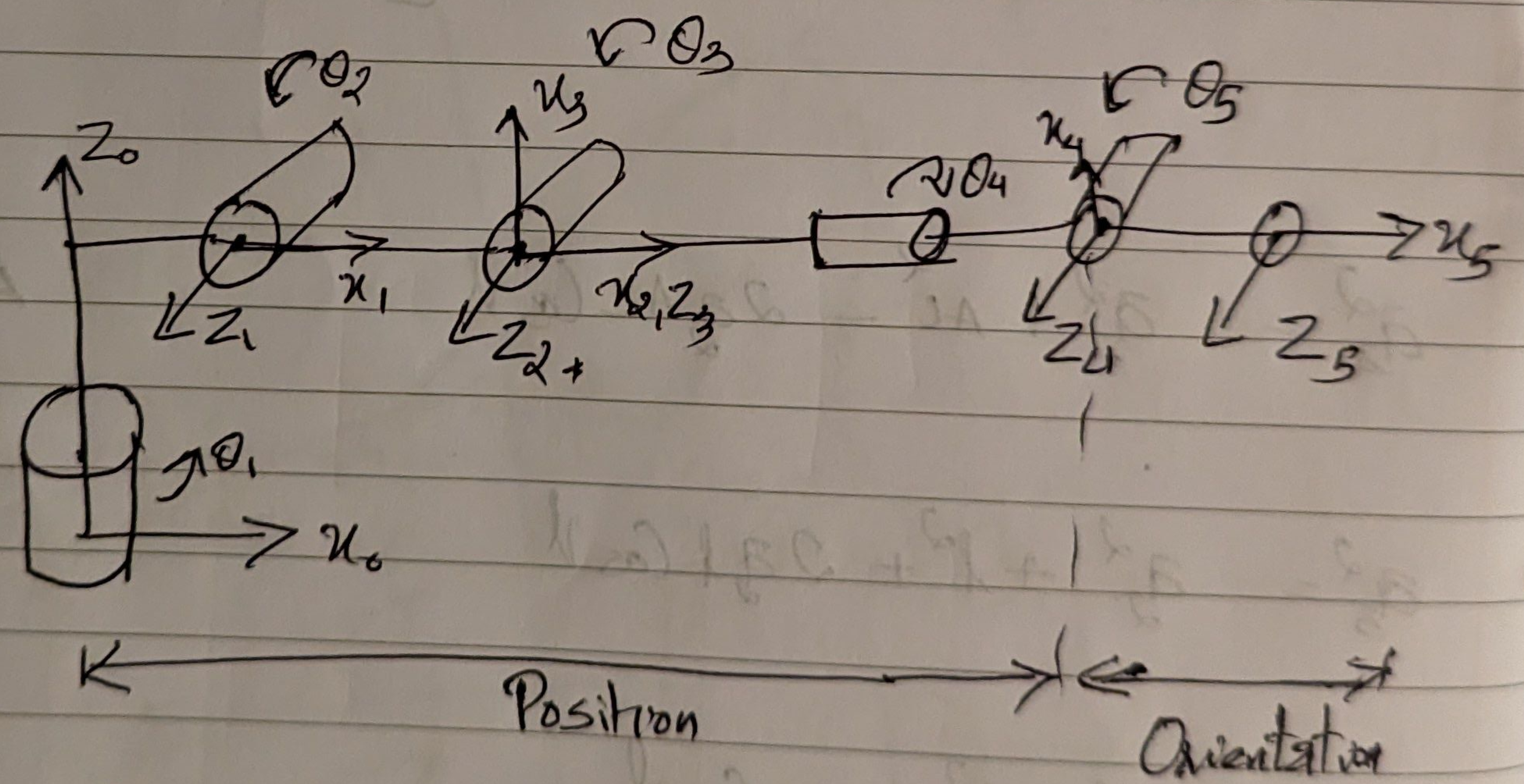
Inverse kinematics

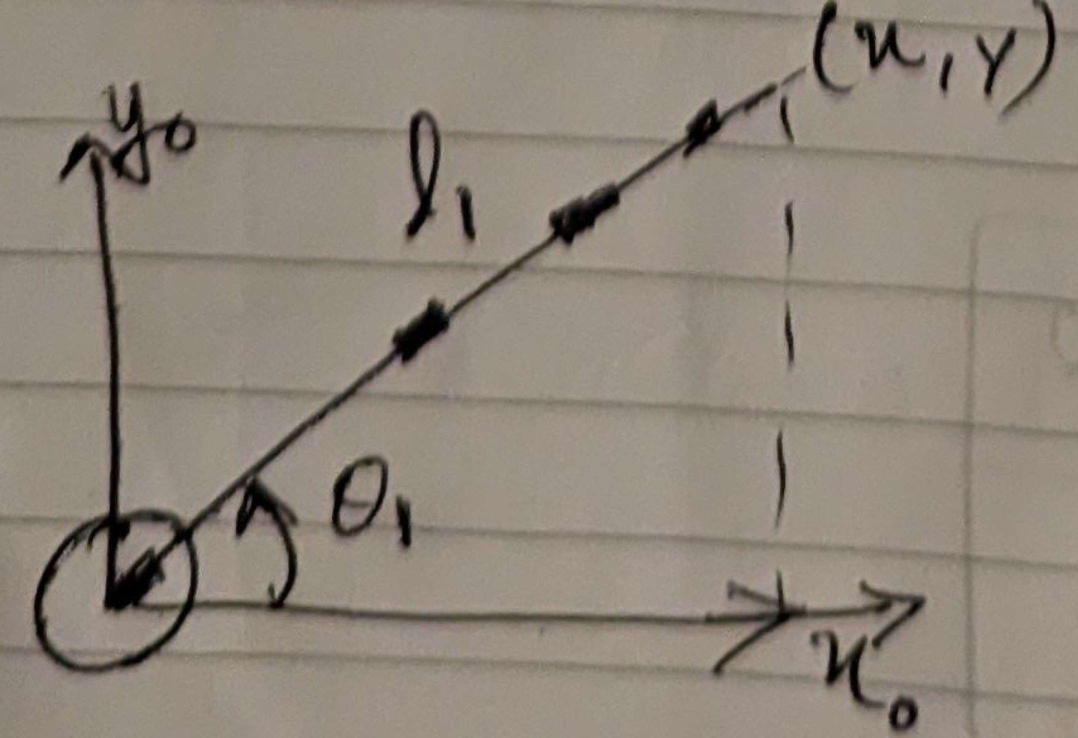
Split the manipulator to two parts.

1. For position
2. For orientation.

Frame : $0 \rightarrow 3 \quad \Rightarrow$ Position
$3 \rightarrow 5 \quad \Rightarrow$ Orientation

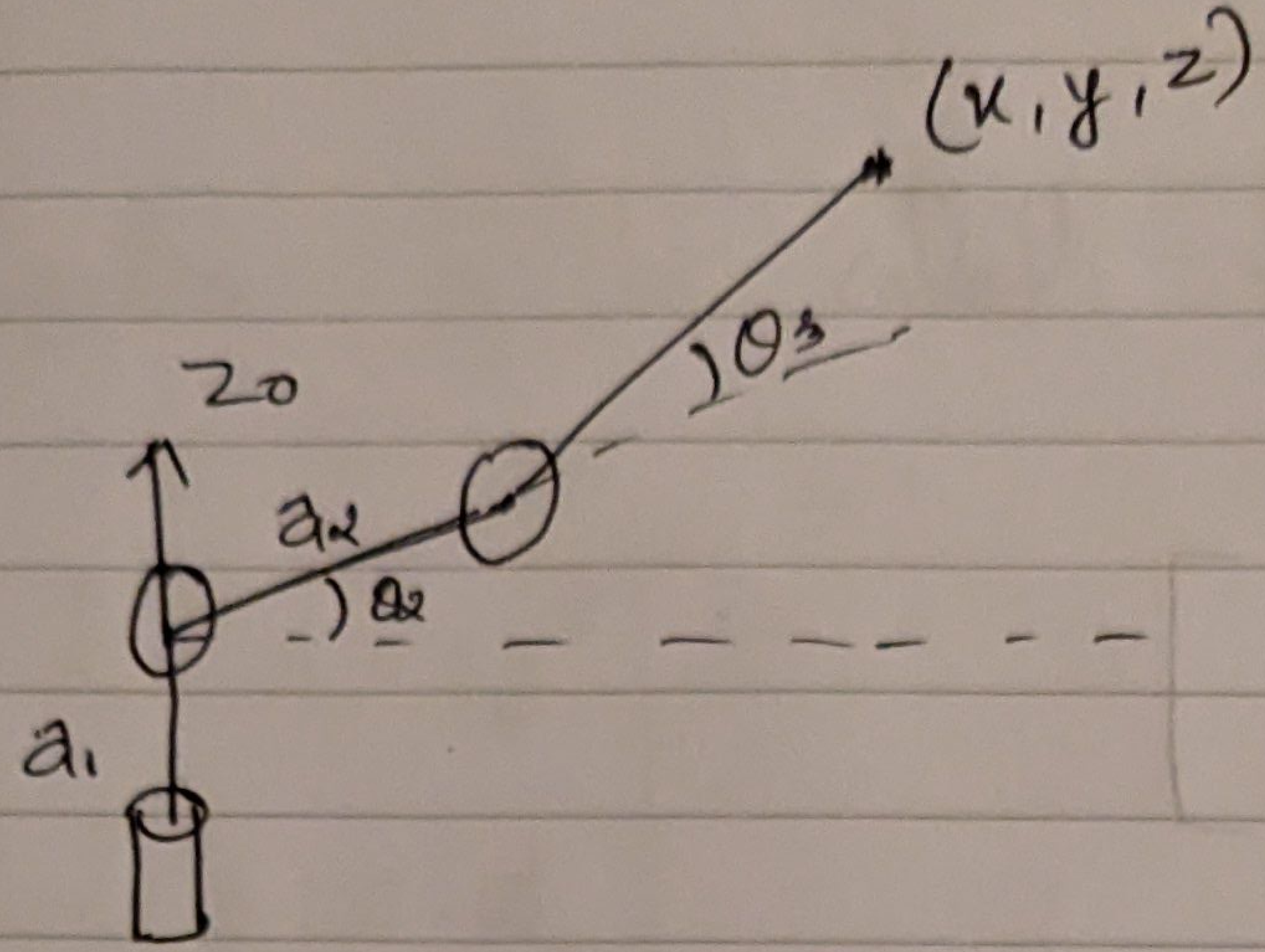


Top-View:

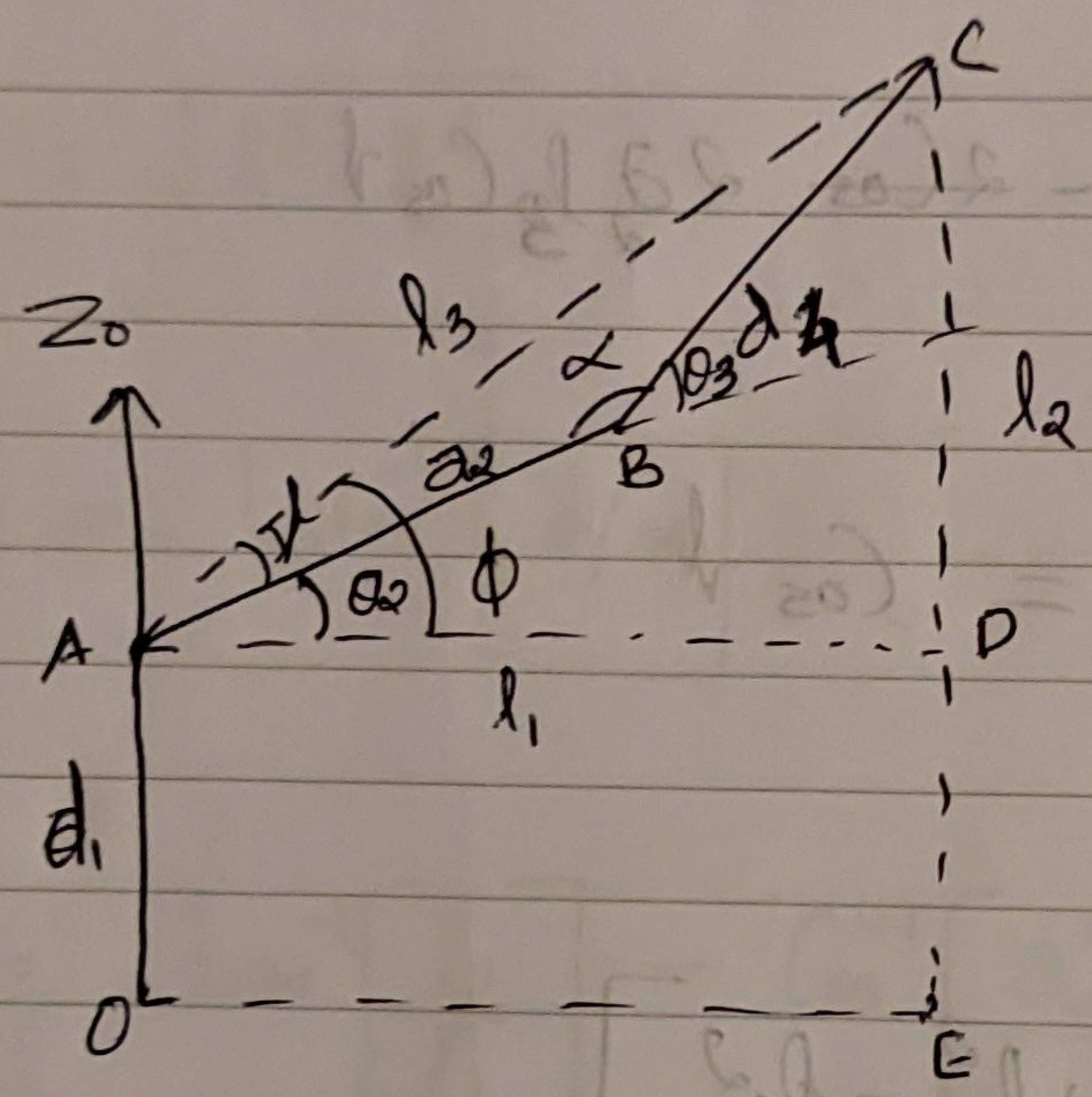


$$\theta_1 = \tan^{-1}\left[y/x \right]$$

$$l_1 = \sqrt{x^2 + y^2}$$



Side view



$$AC^2 = a_2^2 + d_4^2 - 2 a_2 d_4 \cos(\alpha)$$

$AD = l_1$; $CD = l_2 = z - d_1$; $l_3 = \sqrt{l_1^2 + l_2^2}$

$\alpha = 180 - \theta_3$

$$\Rightarrow \cos\theta_3 = \frac{l_3^2 - a_2^2 - d_4^2}{2 a_2 d_4}$$

$$\theta_3 = \cos^{-1}\left[\frac{l_3^2 - a_2^2 - d_4^2}{2 a_2 d_4}\right]$$

$\theta_2 = \phi - \vartheta$

$\phi = \tan^{-1}\left[\frac{l_2}{l_1}\right]$

$d_4^2 = a_2^2 + l_3^2 - \cancel{2\cos}\ 2a_2 l_3 \cos\vartheta$

$\frac{a_2^2 + l_3^2 - d_4^2}{2a_2 l_3} = \cos\vartheta$

$\vartheta = \cos^{-1}\left[\frac{a_2^2 + l_3^2 - d_4^2}{2a_2 l_3}\right]$

$\theta_2 = \phi - \vartheta$

Equations to be coded for position:

$$l_1 = \sqrt{x^2 + y^2}$$

$$l_2 = z - d_1$$

$$l_3 = \sqrt{l_1^2 + l_2^2}$$

$$\theta_1 = \tan^{-1}\left[y/x \right]$$

$$\theta_2 = \tan^{-1}\left[\frac{l_2}{l_1}\right] - \cos^{-1}\left[\frac{a_2^2 + l_3^2 - d_4^2}{2 a_2 l_3}\right]$$

$$\theta_3 = \cos^{-1}\left[\frac{l_3^2 - a_2^2 - d_4^2}{2 a_2 d_4}\right]$$

Rotation Matrices:

$$R_1^0 = \begin{bmatrix} c(\theta_1) & 0 & s(\theta_1) \\ s(\theta_1) & 0 & -c(\theta_1) \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_2^1 = \begin{bmatrix} c(\theta_2) & -s(\theta_2) & 0 \\ s(\theta_2) & c(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3^2 = \begin{bmatrix} -s(\theta_3) & 0 & c(\theta_3) \\ c(\theta_3) & 0 & s(\theta_3) \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_4^3 = \begin{bmatrix} c(\theta_4) & 0 & -s(\theta_4) \\ s(\theta_4) & 0 & c(\theta_4) \\ 0 & -1 & 0 \end{bmatrix}$$

$$R_5^4 = \begin{bmatrix} s(\theta_5) & c(\theta_5) & 0 \\ -c(\theta_5) & s(\theta_5) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^0 = \begin{bmatrix} c(\theta_1) \cdot c(\theta_2) & -c(\theta_1) s(\theta_2) & s(\theta_1) \\ s(\theta_1) \cdot c(\theta_2) & -s(\theta_1) s(\theta_2) & -\cos(\theta_1) \\ s(\theta_2) & c(\theta_2) & 0 \end{bmatrix}$$

$$R_3^0 = \begin{bmatrix} c(\theta_1) c(\theta_2) & -c(\theta_1) s(\theta_2) & s(\theta_1) \\ s(\theta_1) c(\theta_2) & -s(\theta_1) s(\theta_2) & -c(\theta_1) \\ s(\theta_2) & c(\theta_2) & 0 \end{bmatrix} \begin{bmatrix} -s(\theta_3) & 0 & c(\theta_3) \\ c(\theta_3) & 0 & s(\theta_3) \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -c(\theta_1) c(\theta_2) s(\theta_3) - c(\theta_1) s(\theta_2) c(\theta_3) & s(\theta_1) & c(\theta_1) c(\theta_2) c(\theta_3) - c(\theta_1) s(\theta_2) s(\theta_3) \\ -s(\theta_1) c(\theta_2) s(\theta_3) - s(\theta_1) s(\theta_2) c(\theta_3) & -c(\theta_1) & s(\theta_1) c(\theta_2) c(\theta_3) - s(\theta_1) s(\theta_2) s(\theta_3) \\ -s(\theta_2) s(\theta_3) + c(\theta_2) c(\theta_3) & 0 & s(\theta_2) c(\theta_3) + c(\theta_2) s(\theta_3) \end{bmatrix}$$

$R_5^0 \Rightarrow$ Desired rotation matrix

$$R_5^3 = \begin{bmatrix} c(\theta_4) s(\theta_5) & c(\theta_4) \cdot c(\theta_5) & -s(\theta_4) \\ s(\theta_4) s(\theta_5) & s(\theta_4) c(\theta_5) & c(\theta_4) \\ +c(\theta_5) & -s(\theta_5) & 0 \end{bmatrix}$$