MPCS53810 Project - Extension of the Hotelling Problem

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1 Motivation

This project is inspired by both problem 6 of problem set 1 and an interest in chaotic systems like the 3 body problem.

2 Direction

We wish to extend the Hotelling problem to the $\mathbb{R}^2$ case, focusing on "simple", symmetric shapes (circles, squares) with a possible extension to surfaces of shapes in $\mathbb{R}^3$ such as spheres, with n players. It is unlikely that any PSNEs will exist for the n case in $\mathbb{R}^2$ or beyond. Even if this is not true, we want to find out if we can set the shape, number, and starting points to cause orderly patterns in each player trying to optimize their share of the shape. We want to see how changing these conditions will impact the order or chaos of the system. There are also some other cases such as multiple points on the surface of a sphere that we could study or even solve.

3 Goals

Simulate a system of n shops that start at predefined locations in a closed, convex, compact subset of $\mathbb{R}^2$, and model their interactions based on a "regularized" best response function (i.e. a function that restricts movement to locations "near" to the current location). Identify whether shops settle at any location indicating a PSNE, or if positions continue to evolve with time. More importantly, we wish to observe if shop motion follows well-defined paths or if the motion is chaotic, and determine what criteria is required for paths to be well-defined (if there is such criteria).