund contributions take care of the remaining amount not covered by the initial margin. The equity of a CCP is at risk only after exhausting the default fund contributions of all members.

Uses of Margin Accounts in Other Situations

A margin account can be used between a trader and a broker. If the broker is not a member of a CCP, it will have to use an entity that is a member giving rise to a margin account between the broker and the member.

Unlike margin accounts between CCPs and their members, a margin account between a broker and a retail trader comprises both an initial and a maintenance margin.

The balance in the margin account should not fall below the maintenance margin.

Options on Stocks

To hedge against future liabilities, traders with short positions in an option need to post margin with CCPs.

Calculating Margin for an Option

The margin on a **short call option** is the maximum between the following two values:

- 100% of the value of the option + 20% of the share price minus the amount the option is out of money, if any; or
- 100% of the value of the option + 10% of the share price

The margin on a **short put option** is the maximum between the following two values:

- 100% of the value of the option + 20% of the share price minus the amount the option is out of money, if any; or
- 100% of the value of the option + 10% of the strike price

Short Sales

Short stocks are stocks that are borrowed and sold and thereafter repurchased and returned to the account they were borrowed from.

To short a stock, a margin of 150% at the time of the trade is required. The margin account varies with stock price changes. If stock prices increase, the margin account reduces, and vice versa.

A trader who would like to trade 1000 shares with a stock price of $20 per share would be needed to first post an initial margin of 150% × 20,000 = 30,000. The proceeds from the sale ($20,000) belong to the trader, while the additional $10,000 is the margin account.

Note that the margin account still belongs to the trader. It is payable plus interests on the balance. However, if prices increase beyond the margin account, the position is closed out.

Buying on Margin

This occurs when a trader borrows funds from a broker and uses the funds to buy assets. To be able to borrow funds, the trader must deposit a margin of 50% of the value of the assets. The remaining amount will then be provided by the broker, who will keep the assets as security. The difference between the value of the shares and the amount loaned to the trader by the broker forms the balance in the margin account.

The minimum margin balance as a percentage of the value of the shares is known as the maintenance margin. A margin balance should not drop below the maintenance margin. If it drops below the maintenance margin, the trader will be required to provide additional margin, failure to which the broker will sell the shares.

Over-the-Counter Trading vs. Exchange Trading

The over-the-counter market is a decentralized trading platform without a central physical location, where market participants use a host of communication channels to trade with one another without a formal set of regulations. The communication channels commonly used include telephone, email, and computers. OTC trading is facilitated by a **derivatives dealer** who usually is a major financial institution specialized in derivatives. Participants in an OTC Market are either dealers or end-users. Dealers enter into derivative transactions in a bid to satisfy end users.

In an OTC market, it's possible for two participants to exchange products/securities privately without others being aware of the terms, including the price. OTC markets are much less transparent than exchange trading. Since the derivatives are not standardized, they can be customized to meet the needs of the end-user.

Stocks traded in an OTC market could belong to a small company that's yet to satisfy the conditions for listing on the exchange. The OTC market is also popular for large trades.

Advantages of OTC Markets over Exchanges

- There are fewer restrictions and regulations on trades.
- The participants have the freedom to negotiate deals.
- It's cost-effective for corporates as service costs lower.
- There's better information flow between a market maker and the customer. Thanks to one-on-one contact.

Disadvantages of OTC Markets Compared to Exchanges

- There's increased credit risk associated with each OTC trade. The probability of the occurrence of credit risk is dependent on the life of the contract. As the life of the contract increases, the end-user may get more exposed to financial constraints that may prevent him/her from being able to honor his/her end of the bargain. The market variables that influence the price of the derivatives are also more likely to increase.
- Less transparency.

Risks Associated with OTC Markets

Systemic Risk

Systemic risk refers to a market-wide event that would originate from an initial park only to trigger a chain reaction that could devastate the financial markets. Such a spark could be the failure of a player considered “too big to fail.”

According to Ben Bernanke, an American economist, a too-big-to-fail firm is one whose size, complexity, interconnectedness, and critical functions are such that, should the firm go unexpectedly into liquidation, the rest of the financial system and the economy would face severe adverse consequences.

Counterparty Risk

As we’ve seen before, counterparty risk is the risk that a counterparty in a derivatives contract will, either willingly or unintentionally, default on contractual obligations.

Some of the measures taken to mitigate these risks in OTC markets include:

Bilateral Netting

This refers to the offsetting of positions between counterparties due to the fungibility of the contracts. Suppose, for instance, that Bank X bought $50 million worth of euros from Y. At the same time, it sold an otherwise identical contract to Bank A in the amount of $40 million. If the two trades had gone through the same clearinghouse, the net exposure of Bank X would only be $10 million.

Collateral

Collaterals are similar to margins in the exchange-traded markets. They are posted so as to hedge against credit risk. To calculate the collateral and the type of security to be posted, the net value of outstanding derivatives is used. This is specified in the Credit Support Annex (CSA).

Special Purpose Vehicles

SPVs are subsidiary companies created by the main company to handle a project without putting the parent company at risk. If the parent company encounters any financial complication, the SPV continues its operations as normal and vice versa.

Derivative Product Companies

A derivative product company, DPC, is generally an AAA-rated entity set up by one or more banks to serve as a bankruptcy-remote subsidiary of a major dealer. DPCs trade on behalf of the dealer. In case of any financial trouble, the dealer will be required to offset the losses, failure to which the DPC will be sold to another entity, or the transactions may be closed out at mid-market prices.

DPCs differs from SPVs in that it's separately capitalized to obtain an AAA-rating. The DPC shields external counterparties from the knock-on effects that may play out when the DPC parent fails.

DCPs offer flexibility and decentralization while still allowing counterparties to enjoy benefits associated with exchange-based trading.

Credit Default Swaps

CDSs have been designed to reflect an insurance contract. The buyer pays a regular premium to the seller, who then pays the seller in case of the occurrence of a specified event. Companies called Monolines have been set up to specifically offer credit protection using default swaps. Some insurance companies equally offer credit protection alongside their insurance contracts.

Question

A trader wants to purchase 10,000 shares of YYB on margin. The current market price of YYB is $40 per share. How much should the trader part with in order to be able to purchase on margin?

A. $600,000
B. $400,000
C. $200,000
D. $0

The correct answer is **C**.

A trader needs an initial margin of 50% of the value of the assets to be purchased.

Option A is incorrect. To short a stock, a margin of 150% at the time of the trade is required, but only in the case of shorting a stock.

Reading 30: Central Clearing

After completing this reading, you should be able to:

- Provide examples of the mechanics of a central counterparty (CCP).
- Describe the role of CCPs and distinguish between bilateral and centralized clearing.
- Describe the advantages and disadvantages of the central clearing of OTC derivatives.
- Explain regulatory initiatives for the OTC derivatives market and their impact on central clearing.
- Compare margin requirements in centrally cleared and bilateral markets and explain how margin can mitigate risk.
- Compare and contrast bilateral markets to the use of novation and netting.
- Assess the impact of central clearing on the broader financial markets.
- Identify and explain the types of risks faced by CCPs.
- Identify and distinguish between the risks to clearing members as well as non-members.

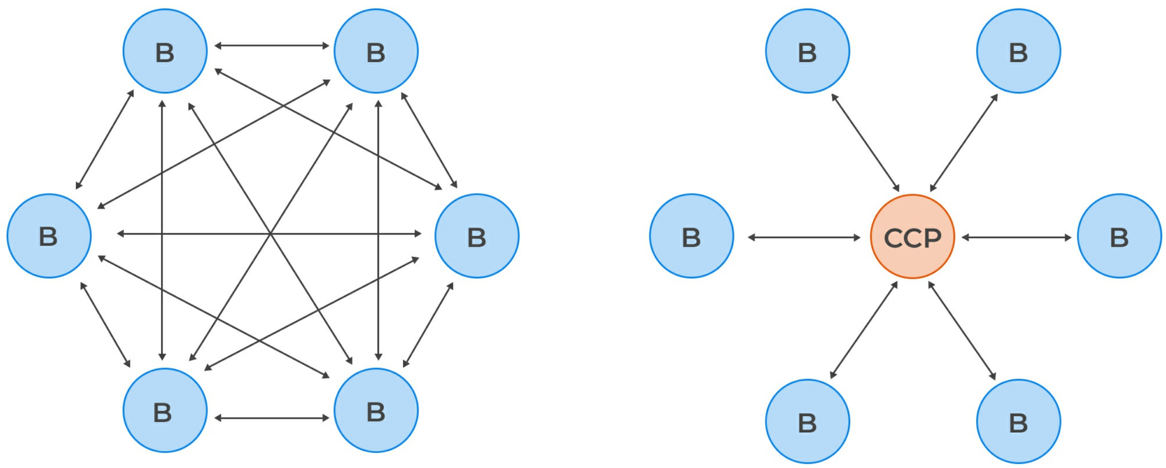
Mechanics of a Central Counterparty

Clearing refers to the use of a central counterparty (CCP) to mitigate risks associated with the default of a trading counterparty.

CCP clearing means that a CCP becomes the **legal counterparty** to each trading party, providing a guarantee that it will honor the terms and conditions of the original trade even in the event that one of the parties defaults before the discharge of its obligations under the trade.

Although a bit simplistic, the following view helps to show the role played by CCPs in trading. Each of the six entities denoted B represents a dealer bank.

Illustration of the Role played by Central Counterparties



The first figure shows six traders trading bilaterally, i.e., traders are trading directly with each other. The second diagram illustrates traders trading through a CCP.

CCPs are commonly used in exchange-traded markets. CCPs use various ways to protect themselves from losses that may be incurred if a member defaults. These include:

1. They adjust their initial margin requirements more often according to market price changes.
2. They collect both initial margin requirements and variation margin payments from their members during and at the end of the day. This is to avoid closing out a member's position since a member has only two hours to meet a margin call.

Recent years have seen an increased use of CCPs in the OTC markets.

Differences between CCPs in Exchange-Traded Markets and in OTC Markets

Exchange-Traded Markets	OTC Markets
Most contracts last a few months with only a few that last for more than a year	Contracts last for at least ten years
Average trade is small	Average trade is relatively big
Contracts trade continuously	Contracts trade intermittently making OTC markets less liquid
Variation margin can be directly determined from market prices	Models are needed in order to determine variation margin

Similarities between CCPs in Exchange-Traded Markets and in OTC Markets

1. They are all required to post both initial and variation margin; and
2. They are all required to make default fund contributions.

CCPs can be involved in both mergers and acquisitions.

Operation of CCPs

Regardless of whether they are trading in the OTC markets or in exchange-traded markets, CCPs operate in a similar way.

1. The initial margin is required in both. The initial margin is set in a way that enables the CCP to close out a defaulting member's position in 5 days.
2. Members are equally required to provide variation margin as well as default fund contributions.

If the initial margin does not sufficiently cover the losses, the variation margin will be exhausted before using default fund contributions.

The equity of a CCP will be at risk only upon exhausting the default fund contributions of all members of the CCP.

To cover their costs, CCPs charge per trade. They make profits on the excess initial margin of members. These profits may be shared out among members if the CCP is owned by members.

CCPs May Compete with Each Other!

Advantages of Competition between CCPs

1. Competition provides a variety for members to choose from.
2. Better services are guaranteed since CCPs will want to improve their services so as to stay ahead.

Disadvantages of Competition between CCPs

Margin requirements and default fund contributions may be reduced so as to attract a lot of traders. This will, in the long run, increase credit risks to the CCP.

Regulation of OTC Derivative Markets

Regulations were introduced after the 2007-2008 financial crises. The increased use of derivatives in OTC markets can be attributed to these regulations.

They were formulated mainly to hedge against systematic risk, the risk that one member defaulting could result in possible losses to the other members trading with the defaulted member.

The three major regulations affecting OTC markets as defined by the G-20 Pittsburgh meeting are:

1. All standardized OTC derivatives (plain vanilla interest swap and credit default swap) to be cleared through CCPs. This requirement helps reduce interconnectedness and systemic risk since it creates an environment where traders have less credit exposure to each other.
2. Electronic platforms to be used for trading standardized derivatives. This promotes price transparency and price availability.
3. All trade to be reported to a central trade repository. This regulation is important to regulators since it provided them with information regarding the risks being taken by the

participants in the OTC market.

Conditions to be satisfied before CCPs clear transactions:

1. The product has to be a standard product.
2. The product has to be actively trading to make it easy to close out a defaulting member's position.
3. There needs a valuation model to value the product so as to correctly determine its initial margin and default fund contributions.
4. The product should have historical data. Historical data is useful in determining the product's initial margin requirement.
5. Products that currently meet all these requirements are interest rate swaps and credit default swaps on indices.

In CCPs, the position of the members of the CCPs is transferred to the CCP. Members of the CCP agree to post initial margin and variation margin and in addition, they also agree to make the required default fund contributions.

Novation is the term used to describe the transfer of a contract from one party to another party.

A central counterparty interjects itself between a buyer and a seller through a process called 'Novation' and becomes a seller to the buyer and a buyer to the seller. By novating the trade, the CCP guarantees settlement of the trade even if one of the parties were to default on their obligation, thereby eliminating counterparty risk.

Transactions not done through the CCP are known as **uncleared transactions**.

In the period, 2016-2020, new rules require that both the initial margin and the variation margin must be posted. This differs from the earlier pre-crisis period where OTC traders were only required to post variation margin, and also during this period, OTC markets were cleared bilaterally.

It is possible for traders to trade using different CCPs. In such a scenario, members of a CCP will trade with non-members **bilaterally**. Bilateral clearing involves two parties in a transaction agreeing on how they will be cleared, what netting arrangements they would prefer, and what will be used as collateral in case of any.

Advantages of CCPs

- CCPs have made it easier for dealers and end-users to exit a transaction.
- By making it easier to exit a transaction, CCPs have improved liquidity in financial markets.
- By trading through a CCP, a traded reduces his/her exposure to credit risk. This is because the risk is borne by all members of a CCP and not just by the particular trader, a process called loss mutualization.
- CCPs make it smooth and easy for traders to trade since they take care of all the arrangements to hedge against risk (netting, margining, settlements, and default fund contributions).
- It is easier to perform netting arrangements while trading through a CCP.

Disadvantages of CCPs

- CCPs are prone to both moral hazards and adverse selection. Traders trading through a CCP may be least concerned with the riskiness of their counterparty since the risk will be passed on to the members of the CCP and not solely borne by the trader. This exposes CCPs to moral hazards. It is also likely that traders will prefer trading through the CCP if they consider their counterparty's risk to be high. This exposes the CCP to adverse selection.
- CCPs are adversely affected by market changes.
- CCPs make it difficult for their members to determine the amount of risk they are getting themselves into.

Activities of a CCP

Compared to banks, CCPs are much simpler and more transparent. The activities of a CCP are:

- Admitting new members.

- Determining the initial margin requirement, variation margin requirement, and default fund contributions.
- Valuing transactions of their members.
- Managing systems for hedging against risk, for example, netting and margining.

Types of Risks Faced by CCPs

Default Risk

A clearing member may default on one or more transactions. Following a default event, a host of other problems may come up. These include:

- Default or increased distress of other members because of the high default correlation.
- Failed auctions, leaving the CCP with no choice but to impose losses on members.
- Resignations because initial margins and default funds have to be returned to resigning members, the loss could be felt by other members.
- A worsening reputation – a default event would also injure the reputation of other members with close ties to fallen members.

Non-Default Events

Such events include:

- Internal/external fraud.
- Operational losses.
- Investment losses.
- Losses due to litigation.

Note that non-default and default losses may be correlated. The default of a member might cause market disturbance and increase the likelihood of operational or legal problems.

Model Risk

OTC transactions are not standardized; they trade less frequently and their prices are not so transparent. For these reasons, valuation models are needed when determining the initial margin requirement and default fund contributions of traders. However, these valuation models may use subjective assumptions.

Liquidity Risk

Generally, the more liquid an investment is, the lower the returns. CCPs need to consider the constraints of investments before investing so as to ensure that their investments can be easily converted to cash whenever the need arises.

Operational and Legal Risk

Centralization of various functions fosters efficiency, but on the downside, it creates a fertile ground for operational bottlenecks. For example, the CCP may have to contend with frequent system failures due to heavy traffic. What's more, segregation and the movement of margin and positions through a CCP is prone to legal risk, depending on the jurisdiction.

Other risks include custody risk in case of failure of a custodian, wrong-way risk, foreign exchange risk, concentration risk, and sovereign risk.

Risks to Clearing Members and Non-Members

Risks to Members

There are several ways through which a clearing member can experience CCP-related losses:

- Forced allocation
- CCP failure

- Auction costs
- Default fund utilization
- Rights of assessment
- Tear-up

Prior to gaining membership, there are several mechanisms through which a prospective member can assess the risks faced by a member of the CCP. Such mechanisms may involve scrutinizing:

- The membership criteria
- Investment policies
- Default management policies
- Operational capacity
- Capital requirements
- The number of alternative CCPs and their credit ratings
- Initial margin and default fund contributions

Risks to Non-Clearing Members

Non-clearing members who clear indirectly through a CCP are usually faced with different risks, most of which may closely resemble those of clearing members. In addition, however, non-clearing members may have an additional layer of protection:

- If a clearing member defaults, clients may be safe provided their clearing member is in compliance with the CCP's requirements and in good financial health.
- If a clearing member defaults, the CCP may safeguard the interests of non-clearing members through margin segregation and portability.
- Since non-clearing members do not contribute toward the default fund, their exposure to the CCP is indirect.

Lessons Learned from Prior CCP Failures

In the last four decades, we've had several high-profile CCP failures and near-failures. Common sources of these failures include:

- Insufficient margins and default funds
- Large movements in the price of the underlying
- The failure to update initial margin requirements to reflect changing market conditions
- Operational problems associated with large price moves and system-crushing trade volumes
- Liquidity strains

Some of the lessons we can learn from these past failures include:

- Operational risk must be mitigated at all costs. Failure to act is never an option
- Variation margins should be recalculated frequently
- CCPs should have access to external sources of liquidity. They can easily default, not because they are insolvent but simply because they are illiquid in short term.
- CCPs should endeavor to monitor positions continuously and act quickly whenever there are large moves

Question 1

Which of the following is a disadvantage of central clearing in OTC markets?

A. Procyclicality

B. Transparency

C. Loss mutualisation

D. Default Management

The correct answer is **A**.

The following are **advantages** of central clearing in OTC markets:

- Transparency
- Offsetting
- Loss mutualisation
- Legal and operational efficiency
- Default Management
- Improved market liquidity

The following are **disadvantages** of central clearing in OTC markets:

- Moral hazard
- Adverse selection
- Bifurcations between cleared vs. non-cleared
- Procyclicality

Question 2

Which of the following is most likely associated with non-default losses?

A. Failed auctions

B. A worsening reputation

C. Internal/external fraud

D. Resignations

The correct answer is **C**.

A clearing member may default on one or more transactions. Following a default event, a host of other problems may come up. These include:

- Default or increased distress of other members because of the high default correlation
- Failed auctions, leaving the CCP with no choice but to impose losses on members
- Resignations because initial margins and default funds have to be returned to resigning members, the loss could be felt by other members.
- A worsening reputation – a default event would also injure the reputation of other members with close ties to fallen members.

Internal/external fraud is in the category of non-default events. However, non-default and default losses may often be correlated.

Reading 31: Futures Markets

After completing this reading, you should be able to:

- Define and describe the key features and specifications of a futures contract, including the underlying asset, the contract price and size, trading volume, open interest, delivery, and limits.
- Explain the convergence of futures and spot prices.
- Describe the role of an exchange in futures transactions.
- Explain the differences between a normal and inverted futures market.
- Describe the mechanics of the delivery process and contrast it with a cash settlement.
- Evaluate the impact of different trading order types.
- Describe the application of marking to market and hedge accounting for futures.
- Compare and contrast forward and futures contracts.

The Key Features of a Futures Contract

A futures contract is a standardized, exchange-tradable obligation to buy or sell a certain amount of an underlying good at a specified price on a specified date.

Key Features:

- **Exchange-tradable:** Unlike forwards, which trade on OTC markets, futures contracts are traded on an organized exchange with a designated physical location.
- **Standardization:** With respect to forward contracts, specific details about quality to be delivered, price, and delivery date are subjects of negotiation between the buyer and the seller. In futures contracts, however, the choice of expiry dates is limited, and trades have fixed sizes. This standardization paves the way for an active secondary market where

trades can be executed. However, perhaps the most pronounced benefit is increased liquidity.

- **Marking to market:** Since the clearinghouse must monitor the credit risk between the buyer and the seller, it performs daily marking to market. This is the settlement of the gains and losses on the contract on a daily basis. It avoids the accumulation of large losses over time, which can lead to a default by one of the parties.

- **Margins:** Daily settlements may not provide a buffer strong enough to avoid future losses. For this reason, each party is required to post collateral that can be seized in the event of default. The initial margin must be posted when initiating the contract. Suppose the equity in the account falls below the maintenance margin. In that case, the relevant party receives a margin call- a requirement to provide additional funds to restore the margin account to the initial level.

- **Clearinghouse:** The clearinghouse is an interposed party between the buyer and the seller, which ensures the performance of the contract. In essence, futures contracts have no credit risk. Each exchange has a clearinghouse. The clearinghouse splits each trade and acts as the opposite side of each position. It's the buyer to every seller and seller to every buyer. In other words, there is no direct contact between the short and long parties. It's the clearinghouse that **makes** margin calls whenever the need arises. In OTC markets, clearinghouses play a similar role.

- **Position limits:** The number of contracts that a speculator can hold is capped at a certain value by the exchange. The aim is to prevent speculators from having an undue influence in the market.

Long **exposure** in a futures contract means the holder of the position is obliged to buy the underlying instrument at the contract price at expiry. The holder will make a profit if the price of the instrument **goes up**. Conversely, they will make a loss if the price goes down. The long futures position can be entered by a speculator who expects the price to rise.

Short **exposure** in a futures contract means the holder of the position is obliged to sell the underlying instrument at the contract price at expiry. The holder will make a profit if the price of the instrument **goes down**. Conversely, they will make a loss if the price of the underlying rises dramatically.

Open Interest

Open interest refers to the number of existing contracts at any point in time. Consistently, the number of long positions always equals the number of short positions. As such, open interest can be described as the number of net long contracts (short contracts).

The net interest of trade is zero at the beginning of the contract. The open interest slowly rises as the trade progresses to a maximum amount seen before delivering the contract. For instance, consider a futures contract between two parties:

- When both parties take new positions, the open interest increases by one.
- When one party closes out a position while the other takes a new position, the open interest remains constant.
- When both parties close out their respective positions, the open interest decreases by one.

Specification of Contracts

Exchanges should clearly be defined as what is being traded in a futures contract. In a case where there are diverse deliverables, the location and time of delivery should be clearly defined, and the party with the short position has the right to choose. The specifications of the futures contract include:

- The contract size.
- Delivery location.
- Time of delivery.
- The underlying asset.

- The price quotes
- The price limit.
- Position limits.

Size of the contract

Exchanges should determine the size of their contracts to cater to both small (retail) and large (large corporations) scale traders. Compared to an agricultural product, the value of what is delivered for a contract on a financial asset is typically much higher.

Delivery Location

Futures contracts on commodities require the specification of the delivery location while taking into consideration the transportation costs. Note that for some contracts, transportation cost also determines the price of the futures contracts.

Time of Delivery

The time to delivery is usually specified in months. At the close of trading, the price of the futures contract is known as the settlement price. It determines the amount and the trader who gets the variation margin.

The future price can either start above or below the spot price. It, however, converges at the spot price as the period of delivery nears. If the prices do not converge and the futures price is above the spot price, traders can hedge by:

1. Shorting futures,
2. Buying spot assets, and
3. Making the delivery of the contract.

If the price is below the spot price, traders will buy the asset and make the future prices rise towards the spot price. Arbitrage opportunities such as these do not last long in the market since the investors take advantage and disappear quickly.

The Underlying Asset

Typically, futures contracts are written on financial assets such as currencies and commodities such as agricultural products. For financial assets, the definition of underlying is plain and simple. For instance, the CME Group defines the underlying asset of one contract on euros as 125,000 euros.

In the case of commodities, grading (based on quality) must be clearly specified of the commodities to be delivered. For instance, a contract should specify if the actual orange fruits or their concentrates are delivered.

Failing to specify the grading system of commodities by the exchange could cause the contract to be terminated. The trader in a short position can deliver low-quality products, which the trader in a long position can reject.

Price Limits

Price limits are set by the exchange and subject to change from time to time. Price limits within which future prices can move in either direction. These movements are known as limit movements. If the limit movements are exceeded, trading is stopped.

When a limit is reached and results in a price increase, the contract is termed the **limit up.** Conversely, if the limit movement results in a price decrease, the contract is called **limit down.**

The price limits help in preventing huge price movement due to speculation. On the downside, if the price limits result from the new information reaching the market, then the price limits obstruct the determination of the true market prices.

Position Limits

Position limit cabs the size of a position that a speculator can hold in the futures contract. Position limits are meant to prevent speculators' domination, which can result in an unacceptable market influence. Position limits are in tens of thousands of contracts.

Delivery Mechanics

Delivery of the underlying assets rarely happens in the futures markets as traders strive to close out their positions before the contract's maturity.

Assets can, however, be traded at spot markets using the most recent settlement price. Thus, the mechanics of delivery is crucial in futures markets.

The delivery time of a contract varies from one contract to another. The delivery process of the contract is initiated when a trader in a short position issues a notice of intention to deliver to the exchange central counterparty clearing house (**CCP**). The notice includes the number of contracts to be delivered and, in the case of commodity products, the grade of the commodity and location of delivery.

The exchange then chooses one or more traders with a long position to accept the contracts. Typically, traders who have had net long positions are allocated the delivery notices, but sometimes traders are allocated randomly. The members cannot deny the delivery notices and are often given a short period of time to transfer the contracts to other members.

The **first notice day** is the first day when the delivery notice is sent to exchange CCP. The **last notice day** is the last day when submission of the delivery notice to exchange CCP can be done.

The price paid for an asset is defined as the most recent settlement price and sometimes adjusted for the delivery location, grade, warehousing cost, and other factors.

Cash Settlements

Cash settlements save traders' delivery processes and costs. However, regulators try to discourage cash settlements since they resemble a gambling process. Therefore, delivery of physical settlements is preferred when it is possible.

However, CME Group's futures contracts on the S&P 500 are settled in cash. Other contracts settled in cash are contracts that depend on weather and real estate prices.

Market Participants

The futures market participants include:

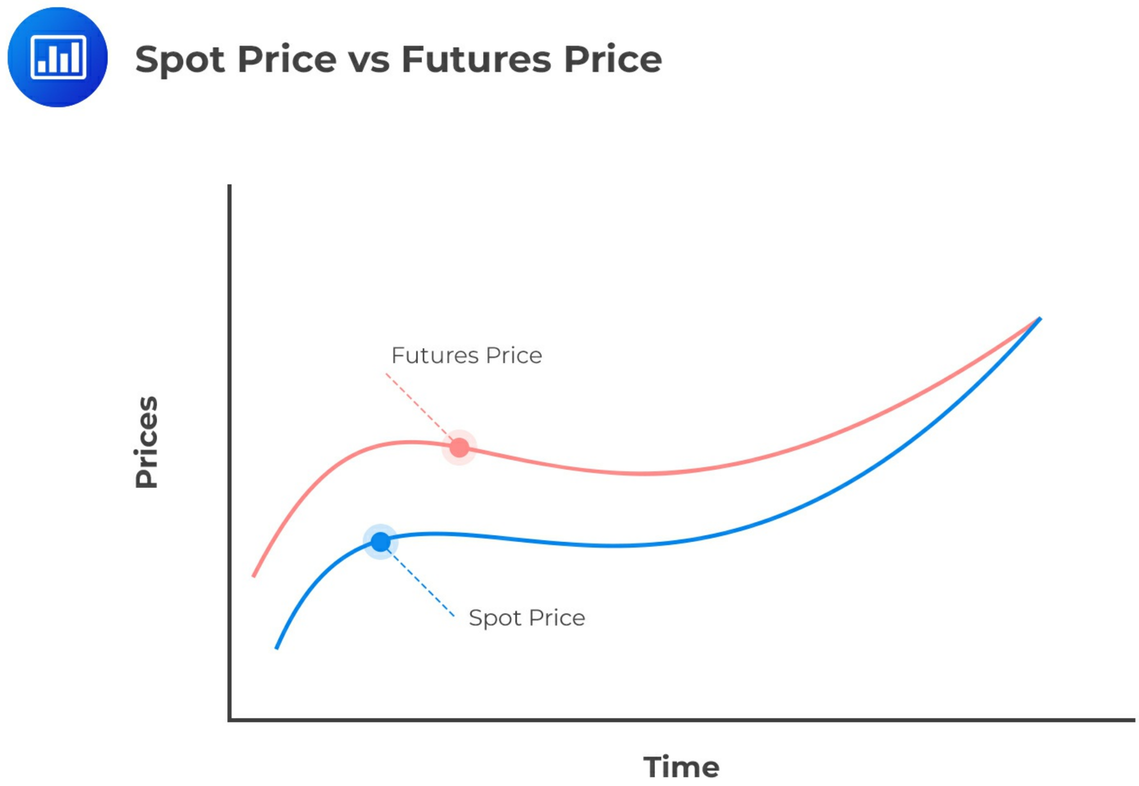
- **Futures Commission Merchants and Introducing Brokers**: They trade on behalf of their clients. Futures Commission merchants also manage clients' funds.
- **Locals**: Traders who trade on their own. Locals and the clients of the futures commission merchants can be classified as:
    - **Scalpels**: They close out future positions within a very short time.
    - **Day traders**: They close out trading positions on the same day they took the position.
    - **Position Traders**: They take positions that reflect market movements over time.

The Convergence of Futures and Spot Prices

The **spot price** is the current market price at which an instrument or commodity is bought or sold for immediate payment and delivery. On the other hand, the futures price is the price of an instrument/commodity today for delivery at some point in the future, called the maturity date. The difference between the two is called the **basis**.

$$\text{Basis} = \text{Spot Price} - \text{Futures Price}$$

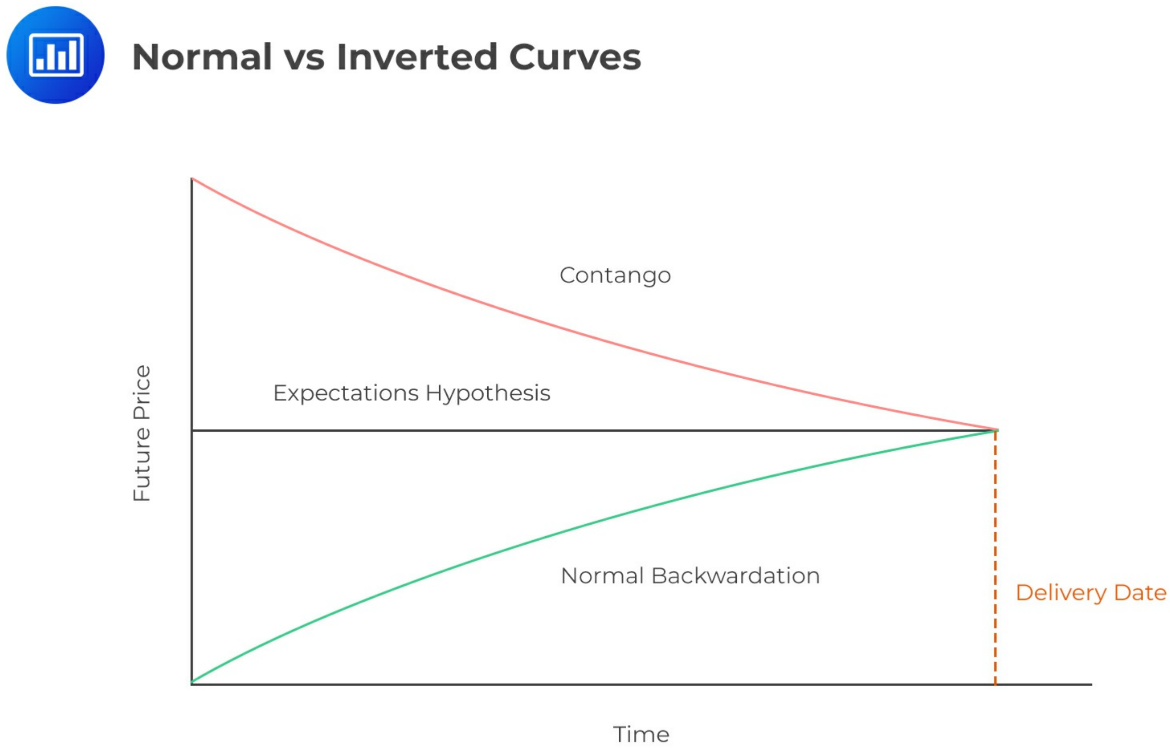
As the maturity date nears, the basis converges toward zero, i.e., the spot price tends towards the futures price. The two rates must be equal as long as no arbitrage opportunities exist on the actual maturity date. At maturity, the futures price becomes the current market price, which is actually the definition of the spot price.



Normal and Inverted Futures Markets

A normal futures market, also known as a **Contango** market, means that futures contracts are trading at a premium to the spot price. For example, suppose the price of a barrel of crude oil today is $50per barrel, but the price for delivery in three months is $53: the market would be in Contango. On the other hand, if crude oil is trading at $50 per barrel for delivery right now, and the three-month contract is trading at $45 per barrel, then that market would be said to be inverted (**backwardation**).

A **normal futures curve** will show a rising slope as the prices of futures contracts rise over time. An **inverted futures curve** will show a falling slope as the prices of futures contracts fall over time.



Terminating a Futures Contract

Traders with short or long positions in futures contracts can terminate them in one of four ways:

- **Delivery**: A short terminates the position by delivering the goods, and the long pays the contract price. This is called delivery. In each exchange, certain conditions must be met before delivery can be executed. Such conditions are contained in the "intention to deliver" file. This method is, however, hardly used.

- **Closeout**: This is a scenario where the futures trader closes out the contract even before the expiry. If a trader has a long position, they will take an equivalent short-term position in the same contract, and both positions will offset each other. Similarly, if a trader has a short position, they will take an equivalent long-term position in the same contract, and both positions will offset each other.

- **Cash settlement**: In this scenario, a trader leaves his position open, and when the contract expires, his margin account will be marked-to-market for P&L on the final day of the contract.

Exchange-for-physicals: In this case, a trader finds another trader who has an opposite position in the same futures contract and delivers the underlying assets to them. This happens outside the designated trading floor, but the traders are obliged to inform the clearinghouse of the transaction immediately afterward.

Placing Orders

A trader uses a futures order or options order to tell his broker exactly what to buy or sell when to do it, and at what price. There are several order types:

- **Market order**: A market order is executed at the best possible price. A market order instructs the executing broker to buy or sell futures contracts immediately at the market price, the best possible price.

- **Limit order**: Limit orders are orders where the trader specifies the price limit. As such, limited orders can only be carried out at a specified price or at an attractive price to the trader. A limit buy order is placed below the current market price, while a limit sell order is placed above the current market price.

- **Stop-loss order**: Limits a trader's loss by making the order a market order once the asset reaches a less favorable price.

- **Stop limit order**: It's an order that specifies both the stop and the limit price. If the spot price is attained, the order will be called a *limit order*. If the spot price equals the limit price, the order will be called a *spot and limit order*.

- **Market if touched order/Board order**: Traders trade at a specified or at a more favorable price. In other words, market if touched orders become market orders if a trade is executed at a specified price or more attractive price.

- **Discretionary/Market not held order**: Trader delays the order with the hopes of finding better prices for the asset.

Duration of Orders

Orders are to be executed by the end of the day, failure to which leads to cancellation. Traders specify other periods of time when the trade is active. **A fill-or-kill order** is an order that is canceled if not executed within a few seconds. An **open order or good-till-canceled** order is an order that is only canceled upon a trader's request; otherwise, the order remains open for the remaining life of the futures contract.

Regulation of Futures Markets

Regulations vary from country to country. The work of these regulators include:

- Ensuring that future markets remain open, giving out licenses to traders.
- Ensuring that the markets operate transparently.
- Ensuring that the markets are financially okay.
- Handling complaints arising from market participants.
- Overseeing the setting up of position limits.

Accounting

Gains and losses from futures markets are accounted for as they occur, a valuation process called **marking to market.** Consider the following example:

Consider an oil company with the fiscal year ending December. The company sells 1000 two-year futures contracts at the end of May when the futures price is $65 per barrel of crude oil. Each contract consists of selling 1,000 barrels of crude oil. Over the period of two years, the following happened:

- The future price of crude oil was $62.5 in the December of the first calendar year.
- The futures price decreases to $61 per barrel in the December of the second year.

- The contract is close out at $64.5 per barrel at the end of May of the third year.

The profit to the oil company is calculated as follows:

First year: (65-62.5)× 1,000 × 1,000 = $250,000

Second year: (62.5-61)× 1,000 × 1,000 = $150,000

Third year: (61-64.5)× 1,000 × 1,000 = -$350,000

Typically, futures are settled daily so that the cash in line with profits earned in the years is accounted for.

Hedging Accounting for Futures

Accounting for the profits and losses when hedging using futures contracts can lead to high earning volatility, which goes against the notion of hedging. Thus, if a company hedges against its position, it must take into consideration the hedging accounting.

The hedging accounting allows the profits (or losses) from the hedging strategy to be recognized simultaneously as the loss (or profits) on the hedged items.

Examples of regulations are: the Financial Accounting Standard Board (FASB) has provided statements FAS 133 and ASC 815, which explains when hedge accounting can and cannot be used by US companies. On the same note, the International Accounting Standards Board (IASB) has given out IAS 39 and IFRS 9.

If we consider the oil company example above and assume that it qualifies for hedge accounting, the gain of $500,000 [=(65-64.5)× 1,000 × 1,000] will be accounted for in the third year.

The rules governing hedging accounting are strict in that the hedge must be entirely documented (for example, with regard to items being hedged and hedging instruments). Moreover, the hedge must be classified as **effective,** implying that and economic activity that is not affected by credit risk must link the hedging instrument and the hedged instrument. Moreover, the efficacy of the hedge must be tested regularly.

Comparison between Futures and Forwards

Similarities

They both are an agreement to trade an item at a later date at a pre-determined price.

Differences

Futures	Forwards
Traded on an exchange.	Traded in an OTC market and are thus more prone to credit risk.
Both financial and non-financial variables can be traded.	Mainly traded on interest rates and foreign exchange.
Trades are settled on a daily basis.	Trades are settled at the end of the life of the asset.
Delivery is rare as traders close out positions before the delivery date.	Actual delivery is made.
The delivery date is specified.	The delivery period is specified and can be a whole month.

Question

A trader wishes to sell different grades of corn. Which of the below statements best describes how the price of the corn should be quoted?

A. Quote prices would be the same price for all corn

B. Quote prices would be different prices for each harvest period

C. The exchange would randomly decide which grade would be a higher/lower price

D. Quote prices would correspond to the quality of the corn

The correct answer is **D**.

Different prices should be quoted for each of the grades available. The prices should further correspond to the quality of the corn. The highest grade should be priced more than the lowest grade.

A is incorrect: The same price should not be quoted for all corn. This is because the price will not be a true reflection of the value of the corn.

B is not the *BEST* answer: The prices should not only be different but also reflect the grade of the corn.

C is incorrect: Randomly deciding on the price of the corn may lead to overpricing low-quality corn and underpricing high-quality corn.

Reading 32: Using Futures for Hedging

After completing this reading, you should be able to:

- Define and differentiate between short and long hedges and identify their appropriate uses.
- Describe the arguments for and against hedging and the potential impact of hedging on firm profitability.
- Define the basis and explain the various sources of basis risk and explain how basis risks arise when hedging with futures.
- Define cross hedging and compute and interpret the minimum variance hedge ratio and hedge effectiveness.
- Calculate the profit and loss on a short or long hedge.
- Compute the optimal number of futures contracts needed to hedge an exposure and explain and calculate the "tailing the hedge" adjustment.
- Explain how to use stock index futures contracts to change a stock portfolio's beta.
- Explain how to create a long-term hedge using a stack and roll strategy and describe some of the risks that arise from this strategy.

Short Hedges and Long Hedges

Short Hedge

A **short hedge** occurs when the trader shorts (sells) a futures contract to hedge against a price decrease in an existing long position. A short hedger already owns the underlying asset or is likely to gain ownership of the asset in the near future, after which they will sell it. When the hedged/underlying asset price decreases, the short futures position realizes a corresponding positive return that **offsets** the loss.

Therefore, a short hedge is suitable when:

- A company owns a certain amount of an asset and plans to sell it at a certain point in the future.
- A company anticipates receiving a certain amount of an asset in the future and intends to sell it.

Consider the following example.

Example: Calculating the Profit or Loss on Short Position

Oelig is an oil refining company and a member of the CME Group. The company will receive 5 million barrels of oil in two months. The company will sell the oil as soon as it receives it. The company knows that due to a large amount of oil it expects to receive, it will lose (gain) a whopping \$50,000 if the price per barrel decreases (increases) by just one cent (i.e., 5,000,000 × 0.01 = 50,000). Although there's a chance of a gain, the company feels this is a risk too big to take.

To hedge the risk of loss, the company enters into a short position. Assume that each traded futures contract involves the purchase or sale of 1,000 barrels of oil. Further, let's assume that Oelig receives the oil during the delivery period of the contract.

Suppose that the current spot price per barrel \$60, and the three-month futures price is 60.50. Now consider the following scenarios:

Scenario 1: At the time of delivery, the spot price per barrel of oil is \$55

Scenario 2: At the time of delivery, the spot price per barrel of oil is \$65

Let's now see what each of the above scenarios would mean for Oelig:

Scenario 1

Assuming the company did not enter the futures contract, the price received for the oil in the market would be \$275m:

$$5,000,000 \times 55 = \$275,000,000$$

In reality, the company is obliged to deliver under the futures contract. The profit made amounts to

$27.5m:

$$5,000(60.5 - 55) \times 1,000 = \$27,500,000$$

If we combine the price received for the oil in the market with the gain on the 5,000 futures contracts, the net price received is $302.5m:

$$\$275,000,000 + 27,500,000 = \$302,500,000$$

This translates to a net price of $ 60.50 per barrel (=$305,500,000/5,000,000). A closer look reveals that this matches the price locked in the futures contracts.

Scenario 2

Assuming the company did not enter the futures contract, the price received from the oil in the market would be $325m:

$$5,000,000 \times 65 = \$325,000,000$$

Under the futures contract, however, the company would be obliged to sell at the futures price of $60.5 per barrel. In these circumstances, the company would make a loss amounting to $22.5m:

$$5,000(60.5 - 65) \times 1,000 = \$22,500,000$$

When this loss is taken into account, the net price received for the oil is again $302.5m:

$$\$325,000,000 - 22,500,000 = \$302,500,000$$

Thus, the net price per barrel is $60.50 (=$302,500,000/5,000,000).

What this example shows, therefore, is that a combination of the market price of an asset at the maturity of a futures contract and the gain/loss on the futures contract should always work out to the futures contract price. It suffices to say that the hedge works as intended when the price of the asset declines between the time the contract is initiated and its maturity. If it increases, the hedger actually makes a loss.

Long Hedge

A **long hedge** occurs when the trader buys a futures contract to hedge against a price increase in an existing short position. A long hedger plans to buy the underlying asset in the future and fears a price rise, triggering a loss. When the hedged/underlying asset price increases, causing a loss, the long futures position realizes a corresponding positive return that offsets the loss in asset value.

Advantages and Disadvantages of Hedging

Advantages

1. **Hedging helps asset holders to lock in a price for their assets.** A corn farmer, for example, who is anticipating a bumper harvest in a few months, can lock in a predetermined price for their corn by taking a short position. By so doing, they eliminate - or at least reduce - the risk of a price decrease.
2. **Hedging helps prospective buyers to lock in a price for the goods they intend to purchase.** Instead of a cereal company waiting to buy corn at the prevailing post-harvest price, the company can lock in a predetermined purchase price by getting into a long futures contract. Even if prices rise dramatically between signing the contract and the maturity date, the company will benefit from a fixed
3. **Hedging makes earnings to be less volatile.** Less volatile earnings attract more investors.

Disadvantages:

I. **Hedging might lock asset holders out of improving market prices.** Although hedging shields asset holders from price declines, it locks them out of increases in value. Even if the asset's price rises, the short futures contract holder must honor all the terms of the contract. They must sell the underlying at the contract price.

II. **Hedging is necessarily not beneficial to a company's** Risk-averse shareholders are assumed to hold diversified portfolios that mitigate specific risks and reduce the effect of systematic risks. Thus, hedging at the company level doesn't necessarily benefit individual shareholders.

III. **Hedging may subject a firm to** In cases where there is negligible risk exposure, hedging might actually increase rather than decrease losses.

Basis Risk

Basis risk is the risk that the value of a futures contract will not move in a normal, steady correlation with the underlying asset price. For example, if the current spot price of gold is $1,500, and the six-month futures price of gold is $1,550, then the basis is $50. Basis risk, in this case, is the risk that between now and the maturity of the contract in six months, the price of gold will fluctuate by more than $50.

A trader is exposed to basis risk if they close out a futures contract before its maturity. The basis is the difference between the spot price and the futures price, and the basis risk the risk associated with the basis at the time of closing out a contract.

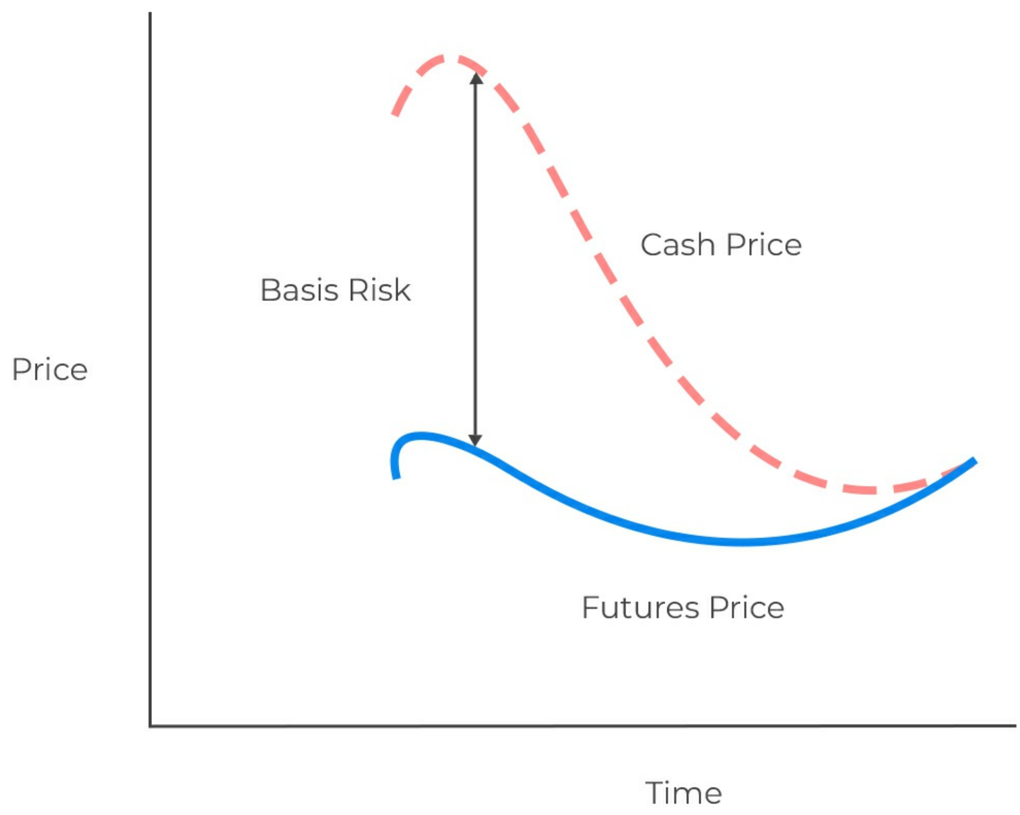
$$\text{Basis} = \text{Spot Price} - \text{Futures Price}$$

If the asset being hedged is different from the underlying asset in the futures contract, Then;

$$\text{Basis} = \text{Spot Price of the hedged asset} - \text{Future price of the underlying asset in the futures contract}$$

The fluctuation in the basis makes hedges less effective than they are meant to be. Between contract initiation and liquidation, the price spread (the difference between the cash and futures price) may narrow or widen.

Illustration of Price Spread



Sources of Basis Risk:

- Imperfect matching between the cash asset and the hedged asset, e.g., hedging jet fuel with motor vehicle fuel.
- Changes in the components of the cost of carry, e.g., interest, storage and safekeeping, and insurance.
- Maturity mismatch, e.g., hedging exposure to physical prices in May with a June futures contract.
- Location mismatch, e.g., hedging crude oil sold in New York with crude oil

futures traded on a Mumbai futures exchange.

To minimize basis risk, choosing the hedging tool that's most correlated with the underlying is imperative.

Basis Risk and Hedging

Short Position

Consider an investor who is due to sell an asset at time t (future), and thus, the investor enters into a short position.

Define the following terms:

F_0 = Future Price at the time of initiation of a contract.

F_t = Future Price at the time the contract is closed out.

S_t = Spot price at the time the contract is closed out.

b_t = Difference difference between the spot price and the futures price at time t (Basis at time $t = S_t - F_t$)

At time t the investor in short position will receive a price of S_t from the asset sold and the profit from the short position in a futures contract will be $F_0 - F_t$. Thus,

$$\begin{aligned}\text{Net Price Received from a Short Hedge} &= S_t + (F_0 - F_t)\\ &= F_0 + (S_t - F_t)\\ &= F_0 + b_t\end{aligned}$$

From the expression above, it easy to see that if the spot price is the same as the futures price when the hedge closed out, then the basis is zero, and thus the net price received is equivalent to futures price ((F_0). On the flip side, if the hedge is closed out before the delivery period and the underlying asset is different from the one delivered, the hedger is subject to basis risk.

Example: Short Hedge and Basis Risk

A farmer will need to sell 200,000 bushels of corn in January. To hedge his position, the farmer uses February CME Group's futures contracts, which are set such that each contract is on 10,000 bushels of corn. At the hedge initiation, the futures price is $3.50 per bushel, and when the hedge is closed out, the futures price was $3.75 per bushel in January.

The spot price for corn sold in January is $3.80 per bushel. What is the net price received per bushel if the farmer closes out his hedge in January?

Solution

If the farmer closes out his position in January, the net price received from the corn is given by:

$$200,000 \times 3.80 + 20 \times 10,000 \times (3.50 - 3.75) = \$710,000$$

Thus, the net price received by the farmer is:

$$\frac{710,000}{200,000} = \$3.55 \text{ per bushel}$$

The result above can be explained as follows: The price of corn increases, so the farmer loses $0.25 (=$3.75-$3.50) per bushel when he closes his futures contracts. Instead, there is a basis of $0.05 ($3.80-$3.75) per bushel, which implies that when the farmer sells the corn at a price of $3.80 per bushel, the basis ($0.05) and the loss ($0.25) combine to reduce the price received per bushel to $3.55 per bushel.

Long Position

Consider an investor who will buy an asset in the future, say at time t. At time t, the investor will pay a price of S_t for the asset received, and the profit on the long position in a futures contract is $F_t - F_0$. Therefore,

$$\begin{aligned}\text{Net Cost of an asset on long hedge} &= S_t - (F_t - F_0) \\ &= F_0 + (S_t - F_t) \\ &= F_0 + b_t\end{aligned}$$

From the above equation, the net price received for the short hedge and the long hedge is both equal to $F_0 + b_t$.

In the context of hedging and basis risk, futures price (F_0) is known when the hedge is initiated; however, the outcomes of hedging depend on uncertainty of the futures' basis (b_t).

Example: Long Hedge and Basis Risk

A US-based company will need to buy GBP 437,500 in March. The company wishes to hedge itself against the exchange rate risk using the CME Group's April futures contract. CME Group trades GBP 62,500 per contract.

At the hedge initiation, the futures price was USD 1.30 per GBP, and at the time of the hedge close out (April), the futures price was USD 1.32 per GBP. However, the spot price in April is USD 1.35 per GBP.

What is the net exchange rate for the company when the hedge is closed out?

Solution

The company needs 7 (=437,500/62,500) CME Group's futures contract to hedge its position. As such, the value of British pounds is given by:

$$437,500 \times 1.35 - [(1.32 - 1.30)] \times 7 \times 62,500 = 581,875$$

Thus, the net exchange rate will be:

$$\frac{581,875}{437,500} = \text{USD } 1.33$$

The above result can be explained as follows:

The exchange rate rises, and hence the hedge improves (deteriorates) the price by USD 0.02 per GBP (=1.32-1.30). However, note that the basis is USD 0.03 (=1.35-1.32) per GBP. The basis amount implies that the company can cash out the futures contract at USD 1.32 per GBP and buy the underlying asset at USD 1.35 per GBP. Given that there is no basis risk, the company will pay the future price of USD 1.30. However, the basis risk increases the futures price to USD 1.33.

Cross Hedging and Optimal Hedge Ratios

There are instances when it may be impossible to find futures contracts on a particular underlying asset. In such scenarios, the hedger may turn to futures on securities that positively correlate with the underlying. This is called **cross hedging**. In other words, cross hedging involves hedging the risk exposure of one asset with the futures contracts in another asset. Since the assets are not entirely identical, there must be enough correlation for the hedge to work. The **Hedge Ratio** is the proportion of position in the futures contracts to the position in the underlying asset.

Cross Hedging Analysis

In cross hedging analysis, the relationship between the changes in spot price and futures price during the life of the hedge is considered. As such, define the following:

ΔS = change in the spot price during the life of the hedge.

ΔF = change in the futures price during the life of the hedge.

The above variables are regressed using the historical data, and that the line of best fit is given by:

$$\Delta S = a + b\Delta F + \epsilon$$

Where a and b are constant, and ϵ is an error term.

Now define the variable *has* the **hedge ratio** such that the proportional change in the value per given unit hedged asset is given by:

$$\Delta S - h\Delta F = a + (b - h)\,\Delta F + \epsilon$$

If we let *h =b,* then the above equation reduces to:

$$\Delta S - h\Delta F = a + \epsilon$$

Optimal Hedge Ratio

The optimal hedge ratio, also called the minimum variance ratio, is the degree of correlation between the underlying asset and the futures contract purchased to hedge financial risks. It is the ratio of the futures position to the spot position.

Define the following variables:

ρ = correlation coefficient between ΔS and ΔF.

σ_S = standard deviation of ΔS.

σ_F = standard deviation of ΔF.

h^0 = Optimal hedge ratio.

Using the result from the cross-hedging analysis, recall that the slope coefficient in a linear model is given by:

$$b = \frac{\text{cov}(\Delta S, \Delta F)}{\sigma_{\Delta F}^2} = \frac{\rho \sigma_{\Delta S} \sigma_{\Delta F}}{\sigma_{\Delta F}^2} = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}}$$

Using the above analogy and the defined variables, it is true that:

$$h^0 = \rho \frac{\sigma_S}{\sigma_F}$$

The last expression implies that the hedge effectiveness is a proportion of ΔS that is hedged away and usually termed as R^2 of the regression. In the case of a linear model with one variable, $R^2 = \rho^2$.

The variables ρ, σ_S, and σ_F are usually determined from the historical data over the life of the hedge. However, short periods are normally used to get more reliable estimates.

Optimal Number of Futures Contracts Needed to Hedge an Exposure

Define the following variables:

Q_A = number of units of the position being hedged.

Q_F = number of units of underlying assets in one futures contract.

N^0 = Optimal number of futures needed for hedging.

The optimal number of futures contracts for hedging a given exposure s given by:

$$N^0 = \frac{h^0 Q_A}{Q_F}$$

Example: Determining the Optimal Number Futures Contracts Needed to Hedge an Exposure

A US airline wishes to hedge its jet fuel using heating oil. From the historical data, it is estimated that the correlation between the monthly price changes of heating oil prices and jet fuel prices is 0.65. The standard deviation of monthly changes in heating oil futures prices is $0.045 per gallon, and that of jet fuel price per gallon is $0.034 per gallon.

The airline has also estimated that it will purchase 2 million gallons of jet fuel in one month. As such, the airline wishes to hedge this position. As per the exchange contract terms, one futures contract is on 50,000 gallons of heating oil.

Calculate the optimal hedge ratio and the number of long contracts required for the airline to hedge its position.

Solution

The optimal hedge ratio is given by:

$$h^0 = \rho \frac{\sigma_S}{\sigma_F} = 0.65 \times \frac{0.034}{0.045} = 0.4911 \approx 49.11\%$$

The number of long contracts needed by the airline to hedge its position is given by:

$$N^0 = \frac{h^0 Q_A}{Q_F} = \frac{0.4911 \times 2,000,000}{50,000} = 19.64 \approx 20$$

Tailing the Hedge

The hedging analysis presented thus far is true when forward contracts are considered. However, if we are to use futures contracts, we ought to make an adjustment called **tailing the hedge** to accommodate the fact that the futures are settled daily (the hedger uses series of daily hedges). Typically, analysts will use the standard deviation of the daily returns rather than the standard deviation of price changes.

Define the following variables:

$\hat{\sigma}_S$ = standard deviation of one-day returns in the spot price, equivalent to the percentage change in the spot price.

$\hat{\sigma}_F$ = standard deviation of one-day returns as provided by the futures price and equivalent to a percentage change in futures.

$\hat{\rho}$ = the correlation coefficient between the one-day spot and futures returns.

S = Spot price.

F = Futures price.

Q_A = number of units of the asset being hedged.

Q_F = number of units of underlying assets on one futures contract.

$V_A = SQ_A$ = value of the position being hedged.

$V_F = FQ_F$ = value of one futures contract.

From the above variables, it easy to see that:

$$\text{Standard deviation of daily spot price change} = \hat{\sigma}_S S$$

and

Thus, the optimal hedge portfolio is given by:

$$h^0 = \hat{\rho}\frac{\hat{\sigma}_S S}{\hat{\sigma}_F F}$$

Consequently, the optimal number of futures contracts needed to hedge an exposure is given by

$$N^0 = \hat{\rho}\frac{\hat{\sigma}_S S}{\hat{\sigma}_F F}\frac{Q_A}{Q_F} = \hat{\rho}\frac{\hat{\sigma}_S V_A}{\hat{\sigma}_F V_F}$$

The optimal hedge ratio and the optimal number of contracts needed to hedge an exposure is estimated as:

$$\hat{h}^0 = \hat{\rho}\frac{\hat{\sigma}_S}{\hat{\sigma}_F}$$

And

$$N^0 = \hat{h}^0\frac{V_A}{V_F}$$

Note: Identify the differences between these formulas and the ones in cross hedging analysis given as:

$$h^o = \rho\frac{\sigma_S}{\sigma_F}$$

Notice that σ_S and σ_F are the standard deviation of changes in S and F, respectively over the life of the hedge, and ρ is the correlation between σ_S and σ_F. On the other hand, $\hat{\sigma}_S$ and $\hat{\sigma}_F$ are the standard deviation of daily returns in S and F, respectively and $\hat{\rho}$ is the correlation coefficient between $\hat{\sigma}_S$ and $\hat{\sigma}_F$.

Example: Tailing the Hedge

A risk manager notices that the standard deviation of daily return on spot price is 1.5% in a given exchange, and the standard deviation of daily return on futures price is 0.9%. The correlation between the daily returns on the spot and futures price is 0.75. The risk manager wishes to hedge assets worth $100,000. The value of one futures contract is $10,000.

What is the optimal hedge ratio and the optimal number of contracts needed by the risk manager to hedge its exposure?

Solution

The optimal hedge ratio is given by:

$$\hat{h}^0 = \hat{\rho}\frac{\hat{\sigma}_S}{\hat{\sigma}_F} = 0.75 \times \frac{0.015}{0.009} = 1.25$$

The optimal contracts are given by:

$$N^0 = \hat{h}^0\frac{V_A}{V_F} = 1.25 \times \frac{100,000}{10,000} = 13$$

Hedging Equity Positions

Stock index futures can be used to alter exposure in the market. In other words, stocks index futures provide a way of increasing or decreasing the market exposure without paying a lot in transaction fees. For instance, consider an investor who is excellent at choosing stocks but has no opinion on the market's future course. The investor will wish to construct a portfolio and then hedge the market's return to earn an excess return of the selected stocks over the market's return.

Example: Hedging Equity Position

An investor has a diversified portfolio of $360,000. The portfolio is set such that it closely tracts FTSE 100. The investor wants to have three-month market exposure. To hedge the investor's position, the investor uses the CME futures contracts on FTSE 100, which is $60 multiplied by the index. The futures price is $3,000.

Assume that the standard deviation of the daily return on the futures price and the standard deviation of the daily return of the asset being hedged is equal. Moreover, the correlation between the daily returns of the spot and futures price is 1.0.

What is the optimal number of contracts to hedge the investor's position?

Solution

Given the information given in the question, we have that:

$$N^0 = \frac{V_A}{V_F} = \frac{360,000}{60 \times 3,000} = 2$$

Adjusting a Stock Portfolio's Beta Using Stock Index Futures

As defined in the capital asset pricing model, beta is a measure of a portfolio's systematic risk. When a trader uses index futures to hedge a position in an equity portfolio, they effectively reduce its systematic risk. As such, hedging is an attempt to reduce a portfolio's beta.

Assume that we want to use S&P 500 futures to hedge a well-diversified portfolio with a beta β. By assuming that the standard deviation of the daily return of the asset being hedged is equal to β multiplied by the standard deviation of the return provided by the futures price and the correlation between the daily return on the asset and the futures price is approximately 1.0, then the futures contracts to hedge a position is given by:

$$N^0 = \beta \frac{V_A}{V_F}$$

Example: Calculating Optimal Contracts Using Stock Index Futures

An investor contracts a portfolio with a beta of 1.45 and worth \$300,000. The six-month index futures are worth \$3,000 per contract. The index futures contract is on \$60 multiplied by the index.

What is the optimal number of contracts required to hedge the portfolio?

Solution

The optimal number of contracts is given by:

$$N^0 = \beta \frac{V_A}{V_F} = 1.45 \times \frac{300,000}{60 \times 3,000} = 1.667 \approx 2$$

The example above assumes that the investor wishes to hedge the beta entirely, that is, to reduce the beta to zero. However, index futures can also be used to modify the portfolio beta.
Let us define the following variables:

β = current portfolio beta.

β^* = target beta.

V_A = value of the position being hedged.

V_F = value of one futures contract.

If $\beta > \beta^*$ the number of contracts that the investor should short is given by:

$$(\beta - \beta^*)\frac{V_A}{V_F}$$

On the other hand, if $\beta < \beta^*$ the number of contracts needed by the investor to take a long position in is given by:

$$(\beta^* - \beta)\frac{V_A}{V_F}$$

Example: Modifying Portfolio Beta

An investor contracts a portfolio with a beta of 1.45 and worth $600,000. The six-month index futures are worth $3,000 per contract. The index futures contract is on $60 multiplied by the index.

The investor wishes to reduce portfolio beta to 0.65. What is the required number of contracts that the investor needs to short?

Solution

Based on the information given in the question, $\beta = 1.45$ and $\beta^* = 0.45$; thus, the required number of contracts is given by:

$$(\beta - \beta^*)\frac{V_A}{V_F} = (1.45 - 0.65)\frac{600,000}{3,000 \times 60} = 2.667 \approx 3$$

The investor, therefore, needs to short three futures contracts to reduce the portfolio beta from 1.45 to 0.65.

Constructing Long-Term Hedges using Stack and Roll Strategy

Hedgers are sometimes confronted with a scarcity of liquid futures contracts for the desired hedge maturities since most liquid futures contracts usually have shorter maturities. To overcome this challenge, the hedgers employ a **stack and roll strategy.**

The stack and roll strategy involves the following:

- Employ a short maturity futures hedge.
- Closeout the hedge just before the delivery and replace it with another futures hedge with a short maturity.
- Close out the new short maturity hedge just before the delivery date and replace it with another short maturity hedge. Repeat the process on and on.

Example: Stack and Roll Strategy

An oil refinery company knows that it will need to sell 1 million barrels of refined oil in 18 months. It is currently 1 January of the first year. Based on the exchange to which the company is registered, the liquid futures contracts are those with maturities of 6 months or less. However, the company wishes to construct a long-term hedge using a stack and roll strategy.

The exchange's refined oil futures contract trade with maturities in February, April, June, July, October, and December. The futures contract is traded on 5000 barrels of oil per contract. To implement its strategy, the company does the following:

- In January of the first year, the company sells 200 futures contracts maturing in July of year 1.
- In June of the first year, the company closes out the futures maturing in July of the first year and sells 200 futures contracts maturing in the February of the second year.
- In January of the second year, the company closes out the futures maturing in the February of the second year. It sells 200 futures contracts maturing in July of the second year.
- In June of the second year, the company closes out the futures maturing in

July of the second year.

The futures price at each subsequent closeout and purchase of futures contracts is as follows:

- The futures contracts are closed out at \$55 per barrel in June of the first year, and the futures contracts are sold at \$56 in the July of the first year.
- The futures contracts in the January of the second year are closed out at \$53 per barrel, and the futures contracts are sold at \$55.50 per barrel in the February of the second year.
- The futures contracts in the June of the second year are closed out at \$57 per barrel, and futures sold at \$54 in the July of the second year when the spot price is \$54.25 per barrel.

What is the total profit(loss) on the short positions of the futures contracts?

Solution

Based on the information given in the question, the profit per contract if given by:

$$5,000\,(56-55)+5,000(55.50-53)+5,000(54-57)=\$2,500$$

Thus, the total gain is:

$$\text{Total Gain}=\frac{1,000,000}{5,000}\times 2,500=\$500,000$$

Cash Flow Considerations

Because of the nature of the daily settlements of futures, discrepancies may arise between the cash flows in a futures contract used for hedging and the ones from the exposure that is being hedged. Therefore, care should be taken to ensure that losses from a futures contract can be easily financed until gains from the hedged positions start to flow in.

Question

Rachel Zane, FRM, is a portfolio manager in charge of a $200 million, well-diversified portfolio that has a beta of 1.5 relative to the Nasdaq-100. The current value of the 3-month Nasdaq-100 Index is 2,500, and the multiplier is 300. Over the next three months, Ms. Zane wants to use the Nasdaq-100 futures to reduce the systematic risk of the portfolio to 1.0. To pull that off, which of the following moves is required?

A. Sell 100 contracts
B. Buy 133 contracts
C. Sell 133 contracts
D. Buy 100 contracts

The correct answer is **C**.

Number of contracts required $=(\beta^* - \beta)\left(\frac{P}{A}\right)$

Where:

β=portfolio beta

β^*=target beta after hedging

P=portfolio value

A=value of futures contract

$$
\begin{aligned}
&= 1.0 - 1.5 \times \frac{200,000,000}{2,500 \times 300} \\
&= -133
\end{aligned}
$$

The negative sign implies 133 contracts need to be sold.

Reading 33: Foreign Exchange Markets

After completing this reading, you should be able to:

- Explain and describe the mechanics of spot quotes, forward quotes, and future quotes in the foreign exchange market and distinguish between the bid and ask exchange rates.
- Calculate bid-ask spread and explain why bid-ask spread for spot quotes may differ from the bid-ask spread for the forward quotes.
- Compare outright (forward) and swap transactions.
- Define, compare and contrast transaction risk, translation risk, and economic risk
- Describe the examples of the transaction, translation, and economic risk and explain how to hedge these risks.
- Describe the rationale for multi-currency hedging using options.
- Identify and explain the factors that determine the exchange rates.
- Calculate and explain the effect of an appreciation/depreciation of a currency relative to a foreign currency.
- Explain the purchasing power parity theorem and use this theorem to calculate the appreciation or depreciation of a foreign currency.
- Explain how the no-arbitrage assumption in the foreign exchange markets leads to the interest rate parity theorem and use this theorem to calculate forward foreign exchange rates.
- Distinguish between covered and uncovered interest rate parity conditions.

The Foreign Exchange Market

The foreign exchange market is a type of market where the parties involved (collection of hedgers and speculators) exchange one currency for another at the specified rates. The market is also called

Forex, Fx, or currency market.

Currency Quotes

The exchange rate can be defined as the number of units of one currency (the quote currency) that are needed to purchase one unit of another currency (base currency). The exchange market is the world's largest market, where all forms of exchange transactions are carried out.

Generally, currency quotes always appear as AAA/BBB or AAABBB, where AAA and BBB are different currencies. The currency to the left of the slash is the base currency, while the currency to the right of the slash is the quote currency.

The base currency (in this case, AAA) is **always equal to one unit**, and the quoted currency (in this case, BBB) is **what that one base unit is equivalent to in the other currency**.

The general convention is to quote base currency/ quoted currency. Currency quotes between USD and another currency is the most common exchange rate. Other quotes, for instance, between GPB and CAD, are called **cross-currency**. In most cases, the USD is the base currency, while the other currency is the quote currency. However, when the US dollar is quoted with the British Pound, the Euro, the Newzealand dollar, and the Australian dollar, then the USD becomes the quote currency.

Example: Interpreting the Currency Quotes

The EUR/USD is quoted as 1.2563. How do we interpret this quote? This quote implies that we need 1.2563 USD to buy one euro.

Bid and Ask Prices

The **bid price** is the price at which the counterparty is willing to buy one unit of the base currency, expressed in terms of the price currency. On the other hand, the a**sk price** is the price at which a counterparty is willing to sell one unit of the base currency, expressed in terms of price currency. For instance, a dealer might quote EUR/USD exchange rate of 1.3849. This quote implies that the dealer is willing to pay 1.3849 USD to buy 1 Euro. Intuitively, we expect the bid price to be slightly less than the offer price because the dealer's goal is to make some cash in every transaction. With that in mind, it is easier to single out the bid price or the offer price given a quote.

Characteristics of Bid-Ask Quotes

I. The ask price should always be higher than the bid price.
II. A market participant requesting the two-sided price quote has the option but not the obligation to transact at either the bid or the ask quoted by the dealer. If a party chooses to transact at the quoted prices, they are said to have "hit the bid" or "paid the offer."

Spot and Forward Exchange Rates

Spot Exchange Rates

A spot exchange rate is the prevailing price level in the market used to directly trade one currency for another, which is delivered at the earliest possible time. The standard delivery time for spot currency transactions is no longer than T+2, after which it will be deemed a forward contract. Spot exchange rates are usually quoted with four decimal places. For instance, the spot-bid for EUR/USD could be stated as 1.1745 and the spot-ask as 1.1747.

Forward Exchange Rates

A forward exchange is a price at which one currency is traded against another at some specified time in the future. The forward exchange rate must respect the arbitrage relationship - the returns from two alternatives, but equivalent investments must be equal (as we will see in covered rate parity).

Forward exchange rates are not quoted with the same base as the spot exchange rates but rather as points that are multiplied by $\frac{1}{10,000}$ then added to the spot exchange rate.

Example: Calculating the Forward Exchange Rates

The following table gives the forward rates as of June 16, 2019. The spot bid and ask rates are 1.1745 and 1.1749, respectively.

Maturity	Bid	Ask
1 month	27.12	28.60
2 months	53.15	54.15
3 months	81.87	83.07
4 months	113.59	114.99
5 months	139.07	140.47

What is the three-month forward bid and ask quotes?

Solution

Recall that forward exchange rates are not quoted with the same base as the spot exchange rates but rather as points that are multiplied by $\frac{1}{10,000}$ then added to the spot exchange rate.

Since we are given the spot bid rate as 1.1745, then the 3-month forward bid rate is:

$$1.1745 + \frac{1}{10,000} \times 81.87 = 1.1745 + 0.008187 = 1.182687$$

Analogously, the 3-month forward ask quote is

$$1.1749 + \frac{1}{10,000} \times 83.07 = 1.1749 + 0.008307 = 1.183207$$

The Bid-Ask Spread

The **bid-ask spread** is the amount by which the offer price exceeds the bid price of a currency in a market. Basically, it is the difference between the highest price that the purchaser is willing to pay and the lowest price that a seller is willing to accept.

Example: Calculating the Bid-Ask Spread

Continuing with the example above, the bid-ask spread for the 3-month forward rate is calculated as:

$$1.183207 - 1.182687 = 0.00052$$

Note that this can also be calculated as:

$$(1.1749 - 1.1745) + (0.008307 - 0.008187) = 0.0004 + 0.00012 = 0.00052$$

That is, we could calculate the bid-ask spread by adding the bid-ask spreads of the spot exchange rate and that of the points.

When the forward exchange rate is less than the spot rate, the points are negative. However, it should be apparent that the magnitude of the negative ask price is less than that of the bid price. This makes sure that the ask price is larger than the bid price. Consider the following example.

Example: Calculating the Forward Exchange Rate

The following table gives the forward rates as of July 8, 2019. The spot bid and ask rates are 1.3184 and 1.3185 respectively.

Maturity	Bid	Ask
1 Month	−9.39	−7.67
2 months	−18.20	−17.29
3 months	−32.10	−30.91
4 months	−40.91	−39.42
5 months	−45.90	−43.95

What is the five-month forward bid and ask quotes?

Solution

The 5-month forward bid quote is:

$$1.3184 + \frac{1}{10,000} \times -45.90 = 1.3184 - 0.004590 = 1.31381$$

Analogously, the forward ask quote is:

$$1.3185 + \frac{1}{10,000} \times -43.95 = 1.3185 - 0.004395 = 1.314105$$

At this point, we could easily calculate the bid-ask spread:

$$1.314105 - 1.31381 = 0.000295$$

Outrights and Swaps

Recall that a forward exchange is a price at which two parties agree to trade one currency against another at some specified time in the future. This is termed as an **outright transaction or outright forward transaction**.

On the other hand, a foreign exchange swap is a type of exchange rate transaction where a currency is bought (sold) in a spot market and then sold (bought) in the forward market. From a different angle, an FX swap is a method of funding an asset transacted in foreign currency by paying the interest due in terms of the domestic currency. An example of an FX swap is where a US-based company funds its Chinese investment by borrowing in USD and buying the Chinese Yuan, and after some time, the company exchanges the money back to USD. By doing this, the company can fund its operation in the Chinese Yuan.

Swap transactions are usually profitable if the foreign currency used will be of more value in the forward market, i.e., more of the local currency will be received for less of the foreign currency at the agreed-upon future date.

For instance, if we use the following table:

Maturity	Bid	Ask
1 month	27.12	28.60
2 months	53.15	54.15
3 months	81.87	83.07
4 months	113.59	114.99
5 months	139.07	140.47

Assume that a US company borrows in USD and buys 1 million EUR today to fund its European operations. At the same time, the company also agrees to sell 1 million EUR for USD within one-month time. In the table above, the points in one month's time are 27.10. This implies that EUR is valuable in the forward market, and thus, the points reduce the net funding cost in USD since more USD is going to be received in a one-month time relative to the amount that would have been received today.

Currency swaps will be discussed in details in chapter 20

Future Quotes

Future quotes are the exchange-traded futures legal contract that stipulates the price in one currency at which another currency can be bought or sold at a future date. A good example is the Chicago Mercantile Exchange (CME) in the US, where diverse futures contracts on exchange rates between USD and other currencies are made.

In the CME setting, USD is always the base currency since investors treat foreign currency as assets value in USD. Assume that the 6-month forward quote for the USD/CAD is 1.2900. The future quote is found by finding the reciprocal of the forward quote. That is:

6-month futures quote is equivalent to the 6-month forward quote is: $\frac{1}{1.2900}$=0.7752 USD per CAD

Estimation of the Foreign Exchange Market (FX) Risk

Firms in the foreign exchange market are exposed to risks. They should, therefore, be keen on the extent to which they could accept risk. Once this is known, the firms should decide whether the risk levels are acceptable and if not, and if not, they should apply appropriate hedging strategies.

Three categories of risk are examined, and these include:

I. Transaction risk
II. Translation risk
III. Economic risk

Transaction Risk

This kind of risk is associated with received and paid capital; it affects the cash flows of a company. Let us look at a simple example:

Assume that an American company imports goods from Kenya, for which it pays in Kenyan shillings. By doing this, the company is faced with USD/KSH risk. That is, if KSH gains strength, then the company will experience little profits if it is required to buy KSH to pay for its services.

In summary, a company buying from a foreign company will be exposed to losses (profits) if: the currency of the foreign company strengthens (loses value), implying that more (less) of the local currency will be needed to purchase one unit of the foreign currency.

Conversely, a company selling to international clients will suffer losses (profits) if the currency of the foreign company weakens (strengthens), implying that more (less) of the received revenue will be needed so as to convert it to the local currency.

Hedging Transaction Risk

Transaction risk is hedged using outright forward transactions and swaps.

In the case of hedging using outright transactions, consider the case of an American company investing in Kenyan Shillings. The company could hedge its position by buying KSH forward, which would hedge the exchange rate paid to Kenyan suppliers while selling USD forward would lock in the FX rate applied to the revenues of USD.

On the other hand, the FX swap is applied when the company owns a foreign company that owns foreign currency that will be used for purchases at a future time to earn interest in its domestic currency. The company would sell the foreign currency in exchange for its domestic currency in the spot market and repurchase it at a stated future time in the forward market.

Translation Risk

This type of risk comes up when an institution’s assets and liabilities are in a foreign currency, which must be valued in the institution’s domestic currency when the financial statements are made. Thus, the institution can experience foreign exchange gains or losses.

As compared to transaction risk, translation risk does not affect the cash flows of a company.

Example: Demonstrating Translation Risk (Investment)

A Canadian company has its investments in the US. At the end of the first year, the company netted

USD 20 million, and the USD/CAD at that time was 1.3200. At the end of the second year, the company's net value remained the same as that of the first year. However, the USD/CAD has changed to 1.2700. At the end of the two years, the company wishes to produce its 2-year financial statements. Intuitively, the company experienced a loss of:

$$(1.32000 - 1.2700) \times 20,000,000 = \text{CAD } 1,000,000$$

Foreign gains or losses can also arise from borrowing in foreign currency. To illustrate this, let us look at an example.

Example: Demonstrating Translation Risk (Borrowing)

Suppose that the Canadian company has a loan of USD 10 million that is supposed to be returned in 10 years (assume that it is paid at par). The interests are paid in USD, implying that the company is prone to transaction risk. Additionally, the company is exposed to translation risk, which is embedded in the repayment of the loan principal.

Given that the USD/CAD at the end of 1st year was 1.3100 and at the end of the 2nd year, USD/CAD had reduced to 1.2800, the value of the loan at the end of the 1st year is:

$$10,000,000 \times 1.3100 = \text{CAD } 13,100,000$$

And at the end of the second year:

$$10,000,000 \times 1.2800 = \text{CAD } 12,800,000$$

Intuitively, the company earns a foreign gain of

$$13,100,000 - 12,800,000 = \text{CAD } 300,000$$

The above results can be attributed to the fact that the USD has weakened over the CAD. On the other hand, had the USD strengthened, the company would have experienced a loss.

Hedging Translation Risk

Translation risk is hedged using the forward contracts on the reporting date to decrease the volatility of profits. The forward contract involves a plan by the concerned firm to sell foreign currency assets or retire foreign currency liabilities at a later time.

Another method of hedging translation is by financing the assets in a country with the borrowed funds in that country. By doing this, the gains (losses) on assets are offset by the losses (gains) on the liabilities.

To see this, suppose that in our example above. Suppose the Canadian company has a USD 20 Million worth of investment. If the company analyzes its position and realizes that the translation is imminent, it could finance its investment with a USD 20 million loan.

It is worth noting the difference between transaction risk and translation risk. We can see that transaction risk has direct effects on the cashflows of a company which is not the case for translation risk. However, the effects of translation risk on the reported earnings of a company can be big.

Also, note that it is only reasonable that we hedge translation risks on one future date. Doing otherwise will be overhedging.

Economic Risk

Economic risk is the risk that the future cash flows of a firm will be affected by the movements in exchange rates.

Economic risk arises from the exchange rate movements and thus is difficult to quantify. Consider a Canadian sales firm in Brazil. If the BRL (Brazilian real) weakens in value relative to the Canadian dollar, the Brazilian customers will see the firm's products as expensive. This will decline the demand for the product or prompts the company's management to reduce the CAD price of its products.

Moreover, economic risk alters with the competitive nature of a company. Consider a company in a given country, Kenya, which has no investment operations overseas. Exchange rate movements might be favorable for a foreign investor who sees Kenya as a desirable place for business. The foreigner's increased business activities could negatively impact the domestic firm's competitive position.

Addressing Economic Risk

Compared with translation and transaction risks, economic risk is not easily quantified. However, possible exchange rate movements should be taken into consideration before essential strategic business decisions are made. For instance, a production firm might decide to move production overseas due to favorable exchange rate movements.

Multicurrency Hedging Using Options

Multinational companies are exposed to many different currencies. Just like any other portfolio, multiple exposures to multiple currencies reduce the FX risk due to diversification. That is, volatility from multiple currency exposures is less than exposure to a single currency.

Companies often prefer options to forward contracts since the options provide downside protection against unfavorable exchange rate movement while allowing a firm to benefit from desirable movements. Therefore, hedging using options involves buying options on individual currencies to cover each adverse exchange rate movement.

Alternatively, a firm might buy an option on a portfolio of currencies to which it is exposed in the over-the-counter market. Such options are basket and Asian options.

Determination of Exchange Rates

Like any other financial asset, currency exchange rates cannot be determined with ultimate precision because they are influenced by supply and demand, which are also affected by other factors. Some of the factors affecting the exchange rates include:

I. Balance of payments and trade flows
II. Monetary policies
III. Inflation

Balance of Payments and Trade Flows

Recall that the balance of payments between the two countries is the difference between the value of exports and imports. We shall demonstrate the effect of balance of payments using an example:

Example

Suppose that the exports from country X to country Y increases. If the exporters exchange the foreign currency (which is their revenues) to domestic currency, the demand for country X currency will increase, strengthening its currency relative to country Y's currency. This causes exports to be more expensive in country Y.

On the other hand, if the imports from country X to Y increase, the currency of country X will weaken relative or that of country Y since the importers will be forced to buy country Y's currency to pay for the goods they are importing. Consequently, the imported goods to country Y become more expensive, lowering the demand for imported goods.

Monetary Policies

The central bank's monetary policy also influences the value of a country's currency. With all other factors held equal, if Country X, say, raises its money supply by 15% while Country Y keeps its money supply at constant, the value of Country X's currency would begin to fall by 15% compared to Country Y's currency. This is due to the fact that 15% more of Country X's currency is being used to buy the same quantity of goods.

Inflation

Inflation gives rise to purchasing power parity; a relationship that allows for theoretical arbitrage opportunities, i.e., a trader can buy goods cheaply from a country with a lower inflation rate and sell them at a higher price in a different country with a higher inflation rate since inflation has negative effects on the exchange rates.

The Purchasing Power Parity

This condition reflects the link between the exchange rates and the difference in countries' inflation rates. The laws of one price state the price of a foreign good x denoted as P_f^x must be the equal price of the similar good in a domestic country, P_d^x, using the spot rate $S_{\frac{f}{d}}$ (We have used the $(\frac{f}{d})$ notation for simplicity). Put mathematically,

$$P_f^x = S_{\frac{f}{d}} \times P_d^x$$

For instance, a product in Canada costs CAD 100. The nominal exchange rate for USD/CAD is 0.76. So, the same product will cost 0.76×100 = USD 76 in the US.

The purchasing power parity amplifies the law of one price to include a broader range of goods and services and not just good x. The law of one price equation transforms into:

$$P_f = S_{\frac{f}{d}} \times P_d$$

where:

P_f-the price level of the foreign country.

P_d-the price level of the domestic country

$S_{\frac{f}{d}}$-nominal exchange rate

Making the $S_{\frac{f}{d}}$ the subject of the formula, we get:

$$S_{\frac{f}{d}} = \frac{P_f}{P_d}$$

Example: Calculating the Spot Exchange Rate Using the Purchasing Power Parity

The inflation rate in the US is 3% per year and 1% per year in Canada. You are also given that the USD/CAD exchange rate is 1.0500. A basket of goods in the US costs USD 100. Assuming that the purchasing power parity holds, what is the new USD/CAD after one year.

Solution

The inflation rate in the US is 3% per year, implying that the price of a basket of goods increases by 3% each. This is analogous to Canada. So, after one year, the basket of goods in the US is:

$$P_d = 1.03 \times 100 = \text{USD } 103$$

The same basket would cost the following in Canada:

$$P_f = 1.05 \times 1.01 \times 100 = \text{CAD } 106.05.$$

According to purchasing power parity,

$$S_{\frac{f}{d}} = \frac{P_f}{P_d} = \frac{106.05}{103} = 1.02961$$

Therefore, the equilibrium in the exchange rates is determined by the ratio of the national price level of the two countries. However, if the transaction cost is largely coupled with the non-tradable nature of some goods, this condition might not hold.

Moreover, if the transaction costs and other trading difficulties are constant, the deviation of the exchange rate is entirely determined by the difference between the inflation rates of the foreign and domestic countries. Mathematically,

$$\%\Delta S_{\frac{f}{d}} \approx \pi_f - \pi_d$$

Where:

$\%\Delta S_{\frac{f}{d}}$=change in the spot exchange rate

π_f=foreign inflation rate

π_d=domestic inflation rate

That is:

Percentage Strengthening of Domestic Spot Rate = Foreign Inflation Rate − Domestic Inflation Rate.

If we go back to our example, the domestic currency weakens by:

$$\%\Delta S_{\frac{f}{d}} \approx \pi_f - \pi_d = 1\% - 3\% = -2\%$$

This implies the domestic spot rate weakened by 2%

Calculating Percentage Appreciation / Depreciation

To calculate the percentage change, one needs to have a clear understanding of the base currency and the quote currency. Let us take an example of the Chinese Yuan (CNY) and South African Rand (ZAR). Suppose that the exchange rate of ZAR/CNY increased from 1.6459 to 1.8356. Therefore, the percentage of appreciation/depreciation of the Chinese Yuan will be:

$$\frac{1.6459}{1.8356} - 1 = -10.33\%$$

The Chinese Yuan depreciated by 10.33% because it used to take 1.6459 CNY to buy one ZAR, but now it has increased to 1.8356 CNY for one ZAR.

Interpretation of the Percentage Appreciation/Depreciation

The depreciation of the Chinese Yuan against the South African Rand can also be expressed as an appreciation of the South African Rand against the Chinese Yuan. The appreciation percentage, in this case, will not be equal to the previous depreciation percentage of -10.33%.

To calculate the percentage appreciation of the South African Rand, we have to invert the exchange rate. To invert a currency exchange rate, we have to divide 1 by the exchange rate. For example, if

$$\text{ZAR/CNY} = 1.6459$$

Then,

$$\text{CNY/ZAR} = \frac{1}{1.6459} = 0.6076$$

Therefore, the appreciation percentage of South African Rand if the exchange rate ZAR/CNY increased from 1.6459 to 1.8356. We have to invert this exchange to CNY/ZAR so that the Chinese Yuan is now the base currency and the South African Rand is the quote currency. That is:

$$\frac{\left(\frac{1}{1.6459}\right)}{\left(\frac{1}{1.8356}\right)} - 1 = \frac{1.8356}{1.6459} - 1 = 11.53\%$$

This represents an 11.53 percent appreciation in the South African Rand against the Chinese Yuan because the CNY/ZAR (as a result of inversion) exchange rate is expressed with the Chinese Yuan as the base currency and the South African Rand as the quote currency. In other words, you now need fewer South African Rands to buy one Chinese Yuan.

Thus, we can see that the appreciation percentage of the South African Rand is different from the Chinese Yuan's depreciation.

Example: Appreciation/Depreciation

A forex trader noticed that the USD/EUR spot rate was 1.2960 and expected to be 1.2863 after one year. Similarly, the CHF/USD spot rate is 0.9587 and is expected to drop to 0.8885. Calculate the euro (EUR) expected appreciation/depreciation against the US dollar over the next year.

Solution

We know that we are dealing with USD/EUR quotes. So, we calculate as:

$$\frac{1.2960}{1.2863} - 1 = 0.007541 = 0.7541\%$$

This was expected because clearly, there was a decrease in USD/EUR, indicating that EUR is appreciating.

Real and Nominal Interest Rates

Nominal Interest Rates

Nominal interest rates are those rates that are listed in the market and show the return that will be earned on a currency. For instance, 5% per year for a given currency of a country implies that 100 units of a currency are anticipated to grow to 105 in one year.

Real Interest Rates

Real interest rate is those rates that are adjusted to accommodate the effects of inflation. The real interest is given by:

$$r_{\text{real}} = \frac{1 + r_{\text{nominal}}}{1 + r_{\text{inflation}}} - 1$$

where

r_{real}=real interest rate

r_{nominal}=nominal interest rate

$r_{\text{inflation}}$=rate of inflation

The above equation is usually approximated as:

$$r_{\text{real}} \approx r_{\text{nominal}} - r_{\text{inflation}}$$

For instance, if the nominal interest rate is 5% and the inflation rate is assumed to be 2%, then the real interest rate is approximated as:

$$r_{\text{real}} \approx 5\% - 2\% = 3\%$$

Or

$$r_{\text{real}} = \frac{1.05}{1.02} - 1 = 0.02941 \approx 2.941\%$$

Note that this is pretty close to the approximation.

Note also that the real and nominal interest rates can both be negative.

The Covered Interest Parity

This is a no-arbitrage condition which states that an investment in a foreign market that is entirely hedged against exchange rate risk should give the same return as a similar investment in a domestic market.

Derivation of Covered Interest Rate Parity

Consider an investor who wishes to start with 1 unit of domestic currency so that he ends up with an amount in foreign currency terms. To achieve this, there are two ways to accomplish this:

I. The trader can invest in the funds in the risk-free foreign rate of interest (i_f) so that the funds grows to $(1+i_f)^T$ at time T. At time *T*, the trader can enter into a forward contract to exchange $(1+i_f)^T$ for foreign currency at a foreign forward rate of exchange $F_{\frac{f}{d}}$ to get:

$$(1+i_f)^T F_{\frac{f}{d}}$$

II. The trader immediately exchange the proceeds to USD and invest in risk-free domestic rate of interest to get $S_{\frac{f}{d}}(1+i_d)^T$

Since we are assuming there is a no-arbitrage condition, then these two investments should give the same result. That is:

$$(1+i_f)^T F_{\frac{f}{d}} = S_{\frac{f}{d}}(1+i_d)^T$$

Rearranging we get:

$$F_{\frac{f}{d}} = S_{\frac{f}{d}}\left(\frac{(1+i_d)^T}{(1+i_f)^T}\right)$$

Where

i_d =The interest rate in the domestic currency or the quoted currency

i_f=interest rate in the foreign currency or the base currency

$S_{\frac{f}{d}}$=current spot exchange rate

$F_{\frac{f}{d}}$=forward foreign exchange rate

In other words, the forward exchange rate should give the same rate involving the spot exchange rate, domestic and foreign risk-free yields either in domestic market instruments or currency-hedged foreign market instruments within the same investment horizon.

For this condition to hold, it is assumed that:

I. There is no transaction cost and
II. The domestic and foreign market instruments should be identical in liquidity, time to maturity, and the default risk.

Note that we have to use the notation f/d, where f stands for the foreign currency and d for the domestic currency. So, f/d can be anything else such as CAD/USD, in which USD is the domestic currency.

Generally given the exchange rate XXXYYY or XXX/YYY then:

$$F_{\frac{XXX}{YYY}} = S_{\frac{XXX}{YYY}}\left(\frac{(1+i_{YYY})^T}{(1+i_{XXX})^T}\right)$$

Where the variables are defined as above.

Example: Covered Interest Rate Parity

The following are interest rates as listed in an interbank market:

Currency	Libor (annualized)	Currency Combinations	Spot Rate
USD	0.30%	USD/EUR	1.6975
EUR	5.00%	JPY/EUR	0.0085
JPY	0.30%	JPY/USD	82.25

A Japanese company investment manager wants to estimate all-in investment returns on a hedged EUR currency exposure if the covered interest parity holds.

Solution

In this question, we do not require any calculations because it is just a matter of intuition. If the covered interest rate parity holds, then the all-in investment for the Japanese company is the fully-hedged EUR Libor, which is equal to a one-year JPY Libor of 0.30%. So, the investment return is simply 0.30% since, according to covered interest rate parity, an investment in a foreign market that is entirely hedged against exchange rate risk should give the same return as a similar investment in a domestic market.

Arbitrage Opportunities in Case the Covered Interest Rate Fails

Assume that a US investor starts with 1 unit of USD to end up with Canadian dollars (CAD). So, the spot rate is quoted as CAD/USD. If the covered rate parity holds then:

$$F_{\frac{CAD}{USD}} = S_{\frac{CAD}{USD}}\left(\frac{(1+i_{USD})^T}{(1+i_{CAD})^T}\right)$$

If

$$F_{\frac{CAD}{USD}} < S_{\frac{CAD}{USD}}\left(\frac{(1+i_{USD})^T}{(1+i_{CAD})^T}\right)$$

Then:

$$
\begin{aligned}
&F_{\frac{CAD}{USD}}(1+i_{CAD})^T < S_{\frac{CAD}{USD}}(1+i_{USD})^T \\
&\Rightarrow \frac{S_{\frac{CAD}{USD}}(1+i_{USD})^T}{F_{\frac{CAD}{USD}}} > (1+i_{CAD})^T
\end{aligned}
$$

This implies that the investor has more CAD than required to pay the borrowed amount; hence he can make a riskless profit.

On the other hand, if

$$F_{\frac{CAD}{USD}} > S_{\frac{CAD}{USD}}\left(\frac{(1+i_{USD})^T}{(1+i_{CAD})^T}\right)$$

Then:

$$F_{\frac{CAD}{USD}}(1+i_{CAD})^T > S_{\frac{CAD}{USD}}(1+i_{USD})^T$$

Therefore, the investor would have more of USD than the borrowed amount in USD and make a riskless profit.

Example: Calculating the Forward Rate Using Covered Interest Parity

Suppose that the risk-free rate of interest in USD and EUR are 3% and 6% per year, respectively. Given that the USD/EUR spot rate is 0.90. What is the 6-month USD/EUR forward rate?

Solution

Using the formula:

$$
\begin{aligned}
F_{\frac{USD}{EUR}} &= S_{\frac{USD}{EUR}}\left(\frac{(1+i_{EUR})^T}{(1+i_{USD})^T}\right) \\
&= 0.90 \times \frac{1.06^{0.5}}{1.03^{0.5}} = 0.9130
\end{aligned}
$$

Note that the spot rate (0.90) is less than the forward rate (0.9130), implying that USD is (theoretically) stronger than the EUR.

Interpretation of Points

When $T < 1$

$$F_{\frac{f}{d}} = S_{\frac{f}{d}} \left(\frac{(1+i_d)^T}{(1+i_f)^T}\right)$$

For simplicity let $F_{\frac{f}{d}} = F$ and $S_{\frac{f}{d}} = S$

The above equation can be approximated as:

$$\frac{F}{S} = \frac{1+i_d T}{1+i_f T}$$

If we subtract 1 from both sides,

$$\frac{F}{S} - 1 = \frac{1+i_d T}{1+i_f T} - 1$$

We get:

$$\frac{F-S}{S} = \frac{1+i_d T - 1 - i_f T}{1+i_f T} = \frac{i_d T - i_f T}{1+i_f T}$$

So,

$$\frac{F-S}{S} = \frac{i_d T - i_f T}{1+i_f T}$$

This can be approximated as:

$$\frac{F-S}{S} \approx (i_d T - i_f T)T$$

In the last expression, F-S expressed as a percentage of the spot rate is equivalent to the number of points divided by 10,000 and it is an approximate value of the interest rate differential at time T.

Example: Calculating the Forward Rate as Points

Suppose that the risk-free rate of interest in USD and EUR are 3% and 6% per year, respectively. Given that the USD/EUR spot rate is 0.90. What is the 6-month USD/EUR forward rate expressed as points?

Solution

In the previous calculation, we had calculated the forward rate as 0.9130. So, the 6-month forward points are:

$$(0.9130 - 0.9000) \times 10,000 = 130$$

If we express in terms of percentage of spot rate, we have:

$$\frac{0.9130 - 0.9000}{0.9000} = 0.014444 = 1.444\%$$

The Uncovered Interest Rate Parity

While covered interest parity is concerned with forward rates and depends on arbitrage arguments, Uncovered interest parity is concerned with exchange rates themselves.

The uncovered interest parity condition postulates that the expected yield from a risky foreign investment must be equal to that of an equivalent domestic currency investment.

It states that the change in spot rate over the investment period should be averagely equal to the difference between the interest rates in two different countries, or simply, the expected appreciation or depreciation should approximately offset the difference in the interest rates.

While using the (f/d) notation (domestic (d) currency as the base currency), assume that an investor has a choice of venturing in one-year domestic market investment and a risky (unhedged) foreign market investment. The uncovered parity condition compels the investor to weigh between the **certain** return from domestic investment and expected return from the risky foreign investment (in terms of foreign currency).

The foreign investment return in domestic currency will be given by:

$$(1+i_f)(1-\%\Delta S_{\frac{f}{d}})-1$$

This can also be represented as:

$$\approx i_f-\%\Delta S_{\frac{f}{d}}$$

Also, the uncovered interest rate parity implies that the anticipated change in the spot rate over the investment period should show the difference between the foreign and domestic interest rates. This is mathematically represented as:

$$\%\Delta S^e_{\frac{f}{d}}=i_f-i_d$$

Where ΔS^e is the future change in the spot rate.

Example: Uncovered Interest Parity

Currencies A (domestic) and B (foreign) have risk-free rates of interest of 2% and 5% respectively. Assuming that the uncovered interest rate parity holds, what percentage would B weaken (strengthen) relative to A?

Solution

According to uncovered interest rate parity

$$\%\Delta S^e_{\frac{f}{d}}=i_f-i_d=5\%-2\%=3\%$$

So, we would expect currency B to weaken by 3% relative to the value of currency A.

Therefore, the assumption brought forward by the uncovered interest rate is that when a country has higher interest rates, its currency will depreciate, which offsets the high yields, bringing the return of the two investments to the same level.

Question

The following are interest rates as listed in an interbank market:

Currency	Libor (annualized)	Currency Combinations	Spot Rate
USD	0.70%	EUR/USD	1.8975
EUR	7.00%	EUR/USD	0.0075
JPY	0.50%	USD/JPY	82.25

Assuming that the Uncovered Interest Parity holds, by how much is the JPY currency expected to change relative to USD over one year?

A. -0.2%

B. 0.2%

C. 0%

D. 0.4%

Solution

The correct answer is **A.**

According to uncovered interest rate parity, the expected change in a spot exchange rate is equivalent to the difference between the interest rates corresponding to each currency (Libors). That is,

$$\%\Delta S^{e}_{\frac{f}{d}} = i_f - i_d = (0.5 - 0.7)\% = -0.2\%$$

Therefore, JPY currency has decreased by 0.2% relative to the USD currency.

Reading 34: Pricing Financial Forwards and Futures

After completing this reading, you should be able to:

- Define and describe financial assets.
- Define short-selling and calculate the net profit of a short sale of a dividend-paying stock.
- Describe the differences between forward and futures contracts and explain the relationship between forward and spot prices.
- Calculate the forward price given the underlying asset's spot price and describe an arbitrage argument between spot and forward prices.
- Distinguish between the forward price and the value of a forward
- Calculate the value of a forward contract on a financial asset that does or does not provide income or yield.
- Explain the relationship between forward and futures prices.
- Calculate the value of a stock index futures contract and explain the concept of index arbitrage.

Financial Assets

A *financial asset* is an asset that derives its value from a particular claim.

Assets held for the purposes of investing are referred to as investment assets. Examples of such assets include stocks and bonds issued by various financial institutions. On the other hand, assets primarily held for the purpose of consumption and not for investment or resale are referred to as non-investment or consumption assets. Examples of such assets include oil, coffee, tea, corn, e.t.c.

In this chapter, we consider three types of assets:

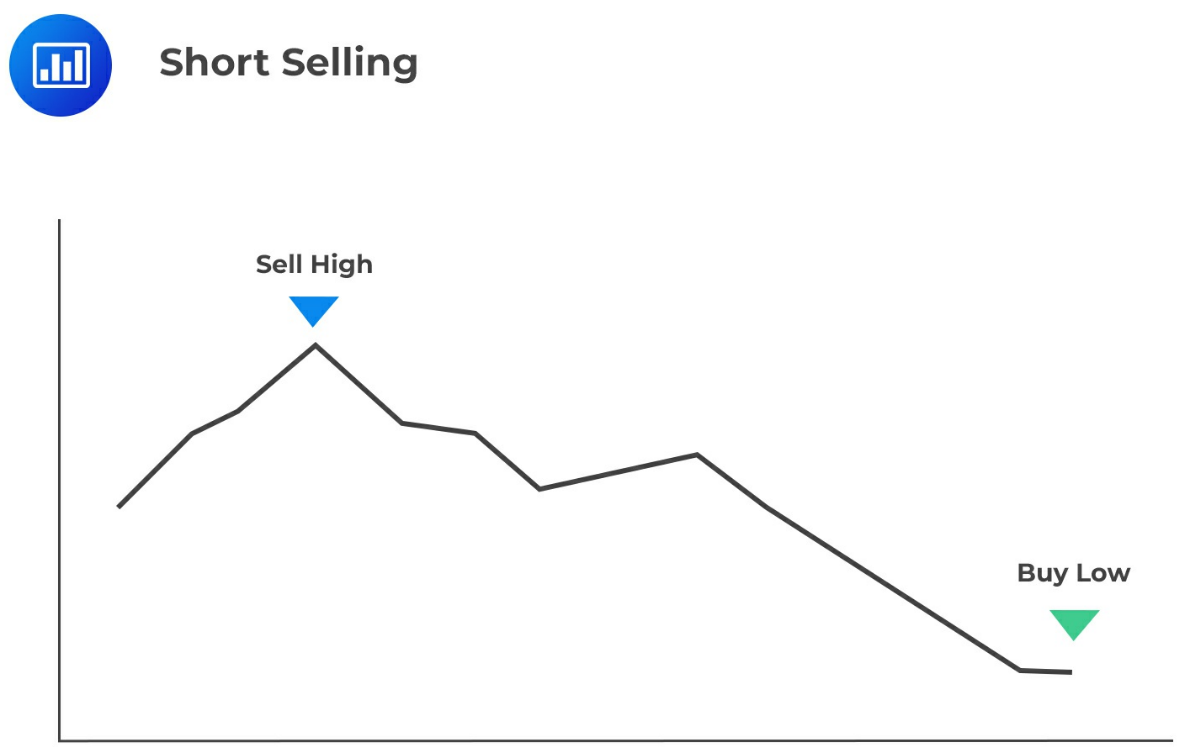
i. Assets providing no income,

ii. Assets providing a known income that is a fixed amount, and

iii. Assets providing a known income that is a percentage of their value.

Short-Selling

Short selling involves the sale of a security not owned by an investor. The investor sells the security but purposes to buy it later. The investor will then realize profits if the price of the security goes down and losses if the price of the security goes up.



Short sales are transacted through a broker. The short seller must deposit some collateral to guarantee the eventual return of the security to the owner. In addition, the short seller is required to pay all accrued dividends, if any, to the lender. Thus, the net profit is equal to:

$$\text{Net profit} = \text{Sale price} - \text{Borrowing price} - \text{Dividend paid}$$

For example, if a trader shorts a stock today at $100, a dividend of $4 is paid next month, and the trader closes the short position the following month at \$90, the net profit will be \$100 – \$90 – \$4 = \$6. His return would normally have been $\frac{\$10}{\$100} = 10\%$, but the dividend that he had to pay to the long position decreased his return to only $\frac{(\$10-\$4)}{100} = 6\%$.

Ignoring brokerage costs and assuming no borrowing fee, the cash flows obtained from a long position should mirror the cash flows from a short position, i.e., a profit of \$200 to a short position trader should mean a loss of \$200 to a long position trader.

This reading examines the relationship between spot and forward prices of assets that provide no income, provide a known income amount, and provide an income that is a percentage of the asset.

Assets that Provide No Income

These assets include treasury bills, stocks that do not provide dividends, and zero-coupon bonds.

Illustration

Consider a financial asset that costs \$50 as of now. The borrowing/lending rate of a financial institute is 5% per year. How can a trader maximize profits if the one-year forward price is:

1. \$70
2. \$40

Example 1: Forward Price of \$70

To make a profit, a trader will have to buy the asset today at USD 50 and then sell it a year later at USD 70.

For that one year, the cost of funding the asset will equal to $(0.05 \times \text{USD } 50) = \text{USD } 2.50$

The profit made will therefore be equal to $\text{USD } 70 - \text{USD } 50 - \text{USD } 2.50 = \text{USD } 17.50$

A forward price greater than USD 52.25 (spot price of the asset today plus the cost of funding the asset in one year) guarantees the trader a profit with zero risks.

Example 2: Forward Price of $40

To make a profit, a trader will have to sell the asset (at USD 50) and enter into a contract to buy it back a year later (at USD 40).

The trader will thus make an initial profit of $10. Buying back the asset generates USD 50 that can be invested at the 5% risk-free interest rate to earn an additional profit of ($0.05\times$ USD 50 = USD 2.50). This gives a total profit of USD$(10.00 + 2.50)$ = USD 12.50.

For profits to be realized, the forward price must be lower than USD 52.25

Assuming that there are no arbitrage opportunities, the forward price should, therefore, be equal to USD 52.25

Suppose that the borrowing and lending rates of a bank are 5.1% and 5.0%, respectively. In a no-arbitrage situation, the first case scenario gives a forward price less than 52.55 (USD $50 \times (1 + 0.051)^1$).

The second case scenario remains unchanged since the lending rate is similar to the interest rate used above. This implies that the forward price should lie between USD 52.55 and USD 52.25.

The Known Income Case

Financial assets that pay a known income may either be bonds that have a known coupon rate or stocks whose dividends are known in advance.

Refer to example 1 above and assume that the assets will provide cash flows of $5 every six months. Assume also that the annually compounded 6 month and one-year interest rates are 4% and 5%, respectively

First Case Scenario: Forward Price of USD 70 and Spot Price of USD 50

The trader buys the asset and purposes to sell it later. After six months, the trader gets $5. The present value of $5 is $4.903. The trader can borrow the present value amount of the loan to be repaid in 6 months-time using the cash flow received at the 6th month.

The remaining $\text{USD}(50 - 4.903) = \text{USD } 45.097$ can then be borrowed at the rate of 5% per year. The total amount required to pay off the loan will then be $\text{USD } 45.097 \times 1.05 = \text{USD } 47.35185$ giving the trader a profit of $\text{USD } 70 - \text{USD } 47.35185 = \text{USD } 22.64815$.

For profits to be realized, the forward price should, therefore, be greater than USD 47.35185.

Second Case Scenario: Forward Price of USD 40

The trader sells the asset and enters into a forward contract to buy it back at USD 40. This can be done by investing \$4.903 from the proceeds of the sale at 4% per year to generate \$5 that will be paid as dividends to the new owner, and the remaining amount $\text{USD } 50 - \text{USD } 4.903 = \text{USD } 45.097$ at 5% per year for one year. This gives a value of $\text{USD } 45.097 \times 1.05 = \text{USD } 47.35185$. The profit is therefore $\text{USD } 47.35185 - \text{USD } 40 = \text{USD } 7.35185$.

For profits to be realized, the forward price should, therefore, be less than USD 47.35185

For profits to be generated, the forward price should be greater than USD 47.35185 for the first case scenario and less than USD 47.35185 for the second case scenario. A no-arbitrage opportunity, therefore, arises when the forward price is USD 47.35185.

The Known Yield Case

Instead of providing the known income as cash, some financial assets provide the income as a percentage of the price of the asset.

Generalization

Defining terms as:

S = Spot Price

F = Forward Price

R = Risk-free interest rate per year compounded annually

Q = Yield

T = Time to maturity

The general formulae for:

1. **The No Income Case**

$$F = S(1 + R)^T$$

If F is greater, to realize profits, traders should buy the assets and sell them in the forward markets. The reverse is true.

2. **The Known Income Case**

$$F = (S - I)(1 + R)^T$$

The spot price, S can be found by making S the subject of the formula;

$$S = I + \frac{F}{(1 + R)^T}$$

If $F > (S - I)(1 + R)^T$, to realize profits, traders should buy the assets and sell them in the forward markets.

If $F < (S - I)(1 + R)^T$, investors can make arbitrage profits by selling the assets and buying the forward contract.

3. **The Known Yield Case**

$$S = \frac{\{F(1 + Q)^T\}}{(1 + R)^T}, F = S\{\frac{(1 + R)}{(1 + Q)}\}^T$$

Valuing Forward Contracts

The price of a forward contract does not necessarily reflect the value of the contract. At the onset of a forward contract, the forward price is calculated as shown above. The value is, however, zero or close to zero. The value of the contract either becomes positive or negative as time passes by and is dependent on asset price changes. The agreed-upon forward price at which the asset will trade will, however, remain constant.

Define K as the forward price of an asset at the time the contract was originally entered, F as the current forward price for the contract, S as the spot price and T as the current time to maturity of the contract.

$$\text{Value of a long forward contract} = \frac{(F - K)}{(1+R)^T} \text{ where } F = S(1+R)^T$$

If an income with a present value of I is to be paid during the remaining life of the contract,

$$\text{Value of a long forward contract} = S - I - \left\{\frac{K}{(1+R)^T}\right\}$$

and

$$\text{Value of a short forward contract} = \frac{(K - F)}{(1+R)^T} \text{ where } F = S(1+R)^T$$

If a yield at the rate of Q is provided,

$$\text{Value of a short forward contract} = \left\{\frac{S}{(1+Q)^T}\right\} - \left\{\frac{K}{(1+R)^T}\right\}$$

Example: Valuing Forward Contracts

Consider a forward contract where the stock price is USD 60, no income is provided, and that the annual rate of interest is 4%. The current forward price is $ 63. If, at some time back, a long forward contract was entered to purchase the stock at $ 65, what is the value of the long forward contract?

Solution

$$\text{Value of a long forward contract} = S - I - \{\frac{K}{(1+R)^T}\}$$
$$= 60 - 0 - \frac{65}{1.04} = -2.5$$

Forwards vs. Futures

Unlike future contracts, which are settled on a daily basis, forward contracts are settled at maturity.

It can be shown that if interest rates are constant (or if they change in a perfectly predictable way), the theoretical no-arbitrage forward and futures prices are the same.
However, in practice, interest rates do vary unpredictably, and futures prices are therefore different from forward prices. This difference is due to the correlations between the returns from the underlying assets and interest rates.
As an example, suppose that the price of an asset is positively correlated with interest rates. If the asset price increases, there will be an immediate gain from a long futures contract. This gain can then be invested at a relatively high-interest rate because (on average) interest rates increase when the asset price increases. If the asset price decreases, there will be an immediate loss on the long futures contract. However, the funds can be financed at a low-interest rate because interest rates (on average) decline when the asset price declines. This makes the long futures contract slightly more attractive than a long forward contract, and the futures price would therefore be slightly higher than the forward price. When the correlation between the return from the underlying asset and interest rate is negative, however, this argument is reversed, and the theoretical futures price is slightly lower than the theoretical forward price.

Another disparity between futures and forwards contracts comes about due to differences in delivery dates. Futures contracts have a range of delivery dates. A trader with a short position will deliver the asset as soon as possible to avoid financing costs if the interest rates charged are higher than the returns from the underlying asset. If the interests are lower, the trader will hold on to the asset as much as possible in order to maximize the income earned on the asset. Forward contracts lack a range of delivery dates.

In as much as forward prices are approximately equal to futures prices, the profits and losses realized from the contracts are not the same. A trader trading with a futures contract will have his/her profits reflected immediately since they are settled on a daily basis. On the other hand, the profits/losses of a forwards contract will be shown in present value terms since forward contracts are settled at maturity.

Index Arbitrage

Index arbitrage is a trading strategy that involves buying the portfolios of a stock underlying an asset in cases where the futures price is greater than the theoretical price and selling them in the futures market. In cases where the theoretical price is greater than the forward price, an investor sells the stock and enters into a long futures position.

Program trading enables a computer to send out all the required trades to an exchange as the futures contract is being traded.

The similarity between the price of a portfolio and an index presents a no-arbitrage opportunity. However, there are some cases where the index does not correspond to the value of a portfolio.

Question 1

Consider a forward contract on a stock index such as the S&P 500. Everything else being constant, which of the following statements is least accurate?

A. The forward price will fall if interest rates rise.

B. The forward price is directly linked to the level of the stock market index.

C. If the time to maturity is increased, the forward price will rise.

D. The forward price will fall if dividend payments on the underlying stocks increase.

The correct answer is **A.**

Increasing the level of interest rates r makes the forward contract more appealing to investors. Thus, the forward price will increase.

Question 2

The one-year U.S.dollar interest rate is 1.5%, and the one-year GBP interest rate is 2.0%. The current $\frac{\text{USD}}{\text{GBP}}$ spot exchange rate is 0.85. Assuming annual compounding, what is the one-year $\frac{\text{USD}}{\text{GBP}}$ forward rate.

A. 0.8825

B. 0.7575

C. 0.8520

D. 0.8542

The correct answer is **D.**

If we assume annual compounding, then:

$$F_0 = S_0 \frac{(1+r)}{(1+R)}$$

Where:

F_0 = forward exchange rate

S_0 = spot exchange rate

r = quoted currency interest rate

R = base currency interest rate

$$F_0 = 0.85 \frac{1.02}{1.015} = 0.8542$$

Exam tip:

All prices (S_0, F_0) are measured in the domestic currency. Unless directed otherwise, you're supposed to apply the indirect quotation methodology in the exam. Under the method, an $\frac{A}{B}$ quote has A as the base currency, and B as the quoted currency. The base currency (in this case, the U.S. dollar) is always equal to one unit (in this case, US$1), and the quoted currency (in this case, the GBP) is what that one base unit is equivalent to in the other currency. That is, 1USD=0.85GBP.

Question 3

The price of a six-month futures contract on an equity index is currently at USD 1,215. The underlying index stocks are valued at USD 1,200. The stocks also pay dividends at a rate of 3%. Given that the risk-free rate is 5%, determine the potential arbitrage profit per contract.

A. $3

B. $12

C. $15

D. $0

The correct answer is **A**.

The fair value of the futures contract, F, is given by:

$$F = S \times \left(\frac{(1+r)}{(1+r^*)}\right)^T$$

Where:

S = current value of the underlying

r^*=rate of dividends

r=risk-free rate

T =time to maturity=$\frac{6}{12}$

$$\begin{aligned} F &= S \times \left(\frac{(1.05)}{(1.03)}\right)^{0.5} \\ &= 1,211.59 \end{aligned}$$

Thus, the actual futures price is too high by $3 (= 1,215 – $1,212).

Reading 35: Commodity Forwards and Futures

After completing this reading, you should be able to:

- Explain the key differences between commodities and financial assets.
- Define and apply commodity concepts such as storage costs, carry markets, lease rate, and convenience yield.
- Identify factors that impact prices on agricultural commodities, metals, energy, and weather derivatives.
- Explain the basic equilibrium formula for pricing commodity forwards.
- Describe an arbitrage transaction in commodity forwards and compute the potential arbitrage profit.
- Define the lease rate and explain how it determines the no-arbitrage values for commodity forwards and futures.
- Describe the cost of carry model and illustrate the impact of storage costs and convenience yields on commodity forward prices and no-arbitrage bounds.
- Compute the forward price of a commodity with storage costs.
- Explain how to create a synthetic commodity position and use it to explain the relationship between the forward price and the expected future spot price
- Explain the relationship between current futures prices and expected future spot prices, including the impact of systematic and nonsystematic risk.
- Define and interpret normal backwardation and contango.

Throughout this chapter, we will assume the daily settlement of futures. This implies that futures and forward contracts will be treated as one and the same thing.

With the exception of a few commodities like gold, most commodities are held as consumption assets and not just as investment assets. Commodity assets are held for the purposes of being used in some way, after which they cease to be available for sale.

Differences Between Commodities and Financial Assets

Commodities	Financial Assets
Storage costs are present.	Negligible storage costs.
Commodities are costly to transport. Prices may reflect the cost of transport.	No transport costs as they are transported electronically.
A higher lease rate when commodities held for investment purposes are borrowed.	Lower fees charged when financial assets are borrowed for shorting.
Returns do not reflect the risk.	Returns reflect the risks.

Types of Commodities

Agricultural Commodities

Agricultural commodities are difficult to store. There is an observable interdependence among agricultural commodities, i.e., livestock feed on plants. As such, they have seasonal prices – low prices at harvest time and high prices as storage costs of the products increases. That is, the prices of agricultural products are seasonal.

The prices of agricultural commodities are influenced by:

- **Political considerations**
- **Market factors:** For example, the presumption of a good harvest may lower the prices
- **Weather conditions:** Extreme weather, e.g., strong winds, can cause destruction to crops, resulting in a decrease in supply, which may lead to an increase in the prices of agricultural commodities.

Metals

Commodities under this category include copper, aluminum, zinc, lead, nickel, platinum, gold, silver, and palladium.

As compared to agricultural commodities, their prices are not seasonal, and metal prices are not affected by the weather. Also, the cost of storing metals is relatively cheaper as compared to that of storing agricultural commodities. Most metals are held purely for investment purposes.

The prices of metals depend on:

- The rate at which new sources of extracting metals are discovered.
- Exchange rates**:** Applicable in metals that are discovered in one country and sold in another country.
- The number of uses of a specific metal.
- Changes in the methods of extraction of the metals
- Government actions.
- Environmental regulations.
- Recycling processes can, at times, affect metal prices.

Energy

Futures contracts are traded on crude oil (which is considered the largest commodity market in the world) and crude oil extracts, natural gas, and electricity.

- **Crude oil**: Available in many grades and has a high global demand. Transportation of crude oil is expensive, making the prices vary regionally.
- **Natural gas**: Used for either heating or generating electricity. Since it is stored below or above the ground, the storage costs are high. The prices of natural gas are seasonal depending on demand. Demand is high during cold seasons and low during hot seasons.

Electricity

Future contracts on electricity are traded in both the OTC and exchange-traded markets. One party of the futures contract receives a specific number of megawatts for a specified period in a specified location at a specified time. Even though futures contracts on electricity exist, they are not traded as actively as the futures contract on crude oil and natural gas.

Electricity differs from other commodities since it is almost not possible to store it. Due to its non-storability, electricity is prone to huge fluctuations in price. The price of electricity mainly depends on:

The price of electricity mainly depends on:

- The price charged at each of the generating stations; and
- High demand. For example, as electricity will be needed for air conditioning and heating in hot or cold seasons, the prices go up.

Weather

Future contracts on weather are traded in both the OTC and the exchange-traded markets.

We have two important weather variables which can be defined as:

HDD (Heating Degree Days) = $\max(0, 65 - A)$

CDD (Cooling Degree Days) = $\max(0, A - 65)$

Where A = 1/2(Highest + Lowest temperature in a day at a specific weather station

Commodities Held for Investment

Despite some commodities having industrial uses, they may be held strictly for investment. Traders owning metals for investment can substitute physically owning the metals to owning futures and forward contracts on the commodities. Such metals have negligible storage costs. They can also be borrowed at a lease rate.

Ignoring lease rates,

$$F = S(1 + r)^T$$

Where T = Time to maturity, and

r = Risk-free rate.

If $F > S(1 + r)^T$, to maximize profits, a trader can buy the investment commodity at the spot prices S and at the same time enter into a forward contract to sell it at maturity T.

If $F < S(1 + r)^T$, to maximize profits, a trader who owns an investment commodity can sell it at a spot price S and enter into a forward contract to buy it at maturity T.

Lease Rate

A lease rate can be defined as the interest rate charged for borrowing the underlying asset.

In the previous chapter, we looked at the forward price formula for the known-yield case, which is given by:

$$F = S\left(\frac{1 + R}{1 + Q}\right)^T$$

Where F is the forward price, S is the spot price, R is the risk-free rate (with annual compounding), and Q is the annual yield.

Now let L be the lease rate so that we have:

$$F = S\left(\frac{1 + R}{1 + L}\right)^T$$

Solving for L, we get:

$$L = \left(\frac{S}{F}\right)^{\frac{1}{T}}(1 + R) - 1$$

Example: Lease Rate

Assume that the spot price of petroleum is USD 1,200 and the 2-year futures price is 1280, and the annually compounded risk-free rate is 5% per year. What is the implied lease rate?

Solution

$$\begin{aligned} L &= \left(\frac{S}{F}\right)^{\frac{1}{T}}(1+R)-1 \\ &= \left(\frac{1200}{1280}\right)^{\frac{1}{2}}(1.05)-1 \\ &= 0.01665 \end{aligned}$$

Convenience Yield

Convenience yield is the additional value that comes with holding the asset rather than having a long forward or futures contract on the asset. A good example of a consumption asset that has a convenience yield is oil. If you hold oil, you'll have the convenience of selling it at a higher price during a shortage. Convenience yield can be considered as the rate of borrowing or the rate that would have been received with physical possession of the asset. It is, thus, arguably, the rate that should be charged to borrow it.

Convenience yield, Y, should satisfy the equation:

$$F = (S+U)\times\left(\frac{1+R}{1+Y}\right)^{T}$$

So that,

$$Y = \left(\frac{S+U}{F}\right)^{\frac{1}{T}}(1+R)-1$$

Where U is the present value of storage costs, F is the forward price, S is the spot price, R is the risk-free rate (with annual compounding) and Y is the convenience yield.

Example 1: Convenience Yield

Assume that the spot price of petroleum is USD 120 per barrel and the 2-year futures price is 100 per barrel, the present value of storing petroleum for 2 years is USD 5, and the annually compounded risk-free rate is 5% per year. What is the implied convenience yield?

Solution

Convenience yield, Y, should satisfy the equation:

$$F = (S + U) \times \left(\frac{1 + R}{1 + Y}\right)^{T}$$

So that,

$$Y = \left(\frac{S + U}{F}\right)^{\frac{1}{T}}(1 + R) - 1 = \left(\frac{120 + 5}{100}\right)^{\frac{1}{2}}(1.05) - 1 = 0.1739 \text{ or } 17.39\%$$

A readily available asset will have **zero convenience yield** as delivery can be made almost immediately. Thus its future price will be obtained by:

$$F = (S + U) \times (1 + R)^{T}$$

In the presence of delivery delays/shortages, convenience yield will be high and:

$$F < (S + U) \times (1 + R)^{T}$$

Example 2: Convenience Yield

From example 1 above, assume that the forward price is unknown and that the convenience yield is 17.39%.

Then, the forward price can be determined using the formula:

$$F = (S + U) \times \left(\frac{1 + R}{1 + Y}\right)^{T} = (120 + 5) \times \left(\frac{1.05}{1.1739}\right)^{2} = USD100$$

Storage Cost

Storage costs are a negative income. Traders incur storage costs of $U(1 + R)^{T}$ for a present value of

U.

Cost of Carry

Cost of carry encompasses the costs of storage, the costs of financing, and the income to be earned on the asset. Remember that financial assets lack storage costs.

Assuming that financial costs are R and the yield Q, the cost of carry will be $\frac{1+R}{1+Q}-1$ which is approximately equal to $R-Q$ (if R and Q are continuously compounded).

As such, the future value of the asset will be the spot price, S, continuously compounded by the difference between the financial costs R and the yield Q multiplied by the time to maturity of the contract:

$$F = Se^{(R-Q)T}$$

for continuously compounded R and Q

In the presence of storage costs,

$$F = Se^{(C-Y)T}$$

where C is the cost of carry and Y is the convenience yield (both expressed with continuous compou

The Relationship Between the Forward Price and the Expected Future Spot Price

Futures prices reflect the spot prices of a commodity in the future. As the maturity of the contract approaches, the futures price converges to the spot prices. Traders take long futures positions to maximize profits if the spot price at maturity is greater than the current spot price and short futures positions if the spot price at maturity is lesser than the current spot price.

However, to ensure that these profits are realized, traders should close out the futures contracts as the time to maturity nears.

Modern Theory

Systematic risk is defined as a risk that is dependent on market factors and cannot be diversified. Unsystematic risk, on the other hand, is a risk that can be diversified.

The Capital Asset Pricing Model (CAPM) argues that the return on investment should exceed the risk-free interest rate provided the systematic risk on a portfolio is positive (positive correlation between the assets returns and the market returns)

In the presence of a negative correlation between the asset and the market returns, the returns on the asset will be less than the market returns.

If there is no correlation between the asset and the market returns, the portfolio is considered to be a well-diversified portfolio and will be considered to have no risk.

The Relationship Between the Current Futures Prices and the Expected Futures Price

Assume that:

P = Present value of the futures time discounted from T to 0 at the risk-free rate

R = Risk-free interest rate compounded annually

T = Time to maturity

F = Futures price of an asset

S = Spot price of an asset

Then,

$$P = \frac{F}{(1 + R)^T}$$

A trader should invest P at the risk-free interest rate so as to get F upon maturity.

To create a long futures position, the trader can invest P at the risk-free interest rate and at the same time enter into a long futures contract to buy F at maturity. The cash flows from this strategy will be –P at time 0 and +St at time T, assuming that St is the spot price at time T.

Suppose E denotes the expected value and X the expected returns compounded annually, the expected cash flow at maturity T is, therefore, E(St):

$$E(S_T) = P(1 + X)^T$$

and we have seen earlier that,

$$P = \frac{F}{(1 + R)^T}$$

Therefore,

$$E(S_T) = F\frac{(1 + X)^T}{(1 + R)^T}$$

This shows that the systematic risk of an investment depends on the correlation between the asset and the market returns.

If the correlation is positive, $X > R$ and thus $E(S_T) > F$.

If the correlation is negative, $X < R$ and thus $E(S_T) < F$.

If there is no correlation, the futures price will equal the expected future spot price.

Note: These results apply to Fx forwards and futures, financial forwards and futures, and commodity futures.

Suppose that the dividends obtained from an index are reinvested in the index, the index will grow at a rate of Q, giving the value of the investment at maturity T as:

$$F = S\frac{(1 + R)}{(1 + Q)}^T (1 + Q)^T = S(1 + R)^T$$

The investor's return will be greater than the risk-free rate since the index is positively correlated to itself. Thus, the expected value of the index at T >F.

Backwardation vs. Contango

Backwardation refers to a situation where the futures price is **below** the spot price. It occurs when the benefits of holding the asset outweigh the opportunity cost of holding the asset as well as any additional holding costs. A backwardation commodity market occurs when the lease rate is greater than the risk-free rate.

Contango refers to a situation where the futures price is **above** the spot price. It is likely to occur when there are no benefits associated with holding the asset, i.e., zero dividends, zero coupons, or zero convenience yield. A contango commodity market occurs when the lease rate is less than the risk-free rate.

Question

The current spot price of a bag of corn is \$10. There exists an active lending market for corn, where the annual lease rate is equal to 8%, the effective annual risk-free rate is equal to 10%, and the $1-$year forward price for corn is \$10.35 per bag. Does arbitrage exist? What's the risk-free profit up for grabs if indeed an arbitrage opportunity is available?

A. No; risk-free profit = \$0
B. Yes; risk-free profit = \$0.35
C. Yes; risk-free profit = \$0.08
D. Yes; risk-free profit = \$0.15

The correct answer is **D**.

An arbitrage position exists if the forward price **is not equivalent** to the expected spot price.

$$\text{Expected spot price in 1 year} = S_0\left(\frac{1+R}{1+\delta}\right)^T$$

Where:

S_0=commodity spot price

r=riskfree rate

δ=lease rate

T =time between today and the future date at which the transaction will occur, i.e, maturity

$$= 10\left(\frac{1.10}{1.08}\right)^1 = 10.19$$

Since 10.35 is greater than 10.19, arbitrage exists.

To take advantage of this opportunity, an arbitrageur can make the following moves:

At initiation,

- Borrow \$10 at the rate of 10%
- Buy a bag of corn at \$10
- Go short on a corn futures contract
- Lend the bag of corn at 8%

At maturity,

- Take back the bag of corn plus proceeds from the lease amounting to $\$0.8 (= 10 \times 1.08 - 10)$
- Deliver the bag of corn; receive \$10.35
- Repay borrowed funds amounting to $\$11 (= 10 \times 1.10)$
- Net profit $= 10.35 + 0.8 – 11 = \$0.15$

Reading 36: Options Markets

After completing this reading, you should be able to:

- Describe the various types, uses, and typical underlying assets of options.
- Explain the payoff function and calculate the profit and loss from an options position.
- Explain the specification of exchange-traded stock option contracts, including that of non-standard products.
- Explain how dividends and stock splits can impact the terms of a stock option.
- Describe the application of commissions, margin requirements, and exercise procedures to exchange-traded options and explain the trading characteristics of these options.
- Define and describe warrants, convertible bonds, and employee stock options.

Types, Uses, and Typical Underlying Assets of Options

The buyer of an option has the right but not the obligation to exercise the option. The maximum loss to the buyer is equal to the premium paid for the option. Note that a trader pays premiums in order to obtain the right to buy or sell an underlying asset at a certain price in the future. On the other hand, the potential gains are theoretically infinite.

To the seller (writer), however, the maximum gain is limited to the premium received after writing the option. The potential loss is unlimited.

Some of the symbols used to represent relevant factors when dealing with options include:

X = strike price

S_t = Price of the underlying asset at time t

C_t = the market value of a call at time t

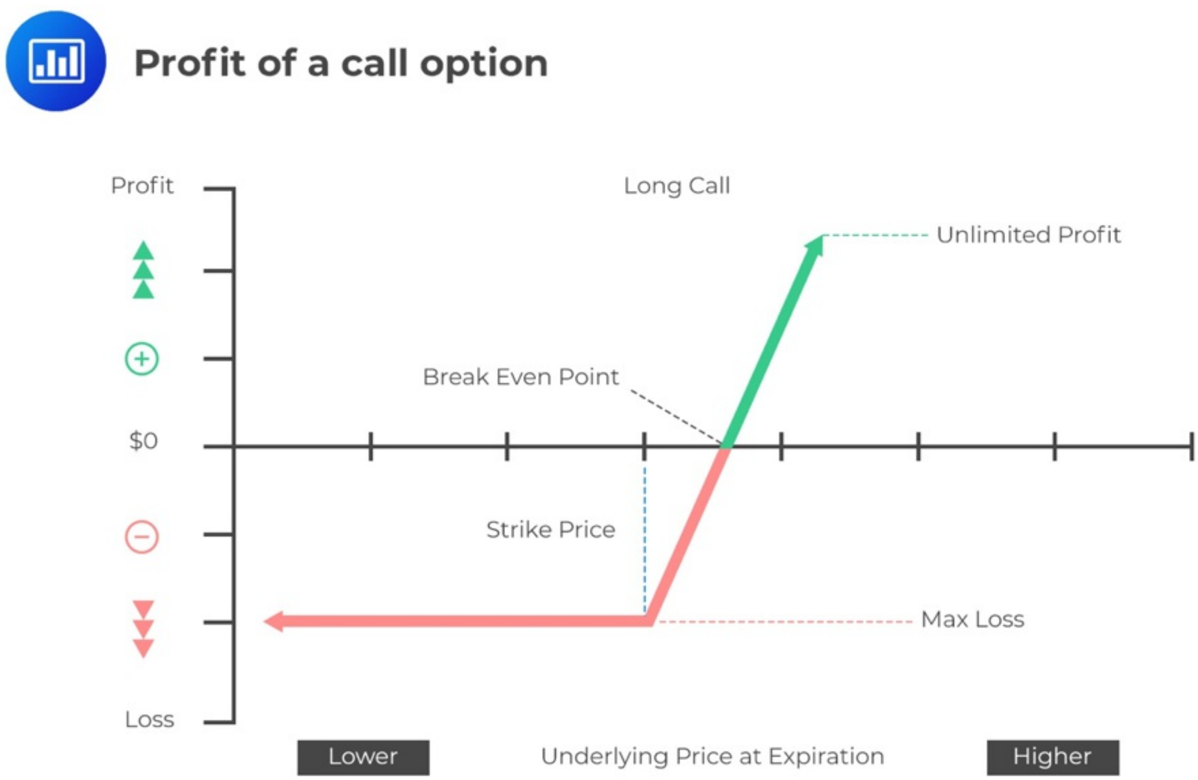
P_t = the market value of put option at time t

t = the time to maturity/expiration of the option

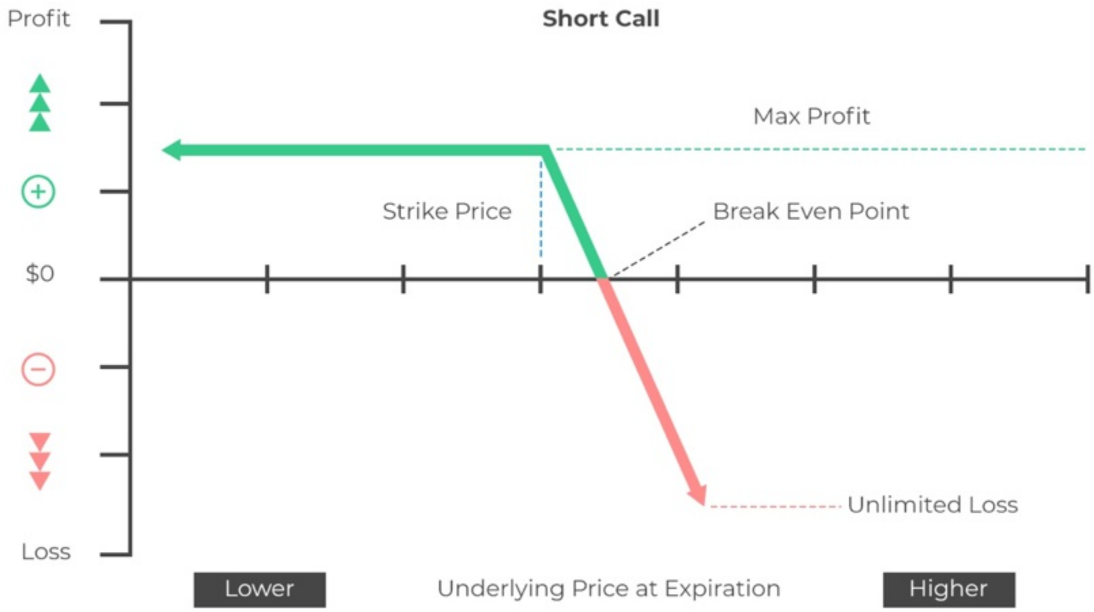
Call Options

A call option gives the owner/holder/buyer the right but not the obligation to buy the underlying stock at a given price on expiry. The buyer is said to hold a long position in the contract, while the seller is said to hold a short position.

When the stock price is less than or equal to the stock price at maturity, the buyer cannot exercise the option because the payoff would be zero. If the stock price is higher than the exercise price at maturity, the buyer will most likely exercise the option. The payoff of the call will be equal to the difference between the market price and the strike price ($S_t - X$).

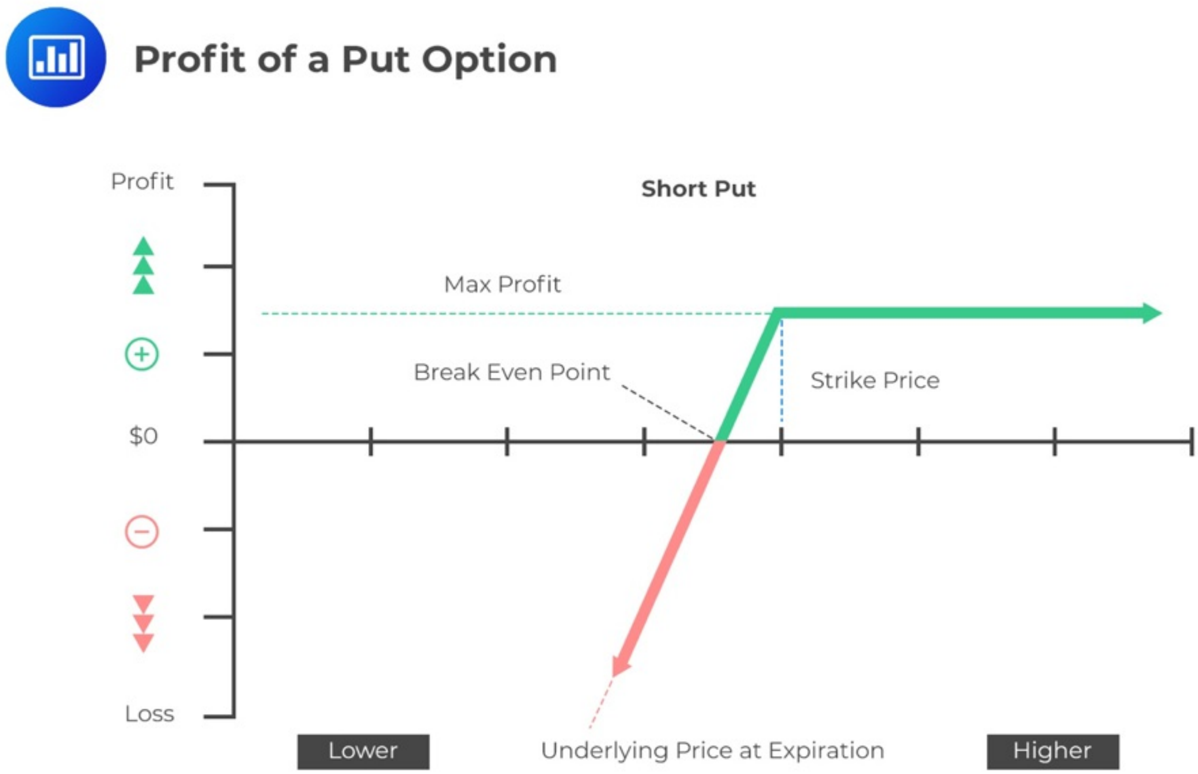


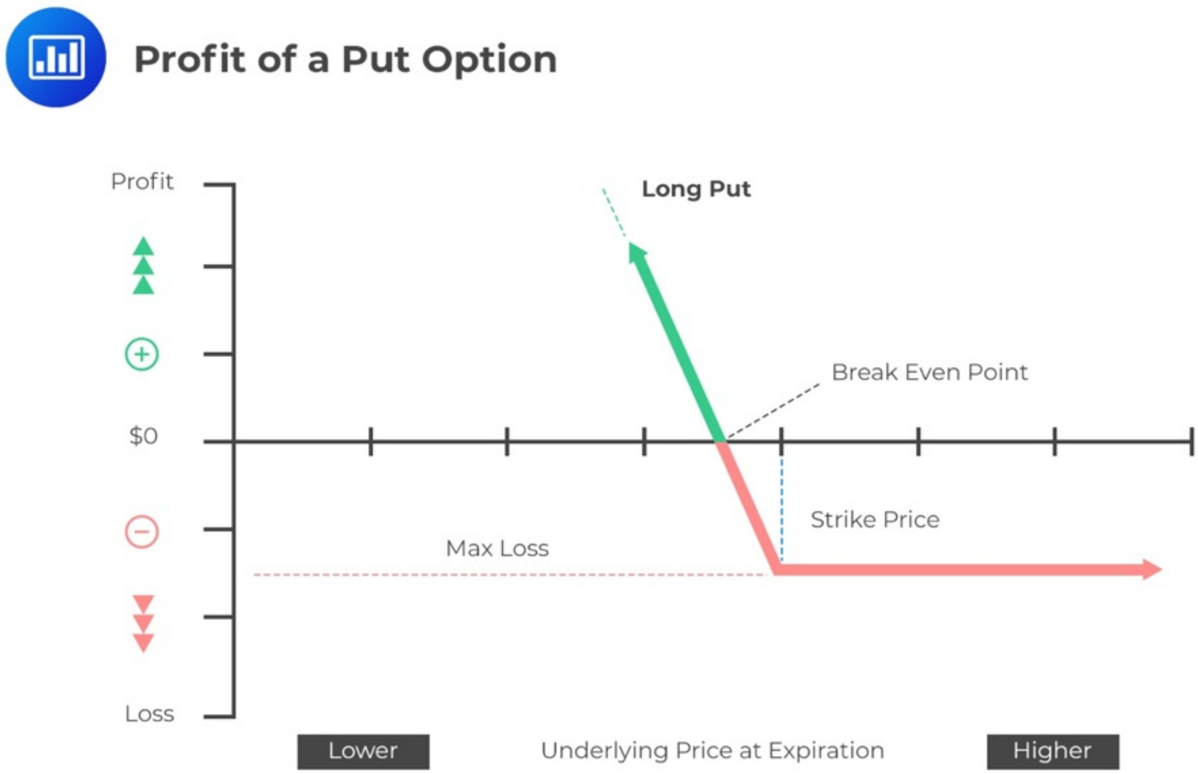
Profit of a call option



Put Options

A put option gives the holder/buyer the right but not the obligation to sell the underlying stock at a specified price. At expiration, the buyer will only benefit if the prevailing market price is less than the exercise/strike price. The payoff is equal to $(X - S_t)$. If the stock stays at X or above, the payoff will be zero.


Profit of a Put Option
Short Put
Profit
Max Profit
Break Even Point
Strike Price
$0
Loss
Lower
Underlying Price at Expiration
Higher



Types of Options

Options that can be exercised at any time, during, and before their maturity/expiration period are known as **American options**. Those that can only be exercised on the expiration/maturity date are known as **European options**. Most exchange-traded options are American options, while most options traded in over-the-counter markets are European options. Since they are only tradable at maturity, European options can easily be analyzed using the Black-Scholes-Merton model. Numerical procedures, for example, binomial trees, are used to analyze American options.

The **exercise date** is the date that has been specified in the contract upon which the contract matures (maturity date). The pre-determined future price at which the underlying asset will be sold is known as the **exercise (or the strike) price**.

Moneyness of Options

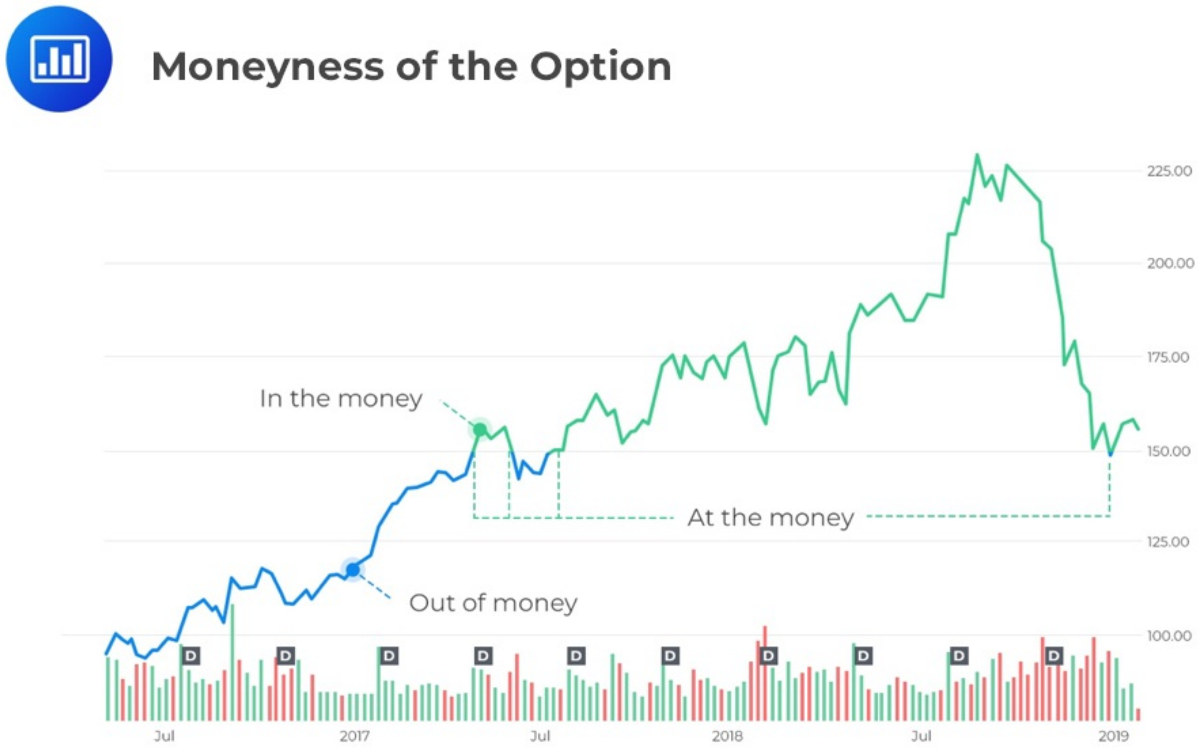
Assume that options were to be exercised today. The option will be said to be:

- in the money, if it gives a positive payoff,
- out of the money, if it gives a negative payoff,
- and at the money, if the payoff is zero.

A call (put) option is said to be in the money if the strike price is lesser (greater) than the asset price at the time of maturity of the contract. If the strike price is greater (lesser) than the asset price at the time of maturity of the contract, a call (put) option is said to be out of the money. When the asset price equals the strike price at the time of maturity, an option is said to be at the money.

Example of the Moneyness of an Option

A great way to visualize this concept is with a graph. Let's say we have a call option on AAPL with a strike price of USD 150. Whenever the price of the underlying (AAPL stock) is above USD, the option is in the money:



Note that for a put option with a strike price of USD 150, it would be the exact opposite – the option would be out of the money anytime the underlying stock is above USD 150.

Profits on Call Options

Example 1: P&L on European Calls

Assume that a trader buys an out-of-the-money European call option with a strike price of \$50 for an asset that is currently selling at \$40. The option expires in 6 months and has a premium of \$5. What are the trader's profits/losses if the price of the underlying asset at maturity is (i) \$40 and (ii) \$60?

For a current asset price of \$40	For a current asset price of \$60
The option will not be exercised. The trader therefore incurs a loss of \$5, the premium paid to secure the option. The seller of the call option earns a profit of \$5.	The option will be exercised. The trader will buy the asset at \$50 and then sell it at \$60. The trader will, as a result, make a profit of \$60 (current price of the asset) - \$50 (strike price) - \$5 (premium paid) = \$5. The seller of the option will suffer a loss of \$5.

Net Loss on Call Options

Sometimes a trader may get a net loss by exercising an option. This usually happens in cases where the current price of the underlying asset is between the strike price and the strike price plus the premium paid. In the above illustration, a trader will suffer a net loss if the asset's current price is between \$50 and \$(50+5) = \$55.

Example 2: Net Loss on European Calls

Suppose that the current market price of the asset in example 1 is \$53. What is the trader’s net loss?

The trader will lose \$5 if he chooses not to exercise the option.

If the option is exercised, the trader gets a negative profit: \$53 - \$50 - \$5 = -\$2.

In as much as the trader suffers a loss by exercising the option, the loss is smaller as compared to the loss obtained if the option is not exercised. In such a scenario, the profits made by the call option seller will also be less than \$5.

Profits on Put Options

Example 3: P&L on European Puts

Assume that a trader buys an out-of-the-money European ***put*** option with a strike price of \$50 for an asset that is currently selling at $40. The option expires in 6 months and has a premium of $5. What are the trader's profits/losses if the price of the underlying asset at maturity is (i) $40 and (ii) $60?

For a current asset price of $40	For a current asset price of $60
If the option will be exercised: The trader's profit will be the $10 difference minus the $5 premium. Since it's a zero-sum game, the option seller will lose $5.	If the option is not exercised: The option buyer will lose the $5 premium while the option seller will gain the $5 premium paid upfront.

Net Loss on Put Options

The trader suffers a net loss if the current market price of the asset is between the strike price minus the premium paid and the strike price, in this case, $45 and $50. For example, if the underlying price at expiration is $49, the put buyer will make $1 from the option being in the money and lose $5 from the option premium paid upfront, for a net loss of $4. This is still better than losing the full option premium of $5!

Payoffs

Denoting the price of the asset at maturity as S_t and the strike price as X, the payoffs from option positions are as shown below.

Long Call: $\max(S_t - X, 0)$

Short Call: $-\max(S_t - X, 0) = \min(X - S_t, 0)$

Long Put: $\max(X - S_t, 0)$

Short Put: $-\max(X - S_t, 0) = \min(S_t - X, 0)$

Intrinsic Value and Time Value

The **intrinsic value** is the value of the option if the option were to be exercised immediately. It is the same mathematical formula as if the payoff of the option was today.

Long Call: $\max(S_t - X, 0)$

Long Put: $\max(X - S_t, 0)$

The **time value** of an option is the difference between the option premium and the intrinsic value:

Option premium = Time value + Intrinsic value

Exchange-Traded Options on Stocks

Options traded in exchanges are American-style options. The largest exchange in the world is the Chicago Board Options Exchange (CBOE). Traders with a short position are randomly allocated (assigned) traders with a long position. A single option contract is the right to trade 100 shares.

Maturity of Stock Options

The CBOE has three cycles of trade (maturity dates):

- Jan Cycle: January, April, July, October
- Feb Cycle: February, May, August, November
- March Cycle: March, June, September, December

The CBOE equally offers weekly options (short-term options) and LEAPS (Long Term Equity Anticipation Securities). LEAPS are simply publicly traded options contracts with expiration dates that are longer than one year.

Strike Prices

The CBOE sets strike prices in different multiples.

Current price of the underlying asset ($)	The strike price will be a multiple of:
5-25	2.5
25-200	5
greater than 200	10

The strike prices of an underlying asset are the three closest prices to the current price of an underlying asset.

Example: Quoted Strike Prices

Assume the price of an underlying asset is $20. The strike prices of the asset that will be listed will be the three closest prices to the current price that are multiples of 2.5. In this case, it would be $17.5, $20, and $22.5.

If the price of the asset moves below $17.5, options with a strike price of $15 will start trading. Conversely, if the price of the asset moves above $22.5, options with a strike price of $25 will start trading. These rules ensure that a lot of options are available for trade.

Options of the same type form a class. Options of a class having a specific maturity date and strike price form an option series.

The Effect of Dividends and Stock Splits

Stock Dividends

Instead of paying cash, a stock dividend involves issuing extra shares to shareholders. For example, if a firm announces a 2% stock dividend, then for every 100 shares held, shareholders will receive 2 more shares.

Exchange-traded options are not usually adjusted for cash dividends. In other words, when a cash dividend occurs, there are no adjustments to the terms of the option contract.

However, when the cash dividend is larger than usual, say, more than 10%, then a committee may be formed by the Options Clearing Corporation to decide if necessary to make adjustments.

Stock Splits

A stock split involves increasing the total number of shares outstanding by issuing more shares to shareholders at a specified ratio. For example, a 2-for-1 stock split means a shareholder will be awarded one more share for every two shares held.

If a stock has a b-for-a stock split, the share price will be reduced by a factor of (a/b). However, this is a theoretical assumption. In reality, the post-split share price can be different. The number of shares will increase by a multiple of (b/a).

The terms of exchange-traded options contracts are adjusted to reflect expected changes in a stock price arising from a stock split.

Non-Standard Products

They include

i. **Flexible exchange (FLEX) Option:** These are exchange-traded options on stock indices, but there's a lot more flexibility. The strike price and expiration dates can be altered if the trading parties so wish.
ii. **ETF options:** These are American-style options that are settled by delivering the underlying shares rather than cash.
iii. **Weekly options:** These are short-term options with a maturity period of roughly 7 days. They are created on a Thursday, with the expiration date being the Friday of the next week.
iv. **Binary options:** Binary options have a fixed payoff in case the option is ITM at expiration.
v. **Credit event binary options (CEBOS):** The CEBOs payoff is triggered when the reference entity suffers a credit event before the option's expiration date.
vi. **Deep out-of-the-money (DOOM) options:** They are designed to only be ITM in the event of a large down price movement in the underlying asset.
vii. **Cliquet Options**: The payoff provided by these options is equal to the sum of the asset's monthly capped returns.

Trading Commissions and Margin Requirements

Most options exchanges use **market makers** to facilitate trading. The market maker will quote bids and offer prices.

A **commission** refers to the fee charged by a broker as a reward for their efforts in facilitating a transaction. Commission costs depend on the size of the trade as well as on the type of broker involved. They reduce the investor's returns.

Options can be **closed out** by taking an offsetting position, just like future markets.

To prevent the options markets from being influenced by just the investors, the CBOE imposes position and exercise limits on traded options:

- **The position limit** refers to the number of contracts an investor can hold on the same side of the market. Long calls and short puts are on one side of the market, while short calls and long puts are on one side of the market.

- **The exercise limit** is the number of contracts that can be exercised within five working days.

In options trading, the term "**margin**" refers to the collateral deposited by the option writer as a form of guarantee that they will honor their contractual obligations. Margin requirements differ from one broker to another and also depend on the nature of the underlying asset. As a general rule, options that mature before 9 months cannot be purchased on margin. Those that mature after 9 months can be purchased by borrowing up to 25% of the purchase price.

There is no margin requirement in case a trader pays cash for the option.

As discussed earlier in chapter 5;

The margin requirement by CBOE on a short call option is the maximum between the following two values:

- 100% of the value of the option + 20% of the share price minus the amount the option is out of money, if any; or

- 100% of the value of the option + 10% of the underlying share price.

The margin on a short put option is the maximum between the following two variables:

- 100% of the value of the option + 20% of the share price minus the amount the option is out of money, if any; or
- 100% of the value of the option + 10% of the strike price.

If the option is on indices, then the 20% in the above formulas is replaced by 15%.

Warrants, Convertibles, and Employee Stock Options

Warrants

Warrants are call options that are issued by a corporation.

The holder of a warrant contacts the issuer in order to exercise the warrant. When the holder of a warrant exercises his/her right to obtain shares in the company, the company issues more stock, after which the warrant holder can now buy the stock at the strike price. The issuances of debts can be made more attractive by the use of warrants.

Convertible Bonds

Convertible bonds are bonds that can be converted to equity using a ratio that has been pre-determined. For example, assume the current share price of a company is USD 30. It could decide to issue 5-year bonds, each with a par value of $100 that can be converted into 10 shares after 2 years. When an investor decides to convert, the company will actually issue more shares in return for the bonds.

Both warrants and convertible bonds are usually traded on exchanges.

Employee Stock Options

A corporation grants employee stock options to its employees.

Employee stock options differ from Exchange-traded option in the following ways:

1. Selling stock options to third parties is not allowed.
2. There is a vesting period that lasts for about 4 years, during which options cannot be exercised.
3. Employees may forfeit their options in case they quit their jobs during the vesting period.
4. Out-of-the-money options may be forfeit in case employees quit their jobs after the vesting period, while in-the-money options may be exercised immediately.

Question

What is the intrinsic value of a put option if the strike price is $63 and the prevailing market price of the underlying is $78?

A. $15

B. -$15

C. $7.5

D. $0

The correct answer is **D**.

$$\begin{aligned}\text{Intrinsic value of a put option} &= \max(0, X - S_t)\\ &= \max(0, 63 - 78) = \max(0, -15) = 0\end{aligned}$$

Reading 37: Properties of Options

After completing this reading, you should be able to:

- Identify the six factors that affect an option's price.
- Identify and compute upper and lower bounds for option prices on non-dividend and dividend-paying stocks.
- Explain put-call parity and apply it to the valuation of European and American stock options, with dividends and without dividends and express it in terms of forward prices.
- Explain and assess potential rationales for using the early exercise features of American call and put options.
- Explain the relationship between options and forward prices.

Six Factors Affecting Option Prices

There are six factors that impact the value of an option:

S = current stock price

K = strike price of the option

T = time to expiration of the option

r = short-term risk-free interest rate over T

D = present value of the dividend of the underlying stock

σ = expected volatility of stock prices over T .

Current Stock Price

The value of all call options increases (decreases) as S increases (decreases). For put options, the value of the put decreases (increases) as S increases (decreases).

The Strike Price of the Option

For call options, the value decreases (increases) as the strike price increases (decreases). For put options, the value increases (decreases) as the strike price increases (decreases).

Time to Expiration

With American-style options, as the time to expiration increases, the value of the option increases. With more time, there are higher chances of the option moving in the money.

- As the time to expiration *increases*, the value of a call option *increases*.
- As the time to expiration *increases*, the value of a put option also *increases*.

However, the same does not apply to European-style options, precisely when the underlying has scheduled dividends. For example, assume we have a two-month call option and a four-month call with the same exercise price K and the same underlying stock. Assume further that a sizeable dividend is expected in three months. The ex-dividend stock price and call price will decrease. As such, the two-month call could actually be more valuable than the four-month call.

Risk-Free Rate over the Lifespan of the Option

Here, the simplest way to think about this is as a rate of return on a stock. Let's say you have the choice between buying a bond worth $1000 or one share of stock priced at $1000. If you know the risk-free rate of interest is 5%, you would expect the stock price to increase by more than 5% on average. Otherwise, why would you buy a share of stock instead of investing in a risk-free bond? Therefore,

- As the time the risk-free rate *increases*, the value of a call option *increases*.
- However, as the risk-free rate *increases*, the value of a put option *decreases*.

Dividends

Payments from an underlying may include dividends. As we've seen previously, immediately after payment of a dividend the stock price falls by the amount of the dividend. However, the benefits of these cash flows to the holders of the underlying security do not pass to the holder of a call option. Therefore,

- As dividends *increase*, the value of a call option *decreases*.
- However, as dividends *increase*, the value of a put option *increases*.

Expected Volatility of Stock Price over Time

Volatility is considered the most significant factor in the valuation of options. As volatility increases, the value of all options increases. Since the maximum loss for the buyer of a call or put option is limited to the premium paid, we can conclude that there are higher chances of the option expiring in the money as volatility increases.

- As volatility *increases*, the value of a call option *increases*.
- As volatility *increases*, the value of a put option *increases*.

Option Pricing Bounds

Let:

c = value of a European call option;

C = value of an American call option;

p = value of a European put option;

P = value of an American put option;

S_T = value of the stock at expiration; and

S_0 = value of the stock today.

A call option gives the holder the right to buy the stock at a specified price. The value of the call is **always less** than the value of the underlying stock. Thus,

$$c \leq S_0 \quad \text{and} \quad C \leq S_0$$

If the value of a call were to be higher than the value of the underlying stock, arbitrageurs would sell the call and buy the stock, earning an instant risk-free profit in the process.

A put option gives the holder the right to sell the underlying stock at a specified price. The value of a put is always less than the strike price. Thus,

$$p \leq K \text{ and } P \leq K$$

If the value of a put were to be higher than the strike price, everyone would move swiftly to sell the option and then invest the proceeds at a risk-free rate throughout the life of the option.

European options can only be exercised at expiration. As such, the value of a European put is always less than the present value of the strike price, that is.

$$p \leq K(1 + r)^{-T}$$

Lower Pricing Bounds for European Calls on Non-Dividend-Paying Stocks

Call options can never be worth less than zero as the call option holder cannot be forced to exercise the option. The lowest value of a call option has a price which is the maximum of zero and the underlying price less than the present value of the exercise price. This is expressed as follows:

$$c + K(1 + r)^{-T} \geq S_0$$

Thus, the lower pricing bound of a European call option is given by:

$$c \geq \max\left(S_0 - K(1 + r)^{-T}, 0\right)$$

Lower Pricing Bounds for European Puts on Non-Dividend-Paying Stocks

A put option has an analogous result. A put option can never be worth less than zero as the option owner cannot be forced to exercise the option. The lowest value of a put option is the maximum of zero, and the present value of the exercise price less the value of the underlying. This is expressed as follows:

$$p + S_0 \geq K(1 + r)^{-T}$$

Thus, the lower pricing bound of a European put option is given by:

$$p \geq \max\left(K(1 + r)^{-T} - S_0, 0\right)$$

American Call Options

Case of No Dividends

American options can be exercised at any time on or before their maturity dates. A key question, however, is whether an investor who owns an American option should take up the offer to exercise early even when the option is deep in the money. Let's look at an example, assuming the stock does not pay dividends.

Example: American Options

A non-paying dividends American call option has a strike price of $50 and expires in three months. The underlying stock has a current price of $80. Assuming no dividends, should the investor exercise this option before expiry?

Looking at the situation, it appears the decision should be straightforward for the investor: exercise the option now, sell the stock immediately, and generate a profit of $30. Exercising the option would appear even more attractive when we consider that each option represents 100 shares of the stock, which means the investor would make a profit of $3,000 from just one option. Even with this profit, the option should not be exercised before maturity if interest rates are **positive**. In order to understand why this is the case, note that the option owner is in either one of two situations.

Situation 1: The investor wants a long position in the stock

Situation 2: The investor does not want a long position in the stock.

If the investor is in Situation 1, they would exercise the option but opt not to sell the stock. It would be best for them to hold the option until it expires. To better understand this, consider two outcomes for Situation 1:

Outcome 1: The stock price is in the money (greater than $50) on the expiry date.

Outcome 2: The stock price is out of the money (less than $50) on the expiry date.

Under Outcome 1, it would be **suboptimal** to exercise the option early, as waiting until the option expires would allow the investor to earn **additional interest** by investing the strike price for two months at the risk-free rate.

How about Outcome 2? If the option is OTM at expiry yet it has already been exercised, the investor would incur a loss of $50 - S, where s is the stock price at the expiry date. If the investor foregoes early exercise and waits until maturity, they would not exercise the option. Instead, they would let it expire worthless and in so doing avoid this loss.

The key point here is that keeping the option unexercised until maturity gives the option holder insurance against the value of the stock falling below $50. As soon as the option is exercised, this insurance is lost.

Shifting our focus to situation 2 where the investor has no motivation to hold the stock, exercising early and selling the stock would appear to be the best decision, but it isn't. The optimal decision would be to sell the option. But why? When the option is exercised immediately, it yields a profit equal to the intrinsic value (which in this case is $30). Selling the option, however, earns a profit of the intrinsic value plus what's known as the time value. The time value is the insurance against the value of the stock falling below $50, as mentioned above.

Formal Proof that a Call Option Should Never be Exercised Early

Let's consider two portfolios an investor can hold until maturity of an American call option

Portfolio 1: A call option plus cash equal to the present value of the strike price.

Portfolio 2: The stock

Under Portfolio 1, the investor can invest the cash amount today at the risk-free rate. By the time the option matures, the cash amount would be equal to the strike price. If the option is in the money at maturity, the investor can use the proceeds from the cash investment to exercise the option at the strike price. This would automatically increase the value of Portfolio 1 to the strike price, but the investor would still have the choice to sell the stock, further increasing the value of the portfolio. If the option is out of the money at maturity, it expires worthless, but the value of the portfolio would still have grown to the strike price thanks to the cash investment.

If follows that in all circumstances, Portfolio 1 would be worth **at least as much** as the stock. On the other hand, Portfolio 2 would be worth the stock price prevailing at maturity of the option. In summary, we can infer that at maturity,

Value of Portfolio 1 ≥ Value of Portfolio 2

This must be true today, otherwise an arbitrage opportunity would arise. If Portfolio 2 is worth more than Portfolio
1 today, an arbitrageur can buy Portfolio 1 and short Portfolio 2. This would create a position that would never lead to a loss and would sometimes lead to a profit.

Today, Portfolio 1 is worth:

Call Price + PV(K)

where PV represents "present value" and K is the strike price.

On the other had, the value of Portfolio 2 today is the stock price, S. We can therefore infer that:

Alternatively,

Call Price ≥ S - PV(K) ....Equation y

We know that the option price can never be negative. Thus,

Call Price > max(S - PV(K), 0)

working under the assumption that interest rates are positive,

K > PV(K)

Equation y above therefore implies that:

Call Price > S - K

Exercising the option today would mean that the call price equals S - K. It follows that the call option should **never be exercised early**.

Employee Stock Options

Employee stock options cannot be sold – they have to be exercised by the employee to which the stock option has been given.

As with all other call options, employee call options on stocks that pay no dividends should not be exercised before maturity.

Effects of Dividends

Dividends reduce the price of a stock. For an American call dividend-paying option to be profitable, the option should be exercised just before the ex-dividend date. However, the option should not be exercised if the dividend is less than $K - K^*$ where K^* is obtained by discounting K from the next ex-dividend date (option maturity) to the current ex-dividend date.

In the presence of dividends (D), the lower bound for a European call option will be adjusted to incorporate dividends:

$$c \geq S_0 - K(1 + r)^{-T} - D(1 + r)^{-T}$$

In the presence of dividends, American call options can be exercised immediately.

American Put Options

Case of No Dividends

Whereas call option holders pay the strike price, put option holders receive the strike price. The decision to exercise an American put option, thus, is dependent on a trade-off between receiving the strike price early so as to reinvest it and benefitting from a very small probability that the stock price will be greater than the strike price at maturity.

Impact of Dividends

Dividends make it undesirable for a put option to be exercised before maturity. In the presence of dividends:

$$p \geq \max(K(1+r)^{-T} + D(1+r)^{-T} - S_0, 0)$$

Options with a longer maturity period are less likely to be exercised before their maturity date as there is enough time for the stock price to move above the strike price.

A put option holder is less likely to exercise the position earlier if the time to maturity increases, the stock price increases, the dividends to be received an increase, and if the interest rate decreases.

Put-call Parity In European Options

Put-call parity states that the price of a call option implicitly informs a certain price for the corresponding put option with the same strike and expiration and vice versa.

In other words, put-call parity is the relationship between the price of a European put option and the price of a European call option, with the same strike price and time to maturity.

Consider the following portfolios:

- Portfolio A: One call option plus an amount of cash equivalent to $K(1+r)^{-T}$
- Portfolio B: One put option plus one share

Since the options are European, they cannot be exercised prior to maturity. Thus, put-call parity demands that the value of the two portfolios today is the same. Expressed mathematically,

$$c + K(1+r)^{-T} = p + S_0$$

Where:

c = value of call option

K = strike price

p = value of put option

S_0 = initial stock price

On the expiration date, the put-call parity is now:

$$c + K = p + S_T$$

because we do not have to use the present value of the bond.

Let's say you own a stock trading at $100, and you also own a put option with an expiration price of $90.

Let's now look at what happens to this two-asset portfolio if the prices at expiration are $80, $89, $110, or $130.

Expiration Price	$80	$89	$110	$130
Stock	$80	$89	$110	$130
Put Option	$10	$1	$0	$0
Portfolio	$90	$90	$110	$130

As you can see from the table above, when you own a put and a stock, you have what is called a protective put. The price can never get below the price floor ($90 in our example), but you still have unlimited profit on the upside.

Now, let's say you own a call with an expiration price of $90, and you also own a zero-coupon, risk-free bond that matures for $90.

With the same expiration prices as the previous table, we now have:

Expiration Price	$80	$89	$110	$130
Call Option	$0	$0	$20	$40
Bond	$90	$90	$90	$90
Portfolio	$90	$90	$110	$130

As we can see from the two tables, the portfolio value at expiration for the same expiration prices is the same whether we own the stock plus the put or the call plus the risk-free bond.

However, before the expiration date, we have to discount the present value of the bond, so the put-call parity is:

$$c + K(1 + r)^{-T} = p + S_0$$

Applying Put-Call Parity

Let's now use an example to illustrate the put-call parity and see how we could exploit arbitrage opportunities in the options market.

Example

A stock currently sells for $51. A 3-month call option on the stock, with a strike price of $50, has a price of $5. Assuming a 10% continuously compounded risk-free rate, determine the price of the associated put option.

Solution

Applying the put-call parity relationship,

$$c + K(1 + r)^{-T} = p + S_0$$

Making P the subject,

$$\begin{aligned} p &= c + K(1 + r)^{-T} - S_0 \\ &= 5 + 50(1.10)^{-0.25} - 51 \\ &= 2.82 \end{aligned}$$

If p is greater than or less than 2.82, there will be arbitrage opportunities.

For example, assume p = 3.50. The following arbitrage opportunities would present themselves:

1. Buy call for $5
2. Short Put to realize $3.50
3. Short the stock to realize $51
4. Invest $49.5 $(= 51 + 3.50 – 5)$ for 3 months, making 50.69(= 49.5(1.10)^{0.25})$

Let S_T be the price of the stock at expiry.

If $S_T > 50$,

- Receive $50.69 from the investment;
- Exercise the call to buy the stock for $50.
- Net profit = $0.69

If $S_T < 50$,

- Receive $50.69 from investment,
- Put exercised by the holder: buy the stock for $50.
- Net profit = $0.69

Put-Call Parity In American Options

Put-call parity is only valid for European options. For American options with the possibility of early exercise, the relationship turns into the equality:

$$S_0 - K \leq C - P \leq S_0 - K(1 + r)^{-T}$$

Effect of Dividends

- When a stock pays a dividend, its value must decrease by the amount of the dividend. This increases the value of a put option and decreases the value of a call option.

- A dividend payment will reduce the lower pricing bound for a call and increase the lower pricing bound for a put.

Lower Bounds of American Options

Option	Minimum value	Maximum value
American call	$C \geq \max(0, S_0 - K(1+r)^{-T})$	S_0
American put	$P \geq \max(0, K - S_0)$	K

Use of Forward Prices

For now, we have only dealt with calls and put options on stocks. Forward prices, for example, on commodities such as oil, can also be used to derive the price of call and put option prices on commodities or other assets that trade with forward contracts. Let's define F as the forward price for a contract maturing at the same time as the options and $F(1+r)^{-T}$ as the present value of F when discounted from the options' maturity at the risk-free rate. Note that K is still the strike price of the option.

The put-call parity relationship is, therefore:

$$c + K(1+r)^{-T} = p + F(1+r)^{-T}$$

Because the put price cannot be negative, a lower bound for a European call price can be deduced as:

$$c \geq F(1+r)^{-T} - K(1+r)^{-T}$$

Similarly, because the call price cannot be negative, the lower bound of the European put price is:

$$p \geq K(1+r)^{-T} - F(1+r)^{-T}$$

Question

A one-year European put option on a non-dividend-paying stock with the strike at USD 50 currently trades at USD 5.55. The current stock price is USD 45. The stock exhibits an annual volatility of 30%. The annual risk-free interest rate is 5%, compounded continuously.

Determine the price of a European call option on the same stock with the same parameters as those of this put option.

A. USD 4.12

B. USD 2.50

C. USD 5.55

D. USD 2.93

The correct answer is **D**.

According to put-call parity,

$$c + K(1 + r)^{-T} = p + S_0$$

Making c the subject,

$$\begin{aligned} c &= p + S_0 - K(1 + r)^{-T} \\ &= 5.55 + 45 - 50(1.05)^{-1} \\ &= 2.93 \end{aligned}$$

Where:

c = value of call option

K = strike price

p = value of put option

S_0 = initial stock price

Reading 38: Trading Strategies

After completing this reading, you should be able to:

- Explain the motivation to initiate a covered call or a protective put strategy.
- Describe principal-protected notes (PPNs) and explain necessary conditions to create a PPN.
- Describe the use and calculate the payoffs of various spread strategies.
- Describe the use and explain the payoff functions of combination strategies.

Options can be arranged in different ways to form different payoff patterns.
Strategies that involve a single option are referred to as **spreads**, while those involving both call and put options are referred to as **combinations**.

Strategies Involving a Single Option

In the previous chapter, we looked at the put-call parity relationship, which is given by:

$$c + PV(K) = p + S_0 \ldots \ldots \ldots \ldots .(i)$$

Where:

c = value of call option

K = strike price

p = value of put option

S_0 = initial stock price

r = risk-free rate

T = time to maturity

PV(K) = present value of the strike price

The above result can also be expressed as:

$S_0 - c = PV(K) - p$...............(ii)

$-p - S_0 = -c - PV(K)$.........(iii)

$c - S_0 = p - PV(K)$............(iv)

The right-hand side of equation (i) shows a combination of a put and an asset.

When you buy a put while holding the underlying asset, it is referred to as a protective put strategy.

The left-hand side of equation (ii) shows an asset plus a short call. This strategy is referred to as a covered call.

Equation iii shows the reverse of a protected put, while equation iv illustrates the reverse of a covered call.

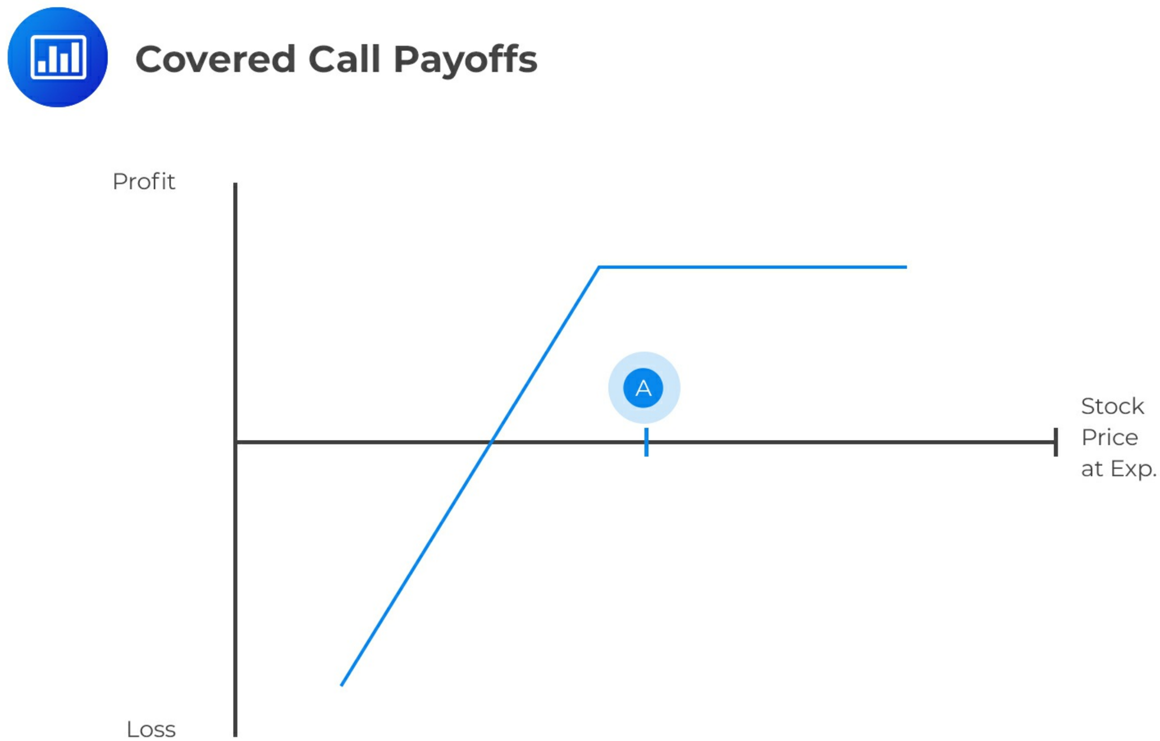
Covered Call

A **covered call** describes a trading strategy where the seller (writer) of a call option also owns the underlying stock. The writer sells call options for the same amount (or less) of stock. If the option is not exercised, the writer gets to keep the premium. If the option is exercised, the writer simply hands the option buyer their shares. In a covered call, the important thing to note is that the investor receives a premium in exchange for giving up any potential profits from high upward movements of the asset price.

A covered call is the opposite of a **naked call**. In the latter, the writer of the call option does not own the underlying stock. However, in the event that the option is exercised, the writer is obligated to buy the shares at the market price and deliver them to the option buyer. A naked call, therefore, has an unlimited risk because the market price can rise unpredictably.

The holder of a covered call can only profit on the stock **up to** the strike price of the options contract. The maximum profit is capped at:

(Strike Price - Stock Entry Price) + Option Premium Received



For example, let's say you've bought a stock at $10, received a $0.50 option premium from selling a $12 strike price call. You'd maintain your stock position as long as the stock price stays below $12 at expiration. If the stock rises to $13, you will only profit up to $12, so your total profit will be $2.5 $(= \$12 - \$10 + \$0.50)$. You'd be forced to give up the extra $0.50.

The maximum loss a covered call holder can incur is equal to:

$$\text{Stock Entry Price} - \$0 + \text{Option Premium Received}$$

That could happen when the stock drops to $0.

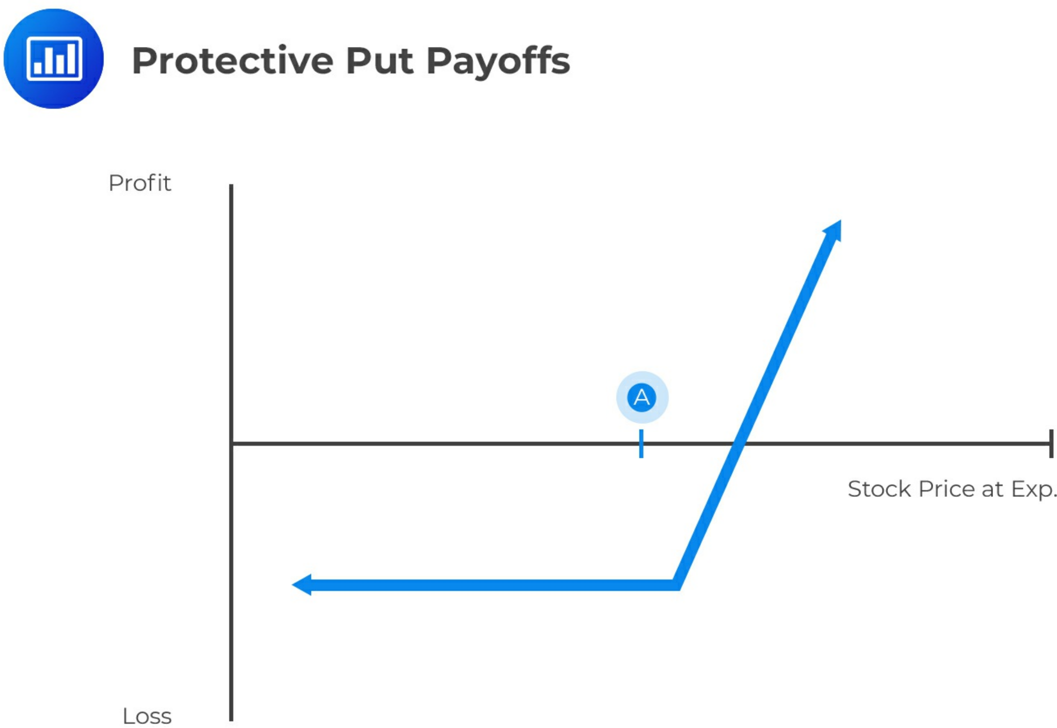
For these reasons, a covered position is taken up to generate cash on a stock that is not expected to increase above the exercise price over the life of the option.

Protective Put Strategy

A **protective put,** also called a put hedge, is a hedging strategy where the holder of a security buys a put to protect themselves against a drop in the stock price of that security.

A protective put has unlimited profit potential. A profit is achieved when the underlying stock price exceeds its purchase price plus the premium paid for the option. On the other hand, the maximum loss is limited by the purchased put option and is equal to the premium paid for buying the put option.

- Max profit = Unlimited
- Max loss = Premium Paid + Purchase Price of Underlying - Put Strike + Commissions Paid.



A protective put is taken by bullish investors worried about near-term uncertainties on a stock.

Principal Protected Notes

A Principal Protected Note (PPN) is a security created from a single option in such a way that the investor makes riskless profits from any gains in the value of the underlying portfolio.

Traders use Principal Protected Notes (PPNs) to hedge against losses while still providing room for potential gains. PPNs are used only in portfolios that provide income to the investor and are more attractive to risk-averse investors.

Illustration: PPNs

Assume that portfolio X comprises of (i) a five-year bond with a face value of USD 1000 and (ii) a five-year call option on a portfolio Y. Portfolio Y currently has a value of USD 1000, with a strike price of USD 1000. Assume also that the five-year interest rate is 10% and is compounded annually.

To benefit from PPNs, the holder of portfolio X should buy the bond now at $\$1000(1+10\%)^{-5} = \$620.92$.

After five years, using the $1000 face value of the bond, the holder of portfolio X can now purchase portfolio Y at a strike price of $1000.

By using PPNs, the holder of portfolio X hedges against any losses associated with a decline in the value of portfolio Y and still benefits from any increase in the value of portfolio Y.

The investor is able to use PPNs by: (i) giving up the interests that would have been earned on the bonds and (ii) not receiving any income from portfolio Y for five years.

Payoffs of Various Spread Strategies

Spread strategies involve either call options solely or put options solely. (Later on, we will see combination strategies, which involve both call and put options.)

Spread strategies include:

i. **Bull Spread:**

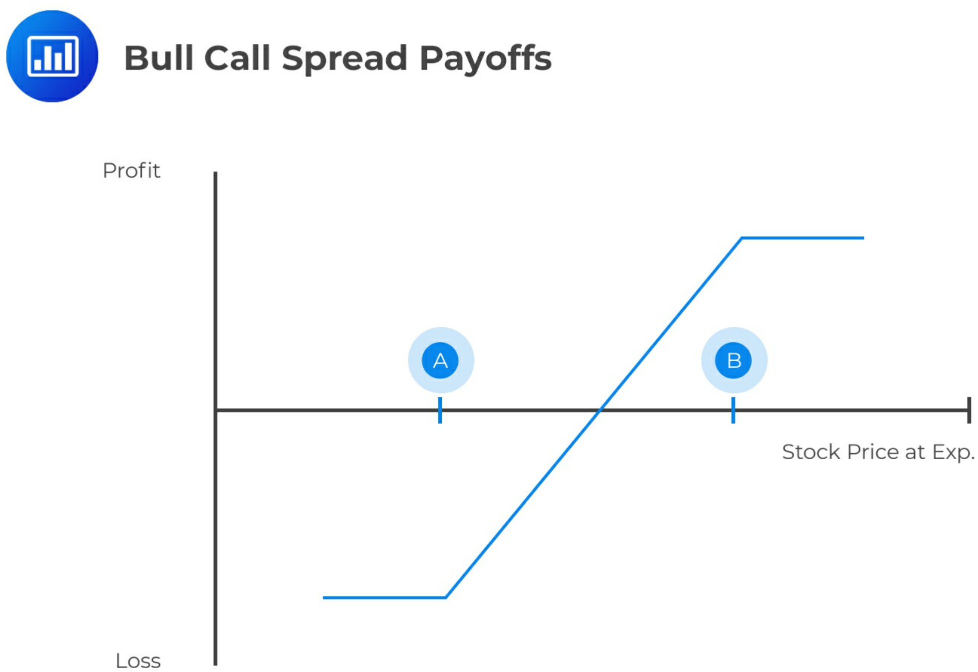
A bull spread is a bullish options strategy designed to take advantage of a moderate rise in the price of the underlying in the near term. In a **bull call spread**, the bullish trader buys a call with a lower strike price and simultaneously sells a call with a higher strike price. The premium received from the sale of the higher strike call subsidizes the premium paid for the purchase of the lower strike call.

Being bullish, the buyer of a bull call spread expects the underlying stock price to rise but remains below the short call's strike.

Example of a bull call spread:

Buy 1 ABC 100 call at	$5.50
Sell 1 ABC 105 call at	$2.0
Net cost	$3.50

The premium paid for the lower strike is higher than the premium received for, the higher strike because the lower strike has higher chances of being attained in the near term.



In a **bull put spread**, the bullish trader buys a put with a lower strike and simultaneously sells a put with a higher strike. The premium received from the sale of the higher strike put is higher than the premium paid for, the lower strike put. As such, the two positions generate a positive net income.

Example of a bull put spread:

Sell 1 ABC 100 call at	$3.50
Buy 1 ABC 95 call at	$1.8
Net benefit	$1.7

The maximum gain, maximum loss, and break-even point for a bull put spread strategy are as follows:

Max profit = Net premium received

Max loss = strike price of short put - strike price of long put - net premium received

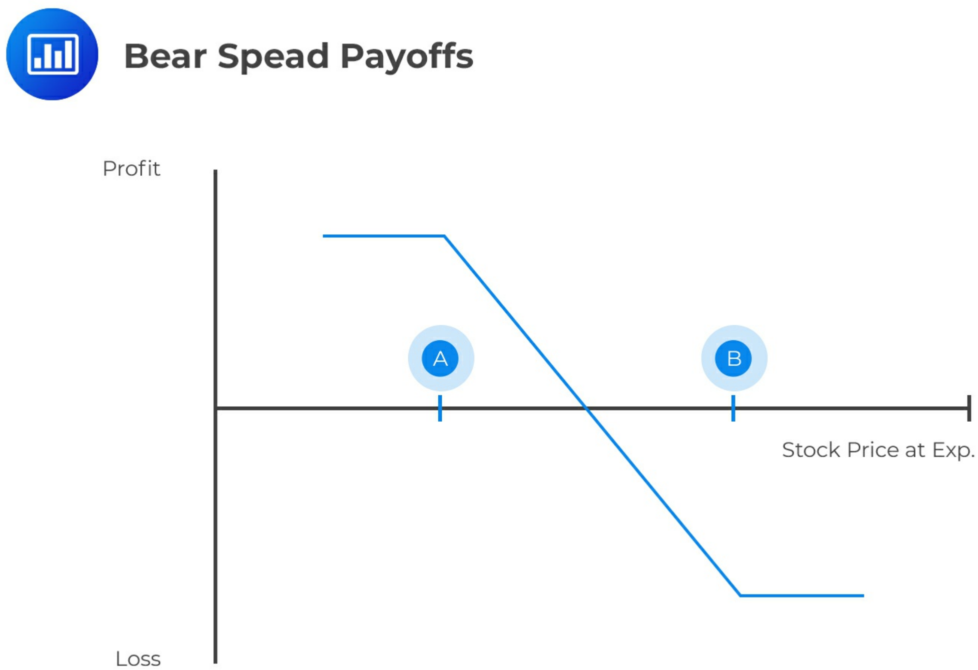
Break-even point = strike price of short put - net premium received

Note: In both call and put spreads, the expiration date, as well as the underlying asset, must be **the same** for both positions.

ii. Bear Spreads:

A **bear spread** is a bearish options strategy designed to take advantage of a moderate decline in the price of the underlying in the near term. A **bear call** spread consists of one long call with a higher strike price and one short call with a lower strike price. On the other hand, a **bear put spread** consists of one long put with a higher strike price and one short put with a lower strike price.

Note that bull spreads involve buying at a lower strike price and selling at a higher strike price. Bear spreads, on the other hand, involve buying at a higher strike price and selling at a lower strike price.



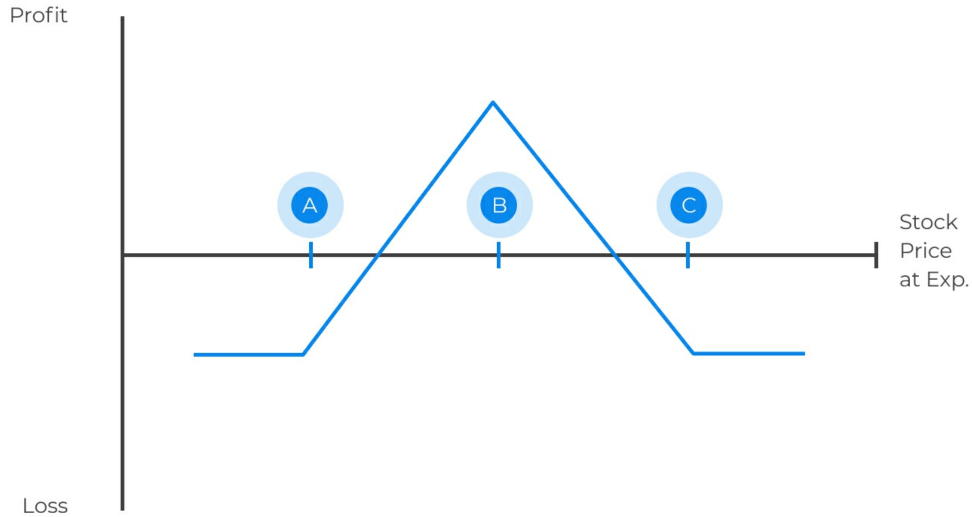
iii. **Box Spreads:**

A box spread is a strategy created from a bull using call options and a bear spread using put options. The strike price and time to maturity of both bull and bear spreads should be the same.

4. Butterfly Spreads:

A butterfly spread is a neutral, limited risk strategy involving various bull spreads and bear spreads. The holder combines four option contracts having the same expiry date at three strike price points, X_1, X_3, $X_2 = \frac{X_1 + X_3}{2}$. Two option contracts are bought – one at a higher strike price and one at a lower strike price - and two option contracts are sold at a strike price in between. A butterfly trader has reason to believe the underlying asset will not move too far away from the current price.

Butterfly Spread Payoffs



The table below shows the payoff from a butterfly spread created from calls, where S_T is the price of the asset.

Range of S_T	Payoff from Long Call with Strike Price X_1	Payoff from Two Short Calls with Strike Price X_2	Payoff from Long Call with Strike Price X_3	Total Payoff
$S_T \leq X_1$	0	0	0	0
$X_1 < S_T \leq X_2$	$S_T - X_1$	0	0	$S_T - X_1$
$X_2 < S_T \leq X_3$	$S_T - X_1$	$-2(S_T - X_2)$	0	$2X_2 - X_1 - S_T = X_3 - S_T$
$S_T > X_3$	$S_T - X_1$	$-(S_T - X_2)$	$S_T - X_3$	$2X_2 - X_1 - X_3 = 0$

Similarly, the payoff from a butterfly spread created from put options can be shown as in the table below:

Range of S_T	Payoff from Long Put with Strike Price X_1	Payoff from Two Short Puts with Strike price X_2	Payoff from Long Put with Strike Price X_3	Total Payoff
$S_T \leq X_1$	$X_1 - S_T$	$-2(X_2 - S_T)$	$X_3 - S_T$	$2X_2 - X_1 - X_3 = 0$
$X_1 < S_T \leq X_2$	0	$-2(X_2 - S_T)$	$X_3 - S_T$	$S_T + X_3 - 2X_2 = S_T - X_1$
$X_2 < S_T \leq X_3$	0	0	$X_3 - S_T$	$X_3 - S_T$
$S_T > X_3$	0	0	0	0

5. Calendar Spreads:

A calendar spread is a trading strategy set up by simultaneously entering a long and a short position on the same underlying asset and at the same strike price, but with different months to expiration. It's a low-risk strategy that's directionally neutral—the holder profits from the passage of time or increase in the underlying's implied volatility. As in a butterfly spread, the holder also believes the stock will have a narrow range of price changes. We have several categories of calendar spreads;

- **A neutral calendar** - when the strike price is close to the current stock price
- **A bullish calendar spread-** has a strike price above the current stock price
- **A bearish calendar spread** has a strike price below the current stock price
- **A reverse calendar spread** – opposite of a calendar spread

6. Diagonal Spreads:

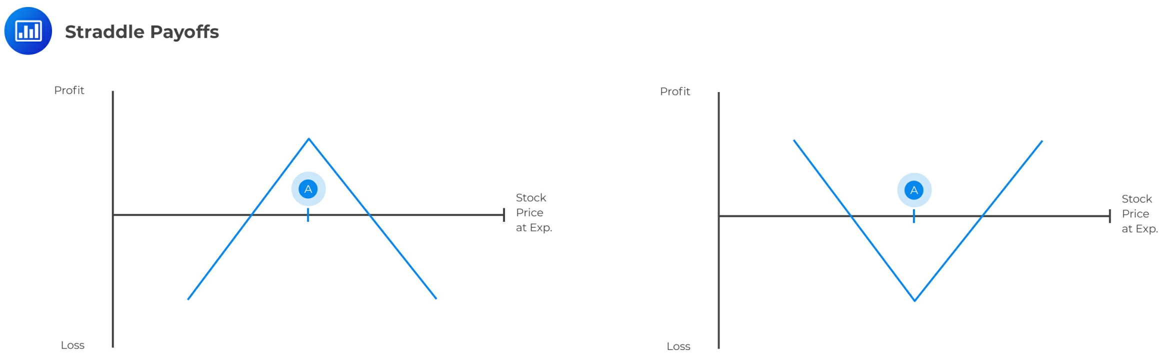
A diagonal spread works much like a calendar spread, but with a little difference; the options in a diagonal spread can have different strike prices in addition to different expirations.

Payoff Functions of Combination Strategies

Combination strategies involve both call and put options.

1. **Straddle:**

A straddle involves two transactions on the same security, with positions that offset one another. A **long straddle** is created by purchasing a call and a put with the same strike price and expiration. A **short straddle** is created by selling a call and a put with the same strike price and expiration. Straddles work much like butterfly and calendar spreads, albeit the losses can be unlimited for short straddles. Long straddles can be appropriate when an investor expects significant movement in the stock price.



Example: Profits for a Long Straddle

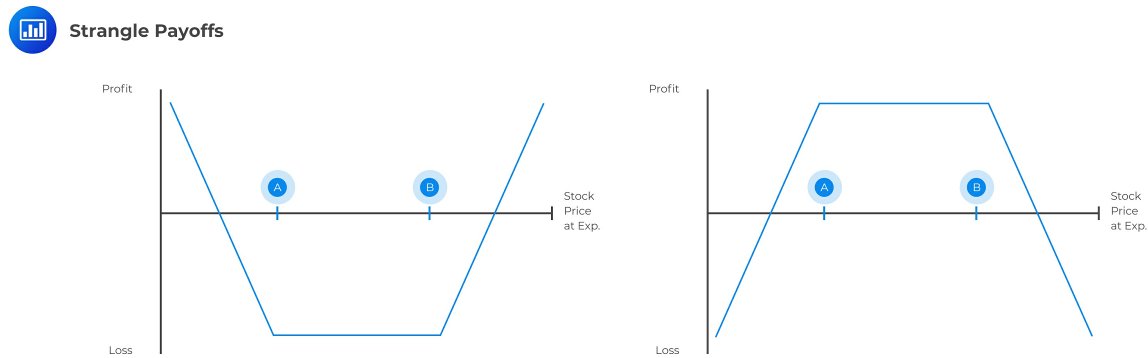
Assume that the price of a six-month European call option is $4.20, and that of a six-month European put option is $3.12. For there to be a profit using a long straddle strategy, the underlying asset price should move by at least $7.32 ($4.20+$3.12).

This implies that, for a strike price of $60, the trader will incur losses if the price of the asset at maturity is between $52.68 and $67.32.

The converse is also true. For a **short straddle** to be profitable, the stock price at expiration would need to be between $52.68 and $67.32.

2. **Strangle:**

Similar to the straddle, a long strangle consists of **a long call** and **a long put option** on the same underlying asset and with the same expiration date. In a strangle, however, the two options have **different exercise prices**. For there to be a profit in a strangle, the asset prices should move further as compared to a straddle. However, a strangle is less costly as compared to a straddle.

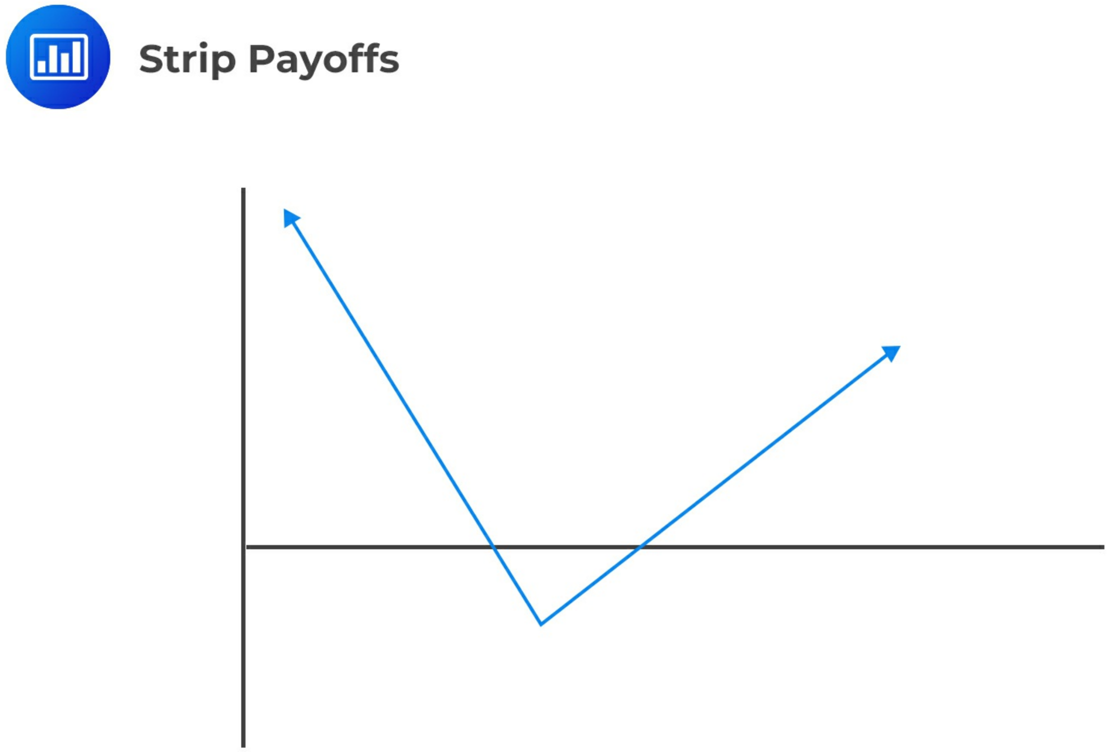


Profits for a Long Strangle

Suppose that the strike price used for the put option is $60 and that used for the call option is $70. The cost of setting up the strangle might be buying a put for $4.20 (strike price of $60) and buying a call for $3.12 (strike price of $70), for a total of $7.32. Profits will be realized if the asset price at maturity is either greater than $77.32 (70+7.32) or less than $52.68 (60-7.32).

3. **Strips:**

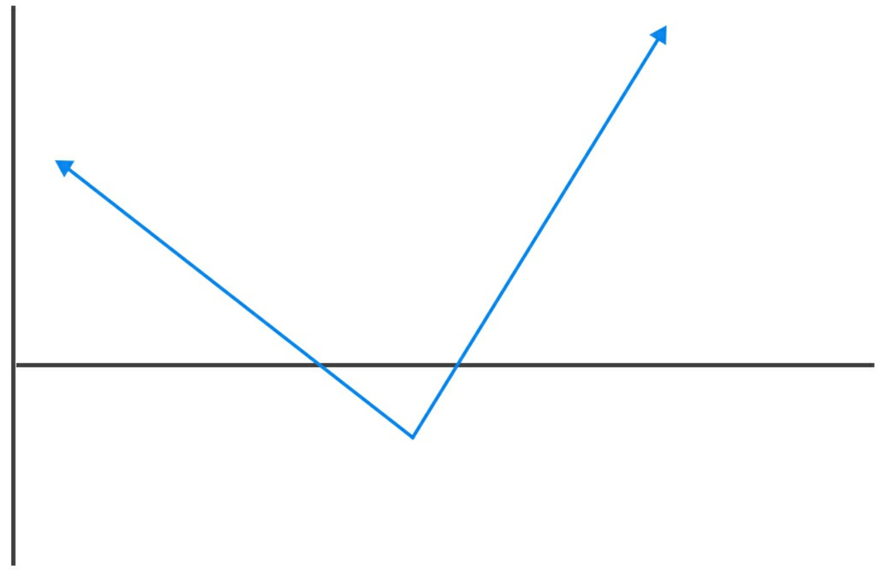
A strip is literally a **long straddle,** but the only difference is that a strip involves the purchase of two puts and one call(instead of one each as in straddle) with the same strike price and expiration. An investor enters into a long strip position when he expects a large move in a stock and considers a decrease in the stock price more likely than an increase.



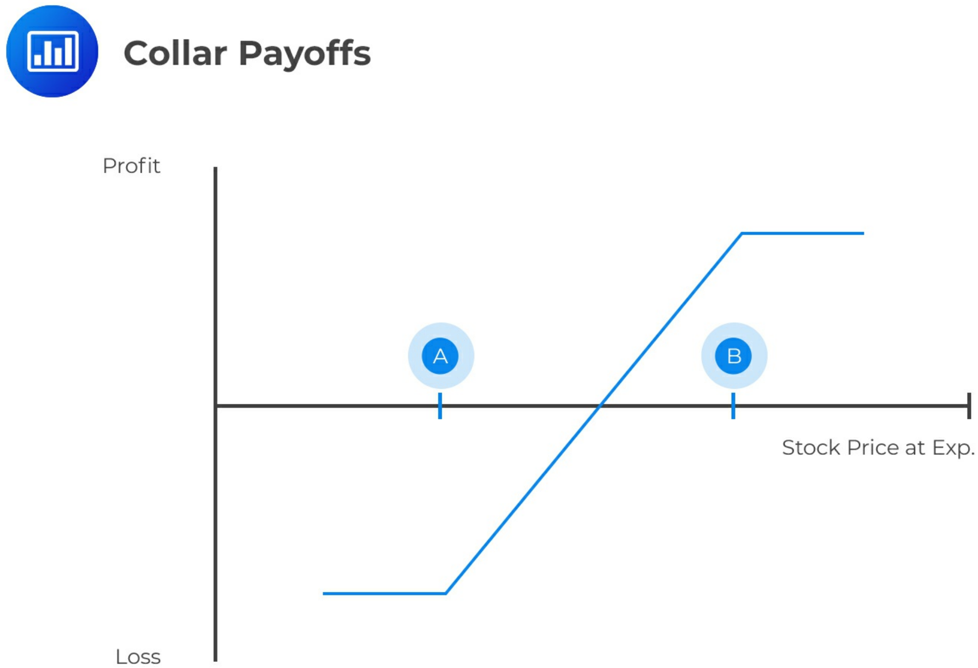
4. **Straps:**

A long strap is a mirror image of a strip strategy. It consists of a **long position in two calls** and **one put** with the same exercise price and expiration date. An investor enters into a long strap position when he expects large moves in a stock but considers an increase in the stock price more likely than a decrease.

Strap Payoffs



5. **Collar:** A collar is a combination of a protective put and a covered call.



Interest Rate Caps and Floors

An **interest rate cap** is a type of interest rate derivative in which the buyer receives payments at the end of each period in which the interest rate exceeds the agreed strike price. The strike rate is also called the cap rate. For example, the buyer could receive payments when the rate exceeds LIBOR + 200bps.

On the other hand, an interest rate floor is a derivative contract in which the buyer receives payments at the end of periods in which the interest rate is below the agreed strike price. For example, the buyer could be entitled to a payment whenever the rate is below LIBOR + 50 bps.

Interest rate caps and floors **provide protection** against fluctuating interest rates.

Practice Questions

Question 1

Which of the following options trading strategies does **not** expose the party to unlimited losses?

A. Naked call

B. Covered call

C. Short strangle

D. Short straddle

The correct answer is **B**.

Options strategies that expose the party to unlimited losses include:

- Naked call
- Short straddle
- Short strangle
- etc.

Question 2

A trader wants to create a long strangle trading strategy with a strike prices of $50 for the call option and $40 for the put option. The prices of four-month European call and put options are $3.12 and $2.87, respectively. At maturity, the stock price has to be greater than which of the following to realize profits?

A. $50

B. $34.01

C. \$53.12

D. \$55.99

The correct answer is **D**.

The question is testing on the use of the long strangle trading strategy, a trading strategy that involves buying a call option with the higher strike price, and buying a put option with the lower strike price.

The strike price for the call (shown as B on the graph) = \$50

The strike price for the put (shown as A on the graph) = \$40

Cost of setting up the long strangle = \$3.12 + \$2.87 = \$5.99

Upper price bound of the asset price at maturity = \$50 + \$5.99 = \$55.99

Lower price bound of the asset price at maturity = \$40 - \$5.99 = \$34.01

Reading 39: Exotic Options

After completing this reading, you should be able to:

- Define and contrast exotic derivatives and plain vanilla derivatives.
- Describe some of the factors that drive the development of exotic products.
- Explain how any derivative can be converted into a zero-cost product.
- Describe how standard American options can be transformed into nonstandard American options.
- Identify and describe the characteristics and payoff structure of the following exotic options: packages, zero-cost products, nonstandard American options, gap, forward start, compound, chooser, cliquet, barrier, binary, lookback, Asian, asset exchange, and basket options.
- Describe and contrast volatility and variance swaps.
- Explain the basic premise of static option replication and how it can be applied to hedging exotic options.

Exotic Derivatives vs. Plain Vanilla Derivatives

Plain vanilla derivatives represent the most basic version of financial derivatives, including futures contracts, forwards, swaps, and over-the-counter (OTC) instruments used in fairly liquid markets. They have a simple expiration date, exercise price and have no additional features. On the other hand, exotic **derivatives** alter the traditional characteristics to create a complex financial instrument that's tailored to meet the specifications of a particular counterparty.

In a plain vanilla derivative, most details are precisely outlined and straightforward. Such details include the initial cost, current market value, expiration date, amounts to be paid, and the cost of the existing position. For exotic derivatives, most of these issues are negotiable.

Some of the reasons behind the development of exotic derivatives include the need to:

- Create a customized hedge that reflects the composition of an entity's underlying assets
- Address tax and regulatory concerns
- Develop products that reflect the direction of future market prices

Conversion of Derivatives into a Zero-cost Product

When two or more derivatives with contrasting features are combined, a **package** is formed. Common packages include a bull, bear, calendar spread, or even a straddle, as discussed in the previous chapter. Through these packages, a trader can create a **zero-cost product**.

Take a collar, for example. The trader combines a long position in a put with a lower strike price and a short position in a call with a higher strike price. If the premium received after selling the call offsets the premium paid for the put, the overall cost of the combined position is reduced to zero.

The option premium for a zero-cost product is not paid up-front. Zero cost products have been customized in such a way that the option premium is payable at maturity as $x(1+r)^t$ where x is the premium that would have been paid now, t is the time to maturity, and r is the interest rate. In this arrangement, the future value of the option premium, $x(1+r)^t$, is exchanged for the option payoff at option maturity.

Transforming a Standard American Option into a Nonstandard American Option

One of the most prominent characteristics of standard American options is the possibility to exercise them **on or before** the expiration date. However, there are certain things that could be done that effectively transform a standard option contract into a non-standard one. These include:

- **Restricting early exercise to only a few specified dates**

For example, a six-month American call could be exercisable **only** on the last day of each month. Such a restriction creates what's called a Bermudan option.

- **Imposing a lock-out period during which the option cannot be exercised**

  For example, a 3-month lockout period could be imposed on a six-month call. That means the holder is not allowed to exercise the option during the first three months of the contract.

- **Having multiple strike prices in different phases of a contract**

  For example, a three-year call could be characterized by strike prices of $30 in the first year, $35 in the second year, and $40 in the final year.

Exotic Options

Gap Call Options

A gap is a European put or call option that option has a strike price, K_1, and a trigger price, K_2. The trigger price determines whether or not the option will have a nonzero payoff. The strike price determines the actual amount of the payoff. The payoff will always be nonzero (positive or negative) for a gap call option as long as the final stock price exceeds the trigger price. For a gap put option, the payoff will always be nonzero as long as the final stock price is less than the trigger price. If $K_1 = K_2$, the gap option payoff will be the same as that of an ordinary option.

When $K_2 > K_1$,

$$\text{Gap call option payoff} = \begin{cases} S_T - K_1 & \text{if} \quad S_T > K_2 \\ 0 & \text{if} \quad S_T \leq K_2 \end{cases}$$

Where:

K_1=strike price

K_2=trigger price

Example 1: Gap Call Option

Let's say $K_1 = 100$ and $K_2 = 105$. This would mean the trigger price exceeds the strike price.

At expiration, we'll have the following payoffs:

Stock Price	96	100	104	105	112
Payoff	0	0	0	0	12

If the trigger price is less than the strike price for a gap call option, negative payoffs are possible.

Example 2: Gap Call Option

Let's say $K_1 = 108$ and $K_2 = 100$. This would mean the trigger price is less than the strike price.

At expiration, we'll have the following payoffs:

Stock Price	96	100	106	108	112
Payoff	0	0	−2	0	4

We can see that between stock prices of 100 and 108 at expiration, the payoff to the call option holder is negative.

Gap Put Options

Traders can also buy and sell gap put options:

$$\text{Gap put option payoff} = \begin{cases} K_1 - S_T & \text{if} \quad S_T < K_2 \\ 0 & \text{if} \quad S_T \geq K_2 \end{cases}$$

If the trigger price is greater than the strike price for a gap put option, negative payoffs could occur.

Cliquet Options

A cliquet option, also know as a ratchet option, comprises a series of options with a forward start date and we have some rules for determining the strike price. For example, a two-year put cliquet option consists of two put options – a one-year put option effective now and a one-year put option that will start one year from today.

Now consider a three-year annuity payment arrangement. Payment will occur at year n (one year option), year n+1 (one year option starting in one year), and at year n+2 (one year option starting in two years).

Forward Start Options

As the words suggest, a forward start option kicks off at some point in the future. For example, today, a trader may purchase a six-month put that will only come into effect three months from today. Forward start in-the-money options are usually used as incentives to boost employee productivity and encourage employee loyalty.It is usually assumed that the option will be at the money at the time the option starts.

Compound Options

A compound option is simply an option on an option, i.e., an option for which the underlying is another option. Thus, a compound option usually has two strike prices and two maturity dates. A compound option can take one of four different forms:

- A call on a call (CoC) gives the investor the right to buy a call option at a set price for a set period of time.
- A call on a put (CoP)gives the investor the right to buy a put option at a set price for a set period of time.
- A put on a call (PoC) gives the investor the right to sell a call option at a set price for a set period of time.
- A put on a put (PoP) gives the investor the right to sell a put option at a set price for a set period of time.

Chooser Options

In a chooser option, the holder is allowed to decide whether it is a call or a put prior to the expiration date. The choice between the two depends in large part on the value of each. Chooser options can be viewed as packages of call options and put options with different strike prices and times to maturity.

Barrier Options

A barrier option is an option whose existence depends upon the underlying asset's price reaching a predetermined barrier level. It can be either:

- A knock-out, implying it expires worthless if the underlying exceeds a certain specified price, effectively limiting profits for the holder but limiting losses for the writer.
- A knock-in, implying it has no value until the underlying reaches a certain specified price.

Binary Options

In a binary option, the payoff is either a fixed monetary amount or nothing at all. Binary options are of two types:

- **Cash-or-nothing option,** which pays a fixed amount of cash if the option expires in-the-money
- **asset-or-nothing option,** which pays an amount equivalent to the value of the stock when the contract is initiated if the option expires in the money.

Suppose an asset-or-nothing binary option has a payoff of $40,000 for an asset price above $10. The payoff will be $0 if the asset price at maturity is $9.99 and $40,000 if the asset price is $10.01.

Lookback Options

A lookback option allows the holder to exercise an option at the most beneficial price of the underlying asset over the life of the option.

Lookback options have two main categories, that is, floating lookback options and fixed lookback options.

Asian Options

In an Asian option, the payoff depends on the average price of the underlying asset over a period of time as opposed to standard options, where the price of the underlying determines the payoff at a specific point in time.

Exotic Options Involving More than One Asset

Asset Exchange Options

As the name suggests, asset exchange options provide room for investors to be able to exchange their assets for another asset. For example, an investor based in the US may exchange his US dollars for Canadian dollars.

Basket Options

A basket option gives the right but not the obligation to buy or sell a basket of securities. The components of the basket could be bonds, stocks, currencies, e.t.c., and maybe specified in advance.

Options Dependent on Volatility

Volatility and Variance Swaps

In a **volatility swap**, volatility is exchanged based on a notional principal. Similarly, a **variance swap** involves the exchange of variance – the square of volatility – based on a notional principal. Volatility and variance swaps do not bet on the price of the underlying.

Variance swaps can be replicated using a collection of puts and calls. They are easier to price compared to volatility swaps.

Hedging Exotic Options

Hedging of exotic options can be done by creating a delta neutral position and rebalancing frequently to maintain delta neutrality. However, some exotic options, such as barrier options, are relatively difficult to hedge. To hedge a barrier option, the portfolio that replicates its boundary conditions must be shorted and unwound when any part of the boundary is reached. The advantage of static options replication is that it does not require frequent rebalancing.

Question

A cash-or-nothing call option has a payout profile equivalent to zero or:

A cash-or-nothing call option has a payout profile equivalent to zero or:

A. The underlying asset price if the value of the asset ends below the strike price.

B. The underlying asset price if the value of the asset ends above the strike price

C. A set amount if the value of the underlying asset ends below the strike price

D. A set amount if the value of the underlying asset ends above the strike price.

The correct answer is **D**.

A cash-or-nothing call option pays a fixed amount as long as the value of the underlying asset is above the strike price at expiration. It differs from a standard call since the payoff does not increase as the underlying's market price soars above the strike price.

Reading 40: Properties of Interest Rates

After completing this reading, you should be able to:

- Describe the various categories of interest rates (Treasury rates, LIBOR, Secured Overnight Financing Rate (SOFR) and repo rates, Swaps, and explain what is meant by the "risk-free" rate).
- Calculate the value of an investment using different compounding frequencies.
- Convert interest rates based on different compounding frequencies.
- Calculate the theoretical price of a bond using spot rates.
- Calculate the Macaulay duration, modified duration, and dollar duration of a bond.
- Evaluate the limitations of duration and explain how convexity addresses some of them.
- Calculate the change in a bond's price given its duration, its convexity, and a change in interest rates.
- Derive forward interest rates from a set of spot rates.
- Derive the value of the cash flows from a forward rate agreement (FRA).
- Calculate zero-coupon rates using the bootstrap method.
- Compare and contrast the major theories of the term structure of interest rates.

What is an Interest Rate?

An interest rate is a return earned by a lender on funds given out to a borrower.

Interest rates are expressed in basis points (1 basis point = 0.01%); which also implies that an interest rate of 2% is equal to 200 basis points.

Categories of Interest Rates

Treasury Rates

Treasury rates are the rates earned by investors in instruments used by a government to borrow in its own currency. These include Treasury bonds and Treasury bills. Treasury rates are considered **"risk-free"** because they have zero risk exposure. That has much to do with the ability of the government to use a range of tools at its disposal to avoid default, including printing of cash and increased taxes. T-bill and T-bond rates are used as the benchmark for nominal risk-free rates.

LIBOR

LIBOR, the London Interbank Offered Rate, is the rate at which the world's leading banks lend to each other for the short term. It's the most widely used benchmark for short-term lending.

Libor rates are compiled from the estimates of unsecured borrowing rates of 16 highly rated global banks. The rates are estimated daily for five different currencies. Libor has seven borrowing periods ranging from one day to one year.

Repo Rates

Repo rates are the implied rates on repurchase (repo) agreements. A repo agreement is an agreement between two parties - the seller and the buyer – where the seller agrees to sell a security to the buyer with the understanding that they (seller) will buy it back later at a higher price. The most common repo transactions are carried out overnight. The credit risk in a repo agreement depends on the term of the agreement as well as the creditworthiness of the seller.

Overnight Interbank Borrowing

The overnight interbank borrowing rate is the rate at which banks borrow from each other. Banks with excess reserves will lend to banks that have shortages. Cash reserves are kept at the central bank and are dependent on the liabilities of a bank. This type of borrowing occurs overnight. The overnight interbank rate, known as the fed funds rate in the US, is the main mechanism through which US monetary policy is channeled.

Swaps

Swaps are used to create long-term interest rates from short-term interest rates.

Assume two traders X and Y. A swap occurs when trader X agrees to pay trader Y a **pre-determined fixed interest** rate of 5% per year compounded quarterly, on a principal of say, $10,000, for five years. In return, trader Y agrees to pay trader X interest at the **three-month Libor rate** on the $10,000 principal for five years. Except for the first Libor rate, the Libor rates are unknown when the swap is agreed upon. In this example, interests will be exchanged every three months.

Risk-Free Rates

Risk-free rates are used in valuing derivatives. Overnight interbank rates determine the risk-free rates using overnight indexed swaps. Even though treasury rates are risk-free rates, they are not preferred since they are artificially low.

Compounding Frequencies

Given an initial investment of A that earns an annual rate R, compounded m times a year for a total of n years, then we can compute the future value, FV, as follows:

$$FV = A\left(1+\frac{R}{m}\right)^{m\times n}$$

In the presence of continuous compounding, then:

$$FV = Ae^{R\times n}$$

Example: Future Value

Suppose USD 1,000 is invested for five years at an interest rate of 4% compounded quarterly per annum.

The future value of the fund after 5 years is given by:

$$FV = A(1 + \frac{R}{m})^{m \times n}$$

$$= 1,000(1 + \frac{0.04}{4})^{20} = \text{USD } 1,220.19$$

Exam tip: For any rate R, the future value with continuous compounding will always be greater than the future value with discrete compounding.

Let R_c be the continuously compounded rate that equates the future value under discrete compounding to the future value under continuous compounding:

$$A(1 + \frac{R}{m})^{m \times n} = Ae^{R_c \times n}$$

$$R_c = m \times \ln(1 + \frac{R}{m})$$

Alternatively, given R_c,

$$R = m\left(e^{\frac{R_c}{m}} - 1\right)$$

Illustration: Different Compounding Frequencies

The table below shows the different future values when $50,000 is compounded at different compounding frequencies at the rate of 5% for three years.

Compounding Frequency	Formula	Future Value
Annual	$50,000(1 + 0.05)^{1 \times 3}$	57881.25
Semi-Annual	$50,000(1 + 0.05/2)^{2 \times 3}$	57984.67
Quarterly	$50,000(1 + 0.05/4)^{4 \times 3}$	58037.73
Monthly	$50,000(1 + 0.05/12)^{12 \times 3}$	58073.61
Daily	$50,000(1 + 0.05/365)^{365 \times 3}$	58091.12
Continuous	$50,000 \times e^{0.05 \times 3}$	58091.71

From the table above, we can see that continuous compounding gives a higher value than discrete compounding. We can also see that the higher the number of compounding frequencies for discrete compounding, the higher the amount

The Usual Conventions

Sometimes, compounding frequencies are determined based on the frequency of payments. Interests of money market instruments – instruments with maturities of less than one year – are expressed as quarterly if payment occurs after three months. If payments occur after six months, they are expressed as semiannual, and as annual if they occur yearly.

Discounting

Discounting is the process of determining the present value of a known future amount.

The general discounting formula is:

$$PV = A(1 + \frac{R}{m})^{-mn}$$

Where:

- PV is the present value of an amount A,
- R is the interest rate compounded with a frequency of m times a year,
- m is the frequency by which R is compounded, and
- n is the number of years.

Also note that $(1 + \frac{R}{m})^{-mn}$ is known as the discount factor.

Example: Present Value

Suppose an amount X is invested for five years at an interest rate of 4% compounded quarterly per annum. If the fund grows to USD 1,500 after 5 years, what is the value of X?

Solution

$$PV = A(1 + \frac{R}{m})^{-mn} = 1,500(1 + \frac{0.04}{4})^{-20} = \$1,229.32$$

The Theoretical Price of a Bond Using Spot Rates

The theoretical price of a bond is given by the present value of all of the bond's cash flows. Assuming each cash flow is associated with a spot discount factor z_j, then:

$$P = [\frac{c}{2} \times \sum_{j=1}^{N} e^{-\frac{z_j}{2} \times j}] + FV(e^{-\frac{z_N}{2} \times j})$$

Where:

P = bond's price

z_j=bond equivalent spot rate corresponding to $\frac{j}{2}$ years on a continuously compounded basis

FV = face value of the bond

N = number of semiannual payment periods

The yield of a bond is the single discount rate that equates the bond's present value to its market price. A bond's par yield is the discount rate that equates the bond's price to its par value.

Example

Consider a $1,000 face value, two-year bond, that pays a coupon rate of 10% semi-annually, and that a spot rate of 11% compounded semi-annually, applies to each cash flow.

The price of the bond, in this case, is given by,

$$P = 50e^{-0.055 \times 1} + 50e^{-0.055 \times 2} + 50e^{-0.055 \times 3} + 50e^{-0.055 \times 4} + 1,000e^{-0.055 \times 4} = \text{USD } 977.15$$

Duration Measures

Duration

Duration, sometimes referred to as **Macaulay duration**, is an approximate measure of a bond's price sensitivity to changes in interest rates.

Duration is expressed in years. For a zero-coupon bond, its duration is simply its time to maturity. For a coupon bond, its duration is shorter than maturity because the cash flows have different weights.

$$\text{Macaulay Duration} = \frac{\sum_{t=1}^{n} PV(C_t)T}{\text{Market Price of Bond}}$$

Where $PV(C_t)$ is the present value of coupon payments at time t, and T is the time to maturity

The formula for Macaulay duration, given continuous compounding, is:

$$\text{Macaulay Duration} = \Sigma_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{P}\right]$$

Where:

t_i=time in years until cash flow c_i is received

y=the continuously compounded yield (discount rate) based on a bond price P

Example: Macaulay Duration

Consider a bond whose current price is USD 120 with a cash flow in one year providing a present value of USD 20 and a cash flow in two years providing a present value of USD 100, the Macaulay duration, in this case, is given by:

$$\frac{20}{120} \times 1 + \frac{100}{120} \times 2 = 1.8333$$

Modified Duration

Modified duration is used in the **absence** of continuous compounding of the yield. If the yield, y, is expressed as a rate compounded n times a year, then:

$$\text{Modified duration} = \left[\frac{\text{Macaulay duration}}{(1+\frac{y}{n})}\right]$$

Note: By yield, we imply the yield to maturity.

Modified duration is based on the concept that bond prices have an inverse relationship with interest rates. When interest rates rise, bond prices fall; when interest rates fall, bond prices rise. Modified duration tells how much a bond's price will rise or fall for a 1% shift in yield to maturity.

Let's say a bond has a modified duration of 5%. What does that imply? Its price will rise by about 5% if its yield drops by 1% (100 basis points), and its price will fall by about 5% if its yield rises by 1%.

Example: Modified Duration

Consider the bond in the previous example, and suppose that the yield with semi-annual compounding is 5 %.

The modified duration is thus given by:

$$\frac{1.8333}{1+\frac{0.05}{2}} = 1.7886$$

Exam tip: Unless given, you must calculate the Macaulay duration to determine the modified duration.

Dollar Duration

The dollar duration, DD, of a bond is a product of its modified duration and its market price. If we use D^* to denote the modified duration, and P_0 to denote the bond's market price, then:

$$DD = D^* \times P_0$$

For illustration, consider the bond in the example above, the dollar duration is given by,

$$120 \times 1.7886 = 214.63$$

Limitation of Duration

Note that the approximate duration relationship is given by:

$$\Delta P = -DP\,\Delta y \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

Which is equivalent to,

$$\frac{\Delta P}{P} = -D\Delta y$$

Where,

P = price of the bond

D = bond's duration

ΔP = change in bond's price

Δy = change in bond's yield

Duration provides a good approximation of the effect of a small parallel shift in the interest rate term structure. However, the above duration relationship cannot provide a good approximation in case the change in the bond yield arises from a non-parallel shift in the interest rate term structure or if the change being considered is large.

This problem is addressed by convexity.

Convexity

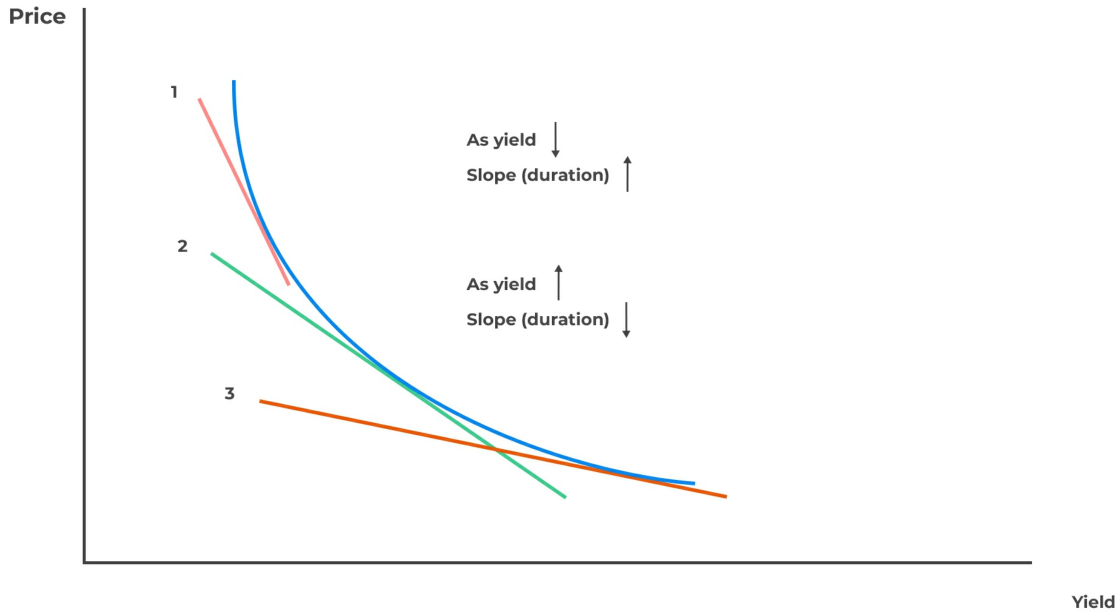
Convexity is a measure of the curvature in the relationship between bond prices and bond yields. It demonstrates how the duration of a bond changes as the interest rate changes. Convexity estimates the amount of market risk affecting a bond or portfolio.

If we let C be the convexity, then equation 1 above can be refined so that we have:

$$\Delta P = -DP\,\Delta y + \frac{1}{2}CP\,(\Delta y)^2$$

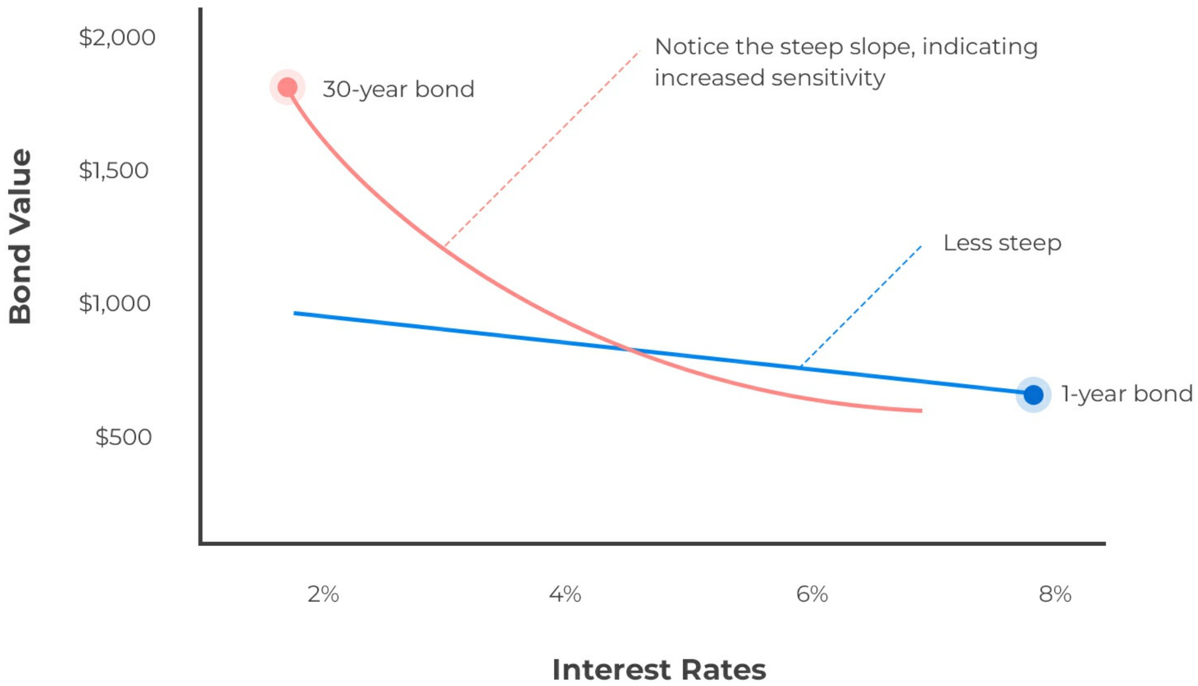
With this equation, relatively large parallel shifts can now be considered.

Convexity - Illustration (1)

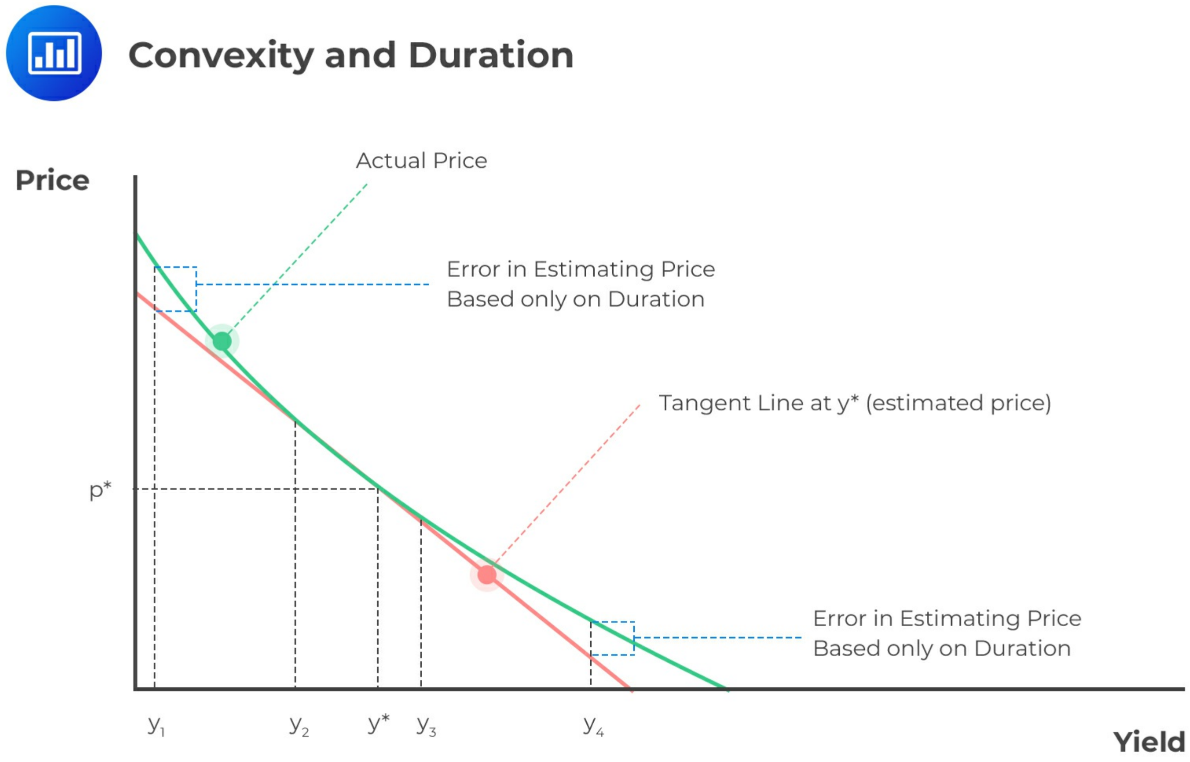


If bond X has a higher convexity than bond Y, what does this imply? All else being equal, bond X will always have a higher market price than bond Y as interest rates rise and fall.

Convexity - Illustration (2)



Duration assumes that interest rates and bond prices have a linear relationship. It's, therefore, a fairly good measure of exactly how bond prices are affected by small changes in interest rates. However, the relationship between bond prices and interest rates is actually non-linear, i.e., convex. This makes convexity a better measure of risk, especially in the presence of large and frequent fluctuations in interest rates.



For the purpose of the percentage change in price triggered by convexity, i.e., the price change not explained by duration, we must calculate the **convexity effect**.

$$\text{Convexity effect} = \frac{1}{2} \times \text{Convexity} \times \Delta y^2$$

Exam tip: Convexity is always positive for regular coupon-paying bonds

Calculating the Change in a Bond's Price Given Its Duration, Its Convexity, and a Change in Interest Rates

Combining duration and convexity results in a far more accurate estimate of the change in the price of a bond given a change in yield.

$$\begin{aligned}\text{Change in bond's price} &= \text{Duration effect} + \text{Convexity effect}\\ &= [-\text{Duration} \times \text{Price} \times \text{Change in yield}]\\ &+ [\frac{1}{2} \times \text{Convexity} \times \text{Price} \times (\text{Change in yield})^2]\end{aligned}$$

Example: Change in a Bond's Price

A portfolio manager has a bond position worth 100 million. The position has a modified duration of 3 years and a convexity of 60. Assuming that the term structure is flat, by how much does the value of the position change if interest rates increase by 25 basis points?

Solution

$$\begin{aligned}\text{Change in bond's price} &= \text{Duration effect} + \text{Convexity effect}\\ &= [-\text{Duration} \times \text{Price} \times \text{Change in yield}]\\ &+ [\frac{1}{2} \times \text{Convexity} \times \text{Price} \times (\text{Change in yield})^2]\\ &= -3 \times 100,000,000 \times 0.0025\\ &+ [0.5 \times 60 \times 100,000,000 \times 0.0025^2]\\ &= -731,250\end{aligned}$$

With every increase in interest rates of 25 basis points, the bond's price will decrease by $731,250

Deriving Forward Rates from a Set of Spot Rates

y_n, the n-year spot rate, is a measure of the average interest rate over the period from now until n years' time.

The forward rate, $f_{t,t+r}$, is a measure of the average interest rate between times t and $t+r$. It's the interest rate agreed today ($t = 0$) on an investment made at time $t > 0$ for a period of r years.

The one-year forward rate, $f_{t,t+1}$, is therefore the rate of interest from time t to time $t+1$. It can be expressed in terms of spot rates as follows:

$$1 + f_{t,t+1} = \frac{(1 + y_{t+1})^{t+1}}{(1 + y_t)^t}$$

If we want to find the forward rate for a period greater than 1-year, e.g. 2-year forward rate, then we will use the formula:

$$(1+f_{t,n})^{n-t} = \frac{(1+y_n)^n}{(1+y_t)^t}$$

Where y_n, the n-year spot rate and y_t, the t-year spot rate. For example, consider a case where we want to calculate the forward rate between year 2 and year 4, that is, $f_{2,4}$. In such a case, we will use the formula,

$$f_{2,4}^{4-2} = \frac{(1+y_4)^4}{(1+y_2)^2}$$

The above results for 1-year forward rate can be generalised as follows,

Step 1: Use the formula:

$$1 + F = \frac{V_2}{V_1}$$

Where V_1 is the value to which a dollar grows by time T_1 and V_2 is the value to which a dollar grows by T_2.

Step 2: Calculate the interest rate that equates the value of one dollar at time T_1 to the value of one dollar at time T_2.

Example: Semiannual Forward Rates

Assume that the six-month spot rate is 0.05, while the one-year spot rate is 0.058. Calculate the forward rate for the period between six months and one year.

To get the forward rate, we take $\frac{V_2}{V_1}$:

$$V1 = (1 + \frac{0.05}{2}) = 1.025$$
$$V2 = (1 + \frac{0.058}{2})^2 = 1.058841$$

$\frac{V_2}{V_1}$ is therefore $\frac{1.058841}{1.025} = 1.033$.

The interest rate at which one dollar has grown between the two periods is $1.033 - 1 = 0.033$.

If the forward rate is expressed with semiannual compounding:

$$\begin{aligned} F/2 &= 0.033 \\ F &= 0.066 \end{aligned}$$

Therefore the forward rate is 6.6% expressed with semiannual compounding

Annual Forward Rates

Let's say you have the following spot rates table:

Year	1	2	3	4
Spot Rate	1.2%	1.5%	1.9%	2.4%

The one-year forward rate between years 1 and 2 is:

$$\begin{aligned} 1 + f_{1,2} &= \frac{V_2}{V_1} = \frac{(1+1.5\%)^2}{(1+1.2\%)^1} = 1.018 \\ f_{1,2} &= 1.8\% \end{aligned}$$

In the same vein, we can calculate the one-year forward rate between years 3 and 4:

$$\begin{aligned} 1 + f_{3,4} &= \frac{V_4}{V_3} = \frac{(1+2.4\%)^4}{(1+1.9\%)^3} = 1.039 \\ f_{3,4} &= 3.9\% \end{aligned}$$

Continuous Compounding

Take the example shown above. If the interest rate is expressed with continuous compounding:

$$\begin{aligned} V_1 &= e^{R_1 T_1} \\ V_2 &= e^{R_2 T_2} \end{aligned}$$

The forward rate, F, with continuous compounding between time T_2 and time T_1 is thus:

$$F = e^{F(T_2 - T_1)} = \frac{V_2}{V_1} = e^{R_2 T_2 - R_1 T_1}$$

Rearranging the above gives the forward rate F as:

$$F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

Example: Continuous Compounding

Assume the data in the following spot rates table is continuously compounded:

Year	1	2	3	4
Spot rate	1.4%	1.8%	2.1%	2.6%

Calculate the continuously compounded forward rate between years 1 and 2.

$$\begin{aligned} F &= \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1} \\ &= \frac{0.018(2) - 0.014(1)}{2 - 1} \\ &= 2.2\% \end{aligned}$$

As you can see, when dealing with continuous compounding, the spot rate is simply multiplied by the time period instead of using $(1 + y_t)^t$.

Forward Rate Agreements

A forward rate agreement is an agreement between two parties to lock in an interest rate for a specified period of time starting on a future settlement date, based on a notional amount. The buyer of a forward rate agreement enters into the contract to protect himself from any future increase in interest rates. The seller, on the other hand, enters into the contract to protect himself from any future decline in interest rates. If Firm A and Firm B enter into a forward rate agreement by agreeing on an interest rate R_K, the cash flows will be:

$$\begin{aligned} &\text{Firm A: } L(R_K - R_M)(T_2 - T_1) \\ &\text{Firm B: } L(R_M - R_K)(T_2 - T_1) \end{aligned}$$

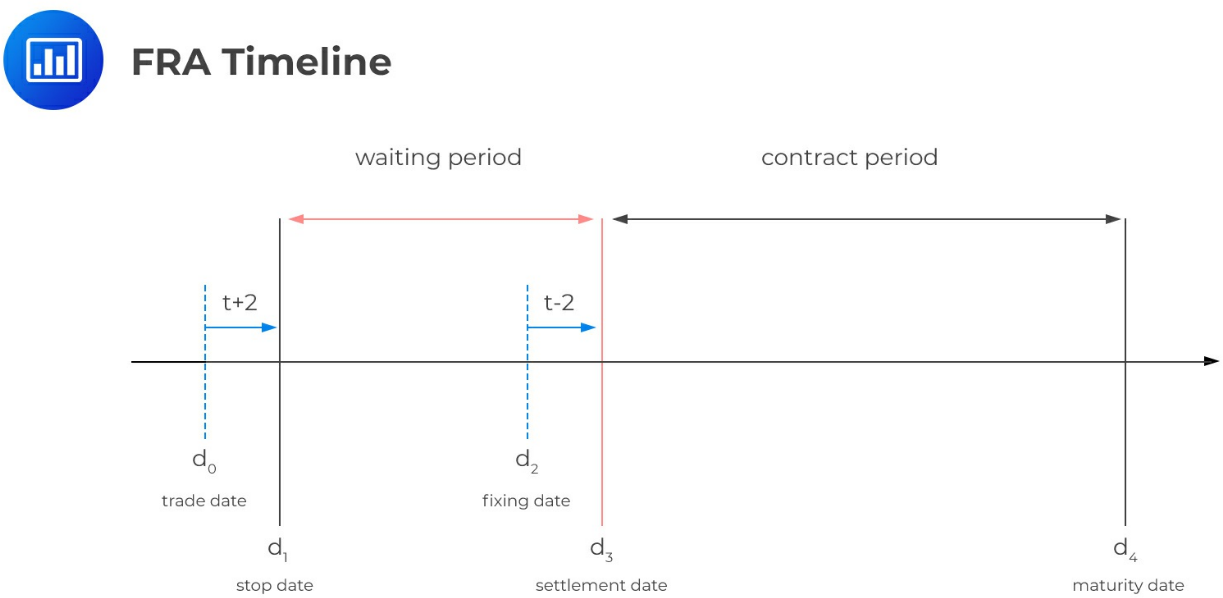
Where:

L = principal amount

R_K = interest rate agreed to in the FRA

R_M = actual interest rate observed between T1 and T2

FRAs are cash-settled on the settlement date – the start date of the notional loan or deposit. The interest rate differential between the market rate and the FRA contract rate determines the exposure to each party. It's important to note that as the principal is a notional amount, there are no principal cash flows.



As time passed, the agreed fixed rate R_K remains the same but the forward LIBOR rate R_F is likely to move in either direction.

Therefore, the value of the forward contract to both parties will be:

$$\text{Firm A: } V_{FRA} = L(R_K - R_F)(T_2 - T_1)e^{-R_2T_2}$$
$$\text{Firm B: } = L(R_F - R_K)(T_2 - T_1)e^{-R_2T_2}$$

Where:

V_{FRA} = value of the forward contract

R_2 = the continuously compounded risk-free rate for a maturity T_2

Example: Forward-Rate Agreements

A German bank and a French bank entered into a semiannual forward rate agreement contract where the German bank will pay a fixed rate of 4.2% and receive the floating rate on the principal of €700 million. The forward rate between 0.5 years and 1 year is 5.1%. If the risk-free rate at the 1-year mark is 6%, then what is the value of the FRA contract between the two banks?

$$V_{FRA} = L(R_K - R_F)(T_2 - T_1)e^{-R_2 T_2}$$
$$V_{FRA} = €700\text{million}(5.1\% - 4.2\%)(0.5\text{years})e^{-0.06*1\text{year}}$$

Zero Rates

A zero-coupon interest rate is also known as zero-rate or spot rate. This is the interest rate that is applied when an investor receives the total return, i.e, interest, and principal at the maturity of the bond.

Bonds with maturities of less than one year provide zero rates directly since they provide total return, (interest and principal) at maturity. Bonds with maturities of more than one year, provide regular coupon payments prior to their maturity. Therefore, zero rates for such bonds have to be calculated.

Consider the following example for illustration,

Example: Zero Rates

Suppose the zero-coupon interest rates (semi-annually compounded) for maturities of 0.5, 1.0, and 1.5 years are 2.0%, 2.5% and 3.0%, respectively. Now consider a USD 1,000 face value, two-year bond that currently trades at 1020, and pays semi-annual coupons of 8%. If the two-year zero-coupon interest rate is R, then, R can be solved using the following equation:

$$\frac{40}{\left(1+\frac{0.02}{2}\right)^{1}}+\frac{40}{\left(1+\frac{0.025}{2}\right)^{2}}+\frac{40}{\left(1+\frac{0.03}{2}\right)^{3}}+\frac{1040}{\left(1+\frac{R}{2}\right)^{4}}=1,020$$

$$\Rightarrow R = 7.18\%$$

Theories of the Term Structure

Market Segmentation Theory

This theory argues that different maturity periods on interest-bearing instruments attract different traders. Traders interested in short-term maturity instruments will influence short-term maturity rates. The same is true for traders interested in medium- and in long-term maturity instruments.

Since traders do not focus on only one part of the interest rate term structure, this theory is considered to be unrealistic.

Expectations Theory

This theory argues that the interest rate term structure reflects the future market expectations of interest rates.

The interest rate term structure will have long-maturity rates being higher than short-maturity rates (upward sloping curve) if the market expects interest rates to rise. The opposite is true if the market expects interest rates to fall.

Liquidity Preference Theory

This theory argues that if the expectations theory holds, most investors will prefer short-term investments to long-term investments. This is because of liquidity considerations – the funds invested in short-term investments will be available earlier to meet any need.

Liquidity considerations create a mismatch between borrowers and lenders as it makes borrowers want to borrow for longer periods and lenders want to lend for shorter periods. Financial intermediaries attempt to fix this mismatch, and in so doing, supply and demand are matched at both maturities.

Question

A portfolio manager has a bond position worth CAD 200 million. The position has a modified duration of six years and a convexity of 120. Assuming that the term structure is flat, by how much does the value of the position change if interest rates increase by 50 basis points?

A. CAD -5,700,000

B. CAD -5,000,000

C. CAD -6,000,000

D. CAD -54,000,000

The correct answer is **A**.

$$
\begin{aligned}
\text{Change in bond's price} &= \text{Duration effect} + \text{Convexity effect} \\
&= [-\text{Duration} \times \text{Price} \times \text{Change in yield}] \\
&+ [\frac{1}{2} \times \text{Convexity} \times \text{Price} \times (\text{Change in yield})^2] \\
&= -6 \times 200,000,000 \times 0.005 \\
&+ [0.5 \times 120 \times 200,000,000 \times 0.005^2] \\
&= -5,700,000
\end{aligned}
$$

With every increase in interest rates of 50 basis point, the bond's price will decrease by $5.7 million.

Reading 41: Corporate Bonds

After completing this reading, you should be able to:

- Understand what a bond is and describe the methods of issuing bonds.
- Describe the features of bond trading and explain the behavior of bond yield.
- Describe a bond indenture and explain the role of the corporate trustee in a bond indenture.
- Define high-yield bonds and describe types of high-yield bond issuers and some of the payment features unique to high yield bonds.
- Differentiate between credit default risk and credit spread risk.
- Describe event risk and explain what may cause it in corporate bonds.
- Describe the different classifications of bonds characterized by the issuer, maturity, interest rate, and collateral.
- Describe the mechanisms by which corporate bonds can be retired before maturity.
- Define recovery rate and default rate, differentiate between an issue default rate and a dollar default rate.
  and describe the relationship between recovery rates and seniority.
- Evaluate the expected return from a bond investment and identify the components of the bond's expected return.

A bond is a debt instrument issued by a borrower (bond issuer) to a lender (bondholder) in an attempt to raise money.

Issuance of Bonds

The issuance of bonds can take place in the following ways:

- **Private Placement**: Bonds are sold to a small number of large institutions, such as pension funds. Such bonds are not tradable; or

- **Public Issue-bonds**: Bonds are sold publicly. Investment banks will buy bonds from the bond issuer and resell them to investors at a higher cost to make profits. Bonds issued in this manner can be traded from one person to another in the OTC markets.

Advantages of a Private Placement

1. Non-public issuances are not rated. As a result, rating agencies are not needed in a private placement.
2. The cost of issuing a private placement is lower as compared to that of a public issue.
3. The registration requirements needed to issue a private placement are fewer than those needed in a public issue.

Bond Trading

The yield of a bond is the return earned on a bond, assuming that the interest and principal of the bond are paid as promised:

$$\text{Yield of a corporate bond} = \text{Risk-free return} + \text{Credit spread}$$

Where:

The risk-free return is the return that would have been earned on a similar risk-free instrument, such as a government bond.

The credit spread is the return paid to an investor to compensate him/her for the risk of default inherent in a corporate bond. The credit spread increases with an increase in the maturity period. As the maturity period increases, the possibility of the bond issuer facing financial difficulties increases. A higher credit risk results in a higher yield, so more risk and potentially more return for the investor.

The yield of a bond also features its liquidity. Liquidity is the ability to convert an asset into cash within a reasonable time period. A decrease in the bond's liquidity leads to an increase in the bond yield. As the liquidity decreases, investors will need a higher yield to compensate them for liquidity risk.

High-Yield Bonds

High-yield bonds are bonds rated below investment-grade by rating agencies - a rating below "BBB" from S&P and below "Baa" from Moody's.

Since they carry more default risk, they must pay a higher yield than investment-grade bonds. They are usually issued by startups or firms with high debt ratios. However, just because high-yield bonds are not investment-grade doesn't necessarily mean they are a no-go zone. Failure is not a certainty. As such, "junk" bonds – as they are often called – can offer excellent returns to investors. In most cases, junk bonds do not fail.

There are several types of high-yield bonds:

- **Story bonds**: issued to fund a specified venture project.
- **Fallen angels**: bonds that were once investment-grade but which have since been downgraded following negative impact events.

Bond Indenture

The bond indenture, also known as the **trust deed**, refers to the official document that outlines the terms of the contract, including the obligations of the issuer and the rights of bondholders. Being representative of a binding contract, the indenture is a well-detailed document crafted by legal experts. For this reason, it's usually in the best interests of the bondholder to seek the services of a **corporate trustee** to interpret the language therein.

A Corporate Trustee's role is to act in the best interests of investors by being an **independent supervisor** of the security. All bond issues over $5 million and sold in interstate commerce must have a corporate trustee. All trustees are required to be professionally competent with no competing interests with their clients.

Credit Default Risk vs. Credit Spread Risk

Credit default risk is the risk that the bond issuer will not make timely payments of interest and principal as obligated within the bond's indenture framework.

Credit default risk is usually evaluated using credit ratings issued by rating agencies like Moody's and Standard & Poor's. The agencies assign a symbol to the rating, e.g., AAA for bonds with the lowest credit risk and C for bonds whose default is imminent.

Credit spread risk is the risk of loss in the value of a bond arising from changes in the level of credit spreads used in the marking to market. (Credit spread refers to the difference between a bond's yield and the yield of a Treasury security with a comparable maturity.)

Macroeconomic as well as issuer-specific factors determine credit spread risk.

Some of the macroeconomic factors include:

- Level and slope of the Treasury yield curve.
- Business cycle.
- Consumer confidence.

Issuer-specific factors include:

- The issuer's future prospects and production outlook.
- The issuer's current financial position.

A measure commonly used to assess credit spread risk is spread duration – the change in the value of a bond for a 1% (100 basis points) change in credit spread, assuming the yield of the underlying treasury security is constant.

Event Risk

Event risk refers to the risk that an unexpected event will negatively impact a company's financial position. Such an unexpected event could take the form of a natural disaster, hostile takeover, restructuring, recapitalization, or even a large-scale share repurchase program.

Any of these events can drastically change a firm's capital structure and reduce the creditworthiness of outstanding bonds and their value. In a bid **to protect** bondholders from such eventualities, a firm may include a **poison put** in the indenture. A poison put gives bondholders the right to redeem a bond before maturity, at or above par value, in the event that the firm suffers a hostile takeover. The poison put may also cover the other unexpected events listed above.

Classification of Bonds

1. By Categories of Issuers

Issuer	Example
Utilities	Gas and Water
Transportation Companies	Truck Companies and Airlines
Financial Instutitutions	Banks and Insurance Companies
Industrials	Retailing and Manufacturing Companies
Internationals	Foreign Governments

2. By Maturity

A bond's maturity is the date on which the principal amount of a bond – the "par value" – is to be paid in full, including any accrued interest. A bond's maturity is set when it's issued. Generally, bonds that mature in 1-3 years are short-term; those maturing in 4-10 years are said to be medium-term. Long-term bonds mature in more than 10 years.

3. By Interest Rates

Straight-Coupon Bonds

Straight-coupon bonds pay a fixed interest rate for the entire life of the issue. In the U.S., bonds typically pay interest twice a year. In Europe, most straight-coupon bonds pay interest annually. At maturity, the amount paid consists of the interest earned in the final period plus principal.

Floating-Rate Bonds

Just like straight-coupon bonds, floating-rate bonds pay interest but based on a non-constant rate. For example, the interest payment at each payment date may be tied to the LIBOR rate on that date.

Participating Bonds

Participating bonds have a minimum interest rate but may pay more if the issuer's profits increase.

Income Bonds

Income bonds pay at most the specified interest rate but may pay less if the issuer's profits decline.

Zero-Coupon Bonds

Zero-coupon bonds do not pay any interest. At maturity, the issuer pays the par value of the bond. Bondholders earn a capital gain by purchasing the bond at a discount to the face value.

$$\text{Original-issue discount(OID)} = \text{Face value} - \text{Offering price}$$

Zero-coupon bonds have **zero reinvestment risk**. The bondholder doesn't have to contend with the issue of reinvesting cash interest payments because there aren't any. The downside, however, is that the bondholder may still be required to pay tax on the accrued interest even though no cash is actually received. In other instances, the par value received at maturity is subject to capital gain tax.

4. By Collateral

Mortgage bonds have a security, such as real property, underlying the issue. The bondholders have the first mortgage lien on the properties of the issuer. As secured bonds, the rate of interest payable may be less than that payable on unsecured bonds.

Collateral trust bonds are secured by a range of financial assets, including stocks, notes, bonds, or similarly ranked securities owned by the issuer. The issuer is usually a holding company, and the collateral consists of claims on their subsidiaries.

Equipment trust certificates (ETCs) are debt instruments that allow the borrower to take possession of an asset and put it to use while paying for it over time. The trustee purchases the asset/equipment and leases it to the borrower, who pays rent on the equipment. The rent is then passed on to the holder of the ETC.

Debentures are unsecured bonds only issued by highly rated institutions. As a result, the interest rate payable is usually higher than that in secured bonds. However, if the issuer has no outstanding secured bonds, debentures have a claim on all of the issuer's assets along with those of guarantors. If the issuer has secured debt, the debenture holder has a claim on all assets not backing the secured debt.

Subordinated debenture bonds are bonds that rank the lowest on the list of creditors in the event of a winding-up. They rank below debentures and unsecured debt. As a result, the issuer has to pay a higher interest rate.

Convertible debentures are unsecured bonds that give the holder the right to convert the bond into common stock. This right to convert is a benefit to the holder and therefore reduces the interest rate paid. However, the issuer has to contend with a dilution of their stock in the event the bondholder exercises their right to convert.

Guarantee bonds are bonds issued by one company but guaranteed by another company.

Mechanisms by Which Corporate Bonds Can Be Retired Before Maturity

A bond's indenture may allow for the **early retirement** of a bond. This means the issuer pays out cash and removes the bond from its balance sheet before the scheduled maturity date. The longer the maturity of the bond, the more time the issuer has to retire the bond issue.

Some of the reasons why an issuer might decide to retire a bond early include:

- To take advantage of lower interest rates. If the current borrowing cost is significantly lower than the rate agreed in the contract, the issuer might retire the bond and replace it with a cheaper bond;

- To get rid of restrictive terms/conditions;
- To increase shareholder value; or
- To alter the firm's capital structure.

Corporate bonds can be retired in two main ways, namely:

i. Mechanisms included in the bond's indenture; or
ii. Mechanisms not included in the bond's indenture.

i. Mechanisms Included in the Bond's Indenture

Call Provisions: Call provisions are basically call options on the bond. The provisions give the issuer the right to purchase the outstanding debt at a fixed price either in whole or in part prior to maturity. There are two types of call provisions:

- **Fixed-price call:** In a fixed-price call, the issuer can call back the bond at various points in time, but the price paid at each point is specified in the indenture. Normally, the price gradually declines as the bond's maturity nears. There may also be provisions that make it impossible to call a bond in its early years.

- **Make-whole call:** Make-whole call provisions use the prevailing market price as the call price subject to a floor price equivalent to the bond's par value. All futures cash flows are discounted based on the current yield of comparable-maturity Treasury securities plus a premium.

Sinking Fund Provision

A sinking fund provision retires a bond periodically/systematically rather than retiring the entire issue at once. The terms of the provision are clearly outlined in the indenture. For example, if a bond has a principal of **$60 million** and **20 years** to maturity, a sinking fund provision may seek to retire the bond in chunks of **$15 million** at **5-year intervals**.

Maintenance and Replacement Fund

This mechanism is used by electric utility companies that retire their bonds to maintain and repair the pledged collateral.

Redemption Through the Sale of Assets

Release-of-property and substitution-of-property clauses are found in most secured bond indentures because bondholders want the integrity of the collateral to be maintained.

ii. Mechanisms Excluded from the Bond's Indenture

Tender offers: In this method, the issuer sends a tender offer declaring its intention to buy back its debt issue. A circular is sent out to all bondholders outlining the finer details of the offer, including the price at which the issuer is willing to execute the offer.

Issuer Default Rate vs. Dollar Default Rate

There are two ways in which default can be measured: by the raw number of issuers that defaulted or the dollar amount of issues that defaulted. This leads us to two types of default rates.

The issuer default rate is the number of issuers that defaulted over a year divided by the total number of issuers at the beginning of the year. It only looks at the number of defaults that have occurred as a proportion of the total number of issues made in a year. It doesn't dig deeper to establish the dollar amount of loss arising from any of the default events.

The dollar default rate is the par value of all bonds that defaulted in a given calendar year divided by the total par value of all bonds outstanding during the year.

Recovery Rates

The recovery rate refers to the percentage amount recovered by a bondholder following a default event. The loss given default, LGD, is the amount that a bondholder stands to lose in the event of default. It's given by:

$$\text{LGD} = 1 - \text{Recovery rate}$$

For example, if the recovery rate on an issue is 60%, the loss given default is 40%. On a $100 million debt instrument, the estimated loss following a default event would be $40 million.

Bonds with higher seniority have **higher** recovery rates because they take precedence in the event of a winding-up.

Question

A bond paying the Fed funds rate plus 4 percent in semiannual payments would be best known as a (an):

A. Income bond.

B. Straight-coupon bond.

C. Floating-rate bond.

D. Collateral trust bond.

The correct answer is **C**.

A floating-rate note (FRN) is a debt instrument with a variable interest rate. The interest rate for an FRN is tied to a benchmark rate—in this case, the Feds fund rate.

Reading 42: Mortgages and Mortgage-Backed Securities

After completing this reading, you should be able to:

- Describe the various types of residential mortgage products.
- Calculate fixed-rate mortgage payment and its principal and interest components.
- Describe the mortgage prepayment option and the factors that influence prepayments.
- Summarize the securitization process of mortgage-backed securities (MBS), particularly the formation of mortgage pools, including specific pools and TBAs.
- Describe the process of trading of pass-through agency MBS.
- Explain the mechanics of different types of agency MBS products, including collateralized mortgage obligations (CMOs), interest-only securities (IOs), and principal-only securities (POs).
- Describe a dollar roll transaction and how to value a dollar roll.
- Understand what nonagency mortgage-backed securities are the mechanics of trading them.
- Explain prepayment modeling and its four components: refinancing, turnover, defaults, and curtailments.
- Describe the steps in valuing an MBS using Monte Carlo simulation.
- Define Option Adjusted Spread (OAS) and explain its challenges and its uses.

Types of Residential Mortgage Products

A mortgage is a loan that has a specific piece of property as collateral. In the '70s and '80s, mortgages existed solely in the **primary market** where banks would issue mortgage products to customers who would, in turn, repay the principal plus interest. In modern times, however, mortgage lenders now repackage mortgages for sale in **secondary markets** as securitized investments.

In the secondary market, mortgages are pooled together and repackaged to form **mortgage-backed security**. The principal and interest payments pass through the bank before the bank hands them over to the MBS investor. Such a mechanism is referred to as the **pass-through structure**.

Lien Status

A first-lien mortgage is more desirable than a second-lien mortgage from the perspective of the lender. In the event of liquidation, a first-lien status would give the lender the right to submit the **first claim** on the proceeds of the liquidation process.

Original Loan Term

Long-term mortgages have a maturity period of 30 years, with medium-term ones ranging between 10-20 years. In recent years, however, borrowers increasingly prefer medium-term loans to long-term ones. Most borrowers want to repay their mortgages as soon as possible.

Credit Classification

Prime (A-grade) loans take the top spot as the most desirable loans from the lender's perspective. They are associated with low rates of delinquency and default thanks to low **loan-to-value** ratios, typically far less than 95%. Borrowers are individuals with stable and sufficient income.

Sub-Prime (B-grade) loans have higher rates of default and delinquency compared to prime loans. They are associated with loan-to-value ratios of 95% or more. Borrowers may be individuals with lower income levels and marginal/poor credit histories.

Alternative A-loans lie in between prime and subprime loans. In essence, they are prime loans, but certain characteristics make them riskier than prime loans. For instance, there may be less documentation available to support income levels, however impressive the actual figures might be.

Interest Rate Type

Fixed-rate mortgages are associated with a fixed rate of interest up to maturity.

Adjustable-rate mortgages are associated with a floating rate of interest. For example, the rate could be LIBOR + 100 bps. In such an arrangement, the rate would change every six months.

Fixed-Rate Mortgages: Principal and Interest Payments

A fixed-rate mortgage is a mortgage loan that has a fixed interest rate for the entire term of the loan. It is based on the creditworthiness of the borrower and uses residential real estate as collateral. The level of interest depends in part on the creditworthiness of the borrower, whereby the riskier the borrower, the higher the interest rate.

Other notable characteristics of fixed-rate mortgages include:

- It consists of equal payments over the life of the mortgage.
- A loan is amortized over its term such that each scheduled payment goes toward the settlement of both principal and interest.
- As the loan matures, the amortization schedule works in such a way that the borrower pays more principal and less interest with each payment.

Calculating the Amount of Each Payment

To determine the amount of each scheduled payment, PMT, we customize the formula for the present value of an annuity.

$$\text{Principal} = \text{PMT}\,\frac{1-(1+r)^{-n}}{r}$$

Where:

r = monthly interest rate (annual rate/12)

n = total number of months

Making PMT subject of the formula,

$$PMT = \frac{\text{Principal}}{\frac{1-(1+r)^{-n}}{r}}$$

Example: Calculating a Payment

Consider the following loan:

- Loan amount: $250,000
- Annual rate of interest: 4.5
- Term: 10 years
- Start date: 01/01/2019

What is the remaining principal at the end of each of the first 3 months?

Solution

r = 0.045/12 = 0.00375, n = 12 × 10 = 120

$$PMT = \frac{\text{Principal}}{\frac{1-(1+r)^{-n}}{r}}$$

$$= \frac{250,000}{\frac{1-(1+0.00375)^{-120}}{0.00375}}$$

$$= 2,590.96$$

On a financial calculator,

N = 120; I/Y = 0.375 (0.045/12); PV = -250,000; FV = 0; CPT => PMT = 2,590.96

We can now create an amortization schedule:

Month 1

Interest $= 0.00375 \times 250,000 = 937.50$

Repayment of principle will therefore be $2,590.96 - 937.50 = 1,653.46$

The remaining principal at the beginning of the second month = $250,000 - 1,653.46 = 248,346.54$

Month 2

Interest = $0.00375 \times 248,346.54 = 931.30$

Repayment of principal will therefore be $2,590.96 - 931.30 = 1,659.66$

The remaining principal at the beginning of the third month = $248,346.54 - 1,659.66 = 246,686.88$

Month 3

Interest = $0.00375 \times 246,686.88 = 925.08$

Repayment of principal will therefore be $2,590.96 - 925.08 = 1,665.88$

The remaining principal at the beginning of the fourth month = $246,686.88 - 1,665.88 = 245,021$

	Month 1	Month 2	Month 3	
Total Payment	\$2,590.96	\$2,590.96	\$2,590.96	<< Equal
Principal	\$1,653.46	\$1,659.66	\$1,665.88	<< Increasing
Interest	\$937.50	\$931.30	\$925.08	<< Decreasing
Loan Balance	\$248,346.54	\$246,686.88	\$245,021.00	<< Decreasing

Note 1: Interest payable is based on the amount of loan outstanding. Therefore, we will always see an increase in the principal paid on each payment.

Note 2: The loan balance only decreases by the principal amount on each payment since the interest payable portion of the payment is paid to the financial institution issuing the loan.

Calculating a Payment using the Remaining Cash Flows

The outstanding principal can also be calculated by discounting the remaining cash flows using the following formula:

$$\text{Outstanding principal} = \frac{\text{PMT}}{\text{r}} \times \left(1 - \frac{1}{(1+\text{r})^{\text{n}}}\right)$$

Assuming that the mortgage in the previous example has five years left, the outstanding principal (in USD) assuming that there have been no prepayments is:

$$\frac{2,590.96}{0.00375} \times \left(1 - \frac{1}{(1+0.00375)^{5\times 12}}\right)$$

$$= 690,922.67 \times 0.2011$$

$$= 138,944.5483$$

Factors that Determine the Value of an MBS

Weighted Average Maturity

Weighted average maturity (WAM) is the weighted average amount of time until the maturities on mortgages in an MBS. To compute WAM,

i. Compute the percentage value of each mortgage or debt instrument in the portfolio. This is achieved by adding the current principal value of all the mortgages together and then calculating each mortgage percentage compared to the total value.
ii. Multiply each percentage by the number of years to maturity.
iii. Add together the subtotals.

Example of Weighted Average Maturity

A mortgage-backed portfolio includes four mortgage investments as follows:

- Mortgage 1: $100,000 in current value, maturity in 5 years
- Mortgage 2: $10,000 in current value, maturity in 2 years
- Mortgage 3: $50,000 in current value, maturity in 6 years
- Mortgage 4: $40,000 in current value, maturity in 3 years

Determine the WAM of the portfolio.

Solution

Total value of portfolio = \$100,000 + \$10,000 + \$50,000 + \$40,000 = \$200,000

We then compute the percentage value of each mortgage:

- Mortgage 1: %value = \$100,000/\$200,000 = 50%
- Mortgage 2: %value = \$10,000/\$200,000 = 5%
- Mortgage 3: %value = \$50,000/\$200,000 = 25%
- Mortgage 4: %value = \$40,000/\$200,000 = 20%

The percentage values of each mortgage are then multiplied by the remaining duration until maturity:

- 50% × 5 years = 2.5 years
- 5% × 2 years = 0.1 years
- 25% × 6 years = 1.5 years
- 20% × 3 years = 0.6 years

The resulting figures are then totaled to produce a WAM of 4.7 years.

Mortgage	Current Value	% Value of Mortgage in Portfolio	% Value×Remaining Duration
Mortgage1	\$100, 000	\$100, 000/\$200, 000 = 50%	50% × 5 years = 2.5 years
Mortgage 2	\$10, 000	\$10, 000/\$200, 000 = 5%	5% × 2 years = 0.1 years
Mortgage 3	\$50, 000	\$50, 000/\$200, 000 = 25%	25% × 6 years = 1.5 years
Mortgage 4	\$40, 000	\$40, 000/\$200, 000 = 20%	20 × 3 years = 0.6 years
Total	\$200, 000		WAM = 2.5 + 0.1 + 1.5 + 0.6 = 4.7 years

How is WAM useful? It helps to determine the interest rate sensitivity of mortgage-backed portfolios. The larger the WAM, the longer the period of exposure to interest rate movements, and the greater the chances of a material effect on portfolio value relative to other investment alternatives.

Weighted Average Coupon

The weighted average coupon (WAC) is the weighted-average interest rate of mortgages that underlie a mortgage-backed security (MBS) at the time the securities were issued. It represents the **average interest rate** of a pool of mortgages with varying interest rates.

To compute WAC,

i. Compute the percentage value of each mortgage or debt instrument in the portfolio. Like under WAM, this is done by adding the current principal value of all the mortgages together and then calculating the percentage of each mortgage compared to the total value.
ii. Multiply each percentage by the gross interest rate of the mortgage
iii. Add together the subtotals

Example of Weighted Average Coupon

A mortgage-backed portfolio includes four mortgage investments as follows:

- Mortgage 1: $100,000 in current value, 5% interest rate
- Mortgage 2: $10,000 in current value, 4% interest rate
- Mortgage 3: $50,000 in current value, 6% interest rate
- Mortgage 4: $40,000 in current value, 3% interest rate

Determine the WAC of the portfolio:

Solution

Total value of portfolio = $100,000 + $10,000 + $50,000 + $40,000 = $200,000

We then compute the percentage value of each mortgage:

- Mortgage 1: %value = $100,000/$200,000 = 50%
- Mortgage 2: %value = $10,000/$200,000 = 5%

- Mortgage 3: %value = $50,000/$200,000 = 25%
- Mortgage 4: %value = $40,000/$200,000 = 20%

The percentage values of each mortgage are then multiplied by their respective interest rates:

- 50% × 5% = 2.5%
- 5% × 4% = 0.2%
- 25% × 6% = 1.5%
- 20% × 3% = 0.6%

The resulting figures are then totaled to produce a WAC of 4.8%.

Modeling the Prepayment Rate

Prepayment is undoubtedly one of the key issues an investor in MBSs would want to keep an eye on. Prepayments speed up principal repayments and also reduce the amount of interest paid over the life of the mortgage. Thus, they can adversely affect the amount and timing of cash flows.

Markets have adopted two main benchmarks that are used to track prepayment risk - the **conditional prepayment rate** (CPR) and the **Public Securities Association** (PSA) prepayment benchmark.

Conditional Prepayment Rate (CPR)

The CPR is a proportion of a loan pool's principal that is assumed to be paid off ahead of time in each period. It measures prepayments as a percentage of the current outstanding loan balance. It is always expressed as a percentage, compounded annually. For example, a 5% CPR means that 5% of the pool's outstanding loan balance pool is likely to prepay over the next year. It is estimated based on historical prepayment rates for past loans with similar characteristics as well as future economic prospects.

The CPR can be converted to a **single monthly mortality rate** (SMM) as follows:

$$\text{SMM} = 1 - (1 - \text{CPR})^{1/12}$$

SMM is, in effect, the amount of principal on mortgage-backed securities that is prepaid in a given month.

Note: This also implies that:

$$\text{CPR} = 1 - (1 - \text{SMM})^{12}$$

Prepayment for a month *i* (in $) = SMM(beginning balance – scheduled principal repayment in month *i*)

Public Securities Association (PSA) Prepayment Benchmark

The Public Securities Association model prepayment benchmark is one of the models used to estimate the monthly rate of prepayment. It is based on the assumption that rather than remaining constant, the monthly repayment rate gradually increases as a mortgage pool ages. The PSA is expressed as a monthly series of CPRs. The model assumes that:

- CPR = 0.2% for the first month after origination, increasing by 0.2% every month up to 30 months
- CPR = 6% for months 30 to 360

A mortgage pool whose prepayment speed (experience) is in line with the assumptions of the PSA model is said to be 100% PSA. Similarly, a pool whose prepayment experience is two times the CPR under the PSA model is said to be 200% PSA (or 200 PSA).

Example of (PSA) Prepayment Benchmark

Compute the CPR and SMM for the 8^{th} and 20^{th} months, assuming 100 PSA and 200 PSA.

Solution

Case 1: Assuming 100 PSA

CPR(month 8) = 8 x 0.2% = 1.6%

100 PSA implies that CPR (month 8) = 1 x 1.6% = 1.6%

SMM = 1 – (1 – CPR)$^{1/12}$ = 1 – (1 – 0.016)$^{1/12}$ = 0.1343%

CPR(month 20) = 20 x 0.2% = 4%

100 PSA implies that CPR (month 20) = 1 x 4% = 4%

SMM = 1 – (1 – CPR)$^{1/12}$ = 1 – (1 – 0.04)$^{1/12}$ = 0.3396%

Case 2: Assuming 200 PSA

CPR(month 8) = 8 x 0.2% = 1.6%

200 PSA implies that CPR (month 8) = 2 x 1.6% = 3.2%

SMM = 1 – (1 – CPR)$^{1/12}$ = 1 – (1 – 0.032)$^{1/12}$ = 0.2706%

CPR(month 20) = 20 x 0.2% = 4%

200 PSA implies that CPR (month 20) = 2 x 4% = 8%

SMM = 1 – (1 – CPR)$^{1/12}$ = 1 – (1 – 0.08)$^{1/12}$ = 0.6924%

Important: CPR and SMM have a **nonlinear** relationship. The implication is that the SMM for 200% PSA does not equal two times the SMM for 100% PSA.

The Securitization Process

To reduce the risk of holding a potentially undiversified portfolio of mortgage loans, many originators (financial institutions) work together to pool residential mortgage loans. The loans pooled together have similar characteristics. The pool is then sold to a separate entity, called a special purpose vehicle (SPV), in exchange for cash. An issuer will purchase those mortgage assets in the SPV and then use the SPV to issue mortgage-backed securities to investors. MBSs are backed by the mortgage loans as collateral.

The simplest MBS structure, a mortgage pass-through, involves cash (interest, principal, and prepayments) flowing from borrowers to investors with some short processing delay. Usually, the issuer of MBSs may enlist the services of a mortgage servicer whose main mandate is to manage the flow of cash from borrowers to investors in exchange for a fee. MBSs may also feature mortgage guarantors who charge a fee and, in return, guarantee investors the payment of interest and principal against borrower default.

Agency Mortgage-Backed Securities

Some agencies (government agencies and government-sponsored enterprises) purchase mortgages from banks and use the cash flows from those mortgages to create mortgage-backed securities that are then sold to investors. By doing so, the agencies enable banks to give out new loans and not keep the loans on their books. The agencies also ensure that banks do not run out of money to give out to potential new homeowners.

To protect their investors against default risk, such agencies charge a fee on their mortgages. The agencies do not, however, protect their investors from prepayment risk. An example of an agency Mortgage-backed security is pass-through security.

Trading of Pass-Throughs

In a pass-through, all investors in the pool get the same return. The return is equivalent to the investor's share of the cash flow in the pool minus the agency fee. Despite being risk-free investments, still, pass-throughs have prepayment risk.

Pass-through agency securities can be traded either as specified pools (traders agree to trade a specified pool at a specified price) or as to be announced (TBAs), which are traded in the forward markets.

Dollar Roll Transactions

A dollar roll transaction is a form of repurchase agreement in which an investor sells mortgage-backed security during one period, called the “front month,” and repurchases it in a subsequent period, called the later or “back month.” In so doing, the investor relinquishes their access to the principal and interest on the loan that is sold. However, the investor receives cash from the sale, which could be reinvested and used to purchase the security later. The aim of the investor is to capitalize on a drop in the price of the MBS by “selling high and buying low.” The trade counterparty benefits in that they do not have to deliver the MBS in the current month and thus get to keep the principal and interest payments that would otherwise be passed through to those securities holders.

A dollar roll works much like selling stocks short.

The price difference between the front month and the back month is known as the drop. When the drop is significant, the dollar roll is said to be "on special." The size of the drop is influenced by:

- Demand for mortgage pass-through securities.
- Holding period (the period between the two settlement dates).
- Funding cost in the repo market.
- The volume of mortgage closings in a mortgage originator’s pipeline.

Dollar rolls work in TBA (to be announced) MBS markets. To be announced is a term that describes forward-settling mortgage-backed securities trades. The term originates from the fact that the specific mortgage-backed security to be delivered to fulfill a TBA trade is not designated at the time the trade is initiated. Rather, trade parties are required to exchange mortgage pool information 48 hours prior to trade settlement. The TBA market treats MBS pools as relatively interchangeable, which increases the MBS market's overall liquidity because thousands of different MBSs with different characteristics can be conveniently traded.

Renowned institutions like Freddie Mac, Fannie Mae, and Ginnie Mae issue mortgage pass-through securities that trade in the TBA market. For example, an investor who just purchased a 30-year 5% Freddie Mac (MBS) pool might sell the MBS 30-year 5% May TBA and buy the MBS 30-year 5% June TBA. In effect, the sale of the May TBA raises cash, while the purchase of the June TBA returns cash.

Example: Dollar Roll Transaction

TBA prices of the Ginnie Mae 3% for January 12 and February 12 settlements are \$102.30 and \$102.10, respectively. The accrued interest to be added to each of these prices is \$0.125. The expected total principal paydown (scheduled principal plus prepayments) is 2% of the outstanding balance, and the prevailing short-term rate is 1%. Also, assume that the actual/360-day convention is applied.

An investor wishes to roll a balance of \$10 million. Determine the value of the roll.

Solution

Proceeds from selling the January 12 TBA are: \$10m × (102.30 + 0.125)/100 = \$10,242,500

Investing these proceeds to February 12 at 1% interest earns interest of: \$10,242,500 × 31/360 × 1% = \$8,820

Purchasing the February TBA, which has experienced a 2% paydown, will cost: \$10m × (1 – 2%) × (102.10 + 0.125)/100 = \$10,018,050

Net proceeds from the roll therefore are: \$10,242,500 + \$8,820 - \$10,018,050 = \$233,270

If the investor does not roll, the net proceeds are the coupon plus principal paydown: \$10m × (3%/12 + 2%) = \$225,000

Value of the roll = net proceeds from the roll – net proceeds without roll: = \$233,270 - \$225,000 = \$8,270

Other Agency Products

1. **Collateralized Mortgage Obligation**: It involves creating tranches – classes of security that bear different amounts of prepayment risk. Assume that there are two tranches, A and B. Tranche A investors finance 60% of the principal while tranche B investors finance 40% of the principal. In this example, tranche B bears most of the prepayment risk. Each tranche will get interest on its outstanding principal; however,

principal payments will only be paid to tranche B after paying tranche A investors.

2. **Interest-Only Securities (IOs)**: Interest payments from mortgage securities will be directed to the interest-only securities.
3. **Principal Only Securities (POs)**: Principal payments from mortgage securities will be directed to the principal-only securities.

Non-Agency Mortgage-Backed Securities

Non-agency mortgage-backed securities are issued by private corporations, like financial institutions, and are not guaranteed by government-sponsored institutions.

Banks sell a mortgage portfolio to a special purpose vehicle (SPV). The SPV then creates securities and passes the cash-flows to the securities it has created. Investors in non-agency mortgage-backed security are not protected against the default risk.

Modeling Prepayment Behavior

Prepayment risk is the risk involved with the premature return of principal on a mortgage. Prepayment effectively renders the borrower free of mortgage obligations. A mortgage prepayment option works much like a call option for the borrower.

Mortgage prepayments take one of two forms:

- Increasing the amount/frequency of payments; or
- Repaying/refinancing the entire outstanding balance.

The four reasons for prepayment are refinancing, turnover, defaults, and curtailment.

Refinancing

As the name suggests, refinancing occurs when the borrower of a loan wishes to refinance a property. Reasons for refinancing include:

- A decrease in interest rates, since refinancing when the interest rates are low reduces the monthly payments of the borrower;
- An increase in the value of the property qualifying the borrower for a higher loan; or
- An improved credit rating of the borrower, enabling him/her to obtain a lower rate.

The incentive function measures the extent of prepayment. The incentive function is equal to $WAC - R$, where WAC is the weighted average coupon, and R is the mortgage rate available to borrowers.

Generally, the prepayment rate increases with a decrease in mortgage rates, and the older a mortgage pool is, the lesser the chances of refinancing.

Turnover

Turnover prepayments are made following the sale of a house. Housing turnover increases at certain periods during the year, e.g., over summer when the weather is favorable.

The rate of turnover payments is usually dependent on the age of a mortgage holder and on his/her geographical location. Borrowers prefer to refinance a significant number of years into the mortgage to minimize penalties and administrative charges that are usually tied to principal outstanding.

Defaults

Defaults occur to agency MBSs. The agency will pay the outstanding loan amount of any defaulter of an agency MBS. This payment is treated as a prepayment.

Curtailments

Curtailments occur when a mortgage holder pays back part of the mortgage earlier. More often, curtailments occur when the mortgage is relatively old/ when the mortgage balance is relatively low.

Monte Carlo Simulation in the Valuation of Mortgage-Backed Securities

A popular method for valuing MBSs is called the Monte Carlo Methodology. The simulation creates thousands of interest paths that the MBS could follow over its life. The process recognizes the fact that there is a probability distribution of the possible outcomes of an MBS. Taking into account multiple interest rate paths is important because interest rates impact repayments and will, therefore, impact the amount and timing of cash flows to the investor.

There are four steps required to value a mortgage security using the Monte Carlo methodology:

Step 1: Simulate short-term interest rate and refinancing rate paths;

Step 2: Project the cash flow on each interest rate path;

Step 3: Determine the present value of the cash flows on each interest rate path;

Step 4: Compute the theoretical value of the mortgage security.

Option-Adjusted Spread

When modeling the value of a mortgage-backed security, the option-adjusted spread (OAS) is the spread that, when added to all the spot rates of all the interest rate paths, will make the average present value of the paths equal to the actual observed market price plus accrued interest. In other words, we purpose to find a single spread such that shifting the paths of short-term rates by that spread results in a model value equal to the market price. OAS is the most popular measure of relative value for mortgage-backed securities.

Mathematically, OLS is determined by the following relationship:

$$\text{Market price} = \frac{\text{PV[path(1)]} + \text{PV[path(2)]} + \ldots + \text{PV[path(n)]}}{\text{n}}$$

Where N = number of interest rate paths

While the LHS of the equation above gives the current market price of the MBS, the RHS of the equation is the Monte Carlo model's output of the average theoretical value of the MBS. The OAS is determined **iteratively; that** is, if the average theoretical value determined by the model is higher (lower) than the MBS market value, the spread is increased (decreased).

We can view the OAS as a measure of MBS returns that takes into account two types of volatility: changing interest rates and prepayment risk.

The OAS should not be confused with a Z-spread. The Z-spread is the yield that equates the present value of the cash flows from the MBS to the price of the MBS discounted at the Treasury spot rate plus the spread. However, it does not include the value of the embedded options (prepayments), which can have a big impact on the present value.

The difference between the Z-spread and the OAS gives the option cost, which we can interpret as a measure of prepayment risk.

Option cost = Zero-volatility spread – OAS

OAS is a byproduct of Monte Carlo simulation; not the traditional value approaches used to value options. This makes it have several limitations:

- It is dependent on some type of prepayment model, e.g., the PSA model. As established earlier, most of these models are based on historical data, which may not always reconcile with actual future results.
- It is subject to all modeling risks associated with simulation
- The process of adjusting interest rate paths is subject to modeling error.
- OAS assumes that the investor holds the securities to maturity, while in reality, most investors hold securities for a finite period

Practice Question

A mortgage-backed portfolio includes four mortgage investments as follows:

- Mortgage 1: $140,000 in current value, 5% interest rate, 5 years remaining duration
- Mortgage 2: $100,000 in current value, 4% interest rate, 6 years remaining duration
- Mortgage 3: $50,000 in current value, 6% interest rate, 3 years remaining duration
- Mortgage 4: $60,000 in current value, 3% interest rate, 2 years remaining duration

The weighted average coupon of the portfolio is closest to:

A. 4.5%
B. 5.1%
C. 4.9%
D. 4.0%

The correct answer is **A**.

The weighted average coupon (WAC) is the weighted-average interest rate of mortgages that underlie a mortgage-backed security (MBS) at the time the securities were issued. It represents the **average interest rate** of a pool of mortgages with varying interest rates.

Total value of portfolio = $140,000 + $100,000 + $50,000 + $60,000 = $350,000

We then compute the percentage value of each mortgage:

- Mortgage 1: %value = $140,000/$350,000 = 40%

- Mortgage 2: %value = $100,000/$350,000 = 28.6%
- Mortgage 3: %value = $50,000/$350,000 = 14.3%
- Mortgage 4: %value = $60,000/$350,000 = 17.1%

The percentage values of each mortgage are then multiplied by their respective interest rates:

- 40% × 5% = 2%
- 6% × 4% = 1.1%
- 3% × 6% = 0.9%
- 1% × 3% = 0.5%

The resulting figures are then totaled to produce a WAC of approx. 4.5%.

Reading 43: Interest Rate Futures

After completing this reading, you should be able to:

- Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation.
- Calculate the conversion of a discount rate to a price for a US Treasury bill.
- Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond.
- Explain and calculate a US Treasury bond futures contract conversion factor.
- Calculate the cost of delivering a bond into a Treasury bond futures contract.
- Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver Treasury bond decision.
- Calculate the theoretical futures price for a Treasury bond futures contract.
- Calculate the final contract price on a Eurodollar futures contract.
- Describe and compute the Eurodollar futures contract convexity adjustment.
- Calculate the duration-based hedge ratio and create a duration-based hedging strategy using interest rate futures.
- Explain the limitations of using a duration-based hedging strategy.

Day Count Conventions

A day count convention dictates how interest accrues over time in a variety of financial instruments, including bonds, swaps, and loans. It determines how interest is calculated at the end of each period. It's usually expressed as a fraction $\frac{A}{B}$. A defines the way in which the number of days between the two days is calculated, usually a notional or 30. B defines the way in which the total number of days in the reference period is measured, usually 360 or 365.

$$\text{Accrued interest} = \text{Coupon} \times \frac{\text{Number of days between dates}}{\text{Number of days in the reference period}}$$

Common Day Count Conventions

Actual/Actual Convention

The actual/actual convention means that the accrued interest is based on the ratio of the actual days elapsed to the actual number of days in the period between coupon payments. For example, suppose we have a bond paying a coupon rate of 10% per annum on a principal of $100 on March 1st and September 1st. This implies a coupon of $5 on each of these dates. If we want to compute the accrued interest as of May 31st, we will have to determine the actual number of days between May 31st and the last coupon date, i.e., March 1st. That's 91 days (= 30 + 30 + 31). The reference period, March 1st to Sept 1st, has 184 actual days. Thus,

$$\text{Accrued interest} = \frac{91}{184} \times \$5$$

The actual/actual is used for **Treasury bonds**.

30/360 Convention

The 30/360 convention follows the same logic but assumes **all** months have exactly 30 days. It's used for **corporate and municipal bonds.**

Based on the 30/360 convention, the above example will give us:

$$\text{Accrued interest} = \frac{89}{180} \times \$5 = 2.4722$$

89 days is obtained by adding 29+30+30 (number of days between March 1st and May 31st). The reference period, March 1st to September 1st, has 180 days(= 29 + 30+ 30+ 30+ 30+ 30+ 1, in March, April, May, June, July, August, and September, respectively).

Actual/360 Convention

The actual/360 convention is used for **money market instruments**, e.g., T-bills, commercial paper, and certificates of deposit. Interests of money market instruments are expressed per year. Suppose that a money market instrument earns an interest of 5% expressed on an actual/360 basis, the accrued interest will be:

$$\text{Accrued interest } = \frac{365}{360} \times \$5$$

Clean Price vs. Dirty Price

The **dirty price** of a bond – also known as the cash price – is the price that includes the present value of all of the bond's cash flows, including the interest accruing on the next coupon payment date. It's the price the issuer of the bond must be paid by the investor in order to dispense with the bond. The dirty price is comprised of the quoted price and accrued interest.

Example: Dirty Price

A bond quoted at 102–20 has accrued $2.54 in interest over the last six months. Determine the dirty price of the bond.

Solution

$$\text{Dirty price } = \text{ Quoted price } + \text{ Accrued interest } = 102 + \frac{20}{32} + 2.54 = \$105.165$$

The **clean price** of a bond is the price that excludes the interest that has accrued since the most recent coupon payment. It's also known as the quoted price.

$$\text{Clean price } = \text{ Dirty price } - \text{ Accrued interest}$$

T-bill Prices

Like other money market instruments, T-bills are issued at a discount to par value on an actual/360-day count basis. The quoted price is, in fact, the discount rate. If P is the quoted price and Y is the corresponding cash price per USD 100 of face value, then the quoted price, P, can be expressed as:

$$P = \frac{360}{n}(100 - Y)$$

Alternatively, you might be asked to compute a T-bill's cash price, in which case you should just make Y the subject of the formula:

$$Y = 100 - \frac{Pn}{360}$$

Example: T-bill Price

A 120-day treasury bill with a cash price of 99 would have a quoted price of:

$$P = \frac{360}{120}(100 - 99) = 3$$

U.S. Treasury Bonds Futures Contracts

Deliverable securities for T-Bond futures contracts are bonds with remaining terms to maturity of 15 years or more. The cash received by the short position in a T-bond futures contract is given by:

$$\text{Cash received} = (QFP \times CF) + AI$$

Where:

QFP = quoted futures price/settlement price

CF = conversion factor

AI = accrued interest since the last coupon date on the bond delivered

The conversion factor is given by:

$$CF = \frac{\text{Discounted bond price} - \text{Accrued interest}}{\text{Face value}}$$

For instance, if a bond has a present value of $125, accrued interest of $5, and $100 face value, then:

$$\text{CF} = \frac{125 - 5}{100} = 1.20$$

Conversion factors for different bonds are issued on a daily basis by the Chicago Board of Trade. A conversion factor is actually the approximate decimal price at which $1 par of a bond would trade if all interest rates are 6% compounded semi-annually.

Cheapest to Deliver Bond

Note that the cost of delivering a bond or note is

$$\text{Market Price} - \text{Price Received}$$

Obviously, the short-position party will choose the bond with the lowest cost to deliver it to the long-position party in line with contractual specifications. Such a bond is referred to as the **cheapest to deliver (CTD)**. Determining the CTB bond is necessitated by a discrepancy between the market price of a security and the conversion factor used to determine the value of the security being delivered. Thus, picking one bond for delivery over another can be advantageous to the short position.

We have seen earlier that the price received for a bond is

$$\text{Cash received} = (\text{QFP} \times \text{CF}) + \text{AI}$$

Where QFP is the quoted futures price/settlement price, CF is the conversion factor, and AI is the accrued interest.

The Cash market price of the bond is

$$\text{Quoted Price} + \text{Accrued Interest}$$

The cheapest to deliver bond is the one for which

$$\text{Quoted Price} - \text{Settlement Price} \times \text{Conversion Factor}$$

is the least.

Note that Settlement price = Quoted futures price, and it should not be confused with the quoted price, which is just the current bond price.

CTB calculations are relevant in all cases where multiple financial instruments can satisfy the contract.

The Impact of the Level and Shape of the Yield Curve on the Cheapest-to-Deliver Treasury Bond Decision

A number of factors can determine the cheapest to deliver bond. For example, if the bond's interest rates are greater than 6 %, then bonds with low coupons and long maturity tend to be the cheapest to deliver. On the other hand, if the bond's interest rates are less than 6 %, then bonds with high coupons and short maturity tend to be the cheapest to deliver.

An upward-sloping yield curve favors long-maturity bonds. On the other hand, a downward-sloping yield curve favors short-maturity bonds.

The Theoretical Futures Price for a Treasury Bond Futures

The theoretical futures price for a Treasury bond futures, F, is given by:

$$F = (S - I)(1 + r)^T$$

Where:

S=spot price of the bond

I=present value of cash flows, i.e., coupons

r=risk-free rate of interest

T =time to maturity

When r is expressed with continuous compounding, the above formula becomes

$$F = (S - I)\, e^{rT}$$

Example: Cheapest to Deliver Bond

Under the terms of a futures contract, a bond will be delivered in 250 days. Assume that the last coupon of the bond was paid 40 days, and the next coupon will be paid in 143 days. If the risk-free rate is 5% with continuous compounding and the bond pays a coupon of 8% compounded semi-annually, what is the dirty futures price of the bond if the clean price is USD 107.00 and the conversion factor is 1.0400?

Solution

We can derive the bond's spot price from the futures price formula.

We know that,

$$F = (S - I)e^{rT}$$

Thus,

$$S = 107.00 + \left(\frac{40}{183}\right) \times 4 = 107.8743$$

since the next coupon is \$4 per \$100 face value and 40 of the 183 days between coupon payments have passed.

$$I = 4e^{-0.05 \times \frac{143}{365}} = 3.9224$$
$$T = \frac{250}{365} = 0.6849 \text{ years}$$

So,

$$F = (107.8743 - 3.9224)e^{0.05 \times 0.6849} = 107.5734$$

If the coupon after the next one is paid in 300 days (50 days after delivery), the clean price can be obtained by subtracting the accrued interest from the dirty price:

$$\text{Dirty price} = 107.5734$$
$$\text{Accrued interest} = \frac{107}{157} \times 4 = 2.7261$$

Where 157 is obtained by subtracting 143 (the last coupon payment date) from 300 (the date when the next coupon payment will be made); and

107 represents the number of days of the 157 days between payments that will have passed (50 days will be remaining to the next coupon payment).

$$\begin{aligned}\text{Clean futures price} &= \text{Dirty price} - \text{Accrued interest}\\ &= 107.5734 - 2.7261 = 104.8473\end{aligned}$$

To get an estimate price of the futures contract price, we divide the clean futures price by the conversion factor which was given in the question:

$$\text{Futures price} = \frac{104.8473}{1.04} = 100.8147$$

The Final Price of Eurodollar Futures Contracts

Eurodollars are U.S. dollars deposited in banks outside the United States. Eurodollar futures provide a valuable tool for hedging fluctuations in short-term U.S. dollar interest rates. These types of futures have a maturity term of 3 months and largely reflect market expectations for that period.

The final price of a Eurodollar futures contract is determined by LIBOR on the last trading day. Eurodollar futures contract settle in cash and are based on a Eurodollar deposit of $1million.

The minimum price change is one “tick,” which is equivalent to one interest rate basis point = 0.01 price points = $25 per contract.

$$\text{Eurodollar futures price} = \$10,000\,[100 - (0.25)\,(100 - Z)]$$

Where Z = quoted price for a Eurodollar futures contract

For example, if the quoted price Z is 98.5, then:

$$\text{Eurodollar futures price} = \$10,000\,[100 - (0.25)\,(100 - 98.50)] = \$996,250$$

The three-month forward LIBOR for each contract is 100– Z. In practice, however, daily marking-to-market can result in differences between actual forward rates and those implied by fixtures contracts. To reduce this difference, we use a convexity adjustment:

$$\text{Actual forward rate} \;=\; \text{Forward rate implied by futures} \;-(\frac{1}{2} \times \sigma^2 \times T_1 \times T_2)$$

Where:

T_1 = maturity on the futures contract

T_2 = time to the maturity of the rate underlying the contract (90 days)

σ= annual standard deviation of the change in the rate underlying the futures contract, or 90-day LIBOR

Duration-Based Hedge Ratio

A duration-based hedge ratio is a hedge ratio constructed when interest rate futures contracts are used to hedge positions in an interest-dependent asset, usually bonds money market securities.

The number of futures contracts (N) required to hedge against a given change in yield, (Δy) is:

$$N = -\frac{P \times DP}{FC \times DF}$$

Where:

P = forward value of the fixed-income portfolio being hedged

DP = duration of the portfolio at the maturity date of the hedge

FC = futures contract price

DF = duration of the asset underlying the futures

The negative sign implies that the number of contracts taken up must be the opposite of the original position. If the investor is **short** of the portfolio, for example, they must **long** N contracts to produce a position with zero duration.

Example: Duration-Based Hedge Ratio

A pension fund has a $25 million portfolio of Treasury bonds with a portfolio duration of 6.1. The cheapest to deliver bond has a duration of 4.7. The six-month treasury bond futures price is $127,000. What is the number of futures contracts to fully hedge the portfolio?

Solution

$$\begin{aligned} N &= -\frac{P \times DP}{FC \times DF} \\ &= -\frac{\$25,000,000 \times 6.1}{\$127,000 \times 4.7} = -255 \end{aligned}$$

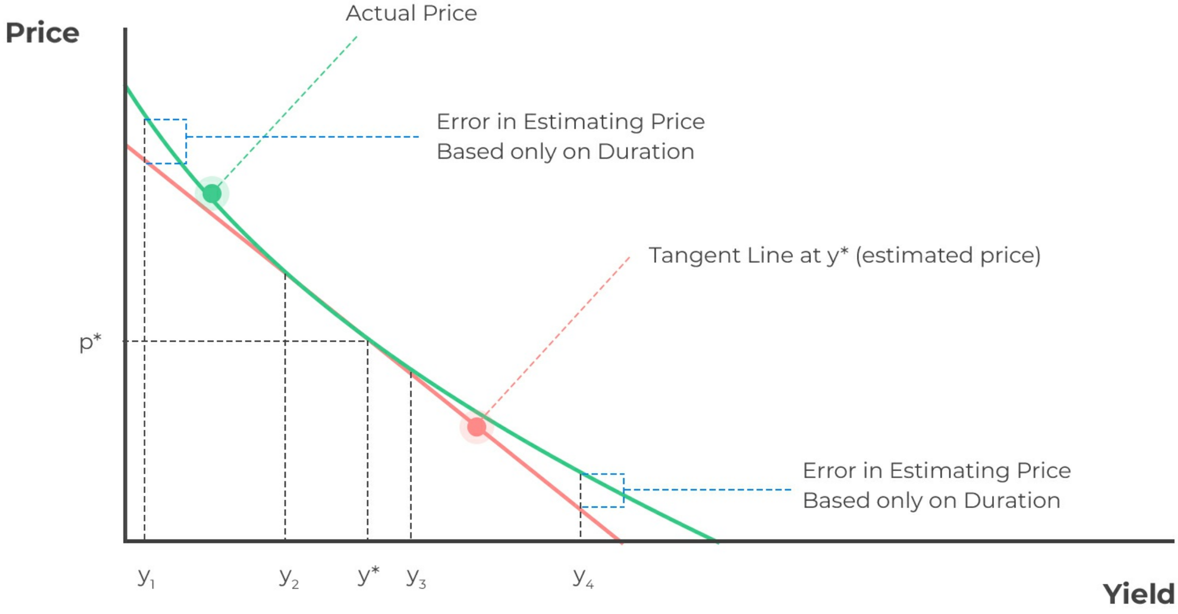
If yields rise over the next 6 months, it’s bad news for the portfolio as it will lose value. Suppose the new value is $23.5 million, then it’s good news in the futures market as the gains can be used to offset the spot market losses.

In fact, if the hedge is executed properly and the yield curve changes are parallel, then it is possible to gain $1.5 million in the derivatives market. The total portfolio value after the hedge is $25 million.

Limitations of a Duration-Based Hedging Strategy

The major limitation of employing a duration-based hedging strategy has much to do with the fact that duration measures are only accurate for small changes in yield. For large changes in yield, the price/yield relationship is not linear but is actually convex. Thus, using the strategy in the face of large moves in yield will result in “underhedging."

Convexity and Duration



Practice Question

Alina Escobar is a junior derivatives analyst at the derivatives investment unit of a financial institution. The company holds a mid-day meeting where managers discuss investment strategies according to recent trends in the market. Escobar's manager asked her to estimate the price of a March Eurodollar futures contract that is quoted as $94.25. Estimate the price that the firm will have to pay if the firm ultimately decides to invest in the March Eurodollar futures contract.

A. $942,500

B. $985,625

C. $991,750

D. $1,050,870

The correct answer is B.

Since the size of one Eurodollar contract is $1 million and the change of one basis point is 25, we will use the following formula to calculate the price of the Eurodollar futures contract:

$$\text{Price of the Eurodollar} = 10,000 \times [100 - 0.25 \times (100–94.25)] = \$985,625$$

Reading 44: Swaps

After completing this reading, you should be able to:

- Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows.
- Describe the role of the confirmation in a swap transaction.
- Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows.
- Explain the role of financial intermediaries in the swaps market.
- Describe the role of the confirmation in a swap transaction.
- Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument.
- Explain how the discount rates in a plain vanilla interest rate swap are computed.
- Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions.
- Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs).
- Explain the mechanics of a currency swap and compute its cash flows.
- Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows.
- Calculate the value of a currency swap based on two simultaneous bond positions.
- Calculate the value of a currency swap based on a sequence of FRAs.
- Identify and describe other types of swaps, including commodity, volatility, credit defaults, and exotic swaps.
- Describe the credit risk exposure in a swap position.

What's an Interest Rate Swap?

An interest rate swap is an agreement to exchange one stream of interest payments for another, based on a specified principal amount, over a specified period of time. The principal in an interest rate swap is known as a notional principal because it is not exchanged. Only interest rates calculated with respect to the notional principal can be exchanged.

Example: Swap Payments

Assume two parties, A and B. Party A, agree to pay Party B a fixed interest rate at 4% per annum, compounded semiannually, on a principal of USD 100,000. Party B, in return, agrees to pay party A interest at the six-month LIBOR (London Inter-Bank Offered Rate). Exchanges occur every six months over a period of three years. The tables below summarize the possible scenarios of parties A and B.

Scenarios for party B: Pays LIBOR and receives a fixed rate.

Time in Years	6-month LIBOR (% per year)	Floating amount paid (USD)	Fixed amount received (USD)	Net cashflow (USD)
0.0	3.00			
0.5	3.20	1500	2000	+500
1.0	3.44	1600	2000	+400
1.5	4.00	1720	2000	+280
2.0	4.30	2000	2000	–
2.5	4.44	2150	2000	−150
3.0	4.70	2220	2000	−220

Note: The exchange takes place one period after the LIBOR is observed. Therefore, the first exchange occurring after six months will occur using the LIBOR rates observed at time zero. The exchange after one year will take place after the LIBOR rates observed at time 0.5 into the contract, and so forth.

At time 0.5, the floating rate amount will be $3\% \times 0.5 \times 100,000 = 1,500$.

The fixed rate amount will be $4\% \times 0.5 \times 100,000 = 2,000$.

As such, the cash flows exchanged will be the netted amount, $2,000 - 1,500 = 500$.

Day Count Issues

The above calculations are just approximations as they do not consider day counts. For more accurate results, it is important to consider day count conventions.

Suppose that the exchanges in the above example take place on 1^{st} January and 1^{st} July. The first cash flow exchange is on 1^{st} July of that year, and the floating rate exchanged will be:

$$\frac{183}{360} \times 3\% \times 100,000 = 1,525$$

Note: 183 was obtained by adding the total number of days between 1^{st} January and 1^{st} July, and the day count convention for LIBOR is actual/360.

It is important to see that 1,525 is 25 more than the approximate value shown in the table (1,500).

Similarly, the fixed rate will be expressed with day count conventions as shown below:

$$\frac{183}{360} \times 4\% \times 100,000 = 2,033.33$$

Again, 2033.33 is USD 33.33 more than the approximate value shown in the table (2,000).

Swap Confirmations

The details of each swap agreement are contained in a document called the **confirmation**. Such documents are drafted by the International Swaps and Derivatives Association (ISDA). Each party must append their signature on the confirmation to show their commitment to the agreement.

The contents of confirmation include the dates when payments will be exchanged, the day count conventions to be used in calculating payments, and the way payments will be calculated.

Quotes

- **Ask quote**: This is the rate a firm is willing to receive to pay LIBOR.
- **Bid quote**: This is the rate a firm is willing to pay to receive LIBOR.
- **Swap rate**: The average of the bid and the ask quotes.

Swaps Based on Overnight Rates

When using an overnight rate (which will replace swaps as discussed in the chapter on Properties of Interest Rates), the floating rate can be obtained using the formula:

$$R = (1 + d_1 r_1)(1 + d_2 r_2)\ldots(1 + d_n r_n) - 1$$

Where:

R is the floating rate,

d is the number of days, and

r is the overnight rate.

Apart from weekends and holidays, d = 1. Weekends and holidays lead to the overnight rate being applied more than once. On a Friday, for example, d=3.

Why Do Firms Trade Interest Rate Swaps?

Interest rate swaps can be used to transform assets into liabilities, or vice-versa, by converting fixed (floating) rates loans and liabilities into floating (fixed) rates.

Illustration 1: Conversion of a Fixed-Rate Liability into a Floating Rate Liability

Assume that the 3-year bid and ask quotes are 3.06% and 3.09%, respectively. A company borrows a bank USD 5,000 at a fixed interest rate of 4%, compounded quarterly. To convert this fixed-rate loan into a floating rate liability, the company can use the three-year bid and ask quotes to enter into a three-year swap with another company. It will then have three sets of cash flows:

1. It will pay 4% on the borrowed USD 5,000 to the bank;
2. It will pay LIBOR on USD 5,000 under the terms of the swap to the swap dealer; and
3. It will receive a fixed rate of 3.06% from the swap dealer on USD 5,000 under the terms of the swap.

The net out interest payment will therefore be:

$$4\% + \text{LIBOR} – 3.06\% = 0.94\% + \text{LIBOR}$$

The company will then have converted a 4% fixed interest rate into a 0.94% + LIBOR floating interest rate.

Illustration 2: Conversion of a Floating Rate Liability into a Fixed Rate Liability

Suppose the above company borrowed the USD 5,000 on a floating interest rate of three months LIBOR plus 0.5% (50 basis points). To convert the floating rate to a fixed rate, the company can use the ask quote of 3.09%. It will then have the below set of cash flows:

1. It will pay LIBOR + 0.5 % on it USD 5,000 borrowings;
2. It will receive LIBOR under the terms of the swap; and
3. It will pay a fixed rate of 3.09% under the terms of the contract.

The net out interest payment will therefore be:

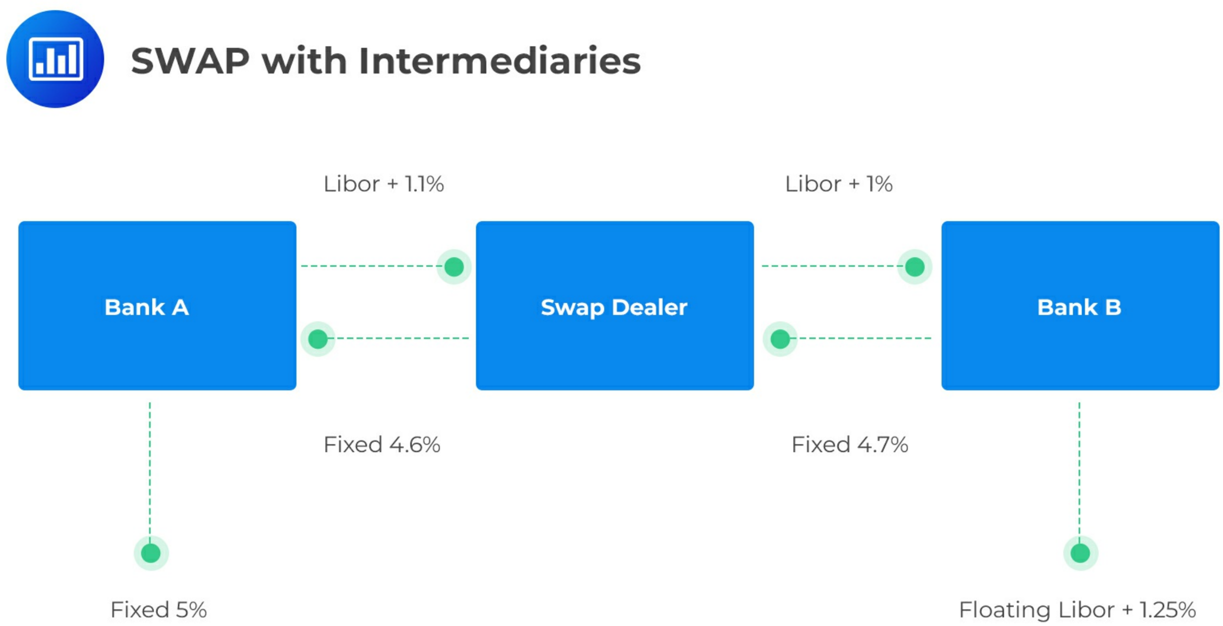
$$\text{LIBOR} + 0.5\% - \text{LIBOR} + 3.09\% = 3.59\%$$

The company will then have converted a floating interest rate of LIBOR + 50 basis points into a fixed interest rate of 3.59%.

Assets with floating (fixed) interest rates can be converted to fixed (floating) interest rates using the same concept.

The Role of Financial Intermediaries in Swap Markets

Just like in other OTC instruments, parties to a swap do not interact one on one. A swap dealer intertwines themselves between the parties taking a commission on the trade.



In most cases, therefore, a swap party stays unaware of the identity of the party in the offsetting position. The swap dealer effectively serves as an **intermediary**.

The Comparative Advantage Argument

Let's look at an example of two firms, A and B.

- A wants to borrow floating
- B wants to borrow fixed

Borrowing costs

Firm	Fixed borrowing	Floating borrowing
A	6%	LIBOR
B	8%	LIBOR + 100bps

From the table, we can see that A can borrow fixed at 6%, and B can borrow fixed at 8%. Also, A can borrow floating at LIBOR, and B can borrow floating at LIBOR + 100bps. However, the difference in borrowing rates for A and B is higher in the fixed market than in the floating market (200bps vs. 100bps). Therefore, A has an **absolute advantage** in both markets but a **comparative advantage** in the fixed market. B, on the other hand, has a **comparative advantage** in the floating market.

When a comparative advantage exists, the implication is that the parties involved **can reduce their borrowing costs** by entering into a swap agreement. The net borrowing savings by entering into a swap is the **difference between the differences**, i.e., $\Delta\text{fixed} - \Delta\text{floating}$.

If we assume that the net borrowing savings are split evenly between the parties, we will divide the total borrowing savings by 2,i.e.,

$$\text{Borrowing savings per party} = \frac{\Delta\text{fixed} - \Delta\text{floating}}{2} = \frac{200\text{bps} - 100\text{bps}}{2} = 50\text{bps}$$

A problem with the comparative advantage argument is that it **assumes the floating rates will remain in force in the long term**. In practice, the floating rate is reviewed at 6-month intervals and may increase or decrease to reflect the borrower's credit risk. It also **assumes zero transaction costs** even when an intermediary is involved in the swap (which is standard practice).

Computing The Discount Rate In A Plain Vanilla Interest Rate Swap

In essence, a swap is a series of cash flows, and therefore its value is determined by discounting all those cash flows to the present (valuation date). The cash flows are discounted using spot rates developed using the swap curve. The curve makes use of the following relationship between forward rates and spot rates, assuming continuous compounding:

$$R_{forward} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

Where:

R_i=spot rate corresponding with T_i years

$R_{forward}$=forward rate between T_1 and T_2

Value of a Plain Vanilla Interest Rate Swap Using the Bond Methodology

In essence, the pay fixed, receive floating party has a long position in a floating rate (since it's an inflow) and a short position in the fixed-rate (since it's an outflow). The pay floating, receive fixed party has a short position in the floating rate (since it's an outflow) and a long position in the fixed-rate (since it's an inflow).

If we denote the value of the swap as V_{swap}, the present value of fixed-leg payments as P_{fix}, and the present value of floating-leg payments as P_{flt}, then:

To the pay fixed, receive floating,

$$V_{swap} = P_{flt} - P_{fix}$$

To the pay floating, receive fixed,

$$V_{swap} = P_{fix} - P_{flt}$$

The important thing to note here is that the two positions are mirror images of each other.

Currency Swap

A currency swap works much like an interest rate swap, but there are several key differences:

- A currency swap involves the exchange of both principal and interest rate payments in different currencies.

- Currency swaps use the spot exchange rate.
- Because the principals in a currency swap are in different currencies, they are exchanged at the inception of the swap. This ensures the principals have equal value using the spot exchange rate.
- There's no netting of payments in a currency swap again because the payments are not in the same currency.
- The two sets of cash flows in a swap are known as legs.

Currency swaps can be used to:

- Transform a liability in one currency into a liability in a different currency
- Transform an investment in one currency into an asset in another currency

Two companies can also get into a currency swap to exploit their comparative advantages regarding borrowing in different currencies. For example,

- Firm X can borrow in $ at 6%, or in £ at 4%
- Firm Y can borrow in $ at 4.5%, or in £ at 3.2%

If X wants to borrow £, and Y wants to borrow $, the two may be able to able to save on their borrowing costs. That could happen if each borrows in the market in which they have a comparative advantage and then swapping into their preferred currencies for their liabilities.

Valuation of a Fixed for Fixed Currency Swap

Assume that USD 5,000 at a fixed rate of 3% is being received in exchange for 4,000 Euros at a fixed rate of 2.5%. Payments are exchanged every year for three years, and interest rates are annually compounded.

There are two legs present in this example, a USD leg and a EURO leg.

Interest for the USD leg $= 0.03 \times 5,000 = 150$.

Interest for the Euro leg $= 0.025 \times 4,000 = 100$.

Time (years)	USD Cash flow	Euro Euro Cash flow
0	−5,000	+4,000
1	+150	−100
2	+150	−100
3	+150	−100

Assume that a year after the first exchange occurs, the risk-free rate for all maturities in USD is 4.5% and 3.5% in Euros. Assume also that 1 Euro = 1.15 USD.

To value this swap, we follow the below steps;

1. Value the remaining currency X cash flows in currency X terms (time is measured from the valuation date and not from the start of the swap).
2. Value the remaining currency Y cash flows in currency Y terms (time is measured from the valuation date and not from the start of the swap).
3. Convert the value of the currency Y cash flows to currency X at the current exchange rate.

$$\text{Value in USD} = 150(1.045^{-1}) + 150(1.045^{-2} + 150(1.045^{-3}) = 412.3447$$
$$\text{Value in Euros} = 100(1.035^{-1}) + 100(1.035^{-2} + 100(1.035^{-3}) = 280.1637$$

The value of the swap in USD is therefore:

$$\text{Value of the Swap} = 412.3447 - 280.1637 \times 1.15 = 90.156445$$

Other Types of Currency Swaps

- **Floating for fixed**: A floating rate in one currency is exchanged for a fixed rate in another currency.

- **Floating for floating**: A floating rate in one currency is exchanged for a floating rate in another currency.

These two currency swaps are valued by valuing each leg in their respective currencies. In valuing the floating rate, we assume that the forward rate will be realized.

Other Types of Swaps

Equity swap

In an equity swap, one of the parties commits to making payments reflecting the return on a stock, portfolio, or stock index. In turn, the counterparty commits themselves to make payments based on either a floating rate or a fixed rate.

Swaption

A swaption gives the holder the right to enter into an interest rate swap. It's purchased for a premium whose value is determined by the strike rate specified in the swaption. Swaptions can either be American or European

Commodity Swap

A floating (or market or spot) price based on an underlying commodity is traded for a fixed price over a specified period.

Volatility Swap

Historical volatility observed over a certain period of time is applied to the notional principal in exchange for pre-determined fixed volatility applied on the same notional principal.

Credit Default Swap (CDS)

This insures against default by a company or bond. Similar to a car insurance contract, the buyer of the protection pays the seller fixed payments over a specified period. In case there is no default, the seller does not pay anything and simply receives the protection fixed payments. In case of default, the seller then pays the buyer the notional amount.

Credit Risk

Swaps can give rise to credit risk, especially when no collateral has been posted. The initial pricing of a transaction should, therefore, consider expected credit losses by each party.

Credit risk is greatly eliminated from transactions done through a central counterparty (CCP), and this is because the CCP requires both initial and variation margins to be posted. The margins can then be transferred to the affected party in case of a default.

Practice Question

A steel manufacturing firm recently issued a $500 million fixed-rate debt at 3% per annum to fund an ambitious expansion plan. The chief risk manager at the firm has advised that the firm convert this debt into a floating rate obligation by tapping into the interest rate swap market. In this regard, he has identified four other firms interested in swapping their debt from floating to a fixed rate. The table below provided the various rates at which all the five firms can borrow:

Firm	Fixed rate (%)	Floating rate 6-month LIBOR +
Steel	5.0	2.5
Firm X	4.5	1.0
Firm Y	7.0	4.0
Firm Z	6.5	2.5
Firm T	6.0	3.5

Identify the firm with which the manufacturer stands to yield the greatest possible combined benefit.

A. Firm T

B. Firm Y

C. Firm X

D. Firm Z

The correct answer is **D**.

Firm	Fixed rate	Floating rate LIBOR +	Fixed spread	Floating spread	Possible benefit
Steel	5.0	2.5			
Firm X	4.5	1.0	−0.5	−1.5	1.0
Firm Y	7.0	4.0	2.0	1.5	0.5
Firm Z	6.5	2.5	1.5	0.0	1.5
Firm T	6.0	3.5	1.0	1.0	0.0

The net borrowing savings by entering into a swap is the **difference between the spreads**, i.e., $\Delta\text{fixed} - \Delta\text{floating}$.