Statistics assignment

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2) Q $f(x) = \alpha e^{-\alpha x}$ $x \geq 0$, $\alpha > 0$
Show that $f(x)$ is PDF and find its CDF, $F(x)$.

→ $f(x) = \alpha e^{-\alpha x}$ for $\infty \geq x \geq 0$

$$\int_0^\infty f(x) = \int_0^\infty \alpha e^{-\alpha x} = \left[\frac{\alpha e^{-\alpha x}}{-\alpha}\right]_0^\infty$$

$$= \left[-e^{-\alpha x}\right]_0^\infty$$

$$= e^{-\alpha(\infty)} + e^{-\alpha(0)}$$

$$= 0 + 1$$

$$= 1$$

So, integration of $f(x)$ is 1, it is proved to be a PDF

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The CDF $F(x) = \int f(x) = -e^{\alpha x}$ (Ans).

1) a) Determine appropriate A values.

(i) $P(X=i) = A \cdot i$, $i = 1, 2, \ldots, n$
$= 0$, otherwise

So, $F(i) = Ai$
For a PMF,

$$\sum_{i=1}^{n} F(i) = 1 \Rightarrow \sum_{i=1}^{n} Ai = 1$$

$$\Rightarrow A\left[\frac{n(n+1)}{2}\right] = 1 \Rightarrow A = \frac{2}{n(n+1)}$$

(ii) $P(X=i) = \frac{A}{2^i}$, $i = 1, 2, 3, \ldots$
$= 0$ otherwise

$$f(i) = \frac{A}{2^i}$$

For F to be a ~~PMD~~ PMF,

$$\sum_{i=1}^{n} F(i) = 1 \Rightarrow \sum_{i=1}^{n} \frac{A}{2^i} = 1$$

$$\Rightarrow A\left(\frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n}\right) = 1$$

$$\Rightarrow A\left(\frac{2^n - 1}{2^n}\right) = 1 \Rightarrow A = \frac{2^n}{2^n - 1}$$

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(b) Show that $f(x) = \frac{1}{\pi(1+x^2)^2}$ satisfies the properties of a PDF

$\rightarrow$ $x \in (-\infty, \infty)$

The function is continuous

$$\int_{-\infty}^{\infty} \frac{dx}{\pi(1+x^2)^2} = \frac{1}{\pi}\left[\frac{x}{2(x^2+1)} + \frac{1}{2}\int \frac{dx}{x^2+1}\right]_{-\infty}^{\infty}$$

$$= \frac{1}{\pi}\left[\frac{x}{2x^2+2} + \frac{\tan^{-1}(x)}{2}\right]_{-\infty}^{\infty}$$

$$= \frac{1}{2\pi}\left[0 + \frac{\pi}{2} + \frac{3\pi}{2}\right] = 1.$$

(6.) Find the probability of $X < 45$. Given PDF of X, $f(x) = \frac{2x}{\left(30 + \frac{x^2}{30}\right)^2}$, $x > 0$

$\rightarrow$ For the given PDF $f(x)$, CDF will be,

$$F(x) = \int f(x)\,dx = \int \frac{2x}{\left(30+\frac{x^2}{30}\right)^2}\,dx$$

$$= 30\int \frac{\frac{x}{15}\,dx}{\left(30+\frac{x^2}{30}\right)} = 30\int \frac{du}{u^2}$$

$$= -\frac{30}{u} = \frac{-30}{30+\frac{x^2}{30}}$$

$$\Rightarrow F(x) = \frac{-900}{900+x^2}$$

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Probability of $0 \le X \le 45$,

$$F(45) - F(0) = \frac{-900}{900+(45)^2} + \frac{900}{900+(0)^2}$$

$$= \frac{900}{900} - \frac{900}{900+2025}$$

$$= 1 - 0.307 = 0.693.$$

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8) $f(x) = \begin{cases} \frac{5}{A}, & -\frac{A}{10} \le x \le \frac{A}{10} \\ 0, & \text{otherwise} \end{cases}$

Find A if $P(|X|<2) = 2P(|X|>2)$

→ For the probability density function $f(x)$

$$F(X) = \int f(x)\,dx = \int \frac{5}{A}\,dx = \frac{5x}{A}$$

Given, $P(|X|<2) = 2P(|X|>2)$

$$\Rightarrow \left[\frac{5x}{A}\right]_0^2 = 2\left[\frac{5x}{A}\right]_2^{\infty}$$

$$\Rightarrow \frac{5(2)}{A} - \frac{5(0)}{A} = 2\left[\frac{5(\infty)}{A} - \frac{5(2)}{A}\right]$$

$$\Rightarrow \frac{10}{A} = 2 - \frac{20}{A}$$

$$A = 15.$$

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10) X denotes no. of heads − no. of tails in 3 tosses. The Head is twice as likely to occur as tail.

→ Due to bias, $P(H) = 2P(T)$

$\Rightarrow P(H) = 2(1 - P(H)) \Rightarrow P(H) = \frac{2}{3}$

$P(T) = \frac{1}{3}$

(i) Probability distribution of X:

$$P(X=1) = \begin{cases} P(H,H,T) = \left(\frac{2}{3}\right)^2 \frac{1}{3} = \frac{4}{27} \\ P(H,T,H) = \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{4}{27} \\ P(T,H,H) = \frac{1}{3} \cdot \left(\frac{2}{3}\right)^2 = \frac{4}{27} \end{cases}$$

So, $P(X=1) = \frac{4}{27} + \frac{4}{27} + \frac{4}{27} = \frac{12}{27}$

$$P(X=-1) = \begin{cases} P(T,T,H) = \left(\frac{1}{3}\right)^2 \cdot \frac{2}{3} = \frac{2}{27} \\ P(T,H,T) = \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{27} \\ P(H,T,T) = \frac{2}{3} \cdot \left(\frac{1}{3}\right)^2 = \frac{2}{27} \end{cases}$$

$P(X=-1) = \frac{6}{27}$

$P(X=-3) = P(T,T,T) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$P(X=3) = P(H,H,H) = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

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(ii) Cumulative distribution of x :=

$F(x) = P(X \leq x)$

$\Rightarrow P(X \leq -3) = \frac{1}{27}$

$P(X \leq -1) = \frac{6}{27} + \frac{1}{27} = \frac{7}{27}$

$P(X \leq 1) = \frac{6}{27} + \frac{1}{27} + \frac{12}{27} = \frac{19}{27}$

$P(X \leq 3) = \frac{8}{27} + \frac{19}{27} = 1$

(iii) What is $P(-1 \leq X \leq 3)$

$P(-1 \leq X \leq 3) = \frac{6}{27} + \frac{12}{27} + \frac{8}{27} = \frac{26}{27}$

(iv) $P(X > 0) = P(X=1) + P(X=3)$

$= \frac{12}{27} + \frac{8}{27} = \frac{20}{27}$

(12.) $P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

$P(X=1) = \lambda e^{-\lambda}$

$P(x=2) = \frac{\lambda^2 e^{-\lambda}}{2}$

$P(X=1) = P(X=2)$

$\Rightarrow \lambda e^{-\lambda} = \frac{\lambda^2 e^{-\lambda}}{2}$

$\Rightarrow \lambda = 2.$

Mean = variance = 2.

Date

(14) Given PMF, $P(X=x) = \binom{k+x-1}{k-1} \theta^k \phi^k$,

$\phi + \theta = 1, x = 0, 1, 2, \ldots$

Find mgf, mean & variance.

→ MGF is given by,

$$M(t) = \sum_{k=0}^{\infty} e^{tk} \binom{k+x-1}{k-1} \theta^k \phi^x$$

$$= \phi^x \sum_{k=0}^{\infty} \binom{k+x-1}{k-1} (\theta e^t)^k$$

$$\binom{k+x-1}{k-1} = \frac{(k+x-1) \ldots (x+1) \cdot x}{k!}$$

$$= (-1)^k \binom{-x}{k}$$

$$M(t) = \phi^x \sum_{k=0}^{\infty} (-1)^k \binom{-x}{k} (\theta e^t)^k$$

$$= \phi^x \left[(-1) \binom{-x}{0} (\theta e^t)^0 + (-1)^1 \binom{-x}{1} (\theta e^t)^1 + \ldots \right]$$

Let X be the number of draws upto and including the first occurrence of drawing a defective smartphone.

Let probability of event θ and $\phi = 1-\theta$

So, probability distribution will be

$$p(x) = P(X=x) = \theta\phi^{x-1}$$

So, expression

$$E[X] = \sum_{x=1}^{\infty} x\,p(x)$$

$$= \theta\sum_{x=1}^{\infty} x(1-\theta)^{x-1}$$

ii) The probability mass function of a binomial random variable is

$$P(X=x) = p(x)$$

$$= {}^{n}C_{x}\,\theta^{x}p^{n-x}$$

$$= \frac{n!}{x!(n-x)!}\,\theta^{x}(1-\theta)^{n}(1-\theta)^{-x}$$

$$\Rightarrow P(X=x) = \frac{1.\left(1-\frac{1}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)}{x!}\,(n\theta)^{x}\left(1-\frac{n\theta}{n}\right)^{n}(1-\theta)^{-x}$$

So, for limit $n\to\infty$, & $\theta\to 0$

$$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0,1,2,\ldots$$

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So, Poisson's distribution is a limiting form of Binomial distribution when:-

a) n is large
b) θ is small.
c) $n\theta$ is not too large.

(iii) $\theta = 0.1$

$\phi = 1 - 0.1 = 0.9$

$x = 4$

Let, X = #attempts for transmitting a packet.

So, geometric Distribution will be

$P(X=4) = P(4) = (0.1)(0.9)^3$
$= 0.0729$

Hence, probability of event X is 0.0729

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