ME581 Homework 1

Due: 11:59, 09/07/2022

Instructions:

- **The following problems are to be documented, solved, and presented in a Jupyter notebook.**
- **Save the notebook as a single PDF, then upload and submit the PDF in Gradescope Write your own codes for Bisection, Newton and Secant methods**

Problem 1

a) Plot the function $f(x) = 1 - 2*ln(x)$ to verify the presence of zeros in the interval (1,2). Label the exact location of $p = e^{(1/2)}$.

b) Perform the first five iterations of the **bisection method** and verify that each approximation satisfies the theoretical error bound of the bisection method.

c) Check if the error decreases or increases in each iteration. Plot the error versus number of iterations.

d) As conclusion, list and discuss any two limitations of the bisection method based on the type of function analyzed. Give examples.

Problem 2 1

Approximate 1/48 (up to 5 decimal places) using **bisection method** to the equation $\frac{1}{x} - 48 = 0$. Include plots of:

a) Approximated value vs Iteration number;

b) Absolute error vs. Iteration number.

Problem 3

The equation $x^3 - 15 = 0$ has a root on the interval (2,3), namely $p = \sqrt[3]{15}$.

a) Perform five iterations of **Newton's method.**

b) For $n \geq 1$, **print** the comparison of $|p_n - p_{n-1}|$ with $|p_{n-1} - p|$ and $|p_n - p|$.

c) For $n \geq 1$, **print** computation of the ratio $|p_n - p|/|p_{n-1} - p|^2$ and show that this value approaches $|f''(p)/2f'(p)|$.

Problem 4

For each of the functions given below, use the **Newton's method** algorithm to approximate **all real roots**. Use an absolute tolerance of 10^{-6} as a stopping condition. For each of the roots **plot** the logarithm of the absolute error $|e_n|$ at each iteration n against the logarithm of the absolute error at the previous iteration. Use this plot to calculate the order of convergence.

a) $f(x) = e^x + x^2 - x - 7$

b) $f(x) = x^3 - x^2 - 9x + 4$

c) $f(x) = 1.08 - 1.03x + \ln(x)$

Problem 5

The function $f(x) = 81x^4 + 27x^3 - 9x^2 + 3x - 22$ has a zero at $x = 2/3$. Using starting value of $p_0 = 0$, perform ten iterations of **Newton's method** to approximate the solution.

a) Plot Absolute error versus Iteration number.

b) What is the order of convergence?

c) What is the multiplicity of the zero at $x = 2/3$

Problem 6

It was observed that **Newton's method** provides quadratic convergence towards roots of multiplicity equal to one. How does the **secant method** perform under such circumstances? Each of the following functions has a zero at the specified location. Perform 25 iterations applying the **secant method**. Does the sequence generated by the secant method converge with α ≈ 1.618 or has the order dropped to α = 1?

$(a)\ f(x) = x^2(1 - cos(x))$ has a zero at $x = 0$. Use $p_0 = -1$ and $p_1 = 2.5$.

(b) $f(x) = 81x^4 + 27x^3 - 9x^2 + 3x - 22$ has a zero at $x = 2/3$. Use $p_0 = 0$ and $p_1 = 0.5$.

Problem 7

a) Verify that the equation $x^4 - 18x^2 + 45 = 0$ has a root on the interval $(1,2)$. Perform four iterations of the **Newton's method**, using the starting value $p_0 = 1$. Given that the exact value of the root is $x = \sqrt{3}$, compute the absolute error in the approximations just obtained. What is the apparent order of convergence?

b) Verify that the equation $x^4 - 18x^2 + 45 = 0$ also has a root on the interval $(3,4)$. Perform four iterations of the **secant method**, using the starting values $p_0 = 3\ and\ p_1 = 4$. Compute the absolute error in each approximation, given the exact value of the root, $x = \sqrt{15}$. What is the order of convergence?

c) Explain clearly the difference in the convergence behavior between parts a) and b)?