CS 224 Assignment I

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LAB

The following trip was performed and this map was generated.

we see that the color corresponding to yellow indicates moderate signal strength, green indicates strong signal strength.

- Green: corresponds to

$$-84\,dB_m \leq RSRP_{green}$$

- Yellow: corresponds to

$$-96\,dB_m \leq RSRP_{yellow} < -84\,dB_m$$

- Red: corresponds to

$$RSRP_{red} < -96\,dB_m$$

THEORY

Q1) for BPSK

$$SNR = \frac{(\text{Energy per symbol})\alpha^2}{(\text{Energy in Noise Signal})}$$

$$SNR = \frac{\alpha^2(s_x^2 + s_y^2)}{(\mathbb{E}(n_x^2) + \mathbb{E}(n_y^2))}$$

for bit in 1 $\underline{s} = (A, 0)$

$n_x, n_y \sim \mathcal{N}(0, \frac{N_0}{2})$

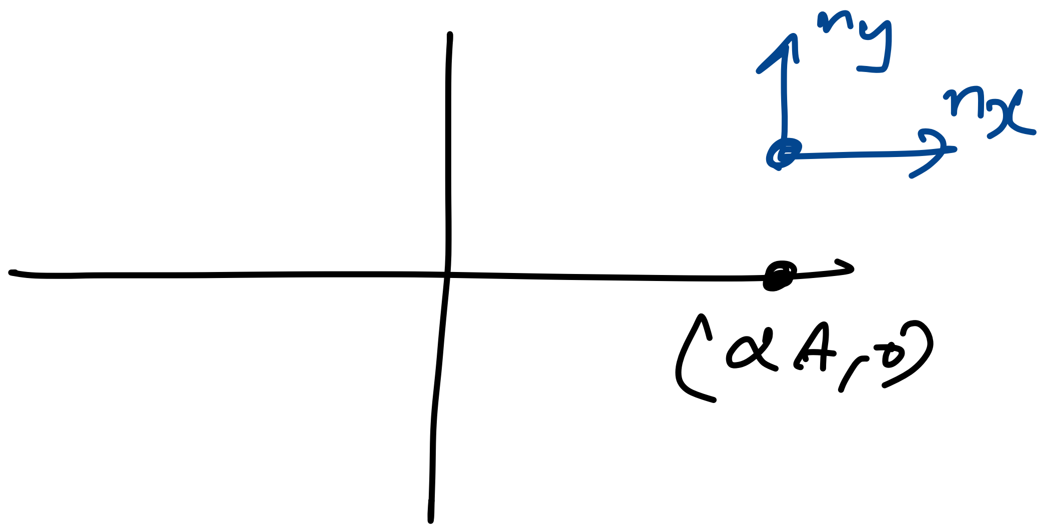
$$\mathbb{E}(n_x^2) - (\mathbb{E}(n_x))^2 = \frac{N_0}{2}$$

$$\mathbb{E}(n_x^2) = \frac{N_0}{2} = \mathbb{E}(n_y^2)$$

$$SNR = \frac{\alpha^2 A^2}{\frac{N_0}{2} + \frac{N_0}{2}} = \frac{\alpha^2 A^2}{N_0}.$$

is the SNR per symbol.

(Signal to Noise Ratio – SNR)



$$\mathbb{P}(\text{error in bit}) = \mathbb{P}(0/1) = \mathbb{P}(1/0)$$

by symmetry.

$$\mathbb{P}(\text{error in bit}) = \mathbb{P}(1)\cdot\mathbb{P}(0/1) + \mathbb{P}(0)\,\mathbb{P}(1/0)$$

$$= \tfrac{1}{2}\cdot\mathbb{P}(n_x \le -\alpha A) + \tfrac{1}{2}\cdot\mathbb{P}(n_x \ge \alpha A)$$

since $\mathbb{P}(n_x \le -\alpha A) = \mathbb{P}(n_x \ge \alpha A)$

we have

$$\mathbb{P}\left(\begin{matrix}\text{error in}\\ \text{bit}\end{matrix}\right) = \mathbb{P}(n_x \ge \alpha A)$$

$$= \frac{1}{\sigma\sqrt{2\pi}}\int_{\alpha A}^{\infty} \exp\left(-\frac{x^2}{2\sigma^2}\right)dx$$

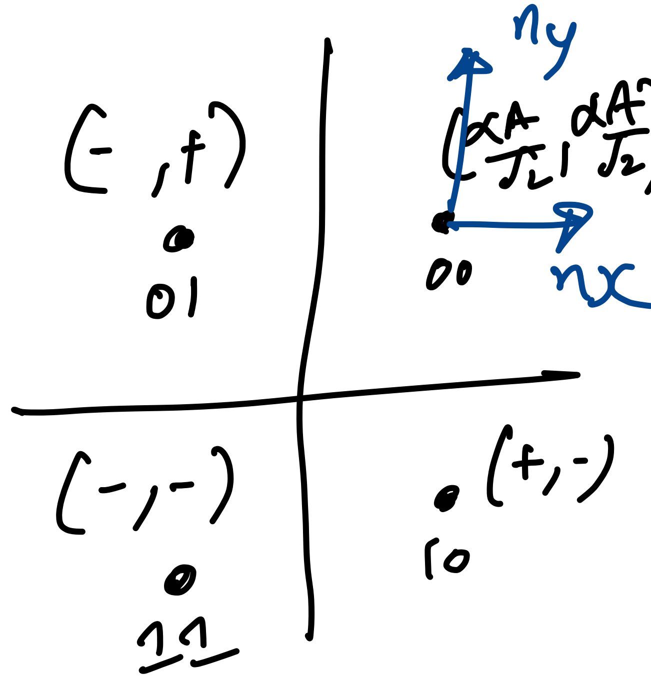
$$= \frac{1}{\sqrt{2\pi}}\int_{\frac{\alpha A}{\sigma}}^{\infty} \exp\left(-\frac{x^2}{2\sigma^2}\right)dx = Q\left(\frac{\alpha A\sqrt{2}}{\sqrt{N_0}}\right)$$

$$\mathbb{P}\left(\text{error in bit}\right) = Q\left(\frac{\alpha A\sqrt{2}}{\sqrt{N_0}}\right)$$

$$= Q\left(\sqrt{2 \cdot SNR}\right)$$

(Q2)

for a bit to be misinterpreted we require that noise in greater than $\frac{\alpha A}{\sqrt{2}}$



$$\mathbb{P}\left(\text{1st bit is wrong}\right)$$

$$= \left[\mathbb{P}(00) + \mathbb{P}(01)\right] \cdot \mathbb{P}\left(\infty \geq n_x \geq \frac{\alpha A}{\sqrt{2}}\right)$$

$$+ \left[\mathbb{P}(10) + \mathbb{P}(11)\right] \cdot \mathbb{P}\left(-\infty \leq n_x \leq -\frac{\alpha A}{\sqrt{2}}\right)$$

due to $N\left(0, \frac{N_0}{2}\right)$ is an even function and $\mathbb{P}(00)$, $\mathbb{P}(01)$, $\mathbb{P}(10)$, $\mathbb{P}(11)$ being equal.

$$\therefore \mathbb{P}\left(\begin{matrix}\text{1st bit}\\ \text{in wrong}\end{matrix}\right) = \left(\frac{1}{4}+\frac{1}{4}\right)\cdot \mathbb{P}\left(n_x \geq \alpha A/\sqrt{2}\right) + \left(\frac{1}{4}+\frac{1}{4}\right)\mathbb{P}\left(n_x \geq \frac{\alpha A}{\sqrt{2}}\right)$$

$$\therefore \mathbb{P}\left(\begin{matrix}\text{1st bit}\\ \text{in wrong}\end{matrix}\right) = \mathbb{P}\left(n_x \geq \alpha A/\sqrt{2}\right)$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{\alpha A/\sqrt{2}}^{\infty} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\frac{\alpha A}{\sqrt{2}\sigma}}^{\infty} \exp\left(-\frac{p^2}{2}\right) dp$$

$$\therefore P\left(\begin{matrix}\text{1st bit}\\ \text{in wrong}\end{matrix}\right) = Q\left(\frac{\alpha A}{\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{\sqrt{N_0}}\right)$$

$$= Q\left(\sqrt{SNR}\right)$$

Similarly probability of second bit in obtained in error.

$$\mathbb{P}' = [\mathbb{P}(00) + \mathbb{P}(10)] \cdot \mathbb{P}(n_y \geq \alpha A/\sqrt{2})$$
$$+ [\mathbb{P}(01) + \mathbb{P}(11)]\, \mathbb{P}(n_y \leq \frac{-\alpha A}{\sqrt{2}})$$

$$\mathbb{P}' = \mathbb{P}(n_y \geq \alpha A/\sqrt{2})$$

as $\mathbb{P}(n_y \geq \alpha A/\sqrt{2}) = \mathbb{P}(n_y \leq \alpha A/\sqrt{2})$

and $\mathbb{P}(00) = \mathbb{P}(01) = \mathbb{P}(11) = \mathbb{P}(10)$

$$\mathbb{P}' = \frac{1}{\sigma\sqrt{2\pi}} \int_{\frac{\alpha A}{\sqrt{2}}}^{\infty} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{\frac{\alpha A}{\sigma\sqrt{2}}}^{\infty} \exp\left(-\frac{p^2}{2}\right) dp \cdot \cancel{\sigma}$$

$$= Q\left(\frac{\alpha A}{\sigma\sqrt{2}}\right) = Q\left(\frac{\alpha A}{\sqrt{N_0}}\right)$$

$$= Q(\sqrt{SNR})$$

Yes, the two probabilities are equal

$$\mathbb{P}\left(\begin{matrix}\text{1st bit}\\ \text{is incorrect}\end{matrix}\right) = \mathbb{P}\left(\begin{matrix}\text{2nd bit}\\ \text{is incorrect}\end{matrix}\right)$$

The value obtained for (1) was

$$Q\left(\sqrt{2SNR}\right) < Q\left(\sqrt{SNR}\right)$$

as $Q(x)$ is a decreasing function of x. as

$$\sqrt{2SNR} > \sqrt{SNR} \Rightarrow Q\left(\sqrt{2SNR}\right) < Q\left(\sqrt{SNR}\right)$$

$$\therefore \mathbb{P}\left[\begin{matrix}\text{error in}\\ \text{a bit for}\\ \text{QPSK}\end{matrix}\right) > \mathbb{P}\left(\begin{matrix}\text{error in}\\ \text{a bit}\\ \text{for BPSK}\end{matrix}\right)$$