Numerical methods

Assignment → 3

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1. If $(4/3)$ is represented approximately by 1.3333 find percentage error.

→ $V_A = 1.3333$

$V_T = 4/3 = 1.33333333$

$$E_p = \left|\frac{V_T - V_A}{V_T}\right| \times 100$$

$$= \frac{1.33333333 - 1.3333}{1.33333333} \times 100$$

$$= 2.5 \times 10^{-3}$$

$$= 0.0025\ \%$$

2. Prove that $\Delta\nabla = \Delta - \nabla$

→ $\Delta\nabla f(x)$

$= \Delta [f(x) - f(x-h)]$

$= \Delta f(x) - \Delta f(x-h)$

$= f(x+h) - f(x) - f(x-h+h) + f(x-h)$

$= f(x+h) - f(x) + f(x-h) - f(x)$

$= \Delta f(x-h)$

$= \Delta f(x) - \nabla f(x)$

$= (\Delta - \nabla) f(x)$

$\Delta\nabla f(x) = (\Delta - \nabla) f(x)$

$\therefore \boxed{\Delta - \nabla = \Delta\nabla}$

3. Find the value of $\Delta^3 f(1)$

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
0	0	3	2	0
1	3	5	2	0
2	8	7	2	0
3	15	9	2	
4	24	11		
5	35			

$$\Delta^3 f(1) = 0$$

4. Round off to 4 sign. figures →

$0.0076581 = 0.007658$

$5.04599 = 5.046$

5. Round off to 3 significant figures →

$7.00254 = 7.002$

$9.85008 = 9.850$

6. Prove that $\Delta \equiv E - 1$

→ $\Delta f(x) = f(x+h) - f(x)$

$= Ef(x) - f(x)$

$= (E-1) f(x)$

$\Delta f(x) = (E-1) f(x)$

$$\boxed{\Delta \equiv E - 1}$$

7. If $f(3)=5$ & $f(5)=3$, then find $f(x)$

→

x	y	Δy
3	5	−2
5	3	

$$u = \frac{x-x_0}{h} = \frac{x-3}{2}$$

$$f(x) = y_0 + u\Delta y_0$$

$$= 5 + \left(\frac{x-3}{2}\right)\times(-2)$$

$$= 5 - x + 3$$

$$= 8 - x. \quad [\text{Ans.}]$$

8. Find value of $\nabla^2 f(2)$.

→

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
0	2	1	9
1	3	10	12
2	13	22	21
3	35	43	
4	78		

$$\nabla^2 f(2) = 22 - 10 = 12.$$

9. Find the value of $\Delta^3 f(0)$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	1	1	8	6
1	2	9	14	
2	11	23		
3	34			

10. Round off to 3 correct decimal places.

→ 2.0055 = 2.006
5.35009 = 5.35

11. If 2/3 is approximated to 0.6667, find relative error.

→ $V_T = 2/3$, $V_A = 0.6667$

$E_A = V_T - V_A = -3.3333 \times 10^{-5}$

$E_R = \frac{E_A}{V_T} = -4.99995 \times 10^{-5}$

12. What is $(\Delta - \nabla) x^2$?

→ $(\Delta - \nabla)x^2 = (x+h)^2 - x^2 - x^2 + (x-h)^2$
$= x^2 + h^2 + 2xh - 2x^2 + x^2 + h^2 - 2xh$
$= 2h^2$

13. If $f(1) = 3$ & $f(2) = -5$, find $f(1.5)$?

→

x	f(x)	Δf(x)
1	3	−8
2	−5	

$u = \frac{1.5 - 1}{1} = 0.5$

$f(1.5) = 3 + 0.5 \times (-8) = 3 - 4 = -1.$

14. Find $\int_0^1 x^2\,dx$ with step length 0.5 using T-rule

→ $\int_0^1 x^2 dx$ $\quad h = 0.5 \quad n = \frac{1-0}{0.5} = 2$

$= \frac{0.5}{2}\left[0 + 2(1 + 0.25 + 2.25) + 4\right]$

$= 2.75$

15.

x	1	2	3	4	5	6
$f(x)$	1	8	27	64	125	216

Find $\Delta^3 f(3)$

→ $\Delta^3 f(3) = \Delta^2\left[f(4) - f(3)\right]$

$= \Delta^2\left[f(5) - 2f(4) + f(3)\right]$

$= f(6) - 3f(5) + 3f(4) - f(3)$

$= 216 - 3\times125 + 3\times64 - 27$

$= 216 - 375 + 192 - 27$

$= 408 - 402 = 6.$

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16. Show the value of $\Delta \cos x$ is $-2\sin\left(x + \frac{h}{2}\right)\sin\frac{h}{2}$.

→ $\Delta f(x) = f(x+h) - f(x)$

$= \cos(x+h) - \cos x$

$= -2\sin\frac{x+h+x}{2}\sin\frac{x+h-x}{2}$

$= -2\sin\left(x + h/2\right)\sin\left(h/2\right).$

17. Taking $h=1$, show that $\Delta(e^x)$ is $(e-1)e^x$?

→ $\Delta(e^x) = e^{x+h} - e^x$

$= e^{x+1} - e^x \quad [h=1]$

$= e^x(e-1)$

18. Taking $h=1$, show that $\Delta(x^3)$ is $3x^2+3x+1$

→ $\Delta(x^3) = (x+h)^3 - x^3$

$= x^3 + 3x^2h + 3xh^2 + h^3 - x^3$

$= 3x^2 + 3x + 1 \quad [h=1]$

19. If $f(0)=3$, $f(2)=7$, then find $f(1)$.

→

x	$f(x)$	$\Delta f(x)$
0	3	4
2	7	

$h=2$

$u = \frac{1-0}{2} = 0.5$

$f(1) = 3 + \frac{1}{2}(u) = 3+2 = 5.$

20.

x	0	1	2	3	4	5
$f(x)$	12	15	20	27	39	52

What is the value of $\Delta^5 f(0)$

→ $\Delta^5 f(0) = \Delta^4 [f(1) - f(0)]$

$= \Delta^3 [f(2) - 2f(1) + f(0)]$

$= \Delta^2 [f(3) - 3f(2) + 3f(1) - f(0)]$

$= \Delta [f(4) - 4f(3) + 4f(2) - 4f(1) + f(0)]$

$= f(5) - 5f(4) + 5f(2) - 5f(2) + 5f(1) - f(0)$

$= 52 - 5\times34 + 5\times27 - 5\times20 + 5\times15 - 12$

$= 52 - 5(34 - 27 + 20 - 15) - 12$

$= 52 - 85 - 12 = -45.$

21. If we take $\pi = 3.14$, instead of $22/7$, find relative error.

→ $V_T = \frac{22}{7}$, $V_A = 3.14$

$E_A = V_T - V_A = 0.00285714$

$E_R = \frac{E_A}{V_T} = 0.000909$

22. Prove that $E\nabla = \Delta$

→ $E\nabla f(x) = E[f(x) - f(x-h)]$

$= f(x+h) - f(x)$

$= \Delta f(x)$

$\boxed{E\nabla = \Delta}$