Assignment-2 on BFS, DFS, MST Algorithms

Name- Rohan Ghosh
ID: 181001001122,
Batch: BCS2B
Techno India University, West Bengal

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1. The incidence matrix of a directed graph $G = (V, E)$ is a $|V| \times |E|$ matrix $B = [b_{ij}]$ such that:

$$b_{ij} = \begin{cases} -1 & \text{if edge } j \text{ leaves vertex } i \\ 1 & \text{if edge } j \text{ enters vertex } i \\ 0 & \text{otherwise} \end{cases}$$

Describe what the entries of the matrix product $B \times B^T$ represent, where B^T is the transpose of B.

Solution 1:

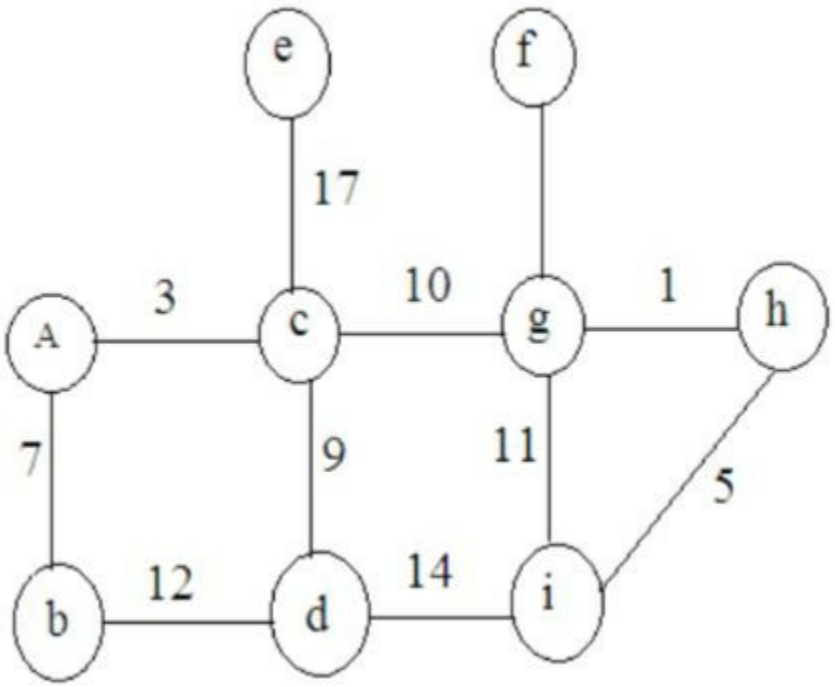
$$BB^T(i, j) = \sum_{e \in E} b_{ie} b^T_{ej} = \sum_{e \in E} b_{ie} b_{ej}$$

- If $i=j$, then $b_{ie}b_{je} = 1$(it is 1 x 1 or (-1) x (-1)) whenever e enters or leaves vertex i , and 0 otherwise.
- If $i \neq j$, then $b_{ie}b_{je} = 1$ when $e = (i,j)$ or $e = (j,i)$, and 0 otherwise.

Thus,

$$B^T_{ij} = \begin{cases} \text{degree of } i = \text{in-degree} + \text{out-degree}, & \text{if } i = j \\ -(\#\text{ of edges connecting } i \text{ and } j), & \text{if } i \neq j \end{cases}$$

2. For this problem, you are to answer some questions about the following graph.



a. In what order are the vertices visited using Depth First Search (DFS)starting from vertex A? (I.e., what is the order of discovery time?) When there is a choice of vertices to visit, use alphabetical order.

Ans: A → b → d → c → e → g → f → h → i

b. In what order are the vertices completed using DFS starting from vertex A?(I.e., what is the order of finishing time?)

Ans: e → f → i → h → g → c → d → b → A

c. In what order are edges added to the Minimum Spanning Tree (MST) using Kruskal's Algorithm? List the edges by giving their endpoints.

Ans: gh, Ac, hi, Ab, cd, cg, fg, ce

d. In what order are edges added to the MST using Prim's Algorithm (growing a single tree) starting from vertex A?

Ans: Ac, Ab, cd, cg, gh, hi, fg, ce

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