Numerical Methods

Assignment → 2

LOKENATH
Page No.
Date

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Batch – BCS 2B

Q. 1. $x_1 - 2x_2 = -1$
$-2x_1 + x_2 = 3$

→ $x_1^{k+1} = -1 + 2 \cdot x_2^{k}$
$x_2^{k+1} = 3 - 2x_1^{k+1}$

Now $(0,0)$ produce
$x_1^1 = -1 + 2(0) = -1$
$x_2^1 = 3 - 2(0) = 3$

Repeated ~~instance~~ iterations produce →

K	0	1	2	3	4	5	6
x_1	0	-1	5	-15	62	-243	922
x_2	0	3	-7	33	-121	489	-1957

The approximation is getting worse hence method diverges.

2. $-x_1 + 4x_2 = 1$ $3x_1 - 2x_2 = 2$

Step 1 $x_1^{k+1} = -1 + 4x_2^{k}$
$x_2^{k+1} = -1 + \frac{3}{2} x_1^{k+1}$

Initially, $x_1^1 = -1 + 4(0) = -1$
$x_2^1 = -1 + \frac{3}{2}(0) = -1$

K	0	1	2	3	4	5	6
x_1	0	-1	-5	-35	-215	-1295	-7775
x_2	0	-1	-85	-53.5	-323.5	-1943.5	-11663.5

The approximation is getting worse, hence diverges.

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Q.3) $x_1 - 2x_2 = -1$, $2x_1 + x_2 = 3$

→ $x_1^{k+1} = \frac{1}{2}\left[3 - x_2^{k}\right]$,

$x_2^{k+1} = \frac{1}{2}\left[-1 + x_1^{k+1}\right]$

K	0	1	2	3	4	5	6
x_1	0	1.5	1.375	1.4062	1.398	1.4005	1.399
x_2	0	0.25	0.1875	0.2031	0.199	0.2002	0.199

Thus, $x_1 = 1.4$, $x_2 = 0.20$ / upto 2 significant digits.

Q.4) $-x_1 + 4x_2 = 1$, $3x_1 - 2x_2 = 2$

$x_1^{k+1} = \frac{1}{3}\left[2 + 2x_2^{k}\right]$, $x_2^{k+1} = \frac{1}{4}\left[1 + x_1^{k+1}\right]$

K	0	1	2	3	4	5
x_1	0	0.666	0.944	0.9907	0.998	0.999
x_2	0	0.416	0.486	0.497	0.499	0.499

Thus, $x_1 = 1.0$ & $x_2 = 0.50$

Q.5) $2x_1 - 3x_2 = -7$, $x_1 + 3x_2 - 10x_3 = 9$

$3x_1 + x_3 = 13$

$x_1^{k+1} = \frac{1}{3}\left[13 - x_3^{k}\right]$, $x_2^{k+1} = \frac{1}{3}\left[7 + 2x_1^{k+1}\right]$

$x_3 = \frac{1}{10}\left[-9 + x_1^{k+1} + 3x_2^{k+1}\right]$

K	0	1	2	3	4	5
x_1	0	4.333	3.967	4.0033	3.999	4.000
x_2	0	5.222	4.978	5.0022	4.999	5.000
		1.099	0.9901	1.0009	0.999	1.000

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Thus, $x_1 = 4.0$, $x_2 = 5.0$, $x_3 = 1.0$.

Q.6 $-4x_1 + 5x_2 = 1$, $x_1 + 2x_2 = 3$

$\rightarrow$ $x_1^{k+1} = \frac{1}{4}\left[-1 + 5x_2^k\right]$, $x_1^1 = \frac{1}{4}\left[-1+5(0)\right] = -0.25$

$x_2^{k+1} = \frac{1}{2}\left[3 - x_1^{k+1}\right]$, $x_2^1 = \frac{1}{3}\left[3+0.25\right] = 1.625$

K	0	1	2	3	4	5	6	7
x_1	0	−0.25	1.781	0.511	1.305	0.508	1.12	0.925
x_2	0	1.625	0.609	1.244	0.847	1.096	0.94	1.030

Thus, $x_1 = 1.0$, $x_2 = 1.0$, The results are getting better, so converges.

Q.7 $4x_1 + 2x_2 - 2x_3 = 0$, $x_1 - 3x_2 - x_3 = 7$,
$3x_1 - x_2 + 4x_3 = 5$.

$x_1^{k+1} = \frac{1}{4}\left[0 - 2x_2^k + 2x_3^k\right]$, $x_2^{k+1} = \frac{1}{3}\left[-7 + x_1^k - x_3^k\right]$

$x_3^{k+1} = \frac{1}{4}\left[5 - 3x_1^k + x_2^k\right]$.

$(0,0,0) \rightarrow$

$x_1^1 = \frac{1}{4}\left[0 - 2(0) + 2(0)\right] = 0$, $x_2^1 = \frac{1}{3}\left[-7 + (0) - (0)\right] = -2.33$

$x_3^1 = \frac{1}{4}\left[5 - 3(0) + (0)\right] = 1.25$

K	0	1	2	3	4	5	6	7
x_1	0	0	1.79	1.70	0.58	0.49	1.205	1.33
x_2	0	−2.33	−2.75	−1.95	−1.50	−1.97	−2.31	−2.06
x_3	0	1.25	−1.6675	0.785	−0.31	−0.44	0.39	0.239.

Date

Thus, $x_1 = 1.3$, $x_2 = -2.0$, $x_3 = 0.30$ converges.

Q.8) Matrix is strictly diagonally dominant →

- $\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$ As, $|a_{11}| > |a_{12}|$, $|a_{22}| > |a_{21}|$

→ Diagonally dominant.

- $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ $|a_{11}| > |a_{12}|$ ~~and true~~ is not true, hence it is not diagonally dominant.

- $\begin{bmatrix} 12 & 6 & 0 \\ 2 & -3 & 2 \\ 0 & 6 & 13 \end{bmatrix}$ $|a_{22}| > |a_{21}| + |a_{23}|$ is not true, hence not dominant diagonally.

- $\begin{bmatrix} 7 & 5 & -1 \\ 1 & -4 & 1 \\ 0 & 2 & -3 \end{bmatrix}$ $|a_{11}| > |a_{12}| + |a_{13}|$,

$|a_{22}| > |a_{21}| + |a_{23}|$,

$|a_{33}| > |a_{31}| + |a_{32}|$.

The matrix is dominant diagonally.