(a) VDP equation reduced to two first order differential equations in terms of the two state variables y and $\mu^{-1}\dot{y}$

$$\frac{d^2y}{dt^2} - \mu\left(1-y^2\right)\frac{dy}{dt} + y = 0 \text{ for } \mu > 0$$

$$y'' - \mu\left(1-y^2\right)y' + y = 0\ ;\ \mu > 0$$

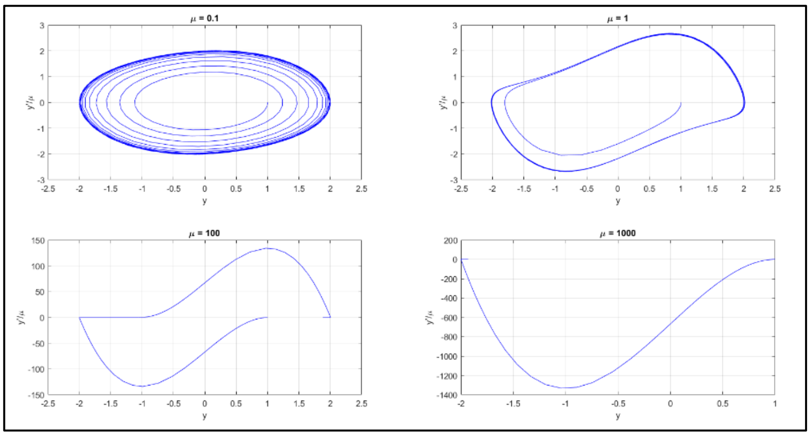
$$\text{Let } y_1 = y \text{ and } y_2 = \frac{y'}{\mu} = \frac{y_1'}{\mu}$$

VDP eqn. reduced to 2 first order ODE's :

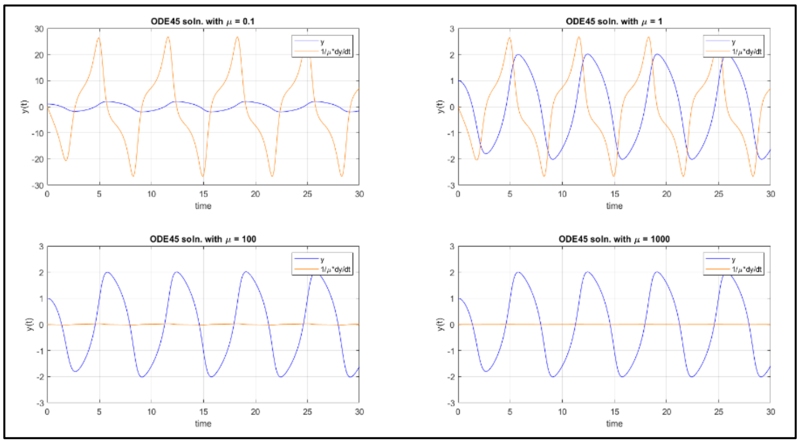
$$\Rightarrow y_1' = \mu y_2 \ \ldots\ldots\ldots\ (i)$$

$$\Rightarrow y_2' = -\frac{y_1}{\mu} + \left(1-y_1^2\right)y_2 \ \ldots\ldots\ (ii)$$

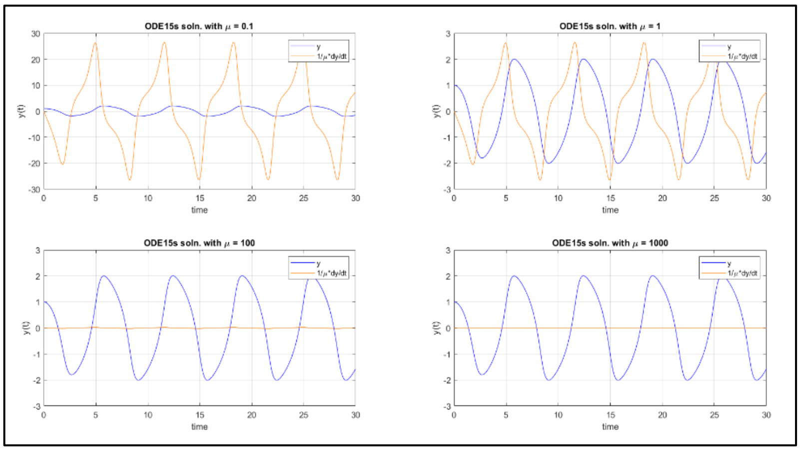
Phase Plane plot for soln. of the VDP equation:



(b) Solution of VDP eqn. solved using ODE45:



Solution of VDP eqn. solved using ODE15s:



(c) Comparison b/w ODE45 and ODE15s:

Condition	*Observation*
$\mu = 0.1,\ 1$	ODE15s is faster for smaller μ values
$\mu = 100,\ 1000$	ODE45 works better when the VDP is more *stiff*, at higher values of μ
Lower iterations (time < 50)	Faster solver depends on the stiffness and damping of the oscillator
High iterations	ODE45 works better in most cases

(d) <u>Analysis of Phase Plane plots of VDP oscillator</u>

- For $\mu = 0.1$, the phase plane plot converges to steady state values, the plot resembles an elliptical shape, which is indicative of *sinusoidal* oscillatory behaviour of the system.
- For higher values of μ, this nature completely breaks down.
- $\mu = 1$, the system seems to converge to a steady state but the shape of the plot is very distorted, resembling a quadrilateral.
- For very high values of $\mu = 100,\ 1000$, the phase plane plot shows instability and does not converge.