PYU33C01, S. Hutzler, 2022

Assignment 3: Numerical solution of an ordinary differential equation

Aim of this exercise is to study the numerical solution of the ordinary differential equation (ODE)

$$f(x,t) = dx/dt = (1+t)x + 1 - 3t + t^2, \tag{1}$$

using three numerical schemes of different accuracy.

The solutions of such an ODE can be understood by plotting the so-called *direction field*. This consists of small arrows, of slope dx/dt (as given by the above ODE), plotted onto grid points (t,x) in the x-t plane. (See for example Chapter 3 in “From Calculus to Chaos”, D. Acheson, Oxford University Press 1997.)

1. Produce such a direction field for t ranging from 0 to 5 and x from -3 to +3 by evaluating dx/dt at 25×25 grid points. Python commands np.meshgrid(,) and ax.quiver(,,,) are useful for this. (You can google how they work.)

2. Pick as a starting value $x(t=0) = 0.0655$ and solve eqn.(1) using the simple Euler method with step size 0.04. (Feel free to use some of the python code presented in the lecture (and available on Blackboard) for solving ODEs.) Plot your numerical solution, together with the direction field. The point $x(0) = 0.0655$ was deliberately chosen to be close to the critical value of $x_c = 0.065923...$ which separates the solutions which eventually tend to $+\infty$ ($x(0) > x_c$), from those which eventually tend to $-\infty$ ($x(0) < x_c$). How does your simple Euler solution behave?

3. Repeat this calculation using both improved Euler method and Runge-Kutta method (step size 0.04 in both cases). (Again feel free to use part of the code provided on Blackboard.) How do your new numerical solutions behave? Reduce the step size to 0.02, what do you observe now? Can you see the benefit of using accurate integration schemes?

Submit a report (as pdf) **via Blackboard** which details your findings, include relevant figures. Submit your Python code as a *.py* file which should also print your name and student number onto the screen when executed.

submission deadline: Friday, November 11, 2022