

CS 182 lecture 18: Latent Variable Models

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Recap: $p(x) = \int p(x|z)p(z) dz$

$$\theta \leftarrow \arg \max_{\theta} \frac{1}{N} \sum_i \mathbb{E}_{z \sim p(z|x_i)} [\log p_{\theta}(x_i|z)] \Rightarrow \theta \leftarrow \arg \max_{\theta} \frac{1}{N} \sum_i \mathcal{L}_i(p, q_i)$$

Variational approximation of $p(z|x_i)$

$q_i(z) = N(\mu_i, \sigma_i)$ should approximate

can bound $\log p(x_i)$

$$\log p(x_i) = \log \int p(x_i|z)p(z) dz$$

$$= \log \int p(x_i|z)p(z) \frac{q_i(z)}{q_i(z)} dz$$

$$= \log \mathbb{E}_{z \sim q_i(z)} \left[\frac{p(x_i|z)p(z)}{q_i(z)} \right]$$

$$\geq \mathbb{E}_{z \sim q_i(z)} \left[\log \frac{p(x_i|z)p(z)}{q_i(z)} \right]$$

$$= \mathbb{E}_{z \sim q_i(z)} [\log p(x_i|z) + \log p(z)] - \mathbb{E}_{z \sim q_i(z)} [\log q_i(z)]$$

$$= \mathbb{E}_{z \sim q_i(z)} [\log p(x_i|z) + \log p(z)] + \mathcal{H}(q_i)$$

maximize this \rightarrow maximize $\log p(x_i)$

"Good" $q_i(z) = D_{KL}(q_i(z) || p(z|x_i))$ should be low.

$$= -\mathcal{L}_i(p, q_i) + \log p(x_i)$$

$$\log p(x_i) = D_{KL}(q_i(z) || p(z|x_i)) + \mathcal{L}_i(p, q_i)$$

min! \leftarrow causal! \rightarrow max!
independent of q_i !

unbiased single estimate of \mathcal{L}_i

$$x_i \rightarrow \mu_i \rightarrow \mu_i + \epsilon \sigma_i = z \rightarrow \theta \rightarrow p_{\theta}(x_i|z)$$

$$\text{VAE objective: } \max_{\theta, \phi} \frac{1}{N} \sum_i \log p_{\theta}(x_i | \mu_{\phi}(x_i) + \epsilon \sigma_{\phi}(x_i)) - D_{KL}(q_{\phi}(z|x_i) || p(z))$$

Conditional models:

$$\mathcal{L}_i = \mathbb{E}_{z \sim q_{\phi}(z|x_i, y_i)} [\log p_{\theta}(y_i|x_i, z) + \log p(z|x_i)] + \mathcal{H}(q_{\phi}(z|x_i, y_i))$$

$$x_i \rightarrow \mu_{\phi}(x_i, y_i) \rightarrow \mu_{\phi} + \epsilon \sigma_{\phi} = z \rightarrow \theta \rightarrow p_{\theta}(y_i|x_i, z)$$

flexible w/ convolutions.

in practice: common for VAE/VAE to ignore latent codes or not compress them at all.

control D_{KL} carefully.

D_{KL} too low $\rightarrow p_{\theta}(x|z) \rightarrow p(x)$. (blurry "average" image outputted)
 D_{KL} too high $\rightarrow q_{\phi}(z|x)$ very far from $p(z)$, "overfit", garbage output

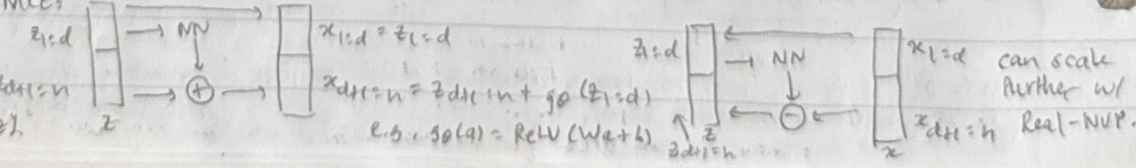
use β parameters to adjust regularizer strength.

Invertible Model: $x = f(z)$

(learn invertible mapping directly)

$$\text{Training Objective: } \mathcal{L} = \max_{\theta} \frac{1}{N} \sum_i \log p(x_i) \rightarrow \max_{\theta} \frac{1}{N} \sum_i \log p(f^{-1}(x_i)) - \log |\det(\frac{df(z)}{dz})|$$

Normalizing Flow model: multiple layers of invertible transformations (layers)



Jensen's inequality
 $\log \mathbb{E}[y] \geq \mathbb{E}[\log y]$
(log is concave)
Entropy ("disorder")
 $\mathcal{H}(p) = -\mathbb{E}_{p(x)} [\log p(x)]$
 $= -\int p(x) \log p(x) dx$

KL-Divergence:

$$D_{KL}(q || p) = \mathbb{E}_{q(x)} [\log \frac{q(x)}{p(x)}]$$

$$= \mathbb{E}_{q(x)} [\log q(x)] - \mathbb{E}_{q(x)} [\log p(x)]$$

$$= -\mathcal{H}(q) + \mathbb{E}_{q(x)} [\log p(x)]$$

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