

FRM Part I Exam

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Questions with Answers - Valuation and Risk Models

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Reading 45: Measures of Financial Risk

Q.939 Billy Marquette has recently joined a small company that provides private commercial jets to royal families, government officials, and directors of big firms. Marquette is a retired commercial pilot with a very basic understanding of finance. On his first day, he is handed a report on risk management measures. The excerpt from the report says “due to volatility in oil prices, the company has a weekly 90% VaR of €20,000”. Which of the following is the most appropriate explanation of the excerpt?

- A. There is a 90% probability that the company will experience a loss of €2,000 on a weekly basis.
- B. There is a 10% probability that the company will experience a loss of €20,000 in any given week.
- C. There is a 90% probability, in any given week, that the company will experience a loss of more than €20,000.
- D. There is a 10% probability, in any given week, that the company will experience a loss in excess of €20,000.

The correct answer is **D**.

VaR is a probabilistic risk measure that measures the potential loss in the value of the portfolio at any given time.

The “weekly 90% VaR” means that we are 90% confident that the company will lose not more than €20,000 in any given week. Put differently, there's a 10% chance that the company will lose more than €20,000.

Q.975 Anshuman, a risk consultant working at Dominic Republic Bank, uses VaR to measure the risks of his bank's positions. He makes the following statements in his consultant report with regard to VaR. Which of the following statement(s) can be accepted by the risk committee of the firm?

- I. VaR is simply the negative of the q_p quantile of the profit and loss (P/L) distribution
- II. VaR is defined contingent on two arbitrarily chosen parameters: a confidence level and a holding or horizon period
- III. VaR not only rises with the confidence level but rises at a decreasing rate

- A. I only.
- B. I and II only.
- C. II and III only.
- D. All of the above.

The correct answer is **B**.

Statement I is correct: VaR is simply the negative of the q_p quantile of the profit and loss (P/L) distribution (where α is the confidence level, and $p = 1 - \alpha$).

Statement II is correct: VaR is defined contingent on two arbitrarily chosen parameters: a confidence level α , which indicates the likelihood that we will get an outcome no worse than our VaR, and which might be any value between 0 and 1; and a holding or horizon period, which is the period of time until we measure our portfolio profit or loss.

Statement III is incorrect: The VaR not only rises with the confidence level but also rises at an **increasing** rate.

Q.978 ANG National Bank intends to use the coherent risk measure to measure the risk of its assets. A risk measure is said to be coherent if it satisfies the properties such as:

- I. Monotonicity
- II. Sub-additivity
- III. Homogeneity
- IV. Translational invariance

- A. I, III & IV only
- B. I, II & IV only
- C. II, III & IV only
- D. All of the above

The correct answer is **D**.

A risk measure summarizes the entire distribution of dollar returns X by one number, $\rho(X)$. There are four desirable properties every risk measure should possess. These are:

- I. **Monotonicity:** If $X_1 \leq X_2$, $\rho(X_1) \geq \rho(X_2)$

Interpretation: If a portfolio has systematically lower values than another, in each state of the world, it must have greater risk.

- II. **Subadditivity:** $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$

Interpretation: When two portfolios are combined, their total risk should be less than (or equal to) the sum of their individual risks. Merging of portfolios ought to reduce risk.

- III. **Homogeneity:** $\rho(kX) = k\rho(X)$

Interpretation: Increasing the size of a portfolio by a factor k should result in a proportionate scale in its risk measure.

- IV. **Translation invariance:** $\rho(X + h) = \rho(X) - h$

Interpretation: Adding cash h to a portfolio should reduce its risk by h . Like X , h is measured in dollars.

Remember that value at risk (VaR) is not a coherent risk measure because it fails the subadditivity test, whereas expected shortfall (ES) is a coherent risk measure.

Q.980 John Adams, a newly recruited junior analyst, is asked to compare expected shortfall and Value at Risk. He jots down the following points in his notes. Which of them are correct?

I. While the expected shortfall (ES) tells what to expect in bad states, i.e., it gives an idea of how bad might it be, Value at Risk tells us nothing other than to expect a loss higher than the Value at Risk itself

II. The expected shortfall-based rule is consistent with expected utility maximization if risks are rankable by a second-order stochastic dominance rule, while a Value-at-Risk-based rule is only consistent with expected utility maximization if risks are rankable by a more stringent first-order stochastic dominance rule

III. The expected shortfall and Value at Risk always satisfy sub-additivity

IV. Finally, the subadditivity of ES implies that the portfolio risk surface will be convex, and convexity ensures that portfolio optimization problems using ES measures, unlike ones that use VaR measures, will always have a unique well-behave optimum

A. I, III & IV only

B. I, II & IV only

C. II, III & IV only

D. All of the above

The correct answer is **B**.

Statement I is correct: While the expected shortfall (ES) tells what to expect in bad states, i.e., it gives an idea of how bad might it be, Value at Risk tells us nothing other than to expect a loss higher than the Value at Risk itself.

Statement II is correct: The expected shortfall-based rule is consistent with expected utility maximization if risks are rankable by a second-order stochastic dominance rule. On the contrary, a Value-at-Risk-based rule is only consistent with expected utility maximization if risks are rankable by a more stringent first-order stochastic dominance rule.

Statement III is incorrect: The expected shortfall always satisfies sub-additivity, while the Value at Risk does not.

Statement IV is correct: Finally, the subadditivity of ES implies that the portfolio risk surface will be convex, and convexity ensures that portfolio optimization problems using ES measures, unlike ones that use VaR measures, will always have a unique well-behaved optimum.

Q.981 Andrew Simons, a risk analyst, is working on risk measures. He is particularly interested in the risk aversion property of risk measures.

Which of the following statement(s) is/are true with regard to the risk aversion property of risk measures?

I. If a user has a 'well-behaved' risk-aversion function, then the weights will rise smoothly, and the rate at which weights rise will be related to the degree of risk aversion: the more risk-averse the user, the more rapidly the weights will rise

II. Expected shortfall is characterized by all losses in the tail region having the same weight implying that the user is risk-neutral between tail-region outcomes

III. In case of Value at Risk (VaR), the weight of the loss associated with a p-value equal to α implies that the user is actually risk-loving

A. I and II only.

B. I only.

C. II and III only.

D. All of the above.

The correct answer is **D**.

Statement I is correct: If a user has a 'well-behaved' risk-aversion function, then the weights will rise smoothly, and the rate at which weights rise will be related to the degree of risk aversion: the more risk-averse the user, the more rapidly the weights will rise.

Statement II is correct: Expected shortfall is characterized by all losses in the tail region having the same weight implying the user is risk-neutral between tail-region outcomes.

Statement III is correct: With Value at Risk (VaR), a larger weight to the loss associated with a p-value equal to α and zero weight to any greater loss implies that the user is actually risk-loving

Q.3325 Which of the following is NOT a property of coherent risk measures?

- A. $Y \geq X \Rightarrow \rho(Y) \geq \rho(X)$
- B. $\rho(X + Y) \leq \rho(X) + \rho(Y)$
- C. $\rho(hX) = h\rho(X)$ for $h > 0$
- D. $\rho(x + n) \leq \rho(X) - n$ for some n

The correct answer is **A**.

Option A gets monotonicity wrong. It should be: $Y \geq X \Rightarrow \rho(Y) \leq \rho(X)$

A risk measure summarizes the entire distribution of dollar returns X by one number, $\rho(X)$. There are four desirable properties every risk measure should possess. These are:

I. **Monotonicity:** If $X_1 \leq X_2$, $\rho(X_1) \geq \rho(X_2)$

Interpretation: If a portfolio has systematically lower values than another, in each state of the world, it must have a greater risk.

II. **Subadditivity:** $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$

Interpretation: When two portfolios are combined, their total risk should be less than (or equal to) the sum of their individual risks. Merging of portfolios ought to reduce risk.

III. **Homogeneity:** $\rho(kX) = k\rho(X)$

Interpretation: Increasing the size of a portfolio by a factor k should result in a proportionate scale in its risk measure.

IV. **Translation invariance:** $\rho(X + h) = \rho(X) - h$

Interpretation: Adding cash h to a portfolio should reduce its risk by h . Like X , h is measured in dollars.

Q.3328 An investment company has a portfolio which has the following ordered performance by historical data. Calculate the expected shortfall $ES_{0.95}$.

Probability	1%	5%	10%	12%	15%
Profit/Loss	-500	-300	-100	-90	-50

- A. 300
- B. 340
- C. 400
- D. 425

The correct answer is **B**.

Given a discrete distribution, the ES is the equivalent of:

$$ES_{\alpha} = \frac{1}{1-\alpha} \sum_{p=0}^{1-\alpha} [p^{\text{th}} \text{ highest loss}] \times [\text{probability of } p^{\text{th}} \text{ highest loss}]$$

At $\alpha = 0.95$,

$$ES_{0.95} = \frac{[(0.01 \times 500) + (0.04 \times 300)]}{0.05} = 340$$

Note: The sum of probabilities in the numerator must sum to $(1 - \alpha)$

Alternative Approach

To calculate the expected shortfall, we must ask ourselves, "If we are in the worst 5% of the loss distribution, what is the expected loss?" The first column of the given table makes it clear that the 5% tail of the distribution is composed of a 1% probability that the loss is 500 and a 4% probability that the loss is 300. Conditional on being in the tail of the distribution, there is, therefore, a 1/5 chance that the loss is 500 million and a 4/5 chance that it is 300. The expected shortfall (in millions of dollars) is, therefore:

$$\left(\frac{1}{5}\right) \times 500 + \left(\frac{4}{5}\right) \times 300 = 340$$

Q.3329 An investment company has a portfolio which has the following ordered performance by historical data. Calculate the expected shortfall, $ES_{0.99}$, i.e., at 99% level of confidence

Probability	1%	5%	10%	12%	15%
Profit/Loss	-500	-300	-100	-90	-50

- A. 168
- B. 400
- C. 460
- D. 500

The correct answer is **D**.

There is only one number 500 beyond 1%, therefore, the average is 500.

Further Explanation

The expected shortfall (also called conditional VaR) is the expected tail loss. It is the average of the worst $100 \times (1 - \alpha)\%$ of losses. For a discrete distribution, ES is derived as:

$$ES_{\alpha} = \frac{1}{1 - \alpha} \sum_{p=0}^{\alpha} (p^{\text{th}} \text{ loss} \times \text{probability of } p^{\text{th}} \text{ loss})$$

In words, to determine the expected shortfall at a level of confidence α , we must find the average of all the outcomes whose probability is less than or equal to $1 - \alpha$.

At a 99% confidence level, the significance level is 1%. To establish the expected shortfall at 5%, we must find the average of all the outcomes whose probability is less than or equal to 1%. In this case,

$$\text{Expected shortfall} = \frac{(0.01 \times 500)}{0.01} = 500$$

Q.3397 A hypothetical portfolio of securities exhibits the following expected losses shown:

Name	Loss (million dollar)	Probability (%)
1	10	40%
2	20	35%
3	50	15%
4	100	5%
5	200	2.5%
6	225	2%
7	250	0.5%

Calculate the expected shortfall at the 95% and 99% confidence level?

- A. ES (95%) = \$225 million; ES (99%) = \$237.5 million
- B. ES (95%) = \$215 million; ES (99%) = \$237.5 million
- C. ES (95%) = \$217.5 million; ES (99%) = \$250 million
- D. ES (95%) = \$225 million; ES (99%) = \$250 million

The correct answer is **B**.

Note that the given data are in expected losses. The expected shortfall (also called conditional VaR) is the expected tail loss. It is the average of the worst $100 \times (1 - \alpha)\%$ of losses. For discrete distribution, ES is derived as:

$$ES_{\alpha} = \frac{1}{1 - \alpha} \sum_{p=0}^{\alpha} (\text{pth loss} \times \text{probability of pth loss})$$

In other words, to determine the expected shortfall at a level of confidence α , we must find the average of all the outcomes whose probability is less than or equal to $1 - \alpha$.

At a 95% confidence level, the significance level is 5%. To establish the expected shortfall at 5%, we must find the average of all the outcomes whose probability is less than or equal to 5%

$$ES_{0.95} = \frac{200 \times 2.5\% + 225 \times 2\% + 250 \times 0.5\%}{5\%} = 215 \text{ million dollars}$$

At a 99% confidence level, the significance level is 1%. To establish the expected shortfall at 1%, we must find the average of all the outcomes whose probability is less than or equal to 1%

$$ES_{0.99} = \frac{225 \times 0.5\% + 250 \times 0.5\%}{1\%} = 237.5 \text{ million dollars}$$

Q.3398 The VaR of a loan portfolio is computed at various confidence levels:

Confidence Level	VaR
95.0%	2%
95.5%	5%
96.0%	6%
96.5%	7%
97.0%	9%
97.5%	10%
98.0%	13%
98.5%	15%
99.0%	20%
99.5%	30%

What is the expected shortfall at the 97.5% confidence level?

- A. 0.1
- B. 0.15
- C. 0.195
- D. 0.2

The correct answer is C.

The expected shortfall at the 97.5% confidence level is computed by averaging all value of risk greater than the 97.5% confidence level.

Expected shortfall at the 97.5% confidence level = $(13\% + 15\% + 20\% + 30\%)/4 = 19.5\%$

Q.3399 Among the given portfolios, which one falls below the Markowitz efficient frontier?

Portfolio	Expected Return	Expected Standard Deviation
A	12%	10%
B	14%	12%
C	15%	12%
D	16%	20%

- A. Portfolio A
- B. Portfolio B
- C. Portfolio B and C
- D. Both Portfolio B and D

The correct answer is **B**.

The efficient frontier represents the set of optimal portfolios that offer the highest expected return for a defined level of risk or the lowest risk for a given level of expected return. Any portfolio that lies below the efficient frontier is sub-optimal because it does not provide enough return for the level of risk.

Although portfolios B and C have the same level of risk, C offers a higher return per unit of risk. B is a sub-optimal allocation since portfolio C, which has the same level of risk, offers a greater return, hence portfolio B must fall below the efficient frontier.

Even though Portfolio D offers the lowest return per unit of risk, there is not enough information to tell whether it falls below the efficient frontier.

Q.3588 Which of the following statement is INCORRECT regarding the efficient frontier?

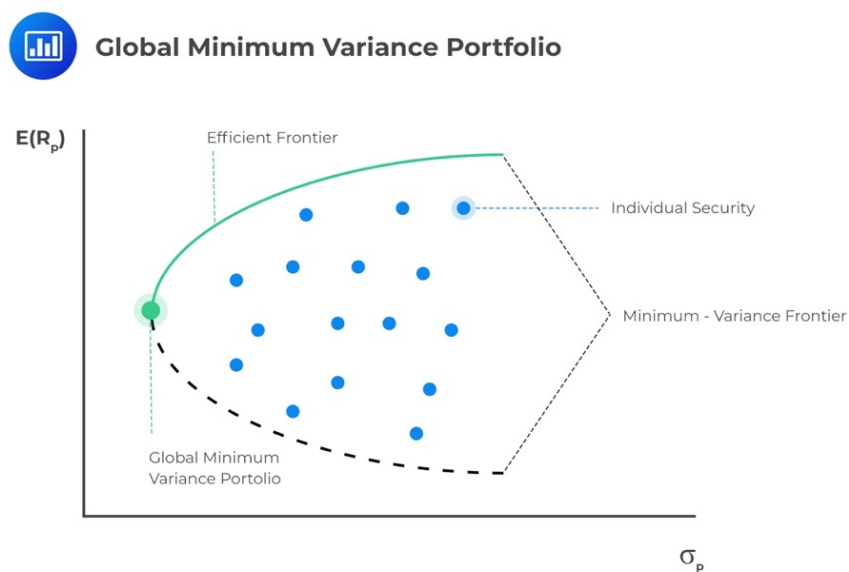
- A. A risk-averse investor will only choose portfolios along the efficient frontier
- B. Portfolios that deliver the greatest return on each level of standard deviation make up the efficient frontier
- C. The graph that shows the efficient frontier has the variance on its Y-axis
- D. None of the above

The correct answer is **C**.

The graph that demonstrates the efficient frontier has the standard deviation at its **X-axis** and the return on its Y-axis.

Option A is a correct statement. A well-known fact is that "presumably, investors are risk-averse; hence, they will choose portfolios on the efficient frontier." Risk-averse investors can still be rational and will choose portfolios on the left-side portion of the curve. Less risk-averse (more risk-neutral) investors will choose portfolios on the right of the curve. But given this theory, ALL rational investors will choose portfolios on the curve.

Option B is a correct statement. Portfolios that deliver the greatest return on each level of standard deviation (or risk) make up the efficient frontier.



Q.4639 A hypothetical portfolio has an annual 1% VaR of \$45,000. Which of the following statements is the **most likely** correct about the portfolio?

- A. The loss over the next year is expected to be at most \$45,000 in 1% of the cases.
- B. There is only a 1% chance that we will gain more than \$45,000 over the next year.
- C. The likelihood of losing more than \$45,000 over the next year is 1%.
- D. The likelihood of losing no more than \$45,000 over the next year is 1%.

The correct answer is **C**.

The VaR gives the maximum amount of loss that can be incurred at a given level of confidence. An annual 1% VaR of 45,000 means that we're 99% confident we'll lose no more than 45,000 over the next year. It would also be correct to say that there's a 1% likelihood (probability) of the loss exceeding \$45,000. Put differently, the probability of losing more than \$45,000 over the next year is 1%.

Option A is incorrect. The loss over the next year is expected to be at most \$45,000 in **99%** of the cases

Option B is incorrect: There is only a 1% chance that we will **lose** more than \$45,000 over the next year

Option D is incorrect: The likelihood of losing no more than \$45,000 over the next year is **99%**

Note: 1% VaR refers to the VaR at the 99% level of confidence.

Q.4640 The investment returns and the corresponding probabilities are given in the following table:

Returns	Probability
20%	0.1
30%	0.3
-10%	0.2
15%	0.3
7%	0.1

What is the standard deviation of the investment returns?

- A. 0.142
- B. 0.154
- C. 0.132
- D. 0.138

The correct answer is **A**.

The variance of the return R is given by:

$$\text{Var}(R) = E(R^2) - [E(R)]^2$$

Now,

$$E(R^2) = 0.1 \times (20\%)^2 + 0.3 \times (30\%)^2 + 0.2 \times (-10\%)^2 + 0.3 \times (15\%)^2 + 0.1 \times (7\%)^2 = 0.04024$$

$$E(R) = 0.1 \times 20\% + 0.3 \times 30\% + 0.2 \times -10\% + 0.3 \times 15\% + 0.1 \times 7\% = 0.142$$

$$\therefore \text{Var}(R) = 0.04024 - (0.142)^2 = 0.020076$$

Thus the standard deviation is given by:

$$\sqrt{0.020076} = 0.1417 = 14.17\%$$

Q.4641 An investor invests his funds in two correlated assets, A and B. The standard deviation of asset A is 20%, and that of B is 15%. The portfolio variance is 2.84%. Given that the investor has three times as much money in asset A than he has in asset B, what is the correlation coefficient between assets A and B?

- A. 0.0962
- B. 0.2133
- C. 0.3994
- D. 0.8078

The correct answer is C.

The variance of a portfolio is given by:

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2\rho w_A w_B \sigma_A \sigma_B$$

Where

w_A : the weight of asset A

w_B : the weight of asset B

σ_A : standard deviation of asset A

σ_B : standard deviation of asset B

ρ : correlation coefficient between asset A and B

Now let the amount invested in asset be B be w_B and thus:

$$\begin{aligned} w_A &= 3w_B \\ w_B + 3w_B &= 1 \\ \therefore w_B &= \frac{1}{4} \Rightarrow w_A = \frac{3}{4} \end{aligned}$$

Now,

$$\begin{aligned} w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2\rho w_A w_B \sigma_A \sigma_B &= 0.0284 \\ \left(\frac{3}{4}\right)^2 \times 0.2^2 + \left(\frac{1}{4}\right)^2 \times 0.15^2 + 2 \times \frac{3}{4} \times \frac{1}{4} \times 0.2 \times 0.15 \times \rho &= 0.0284 \\ \Rightarrow \rho &= \frac{0.0284 - \left(\frac{3}{4}\right)^2 \times 0.2^2 - \left(\frac{1}{4}\right)^2 \times 0.15^2}{2 \times 0.15 \times 0.2 \times \frac{1}{4} \times \frac{3}{4}} = 0.39944 \end{aligned}$$

Q.4642 The losses from a portfolio for one year are normally distributed with a mean of -10 and a standard deviation of 20. What is the value of the one-year 99% VaR?

- A. 35.6
- B. 36.5
- C. 37.5
- D. 39

The correct answer is **B**.

Denote the VaR level by t , then we need:

$$P(X < t) = 0.99$$

(Note we can also use $P(X < t) = 0.01$.)

Standardizing the normal distribution with a given mean and standard deviation, we have:

$$\begin{aligned} P\left(z < \frac{t - (-10)}{20}\right) &= 0.99 \\ \Rightarrow \Phi\left(\frac{t + 10}{20}\right) &= (0.99) \\ \frac{t + 10}{20} &= \Phi^{-1}(0.99) \\ \Phi^{-1}(0.99) &= 2.326 \end{aligned}$$

$$\text{VaR} = -10 + 2.326 \times 20 = 36.52$$

Q.4643 The losses from a portfolio for one year are normally distributed with mean -10 and standard deviation 20. What is the value of the 99% expected shortfall?

- A. 52.85
- B. 37.40
- C. 42.85
- D. 26.43

The correct answer is **C**.

We know that:

$$ES = \mu + \sigma \left(\frac{e^{-\frac{u^2}{2}}}{(1 - X) \sqrt{2\pi}} \right)$$

Now, $U = \Phi^{-1}(0.99) = 2.33$

$$ES = -10 + 20 \left(\frac{e^{-\frac{2.33^2}{2}}}{(1 - 0.99) \sqrt{2\pi}} \right) = 42.85$$

Reading 46: Calculating and Applying VaR

Q.972 After using of the historical simulation method, you have been provided with the following 30 ordered percentage returns for an asset:

[-18, -16, -14, -12, -10, -9, -7, -7, -6, -6, -6, -5, -5, -4, -4, -4, -2, -1, 0, 0, 2, 3, 6, 12, 12, 13, 15, 15, 18, 28]

The value-at-risk (VaR) and expected shortfall (ES), at 90% confidence level, respectively, are closest to:

A. Var: 14; ES: 17

B. Var: 14; ES: 16

C. Var: 12; ES: 16

D. Var: 12; ES: 24

The correct answer is **A**.

VaR can be calculated as the $[(1 - 0.9) \times 30] = 3^{\text{rd}}$ worst observation, which is -14 and hence the VaR is 14.

The ES is the arithmetic average of losses that are **worse** than the VaR. Thus,

$$ES = \frac{(18 + 16)}{2} = 17$$

Q.973 Stuart Broad, a risk analyst working with Macquarie Bank compiles data of 100 simulated percentage returns of the bank's assets:

[-13.33, -12.25, -11.75, -10.66, -8.45, -7.13, -6.48, -5.29 ... 2.89, 3.56, 4.29, 5.38, 6.65, 7.89, 8.54, 9.64, 10.27, 12.28, 13.25]

Using the data, he calculates the expected shortfall (ES) and the value at risk (VAR) of the bank's assets at the 95 percent confidence level using the historical simulation method. What is the expected shortfall and the value at risk computed by Stuart Broad?

- A. Expected shortfall: 12; Value at Risk: 9.45
- B. Expected shortfall: 11.49; Value at Risk: 9.95
- C. Expected shortfall: 12; Value at Risk: 8.45
- D. Expected shortfall: 11.49; Value at Risk: 8.45

The correct answer is **C**.

VaR can be calculated as the $[(1 - 0.95) \times 100] = 5^{\text{th}}$ worst observation, which is -8.45 and hence the VaR is 8.45.

The ES is the arithmetic average of losses that are **worse** than the VaR. Thus,

$$ES = \frac{13.33 + 12.25 + 11.75 + 10.66}{4} = 12$$

Q.982 ANC National Bank handles a portfolio of assets amounting to USD 150 million. Antony Smith, a risk analyst, analyzes the portfolio and observes that the returns are independently identically normally distributed. The annual standard deviation of the portfolio is 0.55. What is the 20 day-Value at Risk at 95 percent confidence assuming 250 trading days in a year?

- A. USD 51.92 million
- B. USD 38.50 million
- C. USD 53.67 million
- D. USD 71.50 million

The correct answer is **B**.

$$\begin{aligned}
 \text{VaR} &= Z \times \text{standard deviation} \times \text{portfolio amount} \\
 \text{Annual VaR} &= 1.65 \times 0.55 \times 150,000,000 = 136,125,000 \\
 \text{Var (T days)} &= \text{1-day VaR} \times \sqrt{T} \dots \dots \text{formula} \\
 \text{1-day VaR} &= \frac{\text{Annual VaR}}{\sqrt{250}} = \$8,609,300 \\
 \text{20-day VaR} &= (\text{1-day VaR}) \times \sqrt{20} = \$38,501,694
 \end{aligned}$$

It's the standard deviation, σ , that determines where you start. If the σ given is annual, you'll start by calculating the annual VaR and then use the square root of the time rule to come up with VaR for shorter periods. Similarly, if given the daily σ , compute the 1-day VaR and then convert that appropriately.

Q.1146 All the following are false with regards to Worst Case Scenario (WCS) measure, EXCEPT:

- A. WCS indicates the number of times portfolio loss exceeds a given limit over a given period.
- B. WCS indicates the maximum a portfolio can lose over a given period.
- C. WCS indicates the probability of losing a given limit over a given period.
- D. WCS indicates the minimum a portfolio can lose over a given period.

The correct answer is **B**.

Worst Case Scenario (WCS) indicates the maximum a portfolio can lose over a given period. For instance, it can indicate that a portfolio may lose a maximum of 10% over 100 trading days, whereas VaR indicates the probability of losing a particular limit over a period.

Q.1147 Plain vanilla European options and forwards are good examples of:

- A. Linear derivatives.
- B. Nonlinear derivatives.
- C. A nonlinear and linear derivative, respectively.
- D. A linear and nonlinear derivative, respectively.

The correct answer is C.

A linear derivative is one whose value is directly related to the market price of the underlying variable. If the underlying makes a move, the value of the derivative moves with a nearly identical margin. In fact, there is a 1:1 relationship between the derivative and the underlying – explaining why linear derivatives are said to be “delta-one” products. However, the delta itself need not always be equal to 1. Examples of linear derivatives include futures and forwards.

A non-linear derivative is one whose value/payoff changes with time and space. Space in this case refers to the location of the strike/exercise price with respect to the spot/current price. The payoff varies with the value of the underlying but also exhibits some non-linear relationship with other variables, including interest rates, dividends, or even volatility. Non-linear derivatives are generally referred to as options. For non-linear derivatives, delta is not constant. Rather, it keeps on changing with the change in the underlying asset. Examples include the Vanilla European option, Vanilla American option, Bermudan option, etc.

Q.2616 Arthur Bell is the portfolio manager at FFF Investments. Recently, he bought 5,000 call options on stocks of one of the local growth-oriented oil refining companies that have never paid dividends. The strike price of the options was \$50. The underlying stock is trading at \$58 and has an annual volatility of return of 33%. Bell estimated the delta of these options to be 0.55. What is the approximate weekly (delta normal) 99% VaR of the position assuming 52 trading weeks in the year?

- A. \$7,725.57
- B. \$17,007
- C. \$6,659.97
- D. \$14,661.22

The correct answer is **B**.

In the delta-normal approach, we first calculate the VaR of the underlying. Then, we use the equation below to revalue the derivative by linear approximation (as the delta multiplied by the VaR of the underlying):

$$\begin{aligned}\text{VaR}_{\text{derivative}} &= \Delta \times \text{VaR}_{\text{underlying}} \\ \text{VaR of the underlying (stock)} &= \sigma \times z_{\text{cl}\%} \times \text{market value}\end{aligned}$$

But first, we have to convert the annual volatility, i.e., σ , into a weekly value:

$$\text{Annual volatility} = \text{weekly volatility} \times \sqrt{52}$$

$$\text{Thus, weekly volatility} = \frac{(33\%)}{\sqrt{52}} = 0.045763$$

$$\text{VaR of the underlying} = 0.045763 \times 2.33 \times \$58 = \$6.18438$$

$$\text{VaR of call option} = 0.55 \times \$6.18 = \$3.40$$

$$\text{VaR of 5,000 call options} = \$3.40 \times 5,000 = \$17,007$$

Note: 2.33 is the standard normal deviate at a 99% confidence level.

Q.2618 An investor is long at a short-term portfolio of at-the-money put options on an underlying asset. The notional value of the portfolio is \$100,000. Assuming that there is one risk factor with a corresponding delta of 0.5, what is the amount of change in the portfolio if the value of the underlying asset changes by 12.5%?

- A. \$5,500.
- B. \$6,250.
- C. \$6,520.
- D. \$7,500.

The correct answer is **B**.

The change in the portfolio value arising from the change in a risk factor is

$$\Delta P = \delta \Delta S$$

Where

ΔP = change on the portfolio

δ = Delta corresponding to a risk factor

ΔS = change in the underlying asset

Thus in this case we have:

$$\Delta P = 0.5 \times 0.125 \times 100,000 = 6,250$$

Q.2619 After a significant increase in the volatility of shares of USY Pharma, Ross Grand, senior portfolio manager, decides to hedge the existing position by buying 300 at-the-money call options. Shares of USY are trading at \$457 and the daily VaR of the underlying at 99% confidence is \$42.59. What is the daily 99% VaR of the options position using the delta-normal method?

- A. \$3,457.99
- B. \$5,791.77
- C. \$4,524.05
- D. \$6,388.50

The correct answer is **D**.

The key point here is that the call options are at the money, so we can assume $\text{delta} = 0.5$. (See the note at the end of the explanation if this is unclear.)

According to the delta-normal method,

$$\begin{aligned}\text{VaR of an option} &= \text{Delta} \times \text{VaR of underlying} \\ \text{VaR of the position} &= 0.5 * \$42.59 * 300 \text{ options} = \$6,388.50\end{aligned}$$

Note: At-the-money call options typically have a delta of 0.5, and the delta of out-of-the-money call options approaches 0 as expiration nears. The deeper in-the-money the call option, the closer the delta will be to 1, and the more the option will behave like the underlying asset. Since the delta of an option ranges between 0 and 1, an at-the-money option is right in between those two numbers.

For put options, at-the-money put options typically have a delta of -0.5, and the delta of out-of-the-money call options approaches 0 as expiration nears. The deeper in-the-money put option, the closer the delta will be to -1,

Q.2623 Ashli More prepares a presentation to the management board on the application of derivatives for hedging risk. She struggles with the classification of linear and non-linear derivatives. Which of the following is an example of linear derivatives?

- A. Futures on stocks, forwards on broad market indices, and plain vanilla European options on bonds.
- B. Interest rate swaps, interest rate caps, and plain vanilla American options on stocks.
- C. Futures on broad market indices.
- D. Futures on stocks and swaptions.

The correct answer is C.

Prices of both futures are changing proportionally to the changes in underlying.
Options A, B, D are incorrect, as they all include options that are non-linear derivatives.

Q.2625 The investment division of a bank is considering an investment of \$100,000,000 in one of the following:

- stocks of ALPHA Plc
- stocks of APPA Corporation
- 20-year bonds of APPA Corporation

Although the department seeks the investment with the highest expected return, to comply with bank's risk policies, the department cannot open a position with a daily 99% VaR higher than \$6,000,000.

The investment opportunities presented above have the following features:

	Expected Annual Return	Expected annual volatility of returns	Modified Duration	Market Price
Stocks of ALPHA	20%	50%	–	\$5 per stock
Stocks of APPA	13%	36%	–	\$30 per stock
Bonds of APPA	8%	10%	3.5	99% per \$100 nominal

Assuming zero daily returns and 252 trading days per year, which investment should the bank choose?

- A. Stocks of ALPHA
- B. Stocks of APPA
- C. Bonds of APPA
- D. None of the above

The correct answer is **B**.

$$\text{VaR of ALPHA's stocks} = 50\% \times \frac{1}{\sqrt{252}} \times 2.33 \times \$100,000,000 = \$7,338,810.18$$

$$\text{VaR of APPA's stocks} = 36\% \times \frac{1}{\sqrt{252}} \times 2.33 \times \$100,000,000 = \$5,283,943.33$$

$$\text{VaR of APPA's bond} = 10\% \times \frac{1}{\sqrt{252}} \times 2.33 \times \$100,000,000 = \$1,467,762.036$$

Both the APPA's bond and APPA's stock have a VaR lower than \$6,000,000, but the bank should choose to invest in APPA's stock since it has a higher return than APPA's bond.

Q.2626 Two managers - X and Y - are looking to establish the 1-day VaR for a long position in an at-the-money call option on a non-dividend-paying stock with the following information: Current stock price: USD 100

Estimated annual stock return volatility: 15%

Current Black-Scholes-Merton call option value: USD 4.80,

Call option delta: 0.5

To compute VaR, manager X uses the delta-normal model, while manager Y opts for the Monte Carlo simulation method for full revaluation. Which manager will estimate a higher value for the 1-day 99% VaR?

- A. Manager Y.
- B. Manager X.
- C. Both managers will have the same VaR estimate.
- D. Insufficient information to determine.

The correct answer is **B**.

The correct answer is B.

Options are nonlinear derivatives, i.e., their value is related to the market price of the underlying variable, but under a convex, non-linear relationship. The payoff of such products varies with the value of the underlying, but also with other elements (interest rates, volatility, dividends, etc.)

The option's price function is convex with respect to the value of the underlying. And for such a non-linear portfolio, the delta-normal model provides only a linear approximation which does not capture the positive effect of this curvature on the portfolio value. It understates the probability of high option values and overstates the probability of low option values.

Therefore, for a long position in the call option, VaR and the expected shortfall under the delta normal model will be extremely high.

On the other hand, for a short position in the call option, the VaR and the expected shortfall under the delta-normal model will be extremely low.

Q.2627 Which of these statements regarding the Structured Monte Carlo (SMC) simulation is INCORRECT?

- A. The SMC assumes normally distributed returns of derivative's underlying.
- B. The SMC assumes that distributions used in the simulation are relevant going forward.
- C. The SMC can generate correlated scenarios for multiple risk factors based on corresponding statistical distributions.
- D. Increasing the number of SMC simulations will always improve the outcome of the simulation.

The correct answer is **D**.

In cases when assumed distributions itself or distribution's parameters are no longer relevant, SMC simulation outcomes will become not reliable and increasing number of simulations will not solve the problem.

Q.3305 Bank A manages interest rate risk by monitoring the VaR using historical data. Bank A collects interest rate returns for 300 days and the data is sorted ascendingly. The lowest 10 interest rate returns are -4.2%, -4.0%, -3.8%, -3.2%, -3.0%, -2.5%, -2.3%, -2.2%, -2.0%, -1.7%. After 30 days, Bank A collects 30 more data points. However, none of these returns is less than -1.7%. What is the change in the 98% VaR as compared to the prior 30 days, assuming that all of the lowest 10 observations are still within the 300-day long historical window?

- A. Unchanged.
- B. VaR has increased by 0.01%.
- C. VaR has increased by 0.08%.
- D. VaR has increased by 0.11%.

The correct answer is **A**.

In general, if there are n ordered observations, and a confidence level $cl\%$, the $cl\%$ VaR is given by the $[(1 - cl\%) n]$ th highest observation. This is the observation that separates the tail from the body of the distribution. For instance, if we have 1,000 observations and a confidence level of 95%, the 95% VaR is given by the $(1 - 0.95)1,000 = 50$ st observation. There are 50 observations in the tail.

Given this information, the 98% VaR, given that $n = 300$ is:

98% VaR = $[(1 - 0.98)300]$ th value = 6th highest observation

Thus, VaR = -2.5%

One important note about this method is that the length of the historical window is fixed, i.e., the oldest observations exit the window as new observations are made. In this case, however, 30 more days have now elapsed (meaning that we have 30 new observations) but there hasn't been a loss big enough to dislodge any of the worst 10 observations made, which we assume are still within the historical window of 300 observations. As such, the 98% VaR will **still** be the 6th highest observation which happens to be -2.5%. In short, the VaR remains unchanged.

Q.3307 Bank A manages interest rate risk by monitoring the VaR calculated using historical data. Bank A collects interest rate returns for 300 days and the data is sorted ascendingly. The lowest 10 interest rate returns are -4.2%, -4.0%, -3.8%, -3.2%, -3.0%, -2.5%, -2.3%, -2.2%, -2.0%, -1.7%. Calculate the 98% VaR.

- A. -2.0%
- B. -2.2%
- C. -2.5%
- D. -2.4%

The correct answer is C.

In general, if there are n ordered observations, and a confidence level $cl\%$, the $cl\%$ VaR is given by the $[(1 - cl\%)n + 1]$ th highest observation. This is the observation that separates the tail from the body of the distribution. In this case, we have 300 observations and a confidence level of 98%; the 98% VaR is given by the $(1 - 0.98)300 = 6$ th observation, that's 2.5%

Q.3313 An option on the INMEX (Mexican) stock index is struck on 2,522 pesos. The delta of the option is 0.6, and the annual volatility of the index is 25%. Using delta-normal assumptions, what is the 10-day VaR of the option at the 95% confidence level? Assume 260 days per year.

- A. 204 pesos
- B. 61.0 pesos
- C. 115.8 pesos
- D. 122.4 pesos

The correct answer is **D**.

As per the delta-normal method, the VaR of a derivative position is given by:

$$\text{VaR}_{\text{Derivative}} = \text{Delta} \times \text{VaR}_{\text{Underlying risk factor}}$$

Thus, the first step is to determine the VaR of the underlying risk factor. i.e., the index. But before that, notice that the question asks for the 10-day VaR, which means it is important that we work out the 1-day VaR and then use the square root of time rule:

$$\begin{aligned}\text{T-day volatility} &= \text{1-day Volatility} \times \sqrt{T} \\ \text{260-day volatility} &= \text{1-day volatility} \times \sqrt{260}\end{aligned}$$

$$\text{Thus, 1-day volatility} = \frac{\text{260-day volatility}}{\sqrt{260}} = \frac{0.25}{\sqrt{260}} = 0.0155$$

Hence,

$$\begin{aligned}\text{1-day VaR of the underlying} &= 2,522 \times 0.0155 \times 1.65 = 64.5 \text{ pesos} \\ \text{10-day VaR} &= \text{1-day VaR} \times \sqrt{(10)} = 64.5 \times \sqrt{(10)} = 204 \text{ pesos}\end{aligned}$$

Finally,

$$\begin{aligned}\text{VaR}_{\text{Derivative}} &= \text{Delta} \times \text{VaR}_{\text{Underlying risk factor}} \\ &= 0.6 \times 204 = 122.4 \text{ pesos}\end{aligned}$$

Note that this questions mixes concepts from the chapters Calculating and Applying VaR and Measuring and Monitoring Volatility.

Q.3314 A futures contract on the S&P 500 is defined as a dollar multiple of the index level. The S&P 500 future traded on the Chicago Mercantile Exchange is defined as a \$250 index. The 1% VaR of the S&P 500 index is 2.45. What is the 1% VaR of the S&P 500 futures contract?

- A. \$61.25
- B. \$512.50
- C. \$612.50
- D. \$1,225

The correct answer is **C**.

Since we already know that the 1% VaR of the S&P 500 index is 2.45, to find the 1% VaR of the S&P 500 futures contract, we simply multiply the contract value by the 1% VAR:

$$(\$250 \times 2.45 = \$612.50).$$

Q.3316 Consider a non-linear portfolio that depends on a share price. Given that delta and gamma of the portfolio are 20 and 2.5, respectively. All else constant, what is the corresponding portfolio change to a stock price increase of \$ 0.5?

- A. \$ 9.45
- B. \$ 10.63
- C. \$ 9.38
- D. \$ 10.31

The correct answer is **D**.

Given the delta and gamma of a portfolio, the change in portfolio is given by :

$$\Delta P = \delta \Delta S + \frac{1}{2} \gamma (\Delta S)^2$$

Where

δ = Delta

ΔS = Change in the share price

γ = Gamma

ΔP = Change in portfolio value. Thus,

$$\Delta P = 20 \times 0.5 + \frac{1}{2} \times 2.5 \times 0.5^2 = 10.3125$$

Q.3318 You have been asked to estimate the VaR of GreenWood Corp. The company's stock is currently trading at USD 308 and the stock has a daily volatility of 1.25%. Using the delta-normal method, the VaR at the 95% confidence level of a long position in an at-the-money put on this stock with a delta of -0.5 over a 1-day holding period is closest to:

- A. USD 3.12
- B. USD 2.15
- C. USD 3.167
- D. USD 4.52

The correct answer is C.

In the case of a linear derivative, VaR scales directly with the underlying risk factor, i.e.,

$$\begin{aligned}\text{VaR}_{\text{linear derivative}} &= \Delta \times \text{VaR}_{\text{underlying risk factor}} \\ \text{VaR} &= \Delta \times 1.645 \times \sigma \times S_0 = 0.5 \times 1.645 \times 0.0125 \times 308 \\ &= 3.1666\end{aligned}$$

Note: Just as the question dictates, this is an estimate: the accurate relationship is non-linear and we are actually omitting the curvature (option gamma). Also, we ignore the negative sign in the solution since a negative amount is implied.

Note 2: This question mixes concepts from the chapters Calculating and Applying VaR and Measuring and Monitoring Volatility.

Q.3320 A market risk manager has gathered historical P&L data for his financial institution over the last 100 days. He intends to determine the VaR and the conditional VaR (CVaR) at 90% level of confidence using the historical simulation method. The worst 15 observation gathered (in million CAD) are:

[-25, -27, -27, -28, -30, -32, -36, -38, -40, -43, -45, -52, -56, -58, -60]

Calculate the VaR and the conditional VaR (CVaR).

A. VaR = 30; Conditional VaR = 46.

B. VaR = 32; Conditional VaR = 47.6.

C. VaR = 36; Conditional VaR = 47.6.

D. VaR = 32; Conditional VaR = 46.

The correct answer is **B**.

We arrange the data in ascending order first:

-60, -58, -56, -52, -45, -43, -40, -38, -36, -32, -30, -28, -27, -25

VaR can be calculated as the $[(1 - 0.9) \times 100] = 10^{\text{th}}$ worst observation, which is -32 and hence the VaR is 32.

Conditional VaR is simply another name for the expected shortfall. The ES is the arithmetic average of losses that are worse than the VaR. Thus,

$$ES = \frac{(60 + 58 + 56 + 52 + 45 + 43 + 40 + 38 + 36)}{9} = 47.6$$

Note: Refer to table 2.3 of your chapter for proof.

Q.3394 A liquid asset K has a profit/loss distribution that's independent and identically distributed. The position has a one-day VaR of \$50,000 at the 95% level of confidence. Estimate the 10-day VaR of the same position at the 99% level of confidence.

- A. \$115,114
- B. \$70,000
- C. \$223,956
- D. \$200,000

The correct answer is C.

The first step should be to convert the one-day VaR at 95% to a one-day VaR at 99%. To do this, recall that:

$\text{VaR}_{95\%}$ uses the upper 5% point of the normal distribution, i.e., 1.645

And the $\text{VaR}_{99\%}$ uses the upper 1% point of the normal distribution, i.e., 2.326

Therefore, to convert the one-day VaR at 95% to a one-day VaR at 99%, we must multiply the former by a scale of $\frac{2.33}{1.645}$

$$\text{One-day VaR}_{99\%} = \frac{2.33}{1.645} \times \$50,000 = \$70,821$$

Next, recall that:

$$\text{VaR}(T \text{ days}) = \text{VaR}(1 \text{ day}) \times \sqrt{T}$$

Therefore,

$$\text{VaR}_{10 \text{ days}} = \$70,821 \times \sqrt{10} = \$223,955.67$$

Q.3395 Peter McLeish is a risk analyst at Quantum Bank. After estimating the 99%, one-day VaR of the bank's portfolio using historical simulation with 900 past days, he is concerned that the VaR is providing too little information on tail losses. He embarks on a deeper examination of simulation results. Sorting the simulated daily P&L from worst to best, he constructs the following table:

P & L Rank	1	2	3	4	5
	-2,000	-1,860	-1,800	-1,720	-1,630
P & L Rank	6	7	8	9	10
	-1,500	-1,400	-1,310	-1,260	-1,190
P & L Rank	11	12	13	13	15
	-1,110	-1,050	-990	-820	-750

Determine the 99%, one-day expected shortfall of the portfolio:

- A. 1,260
- B. 1,653
- C. 1,190
- D. 1,609

The correct answer is **B**.

When dealing with discrete data, let's first find the VaR. VaR can be calculated as the $[(1 - 0.99) \times 900] = 9^{\text{th}}$ worst observation, which is -1,260 and hence the VaR is 1,260.

The ES is the arithmetic average of losses that are worse than the VaR. Thus,

$$ES = (2,000 + 1,860 + 1,800 + 1,720 + 1,630 + 1,500 + 1,400 + 1,310)/8 = 1,652.50$$

Q.4665 Which of the following is **NOT** true about the historical simulation method of estimating VaR?

- A. This method estimates VaR by using a lookback period.
- B. This method assumes that the past performance of a portfolio is a good indicator of the near future.
- C. This method assumes asset price returns and volatility follow a normal distribution.
- D. All of the above.

The correct answer is **C**.

Unlike the Monte Carlo and the parametric methods, the historical simulation method does not need any distributional assumptions to estimate VaR as it uses historical data to perform the estimation.

Option A is incorrect: The historical simulation method's VaR estimates depend on the lookback period and, thus, forecasts are highly dependent on the sample data features. This is the major drawback of the method.

Option B is incorrect: The historical simulation method assumes that the past performance of a portfolio is a good indicator of the near future. It involves reorganizing actual historical returns by putting them in order (from worst to best) and then assumes that the trend will repeat itself, from a risk perspective.

Q.4667 The following are hypothetical ten worst returns for stock TGB from 120 days of data. Find the 1-day 95% VaR and expected shortfall, respectively. -15.72%, -10.92%, -6.50%, -3.56%, -6.90%, -2.50%, -5.30%, -4.31%, -12.12%, -3.45%,

- A. 0.053, 0.1043
- B. 0.056, 0.0958
- C. 0.0431, 0.1149
- D. 0.0431, 0.0958

The correct answer is **A**.

First, we rearrange starting with the worst day, to the least bad day, as shown below:

-15.72%, -12.12%, -10.92%, -6.90%, -6.50%, -5.30%, -4.31%, -3.56%, -3.45%, -2.50%.

In this case, the VaR corresponds to the $(5\% \times 120) = 6^{\text{th}}$ worst day: -5.30%.

This implies ES is the equivalent to an average of the 5 worst returns :

$$ES = \frac{(-15.72\% + -12.12\% + -10.92\% + -6.90\% + -6.5\%)}{5} = -10.432\%$$

So ES is 10.432%

Q.4668 Consider a linear portfolio consisting of short positions in 50 shares, each worth USD 10, and a long position in 150 shares each worth USD 25. What is the relative portfolio change (in USD) of the portfolio if the price of all shares increases by 2%?

- A. 85
- B. -85
- C. 65
- D. -65

The correct answer is **C**.

The change in a linear portfolio is given by:

$$\Delta P = \sum_i n_i \Delta S_i$$

Where

n_i : number of shares of stock i in the portfolio (negative for a short position)

S_i : Price of stock i

So,

$$\begin{aligned}\Delta P &= [-50 \times (0.02 \times 10)] + [150 \times (0.02 \times 25)] \\ &= -10 + 75 = 65\end{aligned}$$

So the change in the portfolio is an increase of USD 65.

Q.4669 In a historical simulation where the risk factor is the stock price, 501-day recent historical data was used to generate 500 scenarios. From the data, the stock price on days 0, 1, 2, 300, ..., 500 are given as USD 20, USD 30, USD 28, USD 26,..., USD 36, respectively. What is the value of the stock price on the 501st day?

- A. 40
- B. 30
- C. 50
- D. 54

The correct answer is **D**.

Assuming that today is 500th day (with a current price of USD 36), we wish to know the stock price on the 501st day. To do this, we use day 0 and 1 change in the stock price from the historical data. Between day 0 and 1, the stock price increase from USD 20 to USD 30. Therefore the stock price on the 501st day is given by:

$$36 \times \frac{30}{20} = \$54$$

Alternatively:

The stock price increased by 50% between day 0 and day 1, therefore we assume that the stock price will also increase by 50% between days 500 and 501, and thus, the stock price on the 501st day is given by:

$$\text{USD } 36 \times 1.50 = \text{USD } 54$$

Q.4670 Given that the one-day standard deviation of a portfolio is 90, what is the 10-day VaR with a 99% confidence level according to the delta-normal model?

- A. 604
- B. 505
- C. 662
- D. 645

The correct answer is **C**.

According to the delta-normal model, we can assume that the mean change in risk factor is zero for a short period of time so that VaR is given by:

$$\text{VaR} = \sigma_P U$$

Where

σ_P : standard deviation of the portfolio

U: point on the normal distribution where X is exceeded

In this case, $\sigma_P=90$ and $U = -2.326$ and thus VaR is given by:

$$90 \times -2.326 = -209.340$$

We know that:

$$\text{VaR}(T, X) = \sqrt{T} \times \text{VaR}(1, X)$$

Thus the 10-day VaR is given by:

$$\sqrt{10} \times -209.340 = -661.99$$

Q.4671 Given that the standard deviation of portfolio change is 90, what is the 10-day expected shortfall with a 99% confidence level according to the delta-normal model?

- A. 640
- B. 759
- C. 650
- D. 749

The correct answer is **B**.

According to the delta-normal model, for a short period of time, we can assume that the mean change in risk factor is zero so that VaR is given by:

$$ES = \sigma_P \left(\frac{e^{-\frac{U^2}{2}}}{(1 - X) \sqrt{2\pi}} \right)$$

Where

σ_P : standard deviation of the portfolio

U: point on the normal distribution where X is exceeded

In this case, $\sigma_P = 90$ and $U = 2.326$ and thus ES is given by:

$$ES = 90 \left(\frac{e^{-\frac{(2.326)^2}{2}}}{(1 - 0.99) \sqrt{2\pi}} \right) = 240.0635$$

But we know that:

$$ES(T, X) = \sqrt{T} \times ES(1, X)$$

Thus the 10-day ES is given by:

$$\sqrt{10} \times 240.06 = 759.1473 \approx 759$$

Q.4674 In the context of the delta-normal model, which of the following statements is **true**?

- I. The delta-normal approach assumes that there is a linear relationship between the portfolio changes and risk factor changes.
- II. The delta normal approach does not consider the curvature of the relationship between the portfolio value and the corresponding risk factors.
- III. Compared to other VaR computation methods, the delta-normal approach is more accurate since it is easily calculated and assumes a normal distribution.

- A. I and II
- B. II only
- C. III only
- D. I and III

The correct answer is **A**.

Statement I is correct: The delta-normal approach assumes that there is a linear relationship between the portfolio changes and risk factor changes and thus works well with linear portfolios.

Statement II is correct: The delta normal approach does not consider the curvature of the relationship between the portfolio value and the corresponding risk factors. Consider the following equation:

$$\Delta P = \sum_i a_i x_i$$

Where when percentage changes are used, $a_i = \frac{\Delta S_i}{S_i}$ and $x_i = \delta_i x_i$ and where actual

changes are considered $a_i = \Delta S_i$ and $x_i = \delta_i$. This equation works well with linear portfolios, but it is an approximation to non-linear portfolios.

Statement III is incorrect: Delta-normal is computationally easy but quite inaccurate compared to other VaR measurement methods. Put more precisely, it may underestimate the occurrence of extreme losses because it relies on the normal distribution.

Q.4675 A risk manager conducts 500 historical simulations to calculate one day, 99% VaR. Which of the following describes the value of VaR in this case?

- A. VaR would be the fifth-worst loss.
- B. VaR would be the average of first, second, third, and fourth-worst losses.
- C. VaR would be the sixth-worst loss.
- D. VaR would be the average of the fifth and sixth loss.

The correct answer is **A**.

For the 500 simulations, when the losses are ordered in descending order, VaR would be the fifth ($=1\% \times 500$) worst loss.

Options B is incorrect: It describes the expected shortfall.

Options C is incorrect: If VaR is a sixth-worst loss, then we would be calculating the one-day 98.8% VaR, which is not what is asked in the question.

Option D is incorrect: This would mean that we want an average of one-day 99% VaR and 98.8% VaR.

Q.4676 The 100-day 99% VaR for a portfolio is 50. What is the corresponding 250-day 99% VaR?

A. 80.54

B. 65.25

C. 79.06

D. 78.54

The correct answer is **C**.

Using the formula

$$\begin{aligned} \text{VaR}(T, X) &= \sqrt{T} \times \text{VaR}(1, X) \\ \Rightarrow \text{VaR}(1, X) &= \frac{\text{VaR}(T, X)}{\sqrt{T}} \end{aligned}$$

We, therefore, need first to calculate the one-day 99% VaR and then translate it to 250-day VaR

$$\text{VaR}(250, 99\%) = \sqrt{250} \times \left[\frac{50}{\sqrt{100}} \right] = 79.06$$

Q.4677 A risk manager realizes over a period of one month, a portfolio he manages increased by USD 5 when the stock price increased by USD 0.1. What is the value of portfolio delta?

- A. 30
- B. 50
- C. 40
- D. 20

The correct answer is **B**.

Greek letter deltas (δ) is defined as:

$$\delta = \frac{\Delta P}{\Delta S}$$

Where ΔS is a small change in risk factors such as stock price and ΔP the corresponding change in the portfolio value.

So,

$$\delta = \frac{5}{0.1} = 50$$

Q.4678 A risk manager estimates 10-day 95% VaR using the delta-model to be USD 50. What is the corresponding one-day expected shortfall at a 95% confidence level?

- A. 66.78
- B. 56.78
- C. 19.82
- D. 94.56

The correct answer is **C**.

Since we are dealing with a short time period, then:

$$\text{VaR} = \sigma_P U$$

$$\text{ES} = \sigma_P \left(\frac{e^{-\frac{U^2}{2}}}{(1 - X) \sqrt{2\pi}} \right)$$

X: confidence level

U : point on the normal distribution where X is exceeded

σ_P : standard deviation of the portfolio change

From the question, we are given that the 10-day 95% VaR is 50, and thus one-day VaR is given by:

$$\frac{50}{\sqrt{10}} = 15.81$$

We have X=95%, then $U = \Phi^{-1}(0.05) = -1.645$.

$$\begin{aligned} \Rightarrow -15.81 &= \sigma_P \times -1.645 \\ \sigma_P &= \frac{-15.81}{-1.645} = 9.612 \end{aligned}$$

Therefore the ES is given by:

$$\text{ES} = \sigma_P \left(\frac{e^{-\frac{U^2}{2}}}{(1 - X) \sqrt{2\pi}} \right) = 9.612 \left(\frac{e^{-\frac{(-1.645)^2}{2}}}{(1 - 0.95) \sqrt{2\pi}} \right) = 19.8220$$

Q.4679 Correlation breakdown is a condition wherein periods of high volatility; correlations tend to be different as compared to normal market conditions. What implication does the correlation breakdown has on VaR and ES?

- A. Calculation of VaR and ES should concentrate on what happens in extreme market conditions.
- B. Calculation of VaR and ES should concentrate on what happens in normal market conditions.
- C. Calculation of VaR and ES should concentrate on what happens in both normal and extreme market conditions.
- D. All of the above.

The correct answer is **A**.

Correlations in a high volatility period are quite different from those of normal market conditions. When calculating VaR or ES, risk managers should determine what will happen in extreme market conditions. Therefore risk managers should try to determine correlations in extreme conditions rather than those in normal market conditions.

Options B, C and D are incorrect since correlation breakdown implies that extreme market conditions should be considered rather than normal market conditions.

Q.4680 Risk factors in a historical simulation of calculating VaR are divided into those where percentage change in the past is used to determine a percentage in the future and those where the actual change in the past is used to define the actual changes in the future. Which of the following risk factors are **NOT** in the same group?

- I. Interest rates and credit spreads
- II. Exchange rates and stock prices
- III. Interest rates and exchange rates
- IV. Stock prices and credit spreads

A. I, II and III

B. III and IV

C. II and IV

D. I and III

The correct answer is **B**.

Risk factors are broadly classified into:

- Those whose past percentage change is to define the future percentage, for example, stock prices and exchange rates.
 - Those whose past actual change is used to define an actual change in the future, for example, interest rates and credit spreads.
-

Q.4681 You have been given the following 30 ordered percentage returns of an asset: [-18,-16,-14,-12,-10,-9,-7,-7,-6,-6,-6,-5,-5,-4,-4,-4,-2,-1,0,0,2,3,6,12,12,13,15,15,18,28] What is the expected shortfall at a 90% confidence level?

- A. 16
- B. 17
- C. 18
- D. 17.5

The correct answer is **B**.

To locate the 10%, we take the third-worst return $(=(1-0.90)*30)$, which is -14. However, recall that VaR need not be represented as a negative.

The ES is the arithmetic average of losses beyond 90%. Thus,

$$ES = \frac{18 + 16}{2} = 17$$

Reading 47: Measuring and Monitoring Volatility

Q.535 Consider the following statements regarding the estimation of volatility:

- I. Under the EWMA model, the weights attached to observations decrease following an exponential pattern as the observations become older
- II. Under the GARCH(1,1) model, the observation's estimated weights decrease following an exponential pattern as the observations become older
- III. Under the GARCH(1,1) model, the long-run average variance rate has some positive weight
- IV. Under the EWMA model, the long-run average variance rate has some positive weight

- A. All the above statements are correct.
- B. Only I, II, and III are correct.
- C. Only I and IV are correct.
- D. None of the above statements are correct.

The correct answer is **B**.

In both the GARCH and EWMA models, weights decline exponentially with time. In addition, the GARCH model has a finite unconditional variance. However, the EWMA model has undefined long-run average variance since $\alpha_1 + \beta$ sum to 1.

Q.536 Using a daily RiskMetrics EWMA model with a decay factor $\lambda = 0.85$ to develop a forecast of the conditional variance, which weight will be applied to the return that is three days old?

- A. 0.1084
- B. 0.0921
- C. 0
- D. 0.153

The correct answer is **A**.

The weight of the last day = $1 - \lambda = 1 - 0.85 = 0.15$

For the day before, the weight is $0.15 * 0.85$ and for 3 days ago, $0.15 * 0.85^2 = 0.1084$

Q.537 Until December 2012, the Kenyan shilling had shown very small historical volatility against the South African Rand. On December 19th, Kenya abandoned the defense of the currency peg. Assuming the data from the close of business on December 19th, which of the following methods of calculating volatility would have shown the greatest jump in measured historical volatility?

- A. 150-day equal weight
- B. 100-day equal weight
- C. Exponentially weighted with a decay factor of 0.92
- D. Exponentially weighted with a decay factor of 0.97

The correct answer is **D**.

The EWMA model for updating variance is given by:

$$\sigma_n^2 = (1 - \lambda)r_{n-1}^2 + \lambda\sigma_{n-1}^2$$

Where:

r_{n-1}^2 = most recent observation of the squared return (on day $n - 1$).

σ_{n-1}^2 = estimate of the variance rate made for the previous day ($n - 1$)

The weight given to the most recent variance estimate is λ and the weight given to the new squared return is $1 - \lambda$.

Thus, the EWMA model with $\lambda = 0.97$ (**Option D**) puts a weight of 0.97 on the most recent variance estimate. This weight is higher than the 0.92 of the EWMA model with $\lambda = 0.92$ (**Option C**). It's also higher than 0.01 (= 1/100) and 0.0067 (= 1/150) of the 100-day (**Option B**) and the 150-day (**Option A**) MAs, respectively.

If there had been a currency peg in place, that implies the exchange rate was fixed. Abandoning such a policy would immediately trigger an increase in the volatility of the shilling vs the SA Rand. The biggest jump would have to involve the volatility measurement method that attaches the greatest weight on the first peg-free day.

Q.538 Given that σ_t^2 is the estimated variance at time t and μ_t is the realized return at time t , select the GARCH(1,1) model that will take the longest time to revert to its mean.

A. $\sigma_t^2 = 0.05 + 0.03\mu_{(t-1)}^2 + 0.91\sigma_{(t-1)}^2$

B. $\sigma_t^2 = 0.03 + 0.03\mu_{(t-1)}^2 + 0.92\sigma_{(t-1)}^2$

C. $\sigma_t^2 = 0.07 + 0.02\mu_{(t-1)}^2 + 0.94\sigma_{(t-1)}^2$

D. $\sigma_t^2 = 0.03 + 0.04\mu_{(t-1)}^2 + 0.91\sigma_{(t-1)}^2$

The correct answer is C.

It's the model with the highest persistence that takes the longest time to revert to its mean.

Persistence is given by:

$$\text{Persistence} = \alpha_1 + \beta$$

Bearing in mind that the general format of GARCH(1,1) models takes the form:

$$h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta h_{(t-1)}$$

The persistence is 0.94, 0.95, 0.96, and 0.95 for models A, B, C, and D respectively.

Q.539 Martin Scholes, FRM, estimates daily variance h_t using the following GARCH model on daily returns r_t :

$$H_t = \alpha_0 + \alpha_1 r_{(t-1)}^2 + \beta h_{(t-1)}$$

Where $\alpha_0 = 0.004$, $\alpha_1 = 0.05$, $\beta = 0.93$ Approximate the long-run annualized volatility (Assume there are 252 trading days in a year).

- A. 0.20
- B. 7.144
- C. 0.45
- D. 0.5

The correct answer is **B**.

In the GARCH(1,1) model, the long-run average variance rate is given by:

$$\begin{aligned} h &= \frac{\alpha_0}{(1 - \alpha_1 - \beta)} \\ &= \frac{0.004}{(1 - 0.05 - 0.93)} = 0.20 \end{aligned}$$

Taking the square root, this gives 0.45 for daily volatility.

To determine the annualized volatility, we have to multiply 0.45 by $\sqrt{252}$, where 252 is the average number of trading days in a year excluding weekends and holidays.

Thus, annualized volatility = $0.45 * \sqrt{252} = 7.144$

Q.540 Consider the GARCH(1,1) model:

$$\sigma_t^2 = \omega + \alpha \mu_{(t-1)}^2 + \beta \sigma_{(t-1)}^2$$

Where $\alpha + \beta < 1$ Which of the following statements is INCORRECT regarding the volatility term structure predicted by the model above?

- A. If we assume that the long-run estimated variance remains unchanged as α and β increase, the volatility term structure predicted by the model reverts to the long-run estimated variance faster.
- B. If we assume that the long-run estimated variance remains unchanged as α and β increase, the volatility term structure predicted by the model reverts to the long-run estimated variance more slowly.
- C. If the current volatility estimate is above the long-run average volatility, we would expect the estimated volatility term structure to be downward-sloping.
- D. If the current volatility estimate is below the long-run average volatility, we would expect the estimated volatility term structure to be upward-sloping.

The correct answer is **A**.

When σ_t is lower than the long-run average, the volatility term structure would go up. Higher persistence represented by $\alpha + \beta$ would mean that mean reversion is slower, not faster.

Q.541 A risk manager at Meridian Bank uses the exponentially weighted moving average technique to model the daily volatility of a security, with λ equal to 0.95. The current daily volatility estimate stands at 1.8%. On a certain day, the security registers a closing price of \$10 and then \$8 the following day. Determine the updated estimate of volatility:

- A. 22.31%
- B. 5.29%
- C. 15.21%
- D. 18.12%

The correct answer is **B**.

The new variance forecast will be given by:

$$\lambda \sigma_{(t-1)}^2 + (1 - \lambda) u_{(t-1)}^2$$

where

σ_n^2 = variance estimate for day n

$\sigma_{(t-1)}^2$ = volatility estimate made at the end of day (t-2) for day (t-1)

$U_{(t-1)}^2$ = most recent daily percentage change = $\ln(\frac{8}{10}) = -22.31\%$ or -0.2231

λ = exponential constant

Thus, the new variance forecast is

$$0.95 * 0.018^2 + (1 - 0.95) * (-0.2231)^2 = 0.0027964805$$

Hence

$$\text{Volatility} = \sqrt{0.0027964805} = 0.0529 \text{ or } 5.29\%$$

Q.542 Consider a portfolio with 60% invested in asset Y and 40% invested in asset Z. The mean and variance of the return on Y are 0 and 49 respectively. The mean and variance of the return on Z are 1 and 84 respectively. Given that the correlation coefficient between Y and Z is 0.4, determine the portfolio volatility.

- A. 43.4
- B. 6.59
- C. 8.5
- D. 23.1

The correct answer is **B**.

$$\begin{aligned}\sigma_p^2 &= 0.6^2 * 49 + 0.4^2 * 84 + 2 * 0.6 * 0.4 * 0.4\sqrt{(49 * 84)} \\ &= 17.64 + 13.44 + 12.32 = 43.40\end{aligned}$$

$$\text{Volatility} = \sqrt{43.40} = 6.59$$

Q.543 A FRM exam candidate uses the EWMA model with a decay factor of 0.90 to model the returns of a security listed on the Japanese Stock Exchange. Determine the weight that will be applied to the return that's 5 days old.

- A. 0.04656
- B. 0.09
- C. 0.06
- D. 0.0656

The correct answer is **D**.

The last day has some weight equivalent to $1 - \lambda = 1 - 0.9 = 0.1$

The day before would have weight equivalent to $(1 - \lambda)\lambda = 0.1 * 0.9 = 0.09$

And for 5 day-old return, the weight would be $(1 - \lambda)\lambda^4 = 0.1 * 0.9^4 = 0.0656$

Note: The lower the value of λ , the faster the rate at which old values are 'forgotten'.

Q.544 The decay factor used in an EWMA model is approximated to be 0.97. In addition, daily volatility is estimated to be 1%. Given that today's stock market return is 3%, determine the new estimate of volatility using the EWMA model.

- A. 0.00027
- B. 0.000124
- C. 0.01114
- D. 0.00567

The correct answer is **C**.

$$\sigma_n^2 = \lambda \sigma_{(n-1)}^2 + (1 - \lambda) u_{(n-1)}^2$$

Where $\lambda = 0.97$ and $u_{(n-1)} = 3\%$

Thus, $\sigma_n^2 = 0.97 * 0.01^2 + (1 - 0.97) * 0.03^2 = 0.000097 + 0.000027 = 0.000124$

$$\sigma_n = 0.01114 \text{ or } 1.11\%$$

Q.545 A generalized autoregressive conditional heteroskedastic (GARCH)(1,1) model has the following parameters: $\omega = 0.000003$; $\alpha = 0.05$; $\beta = 0.94$. Determine the implied long-run volatility level.

- A. 0.0003
- B. 0.07132
- C. 0.02732
- D. 0.01732

The correct answer is **D**.

$$\text{The long-run average variance} = \frac{\omega}{(1 - \alpha - \beta)} = \frac{0.000003}{(1 - 0.05 - 0.94)} = 0.0003$$

$$\text{The long-run volatility} = \sqrt{\left(\frac{\omega}{(1 - \alpha - \beta)}\right)} = \sqrt{0.0003} = 0.01732$$

Q.546 The dollar/sterling exchange rate at 5 P.M. yesterday was 0.78 and the most recent estimate of the daily volatility stands at 0.8%. The EWMA model used in the analysis has $\lambda = 0.9$. If the exchange rate at 5 P.M. today proves to be 0.775, find an estimate of the new daily volatility.

- A. 0.007857
- B. 7.855E-05
- C. 6.4E-05
- D. 6.17E-05

The correct answer is **A**.

The proportional daily change is $\ln(0.775/0.78) = -0.00643$.

The current daily variance estimate = $0.008^2 = 0.000064$.

$$\begin{aligned}\text{New daily variance estimate} &= 0.9 * 0.000064 + 0.1 * 0.00643^2 \\ &= 0.0000576 + 0.000004109 = 0.0000617\end{aligned}$$

$$\text{New daily volatility} = \sqrt{0.0000617} = 0.007857 \text{ or } 0.7857\%$$

Q.547 An analyst has interest in two assets, A and B. At the close of business yesterday, these assets had daily volatilities of 1.3% and 2.0% respectively. In addition, the assets were priced at \$40 for A and \$80 for B as at the close of business yesterday, and the estimated correlation coefficient between the two assets stood at 0.25. The EWMA model used by the analyst had $\lambda = 0.95$. Compute an estimate of the covariance between A and B.

- A. 0.000065
- B. 0.0006
- C. 2.60005
- D. 0.05

The correct answer is **A**.

Recall that:

$$\text{Correlation coefficient} = \frac{\text{Cov}(A, B)}{(\text{Volatility A} * \text{Volatility B})}$$

This means that if we make the covariance the subject of the formula, we get:

$$\begin{aligned}\text{Cov}(A, B) &= \text{Correlation coefficient} * \text{Volatility A} * \text{Volatility B} \\ &= 0.25 * 0.013 * 0.02 = 0.000065\end{aligned}$$

Q.548 An analyst has interest in two assets, A and B. At the close of business yesterday, these assets had daily volatilities of 1.3% and 2.0% respectively. In addition, the assets were priced at \$40 for A and \$80 for B as at the close of business yesterday, and the estimated correlation coefficient between the two assets stood at 0.25. The EWMA model used by the analyst had $\lambda = 0.95$. Assuming that the prices of the assets today are \$40.5 and \$80.5, update the correlation coefficient.

- A. 0.00625
- B. 0.01954
- C. 6.56E-05
- D. 0.2589

The correct answer is **D**.

Recall that:

$$\text{Correlation coefficient} = \frac{\text{Cov}(A, B)}{\text{Volatility A} * \text{Volatility B}}$$

This means that if we make the covariance the subject of the formula, we get:

$$\begin{aligned}\text{Cov}(A, B) &= \text{Correlation coefficient} * \text{Volatility A} * \text{Volatility B} \\ &= 0.25 * 0.013 * 0.02 = 0.000065\end{aligned}$$

The return for A is $0.5/40 = 0.0125$

Similarly, the return for B is $0.5/80 = 0.00625$

The new covariance estimate $0.95 * 0.000065 + 0.05 * 0.0125 * 0.00625 = 0.00006566$

For asset A, the new variance estimate $= 0.95 * 0.013^2 + 0.05 * 0.0125^2 = 0.0001684$

Therefore, new volatility $= \sqrt{0.0001684} = 0.01298$

For asset B, the new variance estimate $= 0.95 * 0.02^2 + 0.05 * 0.00625^2 = 0.000382$

Therefore, new volatility $= \sqrt{0.000382} = 0.01954$

The new correlation estimate $= \frac{0.00006566}{(0.01298 * 0.01954)} = 0.2589$

Q.549 GARCH(1,1) models can be used to estimate the volatility of asset returns if and only if:

- A. $\alpha > \beta$
- B. $\alpha < \beta$
- C. $\alpha + \beta = 0$
- D. $\alpha + \beta < 1$

The correct answer is **D**.

For stability to be achieved, $\alpha + \beta$ must be less than 1. Otherwise, the model would be unstable.

Q.556 Robert Kelly, FRM, uses the EWMA model to carry out daily updates of correlation and covariance rates between two random variables X and Y. The weight for the most recent covariance on day $n - 1$ is 0.80. The correlation estimate between X and Y on day $n - 1$ is 0.6. In addition, on day $n - 1$, X and Y have estimated standard deviations of 0.013 and 0.019 respectively. Also, the percentage change on day $n - 1$ for variables X and Y are 2% and 1% respectively. Calculate the updated covariance between X and Y on day n.

- A. 0.0125
- B. 0.41
- C. 0.0001586
- D. 0.0001482

The correct answer is C.

The exponentially weighted moving average (EWMA) model uses the following formula to compute the updated covariance between 2 random variables X and Y at time n:

$$\text{Cov}_n = \lambda \times \text{cov}_{n-1} + (1 - \lambda)X_{n-1} \times Y_{n-1}$$

Where:

λ = weight of the most recent covariance on day $n - 1$

X_{n-1} = percentage change for variable X on day $n - 1$

Y_{n-1} = percentage change for variable Y on day $n - 1$

We know that $\text{Cov}_{xy} = \rho_{xy} * \sigma_x \sigma_y$

$$\text{Cov}_{n-1} = 0.6 * 0.013 * 0.019 = 0.0001482$$

$$\text{Which gives: } \text{Cov}_n = 0.8 * 0.0001482 + 0.2 * 0.02 * 0.01 = 0.0001586$$

Q.557 Robert Kelly, FRM, uses the EWMA model to carry out daily updates of correlation and covariance rates between two random variables X and Y. The weight for the most recent covariance on day $n - 1$ is 0.80. The correlation estimate between X and Y on day $n - 1$ is 0.6. In addition, on day $n - 1$, X and Y have estimated standard deviations of 0.013 and 0.019 respectively. Also, the percentage change on day $n - 1$ for variables X and Y are 2% and 1% respectively. Compute the updated correlation between X and Y

- A. 0.1586
- B. 0.2152
- C. 0.3088
- D. 0.6152

The correct answer is **D**.

We know that $\rho_{xy} = \frac{\text{Cov}_{xy}}{\sigma_x \sigma_y}$

$$\text{Updated variance of X} = \lambda \sigma_{X,n-1}^2 + (1 - \lambda) X_{n-1}^2 = 0.8 * 0.013^2 + 0.2 * 0.02^2 = 0.0002152$$

$$\text{Updated variance of Y} = \lambda \sigma_{Y,n-1}^2 + (1 - \lambda) Y_{n-1}^2 = 0.8 * 0.019^2 + 0.2 * 0.01^2 = 0.0003088$$

The updated covariance between 2 random variables X and Y at time n:

$$\text{Cov}_n = \lambda \times \text{cov}_{n-1} + (1 - \lambda) X_{n-1} \times Y_{n-1}$$

Where:

λ = weight of the most recent covariance on day $n - 1$

X_{n-1} = percentage change for variable X on day $n - 1$

Y_{n-1} = percentage change for variable Y on day $n - 1$

We know that $\text{Cov}_{xy} = \rho_{xy} * \sigma_x \sigma_y$

$$\text{Cov}_{n-1} = 0.6 * 0.013 * 0.019 = 0.0001482$$

Which gives:

$$\text{Cov}_n = 0.8 * 0.0001482 + 0.2 * 0.02 * 0.01 = 0.0001586$$

Hence the updated correlation coefficient is,

$$\rho_n = \frac{0.0001586}{(\sqrt{0.0002152} * \sqrt{0.0003088})} = 0.615238$$

Q.558 A financial analyst uses daily data to estimate a GARCH (1,1) model as follows:

$$\sigma_n^2 = 0.000002 + 0.16r_{n-1}^2 + 0.74\sigma_{n-1}^2$$

She also has established that the most recent return and variance are 0.04 and 0.02, respectively. Calculate the updated volatility.

- A. 1.51%
- B. 11.11%
- C. 3.33%
- D. 12.27%

The correct answer is **D**.

The Generalized Autoregressive Conditional Heteroskedasticity GARCH(1,1) model is defined as:

$$\sigma_n^2 = \omega + \alpha r_{n-1}^2 + \beta \sigma_{n-1}^2$$

Where:

σ_n^2 = updated variance at time n

ω = long-term average variance rate

α = weight of the most recent squared return

r_{n-1}^2 = most recent squared return

β = weight of the most recent variance rate estimate

σ_{n-1}^2 = most recent variance rate estimate

From the information provided in the question:

$$\sigma_n^2 = 0.000002 + 0.16 \times 0.04^2 + 0.74 \times 0.02 = 0.015058$$

Volatility is given by:

$$\text{Volatility} = \sqrt{\sigma_n^2} = \sqrt{0.015058} = 0.12271 \approx 12.27\%$$

Q.565 A certain analyst uses the EWMA model with $\lambda = 0.9$ to carry out an update of correlation and covariance rates. On day $n - 1$, the observed percentage changes for variables X and Y are 3% and 2% respectively. Historical data puts the correlation estimate between A and Y at 0.54 on day $n - 1$. Furthermore, the estimated standard deviations on day $n - 1$ are 1.2% and 1.4% for X and Y respectively.

Compute the new estimate of the covariance between X and Y on day n.

- A. 9.07205
- B. 0.1234
- C. 0.1416
- D. 0.0001416

The correct answer is **D**.

$$\text{Cov}_n = \lambda \times \text{cov}_{n-1} + (1 - \lambda)X_{n-1} \times Y_{n-1}$$

But first, we must determine Cov_{n-1}

We know that $\text{Cov}_{xy} = \rho_{xy} * \sigma_x \sigma_y$

$$\text{Cov}_{n-1} = 0.54 * 0.012 * 0.014 = 0.00009072$$

And

$$\text{Cov}_n = 0.9 * 0.00009072 + 0.10 * 0.03 * 0.02 = 0.000141648$$

Q.566 A certain analyst uses the EWMA model with $\lambda = 0.9$ to carry out an update of correlation and covariance between the returns of two assets - A and B. The analyst observes that on day $n - 1$, the return on A is 2% and that on B is 3%, and the correlation between A and B is 0.5. In addition, the volatilities of the return on X and Y are 1% and 2%, respectively. Estimate the new coefficient of correlation.

- A. 0.55
- B. 0.62
- C. 0.45
- D. 0.5

The correct answer is **B**.

We have to calculate the estimated updated volatility of each asset as well as the updated covariance between them. Then we will use the relationship:

$$\text{Corr}(A, B) = \frac{\text{Cov}(A, B)}{\sigma_A \sigma_B}$$

The covariance on day n-1 can be calculated as:

$$\text{Cov}_{n-1} = 0.5 \times 0.01 \times 0.02 = 0.0001$$

For day n, the variance is updated as follows:

$$\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1 - \lambda) r_{n-1}^2$$

The variance of A on day n = $0.9 \times 0.01^2 + (1 - 0.9)0.02^2 = 0.00013$; volatility = $\sqrt{0.00013} = 0.0114$

The variance of B on day n = $0.9 \times 0.02^2 + (1 - 0.9)0.03^2 = 0.00045$; volatility = $\sqrt{0.00045} = 0.0212$

For day n, the covariance is updated as follows:

$$\begin{aligned}\text{Cov}(A, B)_n &= \lambda \text{Cov}(A, B)_{n-1} + (1 - \lambda) A_{n-1} B_{n-1} \\ \text{Cov}(A, B)_n &= 0.9 \times 0.0001 + (1 - 0.9) 0.02 \times 0.03 = 0.00015\end{aligned}$$

With the updated values of both the volatilities and covariance,

$$\text{Corr}(A, B)_n = \frac{0.00015}{(0.0114 \times 0.0212)} = 0.62$$

Q.567 The following is a variance-covariance matrix:

$$\begin{bmatrix} 1 & 0 & 0.7 \\ 0 & 1 & 0.7 \\ 0.7 & 0.7 & 1 \end{bmatrix}$$

Determine the correlation rate between variables 2 and 3.

- A. 0.7
- B. 1
- C. 0.49
- D. 0

The correct answer is **A**.

Generally, a 3×3 covariance matrix is given by:

$$\begin{bmatrix} \text{Var}(1) & \text{Cov}(1, 2) & \text{Cov}(1, 3) \\ \text{Cov}(1, 2) & \text{Var}(2) & \text{Cov}(2, 3) \\ \text{Cov}(1, 3) & \text{Cov}(2, 3) & \text{Var}(3) \end{bmatrix}$$

Now,

$$\text{Corr}(2, 3) = \frac{\text{Cov}(2, 3)}{\text{Sd}(1) \times \text{Sd}(2)} = \frac{0.7}{1 \times 1} = 0.7$$

Q.568 Kelvin Klein, a financial analyst, uses daily data to estimate a GARCH (1, 1) model as follows:

$$\sigma_n^2 = 0.000002 + 0.14r_{n-1}^2 + 0.76\sigma_{n-1}^2$$

Kelvin establishes that the estimate of return on day $n - 1$ is 0.02 and the most recent observation on variance is 0.01. Calculate the updated estimate of variance, σ_n^2

- A. 0.066
- B. 0.008
- C. 0.088
- D. 0.017

The correct answer is **B**.

From the information given in the question, we have:

$$\sigma_n^2 = 0.000002 + 0.14 \times 0.02^2 + 0.76 \times 0.01 = 0.007658 \approx 0.008$$

Q.940 A number of risk measures are based on the parametric approach, which assumes that the asset returns are normally distributed. However, mathematicians and statisticians have discovered that in reality, the asset returns deviate from normality. Which of the following options is least likely consistent with the assumption that the asset returns deviate from normality?

- A. Asset returns have fat-tailed distributions which means assets have a higher probability weight in their tails relative to the normal distribution.
- B. Asset returns have skewed distribution, which means that the declines in asset prices are more severe than increases in prices.
- C. Asset returns have unstable parameter values due to varying market conditions.
- D. Asset returns have symmetrical distributions which means they are evenly distributed around the mean returns.

The correct answer is **D**.

Option D is least consistent with the assumption that the asset returns deviate from a normal distribution. Symmetric distribution is the assumption of the normal distribution that indicates that the returns are evenly distributed around it means that also indicates that there is no skewness in asset returns.

Q.941 Donald York is a quantitative analyst at Brooklyn Investments Hub, a tech investment company based in New York. York brings 5 years of experience in quantitative and statistical analysis. In one of his explanatory articles, he mentioned that when the mean and standard deviation of asset returns are the same for any given time period, the distribution of returns is said to be an unconditional distribution. In contrast, if the mean is the same at any time while the standard deviation of the return change over time, the return distribution is referred to as a conditional distribution. Identify the correct option from the following.

- A. York's explanation regarding the unconditional distribution is incorrect.
- B. York's explanation regarding the conditional distribution is incorrect.
- C. York's explanation regarding the conditional and unconditional distribution is incorrect.
- D. York's explanation regarding the conditional and unconditional distribution is correct.

The correct answer is **D**.

Both definitions are correct. An unconditional normal distribution refers to a case when the mean and the standard deviation of the asset returns are the same for any given time period. On the other hand, when the mean is the same at any given day, while the standard deviation of asset returns changes over time, then the distribution is referred to as conditional normal distribution.

Q.942 An analyst is conducting fundamental and technical analysis on Pak-China Trading Co (PCTC) stocks. The analysis takes into account the stock's daily returns based on the mean and volatility of returns. If PCTC returns have a mean of 9.56 bps/day, and a high volatility of 14.25 bps/day and a low volatility of 6.18 bps/day, then determine which of the following distributions fits the most the characteristics of PCTC's return distribution?

- A. Unconditional distribution.
- B. Regime-switching distribution.
- C. Unconditionally lognormal distribution.
- D. Conditional distribution.

The correct answer is **B**.

In a regime-switching distribution, the volatility of returns switches between low volatilities to high volatilities. In this distribution, the volatility does not switch to any volatility between the high and low volatility regimes.

Q.947 Gareth Graham is a senior risk consultant for Poincare Consulting Group. Graham has a strong reputation in the risk managers community, which is why he is frequently invited as a guest speaker at various business schools. During a recent seminar at a reputable business school in Vancouver, Graham mentioned the following comments regarding the cyclical volatility:

- I. It should be considered while analyzing the risk of financial assets that volatility in financial markets is time-varying
- II. While using a historical data model for analyzing volatility, more weight should be put into recent data as opposed to earlier data

Which of the following is correct?

- A. Only statement I is correct.
- B. Only statement II is correct.
- C. Both statements are correct.
- D. None of the statements is correct.

The correct answer is **C**.

Both statements are correct. It should be taken into account while analyzing the risk of assets returns that the volatility in the financial markets is time-varying and sticky. It should also be assumed that the more recent data of asset returns provides more information about future volatility.

Q.948 An analyst is comparing the STDEV or GARCH methodology with that of the RiskMetric® approach for estimating VaR using historical data. He wrote down the following similarities between both methods. Which of the following similarities is incorrect?

- A. Both methods belong to the parametric class of risk assessing models.
- B. Both methods attempt to estimate conditional volatility.
- C. Both methods apply equal weights to all the periods.
- D. Both methods use recent historic data for assessing risk.

The correct answer is **C**.

The standard deviation models apply equal weights to all the windows of past data, while the RiskMetric® and the GARCH approaches apply higher weights on more recent data. The weights decline exponentially to zero as returns become older.

Q.951 Selma Kaya is a junior risk analyst at Galileo Investment Bank. She is interested in estimated the joint density of returns of Algo Corp. and economic growth. Which of the following models should Kaya most likely use?

- A. Risk Metric®.
- B. Black-Scholes model.
- C. GARCH.
- D. Multivariate density estimation (MDE) model.

The correct answer is **D**.

The Multivariate density estimation (MDE) model allows estimating the joint density of returns of assets and other specified variables like the slope term structure, CPI, economic growth, etc.

Q.953 Markus Schmidt is an independent risk consultant at a well-known audit firm. He is currently working as an external risk advisory for George Reed Shipping Inc. During a meeting with the senior management of the shipping company, Schmidt made the following comments:

- I. Using nonparametric is a simple process as it does not impose a specific set of distributional assumptions; rather it uses the historical data directly
- II. Nonparametric methods are better predictors of the future volatility

Which of his comments is/are incorrect?

- A. Comment I only.
- B. Comment II only.
- C. Both comments.
- D. None of the comments.

The correct answer is **B**.

The most efficient predictors of future volatility are not nonparametric models but implied volatility models. Implied volatility models such as the Black-Scholes option pricing model can react better and quicker to current market conditions.

Q.956 If the covariance between Japanese and English interest rates is 0.089, and the variances of interest rates in Japan and England are 17.64% and 10.24%, respectively, then which of the following is closest to the correlation between Japanese and English interest rates?

- A. 4.927
- B. 1.028
- C. 0.6622
- D. 0.8736

The correct answer is **C**.

$$\begin{aligned}\text{Correlation} &= \frac{\text{Covariance}}{(\text{Standard deviation (A)} * \text{Standard deviation (B)})} \\ \text{Correlation} &= \frac{0.089}{(\sqrt{0.1764} * \sqrt{0.1024})} = 0.6622\end{aligned}$$

Q.3301 An investment company uses RiskMetrics to calculate the volatility. The volatility for the previous day is 0.02 and today's return is 10%. What is the updated volatility if $\lambda = 0.97$ is used?

- A. 0.0028
- B. 0.0194
- C. 0.0262
- D. 0.053

The correct answer is **C**.

Adaptive volatility using the RiskMetrics model can be found as:

$$\begin{aligned}\sigma_t^2 &= \lambda \sigma_{t-1}^2 + (1 - \lambda) r_{t-1,t}^2 \\ &= 0.97 * 0.02^2 + 0.03 * 0.1^2 = 0.000688\end{aligned}$$

However, remember that variance is volatility squared.

$$\sigma_t = \sqrt{0.000688} = 0.0262$$

Q.3302 A hedge fund manages risk by calculating future volatility using RiskMetrics™ to calculate the volatility. The volatility of the portfolio today is 3% per day and today's observed return is 1%. The conditional volatility estimate, assuming that $\lambda = 0.9$, is closest to:

- A. 2.86%
- B. 3.49%
- C. 2.50%
- D. 3.21%

The correct answer is **A**.

The volatility according to RiskMetrics is given by:

$$\sigma_n^2 = (1 - \lambda)r_{n-1}^2 + \lambda\sigma_{n-1}^2$$

Where

λ = The weight given to the most recent variance estimate.

r_{n-1}^2 = estimate of most recent squared return

σ_{n-1}^2 = Most recent estimate of variance

From the information given,

$$\sigma_n^2 = (1 - 0.9) 0.01^2 + 0.9 \times 0.03^2 = 0.00082$$

Thus the volatility is

$$\sqrt{0.00082} = 0.02863 \approx 2.86\%$$

Q.3303 A hedge fund manages risk by calculating future volatility using historical standard deviation. The portfolio performance in the past 5 days are 2% (n-1), 4% (n-2), 6% (n-3), 2% (n-4), and 10% (n-5), respectively. The hedge fund uses the historical standard deviation (moving average) method to calculate volatility. What is the volatility estimate?

- A. 4.00%
- B. 5.66%
- C. 3.40%
- D. 2.80%

The correct answer is **B**.

$$\begin{aligned}\sigma_n^2 &= \frac{1}{m} \sum_{i=1}^m r_{n-i}^2 \\ &= \frac{1}{5}(0.02^2 + 0.04^2 + 0.06^2 + 0.02^2 + 0.1^2) \\ &= 0.0032\end{aligned}$$

$$\text{Volatility} = \sqrt{0.0032} = 0.0565685$$

Q.3304 Suppose that λ is 0.97. Using the EWMA approach, what is the weight applied to the squared return on day $n - 3$?

- A. 0.4
- B. 0.97
- C. 0.03
- D. 0.06

The correct answer is **C**.

Let w_0 be the weight applied to the most recent return (i.e., the return on day $n - 1$). The weight for the squared return on day $n - 3$ is

$$w_0\lambda^2 = (1 - \lambda)\lambda^2 = (1 - 0.97) \times 0.97^2 = 0.028227 \approx 0.03$$

Q.3308 For a certain asset, the expected one-period volatility is 0.002. If the speed of the reversion parameter is 0.7, then what is the two-period volatility?

- A. 0.002
- B. 0.0024
- C. 0.0115
- D. 0.0587

The correct answer is **B**.

With mean reversion ($b < 1$), the single-period volatility is σ .

The two-period volatility is

$$\sqrt{1 + b^2} \times \sigma$$

Therefore,

$$0.002 \times \sqrt{1 + 0.7^2} = 0.002 \times 1.2207 = 0.0024$$

Q.3310 For a certain asset, the expected one-day volatility 0.002. What is the expected volatility for 30 days assuming non-predictability and the volatility being constant?

- A. 0.002
- B. 0.005
- C. 0.011
- D. 0.021

The correct answer is **C**.

Here, we simply use the square root rule:

$$\text{T-day volatility} = \text{1-day volatility} \times \sqrt{T}$$

$$\text{30-day volatility} = 0.002 \times \sqrt{30} = 0.002 \times 5.48 = 0.011$$

Q.3377 The parameters of a generalized autoregressive conditional heteroskedastic (GARCH)(1,1) model are $\alpha = 0.05$, $\beta = 0.91$. Long-run volatility is 0.80%. If estimated daily volatility is 2% and recent stock return is 3%, compute the new estimated volatility using the GARCH (1,1) model.

- A. 0.04%
- B. 2.50%
- C. 1.50%
- D. 2.03%

The correct answer is **D**.

$$\gamma = 1 - \alpha - \beta = 1 - 0.05 - 0.91 = 1 - 0.96 = 0.04$$

$$\text{Weighted long-run variance} = \omega = \gamma V_L = 0.04 * (0.008)^2 = 0.00000256$$

New variance is given by:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

$$\text{Variance} = 0.00000256 + 0.05 * 0.03^2 + 0.91 * 0.02^2 = 0.00041156$$

$$\text{Volatility} = \sqrt{0.00041156} = 0.0203 = 2.03\%$$

Q.3378 The decay factor of exponentially weighted moving average (EWMA) model is estimated to be 0.95. If the estimated daily volatility is 2% and the recent stock return is 3%, compute the new estimated volatility using the EWMA model.

- A. 2.06%
- B. 2.03%
- C. 0.04%
- D. 2.50%

The correct answer is **A**.

New variance is given by

$$\sigma_n^2 = (1 - \lambda)u_{n-1}^2 + \lambda\sigma_{n-1}^2$$

$$\begin{aligned}\text{Variance} &= 0.05 * 0.03^2 + 0.95 * 0.02^2 = 0.000425 \\ \text{Volatility} &= \sqrt{0.000425} = 2.06\%\end{aligned}$$

Q.3379 When parameters of a generalized autoregressive conditional heteroskedastic (GARCH)(1,1) model are set to $\omega = 0, \alpha = 1 - \lambda, \beta = \lambda$, the GARCH (1, 1) model reduces to a (an):

- A. Generalized volatility model
- B. EWMA model
- C. ARCH (1)
- D. None

The correct answer is **B**.

GARCH (1, 1) model:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

Put $\omega = 0, \alpha = 1 - \lambda, \beta = \lambda$
 , the above model reduces to:

$$\sigma_n^2 = (1 - \lambda)u_{n-1}^2 + \lambda\sigma_{n-1}^2$$

Q.3380 The parameters of a generalized autoregressive conditional heteroskedastic (GARCH)(1,1) model are ω, α, β , and γ . Which of the following is the necessary condition for estimating volatility using GARCH (1, 1)?

- A. $\alpha > \beta$
- B. $\gamma < 0$
- C. $\alpha + \beta < 1$
- D. $\alpha + \beta + \gamma > 1$

The correct answer is C.

$$\begin{aligned}\alpha + \beta + \gamma &= 1 \\ \alpha + \beta < 1 \quad \text{so that} \quad \gamma &> 0\end{aligned}$$

Q.3383 An analyst is trying to update the estimated covariance by using the exponentially weighted moving average (EWMA) model with $\lambda = 0.91$. The analyst has gathered the following relevant data.

- Estimated standard deviation on day $n - 1$ for variables X : 2%
- Estimated standard deviation on day $n - 1$ for variables Y : 3%
- The correlation between them: 0.8
- The percentage change on day $n - 1$ for variable X : 2.5%
- The percentage change on day $n - 1$ for variable Y : 3.5%

What is the updated estimated covariance between them?

- A. cannot be estimated
- B. 0.00051000
- C. 0.00048000
- D. 0.00051555

The correct answer is **D**.

Covariance between X and Y on day $n - 1$:

$$\text{Cov}_{n-1} = 0.8 * 0.02 * 0.03 = 0.00048$$

Updated covariance using EWMA:

$$\begin{aligned}\text{Cov}_n &= \lambda \text{Cov}_{n-1} + (1 - \lambda) X_{n-1} Y_{n-1} \\ &= 0.91 * 0.00048 + 0.09 * 0.025 * 0.035 \\ &= 0.00051555\end{aligned}$$

Q.3384 An analyst is trying to update the estimated covariance by using the exponentially weighted moving average (EWMA) model with $\lambda = 0.91$. The analyst has gathered the following relevant data.

- Estimated standard deviation on day $n - 1$ for variables X : 2%
- Estimated standard deviation on day $n - 1$ for variables Y : 3%
- The correlation between them: 0.8
- The observed return on day $n - 1$ for variable X : 2.5%
- The observed return on day $n - 1$ for variable Y : 3.5%

Given that the new Covariance between X and Y is 0.00051555, what is the updated estimated correlation between them?

- A. 0.825
- B. 0.800
- C. 0.910
- D. None

The correct answer is **A**.

Compute updated standard deviation using EWMA.

$$\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1 - \lambda) u_{n-1}^2$$

Updated variance for X = $0.91 * 0.02 * 0.02 + 0.09 * 0.025 * 0.025 = 0.00042025$

Updated standard deviation for X = $\sqrt{0.00042025} = 0.0205$

Updated variance for Y = $0.91 * 0.03 * 0.03 + 0.09 * 0.035 * 0.035 = 0.00092925$

Updated standard deviation for Y = $\sqrt{0.00092925} = 0.030483602$

$$\text{Updated correlation} = \frac{\text{Cov}(X, Y)}{[\text{SD}(X) * \text{SD}(Y)]} = \frac{0.00051555}{0.0205 * 0.030483602} = 0.824993729 = 0.825$$

Q.3385 Suppose that an analyst has gathered the following relevant data for estimating updated covariance using the EWMA model:

- Estimated standard deviation on day $n - 1$ for variables X : 2%
- Estimated standard deviation on day $n - 1$ for variables Y : 3%
- The correlation between them: 0.8
- The percentage change on day $n - 1$ for variable X : 25%
- The percentage change on day $n - 1$ for variable Y : 35%

Assuming that we choose $\lambda = 0.96$, what is the updated estimated covariance between variables X and Y using the EWMA model?

- A. 0.00041
- B. 0.00048
- C. 0.08402
- D. 0.003961

The correct answer is **D**.

$$\text{Cov}_{n-1} = 0.8 \times 0.02 \times 0.03 = 0.00048$$

Updated covariance using EWMA model is given by:

$$\begin{aligned}\text{Cov}_n &= (\lambda)\text{Cov}_{n-1} + (1 - \lambda)X_{n-1}Y_{n-1} \\ &= 0.96 \times 0.00048 + 0.04 \times 0.25 \times 0.35 \\ &= 0.0039608 \approx 0.003961\end{aligned}$$

Q.3386 The parameters of a generalized autoregressive conditional heteroskedastic (GARCH)(1,1) model are $\alpha = 0.05$; and $\beta = 0.91$. The long-run average variance rate is 0.80%. The analyst has gathered the following relevant data for estimating updated covariance using the GARCH(1, 1) model.

- Estimated standard deviation on day $n - 1$ for variables X : 2%
- Estimated standard deviation on day $n - 1$ for variables Y : 3%

- The correlation between them: 0.8
- The return observed on day $n - 1$ for variable X : 2.5%
- The return observed on day $n - 1$ for variable Y : 3.5%

What is the updated estimated correlation between the two variables using EWMA model with $\lambda = 0.96$?

- A. 0.8840
- B. 0.4500
- C. 0.5351
- D. 0.9270

The correct answer is C.

Updated standard deviation using the GARCH model:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

$$V_L = \frac{w}{1 - \alpha - \beta}$$

$$\Rightarrow w = V_L \times (1 - \alpha - \beta) = 0.80\% \times (1 - 0.05 - 0.91) = 0.00032$$

$$\text{Updated variance for X} = 0.00032 + 0.05 * 0.025 * 0.025 + 0.91 * 0.02 * 0.02 = 0.00071525$$

$$\text{Updated standard deviation for X} = \sqrt{0.00071525} = 0.026744158$$

$$\text{Updated variance for Y} = 0.00032 + 0.05 * 0.035 * 0.035 + 0.91 * 0.03 * 0.03 = 0.00120025$$

$$\text{Updated standard deviation for Y} = \sqrt{0.00120025} = 0.034644624$$

Covariance between X and Y on day $n - 1$

$$\text{Cov}_{n-1} = 0.8 \times 0.02 \times 0.03 = 0.00048$$

Updated covariance using EWMA model is given by:

$$\begin{aligned} \text{Cov}_n &= (\lambda)\text{Cov}_{n-1} + (1 - \lambda)X_{n-1}Y_{n-1} \\ &= 0.96 \times 0.00048 + 0.04 \times 0.025 \times 0.035 \\ &= 0.0004958 \end{aligned}$$

$$\begin{aligned}
 \text{Updated correlation} &= \frac{\text{Cov}(X,Y)}{[\text{SD}(X) * \text{SD}(Y)]} \\
 &= \frac{0.0004958}{0.026744158 \times 0.034644624} \\
 &= 0.5351
 \end{aligned}$$

Q.4659 If the daily volatility of the price of gold is 0.3% in a given year, and assuming that a year has 252 trading days, what is the annualized volatility of the gold price? (Assume that there are 252 trading days in a year)

- A. 0.0467
- B. 0.0356
- C. 0.0476
- D. 0.0120

The correct answer is **C**.

Using the scaling analogy, the corresponding annualized volatility is given by:

$$\sigma_{\text{annual}} = \sqrt{252} \times \sigma_{\text{daily}} = \sqrt{252} \times 0.003 = 0.047624$$

Q.4661 A stock market investor records the stock price for five consecutive days as 20.20, 20.00, 21.20, 21.00, and 23.30. Estimate the daily volatility using the stock price returns?

- A. 10.25%
- B. 5.6%
- C. 11.45%
- D. 12.56%

The correct answer is **B**.

The variance rate is given by:

$$\sigma_n^2 = \frac{1}{m} \sum_{i=1}^m r_{n-i}^2$$

Recall that return are calculated by:

$$r_i = \frac{S_i - S_{i-1}}{S_{i-1}}$$

The following table gives the consecutive returns:

Day	Stock Price	r_i
1	20.2	–
2	20.0	–0.0099
3	21.2	0.06
4	21.0	–0.00943
5	23.3	0.10952

Now,

$$\sigma_n^2 = \frac{1}{m} \sum_{i=1}^m r_{n-i}^2$$

$$\frac{1}{5} [(-0.0099)^2 + (0.06)^2 + (-0.00943)^2 + (0.10952)^2] = 0.0031563$$

The volatility is the square root of the variance rate:

$$\therefore \sigma_n = \sqrt{0.0031563} = 0.056 = 5.6\%$$

Note: Your textbook in section 3.3 says the following: "The usual formula for calculating standard deviations from sample data would give the volatility estimated for day n from the return on the m previous days [using m-1]. In risk management, we usually simplify this formula [by replacing] m - 1 by m."

Also, it gives an example on which you can see that we use 1/m as the actual number of **days** and not the number of returns.

Q.4662 Consider a GARCH (1,1) model with $\omega=0.00005$, $\alpha=0.025$, and $\beta=0.90$, what is the value of long-run average volatility?

- A. 0.0245
- B. 0.0258
- C. 0.0051
- D. 0.00735

The correct answer is **B**.

GARCH(1,1) model is given by

$$\sigma_n^2 = \alpha r_{n-1}^2 + \beta r_{n-1}^2 + \gamma V_L$$

Where

V_L : long-run average variance rate

α : weight given to the most recent squared returns

β : weight given to the previous variance rate estimate

γ : weight given to long-run average variance rate

V_L can be defined as:

$$V_L = \frac{\omega}{\gamma} = \frac{\omega}{1 - \alpha - \beta} = \frac{0.00005}{1 - 0.925} = 0.0006667$$

Therefore, the long-run average volatility is given by:

$$\sqrt{0.0006667} = 2.58\%$$

Q.4663 The current volatility for stock prices is estimated to be 4% per day, and the corresponding long-run average volatility is 3%. Assuming the GARCH (1,1) model with $\omega=0.00005$, $\alpha=0.025$, and $\beta=0.90$, what is volatility estimate in 50 days?

- A. 0.0009
- B. 0.0393
- C. 0.0302
- D. 0.0275

The correct answer is C.

According to GARCH (1,1), the expected variance rate on day t is given by:

$$\sigma_{n+t}^2 = V_L + (\alpha + \beta)^t (\sigma_n^2 - V_L)$$

Where

V_L : long-run average variance rate

α : weight given to the most recent squared returns

β : weight given to the previous variance rate estimate

σ_n^2 : current volatility estimate

So in this case,

$$\sigma_{n+50}^2 = 0.03^2 + (0.025 + 0.90)^{50} (0.04^2 - 0.03^2) = 0.0009142$$

Expected volatility is the square root of the expected variance rate:

$$\sigma_{n+50} = \sqrt{0.0009142} = 3.024\%$$

Q.4664 An investor notes that the closing stock price for asset A yesterday was USD 50, with a corresponding volatility of 2.5% per day. Similarly, the closing stock price for asset B was USD 30, with a corresponding volatility of 1.5% per day. Today, the stock price for asset A closed at USD 45,

and that of B closed at USD 35. The correlation coefficient between the stocks A and B on close of trading yesterday was 0.55.

Using the EWMA model with $\lambda=0.85$, what is the updated correlation coefficient between stocks A and B?

- A. 0.64
- B. -0.64
- C. 0.78
- D. -0.78

The correct answer is **D**.

According to the EWMA model,

$$s_n^2 = (1-\lambda)r_{n-1}^2 + \lambda s_{n-1}^2$$

For stock A, a return is $-0.10 (= \frac{45-50}{50})$ and that of B is $0.1667 (= \frac{35-30}{30})$. Therefore, updated volatility for stock A is:

$$s_{An} = \sqrt{(1-0.85) \times (-0.10)^2 + 0.85 \times 0.025^2} = 0.04507$$

And that of B is:

$$s_{Bn} = \sqrt{(1-0.85) \times (0.1667)^2 + 0.85 \times 0.015^2} = 0.06603$$

Now using the formula:

$$\text{Corr}(A, B) = \frac{\text{Cov}(A, B)}{s_A s_B}$$

$$\text{Cov}(A, B) = \text{Corr}(A, B) \times s_A s_B$$

So the covariance yesterday was:

$$\text{Cov}(A_{n-1}, B_{n-1}) = 0.55 \times 0.025 \times 0.015 = 0.00020625$$

The covariance is updated using the formula:

$$\text{cov}_n = \alpha \text{cov}_{n-1} + (1-\alpha)x_{n-1}y_{n-1}$$

$$\begin{aligned}\alpha \text{Cov}(A_n, B_n) &= 0.85 \times 0.00020625 + (1 - 0.85) \times -0.10 \times 0.1667 \\ &= -0.002325\end{aligned}$$

Using the formula $\text{Corr}(A, B) = \frac{\text{Cov}(A, B)}{s_A s_B}$ once again, the new correlation is given by:

$$\text{Corr}(A_n, B_n) = \frac{-0.002325}{0.04507 \times 0.06603} = -0.7813$$

Reading 48: External and Internal Credit Ratings

Q.1041 Which of the following statements is INCORRECT with regard to credit ratings?

- A. A credit rating represents the agency's opinion about the creditworthiness of an obligor with respect to a particular debt security or other financial obligation.
- B. Credit rating also applies to an issuer's general creditworthiness.
- C. There are generally two types of assessment corresponding to different financial instruments: short term and long term.
- D. Credit ratings from different agencies convey the same information.

The correct answer is **D**.

Issuer-specific credit ratings represent the agency's opinion about the creditworthiness of an obligor with respect to a particular debt security or other financial obligation.

Issuer credit rating also applies to an issuer's general creditworthiness.

There are generally two types of assessment corresponding to different financial instruments: short-term and long-term.

Credit ratings from various agencies normally convey differentiated information. For instance, Standard & Poor's perceives its ratings primarily as an opinion on the likelihood of default of an issuer, whereas Moody's ratings tend to reflect the agency's opinion on the expected loss on a facility.

Q.1042 Which of the following statements is NOT true regarding the rating process?

- A. The criteria according to which any assessment is provided are very strictly defined and constitute the intangible assets of rating agencies.
- B. The rating agency reviews qualitative as well as quantitative factors and compares the company's performance with that of its peers.
- C. The issuer is notified of the rating and the major considerations supporting it before it is discussed by the rating committee.
- D. When a rating is put on a credit watch list, a comprehensive analysis is undertaken.

The correct answer is C.

The criteria according to which any assessment is provided are very strictly defined and constitute the intangible assets of rating agencies. The rating agency reviews qualitative as well as quantitative factors and compares the company's performance with that of its peers. The issuer is notified of the rating and the major considerations supporting it *only after* the committee discusses the lead analyst's recommendation before voting on it. When a rating is put on a credit watch list, a comprehensive analysis is undertaken.

Q.1043 The rating "outlook" provides information about the:

- A. Rating trend.
- B. Loss severity independent of probability of default.
- C. Loss severity given the probability of default.
- D. Probability of default.

The correct answer is A.

The rating "outlook" provides information about the rating trend. A positive outlook means that there is some potential upside conditional to the realization of current assumptions regarding the company. On the flip side, a negative outlook suggests that the creditworthiness of the company follows a negative trend.

Options B, C, and D are all incorrect since outlooks only indicate the most likely direction of a rating over the medium term. Outlooks do not explain anything to do with the probability of default.

Q.1044 Which of the following factor is part of the quantitative analysis of rating of an industrial company?

- A. Business fundamentals.
- B. Operations and cost control.
- C. Financial ratios.
- D. Both A and B.

The correct answer is **C**.

For industrial companies, the rating analysis is commonly split between business reviews (firm competitiveness, quality of the management and of its policies, business fundamentals, regulatory actions, markets, operations, cost control, etc.) and quantitative analyses (financial ratios, etc.).

Q.1045 The rating of an issuer provided by a rating agency is a (an):

- A. Mere opinion on the issuer or securities issued by the issuer.
- B. Recommendation to purchase, sell, or hold any types of security.
- C. Indicator of the issuer's creditworthiness and also gives the price or relative value of specific securities.
- D. Recommendation based on quantitative analyses and business reviews.

The correct answer is **A**.

The agencies persistently emphasize that their ratings are mere opinions.

Option B is incorrect: Ratings do not constitute any recommendation to purchase, sell, or hold any type of security.

Option C is incorrect: A rating in itself indeed says nothing about the price or relative value of specific securities.

Option D is incorrect: These opinions (not recommendations) are based on quantitative analyses and business reviews.

Q.1046 Which of the following statements is true with regard to the relationship between ratings and probabilities of default?

- A. Across all industries, the number of defaults monotonically increases as we move down the credit rank.
- B. A given rating is meant to be forward-looking; it is devised to pinpoint a precise probability of default.
- C. Ex-post information such as that provided in default tables or transition matrices does guarantee to provide ex-ante insights regarding future probabilities of default or migration.
- D. Both A and B.

The correct answer is **A**.

Rating agencies publish on a regular basis table reporting observed default rates per rating category, per year, per industry, and per region. They show that ratings tend to have homogeneous default rates across industries. This implies that in every industry, the number of defaults increases as we move down the ratings order. However, It is important to bear in mind that for a given rating category, default rates can vary from industry to industry (e.g., a higher percentage of banks with a given rating, say BBB, will default when compared with firms in other industries with the same rating).

Option B is incorrect: Although a rating is meant to be forward-looking, it is not devised to pinpoint a precise probability of default, but rather to point to a broad risk bucket.

Option C is incorrect: Transition matrices serve as indicators of the likely path of a given credit at a given horizon. Ex-post information such as that provided in default tables or transition matrices **does not** guarantee to provide ex-ante insights regarding future probabilities of default or migration.

Q.1047 Through-the-cycle ratings issued by the rating agencies do NOT:

- A. Because of their low volatility, ttc ratings help financial institutions to better manage customers.
- B. Mitigate the effect of cycles on ratings by incorporating the effect of an “average cycle” in their scenarios.
- C. Fluctuate much with temporary changes in microeconomic conditions.
- D. Indicate probabilities of default over a wide time horizon..

The correct answer is C.

Through-the-cycle ratings issued by the rating agencies factor in the effect of cycles, though they recognize it is not always easy to anticipate them and though cycles are not fully repetitive in terms of duration, magnitude, and dynamics. Analysts try to mitigate the effect of cycles on ratings by incorporating the effect of an "average cycle" in their scenarios.

Ratings do not pinpoint a specific PD at a given horizon. They are broad indicators

A through-the-cycle rating does not fluctuate much with temporary changes in macroeconomic conditions since they are already factored in the rating.

Q.1048 Which of the following statement is most likely to be true with regard to the impact of a rating downgrade/upgrade on the price of bonds/stocks?

- A. A rating downgrade is somewhat likely to increase the price of a bond.
- B. A rating downgrade is likely to decrease the price of a bond.
- C. A rating upgrade is unlikely to increase the price of the stock since the price only reflects the earnings expectations of investors.
- D. A rating downgrade is unlikely to decrease the price of a stock since the price largely reflects earnings expectations among consumers.

The correct answer is **B**.

A rating downgrade is likely to decrease the price of the bond. For instance, a rating agency may downgrade a bond issue's rating from BBB to BB because of the fall in the firm's debt repayment ability, for example, the deterioration in the current ratio. This will increase the bond's yield substantially and its price will fall. In this particular case, the bond will go from investment-grade to below investment-grade, which will force many pension funds to sell the bond since some may only be allowed to hold investment-grade securities; this will put more pressure on the bond's already low price. A rating upgrade works in the opposite direction and is likely to increase the price of the bond. A rating upgrade is somewhat likely to increase the price of the stock to reflect improved investor expectations. A rating downgrade is likely to decrease the price of the stock, reflecting a dip in investor confidence.

Q.2810 The CRO of an investment bank is reviewing the internal rating assessment policies. He notices that the bank is using the through-the-cycle approach to rate the borrowers. He is concerned about the effectiveness of the current approach during recessions and asks to compare it with the at-the-point-in-time approach. Which of the following statements is correct?

- A. During recessions, the through-the-cycle approach tends to over-estimate risk during recessions.
- B. During recessions, the at-the-point-in-time approach tends to over-estimate risk during recessions.
- C. During recessions, both through-the-cycle and at-the-point-in-time approaches tend to over-estimate risk.
- D. During recessions, both through-the-cycle and at-the-point-in-time approaches tend to under-estimate risk.

The correct answer is **B**.

The at-the-point-in-time approach assesses the credit quality over the near term and tends to amplify the effect of the business cycle. During recessions, the at-the-point-in-time approach tends to overestimate risk.

On the other hand, through-the-cycle internal ratings try to evaluate the permanent component of default risk. Unlike point-in-time ratings, they are said to be nearly independent of cyclical changes in the creditworthiness of the borrower. They are not affected by credit cycles, i.e. they are through the cycle. As a result, they are less volatile than at-the-point ratings and are valid for a much longer period (exceeding one year).

Q.2811 Greg Teller, a credit risk analyst, was requested by the CRO to check an internal rating transition matrix prepared by an intern. The matrix is based on actual rating migrations over the last ten years. The bank has ratings of A, B, C, and D, with A representing the highest credit quality and D representing a default. The bank currently has a rating of C. The intern prepares the following table:

Annual Rating Transitions (% , Average Annual)

	A	B	C	D
A	95.00	3.00	2.00	–
B	2.00	89.00	5.00	2.00
C	–	7.00	83.00	10.00

After a short review, Teller makes the following statement to the CRO: Statement 1: "The internal rating transition matrix is correct." He also decides to incorporate the findings from the matrix in the conclusion of his research report for LLL Construction (the only C-rated borrower of the bank). He includes the following sentence in the conclusion of the report: Statement 2: "The risk management department recommends creating significant loan loss provisions for LLL's facility as it has a 10% chance of default with the current rating and a 0% chance of improvement to an A rating over the longer term." Are Teller's statements correct?

- A. Statement 1 is correct while statement 2 is incorrect.
- B. Statement 1 is incorrect while statement 2 is correct.
- C. Statement 1 is correct, and statement 2 is also correct.
- D. Statement 1 is incorrect, and statement 2 is also incorrect.

The correct answer is **D**.

Statement 1 is incorrect. The rows of the internal rating transition matrix should sum up to 100. In the matrix prepared by the intern, the sum of the probabilities in the second row is 98.

Statement 2 is incorrect. Based on the internal rating transition matrix, LLL Construct has no chance of a rating improvement to A in one year, but there is a non-zero probability of an improvement over the longer term.

Additional explanation:

The transition matrix shows 1-year transition probabilities given a particular starting point. The matrix is interpreted from left to right. This means A, B, and C, as shown on the left side, indicate the possible starting points.

So, if the bank is currently rated C, a move to A in exactly one year has a zero probability as indicated by the blank probability representing the CA intersection.

Statement 2 is partly correct because, at C, the bank indeed has a 10% chance of transitioning to D (default). However, the final part of the statement is incorrect, and here's why:

If we consider the long-term, or rather a period greater than one year, there's a non-zero chance of an A rating at some point in the future. For example, in just two years, the bank can move from C to B with P(7%), then B to A with P(2%).

Q.3436 Simon Bryan, FRM, is scrutinizing historical migration tables published by S&P's and Moody's. Which of the following statements would possibly appear under "additional information" below such tables?

- A. We should expect to see the highest level of rating stability in the intermediate term (five-year time frame). Risk ratings will tend to have changed more at both the one- year and ten-year horizons.
- B. We should expect to see the highest level of rating stability during the one-year timeframe. This stability will decline at both the five-year time frame and even more so at the ten-year horizon.
- C. We should expect to see the greatest amount of credit rating stability over long periods of time (e.g., ten years). Credit ratings will tend to change more during shorter periods of time.
- D. We should expect to see credit ratings change by about the same amount over time. The ratings transition matrix shows approximately the same figures for the one-year, five-year, and ten-year time horizons.

The correct answer is **B**.

The Rating Transition Matrix tables developed by renowned rating agencies show that credit ratings are their most stable over a one-year horizon and that stability decreases with longer horizons.

Q.3437 ANEX Financials (AF), a U.S. based firm, has just issued a two-year zero-coupon bond currently rated AA. Market analysts expect that one year from now:

- The probability that the rating of AF remains at AA is 90%
- The probability that the rating of AF is downgraded to A is 5%
- The probability that AF is upgraded to AAA is 5%

The risk-free rate is flat at 2%, and credit spreads for AAA-, AA-, and A-rated debt are flat at 40, 60, and 100 basis points, respectively. All rates are compounded annually. What is the best approximation of the expected value of the zero-coupon bond a year from today?

- A. 97.6
- B. 97.5
- C. 97.7
- D. 97.4

The correct answer is **B**.

In a year's time, the bond will be a 1-year zero-coupon bond.

The bond has different expected values in each of the three scenarios outlined above. Precisely,

$$P_{AAA} = \frac{100}{(1 + 0.02 + 0.004)^1} = 97.66$$

$$P_{AA} = \frac{100}{(1 + 0.02 + 0.006)^1} = 97.47$$

$$P_A = \frac{100}{(1 + 0.02 + 0.01)^1} = 97.09$$

Note that as the ratings deteriorate, so does the value of the bond.

Expected value = $5\% \times 97.66 + 90\% \times 97.47 + 5\% \times 97.09 = 97.46$

Note: 1% = 100 basis points

Q.3438 Rating agencies make efforts to incorporate the effects associated with an economic cycle in their ratings. Although this practice is generally valid, it may lead to:

- A. Underestimation of the probability of default in an economic recession
- B. Overestimation of the probability of default in an economic recession
- C. Underestimation of the probability of default in an economic expansion
- D. Divergence of the interests of agency analysts and those of management

The correct answer is **A**.

Since ratings are generally produced with an eye on a long-term period, they must take into account any economic/industrial cycle on the horizon. Rating agencies make efforts to incorporate the effects associated with an economic cycle in their ratings. Although this practice is generally valid, it can lead to underestimation or overestimation of default if the predicted economic cycle doesn't play out exactly as expected. In particular, a firm's probability of defaulting during a severe downturn may be underestimated based on the given rating.

Q.4682 Given a constant hazard rate of 0.02, what is the survival probability until year 3?

- A. 0.9674
- B. 0.9418
- C. 0.9518
- D. 0.942

The correct answer is **B**.

The survival probability to time t is given by;

$$\exp(-ht)$$

Where h is the hazard rate

Then, the survival probability to year 3 is given by;

$$\exp(-0.02 \times 3) = 0.9418$$

Q.4683 Suppose a firm has a debt of \$20 million. If the recovery rate is 80%, and that there is a 0.03 chance that the loan will default, what is the expected loss when the loan defaults?

A. 0.12m

B. 0.15m

C. 0.09m

D. 0.06m

The correct answer is **A**.

Since the recovery rate is 80%, the loss given default (LGD) is

$$\text{LGD} = 100\% - 80\% = 20\%.$$

The expected loss on the loan is, therefore, :

$$\text{EL} = \text{PD} \times \text{LGD} \times \text{EAD} = 0.03 \times 0.20 \times \$20 \text{ million} = \$0.12 \text{ million}$$

Q.4684 Consider a firm in which the following information ratios

- i. Working capital to total assets = 0.32
- ii. Retained earnings to total assets = 0.44
- iii. Earnings before interest and taxes to total assets = 0.80
- iv. Market value of equity to book value of total liabilities = 1.2
- v. Sales to total assets = 1.8

What is the Altman's Z-score for the firm?

- A. 6.235
- B. 6
- C. 6.1582
- D. 6.2543

The correct answer is C.

The Altman's Z-score is given by:

$$\begin{aligned} Z &= 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 0.999X_5 \\ &= (1.2 \times 0.32) + (1.4 \times 0.44) + (3.3 \times 0.80) + (0.6 \times 1.2) + 0.999 \times 1.8 \\ &= 6.1582 \end{aligned}$$

Where:

X_1 : Working capital to total assets,

X_2 : Retained earnings to total assets,

X_3 : Earnings before interest and taxes to total assets,

X_4 : Market value of equity to book value of total liabilities, and

X_5 : Sales to total assets.

The firm in this case, is **not likely** to default since Z-score is greater than 3.

Q.4685 Suppose the hazard rate for the first three years is 0.01 and the hazard rate for the next three years is 0.02. What is the probability of default between years 3 and 6?

- A. 0.0421
- B. 0.0565
- C. 0.9434
- D. 0.9579

The correct answer is **B**.

To calculate the probability of default between year 3 and year 6, we first calculate the average hazard rate which equals to $\frac{0.01+0.02}{2} = 0.015$

The probability of default between years 3 and year 6 is therefore given by:

$$\begin{aligned} PD_{3,6} &= \text{default during the first 3 years} - \text{default during the first 6 years} \\ &= [\exp(-0.01 \times 3)] - [\exp(-0.015 \times 6)] \\ &= 0.0565 \end{aligned}$$

Q.4686 Suppose that company XYZ has a debt value of \$100m and that the value of its assets is \$120m. What is the value of the equity at that future point in time?

- A. 0
- B. 20m
- C. min(20m,0)
- D. 83.33m

The correct answer is **B**.

Let v be the value of the company's assets and let d be the value of the debt. The value of equity is given by:

$$\begin{aligned} \max(v - d, 0) &= \max(120 - 100, 0) \\ &= \max(20, 0) = 20 \end{aligned}$$

Q.4687 Suppose v is the value of the asset, and d is the value of the debt, the firm defaults when:

- A. $v < d$
- B. $v > d$
- C. $v = d$
- D. $\max(v - d) > d$

The correct answer is **A**.

The firm defaults when the value of its assets is less than the value of its total debt.

Reading 49: Country Risk

Q.1029 Armenia Bank has opened a new branch in a country where political risk is substantial. The board of the bank would like to get a report from the risk department on political risks likely to be faced in other countries. The risk department, while presenting the report, makes the following statements:

Statement I: Democratic countries are always less risky than dictatorship countries

Statement II: The chaos of democracy does create more discontinuous risk (policies that change as governments shift), and dictatorships create more continuous risk

Statement III: Corruption is an implicit tax on income (that does not show up in conventional income statements as such) that reduces the profitability and returns on investments for businesses in that country directly and for investors in these businesses indirectly

Statement IV: Countries that are in the midst of physical conflicts, either internal or external, will expose investors/businesses not only economic costs but also physical costs

Which of these statements are true?

A. II, III & IV only.

B. II & III only.

C. III & IV only.

D. All of the above.

The correct answer is C.

Statements I & II are incorrect. Democratic countries are less or more risky than their authoritarian countries. The chaos of democracy does create more continuous risk (policies that change as governments shift), dictatorships create more discontinuous risk.

Statements III & IV are correct. Corruption is an implicit tax on income that reduces the profitability and returns on investments for businesses in that country directly and for investors in these businesses indirectly. Countries that are in the midst of physical conflicts, either internal or external, will expose investors/businesses not only economic costs but also physical costs.

Q.1030 Simon Fernando, a country risk intern working at Cross Country Ratings Limited, is preparing a report on services measuring country risk. He lists out the following details in his report:

Statement I: The country scores used by different services are standardized

Statement II: A country with a risk score of 80 in the PRS scoring mechanism is twice as safe as a country with a score of 40

Statement III: Country risk scores are more useful for measuring relative risk than for ranking the countries.

Which of these statements are correct?

- A. I only.
- B. II & III only.
- C. All of the above.
- D. None of the above.

The correct answer is **D**.

Statement I is incorrect: The country scores are not standardized, and each service uses its own protocol.

Statement II is incorrect: A country with a risk score of 80 in the PRS scoring mechanism is safer than a country with a risk score of 40, but it would be dangerous to read the scores to imply that it is twice as safe.

Statement III is incorrect: Country risk scores are more useful for ranking the countries than for measuring relative risk.

Q.1031 An analysis of country defaults has shown that:

- I. Countries have been more likely to default on sovereign bonds issued than bank debt owed
- II. In dollar value terms, Sub-Saharan countries have accounted for much of sovereign defaulted debt in the last 50 years
- III. As per Moody's, countries are increasingly defaulting on foreign currency debt
- IV. Defaults since the 1960s have been more likely on foreign currency debt than on foreign currency bonds

Which of the above statements are correct?

- A. II, III & IV only.
- B. I & II only.
- C. III & IV only.
- D. All of the above.

The correct answer is **C**.

Statements I & II are incorrect. Countries have been more likely to default on bank debt owed than on sovereign bonds issued, in dollar value terms. Latin American countries have accounted for much of sovereign defaulted debt in the last 50 years.

Statements III & IV are correct. As per Moody's, countries are increasingly defaulting on foreign currency. Defaults since the 1960s have been more likely on foreign currency debt than on foreign currency bonds.

Q.1032 Country ABC recently defaulted on local currency and foreign currency sovereign debt. One of the economists of the country writes an article in the leading business weekly of the country and lists out the short term and long term effects of defaulting on debt:

- I. Default has a negative impact on real GDP growth of between 0.5% and 2%
- II. Default does affect a country's long-term sovereign rating and borrowing costs
- III. Sovereign default makes banking systems more robust
- IV. Sovereign default also increases the likelihood of political change

Which one of them is correct according to research materials available on the subject?

- A. II, III & IV only.
- B. I & II only.
- C. I, II & IV only.
- D. All of the above.

The correct answer is C.

Statement I is correct. Default has a negative impact on real GDP growth of between 0.5% and 2%.
Statement II is correct. Default does affect a country's long-term sovereign rating and borrowing costs. One study of credit ratings in 1995 found that the ratings for countries that had defaulted at least once since 1970 were one to two notches lower than otherwise similar countries that had not defaulted. In the same vein, defaulting countries have borrowing costs that are about 0.5 to 1% higher than countries that have not defaulted.

Statement III is **incorrect**. Sovereign default can make banking systems **more fragile**. A study of 149 countries between 1975 and 2000 indicates that the probability of a banking crisis is 14% in countries that have defaulted, an eleven percentage-point increase over non-defaulting countries.

Statement IV is correct. Sovereign default also increases the likelihood of political change. A study of devaluations between 1971 and 2003 finds a 45% increase in the probability of change in the top leader (prime minister or president) in the country and a 64% increase in the probability of change in the finance executive (minister of finance or head of central bank).

Q.1033 McGrath, a University student, is working on an article titled "Factors Determining Sovereign Default Risk." He sources data from around the world on sovereign default and analyzes the data. Finally, he compiles his findings based on his understanding and analysis of the data. He seeks your help in verifying the accuracy of his findings.

Statement I: Income tax-based systems generate more volatile revenues than sales tax (or value-added tax systems)

Statement II: The decision to default is as much a political decision as it is an economic decision

Statement III: Autocracies are more likely to default than democracies

Statement IV: The independence and power of the central bank will also affect assessments of default risk

Which of these statements are accurate?

A. II, III & IV only.

B. I & II only.

C. I, II & IV only.

D. All of the above.

The correct answer is **D**.

Income tax-based systems generate more volatile revenues than sales tax (or value-added tax systems). The decision to default is as much a political decision as it is an economic decision. Autocracies (where there is less worry about political backlash) are more likely to default than democracies. The independence and power of the central bank will also affect assessments of default risk.

Q.1034 An analysis of the sovereign ratings provided by different rating agencies reveals that:

- I. For the most part, there is a consensus among the rating agencies in the ratings, but there can be significant differences on individual countries
- II. Sovereign ratings change over time but far less than corporate ratings do
- III. Sovereign ratings change little on an annual basis for higher-rated countries compared to lower-rated countries
- IV. Rating agencies assess risk at the broader regional level and have been accused of regional biases

Which of these are correct statements?

- A. II, III & IV only.
- B. I & II only.
- C. I, II & IV only.
- D. All of the above.

The correct answer is **D**.

For the most part, there is a consensus among the rating agencies in the ratings, but there can be significant differences in individual countries. Sovereign ratings change over time but far less than corporate ratings do. Sovereign ratings change little on an annual basis for higher-rated countries compared to lower-rated countries. Rating agencies assess risk at the broader regional level. One of the criticisms that rated countries have mounted against the rating agencies is that they have regional biases, leading them to underrate entire regions of the world (Latin America and Africa).

Q.1035 Three economists participate in a discussion on 'local and foreign currency sovereign ratings' on live television. The following are the opinions expressed by these economists on this topic:

Economist 1: The differential between foreign and local currency ratings is primarily a function of monetary policy independence

Economist 2: Countries that maintain floating rate exchange regimes and fund borrowing from deep domestic markets will see local currency ratings converge on foreign currency ratings

Economist 3: For the most part, local currency ratings are at least as high or higher than the foreign currency rating. There are, however, notable exceptions, where the local currency rating is lower than the foreign currency rating.

Which economist(s) made (an) accurate statement(s)?

- A. Only Economist 3.
- B. Only Economists 1 and 3.
- C. Only Economist 1.
- D. All three economists.

The correct answer is **B**.

Economist 1 is correct. The differential between foreign and local currency ratings is primarily a function of monetary policy independence.

Economist 2 is incorrect. Countries that maintain floating rate exchange regimes and fund borrowing from deep domestic markets will have the **largest differences between local and foreign currency ratings**, whereas countries that have given up monetary policy independence, either through dollarization or joining a monetary union, will see local currency ratings converge on foreign currency ratings.

Economist 3 is correct. For the most part, local currency ratings are at least as high or higher than the foreign currency rating, for the obvious reason that governments have more power to print more of their own currency. There are, however, notable exceptions where the local currency rating is lower than the foreign currency rating. In March 2010, for instance, India was assigned a local currency rating of Ba2 and a foreign currency rating of Baa3.

Q.1036 Each rating agency has its own system for estimating sovereign ratings, but the processes share a great deal in common. With regard to sovereign ratings provided by rating agencies and the processes followed by them, identify the correct statements.

- I. A sovereign rating is focused on the creditworthiness of the sovereign country to private creditors and not to official creditors
- II. Rating agencies also vary on whether their rating captures only the probability of default or also incorporates the expected severity
- III. The ratings are decided by a vote of the committee
- IV. News of a political coup or an economic disaster can lead to a rating review not just for the country in question but also for surrounding countries

A. II, III & IV only.

B. I & II only.

C. I, II & IV only.

D. All of the above.

The correct answer is **D**.

A sovereign rating is focused on the creditworthiness of the sovereign country to private creditors (bondholders and private banks) and not to official creditors (that includes the World Bank, the IMF, and other entities). Rating agencies also vary on whether their rating captures only the probability of default or also incorporates the expected severity. The ratings are decided by a vote of the committee. News of a political coup or an economic disaster can lead to a rating review not just for the country in question but also for surrounding countries (that may face a contagion effect).

Q.1037 Rating agencies have been criticized for failing investors on several counts in the case of sovereign ratings. Which of the following are accurate criticisms faced by rating agencies?

I. Rating agencies have been accused of being far too optimistic in their assessments of corporate rating as compared to sovereign ratings

II. When one rating agency lowers or raises a sovereign rating, other rating agencies seem to follow suit

III. Rating agencies take too long to change ratings, and these changes happen too late to protect investors from a crisis

IV. Once a market is in crisis, there is the perception that rating agencies sometimes overreact and lower ratings too much, thus creating a feedback effect that makes the crisis worse

A. II, III & IV only.

B. I & II only.

C. I, II & IV only.

D. All of the above.

The correct answer is **A**.

Statement I is incorrect: Rating agencies have been accused of being far too optimistic in their assessments of both corporate and sovereign ratings. While the conflict of interest of having issuers pay for the rating is offered as the rationale for the upward bias in corporate ratings, that argument does not hold up when it comes to sovereign ratings, since the issuing government does not pay rating agencies.

Statement II is correct: When one rating agency lowers or raises a sovereign rating, other rating agencies seem to follow suit.

Statement III is correct: Rating agencies take too long to change ratings, and that these changes happen too late to protect investors from a crisis.

Statement IV is correct: Once a market is in crisis, there is the perception that rating agencies sometimes overreact and lower ratings too much, thus creating a feedback effect that makes the crisis worse.

Q.1038 Market interest rates and market-based default spreads play an important role in understanding sovereign ratings. Which of the following statements are true?

- I. Market-based spreads are more dynamic than ratings, with changes occurring in real time
- II. Market-based default measures tend to be far more volatile than ratings and can be affected by variables that have nothing to do with default
- III. The sovereign bond market leads rating agencies, with default spreads usually climbing ahead of a rating downgrade and dropping before an upgrade
- IV. Notwithstanding the lead-lag relationship, a change in sovereign ratings is still an informational event that creates a price impact on the sovereign bonds at the time that it occurs

- A. II, III & IV only.
- B. I & III only.
- C. I, II & IV only.
- D. All of the above.

The correct answer is **D**.

There is a strong correlation between sovereign ratings and market default spreads. Market-based spreads are more dynamic than ratings, with changes occurring in real-time. Market-based default measures tend to be far more volatile than ratings and can be affected by variables that have nothing to do with default. Liquidity and investor demand can sometimes cause shifts in spreads that have little or nothing to do with default risk. The sovereign bond market leads rating agencies, with default spreads usually climbing ahead of a rating downgrade and dropping before an upgrade. Notwithstanding the lead-lag relationship, a change in sovereign ratings is still an informational event that creates a price impact at the time that it occurs.

Q.1039 Bank ABC relies on credit default swaps to assess the default risk of sovereign bonds/debt. Which of the following statements are true with regard to the relationship between Credit Default Swaps and default risk?

- I. Changes in CDS spreads lead to changes in sovereign bond yields and sovereign ratings
- II. The CDS market is quicker or better at assessing default risks than the government bond market, from which default spreads can be extracted
- III. The exposure to counterparty and liquidity risk, endemic to the CDS market, can cause changes in CDS prices that have little to do with default risk
- IV. The narrowness of the CDS market can make individual CDS susceptible to illiquidity problems, with a concurrent effect on prices

- A. I, III & IV only.
- B. I & III only.
- C. I, II & IV only.
- D. All of the above.

The correct answer is **A**.

Statement II is incorrect. It is not clear that the CDS market is quicker or better at assessing default risks than the government bond market, from which we can extract default spreads.

All other statements are correct. Changes in CDS spreads lead to changes in the sovereign bond yields and in sovereign ratings. The exposure to counterparty and liquidity risk, endemic to the CDS market, can cause changes in CDS prices that have little to do with default risk. The narrowness of the CDS market can make individual CDS susceptible to illiquidity problems, with a concurrent effect on prices

Q.1040 Country XYZ chooses to default in local currency. Which of the following may NOT be a compelling reason to default in local currency?

- A. Following Gold standard in the decades prior to 1971.
- B. Shared currency.
- C. Foreign currency debt funding local currency assets.
- D. Local currency debt funding foreign currency assets.

The correct answer is **D**.

In most cases, debt in local currency has been considered safer than debt in foreign currency.

Option A is incorrect: In the decades prior to 1971, countries following the gold standard had to back up their currency with gold reserves putting a limit on how much currency could be printed.

Option B is incorrect: Countries following shared currency give up the power to control how much of the currency they could print in return for a common market and the convenience of a common currency.

Option C is incorrect: When countries have foreign currency debt funding local currency assets, they may choose to default if printing more local currency pushes up inflation and devalues the local currency which leads to substantial losses in the value of assets while liabilities remain unchanged.

Q.2807 Aram Stone recently graduated from one of the most renowned German universities. During his time as an undergrad, Stone developed a unique algorithm that could completely change the development of Artificial Intelligence. Stone wants to patent his idea and initiate a startup, but he did not yet decide on the exact location. To evaluate legal risks, Stone found the rating presented in the table below:

Region	Overall Property Rights	Legal Property Rights	Physical Property Rights	Intellectual Property Rights
Central/Eastern Europe	4.78	4.64	5.47	4.22
Asia Oceania	4.77	4.42	5.44	4.44
Middle East & North Africa	4.76	4.61	5.42	4.26
Latin America	4.57	4.23	5.23	4.25
Africa	4.53	4.26	5.17	4.16
best protection <-> highest scores				

In which of the below regions should Stone register his company and patent if his main concern is the protection of his algorithm?

- A. Central/Eastern Europe
- B. Asia & Oceania
- C. Middle East & North Africa
- D. Latin America

The correct answer is **B**.

Since Stone developed a unique algorithm that could completely change the development of Artificial Intelligence, and that his main concern is the protection of his algorithm, then he should register his company in a region with the highest rating of Intellectual Property Rights protection. In this case, he should choose Asia & Oceania.

Q.3433 Mendoza Valeria, FRM, works as a risk analyst at a Mexican conglomerate. She has been asked to evaluate USD-based bond investments issued by four different companies. According to her employer's guidelines, the firm has a strict policy of only investing in companies with investment-grade ratings on both the S&P rating scale and Moody's. In addition, the firm only invests in countries with favorable sovereign risk quality. Assuming the company is located in the paired country, which (country, company) pair, as outlined below, would be the most appropriate investment?

Country	Import Ratio	Debt Service Ratio	Company	S&P Rating	Moody's Rating
Jolly World	42%	300%	Brighter World Ltd.	A	Ba
Pluto	18%	30%	Green Leaf Corp	A	Aa
Northern Lights	8%	250%	Eastbrom Financial	BB	Baa
Norfork	30%	50%	Helsinki Inc.	BBB	Ba

- A. (Norfork, Helsinki Inc.)
- B. (Northern Lights, Eastbrom Financial)
- C. (Pluto, Green Leaf Corp)
- D. (Jolly World, Brighter World Ltd.)

The correct answer is C.

An investment-grade bond has S&P rating BBB or above or Moody's ratings Baa or above.

Import ratio is the ratio of total imports of a country to that country's total foreign exchange (FX) reserves. The larger the import ratio, the higher the probability of default. This could possibly lead to a rescheduling of payments.

The debt service ratio is the ratio of debt service payments (principal + interest) of a country to that country's export earnings. The lower the ratio, the healthier (less risky) a country is deemed to be.

As you can see above, choice C ticks all three boxes.

Q.3434 Credit rating agencies like S&Ps and Moody's issue two different credit ratings for countries with an appetite for debt. These are the local currency debt rating and the foreign currency debt rating. Historically, it has been observed that defaults on local-currency-denominated debt are less frequent than foreign-currency-denominated debt.

What's the main reason behind this observation?

- A. Local currency debt has a lower spread compared to that of foreign debt
- B. This is a statistical anomaly - ideally, defaults rates in the two categories of debt should be more or less equal
- C. Foreign currency-denominated debt has fewer political ramifications than local currency-denominated debt, making it easier to deal with the consequences from the perspective of a country's leadership
- D. Unlike local currency obligations, foreign currency obligations cannot be settled via monetary expansion

The correct answer is **D**.

The main reason why we haven't witnessed too many local-currency-denominated debt defaults is down to the ability of governments to print money to settle such obligations. No government has printing rights over foreign currencies, and thus monetary expansion is not a viable option in the case of foreign debt. The lower default rate on local-currency-denominated debt is consistent with the observation that credit ratings are higher. This effectively rules out statistical errors.

Option A is incorrect: Local currency debt has a lower spread compared to that of foreign debt, but this is not the main reason why we have more defaults under the latter.

Option C describes a possible reason as to why a government might default on foreign debt, but this only happens in autocratic countries where the leadership enjoys near-dictatorial powers and security of tenure.

Q.3435 Credit ratings have over the years been put to task for issuing credit ratings that do not accurately capture the risk associated with foreign debt owned by different countries around the world.

Which of the following statements is false regarding weaknesses of rating agency sovereign debt ratings?

- A. Ratings are often reactive to real life happenings on the lending market
- B. Rating agencies exhibit some interdependence while issuing credit ratings
- C. Rating agencies use government-provided data to model default risk and come up with a credit rating
- D. None of the above

The correct answer is **D**.

Option A is a true statement: Rating agencies have been found to be reactive rather than proactive, which means they do not properly execute the advisory role they are meant to.

Option B is a true statement: Besides being under the influence of the political class, rating agencies are also not as independent as they should be. This plays out in instances where one agency issues a ratings downgrade on a country and others soon follow suit, without independent analysis of the prevailing situation.

Option C is a true statement: The data that the agencies use to rate sovereign ratings generally come from the governments themselves. As such, there is the potential for governments holding back bad news and revealing only good news which, in turn, may explain the upward bias in sovereign ratings.

Reading 50: Measuring Credit Risk

Q.561 The following is a structure of one-factor models between normally distributed variables, U_i :

$$U_i = a_i F + \sqrt{1 - a_i^2} Z_i$$

Which of the following is NOT a property of the above model?

- A. Every U_i has a standard normal distribution with mean = 0 and standard deviation = 1.
- B. Every Z_i is uncorrelated with each other.
- C. The constant a_i is between 0 and 1.
- D. F and Z_i have standard normal distributions.

The correct answer is C.

The one-factor model is defined as:

$$U_i = a_i F + \sqrt{1 - a_i^2} Z_i$$

Where F is a common factor for all U_i and Z_i is a component of U_i that is unrelated to the factor F and uncorrelated to each other. The a_i are the parameter values that lie between -1 and +1, that is $a_i \in [-1, +1]$.

The variables F and Z_i have the standard normal distributions, that is, $F \sim N(0, 1)$ and $Z_i \sim N(0, 1)$. Therefore, U_i is a sum of two independent normal distributions, and it is, therefore, a normal variable with a mean of 0 and a standard deviation of 1. The variance of U_i is 1 since F and Z_i are uncorrelated.

Note: The Capital Asset Pricing Model is a good example of the one-factor model.

Q.562 A copula is:

- A. A joint probability distribution between two uniformly distributed random variables.
- B. A joint probability distribution between two or more uniformly distributed random variables which still maintains their marginal distributions.
- C. The product of the marginal distributions of two or more random variables.
- D. A statistical tool that represents a multivariate distribution while still maintaining their individual marginal distributions.

The correct answer is **D**.

A copula is a multivariate distribution which assesses the dependence between the variables by retaining their marginal distributions. The copula is applied in skewed financial market distributions in which the correlation coefficient cannot be applied to assess interdependence.

A copula maps the marginal distribution of each variable to the standard normal distribution, which, by definition, has a mean of zero and a standard deviation of one. Copula correlation models create a joint probability distribution for two or more variables while still preserving their marginal distributions. The joint probability of the variables of interest is implicitly defined by mapping them to other variables whose distribution properties are known.

Q.1051 Yusuf, a research scholar associated with Dale University, presents a report on expected loss to the senior management of Glovsky Bank. He makes the following statement(s) in his report:

Statement I: The expected loss is a certain amount of money a bank is expected to lose over a pre-determined period of time when extending loans to its customers

Statement II: Even though credit loss levels will fluctuate from year to year, there is an anticipated average level of losses over time that can be statistically determined

Statement III: Expected loss must be treated as a foreseeable cost of doing business in the lending business

Statement IV: Expected loss represents the level of losses predicted for the following year based on the economic cycle

Which of these statements are true?

A. I & II only

B. I, II & III only

C. II, III & IV only

D. I, II & IV only

The correct answer is **B**.

Statements I, II & III are true. The expected loss is a certain amount of money a bank is expected to lose over a pre-determined period of time when extending loans to its customers. Even though these credit loss levels will fluctuate from year to year, there is an anticipated average level of losses over time that can statistically be determined. The expected loss must be treated as a foreseeable cost of doing business in the lending business.

Statement IV is false. The expected loss is not the level of losses predicted for the following year based on the economic cycle, but rather the long-run average loss level across a range of typical economic conditions.

Q.1052 Economic losses are determined using certain components. Which of the following is not a component that determines economic losses?

- A. Probability of default.
- B. Exposure amount.
- C. Loss rate.
- D. All of the three components determine Economic loss.

The correct answer is **D**.

Probability of default, exposure amount, and loss rate are the three components that determine economic losses.

The probability of default (PD), describes the probability that a borrower will default on contractual payments before the end of a predetermined period. It is expressed as a percentage.

Exposure amount (EA), also known as exposure at default (EAD), is the loss exposure of a bank at the time of a loan's default, expressed as a dollar amount. It is the predicted amount of loss in the event the borrower defaults.

The loss rate, also known as the loss-given-default (LGD), is the percentage loss incurred if the borrower defaults. It can also be described as the expected loss expressed as a percentage.

Q.1053 American International Bank sanctioned a loan to a corporate client. The following particulars are given in the credit note by the credit analyst of the client:

Exposure amount = 100 USD million

Loss rate = 10%

Probability of default = 20%

What is the expected loss of the loan?

- A. USD 2 million.
- B. USD 20 million.
- C. USD 10 million.
- D. USD 40 million.

The correct answer is **A**.

$$\begin{aligned}\text{Expected loss} &= \text{Probability of default at time H} * \text{Exposure amount at time H} \\ &\quad * \text{Loss rate experienced at time H} \\ &= PD * EA * LR \\ &= 100 * 0.2 * 0.1 \\ &= 2 \text{ million}\end{aligned}$$

Q.1054 Rojan Ortiz, a senior credit risk analyst at Asiana Bank, discusses with his colleague the components of the economic losses. He makes the following statements with regard to the components of the economic losses. Which of the following are true?

Statement I: The loss rate is the fraction of the exposure amount that is lost in the event of default

Statement II: Probability of default is a borrower-specific estimate that is typically linked to the borrower's risk rating

Statement III: Exposure amount and loss rate reflect and model the product specifics of a borrower's liability

Statement IV: Probability of default (PD) is a measure of the likelihood that a counterparty goes into default over a predetermined period of time

A. I & II only.

B. I, II & III only.

C. II, III & IV only.

D. All of the above.

The correct answer is **D**.

All statements are correct. The loss rate is the fraction of the exposure amount that is lost in the event of default, meaning the amount that is not recovered after the sale of the collateral. PD is a borrower-specific estimate that is typically linked to the borrower's risk rating. The remaining two components reflect and model the product specifics of a borrower's liability. Probability of default (PD) is a measure to determine whether a counterparty goes into default over a predetermined period of time

Q.1055 A bank credit risk is preparing a manual on unexpected losses. Which of the following statements can be captured in the manual with regard to unexpected loss?

- I. It is important to price unexpected losses in a loan's interest rate adequately
- II. Unexpected loss in statistical terms is the standard deviation of credit losses, that is, the standard deviation of actual credit losses around the expected loss average
- III. Unexpected loss can be calculated at the transaction and portfolio level
- IV. Unexpected loss is the primary driver of the amount of economic capital required for credit risk

A. I & II only.

B. I, II & III only.

C. II, III & IV only.

D. All of the above.

The correct answer is C.

Statement I is incorrect: Unexpected losses cannot be anticipated and hence cannot be adequately priced for in a loan's interest rate.

Statements II, III & IV are correct. Unexpected loss, in statistical terms, is the standard deviation of credit losses, that is, the standard deviation of actual credit losses around the expected loss average. Unexpected loss can be calculated at the transaction and portfolio level. Unexpected loss is the primary driver of the amount of economic capital required for credit risk.

Q.1056 John Sutton, a newly recent finance graduate working at Asana Finance Ltd., approaches his superior, George Shelton, to understand the differences and similarities between expected losses and unexpected losses?. Gorge makes the following statements:

Statement I: The unexpected loss of a specific loan on a stand-alone basis (i.e., ignoring diversification effects) can be derived from the components of expected loss

Statement II: Expected loss is calculated as the mean of a distribution whereas unexpected loss is calculated as the standard deviation of the same distribution

Statement III: Like expected losses, unexpected losses can also be calculated for various time periods and for rolling time windows across time

Statement IV: Unexpected losses stem from the (unexpected) occurrence of defaults and (unexpected) credit migration whereas expected losses must be treated as the foreseeable cost of doing business in lending markets

Which of these statements are true?

- A. Statements I & II only.
- B. Statements I, II & III only.
- C. Statements II, III & IV only.
- D. All of the above.

The correct answer is **D**.

The unexpected loss of a specific loan on a stand-alone basis (i.e., ignoring diversification effects) can be derived from the components of expected losses. The expected loss is calculated as the mean of a distribution whereas unexpected loss is calculated as the standard deviation of the same distribution. Like expected losses, unexpected losses can also be calculated for various time periods and for rolling time windows across time. Unexpected losses (UL) stem from the (unexpected) occurrence of defaults and (unexpected) credit migration whereas expected losses must be treated as the foreseeable cost of doing business in lending markets.

Q.1057 Which of the following represents the correct relationship between the expected loss, unexpected loss, and the actual loss?

- A. $\text{Expected Loss} = \text{Unexpected Loss} - \text{Actual Loss}.$
- B. $\text{Unexpected Loss} = \text{Expected Loss} - \text{Actual Loss}.$
- C. $\text{Actual Loss} = \text{Expected Loss} + \text{Unexpected Loss}.$
- D. $\text{Actual Loss} = \text{Expected Loss} * \text{Unexpected Loss}.$

The correct answer is **C**.

The expected loss is the amount a bank can expect to lose, on average, over a predetermined period when extending credits to its customers. Unexpected loss is the volatility of credit losses around its expected loss. Thus, the actual loss is the summation of the expected loss and the unexpected loss.

Q.1058 ABX Bank Limited is holding a portfolio of loans. Which of the following, considering a loan at the portfolio level, is NOT part of the contribution of the single unexpected loss to the overall portfolio risk?

- A. The loan's expected loss.
- B. The loan's exposure amount.
- C. The correlation of the exposure to the rest of the portfolio.
- D. None of the above.

The correct answer is **D**.

A loan at the portfolio level is not part of the contribution of a single unexpected loss. The overall portfolio risk is a function of:

- (a) The loan's expected loss (because default probability, loss rate, and exposure amount are all part of the unexpected loss equation)
 - (b) The loan's exposure amount
 - (c) The correlation of the exposure to the rest of the portfolio
-

Q.1059 Neeson, a quantitative analyst, is preparing a model for estimating unexpected losses. He is incorporating appropriate distributions for the components of unexpected losses.

Which of the following are true with regard to the distributions of components of unexpected losses?

I. The probability of default is a binomial distribution

II. The loss rate can take a number of shapes, which results in different equations for the variances of loss rate

III. The binomial distribution understates the variance of the loss rate as compared to the uniform distribution

IV. The uniform distribution assumes that all defaulted borrowers would have the same probability of losing anywhere between 0 percent and 100 percent

A. I & II only

B. I, II & III only

C. II, III & IV only

D. I, II & IV only

The correct answer is **D**.

Statement III is incorrect: The binomial distribution overstates the variance of the loss rate, since when a customer defaults, either all of the exposure amount is lost or nothing. On the other hand, the uniform distribution assumes that all defaulted borrowers would have the same probability of losing between 0% and 100%.

Statements I, II & IV are correct: Since default is a Bernoulli variable, the probability of default is a binomial distribution. Unlike the distribution for the probability of default, the loss rate can take a number of shapes, which results in different equations for the variances of loss rate. Possible distributions are the binomial, the uniform, or the normal distribution, and the uniform distribution assumes that all defaulted borrowers would have the same probability of losing anywhere between 0 percent and 100 percent.

Q.1061 Default correlations play an important role in measuring the marginal contributions of a loan to a loan portfolio. With regard to default correlations for a loan portfolio containing a large number of loans:

- A. Default correlations are very difficult, if not impossible, to observe.
- B. If the loan portfolio contains 'n' loans, $[n(n-1)]/2$ pairwise default correlations need to be estimated.
- C. Default correlations are small, but positive providing considerable benefits to diversification in credit portfolios.
- D. All of the above are true.

The correct answer is **D**.

Default correlations are very difficult, if not impossible, to observe where a portfolio of loans consists of many thousand credits. If the loan portfolio contains 'n' loans, $\frac{[n(n-1)]}{2}$ pairwise default correlations need to be estimated. Default correlations are small, but positive providing considerable benefits to diversification in credit portfolios.

Q.1062 Anston Walsh, a credit analyst at Grant Bank, is entrusted with the task of calculating the economic capital for a portfolio of loans underwritten by the bank. Walsh based his task of computing economic capital on the following assumptions/statements. Which of them are to be considered in the computation to determine the most appropriate amount of economic capital?

Statement I: The amount of economic capital needed is the distance between the expected outcome and the unexpected (negative) outcome at a certain confidence level

Statement II: The crucial task in estimating economic capital is the choice of the probability distribution

Statement III: Credit risks are normally distributed

Statement IV: One distribution often recommended for measuring credit risk is the normal distribution

- A. I & II only.
- B. I, II & III only.
- C. I, III & IV only.
- D. All of the above.

The correct answer is **A**.

Statements I & II are correct. The amount of economic capital needed is the distance between the expected outcome and the unexpected (negative) outcome at a certain confidence level. The crucial task in estimating economic capital is the choice of the probability distribution.

Statements III & IV are incorrect. Credit risks are not normally distributed but highly skewed as the upward potential is limited to receiving at maximum the promised payments and only in very rare events to losing a lot of money. One distribution often recommended for measuring credit risk is the beta distribution which is extremely flexible in the shapes of the distribution it can accommodate.

Q.3074 BYJ commercial bank has \$100 million of retail exposures. The 1-year probability of default averages 2% and the recovery rate averages 60%. If the correlation parameter is estimated at 0.1, what will be the 1-year unexpected loss at 99.9% confidence?

- A. \$7.68 million
- B. \$8.01 million
- C. \$4.32 million
- D. \$12.8 million

The correct answer is **C**.

Assuming the loss distribution is lognormal, the α percentile of the distribution of the default rate logarithm is

$$\alpha\text{percentile for default rate} = N \left(\frac{N^{-1}(PD) - \sqrt{\rho}N^{-1}(1 - \alpha)}{\sqrt{1 - \rho}} \right)$$

Thus, the 99 default rate is given by;

$$\begin{aligned} V(0.999, 1) &= N \left[\frac{\{N^{-1}(0.02) + \sqrt{0.1}N^{-1}(0.999)\}}{\sqrt{1 - 0.1}} \right] \\ &= \frac{-2.05 + \sqrt{0.1} \times 3.09}{\sqrt{1 - 0.1}} = 0.128 \end{aligned}$$

This is showing that the 99.9% worst-case default rate is 12.8%

The 1-year unexpected loss at 99.9% confidence is given by:

$$\begin{aligned} &(WCDR - PD) \times LGD \times EAD \\ &= (0.128 - 0.02) \times (1 - 0.6) \times 100 \\ &= 4.32 \text{ million} \end{aligned}$$

Where:

α is the confidence level

ρ is the correlation between each pair of U_i distributions

N^{-1} is the inverse cumulative normal distribution

WCDR(Worst-Case Default Rate) is the 99.9 percentile of the default rate distribution

LGD is the Loss Given Default(equals one minus the recovery rate).

PD is the Probability of Default

EAD is the total exposure at default (i.e., the sum of the principals of all the loans).

Q.3440 An investor holds a portfolio of \$200 million. This portfolio consists of AA-rated bonds (\$120 million) and BB-rated bonds (\$80 million). Assume that the one-year probabilities of default for AA-rated and BB-rated bonds are 4% and 6%, respectively, and that they are independent. In the event of default, the recovery rate for AA-rated bonds is 65%, and the recovery rate for BB-rated bonds is 40%. Determine the one-year expected credit loss from this portfolio:

- A. \$1,680,000
- B. \$4,560,000
- C. \$4,500,000
- D. \$2,880,000

The correct answer is **B**.

The expected loss of the portfolio is the sum of the expected losses of individual assets.

$$EL = EA \times PD \times LR$$

For AA-rated bonds,
 $EA = \$120,000,000$,
 $PD = 0.04$, and
 $LR = 0.35$
 Thus,

$$EL_{AA} = 120,000,000 \times 0.04 \times 0.35 = \$1,680,000$$

For BB-rated bonds,
 $EA = \$80,000,000$,
 $PD = 0.06$, and
 $LR = 0.6$
 Thus,

$$EL_{BB} = 80,000,000 \times 0.06 \times 0.6 = \$2,880,000$$

$$\text{Portfolio Expected Loss} = \$1,680,000 + \$2,880,000 = \$4,560,000$$

Q.3441 A portfolio consists of two bonds. The credit VaR - as defined by the bondholder - is the maximum loss due to defaults at a confidence level of 99%, over a period of one year. The probability that the two bonds jointly default is 2%, with a default correlation of 25%. The bond value, default probability, and recovery rate are USD 500,000, 5%, and 50% for one bond, and USD 300,000, 3%, and 30% for the other. Determine the expected credit loss of the portfolio:

- A. USD 18,800
- B. USD 12,500
- C. USD 18,424
- D. USD 12,424

The correct answer is **A**.

The joint default probability and the default correlation are nugatory as far as the expected credit loss of the portfolio is concerned. In other words, they do no matter.

The expected loss of the portfolio is simply the sum of the expected losses of individual assets.

$$EL = EA \times PD \times LR$$

For the first bond,

$$EA = \$500,000,$$

$$PD = 0.05, \text{ and}$$

$$LR = 0.5$$

Thus,

$$EL_{AA} = 500,000 \times 0.05 \times 0.5 = \$12,500$$

For the second bond,

$$EA = \$300,000,$$

$$PD = 0.03, \text{ and}$$

$$LR = 0.7$$

Thus,

$$EL_{BB} = 300,000 \times 0.03 \times 0.7 = \$6,300$$

$$\text{Portfolio credit loss} = \$12,500 + \$6,300 = \$18,800$$

Note: The joint probability of default and the default correlation would be important only in the calculation of the unexpected credit loss of the portfolio.

Q.3657 Australian Synergies Finance Limited uses beta distributions to measure credit risks. The company states that the beta distribution helps in predicting the credit losses accurately. With regard to the measurement of credit losses, which of the following statements are true?

- I. The beta distribution is often recommended and is a suitable probability distribution for measuring the credit losses
- II. The beta distribution is especially useful in modeling a random variable that varies between -1 and +1
- III. The shape of the beta distribution can be completely determined by specifying the parameters α and β
- IV. The beta distribution is fully characterized by two parameters: expected loss of the portfolio and unexpected loss of the portfolio

- A. I & II only
- B. I, III & IV only
- C. I, II & IV only
- D. II, III & IV only

The correct answer is **B**.

Statement II is incorrect. The beta distribution is especially useful in modeling a random variable that varies between 0 and c (>0). This is because the beta distribution nearly always produces positive outputs. When modeling credit events, losses can vary between 0 and 100%, so that $c = 1$.

Statements I, III & IV are correct. The beta distribution is often recommended and is a suitable probability distribution for measuring credit losses as credit losses are normally distributed but highly skewed. The shape of the beta distribution can be completely determined by specifying the parameters α and β . The beta distribution is fully characterized by two parameters: expected loss of the portfolio and unexpected loss of the portfolio.

Q.3662 Nicolson Finance has taken credit exposure to two corporate clients. The credit risk characteristics of these two loans have been provided below:

Loan to customer 1: The sanctioned amount is USD 600 million, the exposure amount is USD 540 million, the probability of default over the next year is 2%, and the loss rate is 20% if the customer defaults. Moreover, the standard deviations of the probability of default and the loss rate are 3% and 35%, respectively.

Loan to customer 2: The sanctioned amount is USD 300 million, the exposure amount is USD 200 million, the probability of default over the next year is 1%, and the loss rate is 40% if the customer defaults. The standard deviations of the probability of default and the loss rate are 2% and 20%, respectively.

The correlation between the two loan accounts is 0.5.

What is the risk contribution of customer 1 and customer 2 to the loan portfolio?

- A. Customer 1: 26.705 million and Customer 2: 2.611 million
- B. Customer 1: 27.705 million and Customer 2: 1.611 million
- C. Customer 1: 28.619 million and Customer 2: 2.611 million
- D. Customer 1: 28.619 million and Customer 2: 1.611 million

The correct answer is **A**.

$$UL = EA \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \sigma_{PD}^2}$$

$$\begin{aligned} UL_{\text{customer 1}} &= 540 \times \sqrt{0.02 \times (0.35)^2 + 0.20^2 \times 0.03^2} \\ &= \text{USD } 26.924 \text{ million} \end{aligned}$$

$$\begin{aligned} UL_{\text{customer 2}} &= 200 \times \sqrt{0.01 \times (0.20)^2 + 0.40^2 \times 0.02^2} \\ &= \text{USD } 4.308132 \text{ million} \end{aligned}$$

$$\begin{aligned} UL_{\text{portfolio}} &= \sqrt{(UL_{\text{customer 1}})^2 + (UL_{\text{customer 2}})^2 + 2 \times UL_{\text{customer 1}} \times UL_{\text{customer 2}} \times \text{Correlation}} \\ &= \sqrt{26.924^2 + 4.3081^2 + (2 \times 26.924 \times 4.3081 \times 0.5)} \\ &= 29.316 \text{ million} \end{aligned}$$

In general,

$$\text{Risk contribution of customer X} = UL_{\text{customer X}} \times \left[\frac{UL_{\text{customer Y}} + (\text{Corr}_{X,Y} \times UL_{\text{customer Y}})}{UL_{\text{portfolio}}} \right]$$

$$\begin{aligned} \text{Risk contribution of customer 1} &= 26.924 \times \left[\frac{26.924 + (0.5 \times 4.308)}{29.316} \right] \\ &= \text{USD } 26.705 \text{ million} \end{aligned}$$

$$\begin{aligned} \text{Risk contribution of customer 2} &= 4.308 \times \left(\frac{4.308 + (0.5 \times 26.924)}{29.316} \right) \\ &= \text{USD } 2.611 \text{ million} \end{aligned}$$

Q.4644 A bank has two assets outstanding, denominated in U.S. dollars. The correlation between the two assets is 0.4. Other details are as follows:

	Asset A	Asset B
EA	1,600,000	2,000,000
PD	1%	2%
LR	30%	40%

Calculate the expected loss (EL) of the portfolio.

- A. 22400
- B. 20800
- C. 18200
- D. 20200

The correct answer is **B**.

The expected loss of a portfolio is equal to the summation of expected losses of individual asset. That is,

$$\begin{aligned} EL_P &= \sum EA_i \times PD_i \times LGD_i \\ &= [1,600,000 \times 0.01 \times 0.3] + [2,000,000 \times 0.02 \times 0.4] \\ &= 20,800 \end{aligned}$$

Q.4645 The amount of a loan issued by a bank is \$2 million, with a default probability of 0.1% over a period of one year. If the recovery rate is estimated to be 40%, what is the expected credit loss?

- A. \$800
- B. \$1,000
- C. \$1,200
- D. \$700

The correct answer is **C**.

The credit loss of a single asset is given by:

$$EL = EA \times PD \times LGD$$

But also

$$LGD = 1 - \text{Recovery Rate}$$

So, the expected credit loss is given by:

$$EL = \$2 \text{ million} \times 0.001 \times (1 - 0.4) = \$1,200$$

Q.4646 The amount of a loan issued by a bank is \$2 million, with a default probability of 0.1% over one year. If the recovery rate is estimated to be 40%, what is the standard deviation expected credit loss?

- A. \$37,928.35
- B. \$30,567.65
- C. \$32, 464.54
- D. \$35,890.75

The correct answer is **A**.

The standard deviation of the loss is given by:

$$\begin{aligned}\sigma_i &= \sqrt{p_i - p_i^2} [L_i (1 - R_i)] \\ &= \sqrt{0.001 - 0.001^2} [\$2 \text{ million} (1 - 0.4)] = 0.0379284 = 37,928.35\end{aligned}$$

Q.4647 The Bank of Africa has a portfolio of three \$2 million loans, each with a default rate of 0.5% over one year. If the correlation between the loans is 0.4 and the recovery rate is 40%, what is the mean of the portfolio credit loss?

- A. \$18,000
- B. \$12,000
- C. \$10,000
- D. \$9,000

The correct answer is **A**.

The mean of credit loss of a loan is given by:

$$p_i L_i (1 - R_i)$$

Nevertheless, we are given three loans with the same variables. So the mean of the portfolio is:

$$\begin{aligned} &= 3 \times p_i L_i (1 - R_i) \\ &= 3 \times 0.005 \times 2 \text{ million} \times (1 - 0.4) = \$18,000 \end{aligned}$$

Q.4648 The Bank of Africa has a portfolio of three \$2 million loans, each with a default rate of 0.5% over one year. If the correlation between the loans is 0.4, and the recovery rate is 40%, what is the standard deviation of the portfolio credit loss?

- A. \$426,875
- B. \$38,685
- C. \$84,600
- D. \$196,593

The correct answer is **D**.

The variance of the credit loss of a loan portfolio is given by

$$\sigma_p^2 = n\sigma^2 + n(n-1)\rho\sigma^2$$

Since the loans have equal principal L, recovery rate R and default probabilities, we need to compute the common standard deviation as:

$$\begin{aligned}\sigma_i &= \sqrt{p - p^2} [L(1 - R)] \\ &= \sqrt{0.005 - 0.005^2} [2 \times (1 - 0.4)] = 0.0846\end{aligned}$$

Thus, the variance of the credit loss is given by

$$\begin{aligned}\sigma_p^2 &= n\sigma^2 + n(n-1)\rho\sigma^2 \\ \sigma_p^2 &= 3 \times 0.0846^2 + 3(3-1) \times 0.4 \times 0.0846^2 =\end{aligned}$$

Hence, the standard deviation of the credit loss is given by

$$\sigma_p = \sqrt{0.386852} = 0.196593 = \$196,593$$

Q.4650 Aiden Bank has a \$500 million loan portfolio with a PD of 0.5%. Assuming the Vasicek Model, what is the 99.9 percentile of the default rate if the correlation parameter is 0.25?

- A. 0.1305
- B. 0.0165
- C. 0.1169
- D. 0.0175

The correct answer is **C**.

According to the Vasicek model, the 99.9 percentile of the default rate is given by

$$\begin{aligned}
 \text{99.9 percentile for default rate} &= N \left(\frac{N^{-1}(PD) + \sqrt{\rho} N^{-1}(0.999)}{\sqrt{1-\rho}} \right) \\
 &= N \left(\frac{N^{-1}(0.005) + \sqrt{0.25} N^{-1}(0.999)}{\sqrt{1-0.25}} \right) \\
 &\Rightarrow N \left(\frac{-2.576 + 0.5 \times 3.09}{\sqrt{1-0.25}} \right) = N(-1.1905) = 0.1169 = 11.69\%
 \end{aligned}$$

Q.4652 A bank has a loan portfolio consisting of three loans A, B, and C with standard deviations of 1.25 each. The correlations matrix appears as follows:

	Loan A	Loan B	Loan C
Loan A	1	0	0.3
Loan B	0	1	0.6
Loan C	0.3	0.6	1

Suppose the size of loan A is increased by 1%. Using the Euler's theorem, calculate the contribution of loan A to the total standard deviation.

- A. 0.76
- B. 0.72
- C. 0.80
- D. 0.74

The correct answer is **D**.

According to Euler's theorem,

$$Q_i = x_i \frac{\Delta F_i}{\Delta x_i}$$

Where

Δx_i = small change in x_i

ΔF_i = small change in F_i

Q_i = ratio of ΔF_i to a proportional change $\frac{\Delta x_i}{x_i}$ in x_i

We need to first calculate the total loss on the portfolio.

Given a portfolio of three assets, the portfolio standard deviation is given by:

$$\sigma_p = \sqrt{\sigma_A^2 + \sigma_B^2 + \sigma_C^2 + 2\rho_{AB}\sigma_A\sigma_B + 2\rho_{AC}\sigma_A\sigma_C + 2\rho_{BC}\sigma_B\sigma_C}$$

The current portfolio standard deviation is

$$\begin{aligned} & \sqrt{1.25^2 + 1.25^2 + 1.25^2 + 2 \times 0 \times 1.25 \times 1.25 + 2 \times 0.3 \times 1.25 \times 1.25 + 2 \times 0.6 \times 1.25 \times 1.25} \\ & = 2.738613 \end{aligned}$$

Now if the size of loan A is increased by 1%, then the new standard deviation of loan A is given by

$$1.25 \times 1.01 = 1.2625$$

So, the new portfolio standard deviation is

$$\begin{aligned} & \sqrt{1.2625^2 + 1.25^2 + 1.25^2 + 2 \times 0 \times 1.2625 \times 1.25 + 2 \times 0.3 \times 1.2625 \times 1.25 + 2 \times 0.6 \times 1.25 \times 1.25} \\ & = 2.746048 \end{aligned}$$

The change in portfolio standard deviation = $2.746048 - 2.738613 = 0.007435$

So, that

$$Q_A = \frac{0.007435}{0.01} = 0.7435 \approx 0.74$$

The contributions of loans B and C can be calculated in a similar manner, i.e., increasing their respective standard deviations by 1%, holding all other factors constant.

Q.4653 Credit risk capital for derivatives is challenging to calculate as compared to that of the loans. Which of the following reason(s) makes this statement true?

- A. The exposure at default for the derivative is relatively less certain than it is for the loans.
- B. Derivatives are subject to netting agreements.
- C. All of the above.
- D. None of the above.

The correct answer is **C**.

The exposure at default for the derivative is relatively less certain than that of the loans.

Moreover, derivatives are subject to netting agreements, so that all of the outstanding derivatives with a given counterparty may be considered a single derivative in case of a default, making it difficult to estimate credit risk capital.

Q.4654 Barclays Bank has a \$600 million loan portfolio, and the recovery rate in the event of default is 40%. Assuming the Vasicek Model, the required regulatory capital is \$5 million. The 99.9 percentile for default rate is 0.0188. What is the probability of default for the loan portfolio?

- A. 0.0051
- B. 0.0400
- C. 0.0049
- D. 0.0054

The correct answer is C.

The Basel II capital requirement for banks under the IRB approach is given by

$$(WCDR - PD) \times LGD \times EAD$$

Where WCDR is defined as the worst-case default rate, and it is 99.9 percentile of the default rate distribution defined as in Vasicek model, which in this case is 0.0188.

$$\begin{aligned} \Rightarrow (WCDR - PD) \times LGD \times EAD &= 5 \\ (0.0188 - PD) \times (1 - 0.4) \times 600 &= 5 \\ \therefore PD &= 0.004911 \end{aligned}$$

Q.4655 A bank issues a \$2 million loan, with a default probability of 0.5% over one year. The standard deviation of the expected credit loss is \$35,000. What is the recovery rate?

- A. 0.752
- B. 0.456
- C. 0.656
- D. 0.764

The correct answer is **A**.

The standard deviation of the loss is given by:

$$\begin{aligned}\sigma_i &= \sqrt{p_i - p_i^2} [L_i (1 - R_i)] \\ &= \sqrt{0.005 - 0.005^2} [\$2 \text{ million} (1 - R_i)] = 0.035 \\ \Rightarrow 1 - \left[\frac{0.035}{2 \times \sqrt{0.005 - 0.005^2}} \right] &= 0.7519\end{aligned}$$

Q.4656 A bank issues a \$7 million loan, with a default probability of 0.5% over a period of one year. If the recovery rate is estimated to be 35%, what is the expected credit loss on this loan?

- A. \$23,350
- B. \$22,750
- C. \$23,600
- D. \$22,850

The correct answer is **B**.

The credit loss of a single asset is given by:

$$\begin{aligned}EL &= EA \times PD \times LGD \\ &= \$7 \text{ million} \times 0.005 \times (1 - 0.35) = \$22,750\end{aligned}$$

Q.4657 An American bank recently issued a USD 5 million loan to a business entity, of which USD 2 million is currently outstanding. According to the bank's internal rating model, the business entity has a 0.5% chance of defaulting over the next year. In case that happens, the estimated loss rate is 25%. The probability of default and the loss rate have standard deviations of 7% and 17%, respectively. What is the value of unexpected loss?

- A. \$41,245.45
- B. \$42,461.75
- C. \$40,564.56
- D. \$45,563.45

The correct answer is **B**.

The unexpected loss is given by:

$$UL = EA \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \sigma_{PD}^2}$$

So in our case.

EA =USD 2,000,000

PD =0.5%

LR =25%

$\sigma_{LR} = 0.17$

$\sigma_{PD} = 0.07$

Thus,

$$\begin{aligned}
 &= 2,000,000 \times \sqrt{0.005 \times 0.17^2 + 0.25^2 \times 0.07^2} \\
 &= \$42,461.75
 \end{aligned}$$

Q.4658 The bank of Aides has a portfolio consisting of 100,000 loans, each amounting to \$1 million, and has a 1% probability of default in a year. The recovery rate is 40%, and the correlation coefficient is 0.3. Calculate α , the standard deviation of the loss from the loan portfolio as a percentage of its size.

A. 0.033

B. 0.045

C. 0.056

D. 0.045

The correct answer is **A**.

The α parameter is given by:

$$\alpha = \frac{\sigma \sqrt{1 + (n - 1)\rho}}{L\sqrt{n}}$$

For this case,

$L = \$ 1 \text{ million}$

$\rho = 0.1$

$n = 100,000$

$R = 0.4$

$$\begin{aligned}\sigma &= \sqrt{p - p^2} [L(1 - R)] \\ &= \sqrt{0.01 - 0.01^2} [1(1 - 0.4)] = 0.05970\end{aligned}$$

Therefore,

$$\alpha = \frac{0.05970 \sqrt{1 + (100,000 - 1)0.3}}{1 \times \sqrt{100,000}} = 0.03270$$

Reading 51: Operational Risk

Q.1069 US International Bank is contemplating assessing operational risk for regulatory capital. Which of the following approaches can be used to calculate operational risk?

- I. Basic indicator approach
- II. Standardized approach
- III. Advanced measurement approach
- IV. Internal Ratings Based approach

- A. I, II & III only.
- B. I, III & IV only.
- C. II, III & IV only.
- D. All of the above.

The correct answer is **A**.

Banks have three alternatives for determining operational risk regulatory capital: (i) the basic indicator approach, (ii) the standardized approach, and (iii) the advanced measurement approach.

Q.1070 A bank follows the basic indicator approach for assessing operational risk for regulatory purposes. Which of the following statement(s) is/are NOT true with regard to the basic indicator approach?

- A. Under this approach, operational risk capital was set equal to 15% of the three-year average annual gross income.
- B. Gross income is defined as net interest income.
- C. Net interest income is the excess of income earned on loans over interest paid on deposits and other instruments that are used to fund the loans.
- D. All of the above are true.

The correct answer is **B**.

Under the basic indicator approach, operational risk capital was set equal to 15% of the three-year average annual gross income.

Gross income is defined as net interest income plus non-interest income.

Net interest income is the excess of income earned on loans over interest paid on deposits and other instruments that are used to fund the loans.

Q.1071 Eurasia Bank Limited is following the basic indicator approach for calculating the operational risk amount for the year 2016. The financial details of the bank are given below:

	Income earned (In million USD)	Interest paid	Non-interest income
Year 2015	105	52	18
Year 2014	100	50	20
Year 2013	95	40	16

Based on the original Basel Accord, the bank must hold capital for operational risk for 2016 equal to:

- A. USD 10.60 million.
- B. USD 10.95 million.
- C. USD 11.05 million.
- D. USD 7.90 million.

The correct answer is **A**.

Based on the original Basel Accord, banks using the basic indicator approach must hold capital for operational risk equal to the average over the previous three years of a fixed percentage of positive annual gross income. Gross income is defined as net interest income plus non-interest income.

Net interest income plus non-interest income for the previous three years:

	Income earned (In million USD)	Interest paid	Net Income	Non-interest income	Gross Income
Year 2015	105	52	53	18	71
Year 2014	100	50	50	20	70
Year 2013	95	40	55	16	71

Average gross income over the previous three years: 70.667

Operational risk = 15% of average gross income over the previous three years: 10.60

Q.1072 American International Bank is using the standardized approach for measuring operational risk for regulatory capital. The bank is least likely to:

- I. Have an operational risk management function that is responsible for identifying, assessing, monitoring, and controlling operational risk
- II. Keep track of relevant losses by business line and must create incentives for the improvement of operational risk
- III. Have a well-documented operational risk management system
- IV. Estimate unexpected losses based on an analysis of relevant internal and external data, and scenario analyses

A. I & II only.

B. III & IV only.

C. III only.

D. IV only.

The correct answer is **D**.

A bank which is using the standardized approach for measuring operational risk for regulatory capital must satisfy the following conditions:

- (a) The bank must have an operational risk management function that is responsible for identifying, assessing, monitoring, and controlling operational risk
- (b) The bank must keep track of relevant losses by business line and must create incentives for the improvement of operational risk
- (c) There must be regular reporting of operational risk losses throughout the bank
- (d) The bank's operational risk management system must be well documented
- (e) The bank's operational risk management processes and assessment system must be subject to regular independent reviews by internal auditors. It must also be subject to regular review by external auditors or supervisors or both

To use the AMA approach, the bank must satisfy additional requirements. It must be able to estimate unexpected losses based on an analysis of relevant internal and external data and scenario analyses.

Q.1073 The Basel Committee on Banking Supervision (BCBS) has identified seven categories of operational risk. Which of the following categories are covered by the Basel Committee?

- I. Employment practices and workplace safety
- II. Clients, products, and business practices
- III. Execution, delivery, and process management
- IV. Strategic risk

- A. I, II & III only.
- B. II, III & IV only.
- C. I, II & IV only.
- D. All of the above.

The correct answer is **A**.

The Basel Committee on Banking Supervision has identified seven categories of operational risk: (a) Internal fraud, (b) External fraud, (c) Employment practices and workplace safety, (d) Clients, products, and business practices, (e) Damage to physical assets, (f) Business disruption and system failures, and (g) Execution, delivery, and process management.

Strategic risk is not one of the seven categories of operational risk identified by the Basel Committee on Banking Supervision.

Q.1074 A bank with annual revenues of \$4 billion has incurred a loss of \$200 million on account of operational risk. What would be the losses for a bank with a similar business profile but with revenues of \$12 billion? Assume the exponent for scaling losses is 0.23.

- A. USD 7.76 million
- B. USD 12.76 million
- C. USD 257.5 million
- D. USD 200.00 million

The correct answer is **C**.

$$\begin{aligned}\text{Loss of bank B} &= \left(\frac{\text{Revenue of bank B}}{\text{Revenue of bank A}} \right)^{0.23} * \text{Loss of bank A} \\ &= \left(\frac{12}{4} \right)^{0.23} * 200 \\ &= 3^{0.23} * 200 = 257.5 \text{ million}\end{aligned}$$

Q.1075 Your Canadian Bank has been using the standardized approach for the last three years. The board of directors has recently decided to use the advanced measurement approach for measuring operational risk for regulatory risk from the current year onwards. Considering the change in measurement approach, which of the following process has been adopted from the current year onwards in the risk management department of the bank?

- A. Regular reporting of operational risk losses throughout the bank.
- B. Well-documented operational risk management system.
- C. Regular independent review of operational risk management processes by internal auditors, external auditors, and supervisors.
- D. Estimation of unexpected losses based on an analysis of relevant internal and external data, and scenario analyses.

The correct answer is **D**.

A bank which is using the standardized approach for measuring operational risk for regulatory capital must satisfy the following conditions:

- (a) The bank must have an operational risk management function that is responsible for identifying, assessing, monitoring, and controlling operational risk
- (b) The bank must keep track of relevant losses by business line and must create incentives for the improvement of operational risk
- (c) There must be regular reporting of operational risk losses throughout the bank
- (d) The bank's operational risk management system must be well documented
- (e) The bank's operational risk management processes and assessment system must be subject to regular independent reviews by internal auditors. It must also be subject to regular review by external auditors or supervisors or both

To use the AMA approach, the bank must satisfy additional requirements. It must be able to estimate unexpected losses based on an analysis of relevant internal and external data and scenario analyses.

Q.1076 Bank X is following the advanced measurement approach for measuring operational risk. Which of the following should be the operational risk capital computed for regulatory purposes?

- A. The bank must use 15% of net interest income over the previous three years.
- B. The bank's activities are divided into eight business lines. The average gross income over the last three years for each business line is multiplied by a "beta factor" for that business line, and the result is summed to determine the total capital.
- C. The bank must estimate one-year 99.9% VaRs for the seven categories of operational risks identified by the Basel Committee and then aggregate them to determine a single one-year 99.9% operational risk VaR measure.
- D. The bank must use 15% of net interest income plus non-interest income over the previous three years.

The correct answer is C.

Standardized measure: The bank's activities are divided into eight business lines: corporate finance, trading and sales, retail banking, commercial banking, payment and settlement, agency services, asset management, and retail brokerage. The average gross income over the last three years for each business line is multiplied by a "beta factor" for that business line and the result is summed to determine the total capital.

Advanced measurement approach: Banks must estimate one-year 99.9% VaRs for each combination and then aggregate them to determine a single one-year 99.9% operational risk VaR measure.

Basic indicator approach: The bank must use 15% of net interest income plus non-interest income over the previous three years.

Q.1077 Loss severity and loss frequency are two distributions that are important in estimating potential operational risk losses for a risk type. With regard to these two distributions, which of the following is true?

- A. For loss frequency, the natural probability distribution to use is a Poisson distribution, and for the loss-severity probability distribution, a lognormal distribution is used.
- B. For loss frequency, the natural probability distribution to use is a lognormal distribution, and for the loss-severity probability distribution, a Poisson distribution is used.
- C. For loss frequency, the natural probability distribution to use is a Poisson distribution, and for the loss-severity probability distribution, a normal distribution is used.
- D. For loss frequency, the natural probability distribution to use is a normal distribution, and for the loss-severity probability distribution, a Poisson distribution is used.

The correct answer is **A**.

For loss frequency, the natural probability distribution to use is a Poisson distribution. This distribution assumes that losses happen randomly through time so that in any short period of time Δt , there is a probability $\lambda \Delta t$ of a loss occurring.

For the loss severity probability distribution, a lognormal distribution is used. The parameters of this probability distribution are the mean and standard deviation of the logarithm of the loss.

Q.1078 The Basel Committee on Banking Supervision (BCBS) requires the implementation of the advanced measurement approach to involve some elements. These include:

- I. Internal data
- II. External data
- III. Strategic analysis
- IV. Business environment and internal control factors

- A. I, II & III only.
- B. II, III & IV only.
- C. I, II & IV only.
- D. All of the above.

The correct answer is **C**.

The Basel Committee on Banking Supervision (BCBS) requires the implementation of the advanced measurement approach to involve four elements. These include: (a) Internal data, (b) External data, (c) Scenario analysis, and (d) Business environment and internal control factors.

Q.1080 New Zealand National Bank uses the advanced measurement approach to compute the operational risk capital for regulatory purposes. Noria Franti, a financial controller working at the bank, analyzes the research reports on internal data and external data. She concludes the following from the analysis:

- I. Banks have done a much better job at documenting their operational losses than their credit risk losses.
- II. Credit card frauds are high-frequency, low-severity losses.
- III. When an institution can not use its own data, then external data can be used for the loss severity distribution.
- IV. The loss frequency distribution must be specific to the bank and based on internal data and scenario analysis estimates.

Which of these statements are correct?

- A. I & III only.
- B. III & IV only.
- C. I, II & IV only.
- D. II, III & IV only.

The correct answer is **D**.

Statement II is correct: Credit card losses are high-frequency low-severity losses, meaning that they have high expected loss but relatively low unexpected loss.

Statement IV is also correct: The loss frequency distribution must be specific to the bank and based on internal data and scenario analysis estimates.

Statement III is also correct: When an institution can not estimate loss severity from its own data, then external data can be used for the loss severity distribution.

Statement I is incorrect: Banks have done a much better job at documenting their credit risk losses than their operational losses.

Q.1082 The operational risk team of the Canadian Insurance Group informs the risk committee that the company faces higher risk than predicted while insuring a bank against operational losses because the bank operates recklessly after taking the insurance cover, further increasing the risks it is exposed to. Which of the following clauses/precautions can be taken to mitigate this risk?

I. Deductible in the insurance policy

II. Coinsurance provision

III. Policy limit

IV. Understanding the controls existing within the bank and the losses that have been experienced

A. I & II only.

B. II & III only.

C. I, II & III only.

D. IV only

The correct answer is **C**.

Moral hazard is the risk that the existence of the insurance contract will cause the bank to behave differently than it otherwise would. For example, a bank that insures itself against robberies. As a result of the insurance policy, it may be tempted to be lax in its implementation of security measures, making a robbery more likely than it would otherwise have been.

Insurance companies have traditionally dealt with moral hazard in a number of ways. Typically there is a deductible in any insurance policy. This means that the bank is responsible for bearing the first part of any loss.

Sometimes there is a co-insurance provision in a policy. In this case, the insurance company pays a predetermined percentage (less than 100%) of losses in excess of the deductible.

In addition, there is nearly always a policy limit. This is a limit on the total liability of the insurer. The existence of deductibles, coinsurance provisions, and policy limits are likely to provide an incentive for a bank not to relax security measures in its branches.

Q.1083 Frank Andrews, an operational risk analyst, is interested in using the power law to assess operational risk. Which of the following statement(s) is/are true with regard to the power-law?

- I. The power law holds well for large losses experienced by banks
- II. Loss data and scenario analysis are employed to estimate the power-law parameters using the maximum likelihood approach
- III. When loss distributions are aggregated, the distribution with the heaviest tails tends to dominate
- IV. The loss with the highest alpha defines the extreme tails of the total loss distribution

- A. I & II only.
- B. II & III and IV only.
- C. I, II & III only.
- D. All of the above.

The correct answer is C.

Statements I, II & III are correct: The power law holds well for the large losses experienced by banks. Loss data (internal or external) and scenario analysis are employed to estimate the power-law parameters using the maximum likelihood approach. When loss distributions are aggregated, the distributions with the heaviest tails tend to dominate.

Statement IV is incorrect: The loss with the lowest alpha defines the extreme tails of the total loss distribution.

Q.1085 The existence of an insurance contract causes the bank to behave differently than it otherwise would and increases the risks to the insurance company. This risk is known as:

- A. Adverse selection.
- B. Moral hazard.
- C. Wrong-way risk.
- D. Operational risk.

The correct answer is **B**.

Moral hazard is the risk that the existence of the insurance contract will cause the bank to behave differently than it otherwise would. This changed behavior increases the risks to the insurance company.

Option A is incorrect: Adverse selection describes a situation where the risk seller has more information than the buyer about a product, putting the buyer at a disadvantage. For example, a company providing life assurance may unknowingly attract heavy smokers, or even individuals suffering from terminal illnesses. If this happens, the company effectively takes on many high-risk persons but very few low-risk individuals. This may result in a claim experience that's worse than initially anticipated.

Option C is incorrect: Wrong-way risk is the risk that a counterparty to a company has a higher probability of default when the value of outstanding derivatives is negative to the counterparty, thus positive to the company.

Option D is incorrect: Operational risk refers to the risk of loss resulting from failed processes, people, or systems within an institution.

Q.3442 Which of the following statements about the operational risk framework for banks is *most likely* incorrect?

- A. Under the basic indicator approach, banks must set aside capital equivalent to 15% of the three-year average annual gross income
- B. Under the standardized approach, a bank has to declare its gross income in eight business lines and then use beta factors to work out the amount of capital required in each line.
- C. Banks using the advanced measurement approach must calculate the operational risk capital charge at a 99 percentile confidence interval and a one-year horizon.
- D. According to the Basel committee, operational risk includes legal risk but explicitly excludes reputational and strategic risks.

The correct answer is C.

Banks using the advanced measurement approach must calculate the operational risk capital charge at a **99.9 percentile** confidence interval and a **one-year** horizon.

$$ORC_{AMA} = UL(1 - \text{year}, 99.9\% \text{ confidence})$$

Q.3443 Under the AMA method, insurance can be used to offset up to 20% of the operational risk charge. Which of the following statements about hedging operational risk are valid?

- I. All insurance policies suffer from the problem of moral hazard, but deductibles and coinsurance provisions help to combat this problem
- II. Adverse selection can result in a claim experience that's worse than initially anticipated
- III. A primary disadvantage of insurance as a tool for operational risk management is the limitation of policy coverage
- IV. The scorecard capital allocation method allocates capital to business lines in a firm as guided by the results of a risk survey conducted across the firm
- V. If an operational risk hedge works properly, a firm will avoid damage to its reputation from a high-severity operational risk event

A. All of the above

B. III, and V

C. II, III, and V

D. I, II, III, and IV

The correct answer is **D**.

Statement V is invalid: Even if a firm is well protected from high-severity operational risk events, the news of such events will still take a toll on its reputation.

Statement I is valid: To help keep the behavior of the insured in check, insurers use a host of features that include deductibles, policy limits, and coinsurance provisions.

Statement II is valid: Adverse selection creates a high-risk pool of policies that are likely to result in claims, increasing the cash outflows of the insurer relative to inflows.

Statement III is valid: Most insurance policies have a cover ceiling, meaning there's always a cap on the compensation that could be handed to the policyholder.

Statement IV is also valid: Under the scorecard approach, each unit manager is subjected to a survey which has questions on matters of risk. Each manager's responses are transformed into a quantitative measure to come up with an overall score. This total score represents the unit's exposure to risk.

Q.3444 A risk manager has established that there's a 90% probability that losses over the next year will not exceed \$30 million. Given that the power law parameter is 0.8, what is the probability of the loss exceeding \$10 million?

- A. 22%
- B. 24%
- C. 20%
- D. 23%

The correct answer is **B**.

The power law states that the probability of a random variable x exceeding a value V is given by:

$$p(v > x) = KV^{-\alpha}$$

where:

K is constant,

α is the power law parameter.

$$p(v > x) = KV^{-\alpha}$$

$$0.1 = K(30)^{-(0.8)}$$

$K = 1.5195$ Thus,

$$p(v > x) = 1.5195V^{-0.8}$$

when $x = 10$,

$$\text{Probability} = 1.5195 \times 10^{-0.8} = 0.24$$

Q.4596 One of the major operational risks is compliance risk. Which of the following is/are example(s) of compliance risks?

- A. Money laundering
- B. Terrorism financing
- C. Failure to comply with sanctions
- D. All of the above

The correct answer is **D**.

Compliance risks occur when an institution incurs fines due to knowingly, or unknowingly ignoring the industry's set rules and regulations, internal policies, or standard best practices. Some examples of compliance risks include money laundering, financing terrorism activities, and helping clients to evade taxes.

Q.4597 One of the operational risks is rogue trader risk. To protect itself from rogue trader risk, a bank should make the front office and back office independent of each other. Which one of the following statements distinguishes between the back and front office?

- A. Trading takes place in the front office while record keeping is done in the back office
- B. Record keeping is done in the front office, and trading is done in the back office
- C. The front office is where management works, and the back office is where traders trade
- D. None of the above

The correct answer is **A**.

To protect itself from the rogue trader risk, a bank should make the front office and back office independent of each other. The front office is the one that is responsible for trading, and the back office is the one responsible for the record-keeping and verifications of the transactions.

Q.4598 The average loss frequency of Bank of Africa is estimated to be once every 18 months. What is the probability of three losses in a year for this bank?

A. 0.0234

B. 0.057

C. 0.0254

D. 0.0507

The correct answer is **C**.

We need to find λ which from the question we have:

$$\lambda = \frac{1}{1.5} = 0.6667 \text{ losses per year}$$

Now using the Poisson distribution

$$\Pr(n) = \frac{e^{-\lambda} \lambda^n}{n!}$$
$$\Pr(3) \Rightarrow \frac{e^{-0.6667} 0.6667^3}{3!} = 0.0254$$

Q.4599 Given the mean and the standard deviation of lognormal loss of a ban is 200 and 50 respectively, what is the variance of the logarithm of the loss?

A. 0.0606

B. 0.527

C. 0.069

D. 0.0629

The correct answer is **A**.

We know the mean of the logarithm of the loss is given by:

$$\ln\left(\frac{\mu}{\sqrt{1+w}}\right)$$

Furthermore, the variance is given by:

$$\ln(1+w)$$

Where

$$w = \left(\frac{\sigma}{\mu}\right)^2 = \left(\frac{50}{200}\right)^2 = 0.0625$$
$$\Rightarrow \text{Variance} = \ln(1.0625) = 0.0606$$

Q.4600 Given the mean and the standard deviation of the lognormal loss of a bank is 200 and 50, what is the standard deviation of the logarithm of the loss?

- A. 0.2462
- B. 1.0308
- C. 0.0606
- D. 0.4724

The correct answer is **A**.

We know the mean of the logarithm of the loss is given by:

$$\ln\left(\frac{\mu}{\sqrt{1+w}}\right)$$

Furthermore, the variance is given by:

$$\ln(1+w)$$

Where

$$\begin{aligned}w &= \left(\frac{\sigma}{\mu}\right)^2 = \left(\frac{50}{200}\right)^2 = 0.0625 \\ \Rightarrow \text{Variance} &= \ln(1+w) = \ln(1.0625) = 0.0606 \\ \therefore \text{Standard deviation} &= \sqrt{0.0606} \approx 0.2462\end{aligned}$$

Q.4601 Over the past ten years, the Bank of Yemen has had losses (in million euros) of 3, 6, 10, 50, 72, 101, and 200. What is the approximate amount of loss component of the bank under the SMA approach?

- A. €656 million
- B. €678 million
- C. €756 million
- D. €442 million

The correct answer is C.

Using the formula

$$7X + 7Y + 5Z$$

Where X, Y, and Z are the approximations of the average losses from the operational risk over the past ten years defined as:

- X - an average of all losses
- Y - an average of losses greater than 10 million
- Z - an average of losses greater than 100 million

From the question, the average total loss for the ten years is 442:

$$X = \frac{3 + 6 + 10 + 50 + 72 + 101 + 200}{10} = \frac{442}{10} = 44.2$$

The average losses greater than 10 million is:

$$Y = \frac{50 + 72 + 101 + 200}{10} = \frac{423}{10} = 42.3$$

And the average losses greater than 100 million is:

$$101 + 200 = \frac{301}{10} = 30.1$$

So the loss component is given by:

$$7 \times 44.2 + 7 \times 42.3 + 5 \times 30.1 = 756$$

Q.4602 A manager for the stock trading department suspects that one of his staff has gone rogue. Which of the following key indicators would the manager use to identify the rogue trader?

- A. The trader fails to take long holidays.
- B. The trader would seek for long holidays.
- C. The trader would always report trading transactions to the relevant authorities.
- D. The trader would seek guidance from the relevant bodies before taking a position in stock trading.

The correct answer is **A**.

The trade would fail to take holidays to continue hiding his or her unacceptable transactions.

Options B is incorrect: It contradicts option A.

Options C is incorrect: A trader reporting to the relevant authorities promotes transparency, and hence the trader would not be rogue.

Option D: Seeking guidance promotes transparency.

Q.4603 The estimation of loss distribution is laden with several data issues. Which of the following is **NOT** among them?

- A. Inadequate historical records.
- B. The constant purchasing power of money.
- C. Firm-specific adjustments.
- D. None - all of the above are valid data issues.

The correct answer is **D**.

The estimation of loss distribution is faced with the following data issues:

Inadequate historical records: The data available for operational risk losses – including loss frequency and loss amounts – is grossly inadequate, especially when compared to credit risk data. This inadequacy creates problems when trying to model the loss distribution of expected losses.

Inflation: When modeling the loss distribution using both external and internal data, an adjustment must be made for inflation. The purchasing power of money keeps on changing so that a \$10,000 loss recorded today would not have the same effect as a similar loss recorded, say, ten years ago. When there is constant purchasing power, it implies that there is no adjustment for inflation when using both internal and external data. Inadequate historical records and firm-specific adjustments are among the data issues faced in estimating the loss distribution.

Firm-specific adjustments: No two firms are the same in terms of size, financial structure, and operational risk management. As such, when using external data, it is essential to make adjustments to the data in cognizance of the different characteristics of the source and your bank. A simple proportional adjustment can either underestimate or overestimate the potential loss.

Q.4604 The 90-percentile of a loss distribution is 30. Using the power law with $\alpha = 4$, what is the value of 95-percentile of the loss distribution?

- A. 35.68
- B. 30.45
- C. 25.56
- D. 27.89

The correct answer is **A**.

Recall that the power law is given by:

$$\Pr(v > x) = Kx^{-\alpha}$$

Therefore for the 90-percentile loss distribution, we have,

$$\begin{aligned} 0.10 &= K \cdot 30^{-4} \\ \Rightarrow K &= \frac{0.10}{30^{-4}} = 81,000 \end{aligned}$$

Thus, for the 95-percentile loss, we have to solve the equation:

$$\begin{aligned} 0.05 &= 81,000x^{-4} \\ \Rightarrow x &= \left(\frac{0.05}{81,000} \right)^{-\frac{1}{4}} = 35.68 \end{aligned}$$

Q.4606 Bank A has revenues of USD 50 billion and incurs a loss of USD 300 million. Another bank B has revenues of USD 40 billion. Given that the estimated loss for bank A is 250 million, which of the following is closest to the observed loss for Bank (Assume the scale adjustment is 0.23) B?

- A. USD 263.2 million
- B. USD 200 million
- C. USD 312.5 million
- D. USD 237.5 million

The correct answer is **D**.

Using the scale adjustment:

$$\begin{aligned}\text{Estimated Loss for Bank A} &= \text{Observed Loss for Bank B} \times \left(\frac{\text{Bank A Revenue}^{0.23}}{\text{Bank B Revenue}} \right) \\ \Rightarrow 250 &= \text{Observed Loss for Bank B} \times \left(\frac{50^{0.23}}{40} \right) \\ \text{Observed Loss for Bank B} &= 250 \times \left(\frac{50}{40} \right)^{-0.23} = 237.49\end{aligned}$$

Q.4607 Over the last three years, an American bank earned an interest of USD 300 million and paid interest of USD 150 million on average. The bank's noninterest income over the last three years is USD 600 million on average. Under the indicator method, what is the value of the operational risk capital?

- A. \$75 million
- B. \$250.40 million
- C. \$112.50 million
- D. \$140.50 million

The correct answer is C.

Under the indicator approach, the required operational risk is equivalent to 15% of annual gross income over the previous three years. Also, recall that,

$$\begin{aligned}\text{Gross income} &= \text{Interest earned} - \text{Interest paid} + \text{Noninterest income} \\ &= 300 - 150 + 600 = 750 \\ \therefore \text{Gross Income} &= \text{USD 750 Million}\end{aligned}$$

Thus, the required operational risk capital is

$$15\% \times 750 = 112.5$$

Reading 52: Stress Testing

Q.1086 AIZ bank, a newly set up bank, proposes to use stress testing to measure risk. Which of the following statements are true with regard to stress testing as a risk management tool in banking?

- I. Stress testing is an important risk management tool that banks use as part of their internal risk management and, through the Basel II capital adequacy framework, is promoted by supervisors
- II. Stress testing alerts bank management to unexpected adverse outcomes related to a variety of risks and provides an indication of how much capital might be needed to absorb losses should large shocks occur
- III. While stress tests provide an indication of the appropriate level of capital necessary to endure deteriorating economic conditions, a bank alternatively may employ other actions in order to help mitigate increasing levels of risk
- IV. Stress testing is especially important after long periods of benign economic and financial conditions when fading memory of negative conditions can lead to complacency and the underpricing of risk

- A. I & IV only.
- B. II & IV only.
- C. I, II & IV only.
- D. All of the above.

The correct answer is **D**.

Stress testing is an important risk management tool that is used by banks as part of their internal risk management and, through the Basel II capital adequacy framework, is promoted by supervisors. Stress testing alerts bank management to unexpected adverse outcomes related to a variety of risks and provides an indication of how much capital might be needed to absorb losses should large shocks occur. While stress tests provide an indication of the appropriate level of capital necessary to endure deteriorating economic conditions, a bank alternatively may employ other actions in order to help mitigate increasing levels of risk. Stress testing is especially important after long periods of benign economic and financial conditions when fading memory of negative conditions can lead to complacency and the underpricing of risk.

Q.1087 The financial crisis of 2007-2009 has revealed several weaknesses in organizational aspects of stress testing programs. Which of the following are some of these weaknesses?

- I. Stress testing at some banks was performed mainly at the firm-wide level
- II. At some banks, the stress testing program was a mechanical exercise
- III. While stress testing for market and credit risk had been practiced for several years, stress testing for interest rate risk in banks has emerged more recently
- IV. Stress testing frameworks were usually not flexible enough to respond quickly as the crisis evolved

- A. I & IV only.
- B. II & IV only.
- C. I, II & IV only.
- D. All of the above.

The correct answer is **B**.

Statements I & III are incorrect. Stress testing at some banks was performed mainly as an isolated exercise by the risk function with little interaction with business areas. While stress testing for market and interest rate risk had been practiced for several years, stress testing for credit risk in the banking industry has emerged more recently.

Statement II & IV are correct. At some banks, the stress testing program was a mechanical exercise. Stress testing frameworks were usually not flexible enough to respond quickly as the crisis evolved.

Q.1088 With regard to stress testing methodologies, which of the following statement(s) is/are true?

- I. Stress tests may be performed at varying degrees of aggregation, from the level of an individual instrument up to the institutional level
- II. Stress tests are performed for different risk types including market, credit, operational, and liquidity risk
- III. At the most fundamental level, weaknesses in infrastructure limit the ability of banks to identify and aggregate exposures across the bank
- IV. Unlike most risk management models, stress tests do not use historical statistical relationships to assess risk

- A. II, III & IV only.
- B. I, II & IV only.
- C. I, II & III only.
- D. All of the above.

The correct answer is **C**.

Options I, II & III are correct: Stress tests may be performed at varying degrees of aggregation, from the level of an individual instrument up to the institutional level. Stress tests are performed for different risk types, including market, credit, operational and liquidity risk. At the most fundamental level, weaknesses in infrastructure limited the ability of banks to identify and aggregate exposures across the bank.

Option IV is incorrect: Most risk management models, including stress tests, use historical statistical relationships to assess risk. The financial crisis has again shown that, especially in stressed conditions, risk characteristics can change rapidly as reactions by market participants within the system can induce feedback effects and lead to system-wide interactions.

Q.1089 Scenario selection is very important in measuring the risks of the banks using stress tests. With regard to scenario selection and stress tests prior to the crisis, which of the following statements are true?

- I. Scenarios tended to reflect mild shocks, assume shorter durations and underestimate the correlations between different positions, risk types and markets due to system-wide interactions and feedback effects
- II. Sensitivity tests, which are at the most basic level, generally shock individual parameters or inputs without relating those shocks to an underlying event or real-world outcome
- III. Banks also implemented hypothetical stress tests, aiming to capture events that had not yet been experienced
- IV. Scenarios that were considered extreme or innovative were often regarded as implausible by the board and senior management

A. II, III & IV only.

B. I, II & IV only.

C. I, II & III only.

D. All of the above.

The correct answer is **B**.

Statement I is correct: Scenarios tended to reflect mild shocks, assume shorter durations and underestimate the correlations between different positions, risk types and markets due to system-wide interactions and feedback effects.

Statement II is correct: Sensitivity tests, which are at the most basic level, generally shock individual parameters or inputs without relating those shocks to an underlying event or real-world outcome.

Statement III is incorrect: The scenarios chosen in the stress tests were too moderate and were based on a short period of time. The possible correlations between different risk types, products, and markets were ignored. As such, the stress test relied on the historical scenarios and left out risks from new products and positions taken by the banks.

Statement IV is correct: Scenarios that were considered extreme or innovative were often regarded as implausible by the board and senior management.

Q.1091 The senior management of the African Industrial Development Bank is reviewing the stress program post a severe financial crisis in Africa. With regard to the stress testing program, which of the following is most accurate?

- A. Senior management is ultimately responsible for the overall stress testing program, whereas the risk department is accountable for the program's implementation, management, and oversight.
- B. The Chief Risk Officer is ultimately responsible for the overall stress testing program, whereas the risk department is accountable for the program's implementation, management, and oversight.
- C. The risk committee is ultimately responsible for the overall stress testing program, whereas the risk department is accountable for the program's implementation, management, and oversight.
- D. The board of directors is ultimately responsible for the overall stress testing program, whereas the senior management is accountable for the program's implementation, management, and oversight.

The correct answer is **D**.

The board of directors is ultimately responsible for the overall stress testing program, whereas the senior management is accountable for the program's implementation, management and oversight. Recognizing that many practical aspects of a stress testing program will be delegated, the involvement of the board in the overall stress testing program and of senior management in the program's design is essential.

Q.1092 The senior management of the African Industrial Development Bank is reviewing the stress program post a severe financial crisis in Africa. With regard to the stress testing program, which of the following statements are accurate?

- I. The stress testing program should cover pipeline and warehousing risks. A bank should include such exposures in its stress tests regardless of the probability of being securitized.
- II. A bank should enhance its stress testing methodologies to capture the effect of reputational risk. The bank should integrate risks arising from off-balance-sheet vehicles and other related entities in its stress testing program.
- III. A bank should enhance its stress testing approaches for highly leveraged counterparties considering its vulnerability to specific asset categories or market movements and in assessing potential wrong-way risk related to risk-mitigating techniques.
- IV. The stress testing program should explicitly cover complex and bespoke products such as securitized exposures. Stress tests for securitized assets should consider the underlying assets, their exposure to systematic market factors, relevant contractual arrangements and embedded triggers, and the impact of leverage, particularly as it relates to the subordination level of the issue structure.

- A. II, III & IV only.
- B. I, II & IV only.
- C. I, II & III only.
- D. All of the above.

The correct answer is **D**.

The stress testing program should cover pipeline and warehousing risks. A bank should include such exposures in its stress tests regardless of the probability of being securitized. A bank should enhance its stress testing methodologies to capture the effect of reputational risk. The bank should integrate risks arising from off-balance-sheet vehicles and other related entities in its stress testing program. A bank should enhance its stress testing approaches for highly leveraged counterparties in considering its vulnerability to specific asset categories or market movements and in assessing potential wrong-way risk related to risk-mitigating techniques. The stress testing program should explicitly cover complex and bespoke products such as securitized exposures. Stress tests for securitized assets should consider the underlying assets, their exposure to systematic market factors, relevant contractual arrangements and embedded triggers, and the impact of leverage, particularly as it relates to the subordination level in the issue structure.

Q.1151 A bank has recently launched a fund for retail investors. The risk management team carries out stress testing of the newly launched fund to determine the impact of the fund on the bank's overall capital. Jason Bloomberg, a newly recruited risk manager, observes that the bank has an independent risk management team. He notes that the entire risk assessment and identification process is carried exclusively through stress testing. While examining the stress testing result, John observes that the test produces multiple potential losses under various scenarios. Bloomberg also observes that the inputs from the bank's top economists were taken while modeling the stress testing scenarios. He finds that the stress testing procedures are well documented and no deviation is allowed from the procedure.

The bank's stress testing results produce multiple potential losses. In view of this, select the most appropriate option.

- A. The stress testing result must be actionable.
- B. The stress testing results must be integrated into decision-making, but only at the senior-most level of management.
- C. The stress testing result must accurately specify the exact amount of loss associated with a given variable.
- D. Stress testing produces potential losses and hence no action is required on the results.

The correct answer is **A**.

A stress testing result must be actionable and must feed into the decision-making process at the appropriate management level, including strategic business decisions of the board or senior management.

Q.1152 A bank has recently launched a fund for retail investors. The risk management team carries out stress testing of the newly launched fund to determine the impact of the fund on the bank's overall capital. Jason Bloomberg, a newly recruited risk manager, observes that the bank has an independent risk management team. He notes that the entire risk assessment and identification process is carried exclusively through stress testing. While examining the stress testing result, John observes that the test produces multiple potential losses under various scenarios. Bloomberg also observes that the inputs from the bank's top economists were taken while modeling the stress testing scenarios. He finds that the stress testing procedures are well documented and no deviation is allowed from the procedure.

Inputs from the bank's top economist were considered while developing the scenario for stress testing. In view of this, select the most appropriate statement.

- A. Inputs from economists make the model more robust.
- B. Inputs from all stakeholders such as economists, business managers, fund managers, etc. must be taken into account.
- C. Only inputs from the risk management team in collaboration with the fund manager must be taken into account.
- D. The risk management team must independently design the models.

The correct answer is **B**.

The identification of relevant stress events requires the collaboration of all the stakeholders like traders, economists, fund managers, business managers, etc. Inputs from all stakeholders make the model more robust.

Q.1153 A bank has recently launched a fund for retail investors. The risk management team carries out stress testing of the newly launched fund to determine the impact of the fund on the bank's overall capital. Jason Bloomberg, a newly recruited risk manager, observes that the bank has an independent risk management team. He notes that the entire risk assessment and identification process is carried exclusively through stress testing. While examining the stress testing result, John observes that the test produces multiple potential losses under various scenarios. Bloomberg also observes that the inputs from the bank's top economists were taken while modeling the stress testing scenarios. He finds that the stress testing procedures are well documented and no deviation is allowed from the procedure.

Select the most appropriate statement.

- A. The stress testing procedure must be well documented.
- B. The stress testing procedure must be well documented and no deviation must be allowed.
- C. The stress testing procedure must be well documented and it must also allow the bank to perform flexible and ad-hoc stress tests.
- D. The stress testing procedure should not be documented; only the results should be shown to the managers.

The correct answer is C.

The bank must have a well laid down procedure to carry out stress testing. Proper documentation must however not impede the bank from carrying out flexible and ad-hoc stress tests to identify and respond to emerging risk issues.

Q.1154 A bank has recently launched a fund for retail investors. The risk management team carries out stress testing of the newly launched fund to determine the impact of the fund on the bank's overall capital. Jason Bloomberg, a newly recruited risk manager, observes that the bank has an independent risk management team. He notes that the entire risk assessment and identification process is carried exclusively through stress testing. While examining the stress testing result, John observes that the test produces multiple potential losses under various scenarios. Bloomberg also observes that the inputs from the bank's top economists were taken while modeling the stress testing scenarios. He finds that the stress testing procedures are well documented and no deviation is allowed from the procedure.

Select the most appropriate statement.

- A. Stress testing results must be confidential.
- B. Stress testing result must be used only internally.
- C. Stress testing results may be disclosed to outsiders with sufficient supporting information.
- D. Stress testing must be disclosed only to supervisors.

The correct answer is C.

In a bank, stress tests play an important role in the communication of risk. It must also play an important role in external communication with supervisors. A bank may voluntarily disclose the stress test with outsiders with sufficient background information on the underlying assumptions and the methodologies.

Q.1155 XYZ Bank has multiple branches across the country. The bank has 10 verticals for each of its products headed by 10 Vice Presidents. The VPs report directly to the Chairman of the bank. The bank also has an independent risk management team that reports directly to the Chairman. Each vertical carries out its individual stress tests and submits the reports to the vertical head which then presents them to the Chairman. The stress test procedure indicates three scenarios which must be stress tested in each of the verticals, and all the VPs ensure that the procedure is fully complied with.

The current capital position of the bank indicates no material threat to the viability of the bank. In view of this, the bank's risk management team does not include scenarios that challenge the viability of the bank in the stress tests. The risk management team also suggests independent stress testing of market assets and the funding liquidity. The bank, in its investor presentation, proudly claims to stress test each component of the balance sheet.

Each vertical of the bank carries out stress tests independently. In view of this, select the most appropriate statement.

- A. Individual stress testing is desirable.
- B. Individual stress testing overestimates the risk.
- C. Risk arising due to linkages between the verticals must also be included in the stress testing.
- D. Individual stress testing makes the stress testing process straightforward.

The correct answer is C.

Stress tests should cover a wide range of risks including at the firm-wide level. The bank should be able to integrate effectively, in a meaningful fashion, across the range of its stress testing activities and deliver a complete picture of firm-wide risk. The bank must be able to assess the risk arising due to the linkages between the different verticals.

Q.1156 XYZ Bank has multiple branches across the country. The bank has 10 verticals for each of its products headed by 10 Vice Presidents. The VPs report directly to the Chairman of the bank. The bank also has an independent risk management team that reports directly to the Chairman. Each vertical carries out its individual stress tests and submits the reports to the vertical head which then presents them to the Chairman. The stress test procedure indicates three scenarios that must be stress tested in each of the verticals, and all the VPs ensure that the procedure is fully complied with.

The current capital position of the bank indicates no material threat to the viability of the bank. In view of this, the bank's risk management team does not include scenarios that challenge the viability of the bank in the stress tests. The risk management team also suggests independent stress testing of market assets and the funding liquidity. The bank, in its investor presentation, proudly claims to stress test each component of the balance sheet.

Select the most appropriate statement.

- A. Three scenarios, as indicated in the stress test procedure, is sufficient to assess the risk.
- B. Stress testing must include multiple scenarios.
- C. The stress testing procedure must be flexible and must include forward-looking scenarios.
- D. A minimum of 10 scenarios must be used to perform stress testing.

The correct answer is **C**.

An effective stress testing consists of scenarios along a spectrum of events and severity levels. In addition, the stress testing procedure must be flexible in order to identify hidden vulnerabilities and must be forward-looking. There is no fixed limit on the scenarios which must be used to enhance the stress testing procedure.

Q.1157 XYZ Bank has multiple branches across the country. The bank has 10 verticals for each of its products headed by 10 Vice Presidents. The VPs report directly to the Chairman of the bank. The bank also has an independent risk management team that reports directly to the Chairman. Each vertical carries out its individual stress tests and submits the reports to the vertical head which then presents them to the Chairman. The stress test procedure indicates three scenarios that must be stress-tested in each of the verticals, and all the VPs ensure that the procedure is fully complied with.

The current capital position of the bank indicates no material threat to the viability of the bank. In view of this, the bank's risk management team does not include scenarios that challenge the viability of the bank in the stress tests. The risk management team also suggests independent stress testing of market assets and the funding liquidity. The bank, in its investor presentation, proudly claims to stress test each component of the balance sheet.

Select the most appropriate statement.

- A. In the case of bank XYZ, the risk management team is correct not to include scenarios which challenge the viability of the bank.
- B. In the case of bank XYZ, the risk management team must include scenarios with increased severity but must not challenge the viability of the bank.
- C. In the case of bank XYZ, the stress test must not include scenarios which challenge the viability of the bank.
- D. In the case of bank XYZ, the stress test must include scenarios which challenge the viability of the bank.

The correct answer is **D**.

Stress test scenarios must include scenarios that challenge the viability of the bank. Such stress tests uncover hidden risks and interactions among risks. The global financial crisis has indicated the usefulness of such stress testing so that a backup plan can be put in place in the event such scenarios turn out to be true.

Q.1159 Select the most appropriate statements.

- I. A bank must stress test each component of the balance sheet
- II. A bank must stress test off-balance sheet items
- III. A bank must only stress test liquid, market-related items, either on or off the balance sheet
- IV. The stress test must include only contractual off-balance sheet items

- A. I & II only.
- B. I, II & III.
- C. I, III & IV.
- D. All of the above.

The correct answer is **A**.

The bank's stress must integrate risks arising from balance sheet assets and off-balance sheet items. During the global financial crisis, the banks failed to assess the risk arising from off-balance sheet items such as securitization which proved to be very costly. Furthermore, all off-balance items whether contractual or non-contractual must be included in the stress test.

Q.1160 Which of the following statements is/are accurate?

- I. Supervisors should verify the active involvement of senior management in the stress testing program
- II. Banks must submit firm-wide stress tests to supervisors at regular intervals
- III. Under the Internal Capital Adequacy and Assessment Process (ICAAP), the bank will make use of internal models to assess, quantify and stress test risk drivers
- IV. Stress testing results must not impact the strategic business decisions of the bank

A. II, III & IV

B. I, II & III

C. I & III

D. I & IV

The correct answer is **B**.

According to the principles laid down for supervisors:

- 1. Supervisors should verify the active involvement of senior management in the stress testing program
 - 2. Banks must submit firm-wide stress tests to supervisors at regular intervals
 - 3. Stress testing must form an integral part of the Internal Capital Adequacy Assessment Process (ICAAP) and the bank's liquidity risk management framework
 - 4. Stress testing results **must** impact the strategic business decisions of the bank
 - 5. Under the Internal Capital Adequacy and Assessment Process (ICAAP), the bank will make use of internal models to assess, quantify and stress test risk drivers and factors and the amount of capital required to support them.
-

Q.1161 Which of the following statements is/are correct?

- I. A bank must not disclose the assumptions made during the stress testing to supervisors
- II. Supervisors must not take into account capital freely transferable within banking groups in times of stress
- III. Supervisors must only examine the need of capital for the bank
- IV. Supervisors should review the range of remedial actions envisaged by a bank in response to the results of the stress testing program

- A. I only
- B. IV only
- C. I & III
- D. III & IV

The correct answer is **B**.

According to the principles of supervisors:

- 1. A bank must review the assumptions made during the stress testing to supervisors
 - 2. A bank must take into account capital freely transferable within banking groups in times of stress
 - 3. Supervisors must examine the need for capital and liquidity for the bank
 - 4. Supervisors should review the range of remedial actions envisaged by a bank in response to the results of the stress testing program
-

Q.1162 Select the most appropriate statement.

- A. Supervisors must not determine stress scenarios.
- B. Supervisors must determine certain stress scenarios.
- C. Stress scenarios must be designed exclusively by the bank.
- D. Stress scenarios must not be disclosed.

The correct answer is **B**.

According to the principles laid down for supervisors:

1. Supervisors must review the stress scenarios
2. Supervisors must determine certain stress scenarios for the banks as it enhances the ability of supervisors and the bank to assess the impact of specific stress events

In view of the above two principles, option B is correct.

Q.1163 Supervisors must examine a bank's stress testing results as prescribed in the Basel II framework under:

- A. Pillar I.
- B. Pillar II.
- C. Pillar III.
- D. None of the above.

The correct answer is **B**.

Pillar II of the Basel II framework lays down the **supervisory review** process under which the supervisors must examine a bank's stress testing results as part of the supervisory review.

Under pillar I; capital adequacy requirements - the credit risk of counterparties should be mirrored in a new way to calculate the **minimum capital requirement** in the banking book.

Under pillar III; market discipline - banks are required to disclose **market information** about capital allocation and risks taken. Added pressure is therefore pressed on banks for sound risk management decisions to be made should shareholders and potential investors have more data on those decisions.

Q.1165 All the following are true for stress testing, EXCEPT:

- A. The goal of stress testing is to identify unusual scenarios which are not covered under standard VaR models.
- B. Stress testing considers all scenarios covered under standard VaR models.
- C. Stress testing is helpful in the analysis of events which generally get ignored.
- D. Stress testing is helpful in the analysis of extreme events.

The correct answer is **B**.

Stress testing generally involves analysis of events which are extreme in nature and not covered under standard VaR models. VaR and ES are backward-looking. That is, they assume that the future and the past are the same. This is actually one disadvantage of VaR and ES. On the other hand, stress testing is forward-looking. It asks the question, "what if?".

Q.1166 A bank funds its long-term loans by issuing short-term debt instruments such as commercial papers, NCDs with residual maturity of less than 1 year, deposits, etc. A risk manager wants to stress test the bank's balance sheet to examine its vulnerabilities. The stress test may include which of the following scenarios?

- I. Availability of surplus liquidity
- II. Failure to roll over short-term debt
- III. Increase in short-term interest rates
- IV. Increases in deposits

- A. Only I
- B. II & III
- C. I & IV
- D. III & IV

The correct answer is **B**.

The bank funds its long-term liability by short-term debt. If there is surplus liquidity, the bank would have no problem rolling over its short-term debt. Therefore, the surplus is not a source of stress for a bank's balance sheet. Similarly, an increase in deposits (themselves a liability for the bank) will help the bank to fund (repay or roll over) its other liabilities. Thus, increased deposits will also not cause stress on the bank's balance sheet.

The bank instances which will cause stress on the bank's balance sheet are:

1. The bank is not able to roll over its short-term debt, which will result in a funding gap and will cause stress on the bank's balance sheet.
2. An increase in short-term interest rates will result in an increase in the cost of funds for the bank. This increase may also result in a decrease in short-term liquidity which will then impact the bank's ability to roll over its short-term debt.

Therefore, for stress testing, the suitable scenarios are the inability of the bank to roll over debt and the increase in short-term interest rates.

Q.1167 A fund manager examines the annual return generated by fund A for the last 10 years. The return generated by the fund is furnished in the table below:

Year	Return
2015	+9.45%
2014	-4.45%
2013	+5.34%
2012	-3.35%
2011	+2.45%
2010	-6.56%
2009	+7.41%
2008	-8.83%
2007	+2.33%
2006	+1.32%

The fund manager intends to stress test Fund A for various scenarios. Select the correct option with regards to the stress testing of Fund A.

- A. As the maximum loss during the last 10 year is 8.83%, the stress test scenario for maximum loss must not exceed 8.83%.
- B. As the average loss during the last 10 years is 5.80%, the stress test scenario for maximum loss must be equal to 5.80%.
- C. The stress test scenario for maximum loss must be more than the historical maximum loss posted by the fund.
- D. As the average return generated by the fund for the last 10 years is 0.51%, the stress test scenario for maximum loss must be -0.51%.

The correct answer is **C**.

Stress testing generally involves the analysis of events which are extreme in nature and are not present in historical data. Although the maximum historical loss generated by the fund stood at 8.83%, the stress test scenario for maximum loss must be in excess of 8.83%.

Q.1168 A risk manager examines a portfolio (AUM- \$100 million) and observes that the performance of the fund is dependent on two variables, α and β . The manager wants to carry out a stress test of the portfolio. He defines two scenarios to stress test the portfolio:

- I. The value of α is pushed up by x and the value of β is pushed down by y
- II. The value of α is pushed down by x and the value of β is pushed up by y

After performing the stress test, the risk manager suggests that a contingency fund of \$10 million must be maintained. Select the most appropriate statement.

- A. A contingency fund of \$10 million must be maintained as suggested by the risk manager.
- B. A contingency fund of more than \$10 million must be maintained.
- C. The correlation between the two variables must be considered while performing the stress test.
- D. As stress tests generally involve events which rarely occurs, the contingency fund must not be maintained.

The correct answer is **C**.

The risk manager must consider the correlation between the two variables. The correlation between the two variables will correctly determine the contingency fund required to be maintained.

Q.1170 A fund manager intends to carry out scenario analysis of his portfolio. The portfolio consists of 25% government bonds, 60% global equities, and 15% investment in gold ETFs. The fund manager lists the portfolio's risk factors. He intends to perform a scenario analysis by generating scenarios based on the adverse movement in the portfolio's identified risk factors. Such an approach to scenario analysis is referred to as:

- A. Event-driven scenario analysis.
- B. Portfolio-driven scenario analysis.
- C. Factor push method.
- D. Historical method.

The correct answer is **B**.

Scenario analysis can generate either event-driven or portfolio-driven scenarios. In the case of event-driven scenarios, the scenario is formulated from plausible events that generate movements in the risk factors.

In the case of portfolio-driven scenario analysis, the risk factors of the portfolio are identified and then translated into adverse movements in risk factors.

In the above case, the fund manager first identifies the risk factors and then performs scenario analysis by generating adverse movements in the risk factors identified. Therefore, the fund manager utilizes the portfolio driven approach.

Q.1171 A financial institution should set out clearly stated and understandable policies and procedures governing stress testing, which must be adhered to. The policies and procedures ensure that the stress testing of parts of a financial institution converges to the same point. The policies and procedures should be able to:

- A. Explain the purpose of stress testing.
- B. State the frequency at which the stress testing can be done.
- C. Describe the roles and responsibilities of the parties involved in stress testing.
- D. All of the above.

The correct answer is **D**.

The policies and procedures should be able to:

- Explain the purpose of stress testing;
- Describe the procedures of stress testing;
- State the frequency at which the stress testing can be done;
- Describe the roles and responsibilities of the parties involved in stress testing;
- Provide an explanation of the procedures to be followed while choosing the scenarios;
- Describe how the independent reviews of the stress testing will be done;
- Give clear documentation on stress testing to third parties (e.g., regulators, external auditors, and rating agencies);
- Explain how the results of the stress testing will be used and by whom;
- They were amended as the stress testing practices changes as the market conditions change;
- Accommodate tracking of the stress test results as they change through time; and
- Document the activities of models and the software acquired from the vendors or other third parties.

Q.1172 The general belief that diversification leads to risk reduction was challenged during the Global Financial Crisis of 2007-2009. During the crisis, it was observed that the correlation between different assets increased due to which the concept of diversification failed.

Imagine yourself being a risk manager. In order to assess the likely impact of such events, the most appropriate tool is the:

- A. Scenario analysis.
- B. Sensitivity analysis.
- C. Historical simulation.
- D. Factor push analysis.

The correct answer is **A**.

The event that occurred in the Global Financial Crisis of 2007-2009 can be defined as extreme events; such events can be examined/analyzed with the use of scenario analysis. Different scenarios can be modeled, and an analysis of the likely impact can be carried out.

A sensitivity is the result of alternative assumptions relating to a future situation.

Historical Simulation is a method for calculating VaR using historical data.

Factor push analysis involves examining what happens to the portfolio when you take risk factors to the extreme.

Q.1173 All the following are true for stress testing, EXCEPT :

- A. It is highly subjective.
- B. The events are reported without an attached probability making the result difficult to interpret.
- C. It is not helpful in ensuring the survival of an institution in times of market turmoil.
- D. Implausible scenarios may lead to irrelevant potential losses.

The correct answer is C.

Stress testing requires formulating multiple scenarios; the inputs to these scenarios are highly subjective. In addition, these scenarios are reported without an attached probability which makes estimating the likelihood of occurrence of the scenarios difficult to predict. Furthermore, a large number of scenarios can be generated during the scenario analysis, some of which are implausible and may provide irrelevant potential loss scenarios.

Q.1174 Which of the following is/are correct statements?

- I. The worst case loss never exceeds that predicted by VaR measures
- II. Stress testing is a replacement for traditional VaR measures
- III. Stress testing may lead to a large number of information
- IV. Stress testing allows risk managers to assess the blind spots

- A. Only I
- B. I & II
- C. III & IV
- D. I & IV

The correct answer is C.

Statement I is incorrect: The worst-case loss may exceed that predicted by VaR measures; scenario analysis helps in modeling such scenarios.

Statement II is incorrect. Stress testing complements traditional VaR measures; it cannot be used to replace the traditional VaR measures.

Statement III is correct. Stress testing may lead to a large amount of information. Multiple scenarios may result in large amounts of information during scenario analysis.

Statement IV is correct. Stress testing helps in identifying vulnerability which is generally absent in historical data. Therefore, it helps risk managers identify the blind spots.

Q.2812 One of the key elements of sound governance over stress testing is the governance structure.

Which of the following statements regarding governance structure is incorrect?

- A. The internal audit should provide an independent evaluation of the ongoing performance, integrity, and reliability of the stress-testing activities.
- B. The board of directors should execute the overall stress testing strategy (including establishing adequate policies and procedures, assigning competent staff, etc.).
- C. An institution should have clear and comprehensive stress testing policies, procedures and documentation.
- D. Stress-testing governance should incorporate validation or another type of independent review to ensure the integrity of stress-testing processes and results.

The correct answer is **B**.

The execution of the overall stress testing strategy is the responsibility of the senior management, not the board of directors.

In simple terms, the biggest difference between the board of directors and senior management is that the board makes decisions while the senior management implements the plan.

Q.2813 What is the advantage of reverse stress testing?

- A. By concentrating on different scenarios, reverse stress testing helps to identify the most profitable business lines.
- B. Reverse stress testing does not consider scenarios beyond its normal business expectations and concentrates on issues that could affect business during the normal business operations.
- C. Reverse stress testing does not consider scenarios beyond its normal business expectations and consequently does not require a comprehensive analysis.
- D. By evaluating scenarios and circumstances that would render a business unviable, reverse stress testing identifies potential business vulnerabilities.

The correct answer is **D**.

Reverse stress testing starts from an outcome of business failure and identifies circumstances where this might occur (e.g., potential business vulnerabilities).

Option A is incorrect: Reverse stress testing is not aimed to identify profitable business lines.

Options B and C are incorrect: Reverse stress testing does consider scenarios beyond its normal business expectations.

Q.2814 During which phase of the economic cycle is stress testing most important?

- A. During the beginning of an economic recession.
- B. In the middle of an economic recession.
- C. In the beginning of an economic expansion.
- D. After a long period of economic expansion.

The correct answer is **D**.

Stress testing is especially important after long periods of economic growth when the fading memory of negative conditions can lead to complacency and the underpricing of risk.

Q.3445 Which of the following parties bears the ultimate responsibility for stress testing programs in banks?

- A. The risk management function
- B. Senior management
- C. The board of directors
- D. Internal audit

The correct answer is C.

The board of directors is “ultimately” responsible for a firm’s stress tests. Even if board members don’t immerse themselves into the technical details of stress tests, they should ensure that they stay sufficiently knowledgeable about stress testing procedures and interpretation of results. The members’ engagement is essential for the effective operation of stress testing.

Q.3446 Which of the following options *most accurately* presents a key governance issue that played a critical role in the failure of banks in the lead up to the 2007/2009 financial crisis?

- A. Senior management played little or no role at all in the development and operation of stress testing.
- B. Stress testing reports would be passed up to the boards of directors without first being approved by senior management
- C. Stress testing did not appear to be sufficiently integrated into institutions’ risk management frameworks, nor were test results taken into account during decision making
- D. Stress testing programs lacked clear, well-detailed policies meant to outline the procedure to follow from the start to the end, as well as describing the role played by various employees

The correct answer is C.

The financial crisis of 2007/2009 highlighted a critical lesson in relation to governance and stress testing. Precisely, stress testing did not appear to be sufficiently integrated into institutions’ risk management frameworks. The few that had shown more commitment to stress testing did not examine sufficiently severe scenarios, and test results had little or no consideration in decision making at the top level of management.

Q.3447 According to the CRMPG II report and the Basel committee report produced in the aftermath of the 2007/2009 financial crisis, rigorous stress testing should be a goal of all firms. To make stress testing more productive, firms should consider all of the following *except*:

- A. Identifying a wide range of scenarios that could result in portfolio losses
- B. Simulating the effects of capital problems and illiquidity pressures happening at the same time
- C. Asking risk managers to define and clearly express firm loss tolerance levels
- D. Ensuring that the scenarios tested are in line with the direction and long-term strategy set by the board of directors

The correct answer is C.

The role of "Establishing risk tolerance levels" falls under the mandate of business managers or the board of directors but not the risk managers.

Q.3448 The following statements regarding stress testing and value at risk methods are incorrect, EXCEPT:

- A. From a practical point of view, VaR measures commonly utilize just a few scenarios
- B. Ordinal arrangements are a key feature of VaR methods
- C. For regulatory stress tests, the current period is used as the departure point while generating hypothetical scenarios
- D. While VaR methods reveal the causal risk(s), stress tests do not

The correct answer is C.

For regulatory stress tests, generating hypothetical scenarios uses the current period, not past history, as the point of origin.

Option A is incorrect. From a practical point of view, VaR measures commonly utilize very many scenarios, but stress tests accommodate just a few

Option B is incorrect. Ordinal arrangements form part of stress tests, not VaR measures. The latter measures make use of cardinal probabilities.

Option D is incorrect. While stress tests reveal the causal risk(s), VaR measures do not

Q.3451 Prior to the recent crisis, stress testing was marked by several practices including:

- I. Inadequate firm-wide perspective
- II. Overreliance on sensitivity analysis
- III. Limited recognition of interactive effects
- IV. A lack of overall organizational view

- A. II and III
- B. I and IV
- C. II only
- D. All of the above

The correct answer is **D**.

In the years leading up to the recent crisis stress testing in most firms had all of the above characteristics:

- I. Inadequate firm-wide perspective: Business lines were expected to conduct stress tests on their exposures, which means that there was very little assessment of enterprise-wide exposure and the correlation between various risks.
 - II. Banks used to rely on sensitivity analysis which focuses on the impact of a shock on a single factor while holding other factors constant. By so doing, the banks failed to take into consideration the spillover effect and feedback effects arising from correlations arising from various risk factors.
 - III. The stress testing models used in the recent financial crisis were not equipped to predict the interrelationships among various financial securities.
 - IV. Stress testing was silo-based. Business lines were expected to conduct stress tests on their exposures, which means that there was very little assessment of enterprise-wide exposure
-

Q.3452 Following the 2007/2008 financial crisis, stress testing for securitized products should consider which of the following features?

- I. Contingency funding needs of the issuer
- II. Credit ratings of similar securities on the market
- III. Quality of underlying asset pool
- IV. Subordination level of tranches
- V. Systematic market conditions

A. All of the above

B. III and IV

C. II and V

D. I, III, IV, and V

The correct answer is **D**.

Assuming that the resulting security has the same risk profile as an apparently similar issue on the market is inappropriate. Securitized products are complex and possess different risk characteristics compared to the underlying asset.

Therefore, contingency funding needs of the issuer, quality of the underlying asset pool, and systematic market conditions should be considered.

Q.3453 Which of the following statements related to stress testing and Basel II is correct?

- I. Basel II requires banks to conduct stress tests and assess capital adequacy at least once every month
- II. In line with Basel II, a bank should take into account both its capital and liquidity needs while conducting stress tests

- A. I only
- B. II only
- C. Both I and II
- D. Neither I nor II is correct

The correct answer is **B**.

The Basel II framework does not impose monthly stress tests on banks.

Q.3454 Which of the following statements is (are) true?

Prior to the 2007-2008 credit crisis:

- I. Stress testing was mostly geared towards individual business lines without considering comprehensive firm-wide perspective
- II. Stress testing was primarily focused on historical or hypothetical scenarios

- A. I
- B. II
- C. Both I and II
- D. Neither

The correct answer is C.

Statement I is correct: Until 2007, stress tests were mostly done internally by banks as part of their own risk management. Beginning in 2007, governmental regulatory bodies became interested in conducting their own stress tests to ensure the effective operation of financial institutions. This is also the time when stress testing was refined. Nowadays, common tasks associated with Basel III compliance include: Monte Carlo simulation (including the use of copula methods for credit portfolio simulation), scenario analysis and stress testing, econometrics for procyclical and countercyclical analysis, asset-liability modeling, etc.

Statement II is correct: Historical scenarios were frequently implemented based on a significant market event experienced in the past. Such stress tests were, however, not able to capture risks in new products that have been at the center of the turmoil. Furthermore, the severity levels and duration of stress indicated by previous episodes proved to be inadequate.

Banks also implemented hypothetical stress tests, aiming to capture events that had not yet been experienced. Prior to the crisis, however, banks generally applied only moderate scenarios, either in terms of severity or the degree of interaction across portfolios or risk types. At many banks, it was difficult for risk managers to obtain senior management buy-in for more severe scenarios. Scenarios that were considered extreme or innovative were often regarded as implausible by the board and senior management

Q.3455 Which of the following statements is (are) correct?
Stress testing methods consider inter-correlations between:

- I. funding and market risks
- II. basis and liquidity risks
- III. market and pipeline risks
- IV. reputational and liquidity risks

- A. II only
- B. I, II and IV
- C. II and III
- D. I, II, III and IV

The correct answer is **D**.

- I. It is difficult to liquidate an asset without loss under deteriorating market stress conditions.
 - II. Due to a change in basis, an ineffective hedge can result in significant losses as a result of the unprotected decline in the underlying asset's value, giving rise to liquidity problems.
 - III. Because of deteriorating conditions in the securitization market conditions during the recent financial crisis, banks could not securitize assets and had to keep them in the balance sheet.
 - IV. Due to reputational risk, a bank may stand ready to inject credit or liquidity to a special purpose entity (SPE), putting itself under increasing liquidity pressure.
-

Q.4557 Which of the following statements correctly distinguish between stress testing and expected shortfall?

- I. Expected shortfall is backward-looking but stress testing is forward-looking
- II. Stress testing is backward-looking but expected shortfall is forward-looking
- III. Whereas stress testing is based on the future probability distribution, the expected shortfall is based on past probability distribution.
- IV. Expected shortfall analysis often relatively takes a short time but stress testing takes relatively long periods

- A. I and II
- B. II and III
- C. I and IV
- D. IV only

The correct answer is C.

Statement I is correct: Expected shortfall is backward-looking. In that, it assumes that the future and the past are the same. On the other hand, stress testing is forward-looking. It asks the question, “what if?”.

Statement II is incorrect: It contradicts statement I.

Statement III is incorrect: Stress testing does not involve probabilities, while the expected shortfall is founded on probability.

Statement IV is correct: VaR/ES analysis often takes a short period of time, such as a day, while stress testing takes relatively long periods, such as a decade.

Q.4558 Which of the following is/are **TRUE** about the stressed VaR and stressed Estimated shortfall (ES)?

- I. The data used to calculate stressed Var and stressed ES are drawn from stressful periods, such as the year 2007
- II. Stressed VaR and stressed ES are calculated based on a short period of time
- III. Similar to traditional VaR, stressed VaR could be back-tested

- A. I only
- B. III only
- C. I and II only
- D. All of the above

The correct answer is C.

Statement I is correct: The data used to calculate stressed VaR and stressed ES is obtained from stressed periods. Stressed VaR and Stressed ES might be objectively similar, but the time horizon for the stressed VaR/ES is short (one to ten days), while stress testing considers relatively longer periods.

Statement II is correct: Stressed VaR and stressed ES might be objectively similar to stress testing, but the time horizon for the stressed VaR/ES is short (one to ten days). In contrast, stress testing considers relatively longer periods.

Statement III is incorrect: Conventional VaR can be back-tested while stressed VaR cannot.

It is difficult to back-test stressed VaR because these measures focus on extreme outcomes, which do not have any particular observable frequency.

Q.4559 Which of the following is **NOT** an internally developed stress test scenario?

- A. Historical scenario
- B. Baseline scenario
- C. Ad hoc scenario
- D. None of the above

The correct answer is **B**.

Baseline scenario is one of the regulatory scenarios in Comprehensive Capital Analysis and Review (CCAR), which is a US regulatory stress test on banks with consolidated assets of over USD 50 million.

Options A and C are incorrect: The internally developed stress testing scenarios include the historical scenarios, ad hoc scenarios, and stressing key variables.

Q.4560 A bank carries out regular stress testing to determine its appropriate capital level. During a given year, the bank generates a scenario to assume that the GDP growth rate might decline by 3%. Under the impending circumstances, which type of scenario will the bank *most likely* internally generate?

- A. Ad hoc scenarios.
- B. Historical scenarios.
- C. Stressing key variables scenarios.
- D. None of the above.

The correct answer is C.

A scenario could be built by assuming that a significant change occurs in one or more key variables. One of the key variables, in this case, is the decline of the GDP growth by 3%.

Option A is incorrect: Ad hoc scenarios are developed to reflect the current economic conditions, specific exposures to the financial institution, and the effect of change in government policy.

Option B is incorrect: Historical scenarios are generated by the use of historical data whose all relevant variables are assumed to behave in the same manner as in the past.

Q.4561 Which of the following correctly describes stressed VaR?

- A. There is an X% likelihood that the losses will not exceed the VaR level during a given time T.
- B. If the losses exceed the VaR level at a given time T, then the average loss is equivalent to the stressed VaR level.
- C. If a stressed period is repeated, then there is X% likelihood that over a period of T days, the losses will not exceed the stressed VaR level.
- D. If the losses over a period of T days exceed the stressed VaR level, then the expected loss is equivalent to the VaR level.

The correct answer is C.

The stressed VaR would conclude that if there was a repeat of a stressed period, then there is an X% likelihood that losses over a period of T days will not surpass the stressed VaR level.

Option A is incorrect: It is the description of traditional VaR.

Option B is incorrect: The statement would have described conventional expected shortfall (ES) if it has mentioned that “If the losses exceed the VaR level at given time T, then the average loss is equivalent to ES”.

Option D is incorrect: The statement would have described stressed ES if it had mentioned that “If the losses over a period of T days exceed the stressed VaR level, then the expected loss is equivalent to stressed VaR level.”

Q.4563 The variables stated in the context of scenario analysis are termed as:

- A. Core variables
- B. Key variables
- C. Peripheral variables
- D. Volatility variables

The correct answer is **A**.

The variables stated in the context of the stress testing are termed as core variables, while the remaining variables are termed as peripheral variables whose behavior must be derived from the behavior of the core variables.

Option B is incorrect: Key variables are the important variables from which the stress testing scenarios can be developed (Stress key variables).

Option C is incorrect: Same explanation as in option A.

Option D is incorrect: Volatility variable is one that is affected by outside factors.

Q.4564 While analyzing the stress testing results, analysts should consider the impacts of the stress testing scenarios and also knock-on effects. What is a knock-on effect?

- A. It is an effect due to the way a financial institution responds to an adverse condition.
- B. It is an effect on the way financial institutions implement stress testing.
- C. It is a negative influence from the staff conducting the stress test.
- D. It is an effect that arises due to the involvement of the Board and senior management in the stress testing process.

The correct answer is **A**.

A knock-on effect is a consequence of how financial institutions respond to an adverse scenario, which in most cases, worsens an adverse scenario.

Option B is incorrect: The process of stress testing is not an effect.

Option C is incorrect: The behavior of the stress testing staff is not a knock-on effect since the knock-on effect takes place after the scenario has occurred.

Option D: It is essential for the Board and senior management to be involved in the process of the stress testing for it to be taken seriously. Therefore, it is not a knock-on effect.

Q.4565 In the context of the stress testing, which of the following is **INCORRECTLY** described?

- I. The primary objective of reverse stress testing is to determine how a financial institution can fail
 - II. Stressed VaR gives a high percentile of the distribution of losses over a period of time conditional on the recurrence of a stressed period
 - III. Stressed VaR analyses the results of a selected scenario over a short period of time
 - IV. VaR tells us the minimum amount of loss that could be incurred over a period of T days based on past data.
 - V. Knock-on effects are the secondary effects of an adverse scenario
- A. I, II and III
 - B. II only
 - C. IV only
 - D. IV and V

The correct answer is **C**.

VaR tells us the **maximum** amount of loss that could be incurred over a period of T days based on past data.

Statement I is correct: Reverse stress testing is the process of identifying the point at which a financial institution's business model becomes unviable and triggers failure, and then identifying scenarios and circumstances that might cause this to occur. It seeks to establish the conditions that would lead to a pre-defined outcome.

Statement II is correct: Stressed VaR and stressed ES produces loss distributions and risk measures conditioned on a repeat of a given stressed period.

Statement III is correct: One of the differences between stressed VaR and stress testing is that the former models losses over a short period of time, but stress testing models losses over a longer time horizon. If we were interested in modeling the impact of a repeat of 2008, for example, the stressed VaR would tell us the loss that would not be exceeded over T days at a given level of confidence. Stress testing on the other hand would not concentrate on what would happen during the worst T days. Rather it would consider the impact of the whole of 2008 being repeated.

Statement V is correct: A knock-on effect reflects the impact of how firms (particularly other financial institutions) respond to an adverse scenario. In responding to the adverse scenario, the companies often take actions exacerbating adverse conditions.

Q.4566 In the context of regulatory stress testing in the United States, which of the following is **true**?

- A. Comprehensive Capital Analysis and Review (CCAR) is a stress test performed by the Federal Reserve on banks with consolidated assets of over USD 50 million.
- B. Under the CCAR, banks are required to consider three scenarios: baseline, severe, and an internal scenario.
- C. Under the Dodd-Frank Act Stress Test (DFAST), banks are required to submit a capital plan.
- D. The Dodd-Frank Act Stress Test (DFAST) applies to banks with consolidated assets between USD 10 billion and USD 50 billion.

The correct answer is **D**.

Banks with consolidated assets between USD 10 billion, and USD 50 billion are under the Dodd-Frank Act Stress Test (DFAST).

Option A is incorrect: Comprehensive Capital Analysis and Review (CCAR) is a stress test performed by the Federal reserve on banks with consolidated assets of over USD 50 **billion**.

Option B is incorrect: The four scenarios that banks are required to consider were baseline, adverse, **severely adverse**, and an internal scenario.

Option C is incorrect: Unlike CCAR, DFAST does not require banks to submit capital plans.

Q.4567 A financial institution should have written policies and procedures for stress testing. Which one of the following is **NOT** included in policies and procedures of stress testing?

- A. Responsibilities and roles of the staff conducting the stress testing.
- B. Procedure for defining selection of scenarios.
- C. The description of the consumption of the stress testing results.
- D. The description of the limitation of access to stress testing results by the management.

The correct answer is **D**.

The management is not limited to accessing the results. In fact, the policies and procedures should allow the management to track changes in the stress testing results over time.

Options A, B, and C are incorrect: They are contained in the policies and procedure of the stress testing.

Q.4568 One of the key features of stress test governance is validation and independent review. Which one of the following is **NOT** a function of validation and independent review?

- A. It continuously monitors the results of the stress testing.
- B. Making sure that stress testing is based on the robust theory.
- C. It addresses the qualitative aspects of the stress test.
- D. It defines how stress testing should be carried out in a financial institution.

The correct answer is **D**.

Definition of how the stress test will be carried out in a financial institution is a function of the Board.

Options A, B, and C are incorrect: They are all the functions of the validation and independent review.

Q.4569 Validation and independent reviews is an essential aspect of stress testing governance. Which of the following is/are (a) feature(s) for sufficient validation and independent review?

- I. The reviews should be unbiased
- II. The external models from the vendors and the internal models should be subject to the different reviewers
- III. The Board should ensure that the staff carrying out the stress test have relevant qualifications
- IV. The Board should ensure that the stress testing documentation is of satisfactory level

A. I, II, III and IV

B. I only

C. IV only

D. I and IV

The correct answer is **B**.

Statement I is correct: the reviews should be unbiased and should assure the board that the stress test is in accordance with the policies and procedures of the stress test.

Statement II is incorrect: The external models from the vendors and the internal models should be subject **to the same effective review**.

Statement III is incorrect: Ensuring that the staff carrying out the stress test have relevant qualifications is a function of an **internal audit**.

Statement IV is incorrect: Similarly, ensuring that the stress testing documentation is satisfactory is the responsibility of the **internal audit**.

Q.4570 Before the 2007-2008 financial crisis, the stress tests were faced with numerous shortcomings. Which of the following is **NOT** a feature of the stress test before the financial crisis as observed by the Basel Committee?

- A. The Board and senior management were not adequately involved in stress testing.
- B. Stress testing was taken as a mere mechanical process and did not much impact on decision making.
- C. The risk exposure was over-aggregated hence exaggerating the overall picture of the enterprise-wide view of risks.
- D. The scenarios developed were too moderate and the duration involved was too short.

The correct answer is **C**.

The risk exposures were **never** aggregated to produce an enterprise-wide view of risk impacting a financial institution.

Options A, B and D are incorrect: They were the features of the stress tests before the financial crisis of 2007-2008.

Q.4571 In the context of Basel Committee Stress Testing Principles, choose the **correct** statement(s).

- I. The staff responsible for the stress testing principles should be knowledgeable on the objectives of the stress testing framework
- II. Stress testing models, results, and frameworks should be subject to challenge and regular review
- III. The models used in stress testing should be well justified and documented

A. I and II

B. I and III

C. II and III

D. I, II, and III

The correct answer is **D**.

Statement I is correct: According to the principle that “stress testing frameworks should incorporate an effective governance structure,” the staff mandated to carry out stress testing should know the stress testing framework’s objectives.

Statement II is correct: This is the principle called “stress testing models, results, and frameworks should be subject to challenge and regular review.”

Statement III is correct: According to the principle that “models and methodologies to assess the impacts of scenarios and sensitivities should be fit for the purpose,” the models and the methodologies in a stress test should be adequately justified and documented.

Reading 53: Pricing Conventions, Discounting, and Arbitrage

Q.997 Kristen Haynes, an analyst working at Jahmal Securities, is explaining the different terminologies of prices to a new employee. She makes the following statements about mid-market and full prices per 100 face amount of bonds. Which of these statements are accurate?

Statement I. The mid-market price is an average of the highest price (bid-price) that a buyer is willing to pay and the lowest price, (ask-price) that the seller is willing to accept.

Statement II: The full price is often referred to as the flat or quoted price of the bond

- A. Statement I only
- B. Statement II only
- C. Statements I & II
- D. None of the above

The correct answer is **A**.

Statement I is correct: The mid-market price is an average of the highest bid price that a buyer is willing to pay and the lowest ask price that the seller is willing to accept.

Statement II is incorrect: The full price is the total amount a buyer pays for a bond, which is the sum of the flat or quoted price of the bond and the accrued interest.

Q.998 Joshua Williamson, an associate working at Supreme Bonds, calculates the prices of US Treasury bonds using the law of one price. However, he observes differences between the market price of bonds and the prices predicted by the law of one price. Which of the following may be the reason(s) for the differences in price?

I. Transaction costs

II. Bid-ask spreads in the financing markets

III. It is only in theory that US Treasury bonds are commodities, i.e., fungible collections of cash flows

A. I & II only

B. II & III only

C. I & III only

D. All of the above

The correct answer is **D**.

There are transaction costs in doing arbitrage trades which could significantly lower or wipe out any arbitrage profit. Bid-ask spreads in the financing markets, incurred when shorting securities can also create arbitrage opportunities. An arbitrageur would have to buy securities at higher ask prices and sell at lower bid prices. It is only in theory that U.S. Treasury bonds are commodities, i.e., fungible collections of cash flows. In reality, bonds have idiosyncratic differences that are recognized by the market and priced accordingly.

Q.999 With regard to STRIPS, which of the following statements are true?

- I. STRIPS are created when a particular coupon bond is delivered to the Treasury in exchange for its coupon and principal components
- II. When reconstituting a bond, any C-STRIPS maturing on a particular date may be applied toward the coupon payment of that bond on that date
- III. STRIPS prices are essentially discount factors
- IV. The Treasury not only creates STRIPS but retires them as well

- A. I & II only
- B. II & III only
- C. I & III only
- D. All of the above

The correct answer is **D**.

Zero-coupon bonds issued by the U.S. Treasury are called STRIPS. STRIPS are created when a particular coupon bond is delivered to the Treasury in exchange for its coupon and principal components. Coupon or interest STRIPS are called TINTs, INTs, or C-STRIPS while principal STRIPS are called TPs, Ps, or P-STRIPS.

The Treasury not only creates STRIPS but retires them as well. Upon delivery of the set of STRIPS, the Treasury would reconstitute the bond. It is crucial to note that C-STRIPS are fungible while P-STRIPS are not. When reconstituting a bond, any C-STRIPS maturing on a particular date may be applied toward the coupon payment of that bond on that date. By contrast, only P-STRIPS that were stripped from a particular bond may be used to reconstitute the principal payment of that bond.

This feature of the STRIPS program implies that P-STRIPS, and not C-STRIPS, inherit the cheapness or richness of the bonds from which they came. STRIPS prices are essentially discount factors.

Q.1000 A 50-day US T-bill has a quoted price of 1.60. What is the cash price of the bill?

- A. 94.75
- B. 97.43
- C. 100.23
- D. 99.78

The correct answer is **D**.

Recall that the cash price of a US T-bill is given by:

$$C = 100 - Q \frac{n}{360}$$

where

C = Cash Price

Q = Quoted Price of the T-bill.

n = number of calendar days until the maturity of the Treasury bill

So in this case we have Q =1.60 and n =50 so that:

$$\begin{aligned} C &= 100 - 1.60 \times \frac{50}{360} \\ &= 99.77777 \approx 99.78 \end{aligned}$$

Q.1001 With regard to full price and flat price, which of the following statements are true?

- I. The flat price of the bond per 100 face amount is defined as the full price plus accrued interest
- II. When trading bonds day-to-day, it is more intuitive to track the flat prices and negotiate transactions in those terms
- III. Within a coupon period, the full price of a bond, which is just the present value of its cash flows, increases over time as the bond's payments draw near
- IV. From an instant before the coupon payment date to an instant after it, the full price falls by the coupon payment

- A. I, II & III only
- B. II, III & IV only
- C. I, II & IV only
- D. All of the above

The correct answer is **B**.

Statement I is incorrect. The full or invoice price of the bond per 100 face amount is defined as the quoted price plus accrued interests.

All other statements are accurate. The full price changes dramatically over time even when the market is unchanged, including a discontinuous jump on coupon payment dates, while the flat price changes only gradually over time. Therefore, when trading bonds day-to-day, it is more intuitive to track flat prices and negotiate transactions in those terms.

Within a coupon period, the full price of the bond, which is just the present value of its cash flows, increases over time as the bond's payments draw near.

But from an instant before the coupon payment date to an instant after, the full price falls by the coupon payment: the coupon is included in the present value of the remaining cash flows at the instant before the payment, but not at the instant after.

Q.1002 Day conventions play an important role in determining the accrued interest and value of financial instruments. With regard to day conventions, which of the following statements are true?

I. For most government bonds in the US, the actual/actual day-count convention is used to determine accrued interests

II. In money markets, the actual/360 day-count convention is used

III. In case of corporate bonds and for the fixed leg of interest rate swaps, the 30/360 convention is most commonly used

IV. In case of discount securities and for floating legs of interest rate swaps, the 30/360 convention is most commonly used

A. I, II & III only

B. II, III & IV only

C. I, II & IV only

D. All of the above

The correct answer is **A**.

For most government bonds in the US, the actual/actual day-count convention is used to determine accrued interests. In money markets, for discount securities and floating legs of interest rate swaps, the actual/360 day-count convention is used. In the case of corporate bonds and for the fixed leg of interest rate swaps, the 30/360 convention is most commonly used.

Q.1003 Ronam Ltd. invests in semi-annual US Treasury bonds with face values of USD 1,000 on 15 June 2017. A bond made a coupon payment of USD 40 on February 15, 2017. The next coupon is due on August 15, 2017. If the quoted price for the bond for delivery on June 15, 2017, is USD 1001-16, what is the bond's full price?

- A. USD 1,026.52
- B. USD 1,013.48
- C. USD 1,028.02
- D. USD 1,014.98

The correct answer is C.

We have 120 (13, 31, 30, 31, and 15 days in Feb, March, April, May, and June respectively) between the 1st coupon date (Feb 15, 2017) and the settlement date (June 15, 2017).

We have 61 days from the settlement date to the next coupon payment date (Aug 15, 2017).

Accrued interest using actual/actual day-count- convention:

Accrued interest = $\text{USD } 40 * (120/181) \text{ days} = \text{USD } 26.5193$

Full price = Quoted price + Accrued Interest = $1,001.50 + 26.5193 = \text{USD } 1028.0193$

Note: $1001-16 = 1,001 + 16/32 = 1,001.5$

Q.1004 A \$1,000 par value U.S. corporate bond pays a semiannual 10% coupon. Assume the last coupon was paid 100 days ago and there are 30 days in each month. The accrued interest is closest to:

- A. \$55
- B. \$28
- C. \$25
- D. \$30

The correct answer is **B**.

$$AI = \text{Coupon} \times \frac{\text{Number of days from last coupon to the settlement date}}{\text{Number of days in coupon period}}$$

Thus,

$$AI = \$50 \times \frac{100}{180} = \$27.7778 \approx 28$$

Q.2774 A fixed-income trader summarizes in the table below the prices of Treasury Bonds with semiannual coupon payment. The data is as of 01/01/17.

	Maturity	Coupon~Rate	Price (per \$100 face value)
Tranche 1	30/06/2017	3.5%	99 – 00
Tranche 2	31/12/2017	4%	100 – 16
Tranche 3	30/06/2018	5%	101 – 04

What are the discount factors for 0.5, 1 and 1.5 years?

- A. $d(0.5) = 0.9730$; $d(1) = 0.9662$; $d(1.5) = 0.9393$
- B. $d(0.5) = 0.9551$; $d(1) = 0.9422$; $d(1.5) = 0.9102$
- C. $d(0.5) = 0.9633$; $d(1) = 0.9523$; $d(1.5) = 0.9085$
- D. $d(0.5) = 0.98990$; $d(1) = 0.9782$; $d(1.5) = 0.8787$

The correct answer is **A**.

The cash flows are as follow:

	T(0.5)	T(1.0)	T(1.5)
Tranche 1	101.75	–	–
Tranche 2	2	102	–
Tranche 3	2.5	2.5	102.5

$$\begin{aligned}
 99.00 &= 101.75 * d(0.5) \\
 100.50 &= 2 * d(0.5) + 102 * d(1) \\
 101.125 &= 2.5 * d(0.5) + 2.5 * d(1) + 102.5 * d(1.5)
 \end{aligned}$$

$$\begin{aligned}
 d(0.5) &= 0.9730 \\
 d(1) &= 0.9662 \\
 d(1.5) &= 0.9393
 \end{aligned}$$

Q.2775 Consider a 2-year Treasury Bond that is currently trading on the market at a price of 97.75. The bond has a coupon rate of 5%, which is paid out semiannually.

$d(0.5)$	0.9777
$d(1)$	0.9471
$d(1.5)$?
$d(2)$	0.8845

Given the discount factor structure shown in the table above, $d(1.5)$ is closest to:

- A. 0.9385
- B. 0.9228
- C. 0.9205
- D. 0.9107

The correct answer is **D**.

The cash flows are as follow:

T(0.5)	T(1.0)	T(1.5)	T(2.0)
2.5	2.5	2.5	102.5

$$P_0 = \frac{c_1}{1 + \frac{y}{2}} + \frac{c_2}{1 + \frac{y}{2}} + \frac{F + c_n}{1 + \frac{y}{2}}$$

Where

P_0 = Price of the bond

$c_1, c_2 \dots c_n$ are semi-annual coupon payments

F is the face value

y is the yield on the bond.

Note this question has given us discount factors:

$$\frac{1}{1 + \frac{y_1}{2}}, \frac{1}{1 + \frac{y_2}{2}} \text{ and } \frac{1}{1 + \frac{y_4}{2}}$$

We need to find the discount factor in the third coupon payment. Also note that the Treasury Bond is quoted per 100 face value. Thus

$$\begin{aligned} 97.75 &= 2.5 \times d(0.5) + 2.5 \times d(1) + 2.5 \times d(1.5) + 102.5 \times d(2) \\ &= 2.5 \times 0.9777 + 2.5 \times 0.9471 + 2.5 \times d(1.5) + 102.5 \times 0.8845 \\ \Rightarrow d(1.5) &= 0.9107 \end{aligned}$$

Q.2776 A risk manager is concerned with the pricing of one of the bank's treasury bonds that pays 3% semiannual interest and matures in two years. The discount factors for different maturities are as per the following table:

	Discount factor
d(0.5)	0.9950
d(1.0)	0.9727
d(1.5)	0.9327
d(2)	0.9045

What is the price of the bond?

- A. 93.577
- B. 94.187
- C. 95.847
- D. 96.157

The correct answer is **D**.

The price of the bond is given by:

$$1.5 * d(0.5) + 1.5 * d(1) + 1.5 * d(1.5) + 101.5 * d(2) = 96.157$$

Q.2777 At the end of March 2017, a junior trader at an investment bank was requested to provide information on her portfolio. The portfolio is presented below:

	Maturity	Coupon Rate	Market Price (per 100 Face Value)	Frequency
Bond 1	30/09/2017	2%	99 – 08	semiannual
Bond 2	31/03/2018	4%	101 – 16	semiannual
Bond 3	30/09/2018	5%	105 – 16	semiannual

To revalue the portfolio, the trader uses the following discount factors:

	Discount Factor
$d(0.5)$	0.9991
$d(1)$	0.9799
$d(1.5)$	0.9705

Which of the above bonds is/are trading rich?

- A. Bond 1
- B. Bond 1 and Bond 2
- C. Bond 3
- D. Bond 2 and Bond 3

The correct answer is C.

First, we need to calculate prices based on the trader's discount factors for all three bonds:

$$\begin{aligned}
 \text{Model price of bond 1} &= 101 * d(0.5) = 100.909 \\
 \text{Model price of bond 2} &= 2 * d(0.5) + 102 * d(1) \\
 &= 2(0.9991) + 102(0.9799) = 101.948 \\
 \text{Model price of bond 3} &= 2.5 * d(0.5) + 2.5 * d(1) + 102.5 * d(1.5) \\
 &= 2.5(0.9991) + 2.5(0.9799) + 102.5(0.9705) \\
 &= 104.424
 \end{aligned}$$

Then we need to compare model prices and market prices.

	Market	Model	
Bond 1	99.250	100.909	Cheap
Bond 2	101.500	101.948	Cheap
Bond 3	105.500	104.424	Rich

A bond is said to be trading rich when the quoted market price is greater than the model price. While you'll usually be given the market price, you'll have to calculate the model price.

The model price of the bond equals the present value of its future cash flows, namely its principal plus coupon payment, all times the discount factor for funds to be received. For example, for bond 1 which matures in six months and therefore has just one coupon, model price $= (100 + \frac{2}{2})d(0.5) = (100 + 1)0.9991 = 100.909$. For bond 3 which matures in 1.5 years and therefore has 3 semiannual coupons, model price $= \frac{5}{2} \times d(0.5) + \frac{5}{2} \times d(1) + (100 + \frac{5}{2})d(1.5) = 104.424$

Note that because interest is calculated as per \$100, 2% interest = \$2, 5% interest = \$5 ...etc.

The discount factor for a particular term gives the value today, or the present value of one unit of currency to be received at the end of that term. The discount factor for t years is denoted by $d(t)$. Then, for example, if $d(0.5)$ equals 0.9991 (as given in the question), the present value of \$1 to be received in six months is $1 \times 0.9991 = 0.9991$ cents.

Note that, bond market prices are quoted in dollars and fractions of a dollar. By market convention, the normal fraction used for Treasury security prices is $1/32$. In the quoted market prices, the hyphen (-) separates the full dollar portion of the price from the 32nds of a dollar, which are to the right of the hyphen. Thus the bid quote of 99-08 means \$99 plus $8/32$ of a dollar, or \$99.25, for each \$100 face value of the bond.

Q.2778 Which of the below presented fixed income instruments typically trade rich?

- A. Long-term P-STRIPS.
- B. AAA-rated Corporate bonds.
- C. Short-term C-STRIPS.
- D. Mortgage-backed securities.

The correct answer is C.

Short-term (long-term) C-STRIPS often trade rich(cheap). Recent issues tend to trade at higher prices than otherwise similar issues. Some of this premium is due to the demand for shorts and the resulting financing advantage, that is, the ability to borrow money at less than GC rates when using these bonds as collateral.

Q.2779 A trader prepares a presentation to the investment committee of a bank with a suggestion to invest \$10,000,000 in U.S. Treasury 2 and 3/8s of December 31, 2019, tranche with semiannual coupon payment frequency. The trader wants to invest in this tranche, as the quoted market price is much lower than his estimate of a fair price. To prove the point, the trader includes a table below with market prices of C-STRIPS as of June 30, 2017 (valuation date).

	Maturity	Market Price (per 100 face value)
Tranche 1	31/12/2017	\$99.753
Tranche 2	30/06/2018	\$97.257
Tranche 3	31/12/2018	\$95.012
Tranche 4	30/06/2019	\$94.332
Tranche 5	31/12/2019	\$93.805

What is closest to the price of the Treasury bond?

- A. \$99.507
- B. \$105.209
- C. \$106.235
- D. \$98.079

The correct answer is **A**.

$$\begin{aligned}
 \text{Price of the bond} &= 1.1875 * \frac{99.753}{100} + 1.1875 * \frac{97.257}{100} + 1.1875 * \frac{95.012}{100} \\
 &\quad + 1.1875 * \frac{94.332}{100} + 101.1875 * \frac{93.805}{100} \\
 &= 99.507
 \end{aligned}$$

Further explanation:

The price of a bond is simply equal to the present value of all its future payments (coupons plus face value/redemption value). This is the logic applied here.

We wish to find the price of US 2 3/8s of June 30, 2017. A 2 3/8 bond implies it pays an annual coupon of $2\% + 3/8\% = 2.375\%$. We always compute the price of bonds per \$100 face value.

We are told that the bond pays semiannual coupons. That implies a coupon of $2.375\%/2 = 1.1875\%$ every six months. Per \$100 face value, that's \$1.1875. At redemption, the investor will receive the face value plus the coupon of the last six-month period, i.e. $100 + 1.1875$

Now, we need to discount all these to the present, and that means we need to establish the relevant discount factors. Luckily, we have the prices of C-STRIPS maturing at each of those dates. A C-STRIP is priced at a discount to face value, just like T-bills.

$$d(t) \times \text{face value} = \text{price}; \text{ therefore, } d(t) = \frac{\text{price}}{\text{face value}}$$

Q.3418 The following table presents the characteristics of three different bonds with semiannual coupons and different times to maturity:

Maturity	Coupon	Price
6 months	6.0%	102 – 20
12 months	12%	104 – 08+
18 months	7.5%	98 – 24

If the principal repayment for each bond is \$100, which of the following is closest to the discount factor for 1.5 years?

- A. 0.9964
- B. 0.8823
- C. 0.8865
- D. 0.9920

The correct answer is **B**.

The 6-month bond has cash flows only at maturity. It makes its interest payment of $\$3 \left(\frac{6\%}{2} \times \$100 \right)$ plus the principal repayment of \$100 at $t = 0.5$. To find $d(0.5)$ you should equate the present value (price) of the bond to the cash flows. 102-20 is equivalent to 102 full USD plus 20 fractions of a dollar where total no. of fractions is 32. Hence,

$$102 - 20 = 102 + 20/32 = \$102.63$$

$$\$102.63 = \$103d(0.5)$$

$$d(0.5) = 0.9964$$

The 12-month bond makes payments at $t = 0.5$ and at $t = 1$:

$$104 - 08+ = 104 + 8.5/32 = 104.27$$

Note: A “+” sign at the end of a quote represents half a tick.

$$104.27 = \$6d(0.5) + \$106d(1)$$

$$104.27 = \$6 \times 0.9964 + \$106d(1)$$

$$d(1) = \frac{104.27 - 5.98}{106} = 0.9273$$

Similarly, for the 18-month bond,

$$98.75 = \$3.75d(0.5) + \$3.75d(1) + 103.75d(1.5)$$

$$d(1.5) = \frac{98.75 - 3.74 - 3.48}{103.75} = 0.8823$$

Q.3419 As the chief investment manager of one of your corporate clients, you determine that the use of STRIPS (separate trading of registered interest and principal securities) issued by the U.S. Treasury would help match assets with liabilities at various points in the future. Which of the following statements regarding STRIPS is correct?

- A. Shorter-term STRIPS tend to trade cheap while longer-term STRIPS tend to trade rich
- B. Shorter-term STRIPS tend to trade rich while longer-term STRIPS tend to trade cheap
- C. Shorter-term C-STRIPS tend to trade at a discount
- D. Longer-term C-STRIPS tend to trade at a premium

The correct answer is **B**.

STRIP stands for Separate Trading of Registered Interest and Principal of Securities. A US Treasury coupon can have STRIPS in two distinct securities: The principal security, also known as the P-STRIP, and the detached coupons, also called C-STRIPS.

Shorter-term STRIPS tend to trade rich while longer-term STRIPS tend to trade cheap. Shorter-term C-STRIPS tend to trade at a premium, while longer-term C-STRIPS tend to trade at a discount.

Q.4578 Suppose that the cash price of a US Treasury bill is 90 per 100 of face value. If the bill has 60 days to maturity, what is the quoted price of the Treasury bill?

- A. 60
- B. 61
- C. 54.75
- D. 58.93

The correct answer is **A**.

We find the quoted price by the formula,

$$Q = \frac{360}{n}(100 - C)$$

Where

Q = Quoted price of the Treasury bill.

C = Cash price of the Treasury bill.

n = number of calendar days until maturity of the Treasury bill.

$$Q = \frac{360}{60}(100 - 90) = 6(10) = \$60$$

Q.4579 Company ABC wishes to invest in a 184-day Treasury bill from the US government. The bill is currently issued at the cash price of 98.50. What is the quoted price of the bill?

A. 2.86

B. 2.08

C. 2.93

D. 2.75

The correct answer is **C**.

The quoted price of the US T-bill is given by:

$$Q = \frac{360}{n}(100 - C)$$

Where

Q = Quoted price of the Bill

C = Cash price of the bill

n = number of calendar days until the maturity of the Treasury bill

Then we have:

$$Q = \frac{360}{184}(100 - 98.50) = 2.93$$

Q.4580 Suppose a bond with a par value of 1000 has coupon payments of 10% per annum and a yield to maturity of 5%. If the bond has 4 years to maturity, what is the price of the bond?

- A. 841.51
- B. 1,177.30
- C. 1,259.57
- D. 3,628.22

The correct answer is **B**.

Price of the bond is given by:

$$PV = \sum \left(\frac{C}{(1+y)^n} \right) + \frac{C + FV}{(1+y)^n}$$

Where:

C= coupon payments

y= yield to maturity

n= number of years to maturity of the bond

FV= face value of the bond

Coupon payments=10% of 1000=\$100

$$PV = \frac{100}{1.05} + \frac{100}{1.05^2} + \frac{100}{1.05^3} + \frac{100}{1.05^4} + \frac{1000}{1.05^4} = \$1,177.297 \approx 1,177.30$$

Q.4581 A 20-days US treasury bill has a quoted price of 1.50. What is the cash price?

A. 99.84

B. 99.92

C. 98.5

D. 99.98

The correct answer is **B**.

We can get the cash price of the bill using the following formula.

$$Q = \frac{360}{n}(100 - C)$$

Where:

n= the number of calendar days until maturity of the treasury bill.

Q= the quote price

C= the cash price

$$1.50 = \frac{360}{20}(100 - C)$$

$$C = 100 - 1.50 \times \frac{20}{360} = 99.9166$$

Reading 54: Interest Rates

Q.1005 Grant Bank pays an interest of 8% with interest getting compounded quarterly. The effective annual rate is equal to:

- A. 8%
- B. 8.2%
- C. 8.4%
- D. 8.1%

The correct answer is **B**.

Let's assume that a sum of \$1000 is invested initially and the annual rate is assumed to be x.

Sum at the end of 1 year = $\$1000 * (1 + \frac{x}{100})^1$

As the interest paid by the bank is 8% with quarterly compounding, the aggregate value at the end of one year is given by:

$$\$1000 * (1 + \frac{0.08}{4})^4 = \$1000 * 1.082$$

Hence,

$$\begin{aligned} \$1000 * (1 + \frac{x}{100}) &= \$1000 * 1.082 \\ \Rightarrow 1 + \frac{x}{100} &= 1.082 \\ \Rightarrow \frac{x}{100} &= (1.082 - 1) = 0.082 \\ x &= 8.20\% \end{aligned}$$

Therefore, the effective annual rate = 8.20%

Q.1006 Royal Bank extends a loan of \$1000 to a customer for 2 years. The bank charges interest with half-yearly compounding frequency. If the spot rate for a 2-year loan is 10% per annum, then the amount that the customer needs to pay after 2 years is closest to:

- A. \$1,200
- B. \$1,216
- C. \$1,210
- D. \$1,222

The correct answer is **B**.

$$\text{Amount (after T years)} = \text{Principal} * \left(1 + \frac{(\text{Spot Rate})^{(T * \text{Compounding Frequency})}}{(\text{Compounding Frequency})}\right)$$

Where:

Principal = \$1000

Spot rate = 10%

Compounding Frequency = Half yearly = 2

T = 2 Years

$$\text{Amount (After 2 years)} = \$1000 * \left(1 + \frac{0.10^{2*2}}{2}\right) = \$1,215.51$$

Q.1007 The spot rate for 1 year and 2 years are 10% and 12% respectively. The forward rate for a loan to be given in 1 year for a term of 1 year is:

- A. 11%
- B. 13%
- C. 14%
- D. 12%

The correct answer is **C**.

The relation between spot rates and forward rate can be indicated by the expression below:

$$(1 + \text{Spot rate}_{\text{for } T})^T = (1 + \text{Spot rate}_{\text{for } t})^t * (1 + \text{Forward rate}_{t \text{ to } T})$$

Where

Spot rate for 1 year = 10%

Spot rate for 2 years = 12%

Therefore,

$$\begin{aligned}(1 + 0.12)^2 &= (1 + 0.10)^1 * (1 + f_{1,1}) \\ 1 + f_{1,1} &= \frac{(1 + 0.12)^2}{(1 + 0.10)^1} = 1.14 \\ f_{1,1} &= 0.14 = 14\%\end{aligned}$$

Q.1008 John Marauder observes that two zero-coupon bonds issued by ACC Limited are currently trading at prices given in the table below:

Residual maturity	Price	Face value
1 year	\$91.74	\$100
2 year	\$82.64	\$100

The 1-year and 2-year spot rates are:

- A. 9% and 10% respectively.
- B. 10% and 11% respectively.
- C. 8% and 9% respectively.
- D. 11% and 12% respectively.

The correct answer is **A**.

Zero-coupon bonds are bonds that do not carry coupons. Zero coupons bonds trade at a discount to face value, the discount rates used are the spot rates. Hence the price of a zero-coupon bond can be calculated by using the expression below:

$$\text{Price (zero-coupon bond of maturity } T \text{ years)} = FV * \frac{1}{(1 + S_T)^T}$$

Therefore, using the above expression the spot rates are:

$$\begin{aligned} \$91.74 &= \$100 * \left(\frac{1}{(1 + S_1)} \right)^1 \\ 1 + S_1 &= \frac{100}{91.74} = 1.09 \\ S_1 &= 9\% \\ \$82.64 &= \$100 * \left(\frac{1}{(1 + S_2)} \right)^2 \\ (1 + S_2)^2 &= \frac{100}{82.64} = 1.21 \\ S_2 &= 10\% \end{aligned}$$

Q.1009 John Marauder observes that two zero-coupon bonds issued by ACC Limited are currently trading at prices given in the table below:

Residual maturity	Price	Face value
1 year	\$91.74	\$100
2 year	\$82.64	\$100

ABC Traders Private Limited managers are planning to issue zero-coupon bonds with a maturity of 1 year next year. As per the present market conditions, the price of the zero-coupon bond will most likely be:

- A. \$85.09
- B. \$95.09
- C. \$80.09
- D. \$90.09

The correct answer is **D**.

Zero-coupon bonds are bonds that do not carry coupons. Zero coupons bonds trade at a discount to face value, the discount rates used are the spot rates. Hence the price of a zero-coupon bond can be calculated by using the expression below:

$$\text{Price (zero-coupon bond of maturity T years)} = FV * \frac{1}{(1 + S_T)^T}$$

Therefore, using the above expression the spot rates are:

\$

$$\$91.74 = \$100 * \left(\frac{1}{(1 + S_1)^1} \right)$$

$$1 + S_1 = \frac{100}{91.74} = 1.09$$

$$S_1 = 9\%$$

$$\begin{aligned} \$82.64 &= \$100 * \left(\frac{1}{(1 + S_2)^2} \right) \\ (1 + S_2)^2 &= \frac{100}{82.64} = 1.21 \\ S_2 &= 10\% \end{aligned}$$

The forward rate can be computed using the following expression:

$$(1 + S_{\text{for } T})^T = (1 + S_{\text{for } t})^t * (1 + f_{t \text{ to } T})$$

Where

1 year spot rate = 9%

2 year spot rate = 10%

$$f_{1,1} = f$$

Therefore, the 1 year forward rate is equal to:

$$\begin{aligned} (1 + 0.10)^2 &= (1 + 0.09) * (1 + f_{1 \text{ year rate after 1 year}}) \\ 1 + f_{1 \text{ year rate after 1 year}} &= \frac{(1.1)^2}{1.09} = 1.11 \\ f_{1 \text{ year rate after 1 year}} &= 0.11 = 11\% \\ \text{The 1 year forward rate after 1 year} &= 11\% \end{aligned}$$

Hence the price of the zero-coupon bond issued for 1 year after 1 year will be:

$$\begin{aligned} P_{\text{Zero-coupon bond of maturity } T \text{ years}} &= FV * \frac{1}{(1 + S_T)^T} \\ P &= \frac{\$100}{1.11} = \$90.09 \end{aligned}$$

Q.1010 John Marauder observes that two zero-coupon bonds issued by ACC Limited are currently trading at prices given in the table below:

Residual maturity	Price	Face value
1 year	\$91.74	\$100
2 year	\$82.64	\$100

Select the most appropriate statement.

- A. The market expects the interest rates to fall, therefore the yield curve is downward sloping.
- B. The market expects the interest rate to rise, therefore the yield curve is upward sloping.
- C. The market expects the interest rate to remain constant, therefore the yield curve is flat.
- D. The yield curve is always downward sloping.

The correct answer is **B**.

The current 1-year spot rate = 9%

The 1-year forward rate after 1 year = 11%

Hence, the market expects that the current spot rate of 9% will rise to 11% after one year i.e. the today's forward rates are the estimator of tomorrow's spot rate. Therefore, the market expects the interest rate to rise and the yield curve is upward sloping.

Q.1011 If the term structure of spot interest rates is flat, then the term structure of forward interest rates must be:

- A. Upward sloping
- B. Downward sloping
- C. Flat
- D. Humped shaped

The correct answer is **C**.

The term structure of spot rates is basically the spot rates for different maturities. A flat term structure of spot rates means that the spot rate is the same for different maturities, i.e., the spot rates for 1 year, 2 year, 3 year, etc. are the same.

Let's assume that the spot rates for 1 year and 2 years is equal to r . The one-year forward rate starting in one year is:

$$\begin{aligned}(1 + r)^2 &= (1 + r)^1 * (1 + \text{Forward rate}_{1 \text{ year to } 1 \text{ year}}) \\ 1 + \text{Forward rate}_{1 \text{ year to } 1 \text{ year}} &= 1 + r \\ \text{Forward rate}_{1 \text{ year to } 1 \text{ year}} &= r\end{aligned}$$

As illustrated above, the forward rates will be the same for different maturities. Hence, if the term structure of spot rates is flat, the term structure of forward rates will also be flat.

Q.1012 Jack Mangers observes that the spot rates for 2 years and 3 years are 6% and 8% respectively while the 2y1y (1-year forward rate beginning after 2 years) rate is 12.50%. Select the correct statement(s) from the following.

- I. There is an opportunity to generate riskless profit
- II. There is no opportunity to generate riskless profit
- III. The yield curve is upward sloping
- IV. The yield curve is downward sloping

- A. Both I and IV are correct.
- B. Both II and III are correct.
- C. Both I and III are correct.
- D. Both II and IV are correct.

The correct answer is C.

The 2y1y i.e. 1-year forward rates beginning after 2 years can be computed by the following expression:

$$(1 + \text{Spot rate}_{\text{for } 3})^3 = (1 + \text{Spot rate}_{\text{for } 2})^2 * (1 + \text{Forward rate}_{2 \text{ to } 3})$$

$$\text{Spot rate}_{\text{for } 3} = 8\%$$

$$\text{Spot rate}_{\text{for } 2} = 6\%$$

$$\begin{aligned} (1 + 0.08)^3 &= (1 + 0.06)^2 * (1 + \text{Forward rate}_{2 \text{ to } 3}) \\ 1 + \text{Forward rate}_{2 \text{ to } 3} &= \frac{1.08^3}{1.06^2} = 1.1211 \\ \text{Forward rate}_{2Y \ 1Y} &= 0.1211 = 12.11\% \end{aligned}$$

As implied by the spot rates, the forward rate for 1-year forward rate after 2 years must be equal to 12.11%. However, it is 12.50%. Therefore, there is an opportunity to generate riskless profit due to the interest rate differential. Furthermore, as the forward rates are higher, the yield curve is upward sloping.

Q.1013 Jack Mangers observes that the spot rates for 2 years and 3 years are 6% and 8% respectively while the 2y1y (1-year forward rate beginning after 2 years) rate is 12.50%. Select the

most appropriate option.

- A. As there is no interest rate discrepancy, no transaction can generate riskless profit.
- B. Borrowing funds for 3 years, lending the funds for 2 years and an agreement to lend funds for 1 year after 2 years will generate a riskless profit of \$0.43 per \$100.
- C. Lending funds for 3 years, borrowing the funds for 2 years and an agreement to lend funds for 1 year after 2 years will generate riskless profit of \$0.50 per \$100.
- D. Borrowing funds for 3 years, lending the funds for 2 years and an agreement to lend funds for 1 year after 2 years will generate riskless profit of \$0.70 per \$100.

The correct answer is **B**.

The following steps will generate a riskless profit:

1. Borrow funds for 3 years (let us assume that \$100 is borrowed.)
2. Lend the money for 2 years.
3. After two years lend the money as per the forward rate agreement for 1 year at 12.50%.

Time	Action	Cash flow
0	Borrow USD. 100 for 3 years at 8% Lend the money for 2 years at 6%	+\$100 – \$100
2 years	\$100 is repaid after 2 years The repaid money is then again lent for 1 year as per the agreement at 12.50%	+\$112.36 – \$112.36
3 years	The lent money is repaid with 1 year worth of interest ($\$112.36 \times 1.125$) \$100 initially borrowed for 3 years is repaid	+\$126.41 – \$125.97
Profit = USD. 0.43		

Q.1014 The details of a bond currently trading is given below:

Face value: \$1,000

Coupon: 6%

YTM: 6%

Tenure: 10 years

The price of the bond is:

A. \$1,100

B. 990

C. \$1,000

D. 980

The correct answer is C.

Yield to maturity (YTM) is the discount rate which is used to discount the cash flows of the bond to arrive at its price.

The price of the bond is equal to its face value if and only if the YTM is equal to the coupon rate.

The rate at which the price of the bond is equal to its face value is known as *Par rates*.

Therefore, in the above case, the price of the bond is equal to \$1,000.

Q.1015 A rate of 5% is quoted with continuous compounding. What is the equivalent rate, quoted with monthly compounding?

A. 5.01%

B. 6.03%

C. 6.02%

D. 5.59%

The correct answer is **A**.

If the rates are equivalent, then the following equation must hold:

$$e^R = \left(1 + \frac{R_m}{m}\right)^m$$

Where

R = Continuously compounded rate of interest

R_m = interest rate compounded m-times.

Then in this case we have:

$$\begin{aligned} e^{0.05} &= \left(1 + \frac{R_{12}}{12}\right)^{12} \\ \Rightarrow R_{12} &= 12\left[e^{\frac{0.05}{12}} - 1\right] \\ &\approx 5.01\% \end{aligned}$$

Q.1016 The 5-year and 6-year continuously compounded interest rates are 6.75% and 7.25%, respectively. What is the forward rate of interest between year five and year six?

- A. 9.75%
- B. 8.25%
- C. 7.50%
- D. 8.56%

The correct answer is **A**.

When the rates are quoted with continuous compound, the forwards rate between times T_1 and T_2 is given by:

$$F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

Where

R_1 = Spot rate for maturity T_1

R_2 = Spot rate for maturity T_2

Thus for this case we have:

$$F = \frac{0.0725 \times 6 - 0.0675 \times 5}{6 - 5} = 9.75\%$$

Alternatively, note that the following equation must hold:

$$\begin{aligned} e^{0.0675 \times 5} e^{F \times 1} &= e^{0.0725 \times 6} \\ \Rightarrow F &= (0.0725 \times 6) - (0.0675 \times 5) \\ &= 0.0975 = 9.75\% \end{aligned}$$

Q.2781 In the table below, an analyst has summarized the current swap rates prevailing on the market.

Term in Years	Swap Rate
0.5	2.00%
1.0	2.80%
1.5	3.20%

What is the 1.5-year discount factor?

- A. 0.9533
- B. 0.9625
- C. 0.9725
- D. 0.9804

The correct answer is **A**.

Consider an interest rate swap. If we assumed that the notional amount is exchanged, the fixed leg of the swap would resemble a fixed coupon-paying bond, with fixed leg payments acting like semi-annual, fixed coupons, and the notional amount acting like the principal payment. We can therefore write an equation for each “bond” that equates the present value of its cash flows to its price of par.

$$\begin{aligned}
 (100 + \frac{2}{2}) * d(0.5) &= 100 \rightarrow d(0.5) = 0.9901 \\
 \frac{2.8}{2} * d(0.5) + (100 + \frac{2.8}{2}) * d(1.0) &= 100 \rightarrow d(1.0) = 0.9725 \\
 \frac{3.2\%}{2} * d(0.5) + \frac{3.2\%}{2} * d(1.0) + (100 + \frac{3.2\%}{2}) * d(1.5) &= 100 \rightarrow d(1.5) = 0.9533
 \end{aligned}$$

Q.2783 After a recent FED's announcement, a trader observed significant changes across the whole spot rates curve. The correct spot rates are as follows:

Year	Spot rate
0.5	2.0%
1.0	2.1%
1.5	2.3%
2.0	2.2%
2.5	2.5%

What is the 6-month forward rate in two years?

- A. 0.30%
- B. 1.85%
- C. 2.21%
- D. 3.70%

The correct answer is **D**.

$$f(2.5) = 2 \left[\frac{(1 + \frac{0.025}{2})^{2.5 \times 2}}{(1 + \frac{0.022}{2})^{2 \times 2}} - 1 \right] = 3.70\%$$

Q.2785 Par rates prevailing on the market, are:

Term in years	Par Rates
0.5	1.60%
1.0	2.00%
1.5	2.60%
2.0	3.20%

What is the two-year discount factor? (Assume semiannual coupons.)

- A. 0.9871
- B. 0.9619
- C. 0.9421
- D. 0.9380

The correct answer is **D**.

$(\frac{C_T}{2}) * A_T + d(T) = 1$, where C_T is the T year par rate and A_T is the annuity factor.

Term in years	Discount factor	Comments
0.5	0.9921	$(\frac{C_{0.5}}{2}) * d(0.5) + d(0.5) = 1$
1.0	0.9803	$(\frac{C_{1.0}}{2}) * (d(0.5) + d(1.0)) + d(1.0) = 1$
1.5	0.9619	$(\frac{C_{1.5}}{2}) * (d(0.5) + d(1.0) + d(1.5)) + d(1.5) = 1$
2.0	0.9380	$(\frac{C_{2.0}}{2}) * (d(0.5) + d(1.0) + d(1.5) + d(2.0)) + d(2.0) = 1$

Q.3421 The price of a five-year zero-coupon government bond is \$72.25. The price of a similar six-year bond is \$67.34. The one-year implied forward rate from year 5 to year 6 is closest to:

- A. 7.29%
- B. 7.00%
- C. 6.79%
- D. 6.24%

The correct answer is **A**.

First, we need to find the 6-year and 5-year spot rates, assuming the price is given per \$100 par value. 5-year spot, S_5 :

$$72.25 = 100/(1 + S_5)^5$$

$$S_5 = 6.717\%$$

6-year spot, S_6 :

$$67.34 = 100/(1 + S_6)^6$$

$$S_6 = 6.812\%$$

Then, recall that:

$$(1 + 5 \text{ year spot})^5 * (1 + 1\text{-year forward rate}) = (1 + 6\text{-year spot})^6 \text{ Thus,}$$

$$(1 + 1\text{-year forward rate}) = (1 + 6\text{-year spot})^6 / (1 + 5\text{-year spot})^5$$

$$1\text{-year forward rate} = 1.484978899 / 1.38410182 - 1 = 7.288\%$$

Q.3422 The term structure of swap rates is:

n – year	Swap rate
1 – year	3.5%
2 – year	4%
3 – year	4.5%
4 – year	5%
5 – year	5.5%

The two-year forward swap rate starting in three years is closest to:

- A. 0.5%
- B. 4%
- C. 7.02%
- D. 6.22%

The correct answer is **C**.

First, we compute the accrual of a dollar over three and five (3+2) years:

For $T = 3$, this is $(1 + 0.045)^3 = 1.1412$

For $T = 5$, this is $(1 + 0.055)^5 = 1.3070$

$$(1 + R_T)^T = (1 + R_n)^n (1 + F_{n,T})^{T-n}$$

Where $n < T$

$$1.3070 = 1.1412 \times (1 + F_{3,5})^2$$

$$F_{3,5} = \sqrt{\frac{1.3070}{1.1412}} - 1 = 7.02\%$$

Q.3423 Below is the term structure for swap rates:

Maturity in Years	Swap Rate
1	4.0%
2	4.5%
3	5.0%
4	5.5%
5	6.0%

The 1-year forward swap rate starting in four years is closest to:

- A. 4.0%
- B. 6.0%
- C. 7.0%
- D. 8.0%

The correct answer is **D**.

$$(1 + R_T)^T = (1 + R_n)(1 + F_{n,T})^{T-n}$$

Where $n < T$

First, we compute the accrual of a dollar over four and five years:

For $T = 4$, this is $(1 + 0.055)^4 = 1.2388$

For $T = 5$, this is $(1 + 0.06)^5 = 1.3382$

$$1.3382 = 1.2388 \times (1 + F_{4,5})^1$$

$$F_{4,5} = \frac{1.3382}{1.2388} - 1 = 8.0\%$$

Q.4572 A trader invests \$100 million in a savings account. After two years, the total amount in his account is \$150 million. What is the rate of interest compounded semi-annually?

- A. 20.32%
- B. 21.34%

C. 22.47%

D. 25.32%

The correct answer is **B**.

From the FV formula, we have:

$$FV = PV\left(1 + \frac{i_m}{m}\right)^{mn}$$

Where

FV = future value

PV = present value

m = number of compounds per year

n = number of years

From the formula above, we have:

$$150 = 100\left(1 + \frac{i_2}{2}\right)^4$$

Solving for i_2 , we have:

$$i_2 = 2 \left\{ \left(\frac{150}{100} \right)^{\frac{1}{4}} - 1 \right\} = 0.2134 = 21.34\%$$

An **alternative** to solving this question is by using the financial calculator. The variables are as follows: $N = 4$, $PV = -100$, $PMT = 0$ and $FV = 150$ so that $CPT \Rightarrow I/Y = 10.67\%$.

Therefore, the annual rate of interest is $10.67\% \times 2 = 21.34\%$. This is the **easiest** way!

Q.4573 If the one-year spot rate is 5% and the two years spot rate is 8%, what is the one-year forward rate, one year from now?

A. 0.0555

B. 0.111

C. 0.067

D. 0.134

The correct answer is **B**.

The spot rate and the forward rate have the following relationship:

$$(1 + y_2)^2 = (1 + y_1) \times (1 + f_{[1,1]})$$

Where

y_1 = one-year spot rate

y_2 = two-year spot rate and,

$f_{[1,1]}$ = one-year forward rate, one year from now

$$(1.08)^2 = (1.05)^1 \times (1 + f_{[1,1]})$$
$$f_{[1,1]} = \frac{1.08^2}{1.05} - 1 = 0.111 = 11.1\%$$

Q.4574 1000 is invested in an account that pays an annual nominal interest of 8% compounded quarterly per year. What is the value of the amount in the account after three years?

A. 1259.71

B. 1268.24

C. 1061.21

D. 2518.17

The correct answer is **B**.

$$FV = PV \left(1 + \frac{i_m}{m}\right)^{mn}$$

Where:

FV= the future value

PV=the present value

m= number of compounds in a year

n= number of years

The value of the amount in the account after three years is given by:

$$FV = 1000 \left(1 + \frac{0.08}{4}\right)^{12} = 1268.24$$

Q.4575 Suppose a company is obliged to make annual payments of \$5,000 for the premises it occupies. Payments are due on 1st January 2001, 1st January 2002, and 1st January 2003. If the company wishes to cover these payments by investing a single sum in its bank account that pays 8% per annum compound, what sum must be invested on 1st January 2000?

A. 12,885.48

B. 14,853.44

C. 13,916.32

D. 11,298.56

The correct answer is **A**.

An annuity is a series of annual payments of PMT until the final time T. The value of an ordinary annuity is given by:

$$PV_{\text{annuity}} = PMT \frac{1 - (1 + r)^{-T}}{r}$$

Where:

r = discount rate

This is an annuity ordinary because the first payment would happen a year after January 2000.

The present value of the annuity on 1st January 2000 is given by:

$$5000 \left(\frac{1 - 1.08^{-3}}{0.08} \right) = 12,885.48$$

Q.4576 Spot rates with semi-annual compounding are as below:

Maturity (years)	Spot rates (%)
1.0	2.0
1.5	2.5
2.0	3.0

What the forward rate for the period between time 1.0 and 1.5, expressed annually?

- A. 0.035
- B. 0.0175
- C. 0.07
- D. 0.005

The correct answer is **A**.

The forward rate for the period between time 1.0 and 1.5 is given by:

$$\left(1 + \frac{0.025}{2}\right)^3 = \left(1 + \frac{0.02}{2}\right)^2 \left(1 + \frac{f_1}{2}\right)$$

Solving for f_1 , we have

$$\frac{f_1}{2} = \frac{\left(1 + \frac{0.025}{2}\right)^3}{\left(1 + \frac{0.020}{2}\right)^2} - 1 = 0.0175$$

And therefore, $f_1 = 0.0175 \times 2 = 0.035 = 3.5\%$

Q.4577 Spot rates with semi-annual compounding are as below:

Maturity (years)	Spot rates (%)
1.0	2.0
1.5	2.5
2.0	3.0

What is the forward rate for the semi-annual period between time 1.5 and 2, expressed annually?

- A. 0.045
- B. 0.0225
- C. 0.09
- D. 0.0977

The correct answer is **A**.

The forward rate between time 1.5 and 2.0 will be:

$$\frac{f_2}{2} = \frac{(1 + \frac{0.030}{2})^4}{(1 + \frac{0.025}{2})^3} - 1 = 0.0225$$
$$f_2 = 0.0225 \times 2 = 0.045 = 4.5\%$$

Reading 55: Bond Yields and Return Calculations

Q.1017 Yield-to-maturity is an important measure to describe the pricing of a bond. Which of the following statements are true with regard to yield-to-maturity?

- I. Yield-to-maturity is the single rate such that discounting a security's cash flows at that rate gives that security's market price
- II. When the coupon rate exceeds the yield, the bond sells at a discount to its face value
- III. When the yield exceeds the coupon rate, the bond sells at a premium to its face value
- IV. If the term structure is flat, so that all spot rates and all forward rates equal some single rate, then the yield-to-maturity of all bonds equals that rate as well

- A. I & IV only.
- B. II, III & IV only.
- C. I, II & IV only.
- D. All of the above.

The correct answer is **A**.

Statements I & IV are correct. Yield-to-maturity is the single rate such that discounting a security's cash flows at that rate gives that security's market price. If the term structure is flat, so that all spot rates and all forward rates equal some single rate, then the yield-to-maturity of all bonds equals that rate as well.

Statements II & III are incorrect. When the yield is equal to the coupon rate, the bond sells for its face value. When the coupon rate exceeds the yield, the bond sells at a premium to its face value. When the yield exceeds the coupon rate, the bond sells at a discount to its face value.

Q.1018 Derek Johnson, an analyst at American Bonds Inc., is interested in understanding the components of P&L (Profit & Loss) of bonds. With regards to P&L, which of the following statements are true?

- I. P&L is generated by price appreciation plus cash-carry, which consists of explicit cash flows like coupon payments and financing costs
- II. P&L due to carry is meant to convey how much a position earns due to the fact that, as a security matures, its cash flows are priced at earlier points on the term structure
- III. P&L due to roll-down is meant to convey how much a position earns due to the passage of time, holding everything else constant
- IV. The P&L due to the passage of time excluding cash-carry is called carry-roll-down

- A. I & IV only.
- B. II & IV only.
- C. I, II & IV only.
- D. All of the above.

The correct answer is **A**.

Statements I & IV are correct. P&L is generated by price appreciation plus cash-carry, which consists of explicit cash flows like coupon payments and financing costs. The P&L due to the passage of time excluding cash-carry is called carry-roll-down.

Statements II & III are incorrect. P&L due to carry is meant to convey how much a position earns due to the passage of time, holding everything else constant. P&L due to roll-down is meant to convey how much a position earns due to the fact that, as a security matures, its cash flows are priced at earlier points on the term structure.

Q.1019 Consider the details of bonds currently trading:

Bond	Coupon	YTM	Face Value
Bond A	10%	9%	1000
Bond B	6%	8%	1000
Bond C	5%	5%	1000

Select the most appropriate statements: I. $\text{Price}_{\text{Bond A}} > \1000

II. $\text{Price}_{\text{Bond B}} < \1000

III. $\text{Price}_{\text{Bond C}} = \1000

IV. $\text{Price}_{\text{Bond A}} < \1000

V. $\text{Price}_{\text{Bond B}} > \1000

VI. $\text{Price}_{\text{Bond C}} > \1000

A. II, III and V are correct.

B. I, II and III are correct.

C. IV, V and VI are correct.

D. I, III and V are correct.

The correct answer is **B**.

$\text{Price}_{\text{Bond A}} > \text{Face Value}$ when coupon $>$ YTM

$\text{Price}_{\text{Bond B}} < \text{Face Value}$ when coupon $<$ YTM

$\text{Price}_{\text{Bond C}} = \text{Face Value}$ when coupon $=$ YTM

Hence, options I, II, and III are correct.

Q.1020 All the following statements regarding the yield-to-maturity (YTM) are correct, EXCEPT:

- A. The YTM is the discount rate used to discount the bond cash flows to arrive at the price of the bond.
- B. The YTM is the return realized by the bond investor.
- C. The YTM of a zero-coupon bond is equal to the spot rate.
- D. If $YTM < \text{Coupon}$, the bond trades at a premium.

The correct answer is **B**.

YTM is the discount rate used to discount the bond cash flows to arrive at the price of the bond. The return realized by the bondholder is equal to YTM only when the coupons are reinvested at the yield same as the YTM. Generally, an investor fails to reinvest the coupons at the YTM due to which the return realized by the bondholder is not equal to YTM.

Zero-coupon bonds do not pay coupons. Hence the YTM of a zero-coupon bond is equal to the spot rate.

When $YTM < \text{Coupon}$, then the bond trades at a premium. When $YTM > \text{Coupon}$, then the bond trades at a discount.

Q.1021 Corporate bonds trade at a positive spread to government bonds because:

- A. Corporate bonds are more liquid than government bonds.
- B. Corporate bonds have higher credit risks than government bonds.
- C. Corporate bonds generate higher returns than government bonds.
- D. Corporate bonds pay less coupon than government bonds.

The correct answer is **B**.

The spread at which corporate bonds trade depends mainly on its credit risk. Corporate bonds have a higher credit risk as compared to government bonds. Due to the higher credit risk, the spread between the government bond and corporate bond is positive.

Q.1022 Consider the following details with respect to a bond:

Face value: \$1000

Coupon: 10%

Frequency: Semi-annually

Coupon payment dates: January 1st and July 1st

An investor buys the bond on January 22nd at a price of \$990.23, and sells it on August 3rd at \$1030.34. The gross realized return on the bond investment is:

A. 4.05%

B. 9.1%

C. 8.8%

D. 9.4%

The correct answer is **B**.

January 22nd: -990.23

July 1st: +50 (Coupon payment)

August 3rd: +1030.34

$$\text{Gross Realized Return} = \frac{(\$1030.34 + \$50 - \$990.23)}{\$990.23} = 9.10\%$$

Q.1023 Consider the following details with respect to a bond:

Face value: \$1000

Coupon: 10%

Frequency: Semi-annually

Coupon payment dates: January 1st and July 1st

An investor buys this bond at \$1043.43 on January 1st, 2016 and sells it on January 1st, 2017 at \$995.23. The coupon received is reinvested at a semi-annually compounded rate of 9%. The realized gross holding period return is:

A. 0.39%

B. 5.2%

C. 4.5%

D. 5.6%

The correct answer is **B**.

January 1st 2016: -1043.43

July 1st 2016: \$50 + 50*(1+9%/2) (Coupon payment reinvested for 6 months)

January 1st 2017: +50 (Coupon payment) + \$995.23 (Selling the bond) Thus,

$$\text{Gross Realized Return} = \frac{995.23 + 50 + 50(1.045) - 1043.43}{1043.43} = 5.18\%$$

Q.1024 A corporate bond has a residual maturity of 2 years and pays a 10% coupon annually.

Two zero-coupon bonds are currently trading at the price mentioned below:

Price	Residual maturity
USD 92.38	1 year
USD 84.17	2 years

The price of the corporate bond is closest to:

A. USD 101.01

B. USD 117.66

C. USD 101.82

D. USD 93.41

The correct answer is **C**.

A bond is priced using the spot rates as:

$$\text{Price}_{\text{Bond}} = \frac{C}{(1 + \text{Spot rate}_1)^1} + \frac{(FV + C)}{(1 + \text{Spot rate}_2)^2}$$

$$\text{Spot Rate}_{1 \text{ year}} = \left(\frac{100}{92.38}\right) - 1 = 0.0825 = 8.25\%$$

$$\text{Spot Rate}_{2 \text{ year}} = \left(\frac{100}{84.17}\right)^{\frac{1}{2}} - 1 = 0.0900 = 9.00\%$$

The price of the bond can be calculated as:

$$\begin{aligned} \text{Bond Price} &= \frac{10}{(1 + 0.0825)^1} + \frac{100 + 10}{(1 + 0.09)^2} \\ &= 9.24 + 84.2 + 84.17 = 101.82 \end{aligned}$$

Q.1025 A bond with a residual maturity of 2 years that pays a 10% coupon annually is currently trading at \$102.10. It is also observed that two zero-coupon bonds are currently trading at the price mentioned below:

Price	Residual maturity
\$92.38	1 year
\$84.17	2 years

The transaction which will generate riskless profit is:

- A. Sell the bond and purchase the zero-coupon bonds.
- B. Borrow funds and purchase the zero-coupon bonds.
- C. Riskless profit cannot be generated.
- D. Buy the bond and sell the zero-coupon bonds.

The correct answer is **A**.

A bond is priced using the spot rates as:

$$\text{Price}_{\text{Bond}} = \frac{C}{(1 + \text{Spot rate}_1)^1} + \frac{(FV + C)}{(1 + \text{Spot rate}_2)^2}$$

$$\text{Spot Rate}_{1\text{ year}} = \left(\frac{100}{92.38}\right) - 1 = 0.0825 = 8.25\%$$

$$\text{Spot Rate}_{2\text{ year}} = \left(\frac{100}{84.17}\right)^{\frac{1}{2}} - 1 = 0.090 = 9.00\%$$

The price of the bond can thus be calculated as:

$$\begin{aligned}\text{Price}_{\text{Bond}} &= \frac{10}{(1 + 0.0825)^1} + \frac{(100 + 10)}{(1 + 0.09)^2} \\ &= 9.24 + 8.42 + 84.17 \\ &= 101.82\end{aligned}$$

As can be observed from the above, the price of the bond is > than \$101.82. Hence, the bond is overvalued.

1. As observed, the bond is overvalued. Hence, the investor must sell the overvalued bond.
 2. The funds obtained by selling the bond must be invested in the zero-coupon bonds.
 3. By the above transaction, the investor can make a profit of \$102.10 - \$101.82 = \$0.28 per \$100.
-

Q.1026 A fund manager is looking for an opportunity to invest in sovereign bonds. Country A has recently witnessed a major economic recession and has just averted a default on its foreign debt. On the other hand, country B is a developing economy with a low debt to GDP ratio. The bonds of country A and B trade at a spread of a and b with respect to US Treasury bonds. Select the correct option:

- A. $a > b$
- B. $a < b$
- C. $a = b$
- D. $a \leq b$

The correct answer is **A**.

A riskier bond trades at a higher spread compared to less risky bond. The bonds of country A are riskier than the bonds of country B. Therefore, the bonds of A will be traded at a higher spread as compared to bond B.

Q.1027 The bonds of country A is trading at a spread of x with respect to US Treasury bonds in country B and at a spread of y with respect to US Treasury bonds in country C. A relative value trader wants to generate returns by trading in bonds of country A. If $x > y$, then the trade must be:

- A. To buy the bond in country B and sell it in country C.
- B. To buy the bond in country C and sell it in country B.
- C. To sell the bond in country B and C.
- D. To buy the bond in country B and C.

The correct answer is **A**.

The bond trades at a higher spread in country B as compared to country C. Hence, the price of the bond will be lower in country B as compared to country C. Therefore, the bond must be purchased from country B and sold in country C. The transaction will generate a profit of $(x - y)\%$.

Q.2786 Aram Bauer is considering an investment in fixed income instruments. He is interested in a U.S. Treasury tranche as of December 31, 2019. The tranche pays coupons of 4.5% per year compounded semiannually. The price of this tranche as of December 31, 2016, is 97.124.

What is the yield to maturity of the tranche?

- A. 2.78%
- B. 3.22%
- C. 5.55%
- D. 6.44%

The correct answer is **C**.

To compute a bond's yield to maturity, we use the following formula:

$$p = \frac{C_1}{(1+y)^1} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} \cdots + \frac{F + C_N}{(1+y)^N}$$

Where:

P = price of the bond

C_t = annual cash flow in year t

N = time to maturity in years

y = annual yield (YTM to maturity)

F = face value

$$\frac{2.25}{(1 + \frac{y}{2})^1} + \frac{2.25}{(1 + \frac{y}{2})^2} + \frac{2.25}{(1 + \frac{y}{2})^3} + \frac{2.25}{(1 + \frac{y}{2})^4} + \frac{2.25}{(1 + \frac{y}{2})^5} + \frac{102.25}{(1 + \frac{y}{2})^6} = 97.124$$

Using trial and error, we get $y = 5.554\%$

Alternatively, using a financial calculator with the following inputs:

$N = 6$; $PV = -97.124$; $PMT = 2.25$ ($=4.5/2$); $FV = 100$

We get, $I/Y = 2.777\%$

$$\Rightarrow YTM = 2 \times 2.777 = 5.554\%$$

Q.2787 On Jan 1 2017, Commercial Bank of India issued a six-year bond paying an annual coupon of 6% at a price reflecting a yield to maturity of 4%. As of 31 Dec 2017, interest rates remain unchanged. Holding all other factors constant, and assuming a flat term structure of interest rates, how was the bond's price affected? The price:

- A. Remained constant.
- B. Decreased.
- C. Increased.
- D. Increased, but only in the second half of the year.

The correct answer is **B**.

From the data given, it's clear that the bond's coupon is greater than the yield. As such, the bond must have traded at a premium – implying the price must have been greater than the face value. Provided the yield doesn't change, a bond's price will always converge to its face value. Since the price starts higher, it must decrease.

Q.2788 A trader borrows \$3,000,000 with a term of two years at a simple annual interest rate of 2% from his broker. He purchases at par a bond with a 5% coupon paid annually. The bond matures exactly in 10 years. Two years later, the trader sells the bond at the price of \$101 and repays the loan on an annual basis. Assuming that all of the coupons received are reinvested at the rate of 1.5%, for a period of 1 year, what is the trader's net realized return on the transaction described above?

- A. +7.0000%
- B. +7.0750%
- C. +11.0000%
- D. +11.0750%

The correct answer is **B**.

The net realized return is the return after financing costs have been subtracted

$$\begin{aligned}
 \text{Proceeds from the sale of the bond} &= \$3,000,000 * \frac{101}{100} = \$3,030,000.00 \\
 \text{Coupons received from the bond} &= \$150,000.00 * 1.015 + \$150,000.00 = \$302,250.00 \\
 \text{Interest paid to the broker} &= -\$60,000.00 - \$60,000.00 = -\$120,000 \\
 \text{Net proceeds} &= \$3,030,000.00 + \$302,250.00 - \$120,000 = \$3,212,250.00 \\
 \text{Net realized return} &= \frac{(\$3,212,250.00 - \$3,000,000)}{\$3,000,000} = 7.0750\%
 \end{aligned}$$

Q.2789 Alice Tuck invested her annual bonus in a bond with a face value of \$55,000. The bond pays a 5% coupon semiannually and matures in 10 years. At the purchase date, the bond had a yield to maturity of 7%. Six months later Alice received the coupon and sold the bond at the market yield of 6.5%. What is the net realized return on Alice's transaction?

A. +7.0000%

B. +7.1250%

C. +7.2312%

D. +7.5475%

The correct answer is C.

Investment in the bond = \$47,183.18

(N = 20; I/Y = 3.5%; PMT = 1,375 (=55,000 *5% / 2); FV = 55,000

CPT => PV = - 47,183.18)

Proceeds from the sale of the bond = \$49,220.10

(N = 19; I/Y = 3.25%; PMT = 1,375 (=55,000 *5% / 2); FV = 55,000

CPT => PV = - 49,220.10)

Coupons received from the bond = \$1,375

$$\begin{aligned}\text{Total proceeds} &= \$49,220.10 + \$1,375 = \$50,595.10 \\ \text{Net realized return} &= \frac{(\$50,595.10 - \$47,183.18)}{\$47,183.18} = 7.2312\%\end{aligned}$$

Q.3424 At the start of the year, a bank issues a non-zero-coupon bond maturing in five years. During the year, the following events are recorded:

- I. The bank's leverage ratio increases
- II. The bank's business risk increases
- III. The rate of interest earned on government bonds and T-bills increases

Which of the above-mentioned events would be expected to increase the bond's yield to maturity?

- A. I only
- B. I and II only
- C. III only
- D. I, II, and III

The correct answer is **D**.

If interest rates rise, the corporate bond will be competing with T-bonds that offer more than they were offering at the time of the bond's issue. As a result, the price of the bond will most likely fall. The yield to maturity, however, increases as price decreases. Similarly, an increase in leverage ratio and increased business risk will increase the bank's overall risk in the eyes of investors. The result will be a decrease in price, increasing the bond's yield.

Q.3425 Bank A and Bank B both have a credit rating of BBB. Bank A issues a fixed-rate bond with a 10-year term to maturity, while Bank B issues a similar bond with a 5-year term to maturity. Holding all other factors constant, which of the following statements is *most likely* true?

- A. Bond A has a higher interest rate risk than bond B
- B. Bond B has a higher interest rate risk than bond A
- C. Bond A has a lower coupon rate than bond B
- D. Bond B has a higher coupon rate than bond A

The correct answer is **A**.

The longer the bond's maturity, the greater the risk that the bond's value could be impacted by changing interest rates prior to maturity, which may have a negative effect on the price of the bond. As such, bonds with longer maturities generally have higher interest rate risk than similar bonds maturing in less time.

To compensate investors for this interest rate risk, long-term bonds generally offer higher coupon rates than short-term bonds of the same credit quality.

Q.3426 On Jan 1 2017, a 5-year corporate bond, paying an annual coupon of 8%, was selling at a discount. As of 31 Dec 2017, interest rates remain unchanged. Holding all other factors constant, which of the following relationships holds true? (P_0 represents the price of the bond and YTM is the bond's yield to maturity.)

- A. $P_0 < \text{par}$ and $\text{YTM} < 8\%$
- B. $P_0 < \text{par}$ and $\text{YTM} > 8\%$
- C. $P_0 > \text{par}$ and $\text{YTM} > 8\%$
- D. $P_0 > \text{par}$ and $\text{YTM} < 8\%$

The correct answer is **B**.

That the bond is trading at a discount means the price is lower than par (face value). It also means that the bond's coupon is less than its yield to maturity. The price of the bond will gradually "pull to par" (rise to par value as maturity approaches).

Q.4582 A zero-coupon bond with three years to maturity has a face value of 100. If the current market price of the bond is 88, what is the yield to maturity of the bond?

- A. 12%
- B. 13.6%
- C. 4.35%
- D. 3.79%

The correct answer is **C**.

The yield to maturity of a zero-coupon bond is given by:

$$\text{YTM} = \left(\frac{\text{Face value}}{\text{Current price of the bond}} \right)^{\frac{1}{\text{year to maturity}}} - 1$$

In this case,

$$\text{YTM} = \left(\frac{100}{88} \right)^{\frac{1}{3}} - 1 = 4.35\%$$

Q.4583 An investor buys a two-year 100 par-value bond at \$95 per \$100 face value. The bond pays semiannual coupons at a rate of 5% per annum. Suppose that after six months, the coupon is invested and earns 2% for the next 6 months. After 1 year, the investor decides to cash out and sell the bond at \$97. What is the gross realized return for the investor?

- A. 7.37%
- B. 7.42%
- C. 4.22%
- D. 2.11%

The correct answer is **B**.

$$\text{Gross Realised Return} = \frac{\text{Ending Value} + \text{Coupon} - \text{Beginning Value}}{\text{Beginning Value}}$$

Given that the coupon rate is 5% paid semiannually, then the bond pays coupons of \$2.5 after every 6-months, and since the coupon is invested for the next 6-months at 2%, then we have:

In this case,

$$\text{Gross Realised Return} = \frac{97 + 2.5 + 2.5 * 1.02 - 95}{95} = 7.421\%$$

Q.4584 Suppose that James is offered a bond that pays \$40 per annum in perpetuity. If the discount rate is 6%, what is the bond's expected price?

- A. 600
- B. 666.67
- C. 667.77
- D. 666

The correct answer is **B**.

The price of a perpetual bond is given by,

$$PV_{\text{perpetuity}} = \frac{PMT}{r}$$

Where;

r = the discount rate; and

PMT = the periodic payment.

In this case,

$$PV_{\text{perpetuity}} = \frac{40}{0.06} = \$666.67$$

Q.4585 What is the present value of an annuity that pays \$100 per year at the end of each year for the next five years at an effective rate of 5% per annum?

- A. 435
- B. 432.95
- C. 495
- D. 487.98

The correct answer is **B**.

We can find the present value of the annuity using the formula;

$$PV_{\text{annuity}} = PMT \left(\frac{1 - (1 + r)^{-T}}{r} \right)$$

Where,

PV_{annuity} = present value of the annuity

PMT = periodic premium payment

r = effective discount rate

In this case,

$$PV_{\text{annuity}} = 100 \left(\frac{1 - (1.05)^{-5}}{0.05} \right) = \$432.95$$

Reading 56: Applying Duration, Convexity, and DV01

Q.659 On a graduate-level exam on the subject of fixed income investments, students were asked to define duration in three sentences. One of the students mentioned the following three sentences associated with duration:

- I. The duration of a zero-coupon bond is a measure that tells how long on average the holder of the bond has to wait until the bond is redeemed for its full face value.
- II. Since there are no coupons in a zero-coupon bond, the zero-coupon bond does not have duration.
- III. The duration of a coupon bond is equal to its time to maturity.

Which of the sentences are inconsistent with the definition of duration?

- A. Statements I and II are inconsistent with the definition of duration.
- B. Statements II and III are inconsistent with the definition of duration.
- C. Statements I and III are inconsistent with the definition of duration.
- D. All of the statements are inconsistent with the definition of duration.

The correct answer is **B**.

Statement I is consistent with the definition of duration because the duration of a zero-coupon bond is a measure that tells how long on average the holder of the bond has to wait until the bond is redeemed for its full face value.

Statement II is incorrect because the duration of a zero-coupon bond is typically equal to its time of maturity.

Statement III is incorrect because the duration of a coupon bond is shorter than its time to maturity.

Q.1175 The price of a bond at various rates is given in the table below:

Spot rate	Price
3.45%	95.8680
3.40%	96.0780

The DV01 of the bond is:

- A. 11.04
- B. 1.10
- C. 0.042
- D. 0.906

The correct answer is C.

$$\begin{aligned} \text{DV01} &= -\frac{\Delta P}{10,000 * \Delta y} \\ \Delta P &= 95.8680 - 96.0780 = -0.21 \\ \Delta y &= 3.45\% - 3.40\% = 0.05\% \\ \text{DV01} &= -\frac{-0.21}{10,000 * 0.05\%} = 0.042 \end{aligned}$$

Q.1176 The price of a bond at various rates is given in the table below:

Par rates	price
3.45%	95.8680
3.40%	96.0780
3.35%	96.3210

The duration of the bond is:

- A. 0.4719
- B. 4.7149
- C. -4.7149
- D. 0.04719

The correct answer is **B**.

$$\begin{aligned}\text{Duration} &= \frac{-(P_+ - P_-)}{(r_+ - r_-) * P_0} \\ &= \frac{-(95.8680 - 96.3210)}{(3.45\% - 3.35\%) * 96.0780} = 4.7149\end{aligned}$$

Q.1177 The price of a bond at various rates is given in the table below:

Spot rate	Price
3.45%	95.8680
3.40%	96.0780
3.35%	96.3210

Select the most appropriate statement from the following.

- A. The duration is the same at 3.00% and at 3.40%.
- B. The duration is greater at 3.00% as compared to at 3.40%.
- C. The duration is smaller at 3.00% as compared to at 3.40%.
- D. Duration remains unaffected by the level of interest rates.

The correct answer is **B**.

As the interest rate decreases, the slope of the tangent becomes steeper. This signifies that the change in the price of the bond is higher at lower interest rates. As duration measures the change in the price of the bond with respect to the interest rates, duration will be higher in the case of lower interest rates.

Q.1178 A fund manager has the option to buy the following bonds:

- I. A bond with a coupon of 10% and a tenure of 5 years
- II. A bond with a coupon of 5% and a tenure of 5 years

If the fund manager wants to limit the impact of interest rate changes in his portfolio, the ideal bond(s) to invest in is/are:

- A. The bond with the 10% coupon.
- B. The bond with the 5% coupon.
- C. Both bonds, since they react in a similar manner to interest rate changes.
- D. Both bonds, since the diversification effect will reduce the impact of interest rate changes.

The correct answer is **A**.

The impact of interest rate change in a bond portfolio is measured by the duration of the portfolio. The bond with the higher coupon has smaller duration compared to the bond with the lower coupon. Hence, if the fund manager wants to limit the impact of the interest rate changes on his portfolio, he must invest in the bond with the 10% coupon, as this bond will have a smaller duration compared to the bond with the 5% coupon.

Q.1179 A fund manager has the option to buy the following bonds:

- I. A bond with 10% coupon and a tenure of 15 years
- II. A bond with 10% coupon and a tenure of 10 years
- III.

The fund manager expects the interest rate volatility to increase and wants to compose a portfolio which will help him generate maximum return due to the volatility. The fund manager must buy:

- A. The bond with a tenure of 15 years.
- B. The bond with a tenure of 10 years.
- C. Both, since they react in a similar manner to interest rate volatility.
- D. Both, since the diversification effect will help him generate maximum return.

The correct answer is **A**.

Bond duration is generally considered a direct measure of how much a bond's price will change for an interest rate change of 100 basis points (1 percent). For example, for every 1 percent change in interest rates, a bond with a duration of 10 years will change by 10% in price. Generally, bonds with longer maturities and lower coupons have longer durations. These bonds are more sensitive to a change in market interest rates and thus are more volatile in a changing rate environment. An investor wishing to take advantage of higher volatility should therefore go for the bond with the longer maturity. The investor would effectively be betting that the higher volatility will bring about a fall in interest rates because that's the only way they would benefit. If interest rates rise, bond prices fall, and the investor loses.

It follows that a 10% annual coupon bond that matures in 15 years would have a longer duration and would experience a higher rise in price as interest rates fall than a bond with a 10% coupon that matures in 10 years. Of course, duration is a double-edged sword that cuts both ways. If interest rates were to rise, the value of the 15-year bond would fall more than that of the 10-year bond.

Q.1180 All the following are true for convexity, EXCEPT :

- A. Convexity is the second derivative of the price rate function.
- B. For an option free bond, convexity is always negative.
- C. Convexity explains why the price of a bond falls less and rises more in the case of changes in interest rates.
- D. Convexity enhances the bond's return.

The correct answer is **B**.

Convexity is the second derivative of the price rate function. Due to the convex shape of the price-interest curve of a bond, for a given change in the interest rates, the bond price rises more and falls less.

If an increase in interest rates decreases the price of a bond by x , and an equal decrease of 1% in interest rates increases the price of the same bond by y , then due to convexity, $y > x$.

For an option-free bond, the convexity is always positive. As the increase in bond price is larger in value than the decrease in value for the same yield change, convexity enhances the return of a bond.

Q.1181 Details of portfolio X is given below: Note: Portfolio X consists of bonds A, B, C and D, and the value of each bond is given in the above table. The duration of portfolio X is:

Bond	Value	Duration	Convexity
Bond A	\$120,000	5.453	230.453
Bond B	\$100,000	7.213	350.361
Bond C	\$150,000	2.348	120.714
Bond D	\$130,000	8.190	480.341

A. 5.26

B. 5.59

C. 5.10

D. 5.69

The correct answer is **B**.

Total value of the portfolio = \$120,000 + \$100,000 + \$150,000 + \$130,000 = \$500,000

$$\text{Duration of the portfolio} = \frac{\$120,000 * 5.453 + \$100,000 * 7.213 + \$150,000 * 2.348 + \$130,000 * 8.190}{\$500,000}$$

$$= 5.59$$

Q.1182 Details of portfolio X is given below:

Bond	Value	Duration	Convexity
Bond A	\$120,000	5.453	230.453
Bond B	\$100,000	7.213	350.361
Bond C	\$150,000	2.348	120.714
Bond D	\$130,000	8.190	480.341

Note: Portfolio X consists of bonds A, B, C and D, and the value of each bond is furnished in the above table.

The convexity of portfolio X is:

A. 360.426

B. 386.484

C. 200

D. 286.484

The correct answer is **D**.

$$\begin{aligned}\text{Total value of the portfolio} &= \$120,000 + \$100,000 + \$150,000 + \$130,000 = \$500,000 \\ \text{Convexity of the portfolio} &= \frac{(\$120,000 * 230.453 + \$100,000 * 350.361 + \$150,000 * 120.714 + \$130,000 * 480.341)}{(\$500,000)} \\ &= 286.484\end{aligned}$$

Q.1183 Data on three bonds are given below. Assume the current date is March 31, 2015.

Bond	Maturity	Price	Yield	Duration	Convexity
A	March 31, 2020	110.321	2.32%	4.321	34.09
B	March 31, 2030	109.320	3.23%	9.102	78.32
C	March 31, 2045	103.211	4.11%	18.112	323.11

The fund manager is considering purchasing \$10 million (face value) of bonds B at the cost of \$10.932 million. The fund manager expects the interest volatility to increase and hence wants to maximize his returns. However, another fund manager makes the suggestion that instead of investing in bond B, the fund manager should invest in a combination of bonds A and C. Given that the fund manager has a surplus of \$10.932 million and wants the duration of the portfolio to be equal to that of bond B, the investments in A and C which can create a portfolio with a duration similar to B is:

- A. \$5.34 million in bond A and \$5.592 million in bond C.
- B. \$9.321 million in bond A and \$1.611 million in bond C.
- C. \$7.14 million in bond A and \$3.79 million in bond C.
- D. \$8.12 million in bond A and \$2.812 million in bond C.

The correct answer is C.

Let the investments in A and C be x and y respectively. Then, as per the constraints:

$$x + y = 10.932$$

$$\left(\frac{1}{10.932}\right) * (x * 4.321 + y * 18.112) = 9.102$$

Solving for x and y:

$$\begin{aligned} x &= \$7.14 \\ y &= \$3.79 \end{aligned}$$

Q.1184 Data on three bonds are given below. Assume the current date is March 31, 2015.

Bond	Maturity	Price	Yield	Duration	Convexity
A	March 31, 2020	110.321	2.32%	4.321	34.09
B	March 31, 2030	109.320	3.23%	9.102	78.32
C	March 31, 2045	103.211	4.11%	18.112	323.11

The fund manager is considering purchasing \$10 million (face value) of bonds B at the cost of \$10.932 million. The fund manager expects the interest volatility to increase and hence wants to maximize his returns. However, another fund manager makes the suggestion that instead of investing in bond B, the fund manager should invest in a combination of bonds A and C. The fund manager has a surplus of \$10.932 million and wants the duration of the portfolio to be equal to that of bond B. Given that the fund manager expects increased interest volatility, the fund manager should invest in:

- A. \$10.932 million of Bond B.
- B. \$10.932 million of Bond C.
- C. \$5.12 million of bond A and \$5.81 of bond C.
- D. \$7.14 million of bond A and \$3.79 of bond C.

The correct answer is **D**.

The fund manager must invest in such a bond/portfolio which has the highest convexity, as higher convexity will help the fund manager to maximize return due to interest rate volatility.

Let the investments in A and C be x and y respectively. As per the constraint of having a duration of the portfolio equal to that of bond B (9.102 years):

$$x + y = 10.932$$

$$\left(\frac{1}{10.932}\right) * (x * 4.321 + y * 18.112) = 9.102$$

Solving for x and y:

$$\begin{aligned} x &= \$7.14 \\ y &= \$3.79 \end{aligned}$$

$$\text{Portfolio convexity} = \left(\frac{7.14}{10.932}\right) * 34.09 + \left(\frac{3.79}{10.932}\right) * 323.11 = 134.29$$

As the convexity of the portfolio is higher than the convexity of bond B and the portfolio meets the

duration constraints (as it is equal to bond B), the preferred investment is in a portfolio consisting of bond A and C.

Q.1185 Data on three bonds are given below. Assume the current date is March 31, 2015.

Bond	Maturity	Price	Yield	Duration	Convexity
A	March 31, 2020	110.321	2.32%	4.321	34.09
B	March 31, 2030	109.320	3.23%	9.102	78.32
C	March 31, 2045	103.211	4.11%	18.112	323.11

The return generated by a portfolio consisting of \$7.14 million of bond A and \$3.79 million of bond C in the case the interest rate remains constant is:

A. 0.0232

B. 0.0411

C. 0.0294

D. 0.0151

The correct answer is C.

$$\begin{aligned}\text{The return generated in case the rate remains constant} &= \left(\frac{7.14}{10.932}\right) * 2.32\% + \left(\frac{3.79}{10.932}\right) * 4.11\% \\ &= 2.94\%\end{aligned}$$

Q.2791 Suppose the yield on a zero-coupon bond declines from 5.00% to 4.99%, and the price of the zero increases from \$50.0 to \$51.5. Compute the DV01.

A. \$0.0512

B. \$1.5

C. \$2.5

D. \$0.1

The correct answer is **B**.

$$DV01 = -\frac{\Delta BV}{10,000 \times \Delta y}$$

Where:

ΔBV = change in bond value

Δy = change in yield

$$DV01 = -\frac{51.5 - 50.0}{10,000 \times -0.0001} = 1.5$$

Q.2792 Ted Oster wants to calculate the DV01 of a new position in a bond with a face value of \$1,000,000. The bond was bought today for \$84.102 for \$100 face value. Oster knows that the Macaulay duration is 5.25.

What is the DV01 of the position if the bond has a yield to maturity of 10% per annum?

- A. \$401.39
- B. \$405.40
- C. \$483.47
- D. \$492.73

The correct answer is **A**.

$$\begin{aligned}\text{Modified duration} &= \frac{5.25}{(1 + 10\%)} = 4.7727 \\ \text{DV 01} &= \text{Mod dur} \times 0.0001 \times \text{Bond value} \\ &= 4.7727 \times 0.0001 \times \left(\frac{84.102}{100}\right) \times \$1,000,000 \\ &= \$401.39\end{aligned}$$

Note that the reason why we multiply by 0.0001 is to get the dollar change for a 1% change in yield in terms of basis points.

Q.2794 A trader at an investment bank sold to the corporate client \$10,000,000 worth of non-standardized call options on bonds. The DV01 of the option is 0.1011. The trader wants to hedge the options using the same option's underlying bonds that has a DV01 of 0.2455.

What is the bank's hedging transition?

- A. Sell \$4,118,000 face value of U.S. Treasury 3s of December 31, 2020.
- B. Buy \$4,118,000 face value of U.S. Treasury 3s of December 31, 2020.
- C. Sell \$2,428,000 face value of U.S. Treasury 3s of December 31, 2020.
- D. Buy \$2,428,000 face value of U.S. Treasury 3s of December 31, 2020.

The correct answer is **B**.

$$\text{Hedge ratio} = \frac{0.1011}{0.2455} = 0.4118$$

The hedge fund should buy \$4,118,000 face value of U.S. Treasury 3s of December 31, 2020.

Q.3315 Peter Drury, a risk manager at Capital Bank, is evaluating the price sensitivity of an investment-grade callable bond using the bank's valuation system. The table below gives a breakdown of the bond and the embedded option. The current interest rate environment is flat at 5%.

Interest rate level	Bond value per 100 USD face value	Call option value per 100 USD face value
4.95%	98.2520	1.7480
5.0%	98.0000	1.5000
5.05%	97.7500	1.3225

The DV01 of a comparable bond with no embedded options and with the same maturity and coupon rate as the callable bond is closest to:

- A. 0.09275
- B. 0.08015
- C. 0.1285
- D. 0.07500

The correct answer is **A**.

The call option negatively affects the price of a bond because investors lose future coupon payments if the call option is exercised by the issuer.

The value of a plain-vanilla bond can be given by:

$$\text{Price (plain-vanilla bond)} = \text{price (callable bond)} + \text{price (call option)}$$

Thus, the price of the plain-vanilla bond with no embedded options at a rate of 5.0% would be 99.5000, the price at a rate of 4.95% would be 100.0000, and the price at a rate of 5.05% would be 99.0725.

DV01 is the dollar value change in price (value) of a fixed income instrument, such as a bond, in response to a change in the yield of the instrument. It is given by:

$$DV01 = -\frac{\Delta P}{10,000 \times \Delta Y}$$

where:

ΔP = change in price, and

ΔY = change in yield

Thus

$$DV01 = -\frac{99.0725 - 100}{10,000 \times (0.0505 - 0.0495)} = -\frac{-0.9275}{10} = 0.09275$$

Q.3319 An investment company is exposed to a portfolio of bonds. The duration of the portfolio is 8.83 and the convexity of the portfolio is 6.43. What is the change in the price of the portfolio for a one percentage point increase in the interest rate?

- A. -5.5%
- B. 2.4%
- C. -4.4%
- D. -8.8%

The correct answer is **D**.

For a one basis point of change in the interest rate, the formula for the bond price change is:

$$\begin{aligned}\text{Change in price} &= [-\text{Modified Duration} \times \text{Change in yield}] + \left[\frac{1}{2} \times \text{Convexity} \times (\text{Change in yield})^2\right] \\ &= -8.83 \times 0.01 + \frac{6.43}{2} \times 0.01^2 = -8.8\%\end{aligned}$$

Q.3427 Given the following portfolio of bonds:

Bond	Price	Par Amount Held (USD Million)	Modified Duration
A	101.22	4	2.45
B	85.53	6	4.25
C	115.50	9	7.61

What is the value of the portfolio's DV01 (dollar value of a basis point)?

- A. \$10,960
- B. \$11,000
- C. \$11,060
- D. \$12,600

The correct answer is **C**.

$$\text{Portfolio DV01} = \text{Portfolio Modified duration} \times \text{Portfolio Market Value} \times 0.0001$$

But we have to compute the portfolio modified duration first:

Modified duration of the portfolio = Weighted Average of Modified Duration of Individual Bonds in the

$$= w_1D_1 + w_2D_2 + \dots + w_kD_k$$

where :

$$w_i = \frac{\text{Market value of bond } i}{\text{Portfolio market value}}$$

D_i = Modified duration of bond i

k =total no.of bonds in the portfolio

Based on the table above, these are the values for bonds A, B, and C:

Bond	Value	Weight in the Portfolio
A	$= 101.22 \times \frac{4,000,000}{100} = 4,048,800$	$= \frac{4,048,800}{19,575,600} = 0.21$
B	$= 85.53 \times \frac{6,000,000}{100} = 5,131,800$	$= \frac{5,131,800}{19,575,600} = 0.26$
C	$= 115.5 \times \frac{9,000,000}{100} = 10,395,000$	$= \frac{10,395,000}{19,575,600} = 0.53$
Portfolio	$= 4,048,800 + 5,131,800 + 10,395,000$ $= 19,575,600$	

$$\text{Portfolio Modified Duration} = 0.21 \times 2.45 + 0.26 \times 4.25 + 0.53 \times 7.61 = 5.65$$

Therefore,

$$\text{Portfolio DV01} = 5.65 \times 19,575,600 \times 0.0001 = \$11,060.2$$

Q.3428 The current interest rate environment in a certain developing economy is flat at 5%. A risk manager has compiled the following data regarding a callable bond in two other interest rate environments (all values in USD per USD 100 face value):

Level of interest rate	Callable bond	Call option
4.96%	102.00	2.0893
5.00%	102.4465	2.0255
5.04%	101.2111	2.0021

The convexity of the callable bond is closest to:

- A. -102,608
- B. -51,304
- C. -100,020
- D. -103,000

The correct answer is **A**.

Convexity is the second derivative of the formula for change in bond prices with a change in interest rates.

$$\text{Convexity} = \frac{1}{P} \times \frac{d^2P}{dy^2}$$

This approach can be quite rigorous in terms of computation, so you need to estimate convexity using the formula:

$$\text{Convexity} = \frac{P_{-\Delta y} + P_{+\Delta y} - 2P_0}{P_0 \times \Delta y^2}$$

Where:

$P_{-\Delta y}$ = price estimate if yield decreases by a given amount, Δy

$P_{+\Delta y}$ = price estimate if yield increases by a given amount, Δy

P_0 = initially observed bond price (at the flat rate)

Δy = change in yield in one step, expressed in decimal form (0.04% in this case)

$$\begin{aligned} \text{Convexity} &= \frac{102.00 + 101.2111 - 2 \times 102.4465}{102.4465 \times 0.0004^2} \\ &= -\frac{1.6819}{0.00001639} = -102,608 \end{aligned}$$

Q.3429 A 20-year zero-coupon bond is callable annually at par, starting at the beginning of year 11. Assuming a flat yield curve of 20%, the bond's duration is closest to:

- A. 20 years
- B. 15 years
- C. 10 years
- D. Cannot be determined based on the data given

The correct answer is **A**.

Because this is a zero-coupon bond, it will always trade below par, and the call should never be exercised. Hence, its duration is the maturity of the bond, 20 years.

Q.4609 In the context of the one-factor risk metrics, which of the following is/are correct?

- I. DV01 and effective duration hedging provide protection against small parallel shifts of the term structure
 - II. DV01 and effective hedging protect against large parallel shifts of the term structure
 - III. A combination of the convexity and effective duration protects against large parallel shifts in the term structure
 - IV. Effective convexity hedging protects against small parallel shifts in the term structure
- A. I and IV
 - B. I and III
 - C. III only
 - D. I and II

The correct answer is **B**.

Statement I is correct: DV01 and effective duration hedging provide protection against small parallel shifts of the term structure.

Statement II is incorrect: It contradicts statement I.

Statement III is correct: Using both the duration and convexity in hedging protects against large parallel shifts in the term structure of interest rates.

Statement IV is incorrect: Convexity and the effective duration are combined with hedging a position against large parallel shifts in the term structure.

Q.4611 A bank has a position of USD 2 million with a duration of 10 years. To completely hedge its position, the bank takes a short position of USD 1.4 million in bond B. What is the duration of bond B?

A. 10

B. 11

C. 13

D. 14

The correct answer is **D**.

The position required in the bond to hedge a position is given by:

$$P = -\frac{V D_V}{D_B}$$

Where

V = a value of the position

D_V = duration of the position

D_B = duration of the bond

So in this case,

$$\begin{aligned} P &= -\frac{V D_V}{D_B} \\ \Rightarrow -1.4 &= -\frac{2 \times 10}{D_B} \\ \therefore D_B &= \frac{20}{1.4} \approx 14 \end{aligned}$$

Q.4613 Suppose the yield on a zero-coupon bond declines from 7.00% to 6.95% and the price of the zero increases from \$202.45 to \$203.87. What is the value of DV01?

A. 0.342

B. 0.324

C. 0.284

D. 0.242

The correct answer is **C**.

DV01 is defined as:

$$DV01 = -\frac{\Delta P}{\Delta r}$$

Where

Δr = the size of a parallel shift in the interest rate term structure measured in **basis points**

ΔP = resultant change in the value of the position being considered

$$\Rightarrow DV01 = -\frac{203.87 - 202.45}{(-0.0005) \times 10,000} = \frac{1.42}{5} = 0.284$$

Q.4614 The Macaulay duration of a coupon bond is 10.25. If the yield on the bond is 8% compounded semi-annually, what is the corresponding modified duration for the bond?

- A. 10.25
- B. 9.86
- C. 11.45
- D. 9.54

The correct answer is **B**.

Denoting the bond yield by y , modified duration is calculated by dividing Macaulay duration by $(1 + \frac{y}{2})$. So in this case,

$$\text{Modified duration} = \frac{10.25}{(1 + \frac{0.08}{2})} = 9.86$$

Q.4615 A four-year Treasury bond has a face value of USD 5 million and an annual coupon payment of 8% paid semi-annually. The term structure applicable to the bond is a 10% flat yield. Considering ten basis point changes, what is the effective convexity of the bond?

- A. -13.68
- B. 13.68
- C. -203.67
- D. 203.67

The correct answer is **B**.

Convexity is given by:

$$C = \frac{1}{P} \left[\frac{P^+ + P^- - 2P}{(\Delta r)^2} \right]$$

Where

P = the price of the bond.

P^+ = the value of the position when all rates increase by Δr

P^- = the value of the position corresponding to the decrease of all rates by Δr (measured in decimal)

The price of the bond (in millions) with no spread is USD 4,676,839 million. Using a financial calculator with the variables:

$$\begin{aligned}N &= 8, \\ \frac{I}{Y} &= \frac{10\%}{2} = 5\% \\ PV &= ? \\ PMT &= \frac{0.08 \times 5}{2} = 0.2 \\ FV &= 5 \\ \Rightarrow PV &= 4.676839 \text{ million} \\ \Rightarrow P &= 4,676,839\end{aligned}$$

For an increase in 10 point basis (0.10%) the flat yield is now 10.10% and thus $\frac{I}{Y} = 5.05\%$. Using the financial calculator,

$$P^+ = \text{USD } \$4,661,366$$

For a 10 point basis decrease, the flat yield is now 9.9% and thus $\frac{I}{Y} = 4.95\%$.

$$\begin{aligned}\therefore P^- &= \$4,692,376 \\ \Rightarrow C &= \frac{1}{4,676,839} \left[\frac{4,661,366 + 4,692,376 - 2 \times 4,676,839}{(0.001)^2} \right] = 13.681\end{aligned}$$

Q.4616 A bond portfolio consists of three bonds:

- Bond A worth 9 million with a duration of 4;
- Bond B worth 5 million with a duration of 6; and
- Bond C worth 6 million with a duration of 7.

What is the effective duration of the portfolio?

- A. 7.8
- B. 6.6
- C. 5.4
- D. 5.7

The correct answer is **C**.

Recall that the portfolio duration is calculated as the **weighted sum** of the individual durations where the weight attached to each security is equal to its value as a percentage of total portfolio value.

The total value of the portfolio is $9 + 5 + 6 = \text{USD } 20$ million. The duration of the bond is given by:

$$\frac{9}{20} \times 4 + \frac{5}{20} \times 6 + \frac{6}{20} \times 7 = 5.4$$

Q.4617 A position worth USD 3 million has a duration of 4 and a convexity of 5. What is the estimated change in the position for a five-basis-point increase in all rates?

- A. Increase by USD 5,998.125
- B. Decrease by USD 5,998.125
- C. Increase by USD 5,546.670
- D. Decrease by USD 5,546.670

The correct answer is **B**.

By using convexity and duration, the price change estimation is given by:

$$\begin{aligned}\Delta P &= -DP \Delta r + \frac{1}{2} CP (\Delta r)^2 \\ &= -4 \times 3 \times 0.0005 + \frac{1}{2} \times 5 \times 3 \times (0.0005)^2 \\ &= -0.005998125 = -\text{USD } 5,998.125\end{aligned}$$

Reading 57: Modeling and Hedging Non-Parallel Term Structure Shifts

Q.1148 All the following are assumptions of Key Rate Shifts, EXCEPT:

- A. Rates can be determined as a function of a relatively small number of key rates.
- B. There is a parallel shift of rates across the key rates.
- C. There is a linear shift of rates across the term structure.
- D. The rate of a given term is not affected by its neighboring key rates.

The correct answer is **D**.

The basic idea behind the key rate shifts is the assumption that rates can be determined as a function of a relatively small number of key rates. Furthermore, the key rates also assume that there is a parallel shift/linear shift of rates across the term structure. It also assumes that the rate of the given term is affected by its neighboring key rates. For instance, it assumes that the 5-year rate is a function of 2-year and 3-year rates.

Q.1149 An investor who buys a payer swaption:

- A. Has the right to pay the fixed rate on a swap at some time in the future.
- B. Has the right to pay the floating rate on a swap at some time in the future.
- C. Has the right to receive the fixed rate on a swap at some time in future.
- D. None of the above.

The correct answer is **A**.

A payer swaption gives the owner of the swaption the right to enter into a swap where they pay the fixed leg and receive the floating leg.

Option B is incorrect. A **receiver** swaption gives the owner of the swaption the right to enter into a swap in which they will receive the fixed leg and pay the floating leg.

Options C and D are also incorrect, because they contradict option A.

Q.2605 An analyst has gathered the following data on the prices of a 10-year zero-coupon bond simulated for shifts in three key rates:

	Value
Initial Curve	103.300
2-year Shift	102.755
5-year Shift	102.504
10-year Shift	102.303

What is the key rate duration for a 10-year shift?

- A. 19.61
- B. 23.20
- C. 96.52
- D. 99.70

The correct answer is C.

The key rate duration measures how the value of a debt security or a debt instrument portfolio changes at a specific maturity point along the entirety of the yield curve Mathematically,

$$\text{Key rate duration} = \frac{1}{p} * \left(\frac{\text{change in price}}{0.01\%} \right)$$

where p is the initial price

This can be rewritten as:

$$\text{Key rate duration} = -\frac{1}{p} * (\text{change in price}) * 10,000$$

$$\text{Thus, key rate duration for a 10-year shift} = -\frac{(102.303 - 103.300)}{103.300 * 10,000} = 96.52$$

Q.2606 A risk manager prepares a presentation on the interest rate risk of the bank's bond portfolio. The table below shows the value of the portfolio in case of shifts in key rates by one basis point and corresponding key rate duration.

	Value	Key Rate Duration
Initial Curve	500.425	
2-year Shift	500.227	3.957
5-year Shift	500.201	4.476
10-year Shift	?	28.476
30-year Shift	499.500	18.484
Total		55.393

What is the value of the portfolio in the case of a 10-year shift?

- A. 501.850
- B. 500.043
- C. 499.500
- D. 499.000

The correct answer is **D**.

$$\text{Key rate duration} = -\frac{1}{p} \frac{\partial p}{\partial y}$$

Where:

P = initial bond value/price

∂p = change in bond price

∂y = change in yield

Let the value of the bond portfolio in the case of a 10-year shift be "x"

Thus, the key rate duration with respect to the 10-year shift is calculated as:

$$\begin{aligned}
 \text{Key rate duration} &= -\frac{1}{500.425} \frac{x - 500.425}{0.01\%} \\
 \Rightarrow 28.476 &= -\frac{1}{500.425} \times \frac{x - 50.425}{0.01\%} \\
 \Rightarrow 28.476 \times 500.425 &= -\frac{x - 500.425}{0.01\%} \\
 \Rightarrow 28.476 \times 500.425 \times 0.01\% &= -(x - 500.425) \\
 \Rightarrow 28.476 \times 500.425 \times 0.01\% &= (500.425 - x) \\
 \Rightarrow 500.425 - 28.476 \times 500.425 \times 0.01\% &= x \\
 \Rightarrow x &= 499.000
 \end{aligned}$$

Q.2607 Frank Capper wants to estimate the impact of key rate changes on the value of C-STRIPs. Capper uses 2-year, 5-year, 10-year, and 30-year key rates in his analysis. He also wants to incorporate an unexpected 50 basis point shock of the 10-year rate in the model.

Which of the following rates will be affected by the change of the 10-year rate key rate?

- A. 2-year and 5-year
- B. 30-year rate
- C. 5-year and 30-year rates
- D. All of the rates will be affected

The correct answer is C.

Each key rate affects the yields from the term of the previous key rate (or zero) to the term of the next key rate.

Q.2610 A trader wants to hedge the 2-year and 5-year rates exposure of a portfolio. To perform the hedge, the trader can use Bond 1 and Bond 2 presented below.

	Key Rate '01 (per 100 face amount)	
Hedging Bonds	<u>2-year</u>	<u>5-year</u>
Bond 1	0.0080	—
Bond 2	0.0099	0.0160

	Key Rate '01(\$)	
	<u>2-year</u>	<u>5-year</u>
Fixed Income Portfolio	250	320

What is the face value of Bond 1 required to perform the hedge?

- A. \$200,000
- B. \$650,000
- C. \$950,000
- D. \$1,250,000

The correct answer is **B**.

Let's denote $F(i)$ as the face amount of hedging bond i that we need to sell.

We have the following equations:

$$(1): F(2) * \frac{0.0099}{100} + F(1) * \frac{0.0080}{100} = \$250$$

$$(2): F(2) * \frac{0.0160}{100} = \$320$$

$$\text{From (2)} \rightarrow F(2) = \$2,000,000$$

$$\text{From (1) and (2)} \rightarrow F(1) = \$650,000$$

Q.2611 The risk manager of a regional bank is concerned with possible shocks in short-term rates. He wants to find a transaction that will completely eliminate the 2-year exposure and decrease by half the current 5-year exposure. A trader proposes the following two bonds as hedging instruments:

	Key Rate '01 (per 100 face amount)	
Hedging Bonds	<u>2-year</u>	<u>5-year</u>
Bond 1	0.0010	0.0050
Bond 2	0.0015	0.0025

	Key Rate '01(\$)	
	<u>2-year</u>	<u>5-year</u>
Fixed Income Portfolio	1, 000	4, 000

What is the trader's hedging transaction?

- A. Sell \$60,000,000 of bond 1; Sell \$10,000,000 of bond 2.
- B. Sell \$10,000,000 of bond 1; Sell \$60,000,000 of bond 2.
- C. Sell \$10,000,000 of bond 1; Buy \$60,000,000 of bond 2.
- D. Buy \$60,000,000 of bond 1; Sell \$60,000,000 of bond 2.

The correct answer is **B**.

Let's denote $F(i)$ as the face amount of hedging bond i that we need to sell.

To completely hedge the 2-year exposure, $F(1)$ and $F(2)$ should solve the following equation:

$$F(2) * \frac{0.0015}{100} + F(1) * \frac{0.0010}{100} = \$1000 \dots \dots (1)$$

To decrease by half the 5-year exposure of \$4,000, $F(1)$ and $F(2)$ should solve the following equation:

$$F(2) * \frac{0.0025}{100} + F(1) * \frac{0.0050}{100} = \$2,000 \dots \dots (2)$$

Solving equation (1) and (2), simultaneously, we get, $F(2) = \$60,000,000$ and $F(1) = \$10,000,000$

Q.2612 The head of the trading department of a bank suggests speculating on the interest rate curve by eliminating the exposure of the bond portfolio to long-term rates (10-year and 30-year rates) and increasing the exposure to medium-term rates. The table below represents the key rate '01s of the current portfolio and corresponding hedging instruments.

	Key Rate '01 (per 100 face amount)		
Hedging Bonds	<u>5-year</u>	<u>10-year</u>	<u>30-year</u>
Bond 1	0.0050	—	—
Bond 2	0.0170	0.0250	—
Bond 3	0.0100	0.0300	0.0350

	Key Rate '01(\$)		
	<u>5-year</u>	<u>10-year</u>	<u>30-year</u>
Fixed Income Portfolio	1,000	2,000	1,500

What will be the portfolio's 5-year exposure after hedging 10-year and 30-year exposures?

- A. \$85.70
- B. \$171.43
- C. \$8,570.00
- D. \$17,143.00

The correct answer is **A**.

Let's denote $F(i)$ as the face amount of hedging bond i that we need to sell.

We have the following equations:

$$(1): F(3) * \frac{0.0300}{100} + F(2) * \frac{0.025}{100} = \$2,000$$

$$(2): F(3) * \frac{0.0350}{100} = \$1,500$$

$$\text{From (3)} \rightarrow F(3) = \$4,285,714$$

$$\text{From (2) and (3)} \rightarrow F(2) = \$2,857,143$$

$$\text{Finally, the portfolio's 5-year exposure} = \$1000 - F(2) * \frac{0.0170}{100} - F(3) * \frac{0.0100}{100} = \$85.70$$

Q.2613 The risk manager at a regional bank is trying to interpret the results of an interest rate curve shocks simulation. The table below represents the key rate '01s for the fixed income portfolio of the bank.

	Value	Key Rate '01
Initial Curve	2,000.000	
2-year Shift	1,998.500	1.500
5-year Shift	1,998.300	1.700
10-year Shift	1,999.000	1.000
30-year Shift	1,998.000	2.000
Total		6.200

What is the approximate value of the portfolio in the case of a 5 basis point increase of the 2-year rate and a 10 basis point increase of the 30-year rate?

- A. 1,972.500
- B. 1,980.000
- C. 1,992.500
- D. 1,996.500

The correct answer is **A**.

$$\text{Portfolio value} = 2,000 - (5 * 1.5 + 10 * 2) = 1,972.5$$

Q.2614 Anna White, head of the risk management department of a regional bank, asks one of the interns to analyze the volatility of the bank's medium-term fixed-income portfolio with a value of \$10,000. For the analysis of the portfolio, the intern uses 2-year rates (annual volatility of 20%) and 5-year rates (annual volatility of 30%). He calculates that the key rate '01s for 2-year and 5-year shifts are 2 per \$ 100 and 5 per \$ 100 respectively.

What is the volatility of the portfolio if the correlation between 2-year and 5-year rates is 0.45?

- A. \$155.24
- B. \$171.76
- C. \$22,570.00
- D. \$29,500

The correct answer is **B**.

$$\begin{aligned}
 \text{Portfolio variance} &= \left(\frac{2}{100} \times 0.20 \times 10000\right)^2 + \left(\frac{5}{100} \times 0.30 \times 10000\right)^2 \\
 &\quad + 2 \times \frac{2}{100} \times \frac{5}{100} \times 0.20 \times 10000 \times 0.30 \times 10000 \times 0.45 \\
 &= 29,500 \\
 \text{Portfolio volatility} &= \$29,500^{0.5} = \$171.76.
 \end{aligned}$$

Q.2615 Initially, a fixed income portfolio of an investment bank had the following key rate '01s:

	Value	Key Rate
Initial Curve	10,000	
2-year Shift	9,990	10
5-year Shift	9,985	15
10-year Shift	9,981	19
30-year Shift	9,984	16
Total		60

After a recommendation from the risk management department, a trader completely hedged the 30-year exposure with a bond that had the following characteristics:

Hedging Bond	Key Rate '01 (per USD 100 face value)
Initial Curve	
2-year Shift	–
5-year Shift	–
10-year Shift	3
30-year Shift	4

How much will the bank save, if immediately after the hedge, the interest rate curve experienced a 20 basis point upward parallel shift?

- A. 320
- B. 360
- C. 480
- D. 560

The correct answer is **D**.

To hedge the 30-year exposure, the trader will need to sell \$400($= \$\frac{16}{4} * 100$) face value of hedging bond. The sale of hedging bond will also impact the 10-year exposure by $-\$12(= -\frac{3}{100} * \$400)$.

The portfolio key rate '01s after hedge will become:

Hedging Bond	Key Rate '01
Initial Curve	
2-year Shift	10
5-year Shift	15
10-year Shift	7
30-year Shift	–
Total	32

The portfolio value drop without hedging transaction $= 20 * 60 = 1,200$

The portfolio value drop with hedging transaction $= 20 * 32 = 640$

The hedging transaction will save the bank $560(= 1,200 - 640)$.

Q.3430 Kelvin Mertens, FRM, regularly participates in bond trading in the US. He is using key rate analysis to assess the effect of yield changes on bond prices. He finds that the 20-year yield has increased by 10 basis points. Moreover, this shock decreases linearly to zero for the 30-year yield. What is the effect of this shock on the 26-year yield?

- A. Increase of zero basis points
- B. Increase of six basis points
- C. Increase of ten basis points
- D. Increase of four basis points

The correct answer is **D**.

“Linear decline” implies the decline is by the same amount in each time step. The 10 basis point shock to the 20-year yield is supposed to decline linearly to zero for the 30-year yield. If one assumes a simplistic one basis point effect, the impact of each key rate will be one basis point at each key rate and then a linear decline to the subsequent key rate. Thus, the shock decreases by one basis point per year and will result in an increase of four basis points for the 26-year yield.

Q.3431 The following table provides the initial price of a C-STRIP and its present value after the application of a one basis point shift in four key rates.

	Value
Initial value	26.14485
2-year shift	26.14582
5-year shift	26.14885
10-year shift	26.14885
30-year shift	26.02192

The key rate '01 with respect to the 10-year shift is closest to:

- A. -0.004
- B. -0.04
- C. -4
- D. -0.4

The correct answer is **A**.

$$\text{Key rate '01} = -\frac{\Delta \text{BV}}{10,000 \times \Delta y}$$

Where:

ΔBV = change in bond value

Δy = change in yield (0.01%)

The change in bond value here is measured in reference to the initial bond value.

$$= -\frac{26.14885 - 26.14485}{10,000 \times 0.01\%} = -0.004$$

Q.3432 The following table provides the initial price of a C-STRIP and its present value after the application of a one basis point shift in four key rates.

	Value
Initial value	26.11485
2-year shift	26.11582
5-year shift	26.11885
10-year shift	26.13885
30-year shift	26.01192

The key rate duration with respect to the 30-year shift is closest to:

- A. 39
- B. 51
- C. 38
- D. 10

The correct answer is **A**.

First, determine the corresponding key rate '01:

$$\text{Key rate '01} = -\frac{\Delta BV}{10,000 \times \Delta y}$$

Where:

ΔBV = change in bond value

Δy = change in yield (0.01%)

The change in bond value here is measured in reference to the initial bond value.

$$= -\frac{26.01192 - 26.11485}{10,000 \times 0.01\%} = 0.1029$$

Next, you can now compute the key rate duration using the formula:

$$DV01 = \text{Duration} \times 0.0001 \times \text{Bond value}$$

Thus,

$$\begin{aligned} \text{Duration} &= \frac{DV01}{0.0001 \times \text{Bond value}} \\ &= \frac{0.1029}{0.0001 \times 26.11485} \\ &= 39.41 \end{aligned}$$

Alternatively, recall that:

$$\text{Key rate duration} = -\frac{1}{P} \left(\frac{\partial P}{\partial y} \right)$$

Thus, the key rate duration with respect to the 30-year shift can be calculated as:

$$\text{Key rate duration} = -\left(\frac{1}{26.11485} \right) \times \left(\frac{26.01192 - 26.11485}{0.01\%} \right) = 39.41$$

Reading 58: Binomial Trees

Q.1205 Australian Financial Associates is holding the non-dividend paying stock of Neevan Holdings which is trading at USD 10. The continuously compounded risk-free rate is 5 percent per annum, and the annual standard deviation of the stock is 20 percent. What is the value of a 2-year European call option with a strike price of USD 10 using a two-period binomial model?

- A. USD 1.64
- B. USD 1.48
- C. USD 1.58
- D. USD 1.69

The correct answer is **B**.

To price options in the binomial model, we need:

U = size of the up move factor $= e^{\sigma\sqrt{\Delta t}}$

D = size of the down move factor $= e^{-\sigma\sqrt{\Delta t}} = \frac{1}{U}$

σ is the annual volatility of the underlying asset's returns and t is the length of the step in the binomial model ($t = 1$ in this case).

π_u = probability of an up move $= \frac{e^{r\Delta t} - D}{U - D}$

π_d = probability of a down move $= 1 - \pi_u$

Working with the data provided,

$U = 1.2214$ and $D = 0.8187$

$\pi_u = 0.5775$ and $\pi_d = 0.4225$

Let S represent the price of the stock and f represent the value of the call:

Stock Price	Value of the Call
$S_{uu} = \$10 * 1.2214 * 1.2214 = \14.92	$f_{uu} = \max(\$14.92 - \$10, \$0) = \4.92
$S_{ud} = \$10 * 1.2214 * 0.8187 = \10	$f_{ud} = \max(\$10 - \$10, \$0) = \0
$S_{du} = \$10 * 0.8187 * 1.2214 = \10	$f_{du} = \max(\$10 - \$10, \$0) = \0
$S_{dd} = \$10 * 0.8187 * 0.8187 = \6.70	$f_{dd} = \max(\$6.70 - \$10, \$0) = \0

As the table shows, the value of the call will be positive only when the stock moves upward twice.

The expected value of the call 2 years from now is given by:

$$\begin{aligned}
 &0.5775 \times 0.5775 \times \$4.92 + 0.5775 \times 0.4225 \times \$0 \\
 &+ 0.4225 \times 0.5775 \times \$0 + 0.4225 \times 0.4225 \times \$0 \\
 &= \$1.64
 \end{aligned}$$

$$\text{Value of the option today} = \frac{1.64}{e^{0.05 \times 2}} = \$1.48$$

Q.1207 Chris Fleming, an analyst working at Redberg Financials, constructs binomial trees to price options. With regard to binomial trees for pricing options, which of the following statement(s) is/are true?

- I. The underlying assumption in constructing a binomial tree is that the stock price follows a random walk
- II. In the limit, as the time step increases, the binomial tree model valuation of a European option converges to the Black-Scholes-Merton model valuation
- III. An inspection of a typical binomial tree shows that Delta remains constant during the life of an option
- IV. Constructing binomial trees for valuing options on stock indices, currencies, and futures contracts is very similar to doing so for valuing options on stocks

- A. I, III & IV only
- B. I, II & IV only
- C. II, III & IV only
- D. All of the above

The correct answer is **B**.

Statement I is true: The underlying assumption in constructing a binomial tree is that the stock price follows a random walk.

Statement II is true: In the limit, as the time step increases, the binomial tree model valuation of a European option converges to the Black-Scholes-Merton model valuation

Statement III is not true: An inspection of a typical binomial tree shows that delta **CHANGES** during the life of an option.

Statement IV is true: Constructing binomial trees for valuing options on stock indices, currencies, and futures contracts is very similar to doing so for valuing options on stocks.

Q.1210 David Yung, an analyst working at the New Zealand Bank, uses Girsanov's Theorem to study portfolios in a risk-neutral world and in the real world. The Girsanov's Theorem states that:

- A. When we move from the risk-neutral world to the real world, the expected return from the stock price changes, but its volatility remains the same.
- B. When we move from the risk-neutral world to the real world, the expected return from the stock price remains the same, but its volatility changes.
- C. When we move from the risk-neutral world to the real world, both the expected return from the stock price and its volatility remains the same.
- D. When we move from the risk-neutral world to the real world, both the expected return from the stock price and its volatility change.

The correct answer is **A**.

Girsanov's Theorem states that when we move from a risk-neutral world to the real world, the expected return from the stock price changes, but its volatility remains the same. When we move from a world with one set of risk preferences to a world with another set of risk preferences, the expected growth rates in variable change, but their volatilities remain the same.

Q.1211 Australian Financial Associates is holding the non-dividend paying stock of Neevan holdings which is trading at USD 10. The continuously compounded risk-free rate is 5 percent per year, and the annual standard deviation of the stock is 20%. What is the value of a 2-year European put option with a strike price of USD 10 using a two-period binomial model?

- A. USD 0.5323
- B. USD 2.4356
- C. USD 0.6884
- D. USD 2.3456

The correct answer is **A**.

Year 0	Year 1	Year 2
		$S_{uu} = \$12.214 \times 1.2214 = \14.918
	$S_u = \$10 \times 1.2214 = \12.214	$S_{ud} = \$12.214 \times 0.8187 = \10
$S_0 = \$10$	$S_d = \$10 \times 0.8187 = \8.187	$S_{dd} = \$8.187 \times 0.8187 = \6.703

The option will have a positive value only when the price moves downward twice.

Expected put value of the option in two years = $(0.5775 * 0.5775 * \text{USD } 0) + (0.5775 * 0.4224 * 0) + (0.4224 * 0.5775 * 0) + (0.4224 * 0.4224 * 3.297) = \text{USD } 0.5883$
Value of the option today = $0.5883 * e^{-0.05 \times 2} = 0.5323$

Q.1212 Which of the following statement(s) is/are true with regard to Delta?

- I. Delta is an important parameter in the pricing and hedging of options
- II. Delta is the number of units of the stock we should hold for each option shorted in order to create a riskless portfolio
- III. The construction of a riskless portfolio is sometimes referred to as delta hedging
- IV. The delta of a call option is negative, whereas the delta of a put option is positive

- A. I, II & III only
- B. I, II & IV only
- C. II, III & IV only
- D. All of the above

The correct answer is **A**.

Delta is an important parameter in the pricing and hedging of options. The delta of a stock option is the ratio of the change in the price of the stock option to the change in the price. It is the number of units of the stock we should hold for each option shorted in order to create a riskless portfolio. The construction of a riskless portfolio is sometimes referred to as delta hedging. The delta of a call option is positive, whereas the delta of a put option is negative.

Q.1213 Rose Associates is holding stocks of Xerox limited. The current value of the stock is USD 100 and the current continuously compounded risk-free rate is 3 percent. The stock pays a dividend at a continuous dividend yield of 2 percent. The annual standard deviation of the stock is 9 percent. What is the risk-neutral probability of an up-move and down-move for a 1-year European call option on the stock?

- A. 0.68 and 0.32
- B. 0.78 and 0.22
- C. 1.09 and 0.91
- D. 0.53 and 0.47

The correct answer is **D**.

$$U = \text{size of the up-move factor} = e^{\sigma\sqrt{t}} = e^{0.09\sqrt{1}} = 1.094$$

$$D = \text{size of the down-move factor} = \frac{1}{U} = 0.9139$$

The risk-neutral probabilities of upward and downward movements:

$$\begin{aligned}\pi_u &= \text{Risk-neutral probability of an up-move} \\ &= \frac{(e^{(r-q)t} - D)}{(U - D)} \\ &= \frac{(e^{(0.03-0.02)*1} - 0.9139)}{(1.094 - 0.9139)} = 0.53 \\ \pi_d &= \text{Risk-neutral probability of a down-move} \\ &= 1 - \pi_u = 1 - 0.53 \\ &= 0.47\end{aligned}$$

Where r is the risk-free rate and q is the dividend rate

Q.3400 Willy Smith, FRM, has a two-year European put with $K = \$41$. The current price of the underlying is $\$40$. Over the past year, the stock has exhibited a standard deviation of 20%. The risk-free rate of return is 5%. Compute the value of the put today using a two-step Binomial.

- A. \$2.71
- B. \$3
- C. \$0
- D. \$16.1

The correct answer is **A**.

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.2 \times \sqrt{1}} = 1.22$$

$$d = e^{-\sigma\sqrt{\Delta t}} = e^{-0.2 \times \sqrt{1}} = 0.82$$

$$\pi_u = \frac{e^{rt} - d}{u - d} = \frac{e^{0.05 \times 1} - 0.82}{1.22 - 0.82} = 0.5782,$$

$$\pi_d = 1 - 0.5782 = 0.4218$$

Let S represent the price of the stock and f represent the value of the put

Stock Price	Option Payoff
$S_{uu} = \$40 \times 1.22 \times 1.22 = \59.54	$f_{uu} = \max(\$41 - \$59.54, 0) = \$0$
$S_{ud} = \$40 \times 1.22 \times 0.82 = \40	$f_{ud} = \max(\$41 - \$40, 0) = \$1$
$S_{du} = \$40 \times 0.82 \times 1.22 = \40	$f_{du} = \max(\$41 - \$40, 0) = \$1$
$S_{dd} = \$40 \times 0.82 \times 0.82 = \26.90	$f_{dd} = \max(\$41 - \$26.90, 0) = \$14.10$

The expected value of the put 2 years from now is given by:

$$\begin{aligned} &0.5782 \times 0.5782 \times \$0 + 0.5782 \times 0.4218 \times \$1 \\ &+ 0.4218 \times 0.5782 \times \$1 + 0.4218 \times 0.4218 \times \$14.10 \\ &= \$3 \end{aligned}$$

$$\text{Value of the put today} = \frac{\$3}{e^{0.05 \times 2}} = \$2.71$$

Q.3402 XY Z stock is a non-dividend-paying stock currently priced at $\$108$. According to analysis, the annual standard deviation of returns on XY Z stock is 8% and the risk-free rate on interest,

compounded continuously, is 5.5%. Using a two-period binomial model, compute the value of a 6-month American call option on XYZ stock with a strike price of \$110.

- A. \$2.43
- B. \$7.04
- C. \$2.98
- D. \$4.58

The correct answer is **C**.

$$\text{up move factor} = u = e^{\sigma\sqrt{t}} = e^{0.08 \times \sqrt{0.25}} = 1.041$$

$$\text{down move factor} = d = \frac{1}{1.041} = 0.961$$

$$\text{probability of an up move} = \pi_u = \frac{e^{0.055 \times 0.25} - 0.961}{1.041 - 0.961} = 0.66$$

$$\text{probability of a down move} = 1 - \pi_u = 0.34$$

Let S represent the price of the stock and f represent the value of the call

Stock Price	Option Payoff
$S_u = \$108 \times 1.041 = \112.43	$f_u = \max(112.43 - 110, 0) = 2.43$
$S_d = \$108 \times 0.961 = \103.79	$f_d = \max(103.79 - 110, 0) = 0$
$S_{uu} = \$108 \times 1.041^2 = \117.04	$f_{uu} = \max(117.04 - 110, 0) = 7.04$
$S_{ud} = \$108 \times 1.041 \times 0.961 = \108.04	$f_{ud} = \max(108.04 - 110, 0) = 0$
$S_{du} = \$108 \times 0.961 \times 1.041 = \108.04	$f_{du} = \max(108.04 - 110, 0) = 0$
$S_{dd} = \$108 \times 0.961^2 = \99.74	$f_{dd} = \max(99.74 - 110, 0) = 0$

The \$110 call option is in the money when the stock price finishes at \$117.04 at which time the call has a value of \$7.04. At the end of 3 months(3-month node), the expected payoff on the option in the next 3 months, given an up move up to that point, is:

$$\frac{\$7.04 \times 0.66 + \$0 \times 0.34}{e^{0.055 \times 0.25}} = \$4.58$$

In this case, if the holder of the option chose to exercise early, they'd receive a maximum of $\max(112.43 - 110, 0) = 2.43$. Since $\$2.43 < \4.58 , it would not be optimal to exercise the option early. At the end of 3 months(3-month node), the expected payoff on the option in the next 3 months, given a down move up to that point, is:

$$\frac{\$0 \times 0.66 + \$0 \times 0.34}{e^{0.055 \times 0.25}} = \$0$$

If the holder of the option chose to exercise early (3 months following a down move), they'd receive a maximum of $\max(\$103.79 - 110, 0) = \0 . Again, it would not be optimal to exercise the option early. The value of the option today is:

$$\frac{\$4.58 \times 0.66 + \$0 \times 0.34}{e^{0.055 \times 0.25}} = \$2.98$$

Q.3403 You have been provided the following information for a call option on the stock of VeloMedia:

- Current stock price = \$100
- Strike price = \$100
- Time to maturity = 1 year
- Exponential compounding interest rate = 10%
- Annual standard deviation = 30%

What is the value of a European call option using a two-period binomial tree with two distinct intervals of 6 months?

- A. \$15.38
- B. \$10.21
- C. \$0
- D. \$5.86

The correct answer is **A**.

Binomial parameters:

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.30\sqrt{0.5}} = 1.2363, \quad d = \frac{1}{u} = 0.8089$$

Risk-neutral probability:

$$p = \frac{e^{r \times \Delta t} - d}{u - d} = \frac{e^{0.10 \times 0.5} - 0.8089}{1.2363 - 0.8089} = 0.5671, \quad 1 - p = 0.4329$$

$$X = \$100$$

$$C_{uu} = 52.85, \quad C_{du} = C_{ud} = C_{dd} = 0$$

$$C_u = \frac{p \times C_{uu} + (1 - p) \times C_{ud}}{e^{r \times \Delta t}} = \frac{0.5671 \times 52.85 + (1 - 0.4329) \times 0}{e^{0.10 \times 0.5}} = 28.51, \quad C_d = 0$$

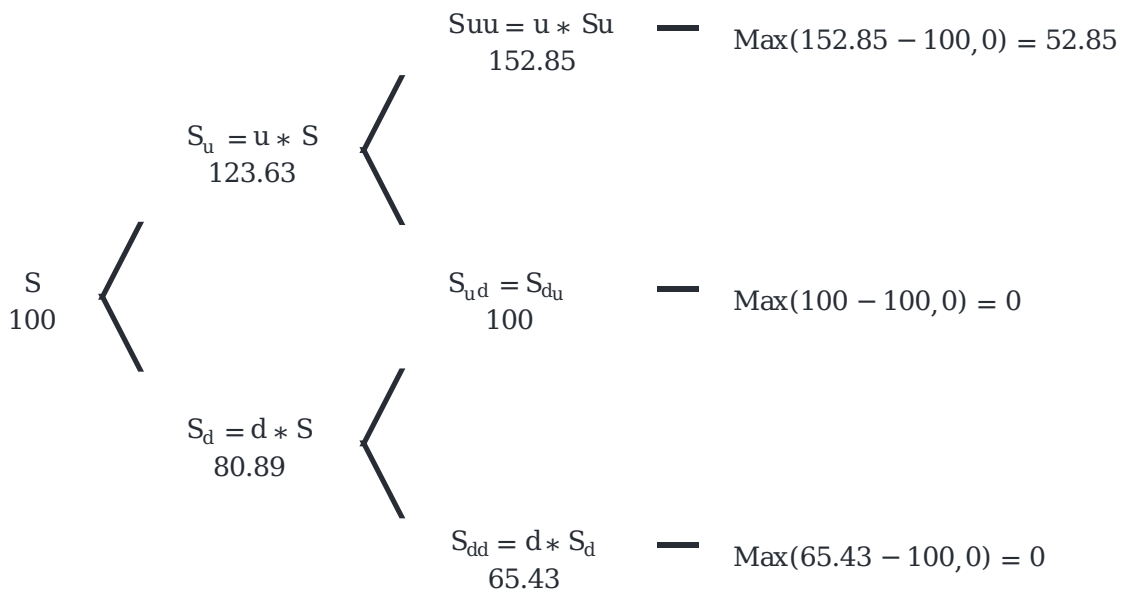
$$C = \frac{0.5671 \times 28.51 + (1 - 0.4329) \times 0}{e^{0.10 \times 0.5}} = 15.38$$

$$u = 1.2363$$

$$d = 0.8089$$

$$p = 0.5671$$

$$1 - p = 0.4329$$



Q.3404 You have been provided the following information for a European call option on the stock of VeloMedia:

- Current stock price = \$100
- Strike price = \$100

- Time to maturity = 1 year
- Exponential compounding interest rate = 10%
- Annual standard deviation = 30%

What is the call option delta at the current date? Use a two-period binomial tree with two distinct intervals of 6 months.

- A. 0
- B. 0.6669
- C. 1
- D. 0.5

The correct answer is **B**.

Binomial parameters:

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.30\sqrt{0.5}} = 1.2363, \quad d = \frac{1}{u} = 0.8089$$

Risk-neutral probability:

$$p = \frac{e^{r \times \Delta t} - d}{u - d} = \frac{e^{0.10 \times 0.5} - 0.8089}{1.2363 - 0.8089} = 0.5671, \quad 1 - p = 0.4329$$

$$X = \$100$$

$$C_{uu} = 52.85, \quad C_{du} = C_{ud} = C_{dd} = 0$$

$$C_u = \frac{p \times C_{uu} + (1 - p) \times C_{ud}}{e^{r \times \Delta t}} = \frac{0.5671 \times 52.85 + (1 - 0.4329) \times 0}{e^{0.10 \times 0.5}} = 28.51, \quad C_d = 0$$

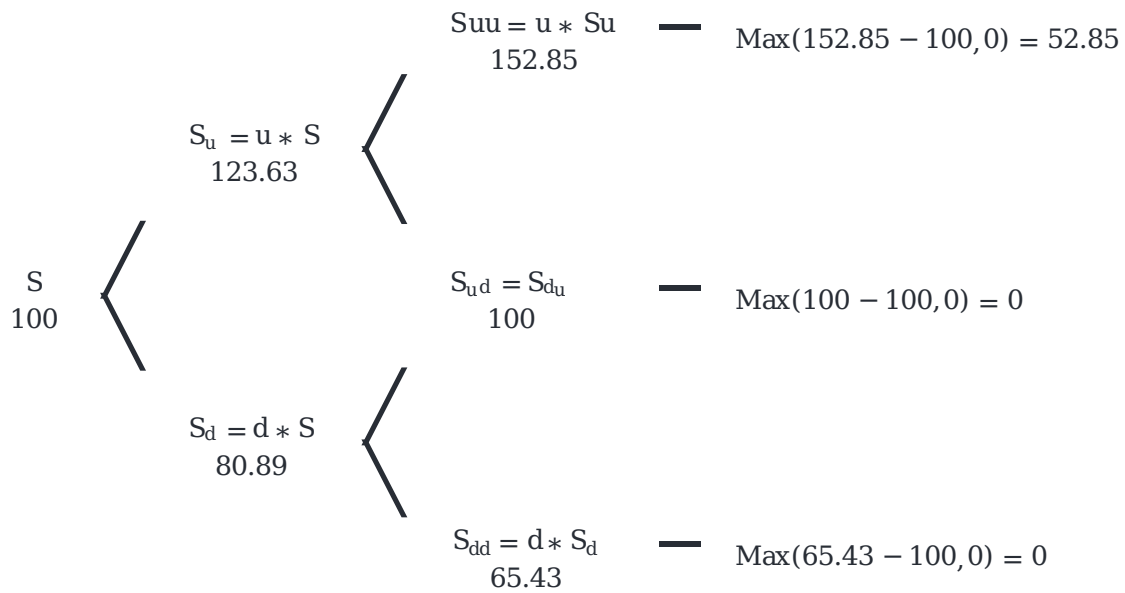
$$C = \frac{0.5671 \times 28.51 + (1 - 0.4329) \times 0}{e^{0.10 \times 0.5}} = 15.38$$

$$u = 1.2363$$

$$d = 0.8089$$

$$p = 0.5671$$

$$1 - p = 0.4329$$



Now, the call option delta at the current date is:

$$\begin{aligned}
 \Delta &= \frac{\Delta C}{\Delta S} = \frac{C_u - C_d}{S_u - S_d} \\
 &= \frac{28.51 - 0}{123.63 - 80.89} = 0.6669
 \end{aligned}$$

Q.3405 You have been provided the following information for a put option on the stock of VeloMedia:

- Current stock price = \$100
- Strike price = \$100
- Time to maturity = 1 year
- Exponential compounding interest rate = 10%
- Annual standard deviation = 30%

Compute the value of the European put option using a two-period binomial tree with two distinct intervals of 6 months.

- A. \$15.38
- B. \$5.86
- C. \$0
- D. \$10

The correct answer is **B**.

$$p_{uu} = p_{du} = p_{ud} = 0, \quad p_{dd} = \max(X - S_{uu}, 0) = \max(100 - 65.43, 0) = 34.57$$

$$P = PV[p^2 p_{uu} + 2p(1-p)p_{ud} + (1-p)^2 p_{dd}] = \frac{p^2 p_{uu} + 2p(1-p)p_{ud} + (1-p)^2 p_{dd}}{R^2}$$

$$P = \frac{0.5671^2 \times 0 + 2 \times 0.5671 \times (1 - 0.5671) \times 0 + (1 - 0.5671)^2 \times 34.87}{e^{0.10 \times 0.5 \times 2}}$$

$$P = \frac{(0.4329)^2 \times 34.57}{e^{0.10}} = 5.86$$

Note: we would arrive at the same answer if we compute the value of the corresponding call (15.38 in this case) and then use the put-call parity to find the value of the put.

$$P + S = C + PV(X)$$

$$P = 15.38 + 100 \times e^{-0.10 \times 1} - 100 = 5.86$$

Q.3406 A call option has a delta of 0.65. What is the put option delta?

- A. -0.65
- B. -0.35
- C. 0.35
- D. None

The correct answer is **B**.

Call option delta = Put option delta + 1

Put option delta = 0.65 - 1 = -0.35

The range of call option delta is always 0 to 1.

The range of put option delta is always -1 to 0.

Q.4697 The current price of a stock is \$40. Its volatility is 10% per annum and the risk-free rate is 5% per annum with continuous compounding. Using a two-step binomial, what is the value of a six-month European call option on the stock with a strike price of \$40?

- A. 1.54
- B. 2.0
- C. 1.58
- D. 1.83

The correct answer is **A**.

In this case,

$$\begin{aligned}u &= e^{\sigma\sqrt{t}} = e^{0.10\sqrt{0.25}} = 1.0513 \\d &= \frac{1}{1.0513} = 0.9512 \\p &= \frac{e^{rt} - d}{u - d} = \frac{e^{0.05 \times 0.25} - 0.9512}{1.0513 - 0.9512} = 0.6132 \\1 - p &= 1 - 0.6132 = 0.3868\end{aligned}$$

Where:

u = size of the up-move factor

d = size of the down move factor

σ = volatility of the stock

p = probability of an up move

$1 - p$ = probability of a down move

Using a two-step binomial,

$$S_{uu} = 44.2093$$

$$S_{ud} = 40$$

$$S_{du} = 40$$

$$S_{dd} = 36.1913$$

Notice that the only time the option is in the money is when two upward price movements lead to an ending price of \$44.2093 and a call value of \$4.2093. The expected value of the option at the end of the second period is the value of the option in each state multiplied by the probability of that state occurring:

$$\text{Expected call value in two periods} = 0.6132 * 0.6132 * 4.2093 = \$1.5828$$

The value of the call option today is the expected value in six months discounted at the risk-free rate of 5%:

$$\text{Call} = \frac{\$1.5828}{e^{0.05(0.5)}} = \$1.5437$$

Q.4698 A 6-month stock currently trading at \$40 pays a continuous dividend of 2%, and the current continuously compounded risk-free rate is 3%. Assuming an annual standard deviation of 3%, and a strike price of 40, what is the value of the put today?

A. 0.32

B. 0.28

C. 0.51

D. 0.52

The correct answer is **A**.

In this case,

$$u = e^{\sigma\sqrt{t}} = e^{0.03\sqrt{0.5}} = 1.02144$$
$$d = \frac{1}{1.02144} = 0.97901$$
$$p = \frac{(e^{(r-q)t}) - d}{u - d}$$

Where:

u= size of the up-move factor

d= size of the down move factor

σ = volatility of the stock

p= probability of an up move

1 – p= probability of a down move

$$p = \frac{e^{(0.03-0.02)0.5} - 0.97901}{1.02144 - 0.97901} = 0.613$$

$$\text{Probability of a down move} = 1 - 0.613 = 0.387$$

Let S represent the stock price, and f represent the value of the put option.

$$S_u = 40.84$$
$$S_d = 39.16$$

The payoffs at the final node are:

$$f_u = \max(40 - 40.84, 0) = 0$$
$$f_d = \max(40 - 39.16, 0) = 0.84$$

The value of the put today is given by:

$$\begin{aligned}f &= (f_u p + f_d (1 - p)) e^{-rt} \\&= (0 \times 0.613 + 0.8396 \times 0.387) e^{-0.03 \times 0.5} = 0.32008\end{aligned}$$

Reading 59: The Black-Scholes-Merton Model

Q.984 Antony Meech, a research analyst working at FinSearch Inc., is preparing a note on lognormal distributions and normal distributions. He notes down the following points on lognormal distribution:

- I. The model of stock price behavior used by Black, Scholes, and Merton assumes that percentage changes in the stock price in a very short period of time are normally distributed
- II. A variable that has a lognormal distribution can take any value between zero and infinity
- III. Like the normal distribution, the mean, median, and mode are all the same in the lognormal distribution

Which of them are correct?

- A. I & II only
- B. II & III only
- C. I, II & III only
- D. I & III only

The correct answer is **A**.

The model of stock price behavior used by Black, Scholes, and Merton assumes that percentage changes in the stock price in a very short period of time are normally distributed. A variable that has a lognormal distribution can take any value between zero and infinity. Unlike the normal distribution, it is skewed so that the mean, median, and mode are all different in the lognormal distribution.

Note that, When the return on a stock over a short period is normally distributed, the stock price at the end of a relatively long period has a lognormal distribution. i.e

- i. stock price; over a short period is normally distributed.
 - ii. stock price; over a relatively long period is lognormally distributed.
-

Q.985 Ricky Gervais, a retired veteran, is holding shares of TMT Limited which are currently trading at USD 100. The volatility of the share is 25 percent per year, and the expected return on the stock is 10 percent for the same period. What is the expected stock price in one year?

A. USD 110.517

B. USD 128.403

C. USD 102.532

D. USD 101.432

The correct answer is **A**.

The expected stock price in one year is given by:

$$E(S_T) = S_0 e^{\mu T} = 100 e^{(.10)} = \text{USD } 110.517$$

Where

μ = expected rate of return.

Q.986 Mike Finova is holding shares of TMT Limited which are currently trading at USD 100. The volatility of the share is 25 percent per year, and the expected return on the stock is 10 percent for the same period. What is the standard deviation of the stock in one year?

- A. USD 787.68
- B. USD 28.07
- C. USD 14
- D. USD 100

The correct answer is **B**.

$$\begin{aligned}\text{The variance in one year} &= \text{Var}(S_T) = S_0^2 e^{(2\mu T)} (e^{\sigma^2 T} - 1) \\ &= 100^2 e^{(2 \cdot 0.1 \cdot 1)} (e^{0.25^2} - 1) \\ &= 10000 * 1.2214 * 0.06449 = 787.68\end{aligned}$$

where

μ = Expected rate of return and,

σ = volatility

Standard deviation of the stock price in 1 year = $\sqrt{787.68} = 28.07$

Q.988 The manager at American Derivatives Limited Hedge Fund proposes to use the Black-Scholes-Merton differential equation to understand the pricing of derivatives dependent on non-dividend paying stocks. Which of the following assumptions with respect to the Black-Scholes-Merton model must be made to get accurate results?

- I. The short-selling of securities is not permitted
- II. There are no riskless arbitrage opportunities
- III. The risk-free rate of interest is known and constant
- IV. Security trading is continuous

- A. I, II & III only
- B. I, III & IV only
- C. II, III & IV only
- D. All of the above

The correct answer is **C**.

The assumptions underlying the Black-Scholes-Merton differential equation are:

1. The stock price follows the process with expected return and standard deviation constant
 2. The short selling of securities with full use of proceeds is permitted
 3. There are no transaction costs or taxes. All securities are perfectly divisible
 4. There are no dividends during the life of the derivative
 5. There are no riskless arbitrage opportunities
 6. Security trading is continuous
 7. The risk-free rate of interest, r , is known and constant
-

Q.989 With regard to the Black-Scholes-Merton Model, which of the following statements are true?

- I. The Black-Scholes-Merton differential equation does not involve any variables that are affected by the risk preferences of investors
- II. The current stock price, time, stock price volatility, and the risk-free rate of interest are involved
- III. The Black-Scholes-Merton differential equation involves the expected return on the stock and, therefore, is dependent of risk preferences
- IV. The Black-Scholes-Merton differential equation is an equation that must be satisfied by the price of any derivative dependent on a non-dividend paying stock

- A. I, II & IV only
- B. I, III & IV only
- C. II & III only
- D. All of the above

The correct answer is **A**.

The Black-Scholes-Merton differential equation is an equation that must be satisfied by the price of any derivative dependent on a non-dividend paying stock. It does not involve any variables that are affected by the risk preferences of investors. The current stock price, time, stock price volatility, and the risk-free rate of interest are involved, and all these are independent of risk preferences.

Point III is incorrect. The Black-Scholes-Merton differential equation is independent of risk preferences if it involved the expected return, μ , on the stock.

Q.990 Bret Lee, a research student, studying at McJohn University, analyzes academic material on regular options, employee stock options, and warrants. After analyzing the data, he prepares a brief to present to his professor. He jots down the following points in the brief:

Statement I: The exercise of a regular call option has no effect on the number of the company's shares outstanding

Statement II: The exercise of warrants leads to the company issuing more shares and selling them to the holder of the warrant at the strike price

Statement III: Exercise of warrants tend to dilute the interest of the existing shareholders as the strike price is usually less than the market price

Statement IV: Exercise of warrants and employee stock options does not have any effect on the number of company's shares outstanding

Which of these statements are accurate?

- A. I, II & III only
- B. I, III & IV only
- C. II, III & IV only
- D. All of the above

The correct answer is **A**.

Statements I, II & III are correct. The exercise of a regular call option has no effect on the number of the company's shares outstanding. If the writer of the option does not own the company's shares, he or she must buy them in the market in the usual way and then sell them to the option holder for the strike price. The exercise of warrants of employee stock options leads to the company issuing more shares and selling them to the holder of the warrant at the strike price.

Statement IV is incorrect. Exercise of warrants and employee stock options tend to dilute the interest of the existing shareholders as the strike price is usually less than the market price.

Q.992 Steyn Associates used implied volatilities in pricing securities instead of historical volatilities. With regard to volatilities, which of the following statement(s) is/are true?

- I. Implied volatilities are the volatilities implied by option prices observed in the market
- II. Historical volatilities are backward-looking, whereas implied volatilities are forward-looking
- III. Traders often quote the implied volatility of an option rather than its price. This is convenient because the implied volatility tends to be less variable than the option price
- IV. The implied volatilities of actively traded options are used by traders to estimate appropriate implied volatilities for other options

- A. I, II & III only
- B. I, III & IV only
- C. II, III & IV only
- D. All of the above

The correct answer is **D**.

Implied volatilities are the volatilities implied by option prices observed in the market. Implied volatilities are used to monitor the market's opinion about the volatility of a particular stock. Historical volatilities are backward-looking, whereas implied volatilities are forward-looking. Traders often quote the implied volatility of an option rather than its price. This is convenient because the implied volatility tends to be less variable than the option price. The implied volatilities of actively traded options are used by traders to estimate appropriate implied volatilities for other options.

Q.993 Stephen Hawking, a trader working at Orange Securities, collects the following data of a 1-year European put and call options on the stock Mango Apparel. The current stock price is USD 120, and the strike price of the option is USD 125. The risk-free rate is 10 percent. If the prices of a European call and European put are USD 10 and USD 15, respectively, then what is the implied dividend yield of the stock Mango Apparel?

- A. 0.11439
- B. 0.045323
- C. 0.10439
- D. 0.055323

The correct answer is **C**.

Put-call parity:

$$\text{Call option} - \text{Put option} = S_0 e^{-qxT} - Ke^{-rxT}$$

Where

S_0 = Initial stock price;

q = implied dividend yield;

r = risk-free rate; and

T = time

$$\begin{aligned} 10 - 15 &= 120e^{-qx1} - 125e^{-0.10 \times 1} \\ e^{-q} &= 0.9009 \\ \ln(0.9009) &= -q \end{aligned}$$

$$\text{Implied dividend yield (q)} = 10.439\%$$

Q.994 Chinese International Bank is analyzing the stock of Jatoka International. One-year European call and put options are written on the stock of Jatoka International which is a non-dividend paying stock. The initial stock price is Yuan 100 and the risk-free rate is 5%. The time to maturity is 1 year, and the strike price is Yuan 125. Furthermore, $N(d_1) = 0.6925$, $N(d_2) = 0.5435$. What are the values of European put and call options (approx.) using the Black-Scholes differential equation?

- A. Call option value is USD 4.626 and Put option value is USD 23.52.
- B. Call option value is USD 23.52 and put option value is USD 4.626.
- C. Call option value is USD 2.626 and Put option value is USD 13.52.
- D. Call option value is USD 13.52 and put option value is USD 2.626.

The correct answer is **A**.

$$\begin{aligned}
 \text{Call value} &= S_0 N(d_1) - X e^{-R_f \times T} N(d_2) \\
 &= [100 \times 0.6925] - \{125 e^{(-0.05)} \times 0.5435\} \\
 &= \text{USD } 4.626 \\
 \text{Put value} &= [X e^{-R_f \times T} \times (1 - N(d_2))] - [S_0 \times (1 - N(d_1))] \\
 &= [125 e^{-0.05} \times (1 - 0.5435)] - [100 \times (1 - 0.6925)] \\
 &= \text{USD } 23.52
 \end{aligned}$$

Q.995 Raheja Financials is holding the stock of Duckworth Limited which is trading at USD 50. A European call option that expires in 3 months with a strike price of USD 51 is available for trading. The annualized standard deviation is 20 percent, and the risk-free rate of interest is 4 percent. What is the value of the European call option using the Black-Scholes-Merton model expiring in 3 months if $N(d_1) = 0.48085$ and $N(d_2) = 0.44116$?

- A. USD 2.422
- B. USD 2.224
- C. USD 1.767
- D. USD 1.259

The correct answer is **C**.

Value of European call option as per the Black-Scholes-Merton Model:

$$\begin{aligned}\text{Call value} &= S_0 N(d_1) - X e^{-R_f T} N(d_2) \\ &= [50 \times 0.48085] - \{51 e^{(-0.04 \times 0.25)} \times 0.44116\} \\ &= \text{USD } 1.767\end{aligned}$$

Q.996 Michael Wong is holding the stock of Duckworth limited which is trading at USD 50. A European put option that expires in 3 months with a strike price of USD 51 is available for trading. The annualized standard deviation is 20 percent, and the continuously compounded risk-free rate is 4 percent. What is the value of the European put option using the Black-Scholes-Merton model expiring in 3 months if $N(d_1) = 0.48085$ and $N(d_2) = 0.44116$?

- A. USD 1.77
- B. USD 1.26
- C. USD 2.19
- D. USD 2.26

The correct answer is **D**.

The value of European put option as per the Black-Scholes-Merton Model is calculated as follows:

$$\begin{aligned}\text{Put value} &= [Xe^{-R_f \cdot T} * (1 - N(d_2))] - [S_0 * (1 - N(d_1))] \\ &= [51e^{-0.04 * 0.25} * (1 - 0.44116)] - [50 * (1 - 0.48085)] \\ &= \text{USD } 2.26\end{aligned}$$

Q.3407 A stock price has an expected return of 10% and a volatility of 30%. The current price is \$30. What is the probability that a European call option on the stock with an exercise price of \$32 and a maturity date in six months will be exercised?

- A. 0.5032
- B. 0.247
- C. 0.4305
- D. 0.008

The correct answer is C.

The required probability is the probability of the stock price being above \$32 in six months' time since a call option is only exercised if the stock price is higher than the strike price. Suppose that the stock price in six months is S_T . The probability distribution of $\ln S_T$ is

$$\ln S_T \sim N \left(\ln 30 + (0.10 - \frac{0.3^2}{2})0.5, 0.3^2 \times 0.5 \right) \sim N(3.429, 0.212^2)$$

The required probability is given by:

$$P(\ln S_T > \ln 32) = 1 - P(\ln S_T \leq \ln 32) = 1 - N\left(\frac{3.466 - 3.429}{0.212}\right) = 1 - N(0.175)$$

Note that:

$$\ln 32 = 3.466$$

From the standard normal distribution tables, $N(0.175) = 0.5695$

Therefore, the required probability is 0.4305

Q.3408 What is the price of a European call option on a non-dividend-paying stock when the stock price is \$68, the strike price is \$65, the risk-free interest rate is 16% per annum, the volatility is 39% per annum, and the time to maturity is three months?

- A. 5.35
- B. 4.85
- C. 8.31
- D. 0.536

The correct answer is **C**.

Recall that the price of European call option is given by:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

Where:

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

In this case,

$$S_0 = 68$$

$$K = 65$$

$$r = 0.16$$

$$\sigma = 0.39 \text{ and,}$$

$$T = 0.25$$

$$d_1 = \frac{(\ln\frac{68}{65} + (0.16 + \frac{0.39^2}{2})0.25)}{0.39\sqrt{0.25}} = \frac{0.1041}{0.195} = 0.5338$$

$$d_2 = d_1 - 0.39\sqrt{0.25} = 0.3388$$

The price of the European call is

$$c = 68 \times N(0.5338) - 65e^{-0.16 \times 0.25} N(0.3388)$$

$$= 68 \times 0.7032 - 65 \times 0.9608 \times 0.6326$$

$$= 8.31$$

Q.3409 Consider a European call option when the stock price is \$20, the exercise price is \$22, the time to maturity is six months, the volatility is 20% per annum, and the risk-free interest rate is 15% per annum. Two equal dividends of \$1 are expected during the life of the option, with ex-dividend dates at the end of two months and five months. What is the value of the option?

- A. \$0.30
- B. \$0.40
- C. \$0.25
- D. \$0.26

The correct answer is **A**.

Note that S_0 is reduced to S by the present value of the dividends payable, but all other variables remain the same

$$S = S_0 - PV$$

Where

$$PV = D_1 e^{-(r)\frac{\Delta t_1}{m}} + D_2 e^{-(r)\frac{\Delta t_2}{m}} = 1e^{-(0.15)\frac{2}{12}} + 1e^{-(0.15)\frac{5}{12}} = 1.9147$$

$$S = \$20 - \$1.9147 = \$18.09$$

$$K = 22,$$

$$T = 0.5,$$

$$\sigma = 0.2$$

$$\begin{aligned} d_1 &= \frac{\ln \frac{S}{K} + [R_f^C + (0.5 \times \sigma^2)] T}{\sigma \sqrt{T}} \\ &= \frac{\ln \frac{18.09}{22} + [0.15 + (0.5 \times 0.2^2)] 0.5}{0.2 \sqrt{0.5}} = -0.7826 \\ d_2 &= d_1 - (\sigma \sqrt{T}) = -0.7826 - 0.2 \sqrt{0.5} = -0.9241 \end{aligned}$$

From statistical tables,

$$N(d_1) = N(-0.7826) = 1 - N(0.7826) = 1 - 0.7831 = 0.2169$$

And

$$\begin{aligned} N(d_2) &= N(-0.9241) = 1 - N(0.9241) = 1 - 0.8223 = 0.1777 \\ C_0 &= [S \times N(d_1)] - [K \times e^{-R_f^C \times T} \times N(d_2)] \\ &= 18.09 \times 0.2169 - 22e^{-0.15 \times 0.5} \times 0.1777 \\ &= 3.9237 - 3.6269 = 0.2968 \approx 0.30 \end{aligned}$$

Q.3410 Consider a European option on a non-dividend paying stock with the following characteristics:

- Current stock price = \$50
- Exercise price = 50
- Continuous compounding interest rate = 8%
- Standard deviation = 34%
- Time to expiration = 2 years

Calculate the price of the call option and its delta using the Black-Scholes-Merton model.

- A. Call = \$12.97; $\Delta = 0.7167$
- B. Call = \$5.57; $\Delta = 0.7167$
- C. Call = \$5.57; $\Delta = -0.2832$
- D. Call = \$12.97; $\Delta = -0.2832$

The correct answer is **A**.

The price of a call option on a non-dividend-paying stock:

$$C = S \times N(d_1) - PV(X) \times N(d_2)$$

$$d_1 = \frac{\ln\left[\frac{S}{PV(X)}\right] + \frac{\sigma\sqrt{T}}{2}}{\sigma\sqrt{T}} = \frac{\ln\left[\frac{S}{X}\right] + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\ln\left(\frac{50}{50}\right) + \left(0.08 + \frac{0.34^2}{2}\right)2}{0.34\sqrt{2}} = 0.5732$$

$$d_2 = 0.5732 - 0.34\sqrt{2} = 0.0923$$

From the standard normal table $N(d_1) = N(0.5732) = 0.7167$ and $N(d_2) = N(0.0923) = 0.5367$

$$\begin{aligned} \text{Value of the call} &= S \times N(d_1) - PV(X) \times N(d_2) \\ &= 50 \times 0.7167 - 50e^{-0.08 \times 2} \times 0.5367 \\ &= 12.9677 \end{aligned}$$

Using put-call parity relation,

$$\text{Value of the put} = C + PV(X) - S = 12.9677 + 50e^{-0.08 \times 2} - 50 = 5.5749$$

$$\text{Delta of call} = N(d_1) = 0.7167$$

$$\text{Delta of put} = -N(-d_1) = -N(-0.5732) = -(1 - 0.7167) = -0.2833$$

Where N is the normal distribution.

Q.3411 Consider a European option on a non-dividend paying stock with the following characteristics:

- Current stock price = \$50
- Exercise price = 50
- Continuous compounding interest rate = 8%
- Standard deviation = 34%
- Time to expiration = 2 years

Calculate the price of a put option and its delta using the Black-Scholes-Merton model.

A. Put = \$12.97; $\Delta = 0.7167$

B. Put = \$5.57; $\Delta = 0.7167$

C. Put = \$5.57; $\Delta = -0.2832$

D. Put = \$12.97; $\Delta = -0.2832$

The correct answer is C.

The price of a put option on a non-dividend-paying stock can be calculated as:

$$P + S = C + PV(X)$$

$$P = S[N(d_1) - 1] - PV(X)[N(d_2) - 1]$$

$$N(d) + N(-d) = 1$$

$$P = -S \times N(-d_1) + PV(X) \times N(-d_2)$$

$$d_1 = \frac{\ln\left[\frac{S}{X}\right] + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

The price of a call option on a non-dividend-paying stock:

$$C = S \times N(d_1) - PV(X) \times N(d_2)$$

$$d_1 = \frac{\ln\left[\frac{S}{PV(X)}\right] + \frac{\sigma\sqrt{T}}{2}}{\sigma\sqrt{T}} = \frac{\ln\left[\frac{S}{X}\right] + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\ln\left(\frac{50}{50}\right) + \left(0.08 + \frac{0.34^2}{2}\right)2}{0.34\sqrt{2}} = 0.5732$$

$$d_2 = 0.5732 - 0.34\sqrt{2} = 0.0923$$

From the standard normal table $N(d_1) = N(0.5732) = 0.7167$ and $N(d_2) = N(0.0923) = 0.5367$

$$\begin{aligned} \text{Value of the call} &= S \times N(d_1) - PV(X) \times N(d_2) \\ &= 50 \times 0.7167 - 50e^{-0.08 \times 2} \times 0.5367 \\ &= 12.9677 \end{aligned}$$

Using put-call parity relation,

$$\text{Value of the put} = C + PV(X) - S = 12.9677 + 50e^{-0.08 \times 2} - 50 = 5.5749$$

$$\text{Delta of call} = N(d_1) = 0.7167$$

$$\text{Delta of put} = -N(-d_1) = -N(-0.5732) = -(1 - 0.7167) = -0.2833$$

Where, N is the normal distribution.

Q.3412 Consider a company with N million shares outstanding, each worth S_0 , that is contemplating issuing M warrants. Each warrant would grant the holder the right to purchase one share with a strike price of X in a year. Assuming the value of a corresponding 1-year European call option is worth C, the cost of issuing the warrants would take which of the following forms?

A. $\frac{C}{N+M}$

B. $\frac{MC}{N+M}$

C. $\frac{MNC}{N+M}$

D. $\frac{NC}{N+M}$

The correct answer is C.

It can be shown the cost of issuing each warrant is given by:

$$\frac{N}{N+M} \times \text{Price of the Warrant}$$

Where

N = Number of existing shares

M = Number of warrants issued

Note that a warrant is defined as options issued by a company on its own stock. So, in this case, the cost of issuing M warrants is given by:

$$M \times \frac{N}{N+M} \times C = \frac{MNC}{N+M}$$

Q.3414 What is the effect of dividends on option prices?

A. Call option prices increase; Put option prices increase

B. Call option prices increase; Put option prices decrease

C. Call option prices decrease; Put option prices increase

D. Call option prices decrease; Put option prices decrease

The correct answer is C.

Because of a dividend, the net stock price decreases in value.

Effective $S = S - PV(\text{Dividend})$

When a stock price decreases, the call price decreases and the put price increases.

Q.4618 A stock has an initial price of \$50, an expected annual return of 20%, and annual volatility of 25%. What is the 95% confidence interval for the stock price at the end of 3 years?

A. $\$22.83 < S_T < \301.27

B. $\$44.18 < S_T < \57.33

C. $\$31.28 < S_T < \57.08

D. $\$35.49 < S_T < \193.77

The correct answer is **D**.

In this case, $S_0=50$, $\mu=0.20$, $\sigma=0.25$, and $T=3$

The logarithm of the stock follows a normal distribution with the following parameters:

$$\ln S_T \sim N \left(\ln S_0 + \left(\mu - \frac{\sigma^2}{2} \right) T, \sigma \sqrt{T} \right)$$

Where:

μ = the expected annual return of the stock.

σ = the annual volatility for the stock.

T = time

S_T = The stock price at time T .

$$\begin{aligned} \ln S_T &\sim N \left(\ln S_0 + \left(\mu - \frac{\sigma^2}{2} \right) T, \sigma \sqrt{T} \right) \\ &= N \left[\ln 50 + \left(0.20 - \frac{0.25^2}{2} \right) 3, 0.25 \times \sqrt{3} \right] \\ \ln S_T &\sim N(4.418, 0.433^2) \end{aligned}$$

The distribution has a mean of 4.418 and standard deviation of 0.433

To obtain the 95% confidence interval for stock price using the above data, we will proceed as

follows:

$$\begin{aligned}\ln S_T &\sim N(4.418, 0.433^2) \\ \ln S_T &= \mu \pm Z_\alpha \times \sigma \\ (\text{In this case, we have } \sigma &= 0.433) \\ \text{and } Z_\alpha &= 1.96 \\ 4.418 - 1.96 \times 0.433 &< \ln S_T < 4.418 + 1.96 \times 0.433 \\ e^{3.56932} < S_T &< e^{5.26668} \\ \$35.49 < S_T &< \$193.77\end{aligned}$$

Q.4619 Suppose the current exchange rate for a currency is 1.25, and the exchange rate volatility is 15%. Calculate the value of a call option to buy 1000 units of the currency in 5 years at an exchange rate of 2.50. The domestic and foreign risk-free interest rates are 1% and 2%, respectively.

- A. 2.9
- B. 3.8
- C. 43.5
- D. 1249.9

The correct answer is **A**.

In this case $S_0=1.25$, $K=2.50$, $r=0.01$, $r_f=0.02$, $s=0.15$, and $T=5$

From BSM pricing formula,

$$C_0 = S_0 e^{-r_f T} \times N(d_1) - K e^{-r T} \times N(d_2)$$

Where:

T = time to maturity

S_0 = current stock price

K = strike price

r = domestic risk-free rate

r_f = foreign risk-free rate

s = volatility of the stock price

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + [r - r_f + (\frac{\sigma^2}{2})]T}{\sigma\sqrt{T}} = \frac{\ln\frac{1.25}{2.50} + [0.01 - 0.02 + \frac{0.15^2}{2}]5}{0.15\sqrt{5}} = -2.05$$
$$d_2 = d_1 - \sigma\sqrt{T} = -2.05 - 0.15\sqrt{5} = -2.39$$

From the standard normal tables,

$$N(d_1) = N(-2.05) = 1 - 0.9798 = 0.0202$$

$$N(d_2) = N(-2.39) = 1 - 0.9916 = 0.0084$$

The value of the call is therefore given by:

$$C_0 = S_0 e^{-r_f T} \times N(d_1) - K e^{-r T} \times N(d_2)$$
$$= 1.25 e^{-0.02 \times 5} \times 0.0202 - 2.50 e^{-0.01 \times 5} \times 0.0084 = 0.0029$$

This is the value of the option to buy one unit of the currency. The value of an option to buy 1000 units is:

$$0.0029 \times 1000 = 2.9$$

Q.4620 The futures price of an asset is USD 40, and the annual volatility of the futures price is 20%. If the risk-free rate is 5%, what is the value of a put option to sell futures in 6 months for USD 45?

- A. USD 2.75
- B. USD 5.52
- C. USD 2.68
- D. USD 4.82

The correct answer is **B**.

In this case,

$F_0=40$, $K=45$, $r=0.05$, $s=0.20$, $T=0.5$

The following formula gives the value of the put option:

$$P_0 = Ke^{-rT} \times N(-d_2) - F_0e^{-rT} \times N(-d_1)$$

Where:

P_0 = value of the put option

K = strike price

s = volatility of the futures price

r = risk-free rate

T = time

Now define:

F_0 = current futures price

Then we have:

$$d_1 = \frac{\ln\left(\frac{F_0}{K}\right) + \frac{Ts^2}{2}}{s\sqrt{T}} = \frac{\ln\left(\frac{40}{45}\right) + \frac{0.5 \times 0.20^2}{2}}{0.20\sqrt{0.5}} = -0.76214$$

Thus,

$$d_2 = d_1 - s\sqrt{T} = -0.76214 - 0.20\sqrt{0.5} = -0.90356$$

The value of the put option is given by:

$$\begin{aligned} P_0 &= Ke^{-rT} \times N(-d_2) - F_0e^{-rT} \times N(-d_1) \\ &= 45e^{-0.05 \times 0.5} \times 0.8159 - 40e^{-0.05 \times 0.5} \times 0.7764 = 5.5197 \end{aligned}$$

Q.4621 The current price of a stock is USD 50. If this price grows to USD 74.59 in two years, what is the realized return on the stock per annum?

- A. 0.2
- B. 0.49
- C. 0.34
- D. 0.22

The correct answer is **A**.

We can derive the formula for calculating the realized return on a stock from the formula for calculating the expected price of a stock, i.e.,

$$E(S_T) = S_0 e^{\mu T}$$
$$\mu = \frac{1}{T} \ln \frac{S_T}{S_0}$$

Where:

μ = realized return on a stock

T = time to maturity

S_0 = initial stock price

S_T = stock price at time T

$$\text{Realized return} = \frac{1}{2} \ln \frac{74.59}{50} = 0.20 = 20\%$$

Q.4622 The following are monthly stock prices in EUR: 21, 35, 40, and 28. From this data, what is the estimated volatility of the log-returns per month?

- A. 0.435
- B. 0.189
- C. 0.402
- D. 0.355

The correct answer is **A**.

Month	Stock price, S_i	$\frac{S_i}{S_{i-1}}$	$X_i = \ln \frac{S_i}{S_{i-1}}$	X_i^2
0	21			
1	35	1.6667	0.5108	0.2609
2	40	1.1429	0.1336	0.0178
3	28	0.7000	-0.3567	0.1272

We calculate volatility of returns per month using the variance formula below:

$$\text{variance} = \frac{1}{n-1} \left\{ \sum X_i^2 - n \left(\frac{\sum X_i}{n} \right)^2 \right\}$$

In this case:

$$\sum X_i = 0.2877, \sum X_i^2 = 0.4059, n=3, \frac{\sum X_i}{n} = \frac{0.2877}{3} = 0.0959$$

Therefore,

$$\text{variance} = \frac{1}{2} \{ 0.4059 - 3 \times 0.0959^2 \} = 0.1892$$

The volatility per month is, therefore $\sqrt{0.1892} = 0.435 = 43.5\%$

Reading 60: Option Sensitivity Measures: The “Greeks”

Q.1186 Fintech Company Inc. is planning to purchase a call option on European Airlines. The continuous dividend yield is 2 percent and the time to maturity is 2 years. If the continuous risk-free rate is 5 percent and $N(d1)$ is 0.45, what is the Delta of the call option?

- A. 0.432
- B. -0.432
- C. 0.864
- D. -0.864

The correct answer is **A**.

The Delta of a call option with a continuous dividend yield is calculated using the formula below:

$$\text{Delta} = N(d1) e^{(-qT)}$$

Where

Continuous dividend yield (q) = 2%

Time to maturity = 2 years

$$\text{Delta} = 0.45 * e^{(-0.02 * 2)} = 0.432$$

Q.1187 John Augustus, an equity analyst at Fintech Inc., is evaluating a portfolio of American Airlines stock and options on the same stock. The portfolio is currently Delta neutral but has a positive Gamma. If Augustus would like to make the portfolio both Delta and Gamma neutral, then Johnson will:

- A. Sell stock of American Airlines and sell Put options on stock of American Airlines.
- B. Buy stock of American Airlines and buy Put options on stock of American Airlines.
- C. Buy stock of American Airlines and sell Put options on stock of American Airlines.
- D. Sell stock of American Airlines and buy Put options on stock of American Airlines.

The correct answer is **A**.

To make the portfolio Gamma neutral, options on the stock of American Airlines are to be sold. Selling put options on American Airlines makes the portfolio Delta positive. To make the portfolio Delta neutral, the stock of American Airlines need to be sold.

Detailed Answer

Here's what you need to have in mind to solve just about every problem of this type.

(I) All long options, calls and puts, are positive Gamma. All short options, calls and puts, are negative Gamma.

(II) The delta value of calls is always positive (somewhere between 0 and 1) and with puts, it's always negative (somewhere between 0 and -1). Stocks effectively have a delta value of 1.

In this case, we have positive gamma; we are long gamma, To attain a gamma-neutral status, we have to go short gamma. i.e., sell options.

Now, look at the choices; only (a) and (c) involve selling options, which are actually put options. We know that puts have a negative delta, so selling them will render the position delta positive (long delta). To attain an equilibrium, we will need to neutralize the "positive" by selling the underlying stock (short delta). If we buy the underlying stock (which has a delta of 1) we will end up even more delta positive.

Thus, choice (a) is correct.

The best way to approach this type of question is via the elimination of choices as we just did. Get rid of the wrong choices based on what you already know.

Q.1188 Trenor Johnson, a portfolio manager, working at Sterile Finances Limited, is analyzing the delta of a portfolio. Which of the following statements is (are) true about the delta of a portfolio?

- A. Call options have negative deltas while put options have positive Deltas.
- B. Delta on options lie between -1 and +1.
- C. The delta of the underlying asset is always zero.
- D. All of the above.

The correct answer is **B**.

The delta for a call option always ranges from 0 to 1 because as the underlying asset increases in price, call options increase in price. Put option deltas always range from -1 to 0 because as the underlying security increases, the value of put options decrease. Therefore, the delta will lie between -1 and +1, while the delta of the underlying asset is always 1. The delta of a put option is negative reflecting an inverse relationship with the price of the underlying.

Q.1189 Which of the following statements is/are true regarding theta?

- I. Theta is a measure of the change in the value of the options portfolio with the passage of time
 - II. A positive theta implies that the portfolio will increase in value as time passes
 - III. Theta increases as the expiration date approaches for at-the-money options
 - IV. Theta increases as an option which is either out of the money or in the money approaches expiration
- A. I & II
 - B. III & IV
 - C. I, III & IV
 - D. All the above

The correct answer is **A**.

Points I and II are correct.

Points III and IV are incorrect. Theta **decreases** as the expiration date approaches for at-the-money options. Theta also **decreases** as an option that is either out of money or in the money approaches expiration.

Q.1191 Which of the following statements is true with regard to Gamma?

- I. Gamma of a portfolio of options on an underlying asset is the rate of change of the portfolio's Delta with respect to the price of the underlying asset
- II. Gamma is the second partial derivative of the portfolio with respect to asset price
- III. If Gamma is highly negative or highly positive, Delta is very sensitive to the price of the underlying asset
- IV. When Gamma is positive, theta tends to be negative

- A. I & II only
- B. I, II & III only
- C. II & III only
- D. All the above

The correct answer is **D**.

Gamma of a portfolio of options on an underlying asset is the rate of change of the portfolio's Delta with respect to the price of the underlying asset. It is the second partial derivative of the portfolio with respect to asset price. If Gamma is highly negative or highly positive, Delta is very sensitive to the price of the underlying asset. When Gamma is positive, theta tends to be negative.

Q.1192 The Vega of the stock of Amazon is 5. If the volatility of the underlying asset increases by 1 percent, what changes will take place in the price of the call option and put option if the maturity and exercise prices of both options remain the same?

- A. The price of the call option increases by 0.05 and the price of the put option increases by 0.05.
- B. The price of the call option decreases by 0.05 and the price of the put option decreases by 0.05.
- C. The price of the call option decreases by 0.05 and the price of the put option increases by 0.05.
- D. The price of the call option increases by 0.05 and the price of the put option decreases by 0.05.

The correct answer is **A**.

The vega of a portfolio of derivatives, V , is the rate of change of the value of the portfolio with respect to the volatility of the underlying asset.

The change in the price of call and put options is: $\text{Vega} * \text{Volatility} = 5 * 0.01 = 0.05$

Q.1193 Which of the following statement is NOT true with regard to Vega?

- I. The Vega of a derivative portfolio is the rate of change of the value of the portfolio with the change in the volatility of the underlying assets
- II. The Vega of a long position is always negative
- III. A position in the underlying asset has a Vega equal to zero
- IV. At-the-money options have the greatest Vega

- A. II and III only
- B. III only
- C. II only
- D. I, II and IV only

The correct answer is **C**.

The Vega of a long position is always positive.

All three other statements are correct. The Vega of a derivative portfolio is the rate of change of the value of the portfolio with the change in the volatility of the underlying assets. A position in the underlying asset has a Vega equal to zero. At-the-money options have the greatest Vega.

Q.1194 Consider a call option on a non-dividend paying stock where the stock price is \$95, the risk-free rate is 5%, the time to maturity is 40 weeks (= 0.7692 years) and $N' = 0.398185$. A 1% increase in the volatility will increase the value of the option by approximately:

- A. -33.176
- B. -0.33176
- C. 33.176
- D. 0.33176

The correct answer is **D**.

$$\begin{aligned}\text{The Vega of a call option} &= S_0 \sqrt{T} N'(d_1) \\ &= \$95 * \sqrt{0.7692} * 0.398185 \\ &= 33.176\end{aligned}$$

Thus, a 1% (0.01) increase in the volatility increases the value of the option by approximately
 $0.01 * 33.176 = 0.33176$

Note that:

Both call and put options have the same value of vega and this value is a positive number

Q.1195 Which of the following statement is true with regard to Rho?

- A. The Rho of a portfolio of options is the rate of change of the value of the portfolio with respect to the interest rate.
- B. The Rho of a portfolio of derivatives is the rate of change of the value of the portfolio with respect to the volatility of the underlying asset.
- C. The Rho of a portfolio of options on an underlying asset is the rate of change of the portfolio's Delta.
- D. The Rho of a portfolio of options is the rate of change of the value of the portfolio with respect to the passage of time with all else remaining the same.

The correct answer is **A**.

The Rho of a portfolio of options is the rate of change of the value of the portfolio with respect to the interest rate.

Q.1196 A stock is currently trading at \$25. The delta of the call option is 0.482. A fund manager buys 100,000 call options with a strike price of \$26.50 on the stock. To maintain a delta neutral position, the fund manager must:

- A. Buy 4,820,000 shares
- B. Sell 4,820,000 shares
- C. Buy 1,000,000 shares
- D. Sell 1,000,000 shares

The correct answer is **B**.

As the fund manager buys 100,000 call options, the delta of the option position is:

$$\delta \text{ of option position} = 0.482 * 100,000 = +48,200$$

To get a delta neutral position, the fund manager needs to sell 4,820,000(= 48,200 × 100) shares since 1 call option typically gives the holder the right to buy 100 shares at strike price.

Q.1197 A stock is currently trading at \$25. The delta of the call option is 0.482. A fund manager buys 100,000 call option contracts on the stock with a strike price of \$29. What action is most likely to be taken by the fund manager to maintain a delta neutral position?

- A. Sell 4,820,000 shares.
- B. Buy 4,820,000 shares.
- C. Buy 48,200 call options.
- D. Sell 48,200 shares.

The correct answer is **A**.

Since the trader has a long position in the call option, he or she should sell the underlying shares to attain a delta-neutral position. But how many shares exactly should he or she sell?

Each long call option has a delta of 0.482. Since the trader has bought 100,000 options, the portfolio delta is $100,000 \times 0.482 = 48,200$. But since each option holds the right to 100 shares, he or she should sell 4,820,000 shares of the underlying. The gain (or loss) on the option will then be offset by the loss(or gain) on the shares.

Scenario One

If the price of the underlying stock increases by \$1 to \$26 per share, we would then expect the option price to increase by about \$0.482 ($= 1 \times 0.482$) per option. The gain on the overall long option position will be:

$$100,000 \times \$0.482 \times 100 = \$4,820,000$$

The loss on the 100,000 shares will be $1 \times \$4,820,000 =$

This would reduce the net gain to zero.

Scenario Two

If the price of the underlying stock decreases by \$1 to \$24 per share, we would then expect the option price to decrease by about \$0.482 ($= 1 \times 0.482$) per option. The loss on the overall short option position will be:

$$100,000 \times \$0.482 \times 100 = \$4,820,000$$

The gain on the 100,000 shares will be $1 \times \$4,820,000 = \$4,80,000$

This would reduce the net loss to zero

Things to Remember

Whereas a call option position comes with positive delta, a short call position comes with negative delta. On the other hand, a long put position comes with negative delta while a short put position comes with positive delta.

Q.1198 A portfolio manager buys 100 APR 45 call option selling for \$3.58 that have a delta of 0.4 and a gamma of 0.1. If the underlying trades downwards by \$1, then the delta of the overall position will now be:

- A. 0.3
- B. 30
- C. 27
- D. 0.27

The correct answer is **B**.

Gamma reflects the change in delta in response to a one-point movement of the underlying stock price.

Our gamma here is 0.1. For every one-point move in the price of the underlying, delta of the corresponding option will change by 0.1. A one point (1 USD) increase in price will prompt a 0.1 increase in delta. Similarly, a one point decrease in price will prompt a 0.1 decrease in delta.

In our case, delta will decrease from 0.4 to 0.3.

But that's delta for just one call option. For 100 options, $\text{delta} = 0.3 * 100 = 30$.

Note that gamma is always a positive number regardless of whether you are buying calls or puts but is effectively negative when you write options.

Note that:

100 APR 45 call option:

100 here represents the number of call options.

45 here means that an investor can exercise the right to buy the stock at 45 per share.

Q.1199 A portfolio of derivatives on a stock has a delta of 2400 and a gamma of -100. Also available for trading is an option on the stock with a delta of 0.5 and a gamma of 0.04. To make the portfolio gamma neutral, the portfolio manager should:

- A. Buy 2,500 options.
- B. Sell 2,500 options.
- C. Buy 1,200 options.
- D. Sell 1,200 options.

The correct answer is **A**.

To create a gamma-neutral position (sometimes called gamma-neutral hedging), the manager must add the appropriate number of options that equals the existing portfolio gamma position. In this case, the existing gamma position is -100, and an available option exhibits a gamma of 0.04, which translates into buying approximately 2,500 options ($100 / 0.04$).

Things to Remember

Gamma-neutral hedging is the construction of options trading positions that are hedged such that the total gamma value of the position is zero or near zero. The goal is to take options combinations that will make the overall gamma value as close to zero as possible. This results in the delta value of the positions remaining stagnant no matter how strongly the underlying stock moves.

Q.1200 A portfolio manager anticipates that the market volatility will increase substantially in the coming days. He observes two call options which are currently being traded in the market:

- I. A call option on stock A, currently trading at \$20 with a strike price of \$30
- II. A call option on stock B, currently trading at \$40 with a strike price of \$42

The portfolio manager wants to derive the maximum benefit from the anticipated market volatility. The preferred investment should be:

- A. Go long on call options on Stock A.
- B. Go long on call options on Stock B.
- C. Go short on call options on Stock A.
- D. Go short on call options on Stock B.

The correct answer is **B**.

The volatility of call options is highest when the stock price approaches the strike price. As the portfolio manager wants to derive benefit from the anticipated volatility, he must go long on call options on Stock B, as the share price is near the strike price of the call option.

Q.1201 Stock A is currently trading at \$40. A three-month futures contract on Stock A is currently trading at \$40.60. Assume the risk-free rate to be 6%.

The delta of the futures contract is:

A. 1.02

B. 1.2

C. 1.12

D. 1.22

The correct answer is **A**.

$$\Delta_{\text{futures contract}} = e^{rT}$$

Where

r = Risk-free rate

T = Time to expiry of the contract

$$\Delta_{3 \text{ months}} = e^{(0.06 * (\frac{3}{12}))} = 1.02$$

Q.1203 A fund manager sells 200,000 call options on stock A, a non-dividend paying stock. The delta of the stock option is 0.45, and the risk-free rate is 6%.

Select the most appropriate statement.

- A. The position can be made delta and gamma neutral by buying 9,000,000 shares of the underlying asset.
- B. The position can be made delta neutral by going short 90,000 shares on the underlying asset.
- C. The position can be made delta neutral by buying 9,000,000 shares of the underlying asset.
- D. The position can be made delta neutral by going short on call options.

The correct answer is C.

$$\Delta_{\text{Call options}} = -200,000 \times 0.45 \times 100 = -9,000,000$$

Since the fund manager has sold call options, he has a short position. Going long 9,000,000 shares will make the position delta neutral.

Q.3415 Which one of the following statements is true regarding option Greeks?

- A. Gamma is greatest for in-the-money options with long maturities
- B. Delta of deep-in-the-money put options tends toward +1
- C. Vega is greatest for at-the-money options with long maturities
- D. When buying at-the-money options, theta tends to be positive

The correct answer is C.

An option's vega becomes less and less the further your option is from the at the money strike. In other words, vega is greatest for at-the-money options.

Option A is incorrect. For in-the-money options, gamma is small.

Option B is incorrect. For in-the-money puts, delta tends toward -1

Option D is incorrect. When buying at-the-money options for long-term keeping, theta is negative.

Q.3417 A trader has a short option position that's delta-neutral but has a gamma of -800. In the market, there's a tradeable option with a delta of 0.8 and a gamma of 2. To maintain the position gamma-neutral and delta-neutral, the most appropriate strategy is to:

- A. Sell 320 options and buy 400 shares of the underlying
- B. Buy 400 options and sell 320 shares of the underlying
- C. Buy 400 options and buy 320 shares of the underlying
- D. Sell 320 options and buy 320 shares of the underlying

The correct answer is **B**.

As it stands, the position is gamma-negative, and therefore the trader has to buy calls to increase gamma back to zero. The number of options that must be added to the existing portfolio to generate a gamma-neutral position is given by:

$$-\left(\frac{\Gamma_p}{\Gamma_T}\right)$$

Where: Γ_p =gamma of the existing portfolio position Γ_T =gamma of a traded option that can be added

$$= -\left(-\frac{800}{2}\right) = 400$$

Buying 400 calls, however, increases delta from zero to 320 ($= 400 \times 0.8$). Therefore, the trader has to sell 320 shares to restore the delta to zero. Positions in shares always have zero gamma.
