ME581 Homework 2
Due: 11:59pm, 09/22/2021

Instructions:

- **The following problems are to be documented, solved, and presented in Jupyter notebooks.**
- **Save the notebooks as PDFs, then upload and submit the PDFs in Gradescope.**
- **Write your own codes for GE method**

Problem 1

A system of equations is represented by the following augmented matrix

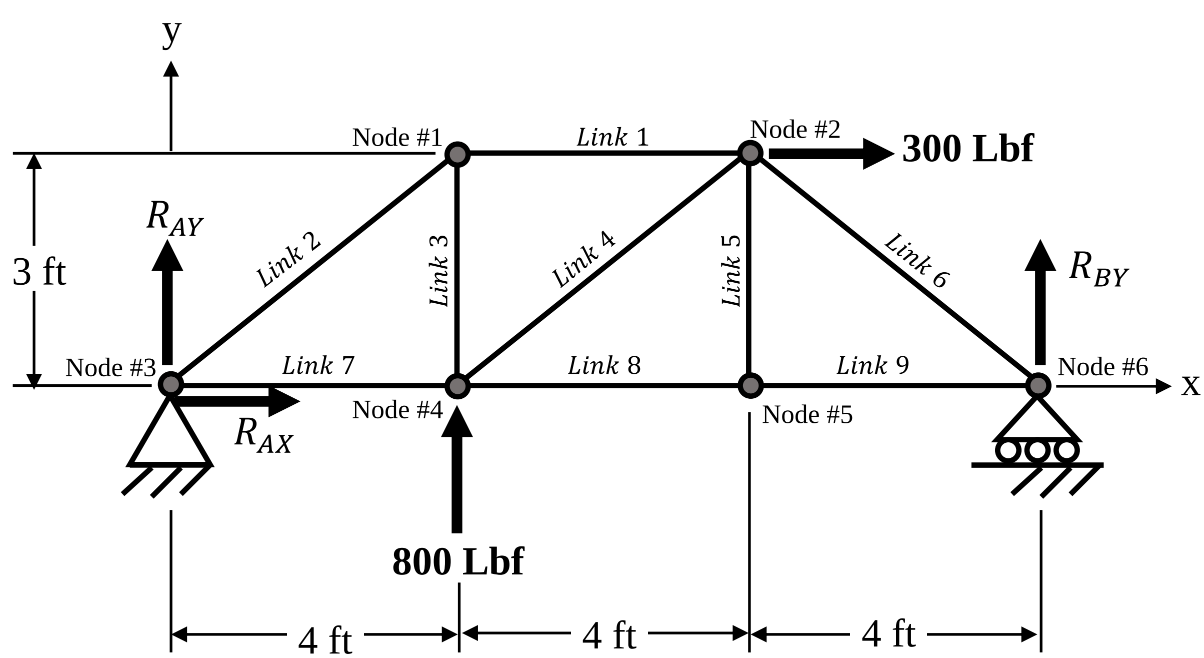
$$\left[\begin{array}{ccccc|c} -9 & 11 & -21 & 63 & -252 & -356 \\ -505 & 506 & -1008 & 3031 & -12117 & -17167 \\ -575 & 576 & -1149 & 3451 & -13801 & -19552 \\ 3891 & -3891 & 7782 & -23345 & 93365 & 132274 \\ 1024 & -1024 & 2049 & -6144 & 24572 & 34813 \end{array}\right]$$

For this system of equations,

(a) Solve the system using Gaussian Elimination with *No* Pivoting,
(b) Solve the system using Gaussian Elimination with Scaled Partial Pivoting.
(c) Compare the solutions obtained in (a) and (b), and discuss.

Report a table with results to 15 significant figures. The exact solution is $[1 \quad -1 \quad 1 \quad -1 \quad 1]^T$.

Problem 2
A plane truss is shown:



Equilibrium requires that the x- and y-components of the forces at the nodes sum to zero. Sum these forces to develop a system of equations $\boldsymbol{AF} = \boldsymbol{b}$, where $\boldsymbol{F}$ is the member- and reaction-force vector defined as

$$\boldsymbol{F} = [F_1\ F_2\ F_3\ F_4\ F_5\ F_6\ F_7\ F_8\ F_9\ R_{AX}\ R_{AY}\ R_{BY}]^T.$$

where F_1-F_8 are the internal forces in links 1-8 respectively and where R_{AX}, R_{AY}, and R_{BY} are the reaction forces applied by the supports as shown in the figure.

(a) Print the resulting matrix A to at least two significant figures.
(b) Print the resulting right-hand vector $\boldsymbol{b}$ to at least two significant figures.
(c) Use Gaussian Elimination with Scaled Partial Pivoting to determine the member and reaction forces $\boldsymbol{F}$ when the truss is subjected to loading on Node 2 and Node 4 as shown in the figure.

Problem 3
Use Gaussian Elimination with Scaled Partial Pivoting in double precision to solve the following 10×10 system of equations

$$A = \begin{bmatrix} 7 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 \\ -1 & 7 & -1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & -1 & 7 & -1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 & 7 & -1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & -1 & 7 & -1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 & -1 & 7 \end{bmatrix}; \quad \boldsymbol{b} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 1 \end{Bmatrix}; \quad \boldsymbol{x} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_9 \\ x_{10} \end{Bmatrix}$$

Problem 4
A linear system of equations $A\boldsymbol{x} = \boldsymbol{b}$ is given by

$$\begin{bmatrix} 2 & 3 \\ 10 & 16 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

The exact solution $\boldsymbol{x}$ and an approximate solution $\widetilde{\boldsymbol{x}}$ are given by

$$\boldsymbol{x} = [5 \quad -3]^T$$

$$\widetilde{\boldsymbol{x}} = [4 \quad -2]^T$$

(a) Compute the matrix inverse A^{-1} by any means.
(b) Compute the error $\boldsymbol{e} = \widetilde{\boldsymbol{x}} - \boldsymbol{x}$.
(c) Compute the residual $\boldsymbol{r} = A\widetilde{\boldsymbol{x}} - \boldsymbol{b}$.
(d) Using the L_∞ norm, compute the relative error $\|\boldsymbol{e}\|_\infty / \|\boldsymbol{x}\|_\infty$.
(e) Using the L_∞ norm, compute the condition number $\kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.
(f) Using the L_∞ norm, compute the relative residual $\|\boldsymbol{r}\|_\infty / \|\boldsymbol{b}\|_\infty$
(g) Compute the multiplicative product of the condition number and the relative residual.
(h) Does the relative error (calculated above) exceed the error bound given by the product of the condition number and relative residual (calculated above)?

Problem 5
Let

$$A = \begin{bmatrix} 52 & 10 \\ 70 & 15 \end{bmatrix}.$$

(a) Find the inverse A^{-1} by any means, then calculate the condition number $\kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

(b) Let $\boldsymbol{b} = [4 \quad 20]^T$. Solve the system $A\boldsymbol{x} = \boldsymbol{b}$.

(c) Let the perturbation $\delta\boldsymbol{b} = [0.01 \quad -0.01]^T$. Solve the perturbed system $A\tilde{\boldsymbol{x}} = (\boldsymbol{b} + \delta\boldsymbol{b})$.

(d) Compare the actual value of the relative change in the solution, $\|\delta\boldsymbol{x}\|_\infty / \|\boldsymbol{x}\|_\infty$, with its theoretical upper bound as predicted by the equation

$$\frac{\|\delta\boldsymbol{x}\|_\infty}{\|\boldsymbol{x}\|_\infty} \leq \frac{\kappa_\infty(A)}{1 - \kappa_\infty(A)(\|\delta A\|_\infty / \|A\|_\infty)} \left(\frac{\|\delta\boldsymbol{b}\|_\infty}{\|\boldsymbol{b}\|_\infty} + \frac{\|\delta A\|_\infty}{\|A\|_\infty} \right)$$

The matrix A is not perturbed here, meaning that $\delta A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Problem 6

Let

$$A = \begin{bmatrix} 8 & 4.5 \\ 6.6 & 1.1 \end{bmatrix} \text{ and } \boldsymbol{b} = \begin{bmatrix} 2.23 \\ 60.1 \end{bmatrix}.$$

(a) Find the inverse A^{-1} by any means, then calculate the condition number $\kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

(b) Solve the system $A\boldsymbol{x} = \boldsymbol{b}$.

(c) Let

$$\delta A = \begin{bmatrix} -0.011 & 0 \\ 0 & 0.018 \end{bmatrix} \text{ and } \delta b = \begin{bmatrix} 0.025 \\ -0.015 \end{bmatrix}.$$

Solve the perturbed system $(A + \delta A)\tilde{\boldsymbol{x}} = (\boldsymbol{b} + \delta\boldsymbol{b})$.

(d) Compare the actual value of the relative change in the solution, $\|\delta\boldsymbol{x}\|_\infty / \|\boldsymbol{x}\|_\infty$, with its theoretical upper bound as defined above.