



✓ Congratulations! You passed!

Next Item



1. In this quiz, you will calculate the Jacobian matrix for some vector valued functions.

1/1 point

For the function $u(x,y)=x^2-y^2$ and v(x,y)=2xy, calculate the Jacobian matrix $J=\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}$.

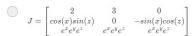
- $J = \begin{bmatrix} 2x & 2y \\ -2y & 2x \end{bmatrix}$

Correct Well done!

- $J = \begin{bmatrix} 2x & 2y \\ 2y & 2x \end{bmatrix}$
- $J = \begin{bmatrix} 2x & -2y \\ -2y & 2x \end{bmatrix}$



 $\begin{array}{ll} 2. & \text{ For the function } u(x,y,z) = 2x + 3y, v(x,y,z) = \cos(x)\sin(z) \text{ and} \\ \\ w(x,y,z) = e^x e^y e^z, \text{ calculate the Jacobian matrix } J = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial v}{\partial z} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial v}{\partial z} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \end{bmatrix}$



Correct

Well done!



3. Consider the pair of linear equations u(x,y)=ax+by and v(x,y)=cx+dy, where a,b,c and d are all constants. Calculate the Jacobian, and notice something kind of interesting!



$$J = \begin{bmatrix} b & c \\ d & a \end{bmatrix}$$

Correct

Well done!

A succinct way of writing this down is the following:

$$\begin{bmatrix} u \\ v \end{bmatrix} = J \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

This is a generalisation of the fact that a simple linear function $f(x)=a\cdot x$ can be re-written as $f(x)=f'(x)\cdot x$, as the Jacobian matrix can be viewed as the multi-dimensional derivative. Neat!

$$J = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$J = \begin{bmatrix} b & c \\ a & d \end{bmatrix}$$



4. For the function $u(x,y,z)=9x^2y^2+ze^x, v(x,y,z)=xy+x^2y^3+2z$ and $w(x,y,z)=cos(x)sin(z)e^y$, calculate the Jacobian matrix and evaluate at the point (0,0,0).

$$J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$



Correct Well done!

$$J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$



In the lecture, we calculated the Jacobian of the transformation from Polar co-ordinates to Cartesian co-ordinates in 2D. In this question, we will do the same, but with Spherical co-ordinates to 3D.

For the functions $x(r,\theta,\phi)=rcos(\theta)sin(\phi),y(r,\theta,\phi)=rsin(\theta)sin(\phi)$ and $z(r,\theta,\phi)=rcos(\phi)$, calculate the Jacobian matrix.

$$J = \begin{bmatrix} r^2 cos(\theta) sin(\phi) & -sin(\theta) sin(\phi) & cos(\theta) cos(\phi) \\ rsin(\theta) sin(\phi) & rcos(\theta) sin(\phi) & rsin(\theta) cos(\phi) \\ cos(\phi) & 1 & rsin(\phi) \end{bmatrix}$$

$$J = \begin{bmatrix} rcos(\theta)sin(\phi) & -rsin(\theta)sin(\phi) & rcos(\theta)cos(\phi) \\ rsin(\theta)sin(\phi) & r^2cos(\theta)sin(\phi) & sin(\theta)cos(\phi) \\ cos(\phi) & -1 & -rsin(\phi) \end{bmatrix}$$

Well done! The determinant of this matrix is $-r^2sin(\phi)$, which does not vary only with θ .

$$J = \begin{bmatrix} r cos(\theta) sin(\phi) & -sin(\theta) sin(\phi) & cos(\theta) cos(\phi) \\ r sin(\theta) sin(\phi) & cos(\theta) sin(\phi) & sin(\theta) cos(\phi) \\ r cos(\phi) & 0 & -sin(\phi) \end{bmatrix}$$