ME581 Homework 4

Instructions:

- **Save the notebook as a single PDF, then upload and submit the PDF in Gradescope**
- **Write your own codes to carry out numerical interpolation and integration**

Problem 1

Interpolation is used heavily to find the thermodynamic properties at given conditions from tabulated data. Use the portion of the given propane table at 20 bar to:

A. Construct the **Lagrange** form of interpolating polynomial. Plot the polynomial for temperatures in the range [60 C°,180 C°]. Use the polynomial to estimate the density (kg/m^3) at temperature = 85, 135 and 163 C°. Print the density.
B. Repeat part (A) with the **Newton** form of interpolation polynomial.

Temperature ($C°$)	**Density (kg/m^3)**
60	46.2342
100	34.5256
140	29.1571
180	25.5715

Problem 2

Evaluate the following integrals:

$$\int_{-\pi}^{\frac{\pi}{2}} x^{-1} \sin x \; dx$$

$$\int_{-1}^{5} (8x^2 - 5x - 1) \; dx$$

$$\int_{0}^{3.5} e^{-x^2} \, dx$$

A. Analytically (the use of any built-in solver is also acceptable)
B. With the application of the **Midpoint Rule**
C. With the application of the **Trapezoidal Rule**
D. With the application of the **Simpson's Rule**

For each of the numerical estimates (B) through (D), determine the absolute error based on (A). Did you expect to find the exact solution?

Problem 3

In mathematics, the error function is defined as:

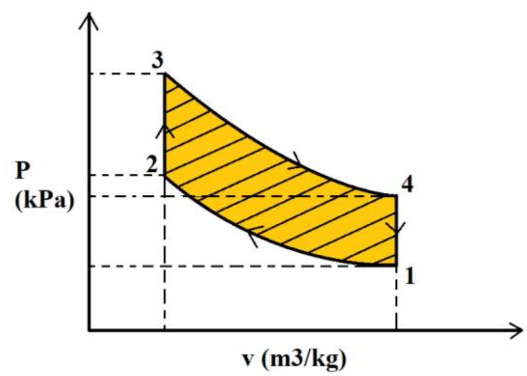
$$f(a) = \left(\frac{3}{\pi^2}\right)^{\frac{3}{4}} \int_0^a e^{-x^2}\, dx$$

A. Estimate **f(1.5)** using the **two-points Gauss quadrature** approach. If the exact value is 0.350498, determine the absolute error.

B. Repeat part (A) with the **three-points Gauss quadrature** approach. (Hint: the weighting factors and function arguments for the three-points Gauss quadrature approach are $\omega_1 = \frac{5}{9}, \omega_2 = \frac{8}{9}, \omega_3 = \frac{5}{9}$ and $x_1 = -\sqrt{\frac{3}{5}}, x_2 = 0, x_3 = \sqrt{\frac{3}{5}}$, respectively).

Problem 4

A turbocharged internal combustion engine working on reversible Otto cycle compress intake air from state 1 (0.15 m^3/kg) to state 2 (0.05 m^3/kg). Heat addition at constant volume combustion increased the pressure 6521.6 kPa reaching state 3, and thereafter the compressed gas expands to state 4 where the pressure 1418.2 kPa and volume 0.15 m^3/kg. The constant volume heat rejection decreased the pressure to 974.3 kPa. The pressure (kPa) was measured at some points as a function of volume (m^3/kg). The data is summarized in the following table:



Compression Stroke:

Volume (m^3/kg)	0.15	0.125	0.1	0.075	0.05
Pressure (kPa)	974.3	1212.6	1584.9	2238.3	3641.1

Expansion Stroke:

Volume (m^3/kg)	0.05	0.075	0.1	0.125	0.15
Pressure (kPa)	6521.6	3895.3	2605.8	1816.2	1418.2

A. Using the data points consigned in the table, construct an interpolation polynomial of the form $p = f(v)$, using the Newton's form. What is the degree of the polynomial? Plot the interpolation function with the data points in the same graph.

B. The work is defined as $w = \int p\,dv$. Obtain the work when the gas is compressed from state 1 to state 2 by numerically integrating the polynomial in part (A) using a **Composite Trapezoidal Rule** with 200 equal-length intervals.
C. The work is defined as $w = \int p\,dv$. Obtain the work when the gas is expanded from state 3 to state 4 by numerically integrating the polynomial in part (A) using a Composite Trapezoidal Rule with 200 equal-length intervals.
D. Find the net work by the cycle kJ/kg. Hint: $W_{net} = \int_3^4 pdv - \int_1^2 pdv$.

Problem 5

Map the region Ω defined by the given four corners of a quadrilateral to the standard region $(-1 \le u \le 1)$, $(-1 \le v \le -1)$. Map the general integral $\iint_\Omega f(x,y)\,dx\,dy$ to the standard region. Evaluate the following integrals using **Two-Point Gaussian Quadrature**. Print the integral value.

A. Integration function: $f(x,y) = (x^3 - y^2)$, Quadrilateral: (0, 0), (5, -1), (4, 5), and (2, 4).
B. Integration function: $f(x,y) = e^{-0.05x^2 - y}$, Quadrilateral: (2, -1), (11, 9), (15, 14), and (1, 5).