Maths Assignment

Name → Rohan Ghosh

Batch → BCS2B

ID. → 18100100112 2

Assignment - 1

(1.) LHS = $f(4)$

RHS = $f(2) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)$

$= f(3) + \Delta f(2) + \Delta^2 (f(1) + \Delta f(1))$

$= f(3) + \Delta f(2) + \Delta^2 f(2)$ $\left[f(x) + \Delta f(x) = f(x+1)\right]$

$= f(3) + \Delta (f(2) + \Delta f(2))$

$= f(3) + \Delta f(3)$

$= f(4) = LHS.$

(2) $\Delta^2\left(\frac{1}{x}\right)$

$= \Delta\left(\frac{1}{x+h} - \frac{1}{x}\right)$ $\quad [\because \Delta f(x) = f(x+h) - f(x)]$

$= \frac{1}{x+2h} - \frac{1}{x+h} - \frac{1}{x+h} + \frac{1}{x}$

$= \frac{1}{x+2h} - \frac{2}{x+h} + \frac{1}{x}$

$= \frac{x^2 + hx - 2(x^2+2x) + x^2 + 2h^2 + 3xh}{x(x+h)(x+2h)}$

$= \frac{2h^2}{x(x+h)(x+2h)}$

(3)

x	$f(x) = \sin x$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
30	0.5	0.0736	−0.0044	−0.0005	0	0.0002
35	0.5736	0.0692	−0.0049	−0.0005	0.0002	
40	0.6428	0.0643	−0.0054	−0.0003		
45	0.7071	0.0589	−0.0057			
50	0.7660	0.0532				
55	0.8192					

For $x = 32$, $x_0 = 30$, $x = 5$, $u = \frac{32-30}{5} = 0.4$

$f(3) = 0.5 + 0.4 \times 0.0736 + \frac{0.4(0.4-1)}{2}(-0.0044)$

$+ \frac{0.4(0.4-1)(0.4-2)}{6} \times (-0.0005) + \dots$

$= 0.52944 + 0.000528 - 0.000032 + 0.000005$

$= 0.529997$ (Ans.)

(4.) Let error = ϵ

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	1	1	30	$160+E$	$200-4E$	$200+10E$
1	2	31	$190+E$	$360-3E$	$420+6E$	$20-10E$
2	33	$221+\epsilon$	$550-2E$	$780+3E$	$440-4E$	
3	$254+\epsilon$	$771-E$	$1330+E$	$1220-E$		
4	1025	2101	2550			
5	3126	4651				
6	7777					

Value of $\Delta^5 y$ should be equal, Sum = 240,
each entry = 120

$20 - 10E = 120$

$E = -10$

$f(3) = 254 + E = 244.$

(5.)

D	A	ΔA	$\Delta^2 A$	$\Delta^3 A$	$\Delta^4 A$
80	5026	648	46	-2	-4
85	5674	688	38	2	
90	6362	726	40		
95	7088	766			
100	7854				

For $x = 98$, $x_0 = 100$, $x = 5$, $u = \frac{-100+98}{5} = -0.4$

$$f(98) = 7854 + (-0.4)\times 766 + \frac{(-0.4)(-0.4-1)}{2}\times 40 + \frac{(-0.4)(-1.4)(-2.4)}{6}\times 2$$

$$= 7854 - 306.4 + 11.2 - 0.448$$

$$= 7558.352 \quad \text{(Ans.)}$$

(E.)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1	2·72	0·60	0·14	0·02	0·01	0·02	−0·07
1·2	3·32	0·74	0·16	0·03	0·03	−0·05	
1·4	4·06	0·90	0·19	0·06	−0·02		
1·6	4·96	1·09	0·25	0·04			
1·8	6·05	1·34	0·29				
2·0	7·39	1·63					
2·2	9·02						

$x = 1·05,\ x_0 = 1,\ h = 0·2,\ u = \dfrac{1·05 - 1}{0·2} = 0·25$

$$f'(1·5) = \frac{1}{0·2}\left[0·6 + (-0·5)\frac{0·14}{2} + \frac{0·54 \times 0·02}{6} + \cdots\right]$$

$$= \frac{1}{0·2}\left[0·6 - 0·035 + 0·0018\right]$$

$$= 2·834 = \frac{dy}{dx}(x = 1·05)$$

$$f^4(1·5) = \frac{1}{(0·2)^2}\left[0·14 + (-0·25)\times 0·02 + 13·75 \times 0·01 \over 24 + \cdots\right]$$

$$= \frac{1}{(0·2)^2}\left[0·14 - 0·015 + 0·0057\right]$$

$$= 3·2675$$

$$f'(1·2) = \frac{1}{0·2}\left[0·74 - \frac{1}{2}\times 0·16 + \frac{1}{3}\times 0·03 - \cdots\right]$$

$$= 3·3125$$

$$\frac{d^2y}{dx^2}(x = 1·20) = \frac{1}{(0·2)^2}\left[0·16 - 0·03 + \frac{11}{12}\times 0·03\right]$$

$$= 3·9375 \text{ (Ans.)}$$

(7)

x	$\sqrt{x}$	$\Delta\sqrt{x}$	$\Delta^2\sqrt{x}$	$\Delta^3\sqrt{x}$	$\Delta^4\sqrt{x}$
15	3.873	0.25	-0.019	0.017	-0.034
17	4.123	0.231	-0.002	-0.017	
19	4.359	0.229	-0.016		
21	4.583	0.213			
23	4.796				

$x = 15,\ x_0 = 15,\ h = 2,\ u = 0$

First derivative of $\sqrt{x} = \frac{1}{2}\left[0.25 - \frac{1}{2}(-0.019) + 0.034 - 0.031\right]$

$= 0.15375$ (Ans.)

Second derivative of $\sqrt{x} = \frac{1}{4}\left[-0.019 - 0.017 + \frac{11}{12}(0.031)\right]$

$= -0.00189$ (Ans.)

(8)

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	15	5	5	-5	15
2	20	10	0	10	
4	30	10	10		
6	40	20			
8	60				

$x = 7,\ x_0 = 8,\ h = 2,\ u = \frac{7-8}{2} = -0.5$

$\frac{dy}{dx} = \frac{1}{2}\left[20 + 0 \times \frac{10}{2} + (-0.75) \times \frac{10}{6} + \cdots\right]$

$= 9.7916$ (Ans.)

For $x = 0,\ x_0 = 0,\ h = 2,\ u = 0$

$\frac{d^2y}{dx^2} = \frac{1}{4}\left[5 - (-5) + \frac{11}{2} \times 15\right] = 5.9375$ (Ans.)

(7) To find value of $\log 2^{1/3}$ from $\int_0^1 \frac{x^2}{1+x^3}\,dx$

$h = 0.25$

→ Since $h = 0.25$ we divide in 4 parts.

x	y
$x_0 = 0$	$y_0 = 0$
$x_1 = 0.25$	$y_1 = 0.0615$
$x_2 = 0.50$	$y_2 = 0.2222$
$x_3 = 0.75$	$y_3 = 0.3956$
$x_4 = 1.00$	$y_4 = 0.5000$

By Simpson's $\frac{1}{3}$rd rule,

$$\int_0^1 \frac{x^2}{1+x^3}\,dx = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2y_2\right]$$

$$= \frac{0.25}{3}\left[(0+0.5) + 4(0.0615+0.3956) + 2\times 0.2222\right]$$

$$= \frac{0.25}{3} \times \left[0.5 + 1.8284 + 0.4444\right]$$

$$= 0.231 \quad \text{(ans.)}.$$

(10) $\frac{\pi}{4} = \int_0^1 \frac{dx}{1+x^2}$, $n = 10$

$h = \frac{1-0}{10} = \frac{1}{10} = 0.1$

$y = \frac{1}{1+x^2}$

x	y	x	y
$x_0 = 0$	$y_0 = 1$	$x_6 = 0.6$	$y_6 = 0.7352$
$x_1 = 0.1$	$y_1 = 0.9901$	$x_7 = 0.7$	$y_7 = 0.6711$
$x_2 = 0.2$	$y_2 = 0.9615$	$x_8 = 0.8$	$y_8 = 0.6098$
$x_3 = 0.3$	$y_3 = 0.9174$	$x_9 = 0.9$	$y_9 = 0.5525$
$x_4 = 0.4$	$y_4 = 0.8621$	$x_{10} = 1.0$	$y_{10} = 0.5$
$x_5 = 0.5$	$y_5 = 0.8$		

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + \cdots + y_9) + y_{10}\right]$$

$$= \frac{0.1}{2}\left[1 + 2\,(0.9901 + 0.9615 + 0.9174 + 0.8621 + 0.8000 + 0.7353 + 0.6711 + 0.6098 + 0.5525) + 0.5\right]$$

$$= \frac{0.1}{2}\left[1 + 14.1996 + 0.5\right] = 0.78498$$

$$= 0.7850 \text{ (approx)}$$

$$\frac{\pi}{4} = \frac{3.14}{4} = 0.7853 \text{ (approx)}.$$

(11.)

x	1	2	3	4	5	6	7	8	9
y	0.2	0.7	1	1.3	1.5	1.7	1.9	2.1	2.3

Problem :- $\int_1^9 f(x)\,dx$, $h = 1$

Area $= \frac{h}{2}\left[f(1) + 2\left(f(2)+f(3)+f(4)+\ldots+f(8)\right) + f(9)\right]$

$= \frac{1}{2}\left[0.2 + 2(0.7+1+1.3+1.5+1.7+1.9+2.1) + 2.3\right]$

$= \frac{1}{2}\left[22.9\right]$

$= 11.45$ unit square

(12.) $\int_0^{\pi/2} \cos x\,dx$ $\qquad n = 6$

$h = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12}$ $\qquad y = \cos x$

x	y
$x_0 = 0$	$y_0 = 1$
$x_1 = \pi/12$	$y_1 = 0.9659$
$x_2 = \pi/6$	$y_2 = 0.8660$
$x_3 = \pi/4$	$y_3 = 0.7071$
$x_4 = \pi/3$	$y_4 = 0.5$
$x_5 = 5\pi/12$	$y_5 = 0.2588$
$x_6 = \pi/2$	$y_6 = 0$

By Simpson's 1/3rd rule,

$$\int_0^{\pi/2} \cos x\,dx = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{\pi}{36}\left[1 + 4(0.9659 + 0.7071 + 0.2588) + 2(0.8660 + 0.5)\right]$$

$$= \frac{\pi}{36}\left[1 + 7.7272 + 2.732\right]$$

$$= \frac{\pi}{36} \times 11.4592 \approx 1.000 \text{ (approx)}$$