Control Systems Lab - Experiment 3
Noise Cancellation

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1 Aim

At the end of this experiment, we aim to achieve the following objectives:

- To design and implement an analog circuit for noise cancellation in headphones
- To achieve an attenuation of 20 dB, when a noise of 100 Hz frequency is applied.
- To design an analog compensator to stabilize the system, i.e. loop shaping of the loop transfer function

2 Procedure

We first calculated the output transfer characteristics from the headphone setup given. Once we had the data, we fed the data into MATLAB and obtained a second order approximation of the system. Now, we needed to decide a compensator $C(s)$ such that at $f = 100$ Hz, the output to noise transfer function attenuates by 20dB
The closed loop system would look like this:

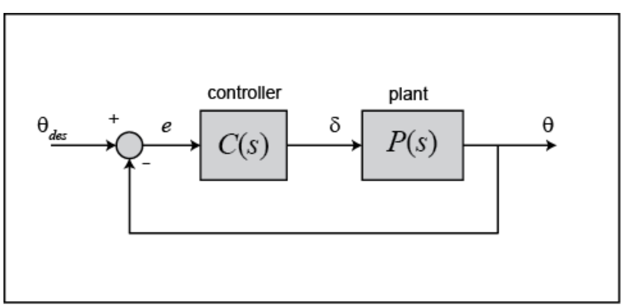

Figure 1: Unity Feedback System for system with Open Loop Transfer function $P(s) = G(s)$ connected to a compensator with open-loop transfer function $C(s)$

Now, we have the output $Y(s)$ and input $X(s)$ with noise $N(s)$ being added at the output node (just at the point where feedback is being taken). Calculating the output transfer functions, we get,

$$Y(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} X(s) + \frac{1}{1 + C(s)G(s)} N(s)$$

Given that we need 20dB attenuation for noise at 100 Hz, we need,

$$20log(\frac{1}{1 + C(s)G(s)}) = -20 \Rightarrow C(s)G(s) = 9 \Rightarrow 20log(\frac{C(s)G(s)}{1 + C(s)G(s)} = 20log(0.9) = -0.91$$

Thus we need a gain of -0.91 in our closed loop input to output transfer function. Also, we need Gain Margin > 5 and Phase Margin $> 30^o$. Plotting the observed values of phase response and gain from the headphone, we first obtain a 10th order (any large order approximation for the transfer function would work) to get $G(s)$. Then, upon observing the bode plot of $G(s)$, we decided to build a compensator $C(s)$ consisting of a lead and a lag compensator.

3 Result

The compensator $C(s)$ would be of the form:

$$C(s) = k\frac{s + a}{s + b}\frac{s + c}{s + d} \tag{1}$$

Upon careful tuning and observing from the bode plot of the closed loop transfer function $\frac{C(s)G(s)}{1+C(s)G(s)}$, we obtain,

- k = 0.0779
- a = 1527
- b = 1312
- c = 1167
- d = 1423

This gives us the desired gain of -0.914 at 100Hz for the closed loop input-output transfer function.

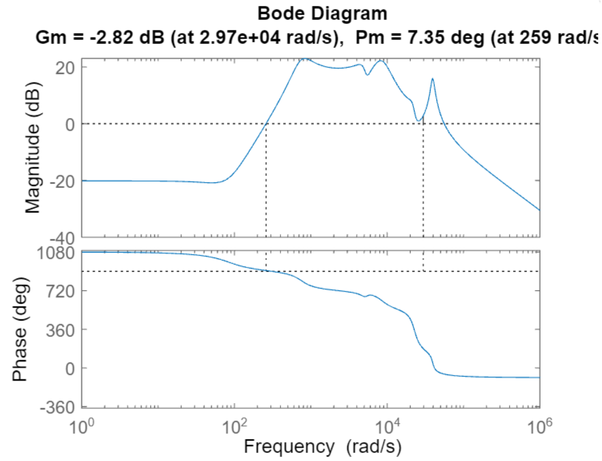

Figure 2: Magnitude Response for the Observed Data

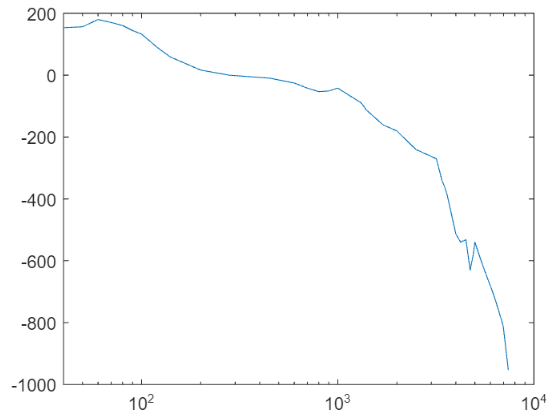

Figure 3: Phase Response for the Observed Data

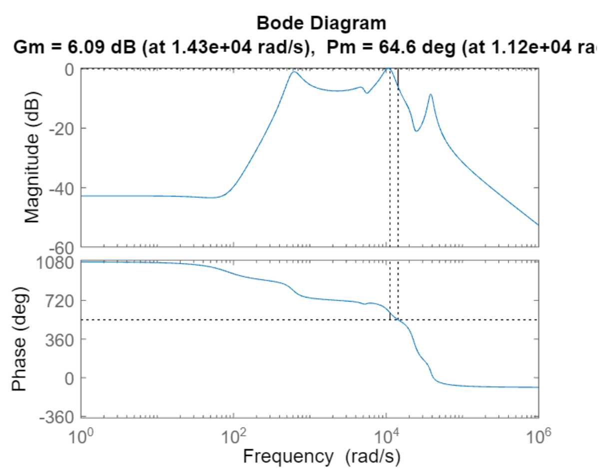

Figure 4: Phase and Magnitude Response after compensation || Observe that the gain at 100 Hz is -0.914 dB

4 Problems faced and their solution

- We initially used a single lag compensator. Using the SISO tool on MATLAB, we tried obtaining a compensator which met the required specifications. We then realised the need to use 2 compensators, a lag and a lead compensator, which upon tuning with the SISO tool on MATLAB gave us the desired compensator.