CS 224M: Assignment - 1

NOTE: This assignment is to be done **individually**. You can discuss the assignment, but the final submitted solution has to be written by each student individually. You can upload your solutions in PDF format (scan of handwritten solution is okay) to Moodle before the deadline.

The assignment is divided into 2 parts, a theory section about signal transmission and then a lab section where you measure how signal strength in wireless networks changes over a spatial area.

Theory - 70 Marks

Consider a communication system that consists of a single transmitter and a single receiver. Assume that the transmitted signal is attenuated and then corrupted by additive white Gaussian noise at the receiver. Denote the point in the two-dimensional constellation diagram (with cosine on X-axis and sine on Y-axis) corresponding to the *transmitted signal* $s(t)$ is $\underline{s} = (s_x, s_y)$. The unit vector on the x-axis is $\sqrt{\frac{2}{T}}\cos(2\pi f_0 t)$ (that is point (1,0)) and the unit vector on the y-axis (that is (0,1)) is $\sqrt{\frac{2}{T}}\sin(2\pi f_0 t)$, where T is the symbol duration (which is $1/f_0$). Let us define the dot-product between two signals $g(t)$ and $h(t)$ as

$$\langle g(t), h(t)\rangle = \int_0^T g(t)h(t)\ dt$$

We will assume that the *transmitted energy of any symbol* $s(t)$ is given by

$$||\underline{s}||^2 := \langle s(t), s(t)\rangle = s_x^2 + s_y^2$$

The signal is attenuated by factor α. Hence the received constellation point[1] is $\underline{r} = (r_x, r_y)$ where $r_x = \alpha s_x + n_x$ and $r_y = \alpha s_y + n_y$ where n_x, n_y are i.i.d. Gaussian random variables with zero mean and variance $\frac{N_0}{2}$, where N_0 is the noise energy per symbol. Note that half the noise energy is in the X-axis direction and the other half in the Y-axis direction, which is why variance is $N_0/2$ in each direction.

We define the signal-to-noise ratio (SNR) per symbol at the receiver as the ratio of the following two quantities: (i) $\alpha^2\times$(average energy per transmitted symbol), and (ii) noise energy per symbol. Average energy per transmitted symbol is just the expected value (mean) of energy of a transmitted symbol.

In the following, derive the required probabilites in terms of the $Q(.)$ function which is defined as

$$Q(z) = \frac{1}{\sqrt{2\pi}}\int_z^\infty \exp(-x^2/2)\ dx. \tag{1}$$

Write your final answers for each probability as a function of SNR per symbol. Show your working for all questions.

[1] In class we considered the received constellation point after amplification. Here we are considering it before amplification.

1. (BPSK) 25 Marks: Suppose the transmitter uses constellation diagram $(-A, 0)$ and $(A, 0)$ to convey bit information 1 and 0 respectively. This means that $\underline{s}$ is chosen as one of these constellation points depending on the value of the bit to be transferred. Derive an expression for the probability of incorrectly detecting the transmitted bit in terms of SNR per symbol. Assume that bits 1 and 0 are transmitted with equal probability.

2. (QPSK) 45 Marks: Suppose the transmitter uses constellation diagram $(A/\sqrt{2}, A/\sqrt{2})$, $(-A/\sqrt{2}, A/\sqrt{2})$, $(-A/\sqrt{2}, -A/\sqrt{2})$ and $(A/\sqrt{2}, -A/\sqrt{2})$, and assigns bits 00, 01, 11, 10 to these points respectively. Assume that all constellation points are transmitted with equal probability. Calculate the probability of the first bit being received in error. Calculate the probability of the second bit being received in error. Are these two probabilities equal? Are they greater than or less than the probability calculated for BPSK above (assuming the same SNR per symbol for BPSK and QPSK)?

Lab - 30 Marks

You may have noticed that sometimes the signal strength (given by the number of bars) shown by your phone is low (for 4G or WiFi). Usually the download data rates and/or voice call quality degrades when signal strength is low.

Cellular providers such as Airtel, Vi, and Jio have to try to ensure that signal strength is good everywhere, to keep their customers happy. They have to place 4G base-stations (called *eNodeB* in LTE parlance) strategically to ensure this. The wireless signals can face attenuation due to

1. Obstacles
2. Multi-path effect

The latter happens when the signal bounces off many objects and cancels each other, that is *destructively interfere* at the phone of the receiver. Because of this, it is not easy to theoretically predict exactly what the signal strength will be at all locations, given the placement of base-stations.

Use one or more *apps* on your mobile phone to record signal strength over some geographical area (try to measure signal strength over a path of at least 1 km), and pictorially represent it on a map. Use colours to say if the signal strength is good or bad etc. In your report state the range of signal strengths (in dBm) which correspond to different colours. Give screenshot(s) of signal strength over some geographical area.

Recommendation for Android

NetMonitor Cell Signal Logging Lite

For this app, under the *Main* tab, you can click on the red button on the top right to record, and then the white square button to stop recording. Each time you do this, a new Session is logged which you can later find under the *Sessions* tab. You can view the details of recorded sessions by going to

Sessions → click on session to view → Map icon to the left of the *dustbin* icon

This should give you a Google Map of signal strength. Clicking on any location of the path on the map will give you a list of measurement points corresponding to that location. Clicking on one of these will give you the raw data and some plots.

NOTE: You can feel free to use any other app if you like. There may be similar apps available for iPhones like Network Analyzer