ME 581
Midterm Exam
Due Monday Oct 24 11:59PM EST

Problem 1 (30 pts)	
Problem 2 (30 pts)	
Problem 3 (40 pts)	

1) The exam will be available at 5:00PM-EST on October 21, 2022 and it is due at 5:00 PM-EST on October 24, 2022.
2) It will be solved using a Jupyter notebook and submitted as a single pdf file in Gradescope.
3) Notes, books, and homework are allowed during the exam.
4) Laptops and calculators are allowed.
5) You can use all the functions that you wrote for previous homework assignments in your exam, so make sure that they are working properly.
6) You do not need a proctor.
7) The midterm exam should be done on your own and should not be discussed with any other person. If you have questions during the exam please email Prof Koslowski (marisol@purdue.edu)

Problem 1 (30 points)

Given $f(x) = 4\,x^4 + \frac{23}{2}\,x^3 + \frac{15}{2}x^2 + \frac{9}{8}x + 3$

(a) Plot $f(x)$ and identify the number of local maxima and minima in the interval [-2,1]

For each local maximum and minimum:

(b) Use Newton's method to find the approximate value. Use a tolerance 10^{-6} to stop the algorithm, i.e. stop when $|x_{i+1} - x_i| < 10^{-6}$

(c) Plot in a graph the absolute error $e_i = |x_{i+1} - x_i|$ versus the iteration number i.

(d) Determine the order of convergence and explain the order that you find.

Problem 2 (30 points)

You can find the root of the function in the interval (1,2)

$$f(x) = x^3 - 3x + 2x^2 - 1$$

using a fixed point iteration with the following two functions:

$$g_1(x) = \sqrt[3]{3x - 2x^2 + 1}$$
$$g_2(x) = (x^3 + 2x^2 - 1)/3$$

a) Starting at x=1.0 perform fixed point iterations with $g_1(x)$ to find the root, use an error tolerance 10^{-6}
b) Starting at x=1.0 perform fixed point iterations with $g_2(x)$, use an error tolerance 10^{-6}
c) Do the sequences generated in a) and b) converge to the fixed point? Explain.
d) What is the condition for convergence of a fixed-point iteration?

Problem 3 (40 points)

The most economical way to invert an $n \times n$ matrix **A** is to solve for **X** in the equations

$$\boldsymbol{A} \cdot \boldsymbol{X} = \boldsymbol{I}$$

where ***I*** is the identity matrix and ***X*** is an $n \times n$ matrix.

Use this method to find $\mathbf{A}^{-1}$ using the conjugate gradient method. Print $\mathbf{A}^{-1}$

$$\boldsymbol{A} = \begin{pmatrix} 1.00 & 0.91 & 0.82 & 0.75 & 0.60 & 0.50 & 0.00 & 0.20 \\ 0.91 & 2.00 & 0.85 & 0.80 & 1.77 & 0.27 & 0.32 & 1.40 \\ 0.82 & 0.85 & 1.50 & 1.90 & 0.83 & 2.10 & 0.67 & 1.51 \\ 0.75 & 0.80 & 1.90 & 1.20 & 0.97 & 0.28 & 0.23 & 4.50 \\ 0.60 & 1.77 & 0.83 & 0.97 & 1.75 & 0.30 & 0.89 & 1.75 \\ 0.50 & 0.27 & 2.10 & 0.28 & 0.30 & 2.00 & 2.21 & 0.79 \\ 0.00 & 0.32 & 0.67 & 0.23 & 0.89 & 2.21 & 3.00 & 4.00 \\ 0.20 & 1.40 & 1.51 & 4.50 & 1.75 & 0.79 & 4.00 & 0.96 \end{pmatrix}$$