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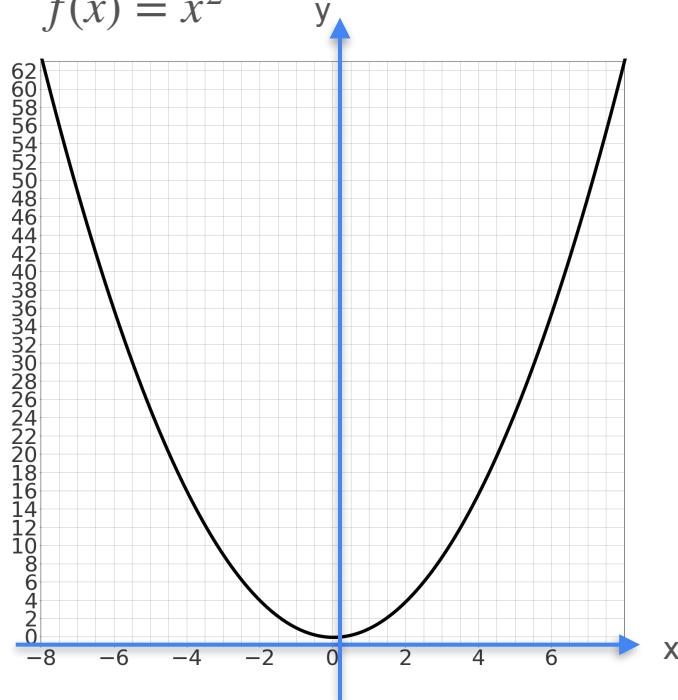
Gradients and Gradient Descent

Tangent planes

Functions of Two Variables

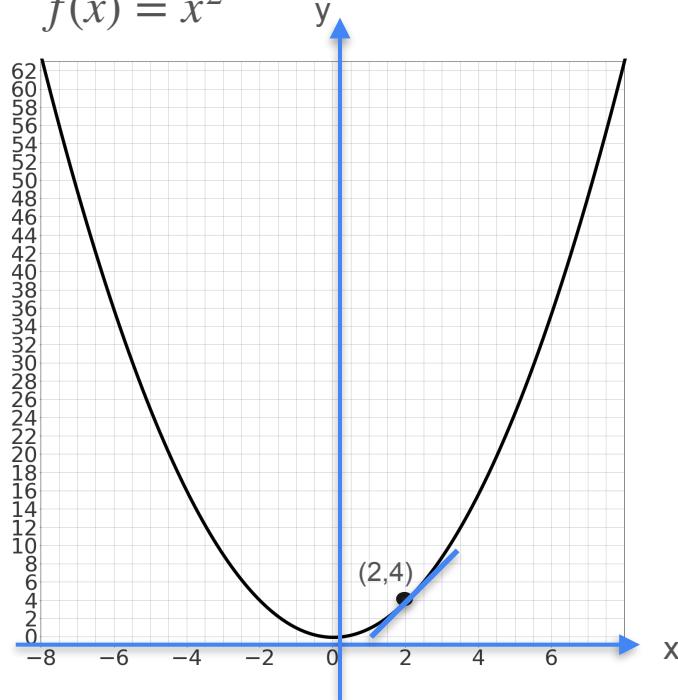
Functions of Two Variables

$$f(x) = x^2$$



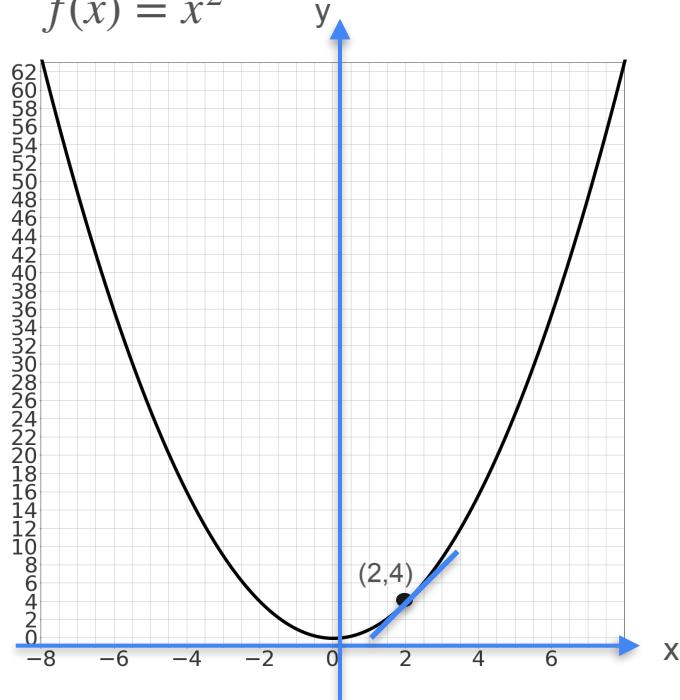
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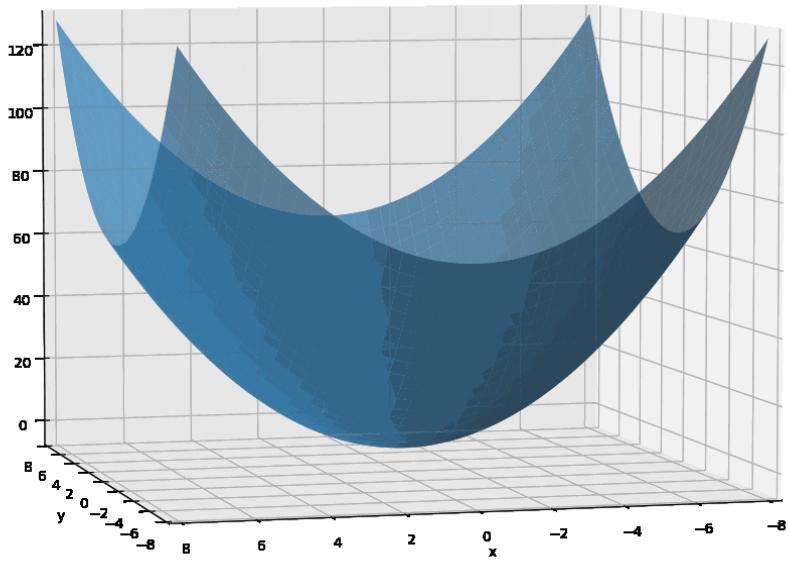


Functions of Two Variables

$$f(x) = x^2$$

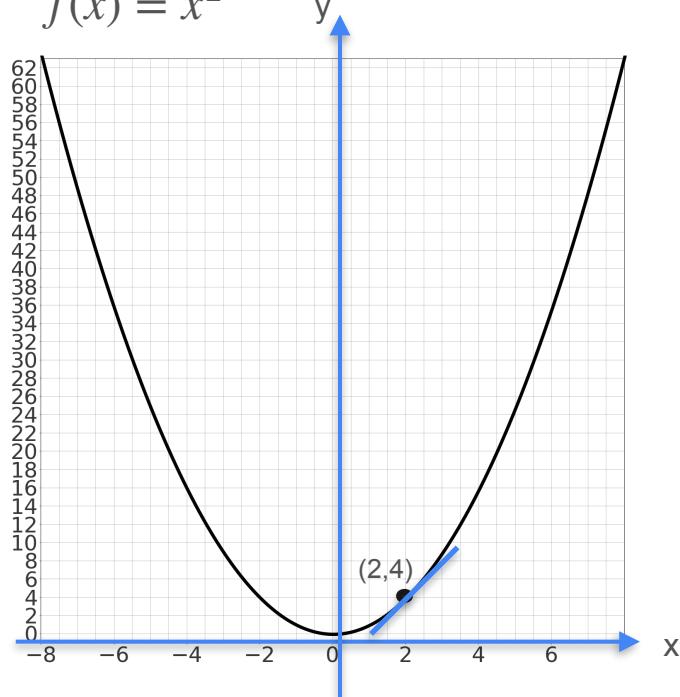


$$f(x, y) = x^2 + y^2$$

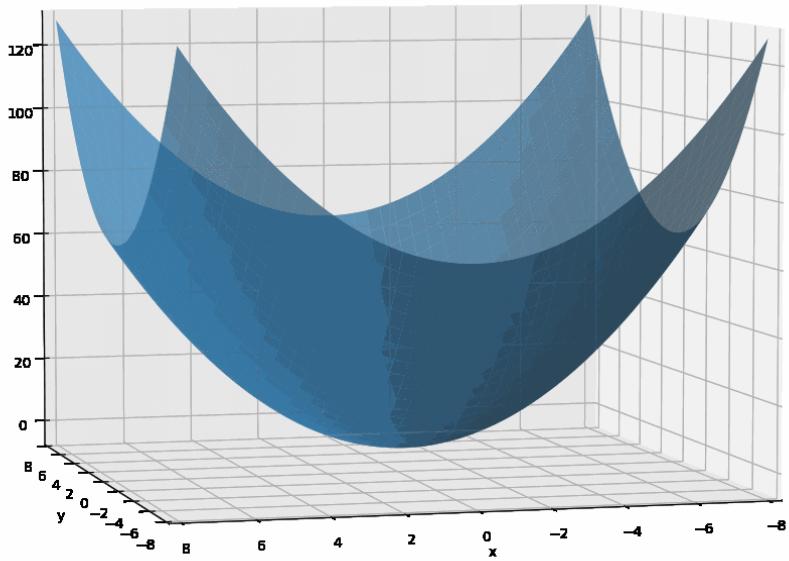


Functions of Two Variables

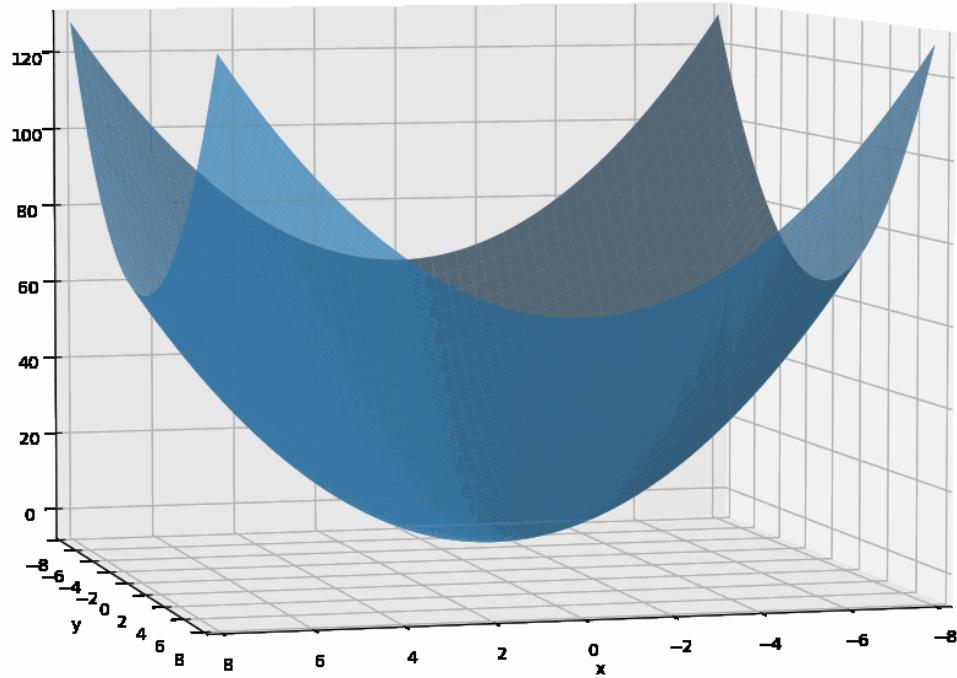
$$f(x) = x^2$$



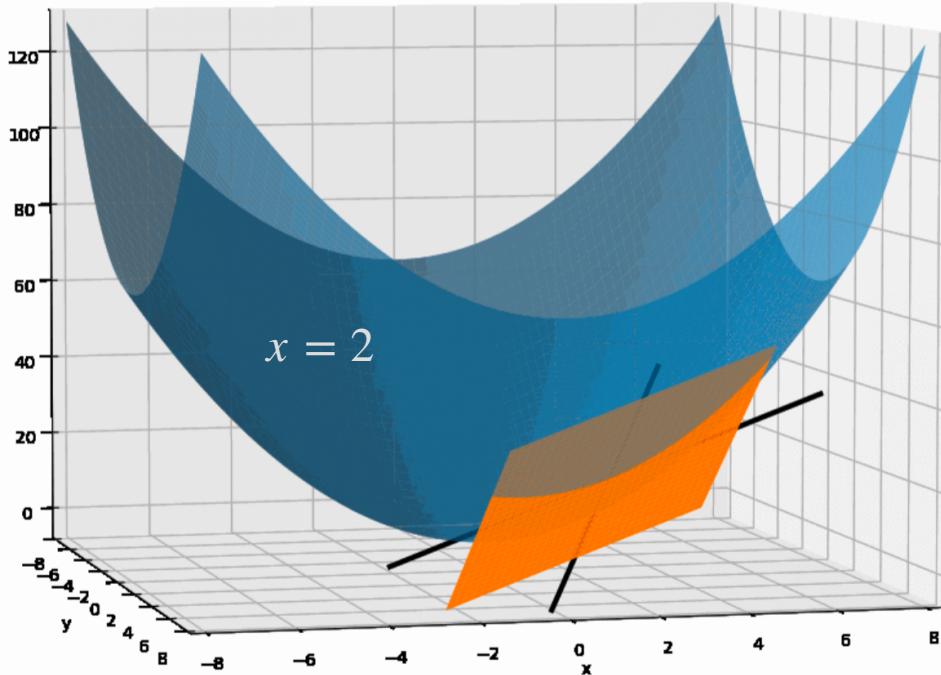
$$f(x, y) = x^2 + y^2$$



Finding the Tangent Plane



Finding the Tangent Plane



Fix $y=4$ $f(x,4) = x^2 + 4^2$

$$\frac{d}{dx} (f(x,4)) = 2x$$

Fix $x=2$ $f(2,y) = 2^2 + y^2$

$$\frac{d}{dy} (f(2,y)) = 2y$$

The tangent plane contains both tangent lines.

Video 2: Introduction to Partial Derivatives

Example with the parabola, show tangent plane and slices

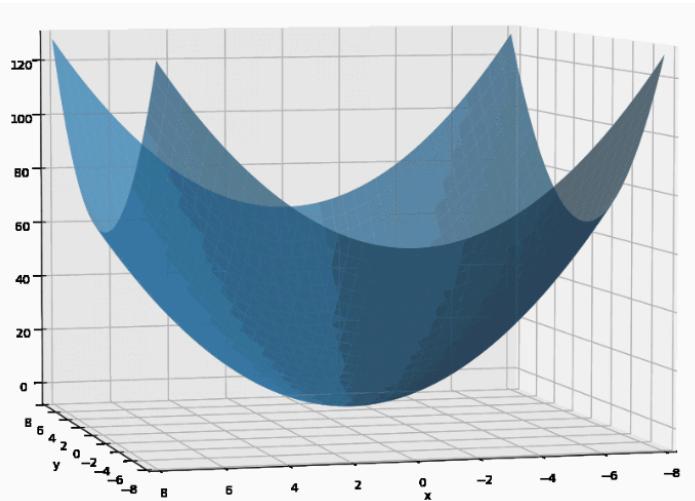


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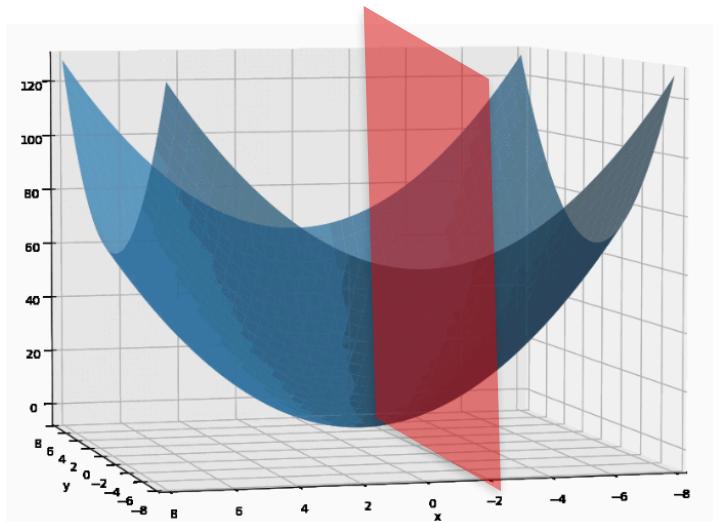
Gradients and Gradient Descent

Partial derivatives - Part 1

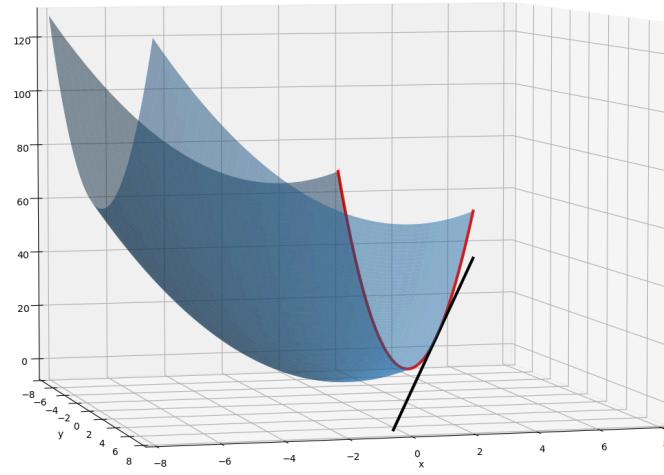
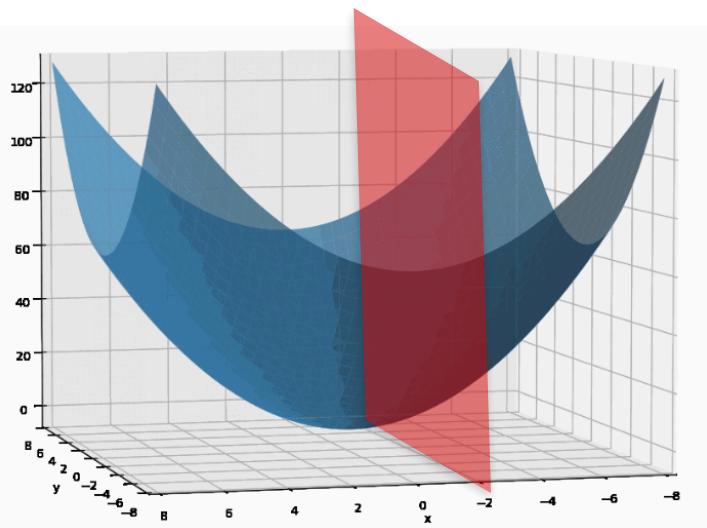
Slicing the Space



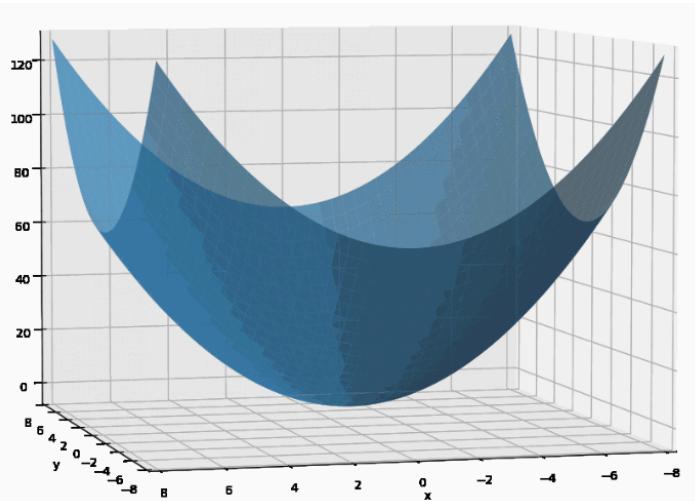
Slicing the Space



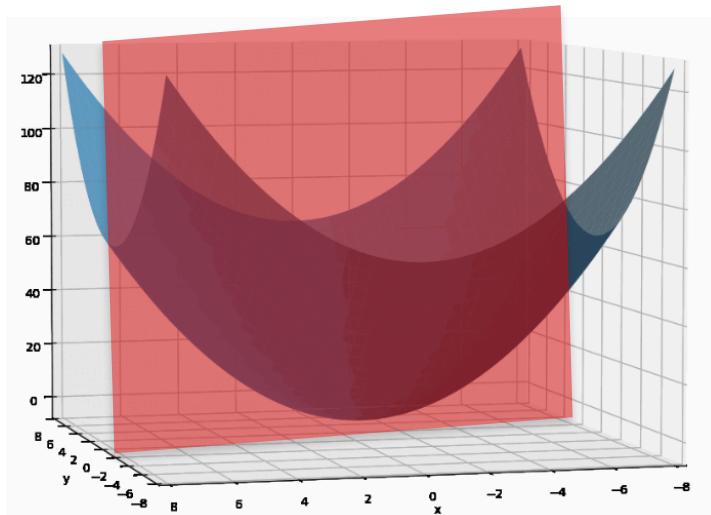
Slicing the Space



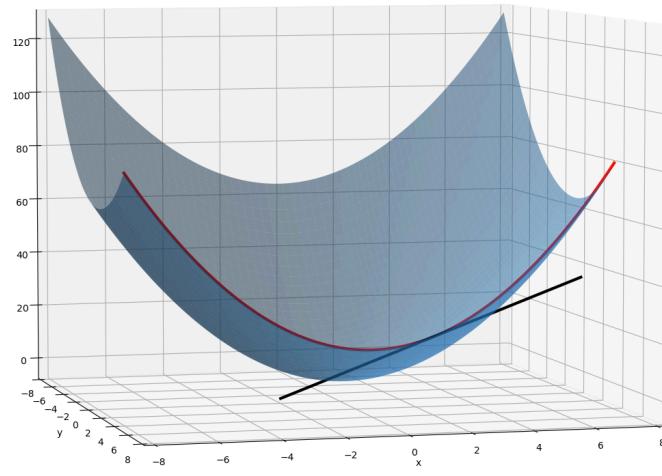
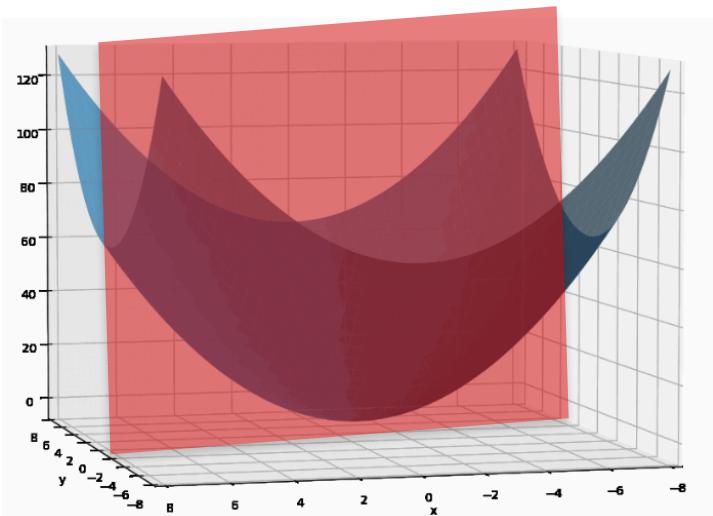
Slicing the Space



Slicing the Space

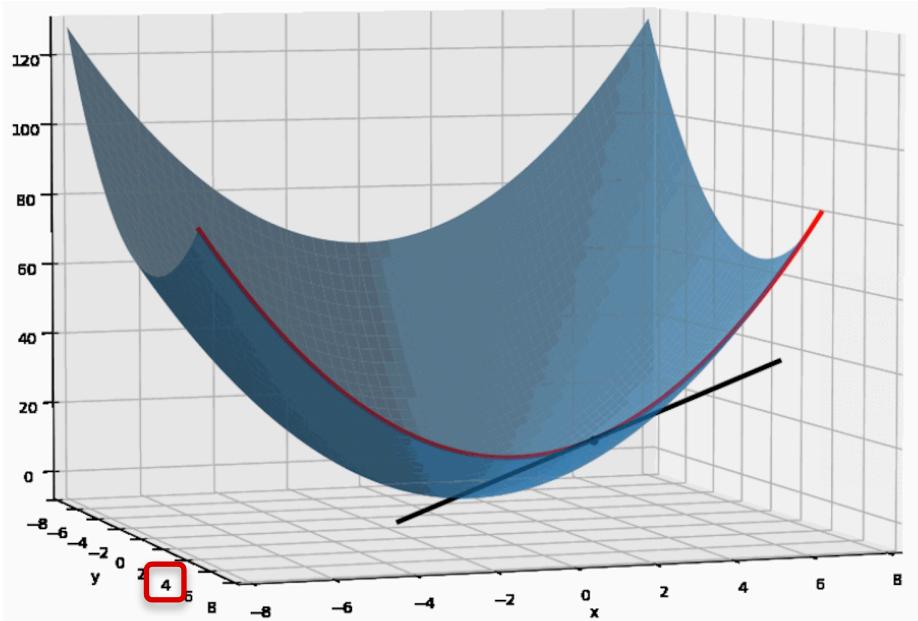


Slicing the Space



Partial Derivatives

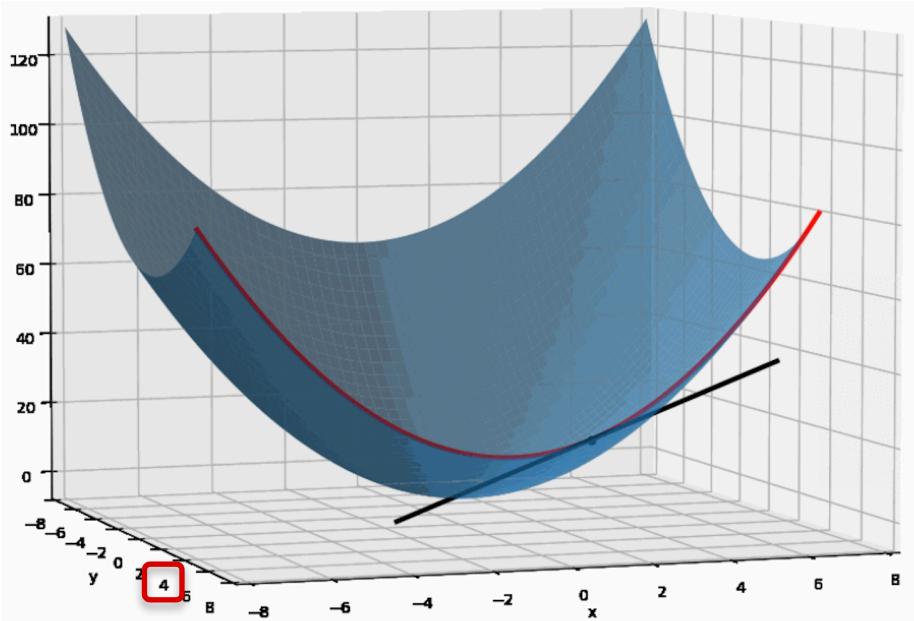
$$f(x, y) = x^2 + y^2$$



Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant

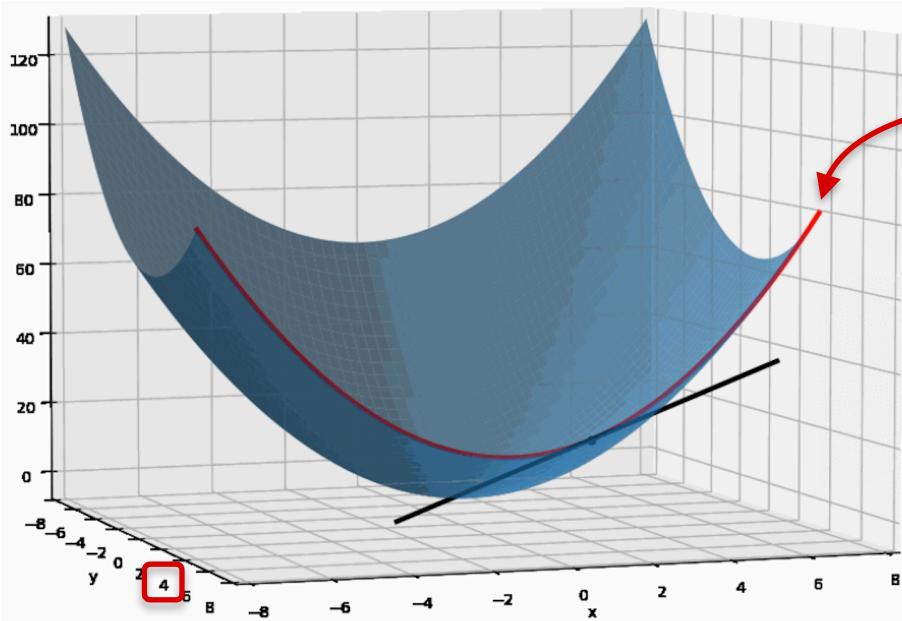


Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant

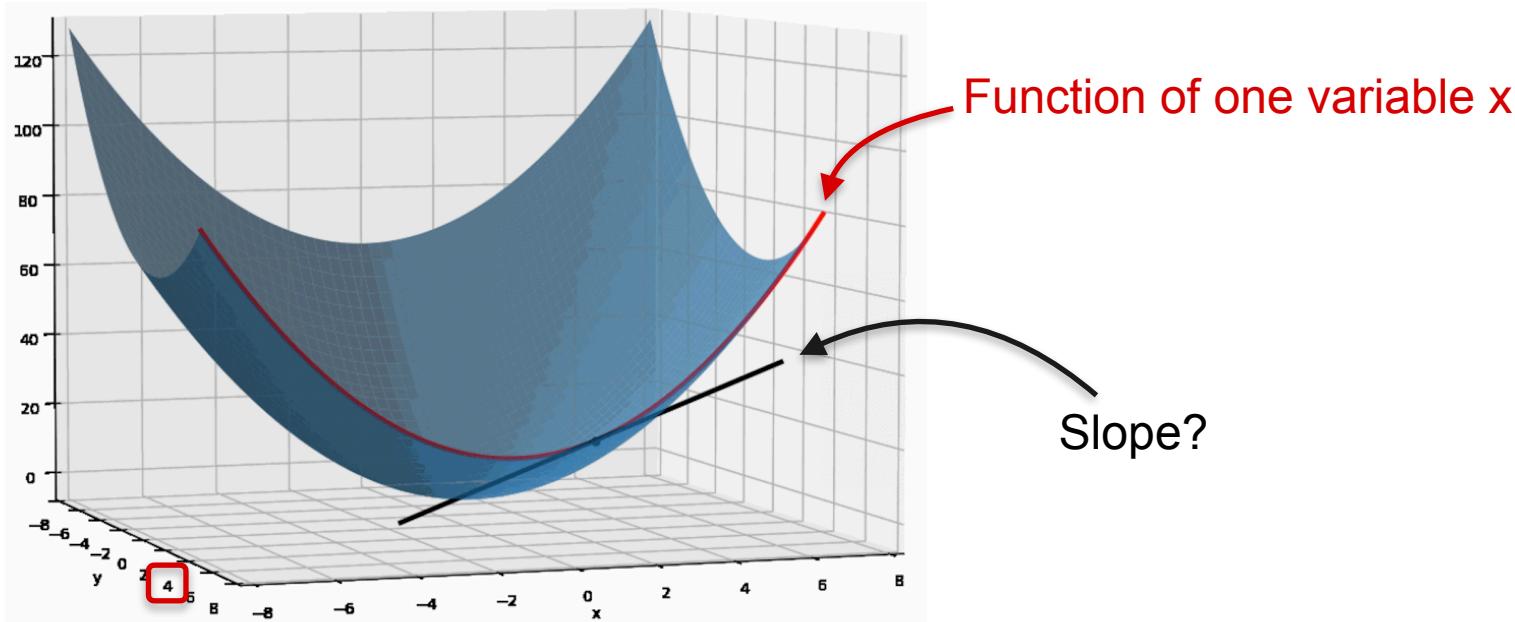
Function of one variable x



Partial Derivatives

$$f(x, y) = x^2 + y^2$$

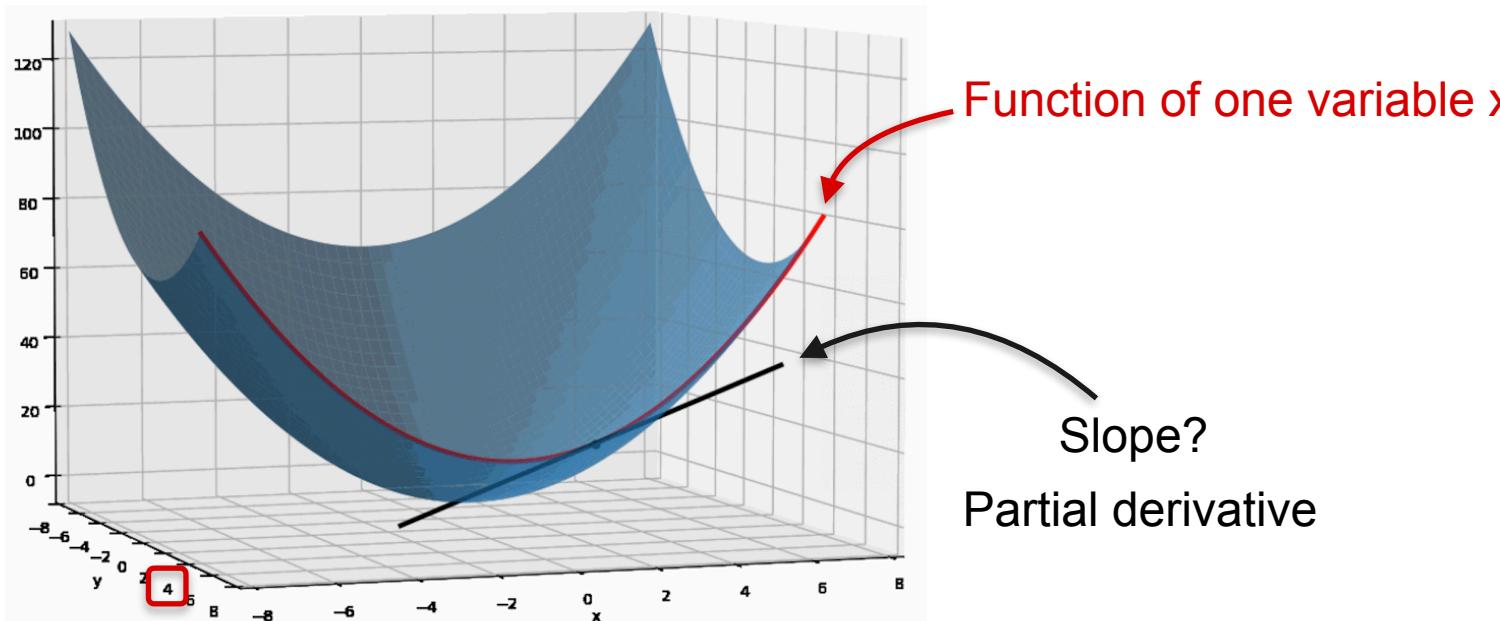
Treat y as a constant



Partial Derivatives

$$f(x, y) = x^2 + y^2$$

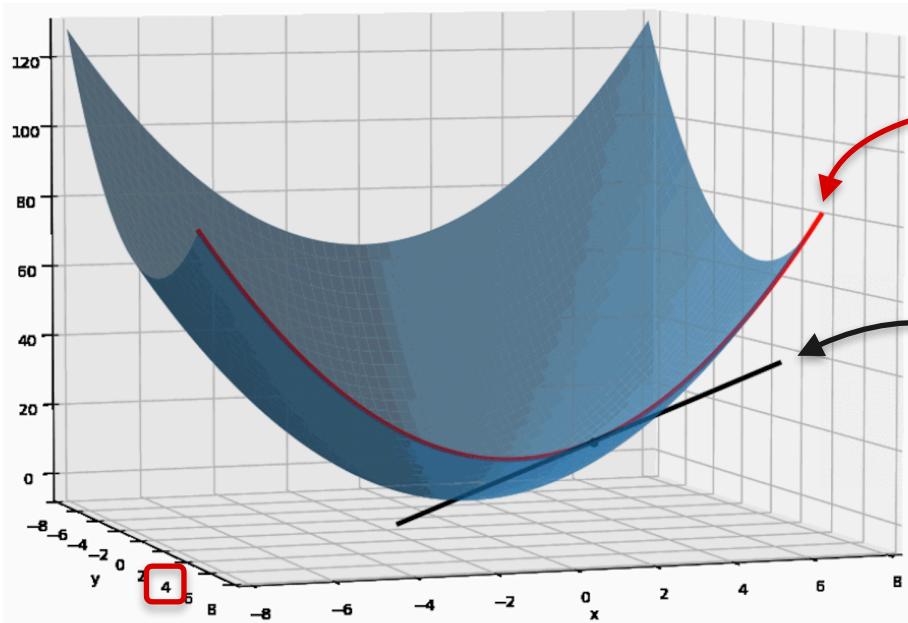
Treat y as a constant



Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



$$f(x, y) = x^2 + y^2$$

Function of one variable x

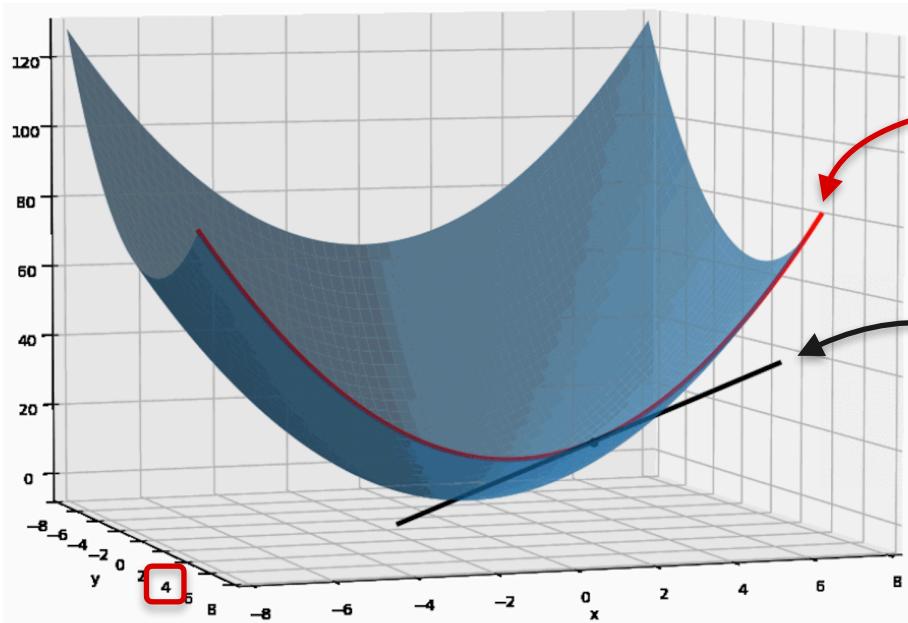
Slope?

Partial derivative

Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



Function of one variable x

Constant

$$f(x, y) = x^2 + \boxed{y^2}$$

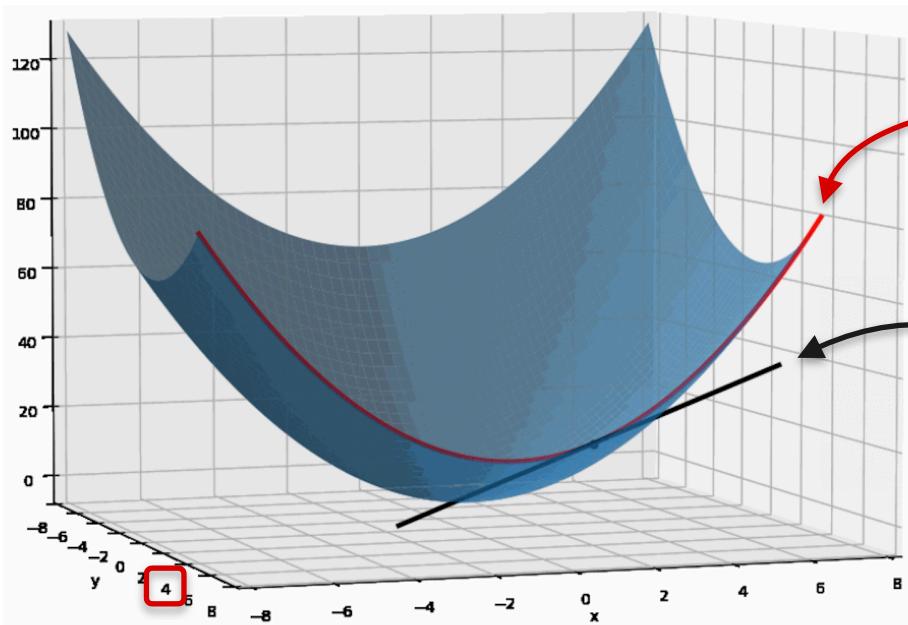
Slope?

Partial derivative

Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



Function of one variable x

Slope?

Partial derivative

Constant

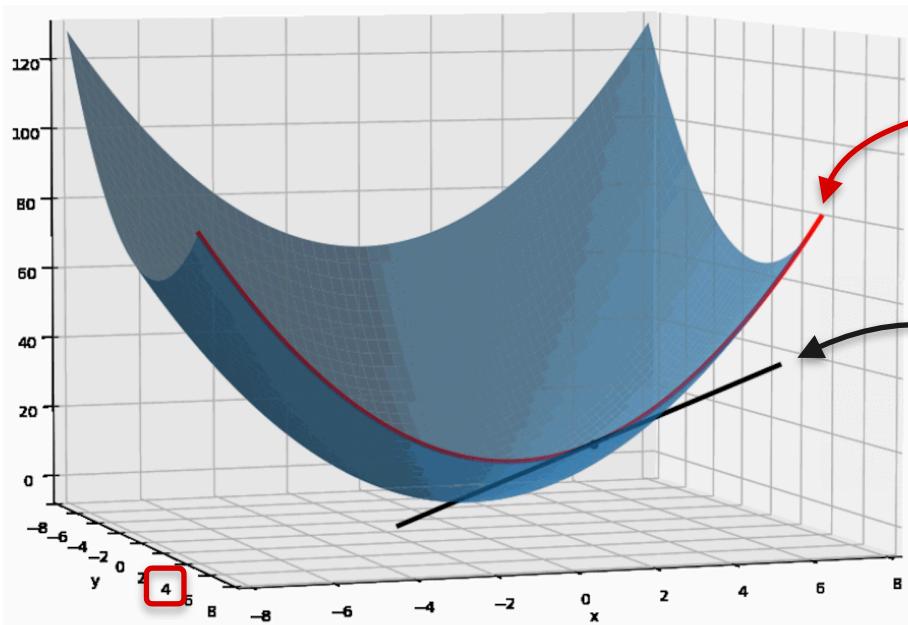
$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = 2x + 0$$

Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



Function of one variable x

Slope?

Partial derivative

Constant

$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = 2x + \boxed{0}$$

Derivative = 0

Partial Derivatives

Partial Derivatives

$$x^2 + y^2$$

Partial Derivatives

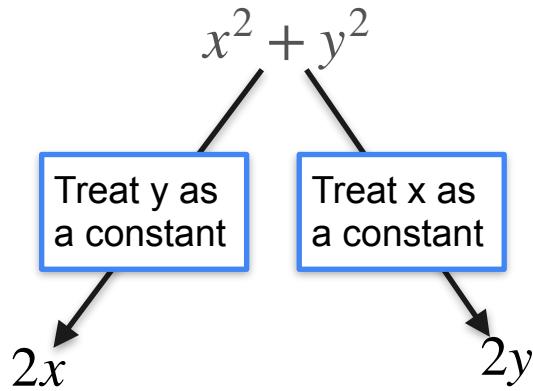
$$x^2 + y^2$$

Treat y as
a constant

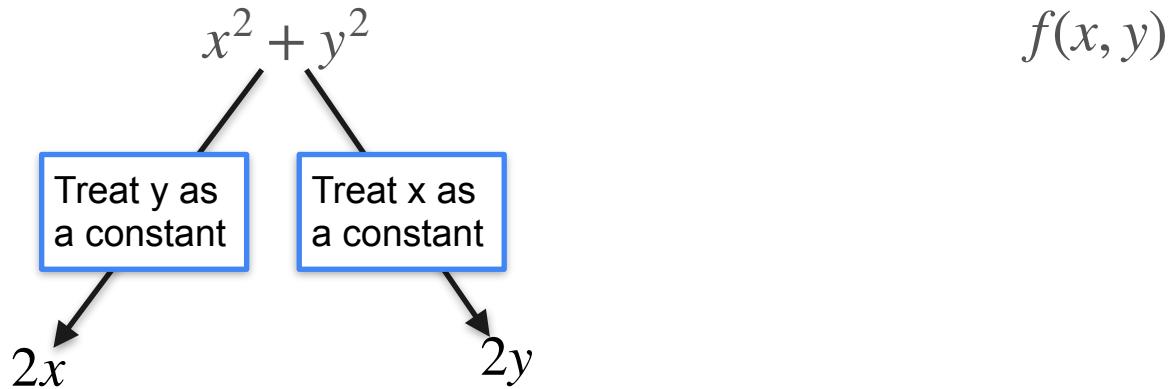
$$2x$$

The diagram illustrates the calculation of the partial derivative of the function $x^2 + y^2$ with respect to x . A blue box contains the text "Treat y as a constant". An arrow points from this box to the term $2x$, indicating that since y is treated as a constant, only the term containing x is differentiated.

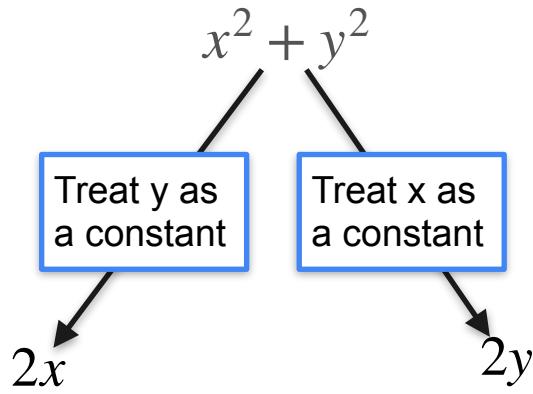
Partial Derivatives



Partial Derivatives

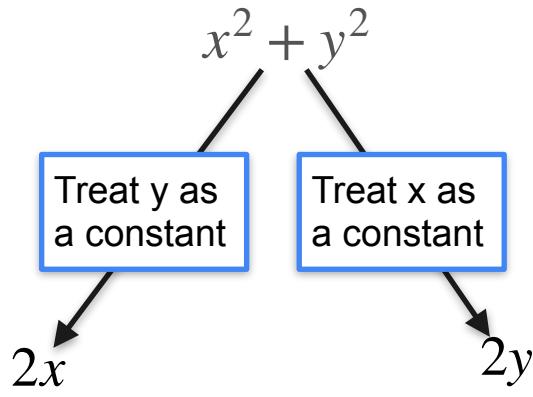


Partial Derivatives



A diagram illustrating the formula for the partial derivative f_x . At the top right is the function $f(x, y)$. An arrow points from this to the formula $f_x = \frac{\partial f}{\partial x}$, which is shown in a large, bold font.

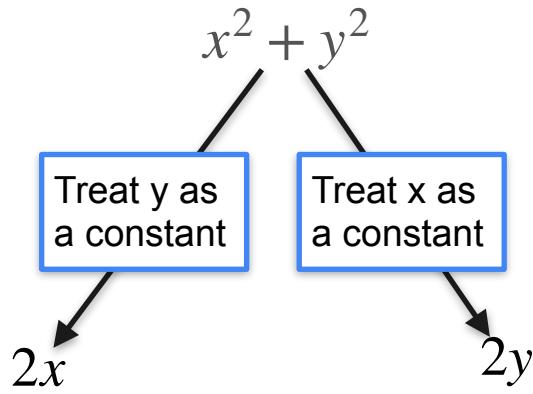
Partial Derivatives



The diagram shows the function $f(x, y)$ at the top, with two arrows pointing downwards to the partial derivative equations below. The left arrow points to the equation $f_x = \frac{\partial f}{\partial x}$, and the right arrow points to the equation $f_y = \frac{\partial f}{\partial y}$.

$$f_x = \frac{\partial f}{\partial x}$$
$$f_y = \frac{\partial f}{\partial y}$$

Partial Derivatives

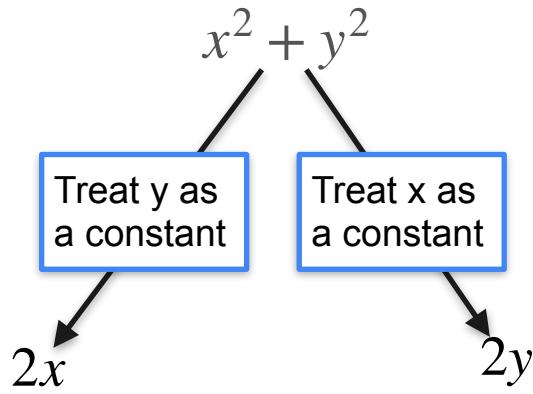


The diagram shows the function $f(x, y)$ at the top, with two arrows pointing downwards to the partial derivative formulas below. The left arrow points to the formula $f_x = \frac{\partial f}{\partial x}$, and the right arrow points to the formula $f_y = \frac{\partial f}{\partial y}$. Below these formulas, the text "Partial derivative of f with respect to x " is written.

$$f_x = \frac{\partial f}{\partial x}$$
$$f_y = \frac{\partial f}{\partial y}$$

Partial derivative of f with respect to x

Partial Derivatives



A diagram illustrating the general concept of partial derivatives. At the top, the function $f(x, y)$ is shown. Two arrows point downwards from it to the partial derivative formulas $f_x = \frac{\partial f}{\partial x}$ and $f_y = \frac{\partial f}{\partial y}$, which are labeled as the "Partial derivative of f with respect to x " and "Partial derivative of f with respect to y ".

Intro To Partial Derivatives

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

Find partial derivatives of f with respect to x and y

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

$$\frac{\partial f}{\partial x} =$$

Find partial derivatives of f with respect to x and y

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

$$\frac{\partial f}{\partial x} =$$

Find partial derivatives of f with respect to x and y

$$\frac{\partial f}{\partial y} =$$

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} = ?$$

TASK

Find partial derivatives of f with respect to x and y

Intro To Partial Derivatives

Partial derivative notation

$$f(x, y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} = ?$$

TASK

Find partial derivatives of f with respect to x and y

Intro To Partial Derivatives

Partial derivative notation

$$f(x, y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} = ?$$

TASK

Find partial derivatives of f with respect to x and y

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

TASK

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} =$$

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

TASK

Find partial derivative of f with respect to x

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} =$$

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

TASK

Find partial derivative of f with respect to x

Step 1:

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} =$$

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} = ?$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} = ?$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2:

Intro To Partial Derivatives

$$f(x, y) = x^2 + \boxed{y^2}$$

$$\frac{\partial f}{\partial x} = ?$$

$$\frac{\partial f}{\partial y} = ?$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$


TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$


TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + 1$$

$$f(x, y) = x^2 + \boxed{y^2}$$


TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + 1$$

$$f(x, y) = x^2 + \boxed{y^2}$$


TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

$$\frac{\partial f}{\partial x} = 2x$$

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

$$\frac{\partial f}{\partial x} = 2x$$

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case x .

$$\frac{\partial f}{\partial y} =$$

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

$$\frac{\partial f}{\partial x} = 2x$$

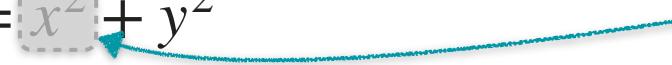
Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case x .

$$\frac{\partial f}{\partial y} =$$

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = \boxed{x^2} + y^2$$


TASK

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case x .

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} =$$

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

$$\frac{\partial f}{\partial x} = 2x$$

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case x .

$$\frac{\partial f}{\partial y} =$$

Step 2: Differentiate the function using the normal rules of differentiation.

Intro To Partial Derivatives

$$f(x, y) = x^2 + y^2$$

TASK

$$\frac{\partial f}{\partial x} = 2x$$

Find partial derivative of f with respect to x

Step 1: Treat all other variables as a constant. In our case x .

$$\frac{\partial f}{\partial y} = 2y$$

Step 2: Differentiate the function using the normal rules of differentiation.



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Gradients and Gradient Descent

Partial derivatives -Part 2

Partial Derivatives (More Examples)

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

TASK

What is the partial derivate of f with respect to x ?

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} = ?$$

TASK

What is the partial derivate of f with respect to x ?

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivate of f with respect to x

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$


$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

Constant coefficient

$$\frac{\partial f}{\partial x} =$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

Constant coefficient

$$\frac{\partial f}{\partial x} = 3$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} = 3$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

Differentiate with respect to x

TASK

Find partial derivate of f with respect to x

$$\frac{\partial f}{\partial x} = 3$$

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

Differentiate with respect to x

TASK

Find partial derivate of f with respect to x

$$\frac{\partial f}{\partial x} = 3(2x)$$

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$

$$\frac{\partial f}{\partial x} = 3(2x)$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$



 treat as constant coefficient

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2$$



treat as constant coefficient

$$\frac{\partial f}{\partial x} = 3(2x)$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} = 3(2x)y^3$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial x} = 3(2x)y^3$$

$$= 6xy^3$$

TASK

Find partial derivate of f with respect to x

Step 1: Treat all other variables as a constant. In our case y .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

TASK

What is the partial derivate of f with respect to y ?

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial y} =$$

TASK

What is the partial derivate of f with respect to y ?

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial y} = ?$$

TASK

What is the partial derivate of f with respect to y ?

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$

$$\frac{\partial f}{\partial y} =$$

TASK

What is the partial derivate of f with respect to y ?

Step 1: Treat all other variables as a constant. In our case x .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3 \text{ } \square y^3$$

$$\frac{\partial f}{\partial y} =$$

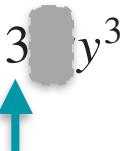
TASK

What is the partial derivate of f with respect to y ?

Step 1: Treat all other variables as a constant. In our case x .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$


$$\frac{\partial f}{\partial y} = 3$$

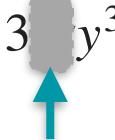
TASK

What is the partial derivate of f with respect to y ?

Step 1: Treat all other variables as a constant. In our case x .

Step 2: Differentiate the function using the normal rules of differentiation.

Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$


$$\frac{\partial f}{\partial y} = 3 \quad \text{[grey box]}$$

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Partial Derivatives (More Examples)

$$f(x, y) = 3 \quad y^3$$


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Partial Derivatives (More Examples)

$$f(x, y) = 3x^2y^3$$



$$\begin{aligned}\frac{\partial f}{\partial y} &= 3(x^2)(3y^2) \\ &= 9x^2y^2\end{aligned}$$

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What is the partial derivate of f with respect to y ?

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Gradients and Gradient Descent

Gradients

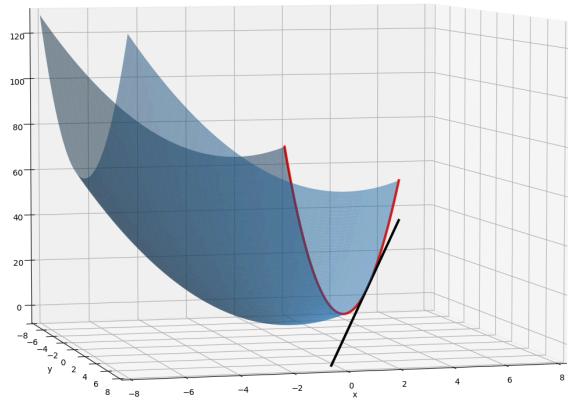
Gradient

$$f(x, y) = x^2 + y^2$$

Gradient

$$f(x, y) = x^2 + y^2$$

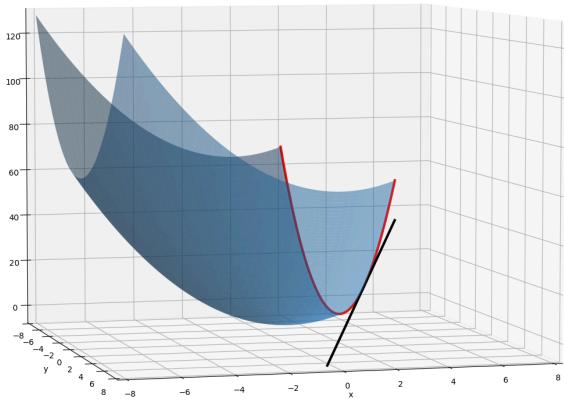
Treat y as a constant



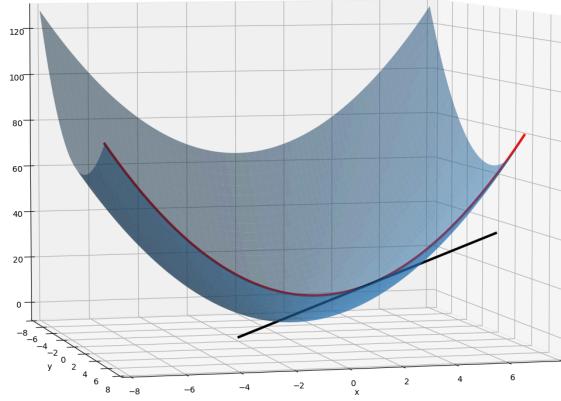
Gradient

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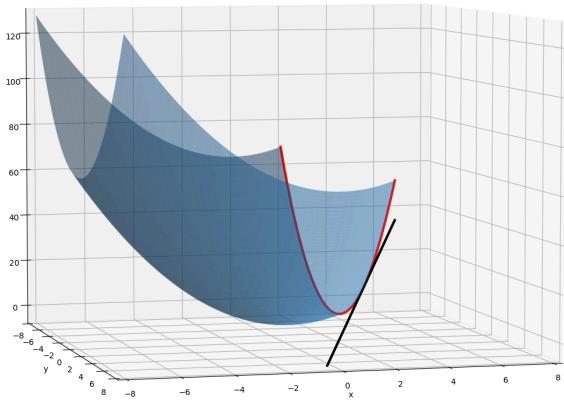
Treat x as a constant



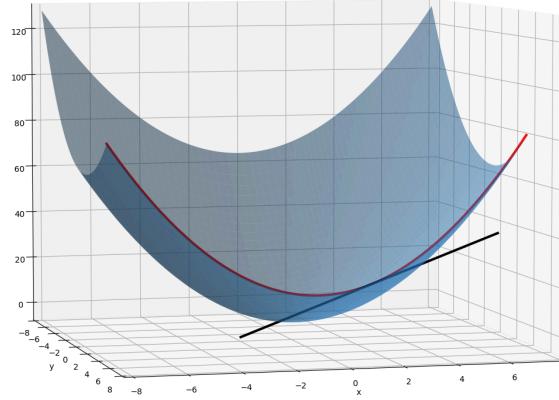
Gradient

$$f(x, y) = x^2 + y^2$$

Treat y as a constant



Treat x as a constant

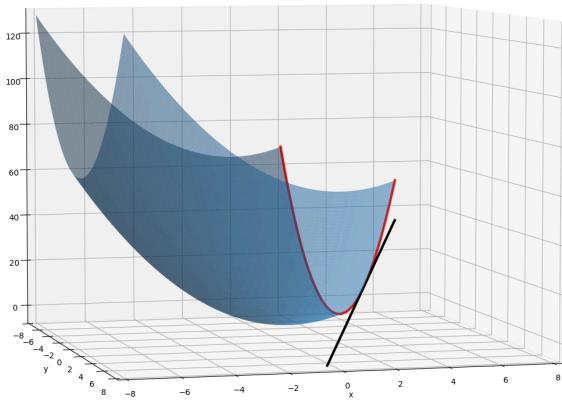


$$\frac{\partial f}{\partial x} = 2x$$

Gradient

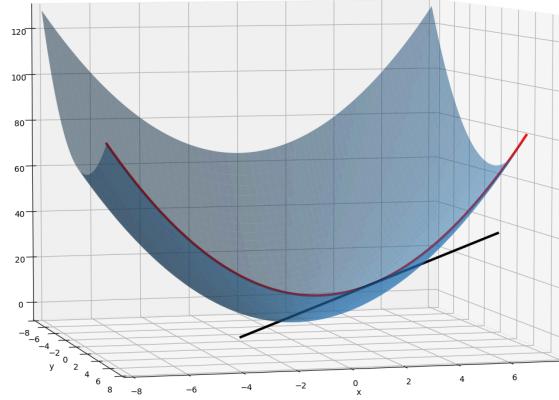
$$f(x, y) = x^2 + y^2$$

Treat y as a constant



$$\frac{\partial f}{\partial x} = 2x$$

Treat x as a constant



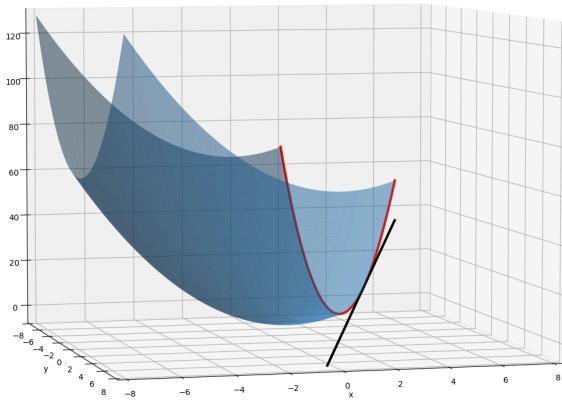
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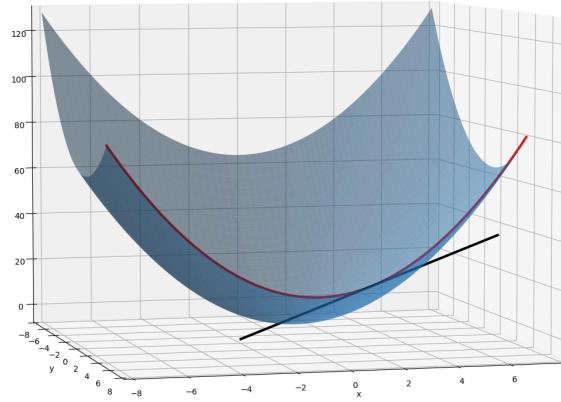
Gradient

Treat y as a constant



$$\frac{\partial f}{\partial x} = 2x$$

Treat x as a constant

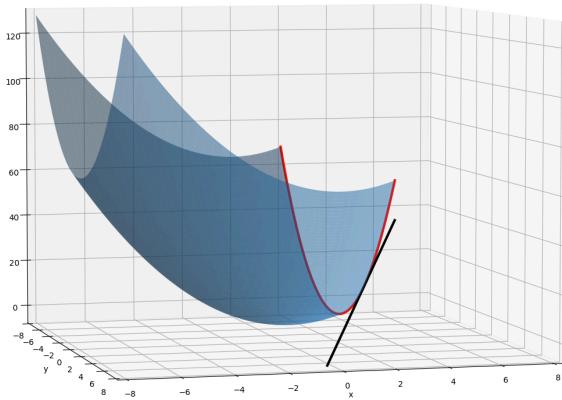


$$\frac{\partial f}{\partial y} = 2y$$

Gradient

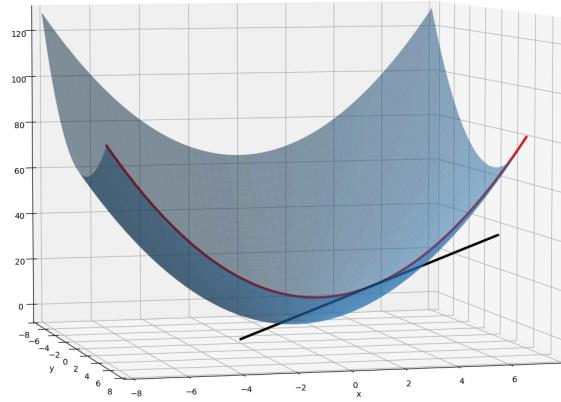
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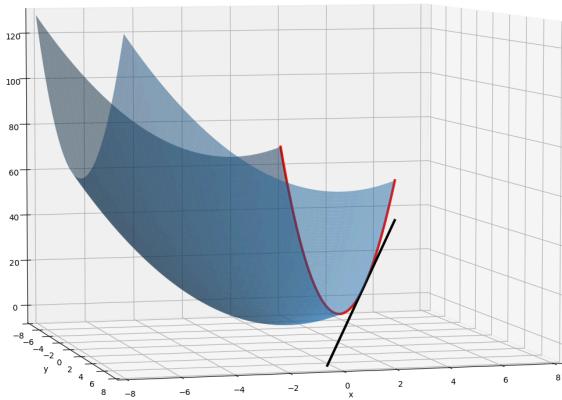
Gradient

$$\begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

Gradient

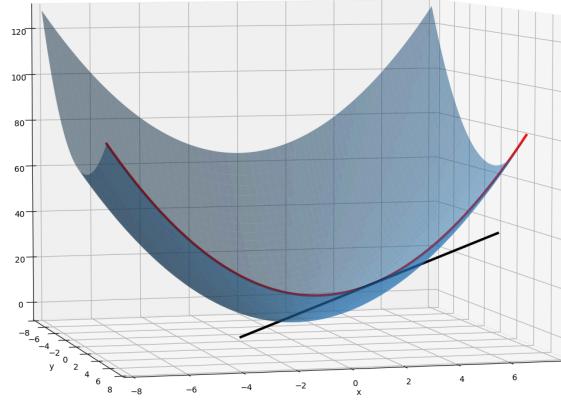
$$f(x, y) = x^2 + y^2$$

Treat y as a constant



$$\frac{\partial f}{\partial x} = 2x$$

Treat x as a constant



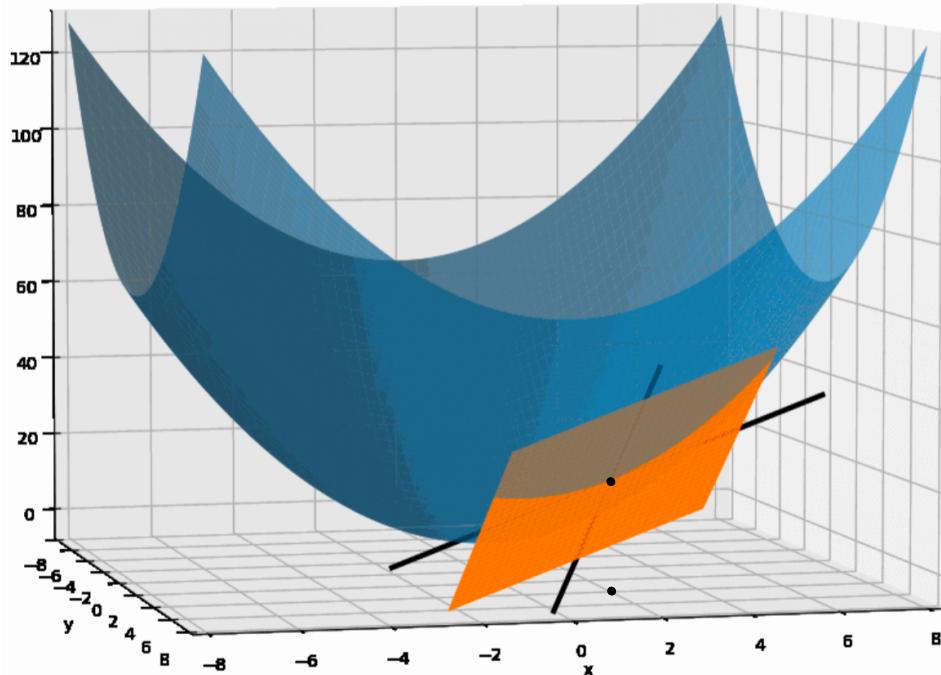
$$\frac{\partial f}{\partial y} = 2y$$

Gradient

$$\begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

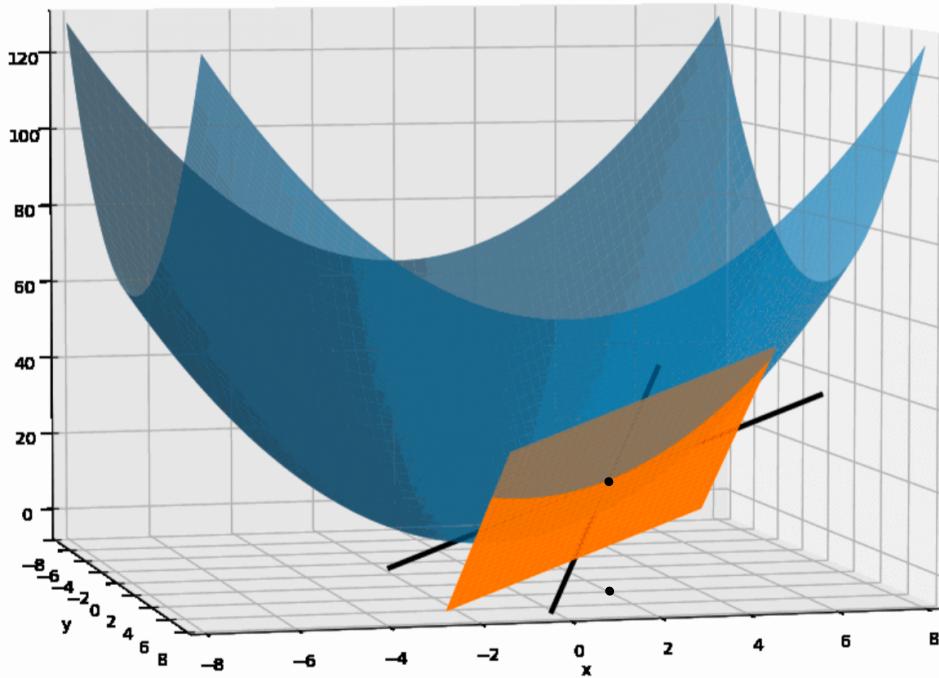
Gradient



$$f(x, y) = x^2 + y^2$$

The gradient of $f(x, y)$ is: $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

Gradient



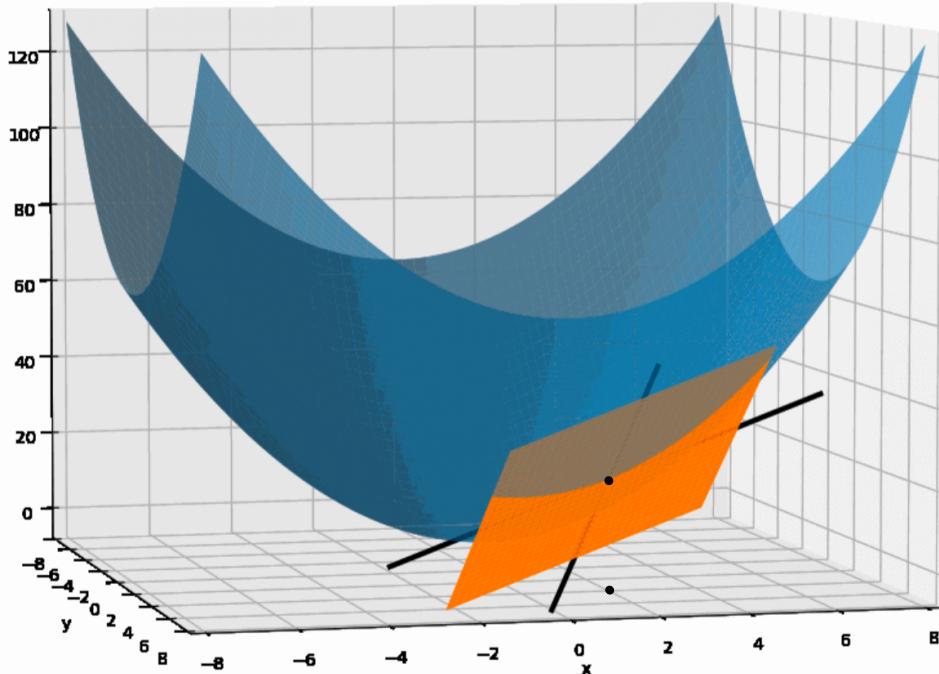
$$f(x, y) = x^2 + y^2$$

The gradient of $f(x, y)$ is: $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

TASK

Find the gradient of $f(x, y)$ at $(2, 3)$

Gradient



$$f(x, y) = x^2 + y^2$$

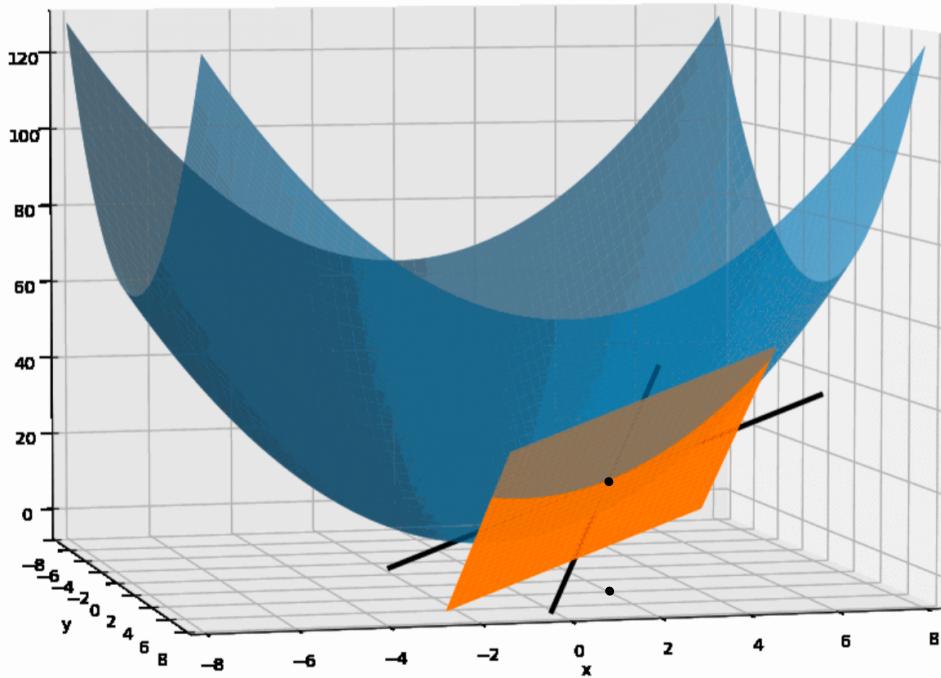
The gradient of $f(x, y)$ is: $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

TASK

Find the gradient of $f(x, y)$ at $(2, 3)$

The gradient of $f(x, y)$ is given as:

Gradient



$$f(x, y) = x^2 + y^2$$

The gradient of $f(x, y)$ is: $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

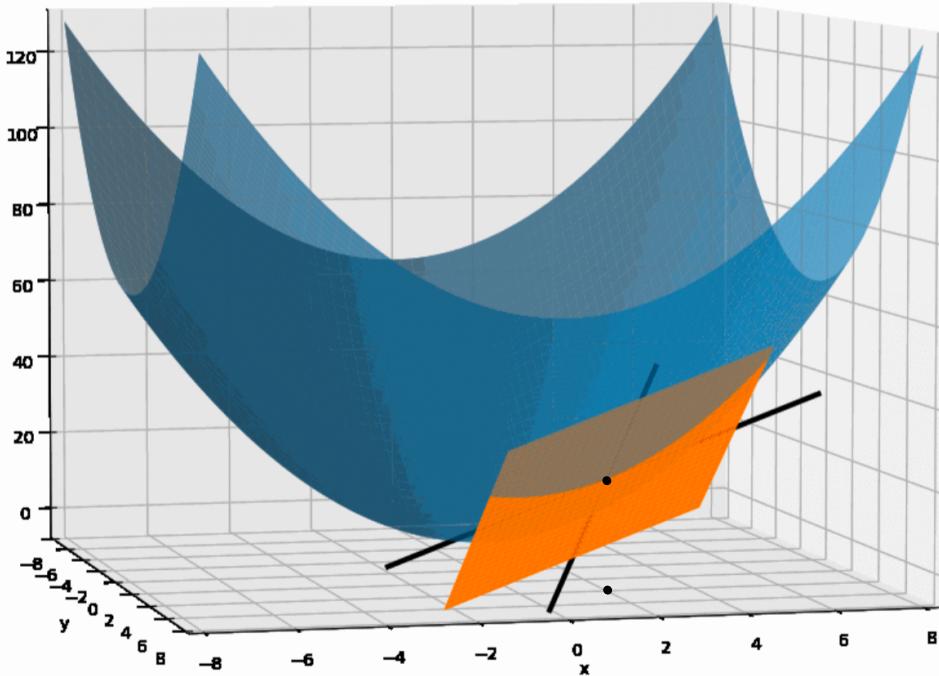
TASK

Find the gradient of $f(x, y)$ at $(2, 3)$

The gradient of $f(x, y)$ is given as:

$$\nabla f = \begin{bmatrix} 2 \cdot 2 \\ 2 \cdot 3 \end{bmatrix}$$

Gradient



$$f(x, y) = x^2 + y^2$$

The gradient of $f(x, y)$ is: $\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

TASK

Find the gradient of $f(x, y)$ at $(2, 3)$

The gradient of $f(x, y)$ is given as:

$$\nabla f = \begin{bmatrix} 2 \cdot 2 \\ 2 \cdot 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$



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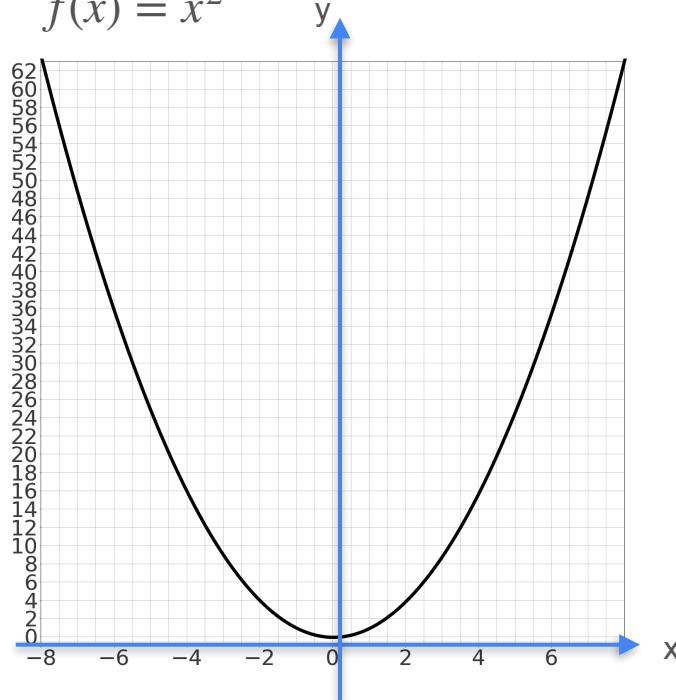
Gradients and Gradient Descent

**Gradients and maxima/
minima**

Functions of Two Variables

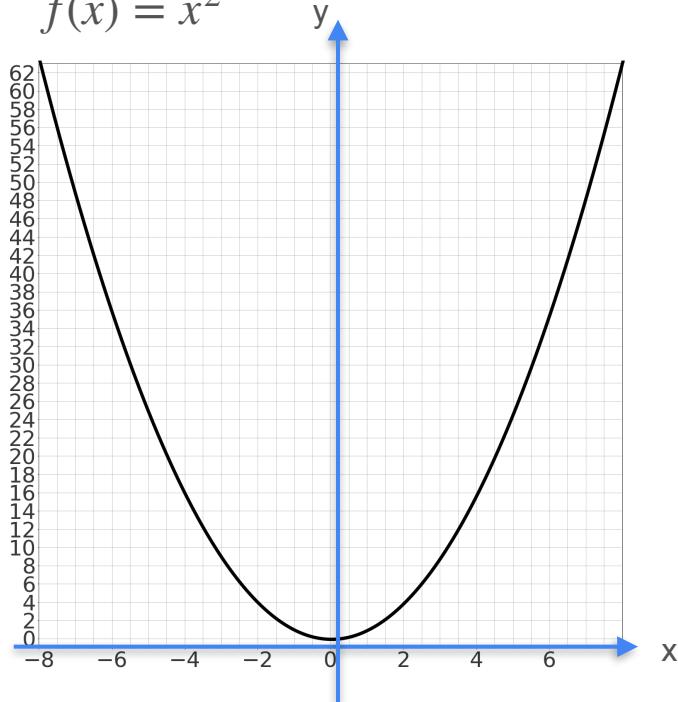
Functions of Two Variables

$$f(x) = x^2$$

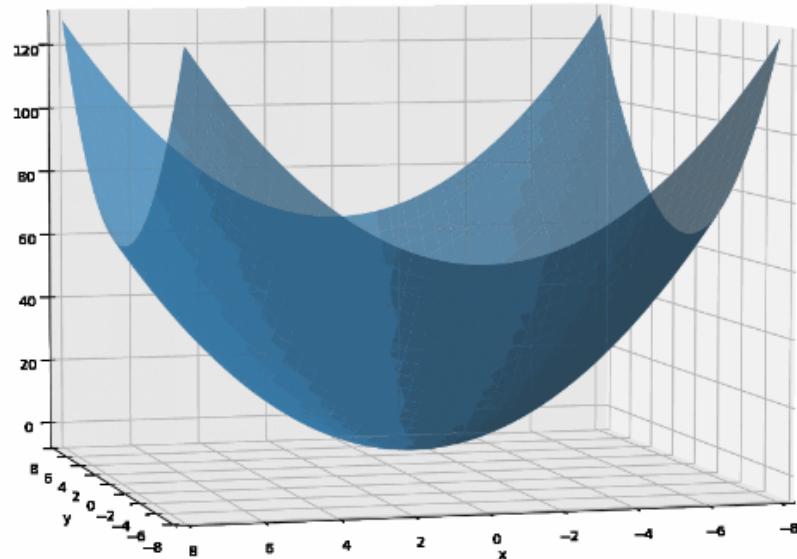


Functions of Two Variables

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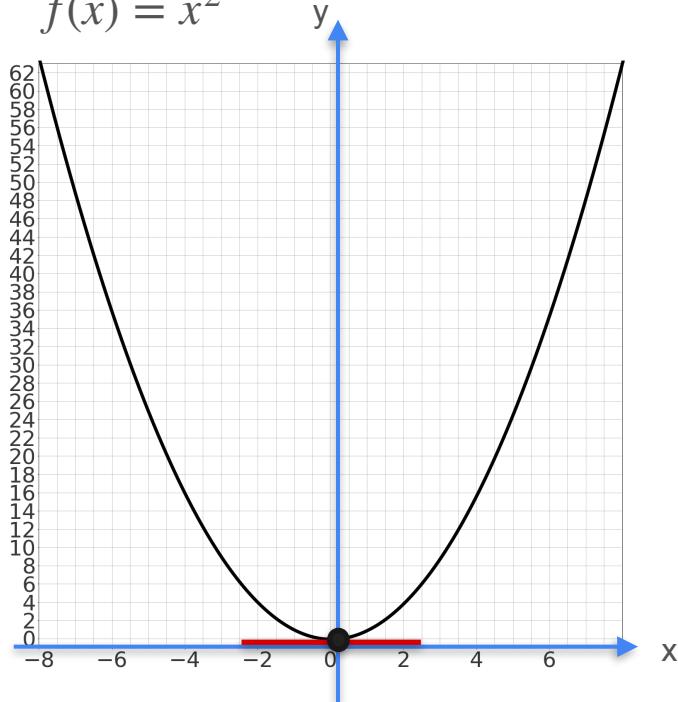


$$f(x, y) = x^2 + y^2$$

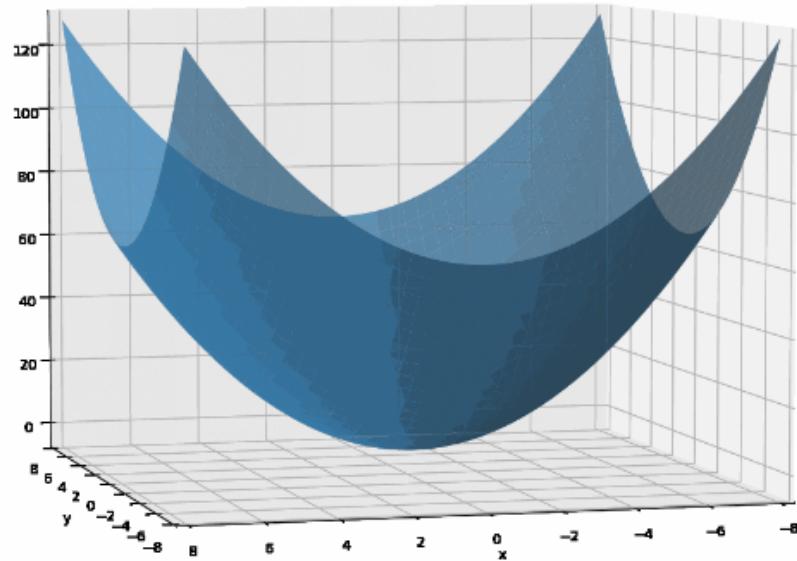


Functions of Two Variables

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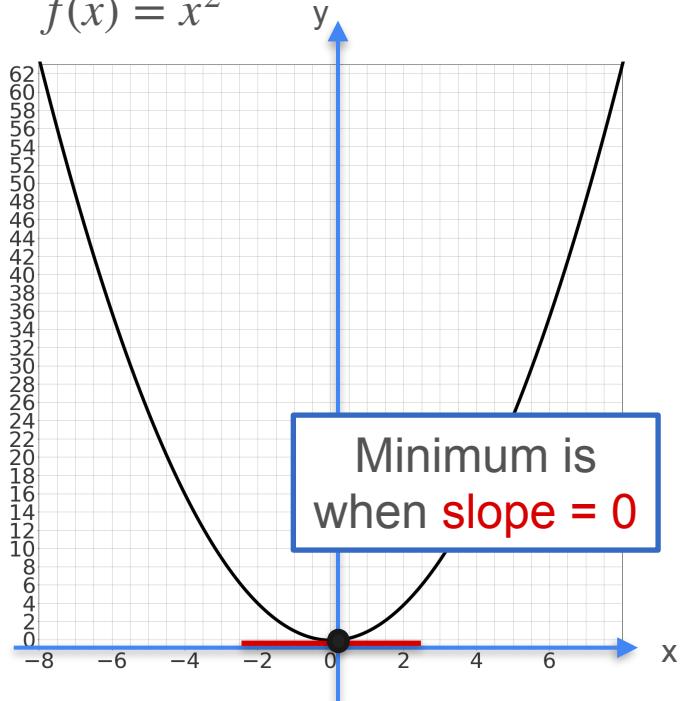


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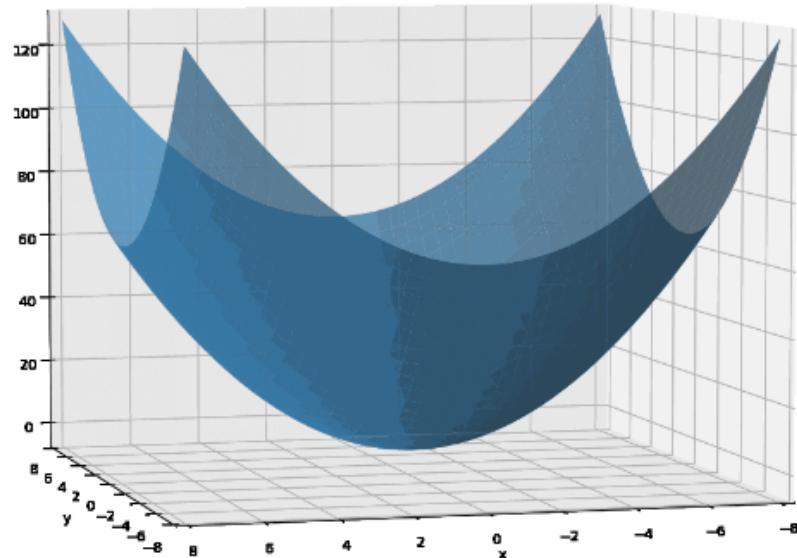


Functions of Two Variables

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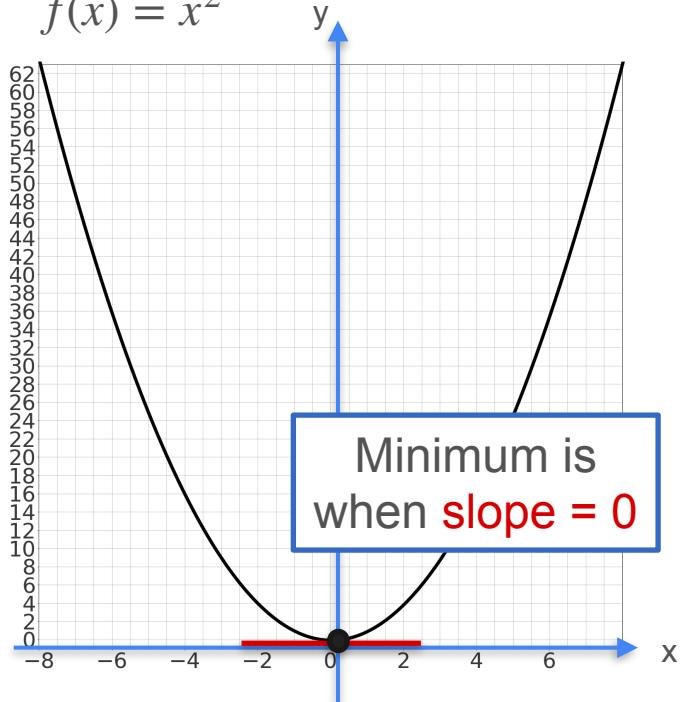


$$f(x, y) = x^2 + y^2$$

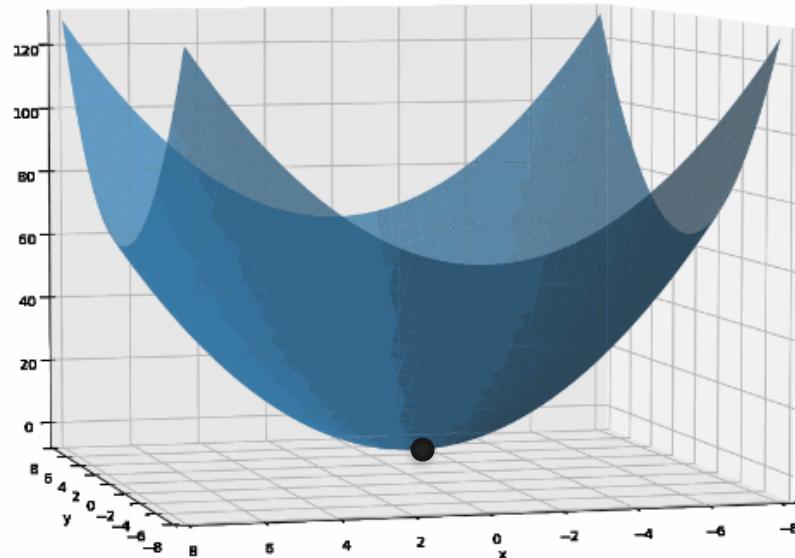


Functions of Two Variables

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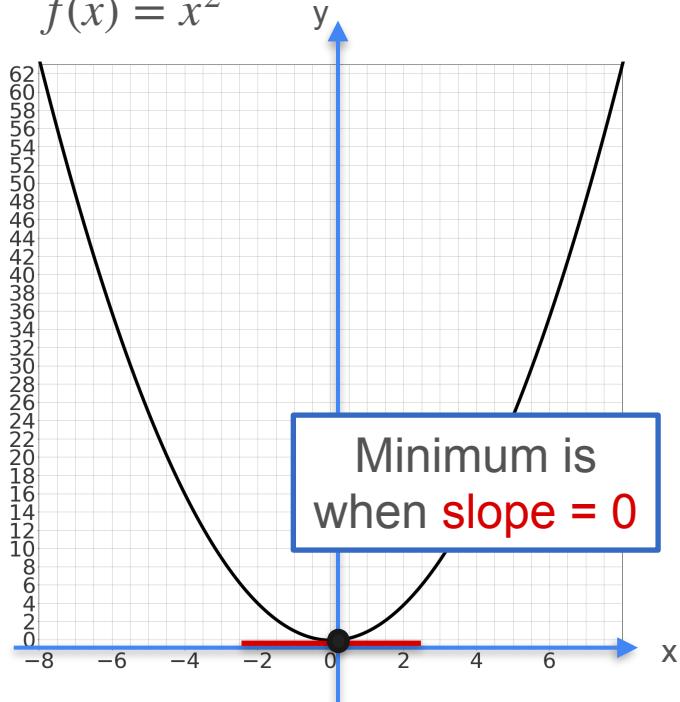


$$f(x, y) = x^2 + y^2$$

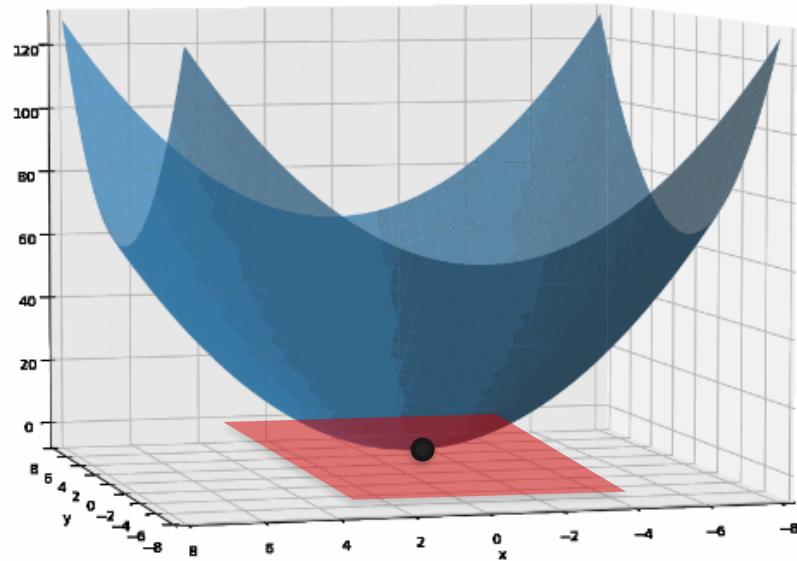


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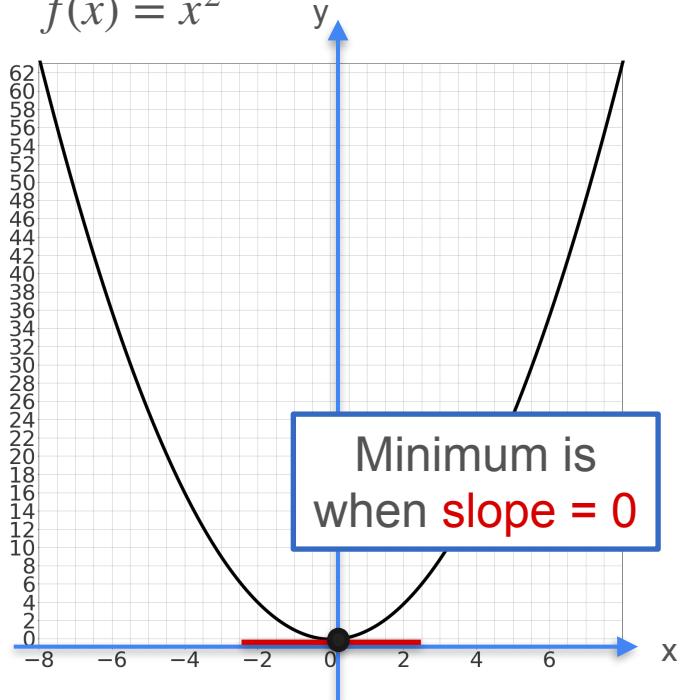


$$f(x, y) = x^2 + y^2$$

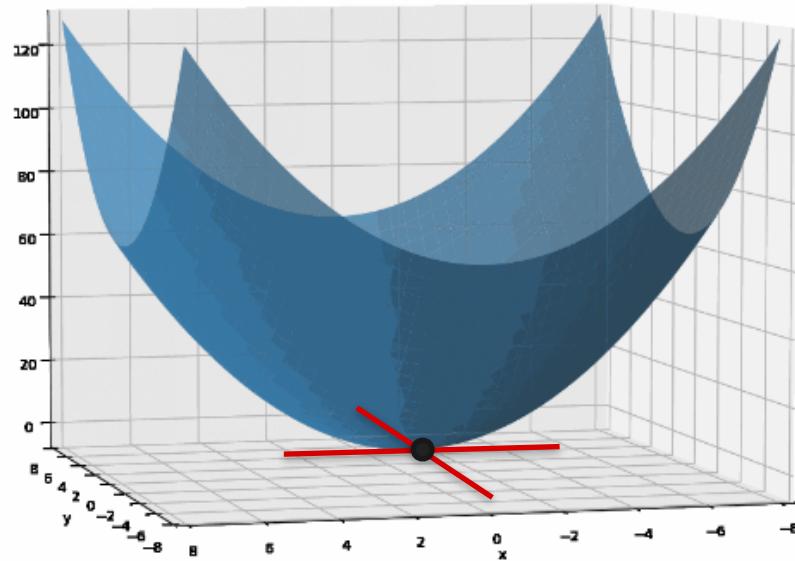


Functions of Two Variables

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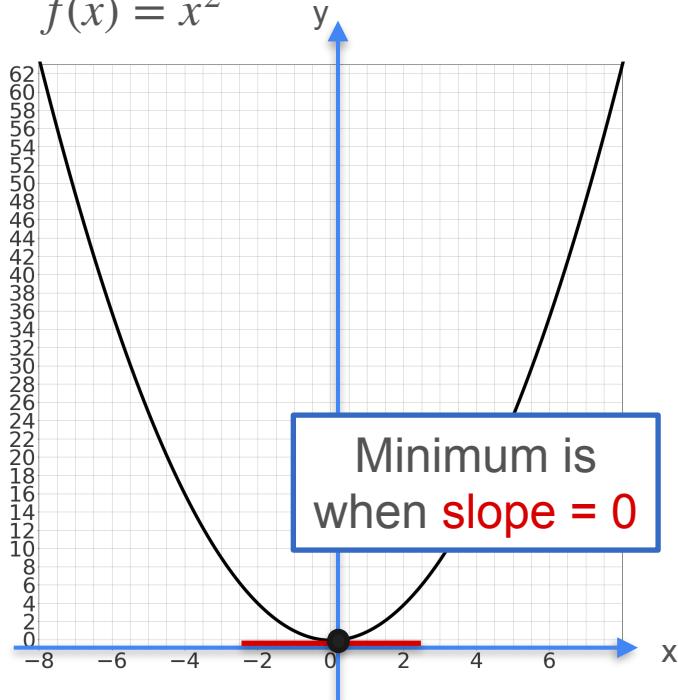


$$f(x, y) = x^2 + y^2$$

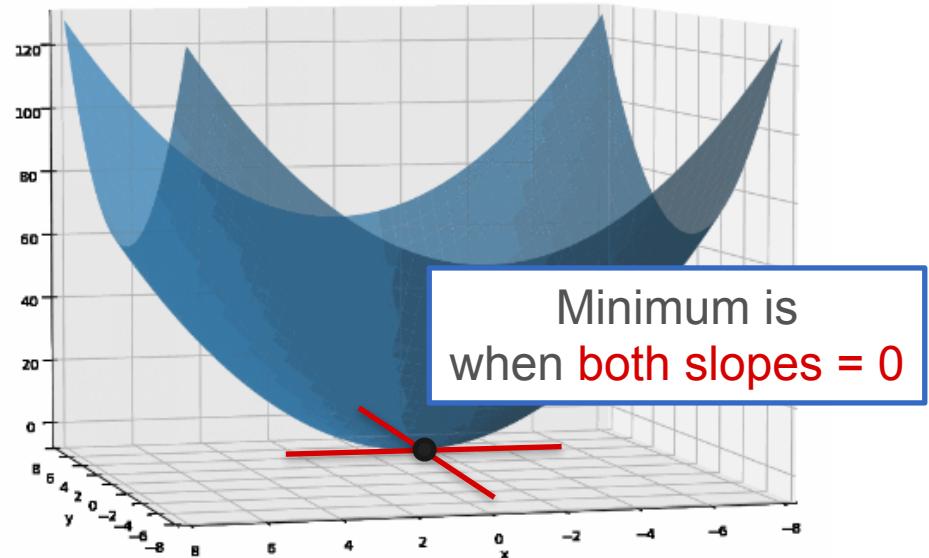


Functions of Two Variables

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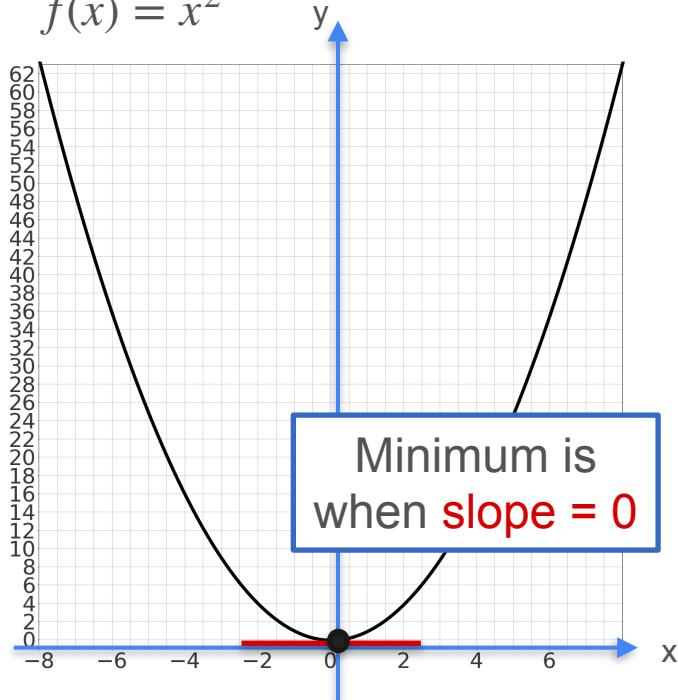


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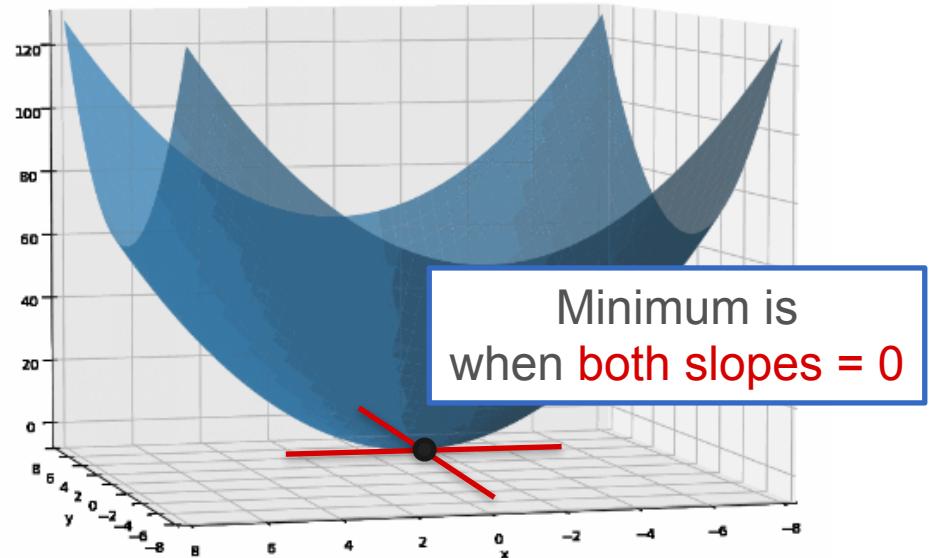


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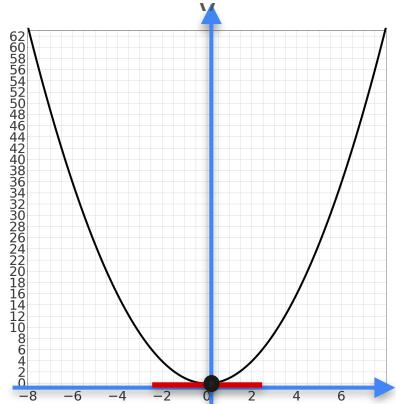


$$f(x, y) = x^2 + y^2$$



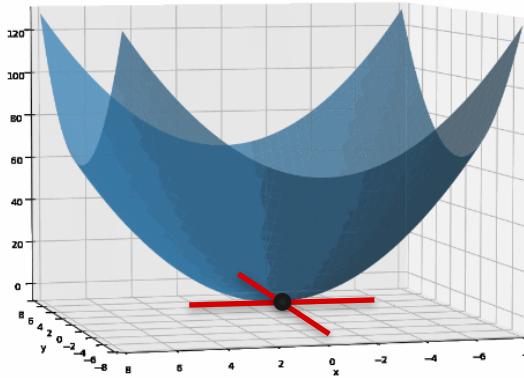
Functions of Two Variables

$$f(x) = x^2$$



Minimum is
when **slope = 0**

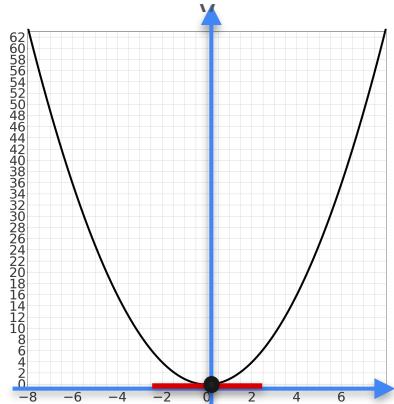
$$f(x, y) = x^2 + y^2$$



Minimum is
when **both slopes = 0**

Functions of Two Variables

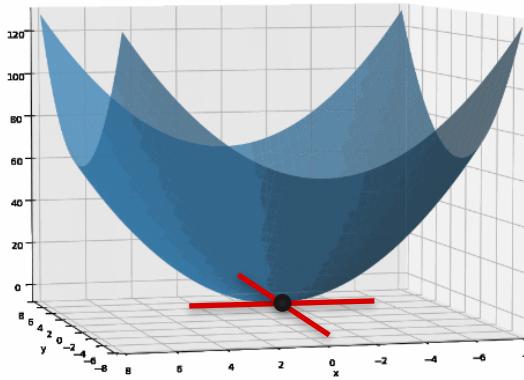
$$f(x) = x^2$$



Minimum is
when **slope = 0**

$$f'(x) = 0$$

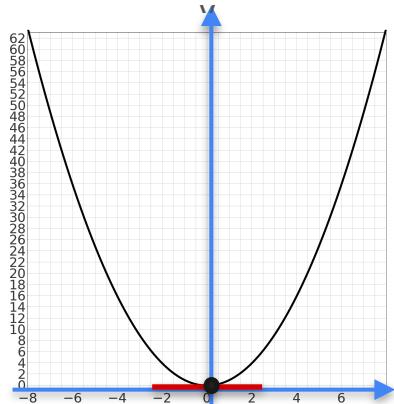
$$f(x, y) = x^2 + y^2$$



Minimum is
when **both slopes = 0**

Functions of Two Variables

$$f(x) = x^2$$

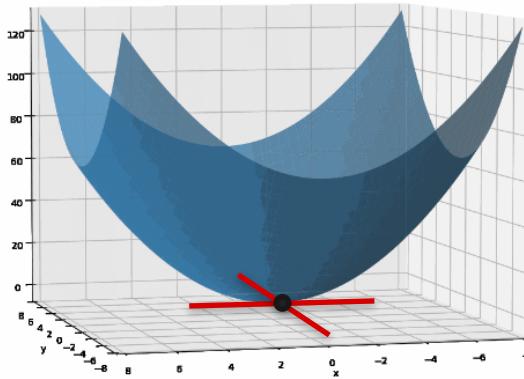


Minimum is
when **slope = 0**

$$f'(x) = 0$$

$$2x = 0$$

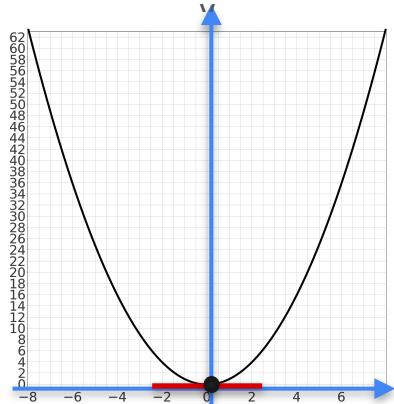
$$f(x, y) = x^2 + y^2$$



Minimum is
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Functions of Two Variables

$$f(x) = x^2$$



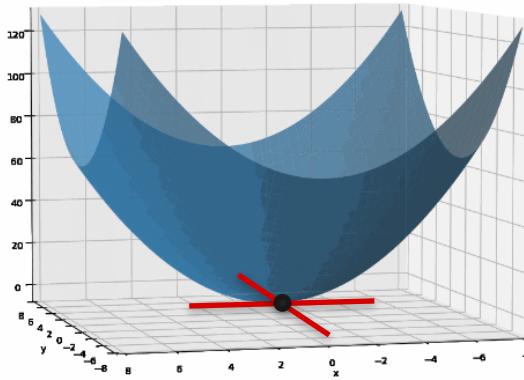
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$$x = 0$$

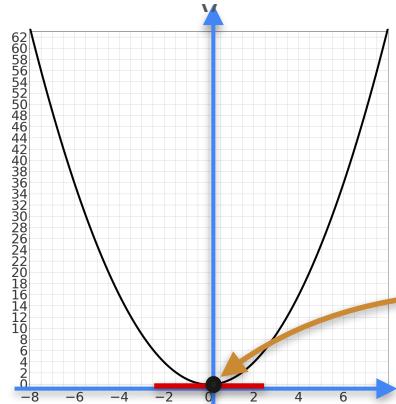
$$f(x, y) = x^2 + y^2$$



Minimum is
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Functions of Two Variables

$$f(x) = x^2$$



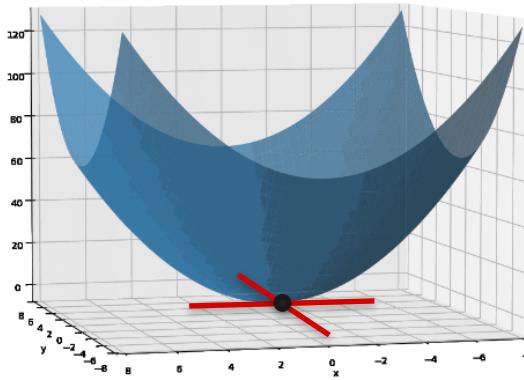
Minimum is
when **slope = 0**

$$f'(x) = 0$$

$$2x = 0$$

$$x = 0$$

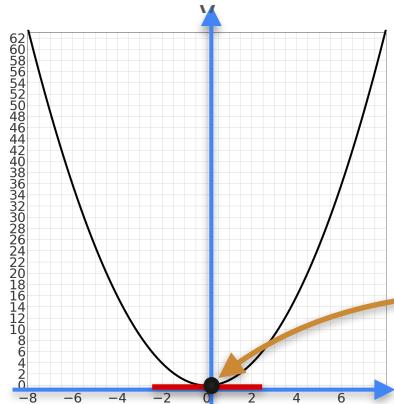
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Minimum is
when **both slopes = 0**

Functions of Two Variables

$$f(x) = x^2$$



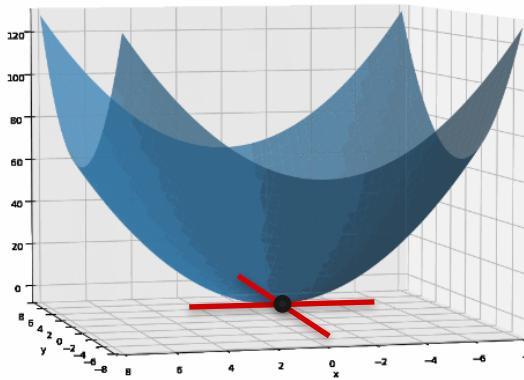
Minimum is
when **slope = 0**

$$f'(x) = 0$$

$$2x = 0$$

$$x = 0$$

$$f(x, y) = x^2 + y^2$$

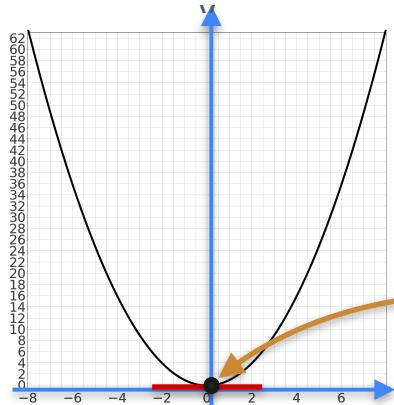


Minimum is
when **both slopes = 0**

$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0$$

Functions of Two Variables

$$f(x) = x^2$$



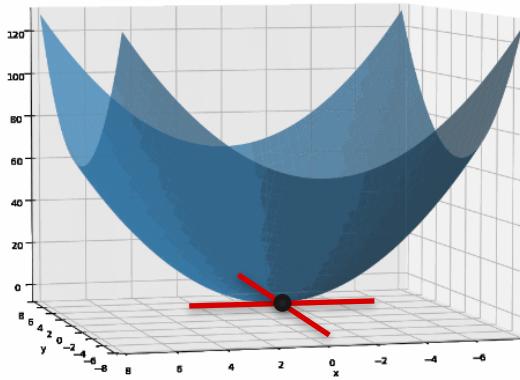
Minimum is
when **slope = 0**

$$f'(x) = 0$$

$$2x = 0$$

$$x = 0$$

$$f(x, y) = x^2 + y^2$$



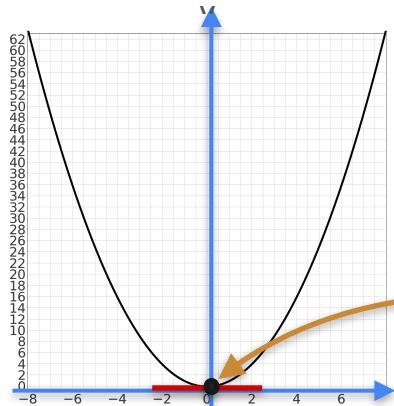
$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0$$

$$2x = 0 \text{ and } 2y = 0$$

Minimum is
when **both slopes = 0**

Functions of Two Variables

$$f(x) = x^2$$



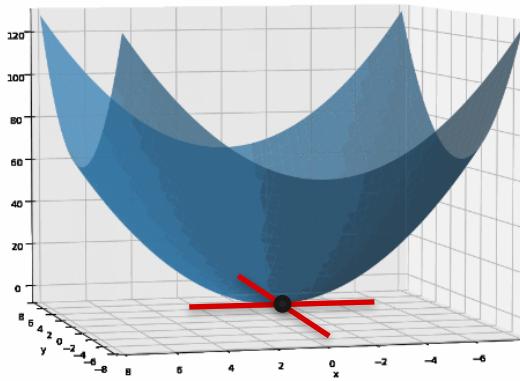
Minimum is
when **slope = 0**

$$f'(x) = 0$$

$$2x = 0$$

$$x = 0$$

$$f(x, y) = x^2 + y^2$$



$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0$$

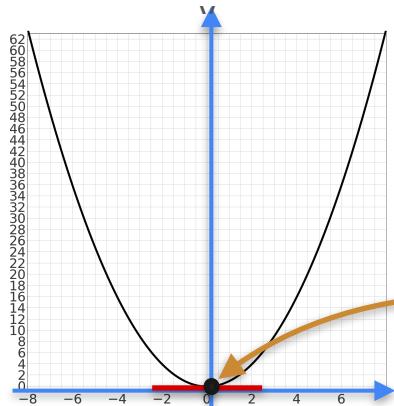
$$2x = 0 \text{ and } 2y = 0$$

$$(x, y) = (0,0)$$

Minimum is
when **both slopes = 0**

Functions of Two Variables

$$f(x) = x^2$$



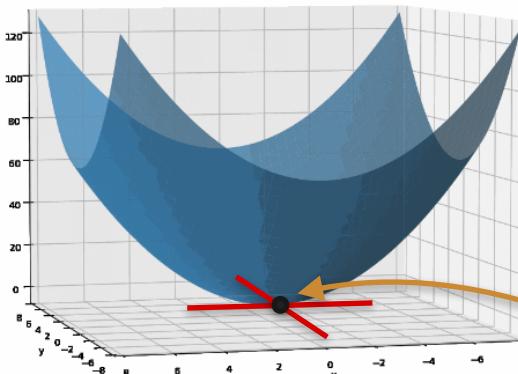
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Minimum is
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$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0$$

$$2x = 0 \text{ and } 2y = 0$$

$$(x, y) = (0,0)$$



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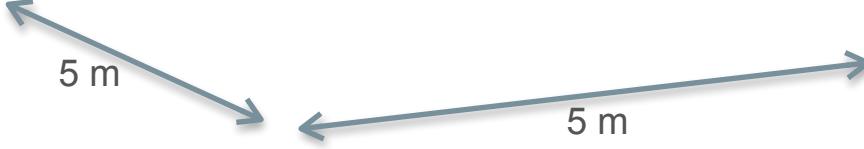
Gradients and Gradient Descent

**Optimization with gradients:
An example**

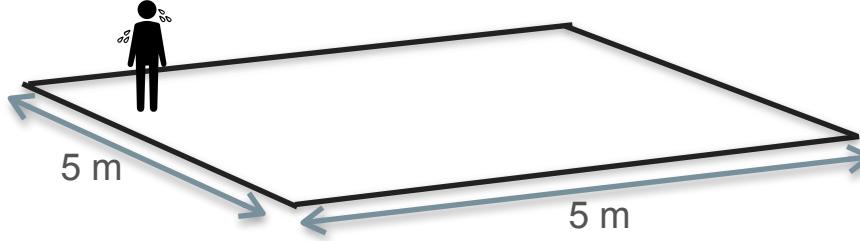
Motivation for Optimization in Two Variables



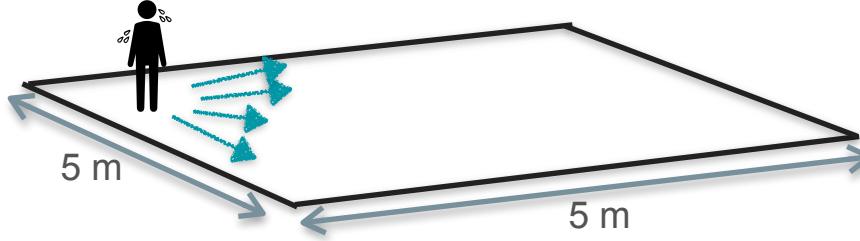
Motivation for Optimization in Two Variables



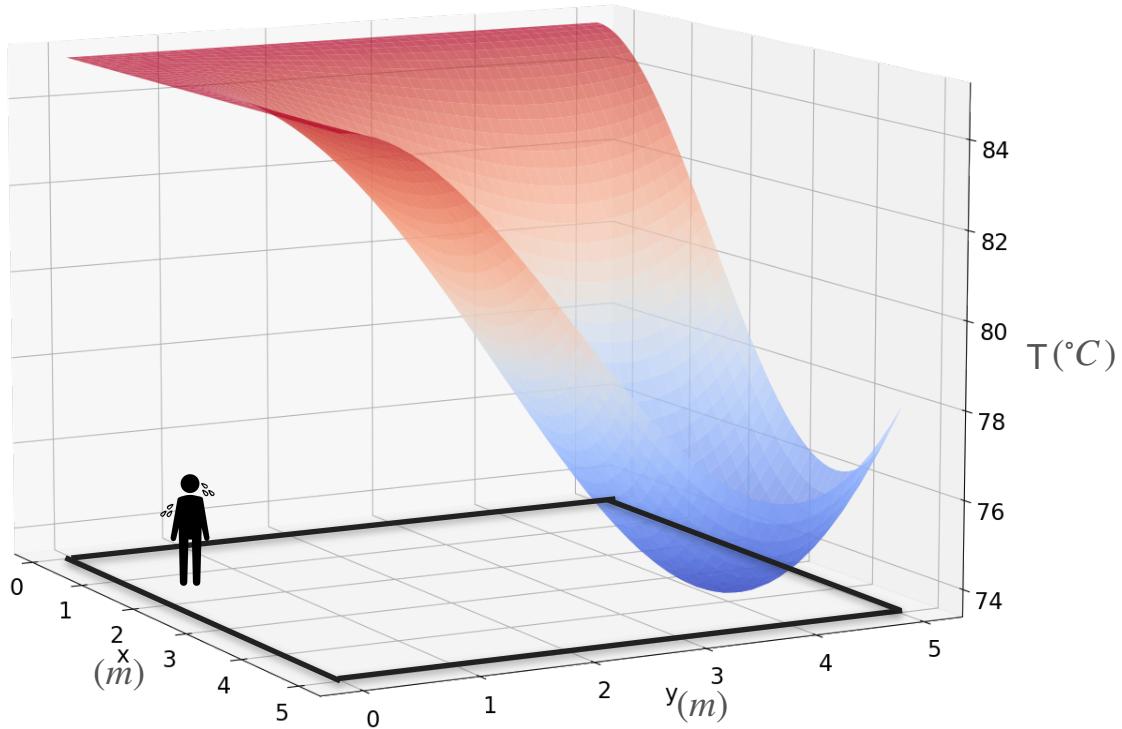
Motivation for Optimization in Two Variables



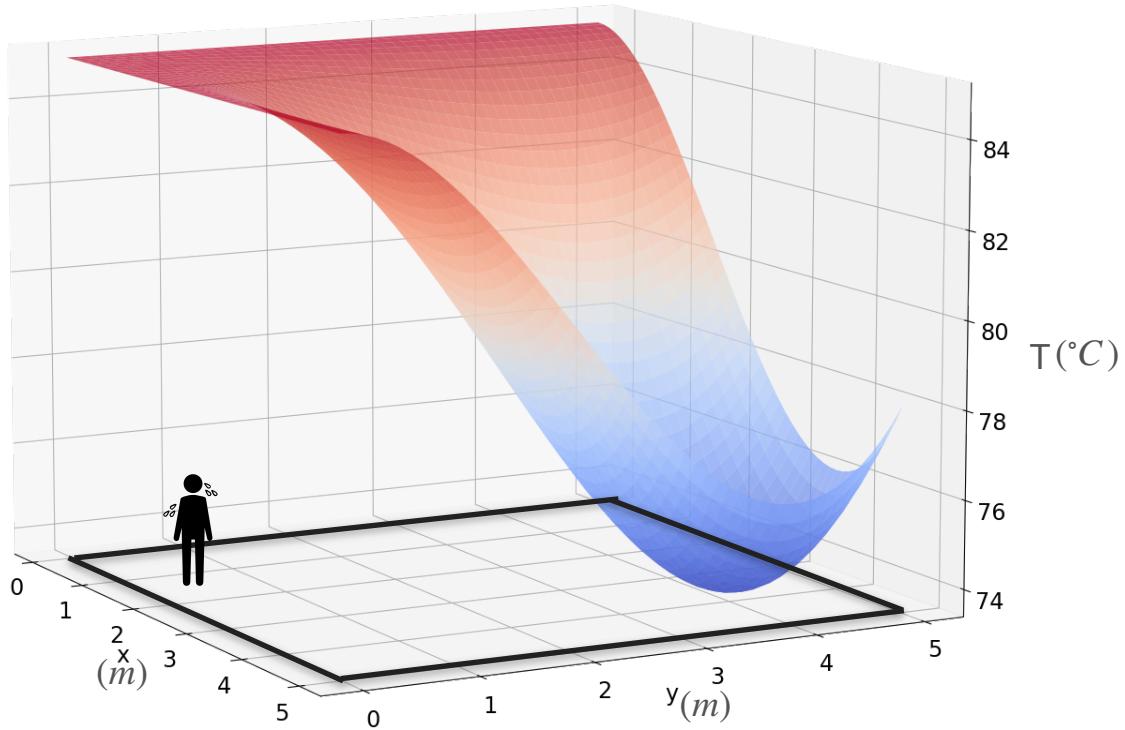
Motivation for Optimization in Two Variables



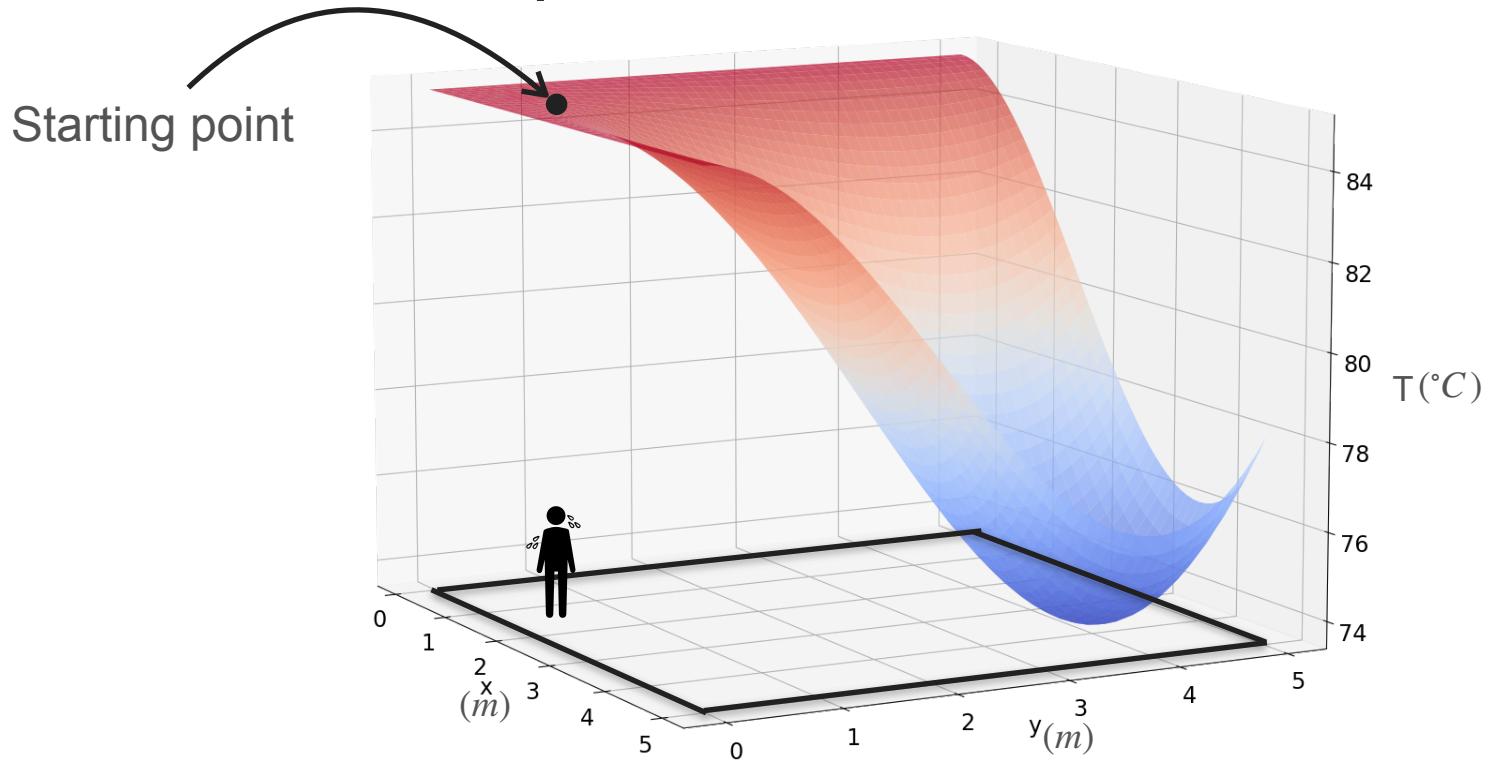
Motivation for Optimization in Two Variables



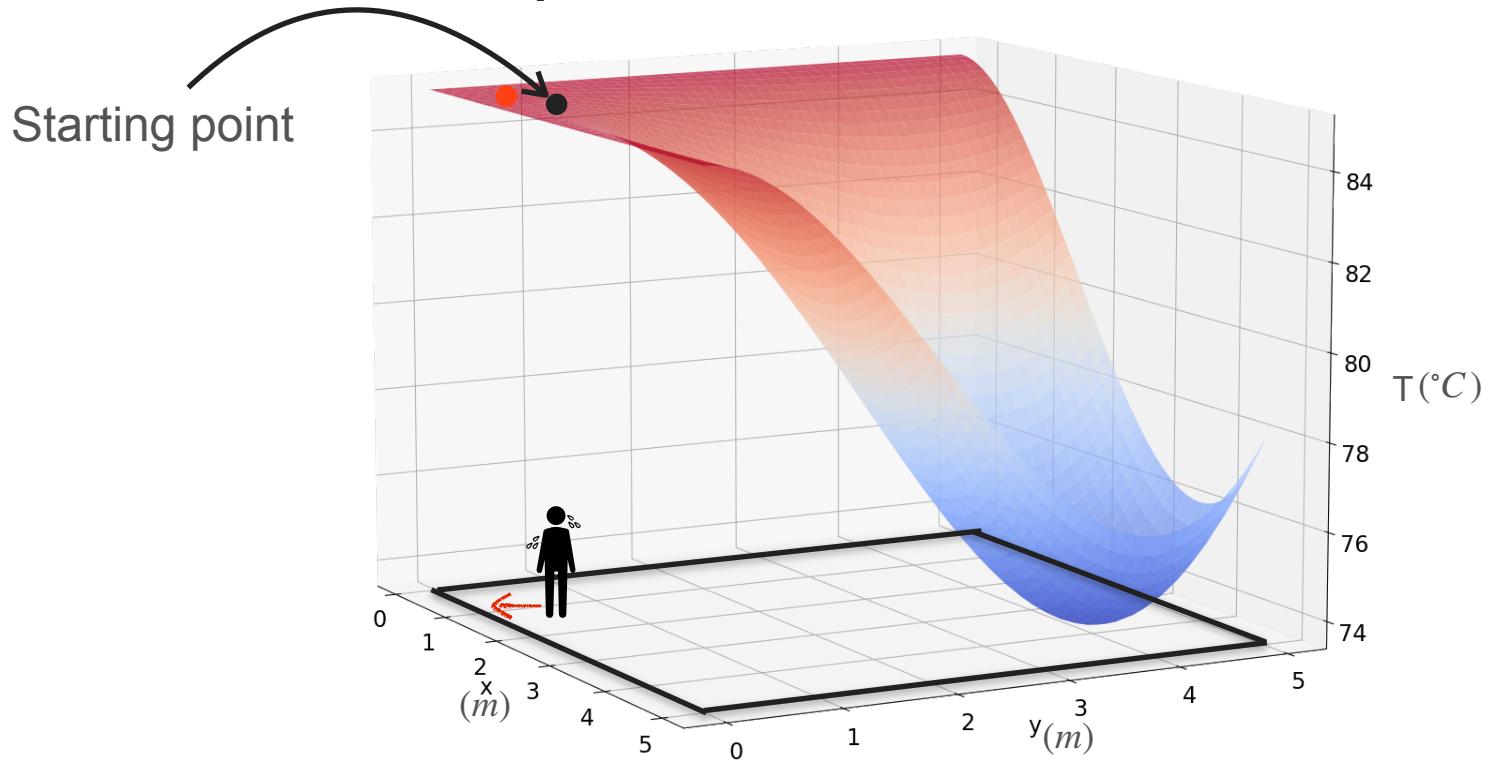
Motivation for Optimization in Two Variables



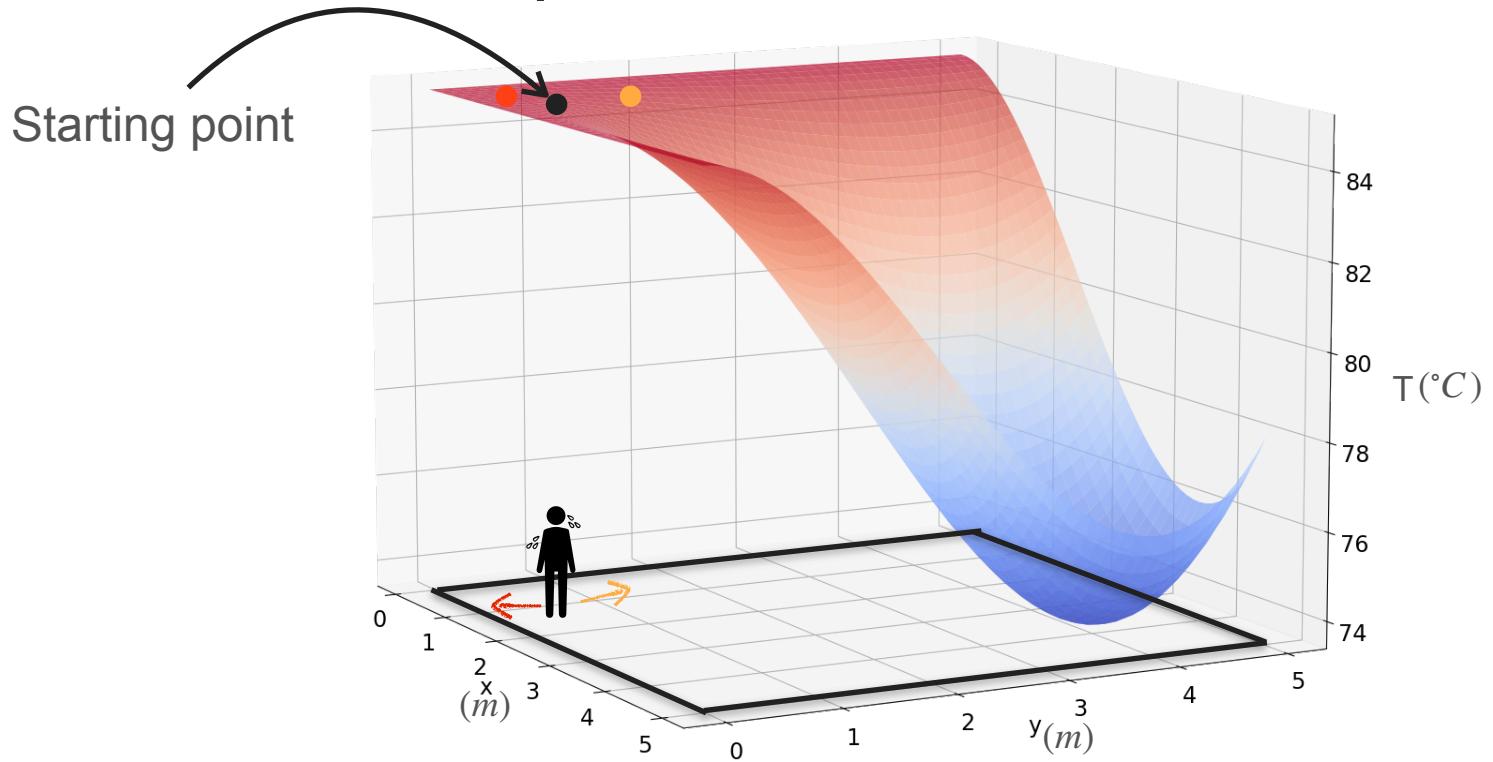
Motivation for Optimization in Two Variables



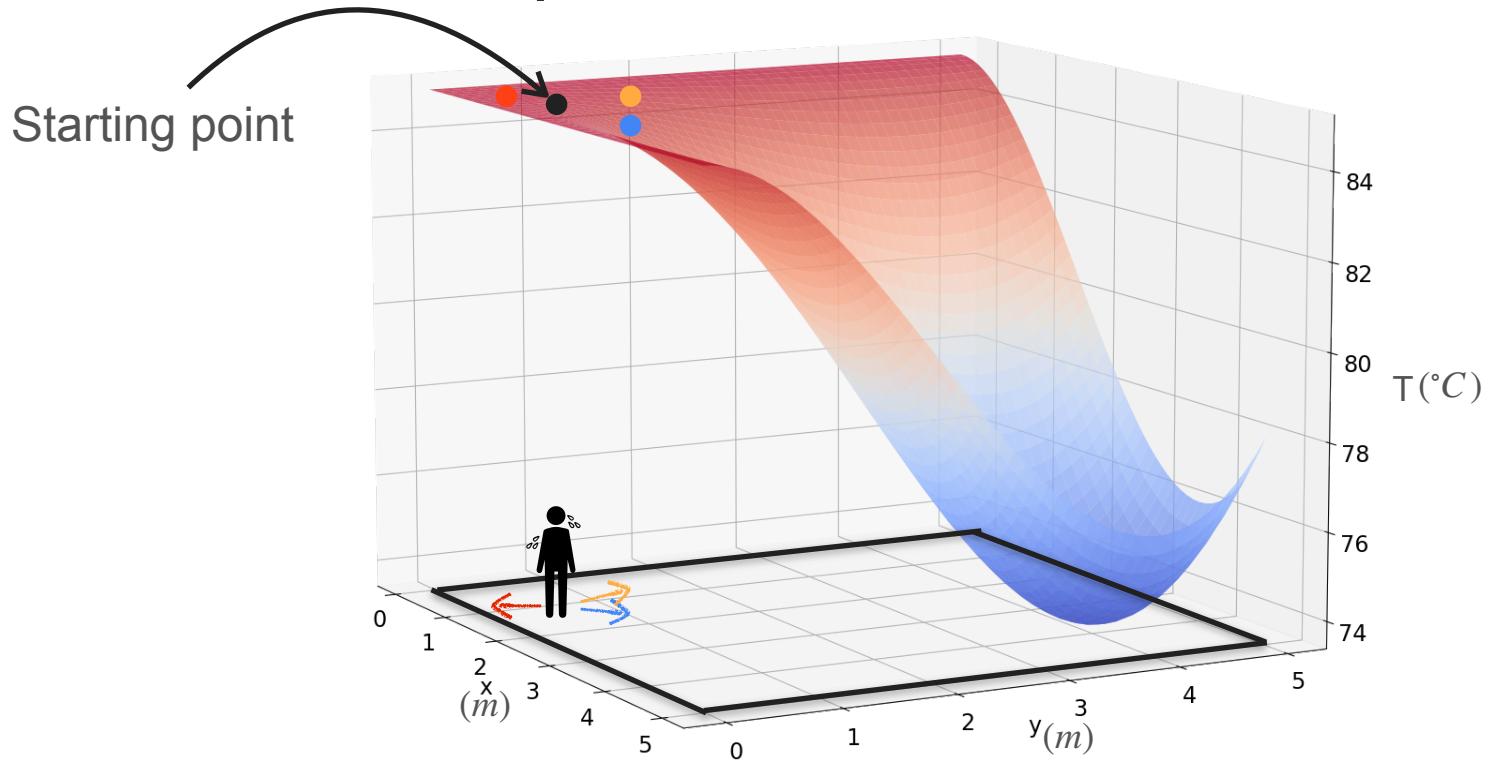
Motivation for Optimization in Two Variables



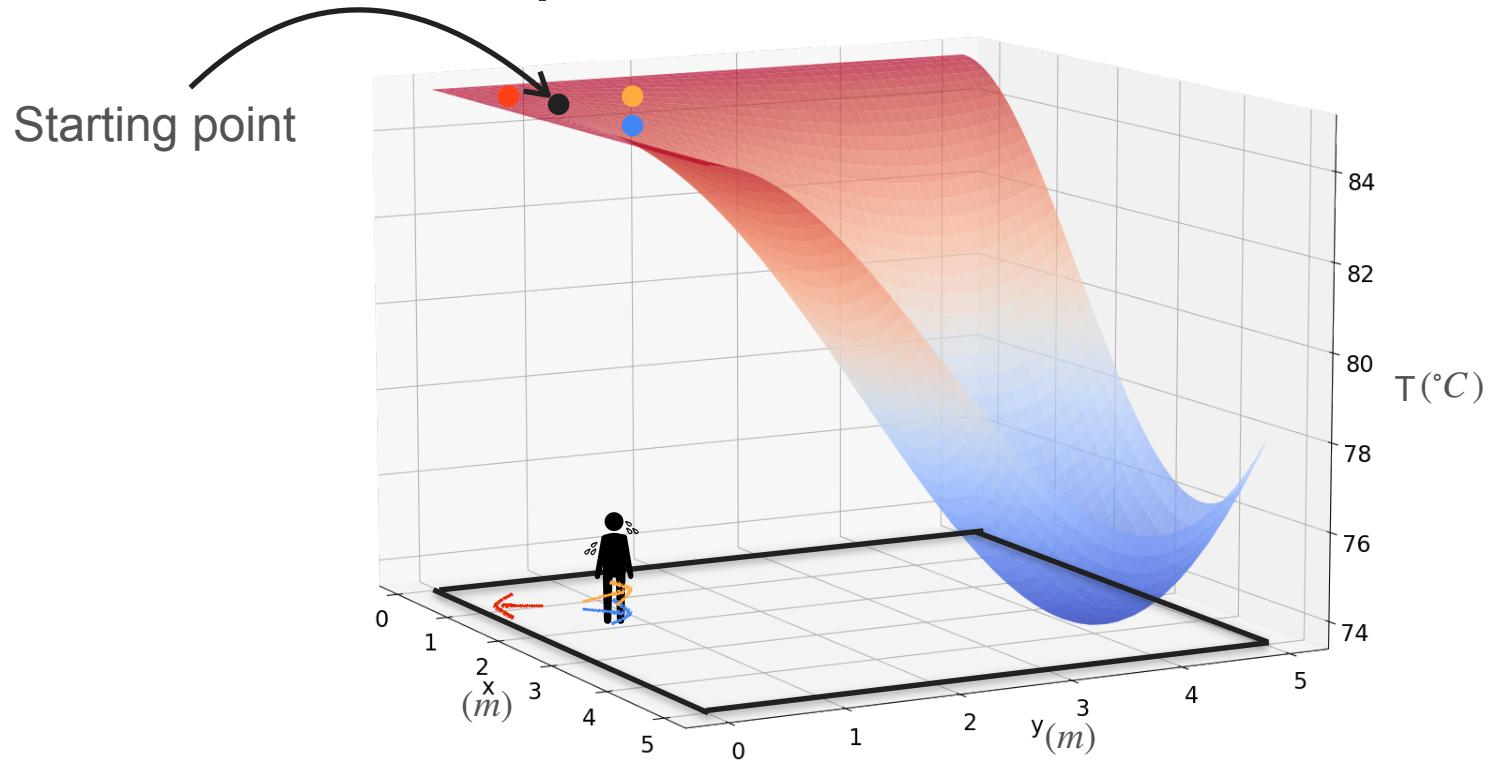
Motivation for Optimization in Two Variables



Motivation for Optimization in Two Variables

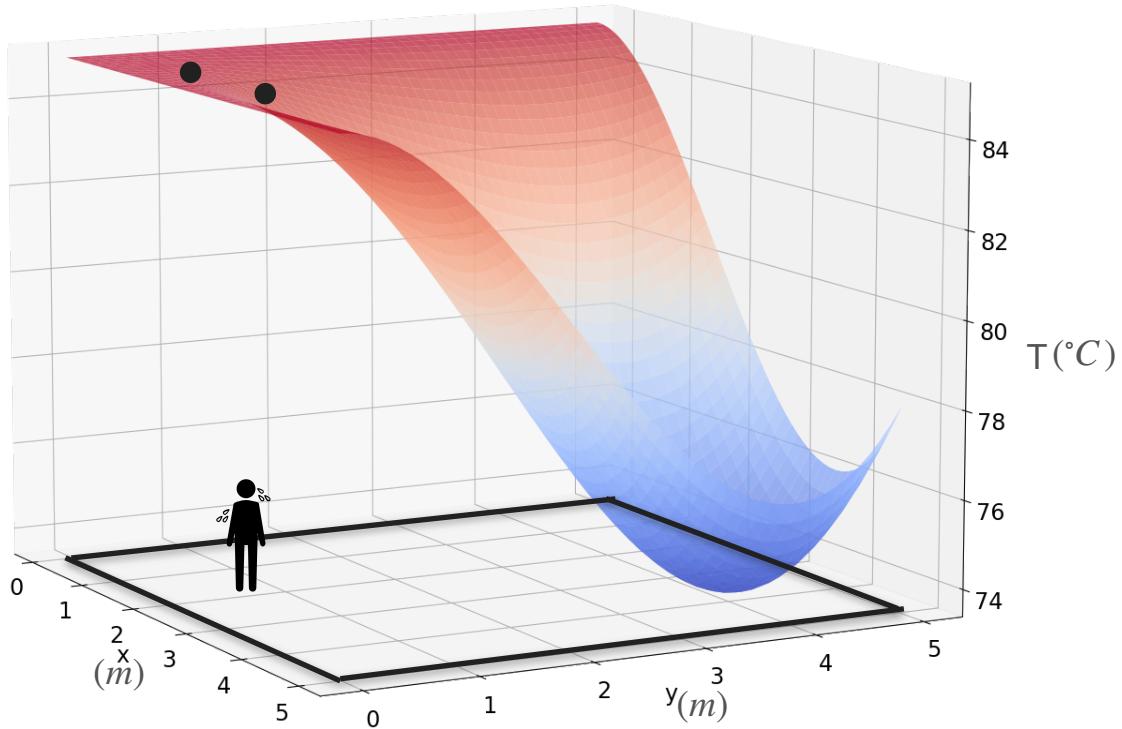


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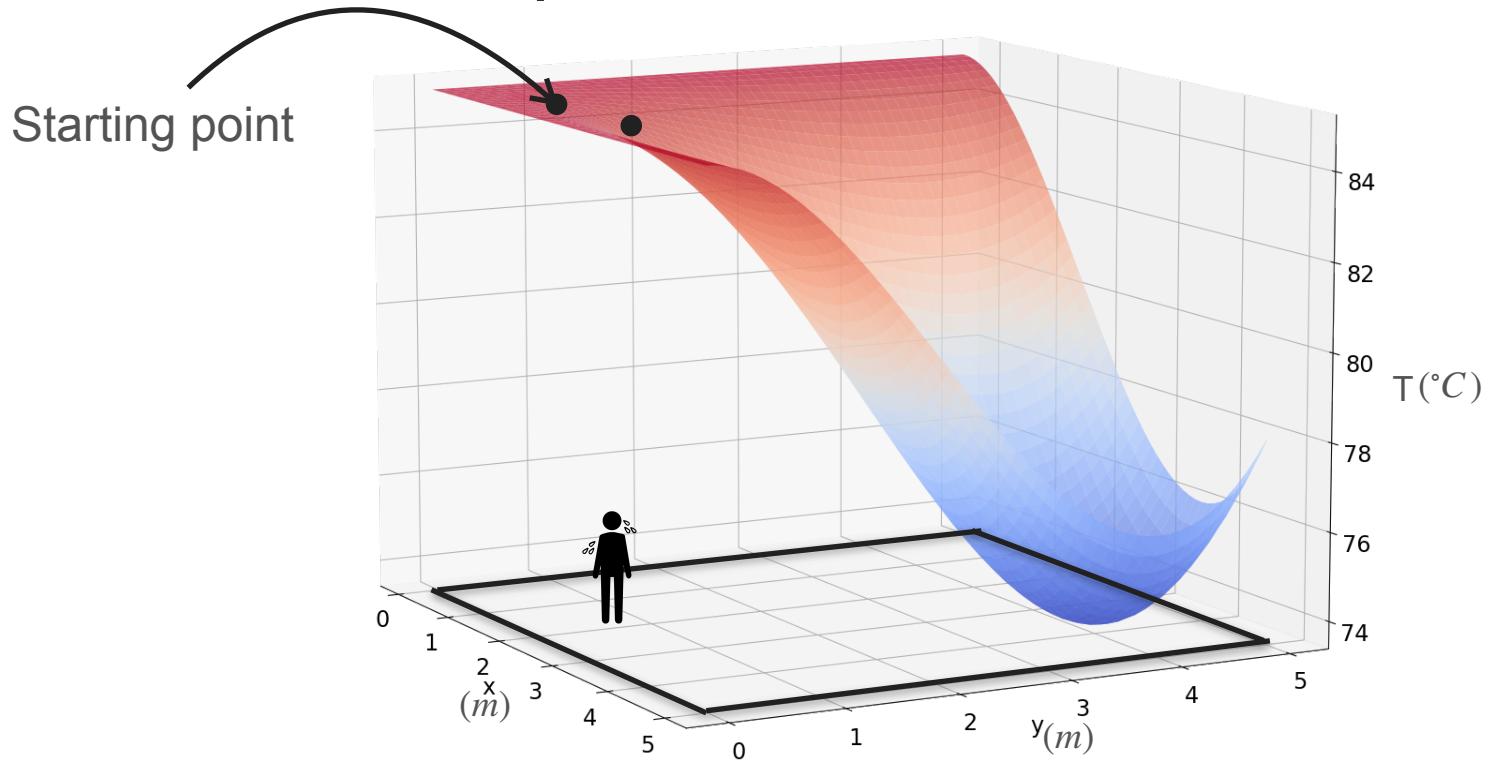


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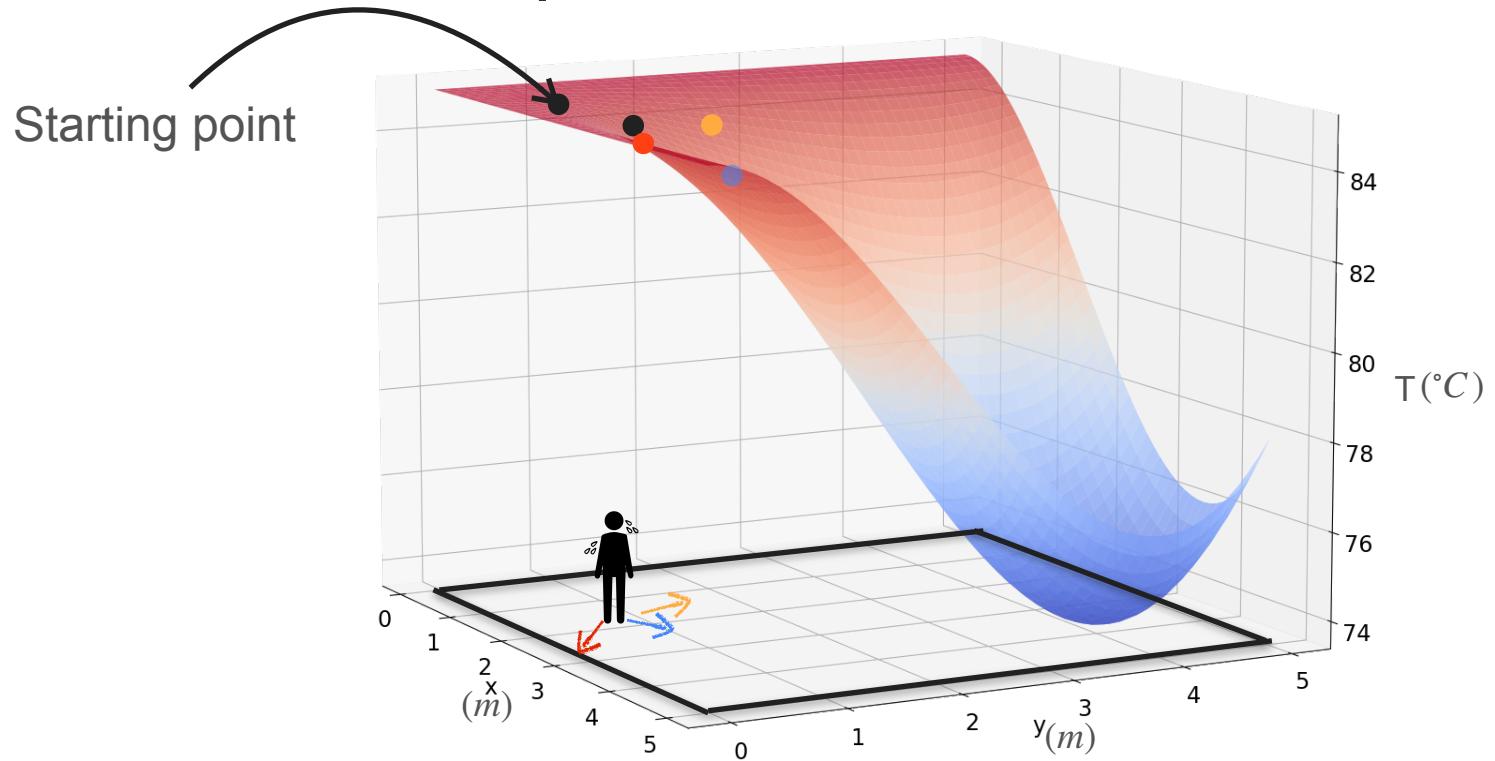
Starting point



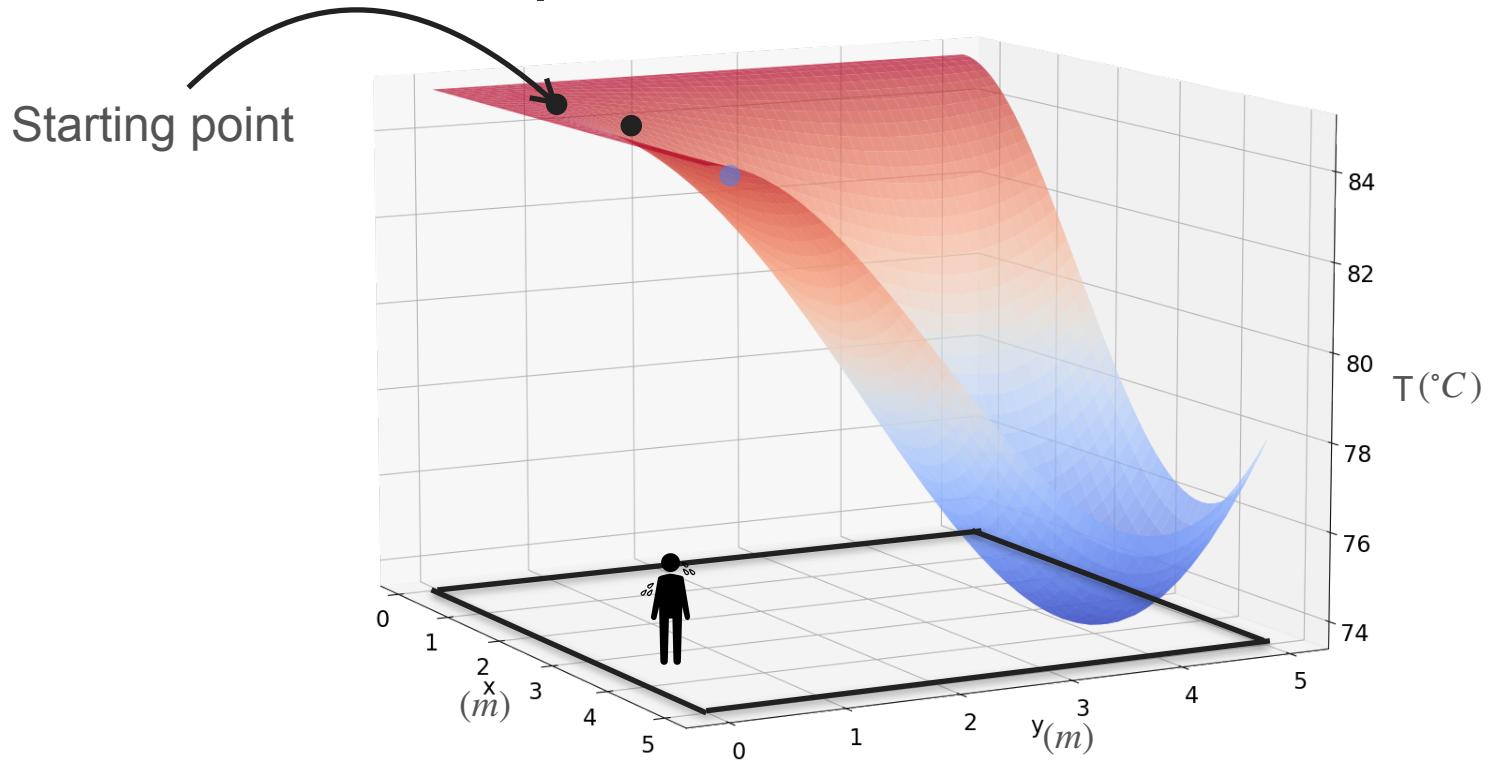
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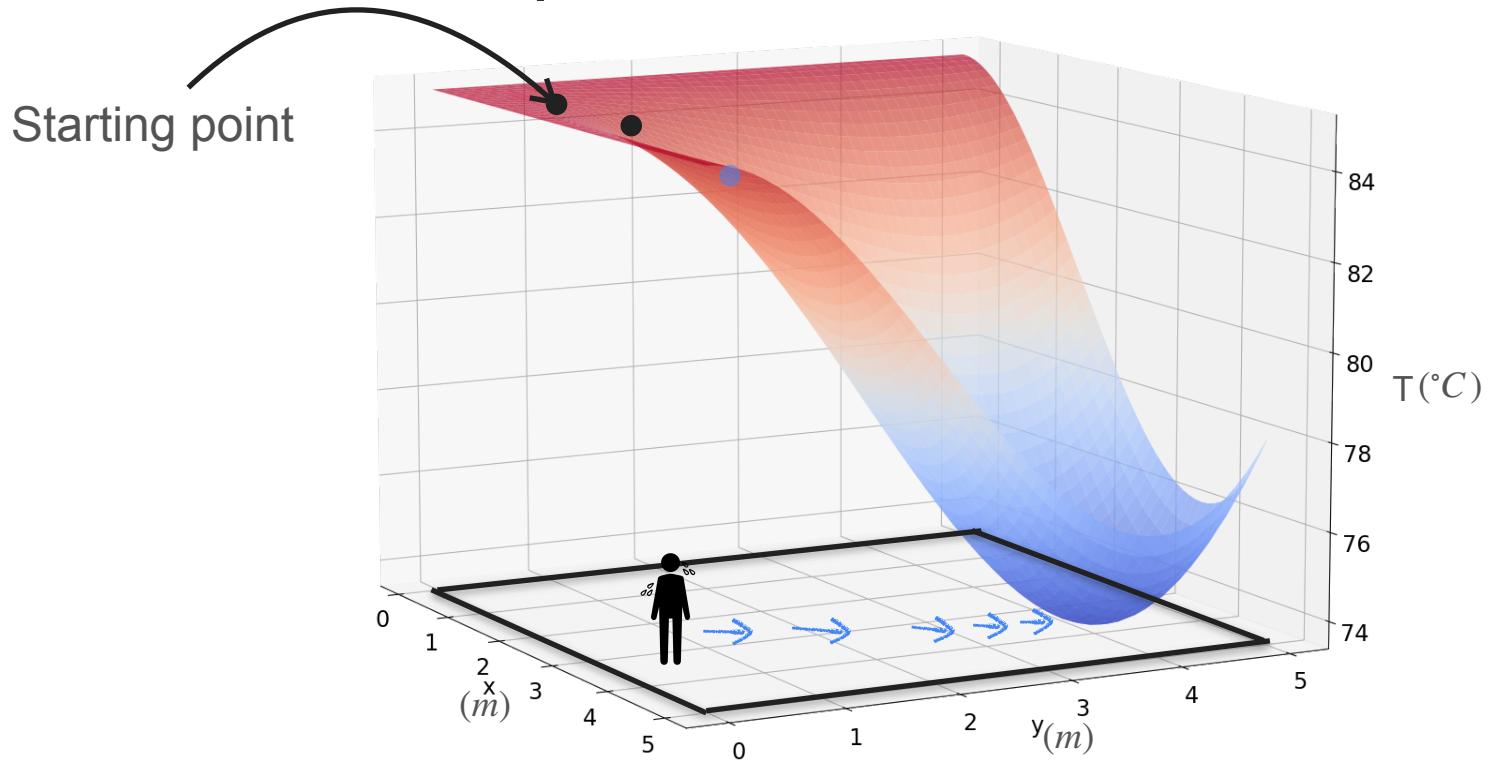
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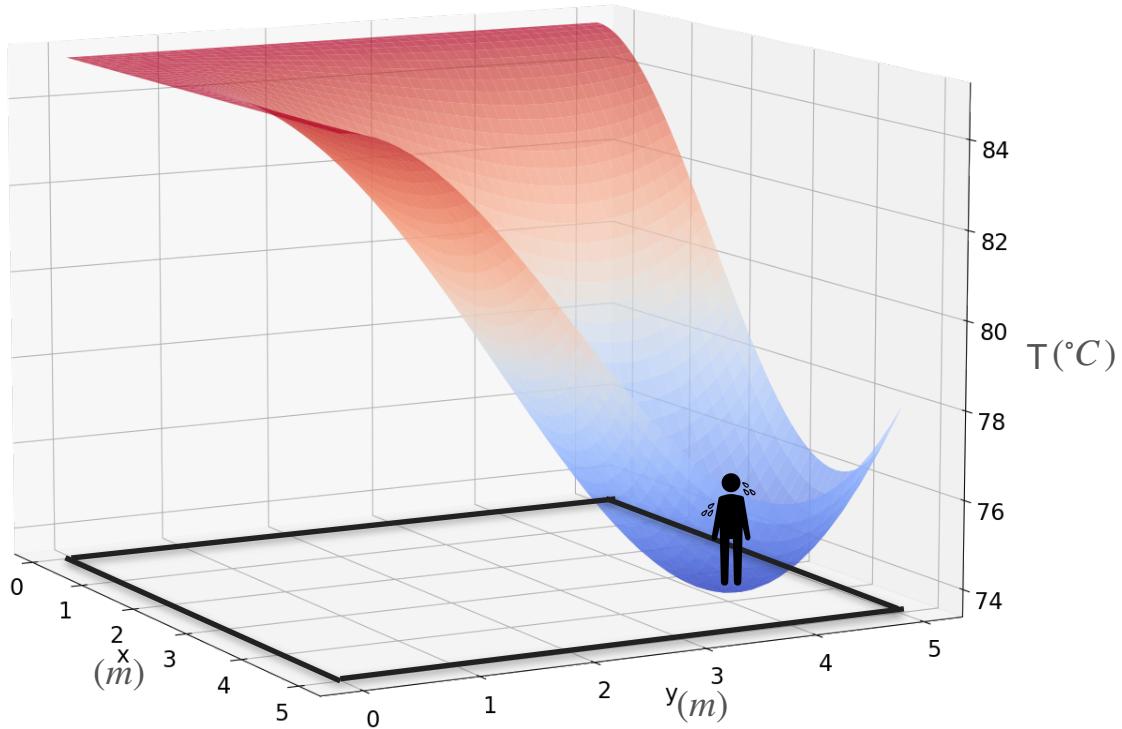
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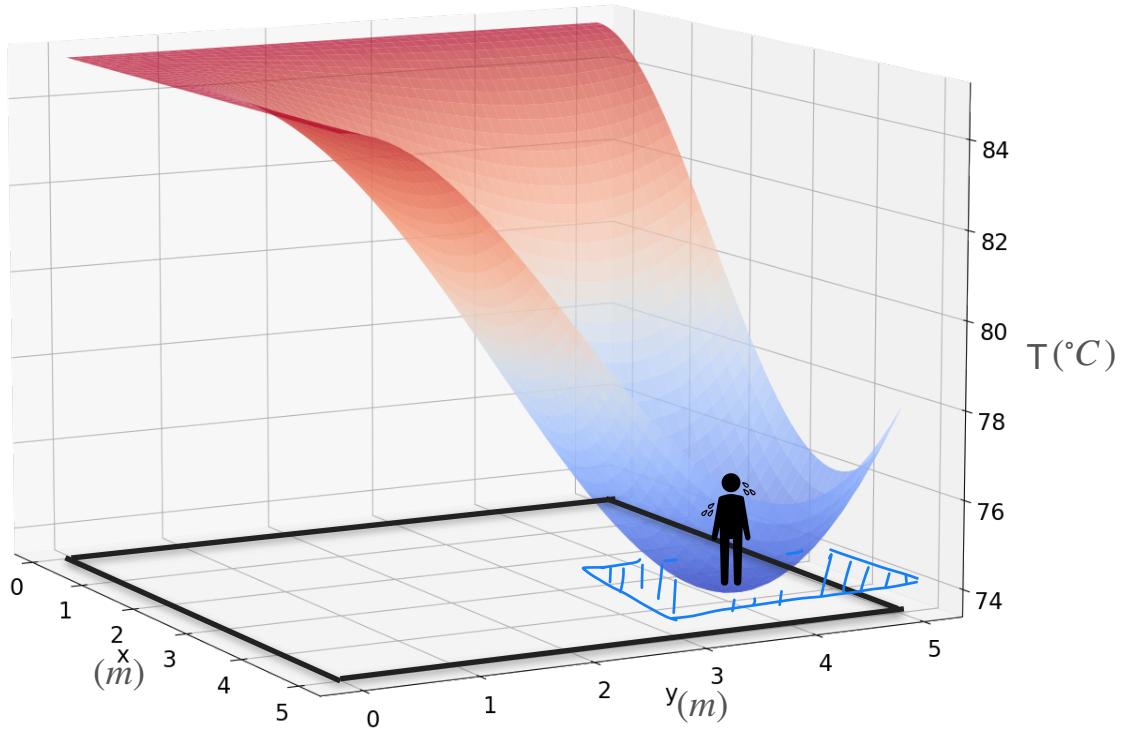
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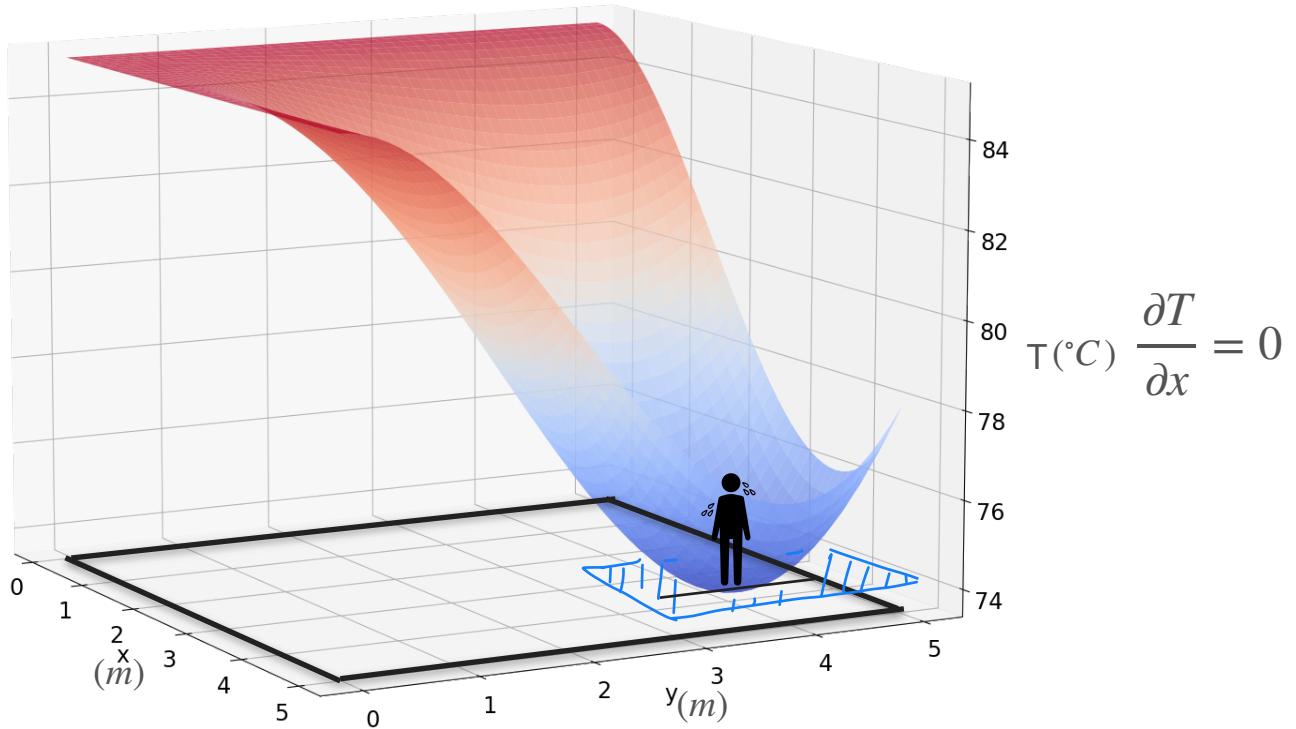
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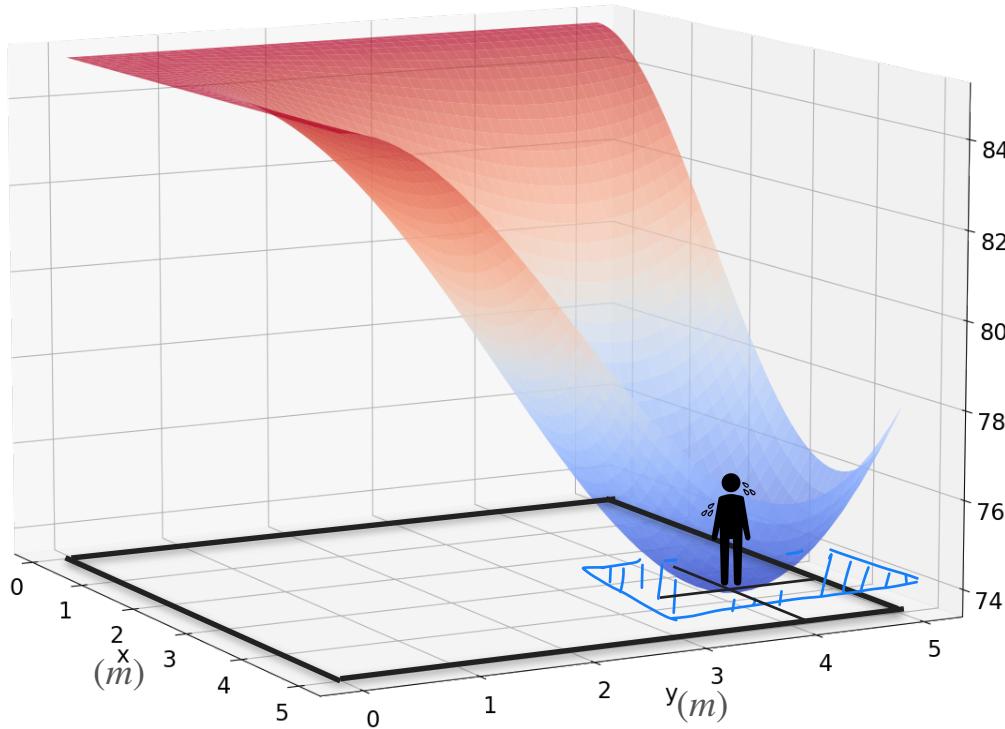
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Motivation for Optimization in Two Variables



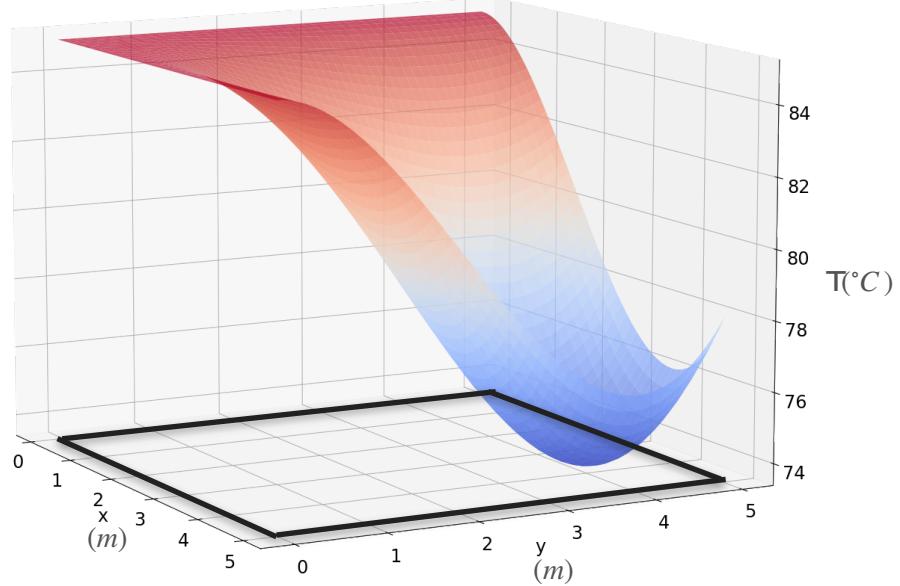
Motivation for Optimization in Two Variables



$$T(\text{°C}) \frac{\partial T}{\partial x} = 0$$

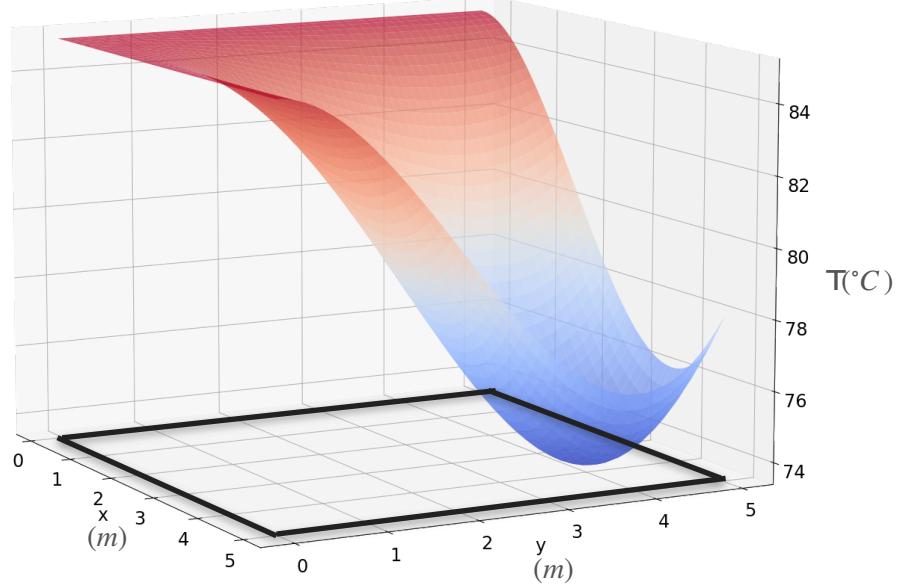
$$\frac{\partial T}{\partial y} = 0$$

Exercise



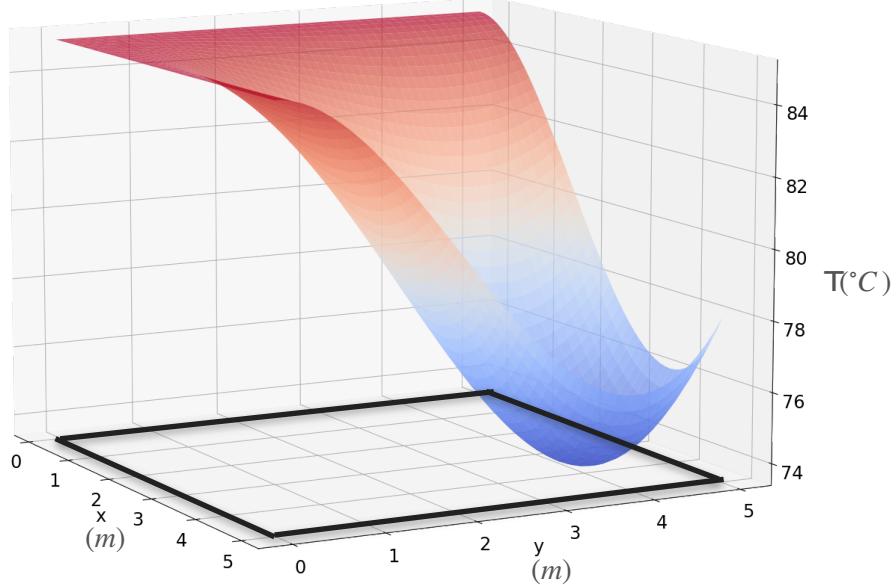
Exercise

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



Exercise

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

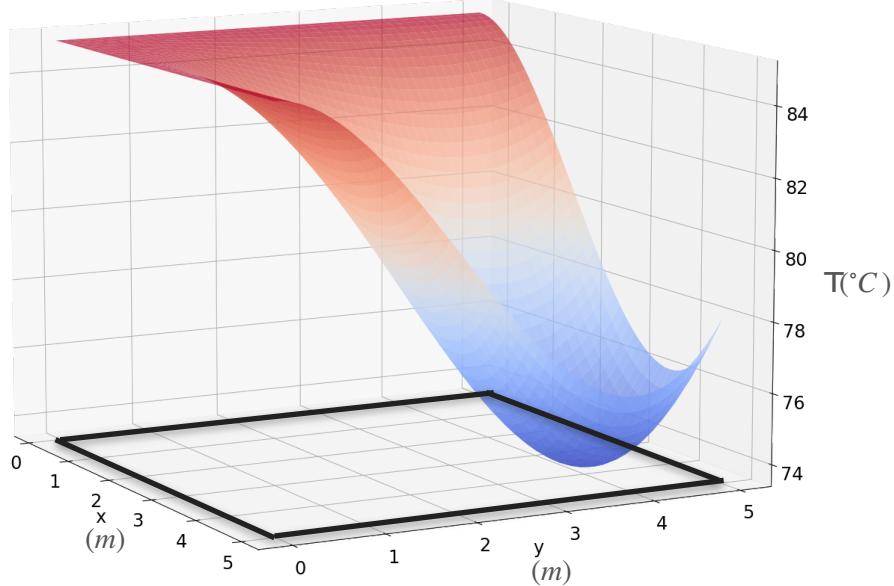


Try and calculate

$$\frac{\partial f}{\partial x}$$

Exercise

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



Try and calculate

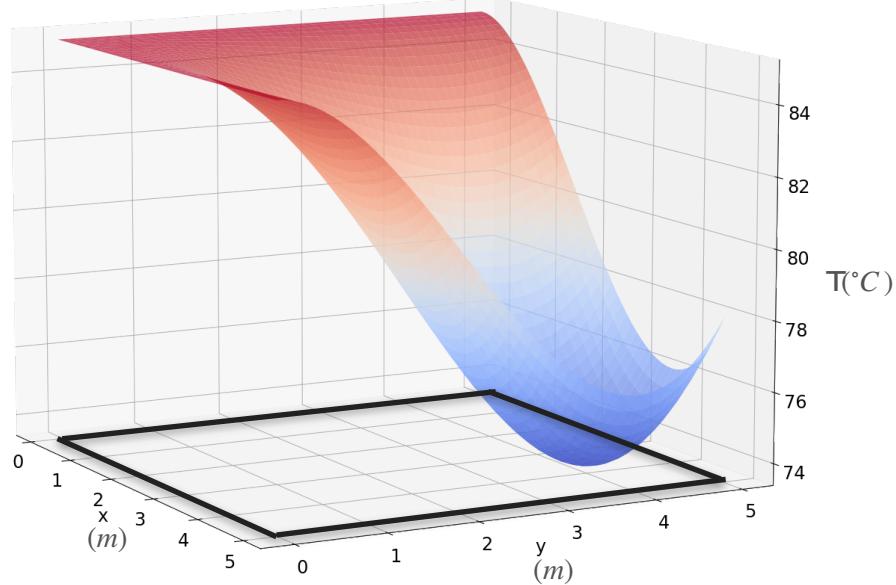
$$\frac{\partial f}{\partial x}$$

and

$$\frac{\partial f}{\partial y}$$

Motivation for Optimization in Two Variables

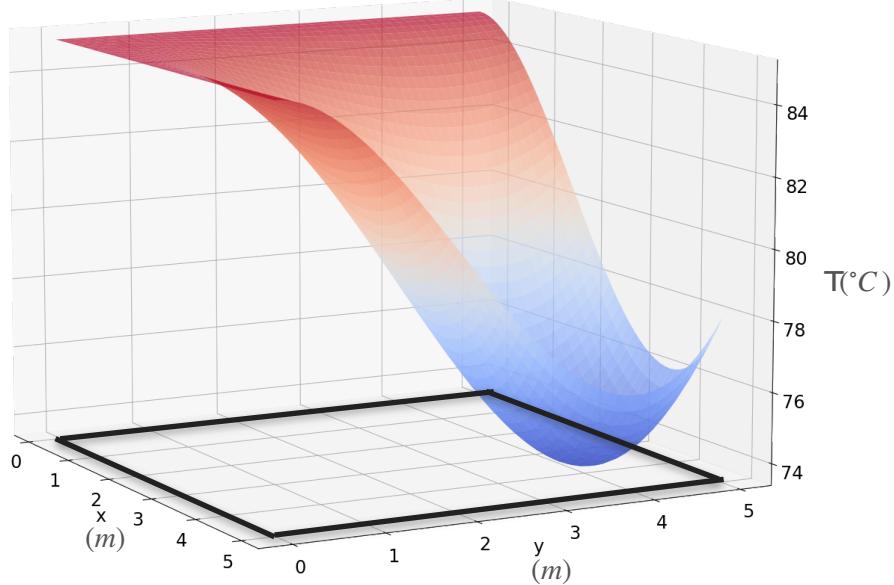
$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

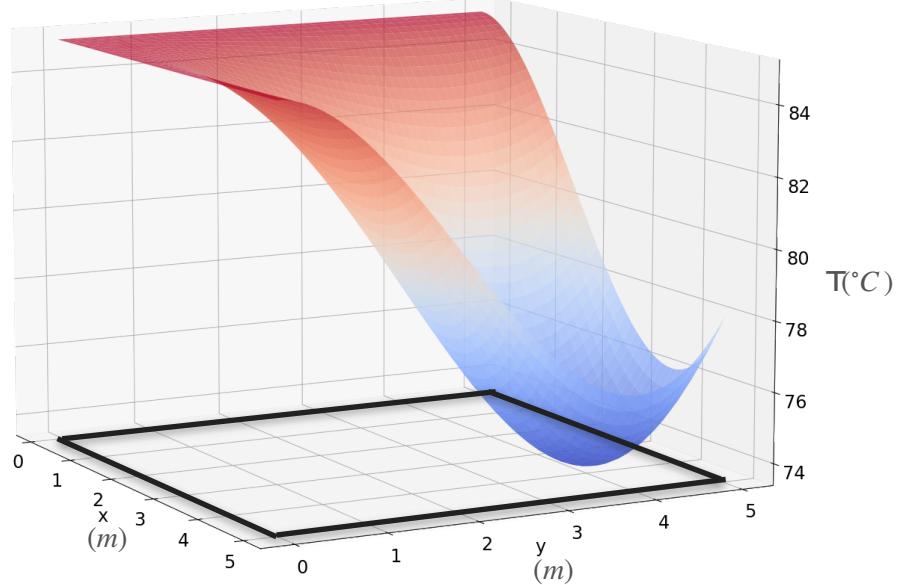
$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6)$$



Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6)$$

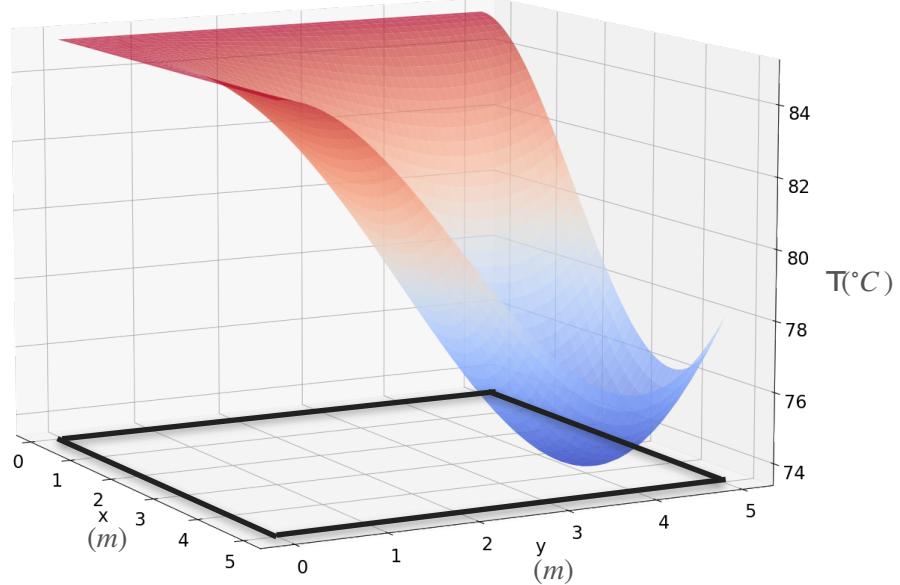


$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12)$$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

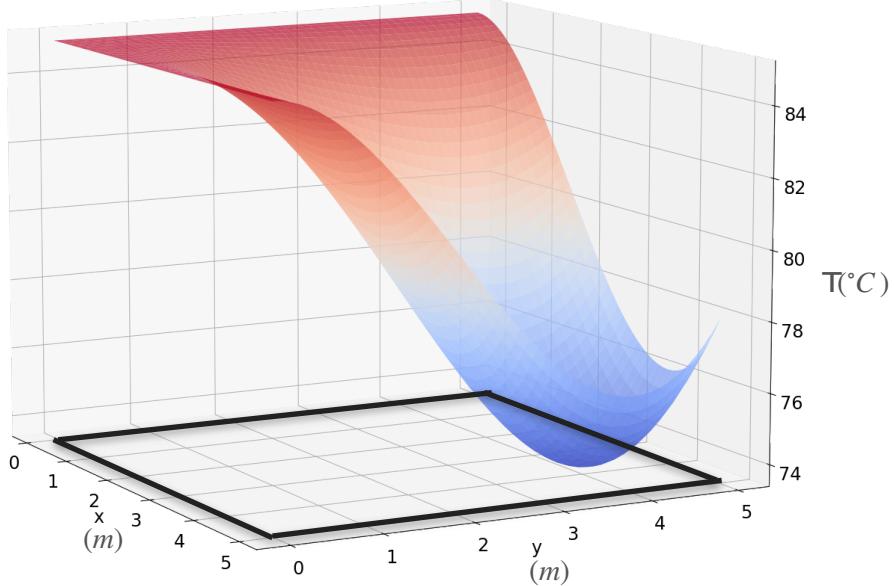
$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$



$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

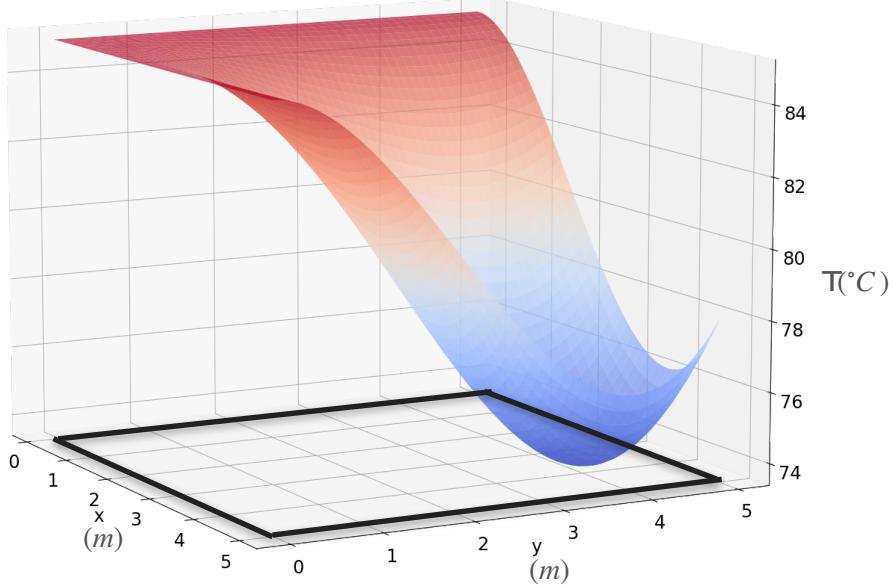


$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

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Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



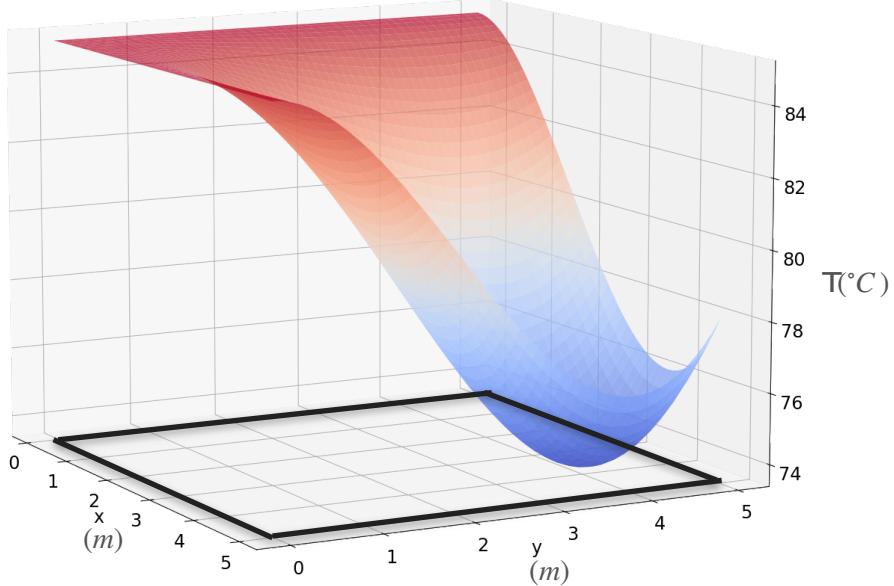
$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

$$x = 0 \quad x = 4$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



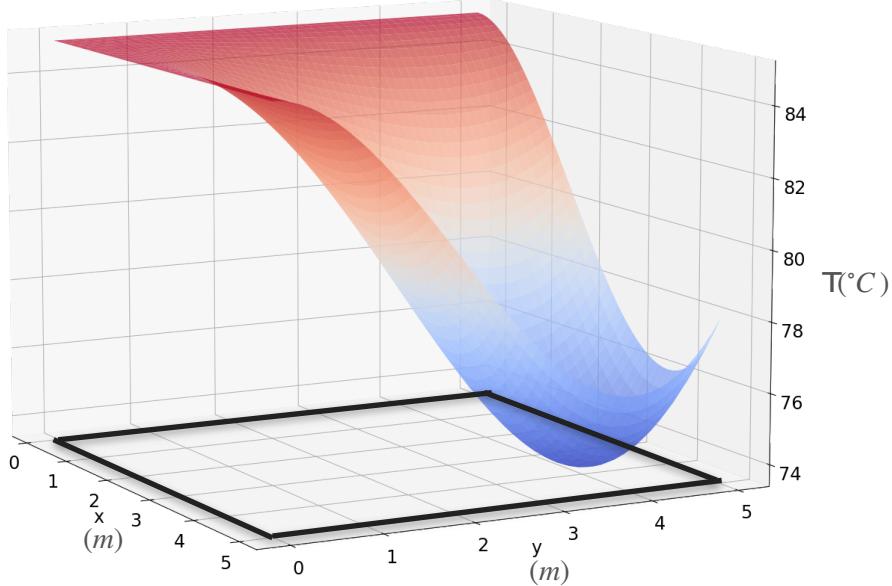
$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

$x = 0$ $x = 4$ $y = 0$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

Motivation for Optimization in Two Variables

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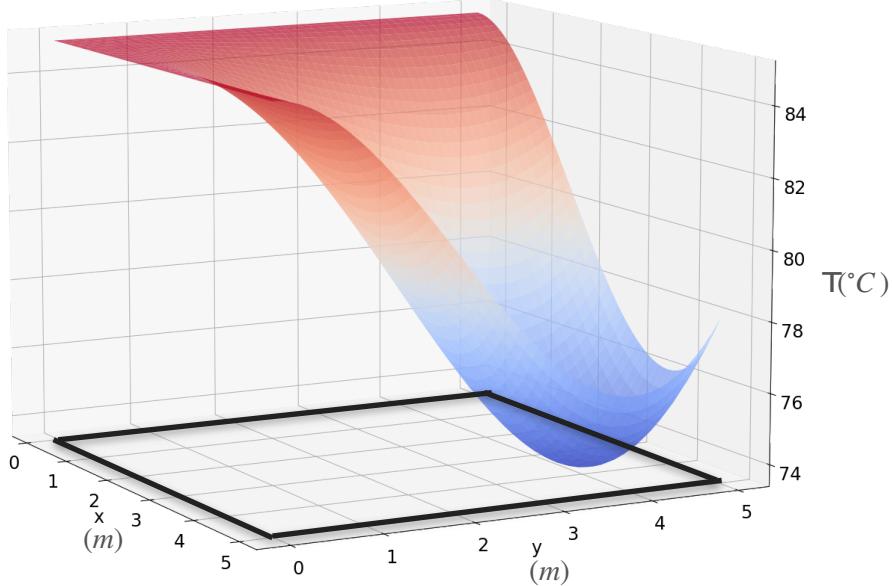
$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

$$x = 0 \quad x = 4 \quad y = 0 \quad y = 6$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

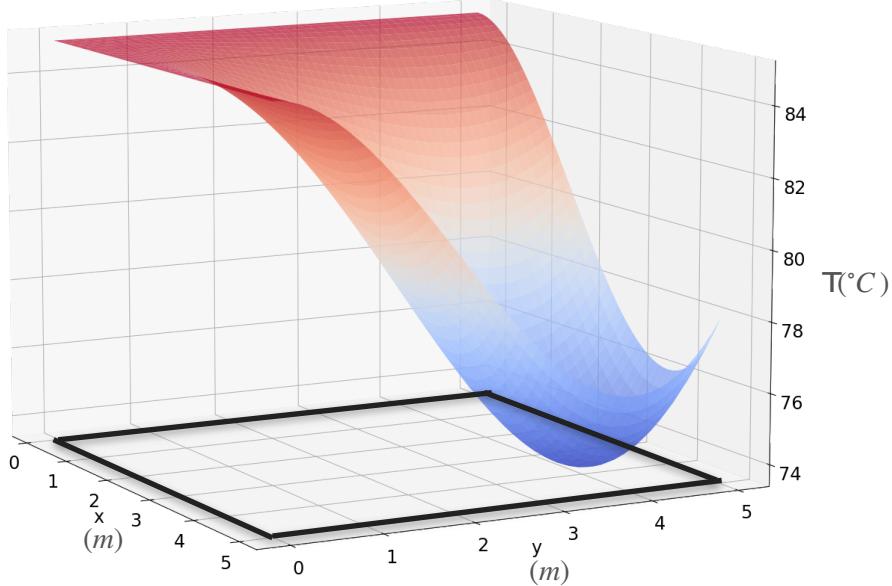
$x = 0$ $x = 4$ $y = 0$ $y = 6$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

$x = 0$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

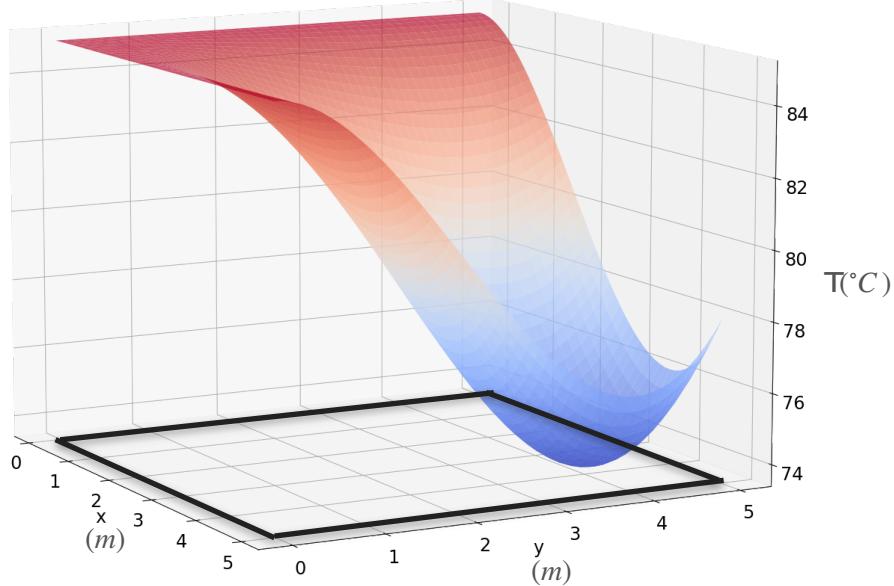
$x = 0$ $x = 4$ $y = 0$ $y = 6$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

$x = 0$ $x = 6$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

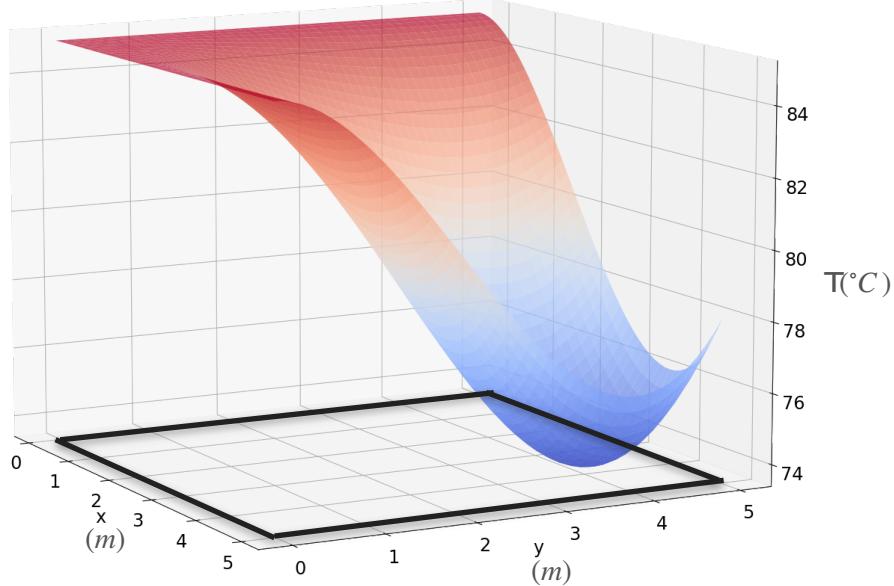
$x = 0$ $x = 4$ $y = 0$ $y = 6$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

$x = 0$ $x = 6$ $y = 0$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6) = 0$$

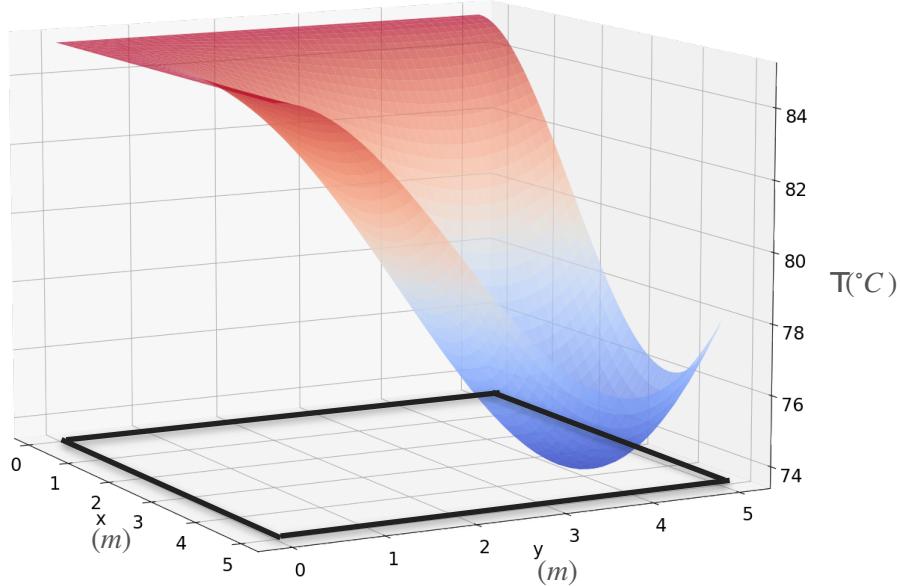
$x = 0$ $x = 4$ $y = 0$ $y = 6$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12) = 0$$

$x = 0$ $x = 6$ $y = 0$ $y = 4$

Motivation for Optimization in Two Variables

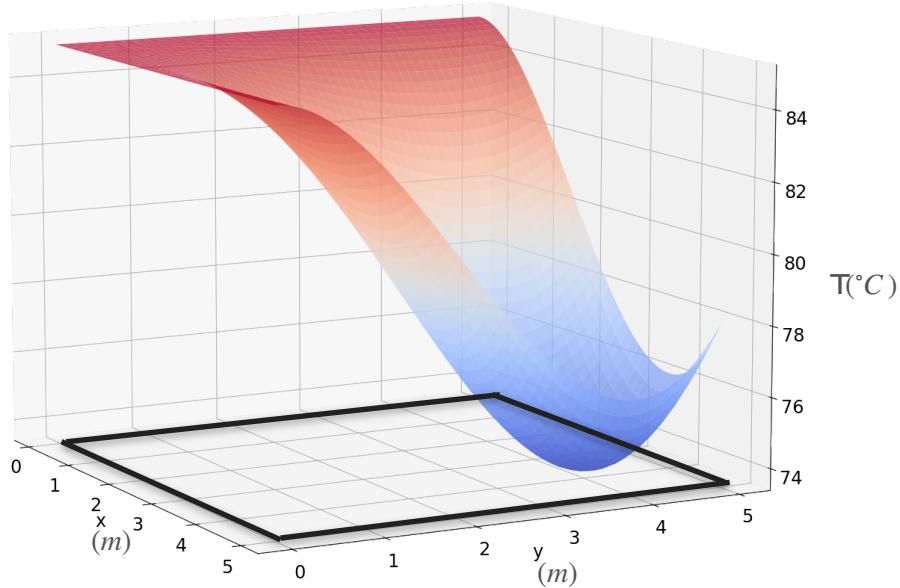
$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



Motivation for Optimization in Two Variables

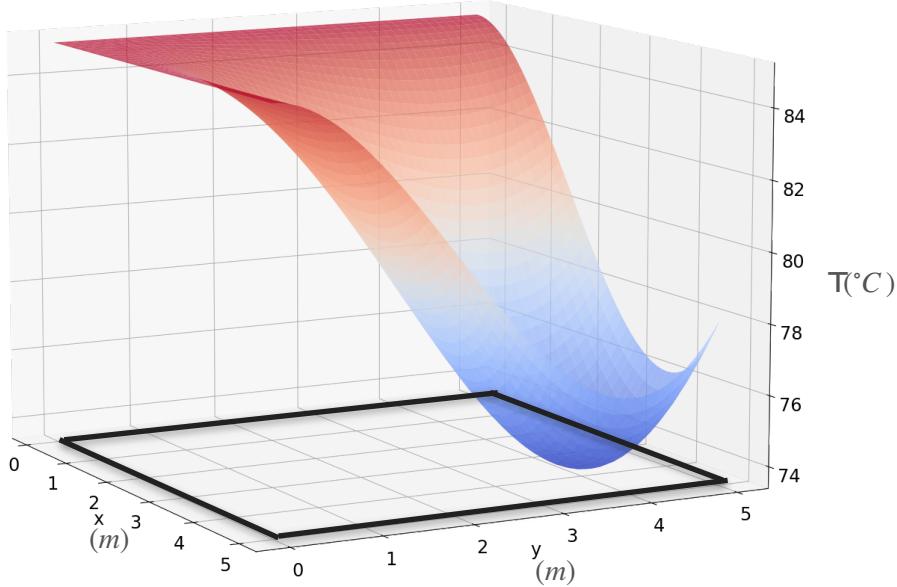
$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

Candidate points for the minima



Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



Candidate points for the minima

$$x = 0$$

$$y = 0$$

$$x = 0, y = 0$$

$$x = 0, y = 4$$

$$x = 0, y = 6$$

$$x = 4, y = 0$$

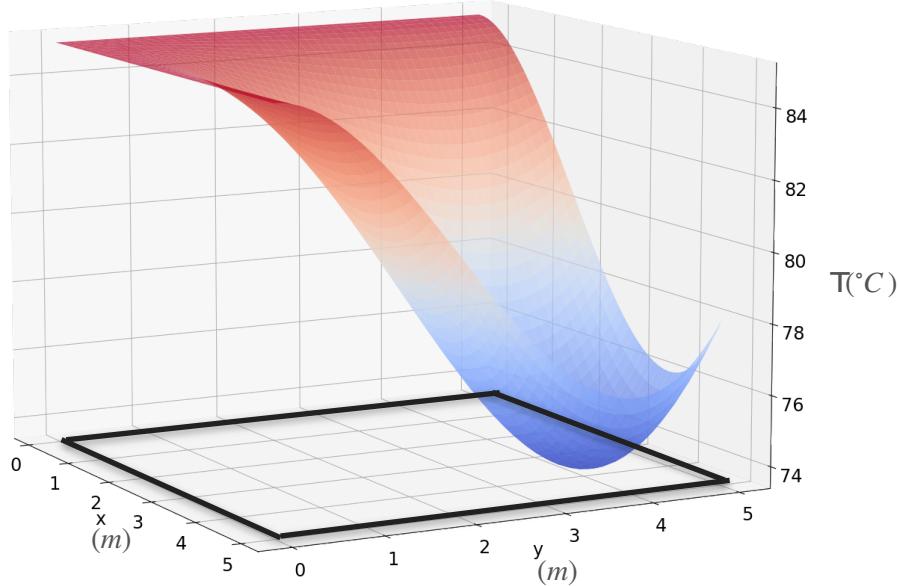
$$x = 4, y = 4$$

$$x = 6, y = 0$$

$$x = 6, y = 6$$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



Candidate points for the minima

$$x = 0$$

$$y = 0$$

$$x = 0, y = 0$$

$$x = 0, y = 4$$

$$x = 0, y = 6$$

Outside

$$x = 4, y = 0$$

$$x = 4, y = 4$$

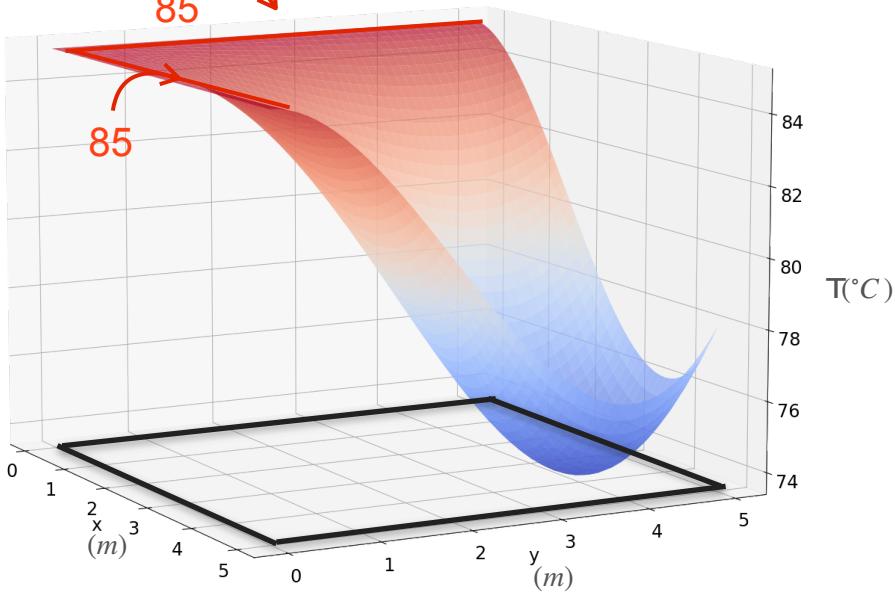
$$x = 6, y = 0$$

Outside

$$x = 6, y = 6$$

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$



Candidate points for the minima

$$\begin{aligned}x &= 0 \\y &= 0\end{aligned}$$

Maxima

$$x = 0, y = 0$$

$$x = 0, y = 4$$

$$x = 0, y = 6$$

Outside

$$x = 4, y = 0$$

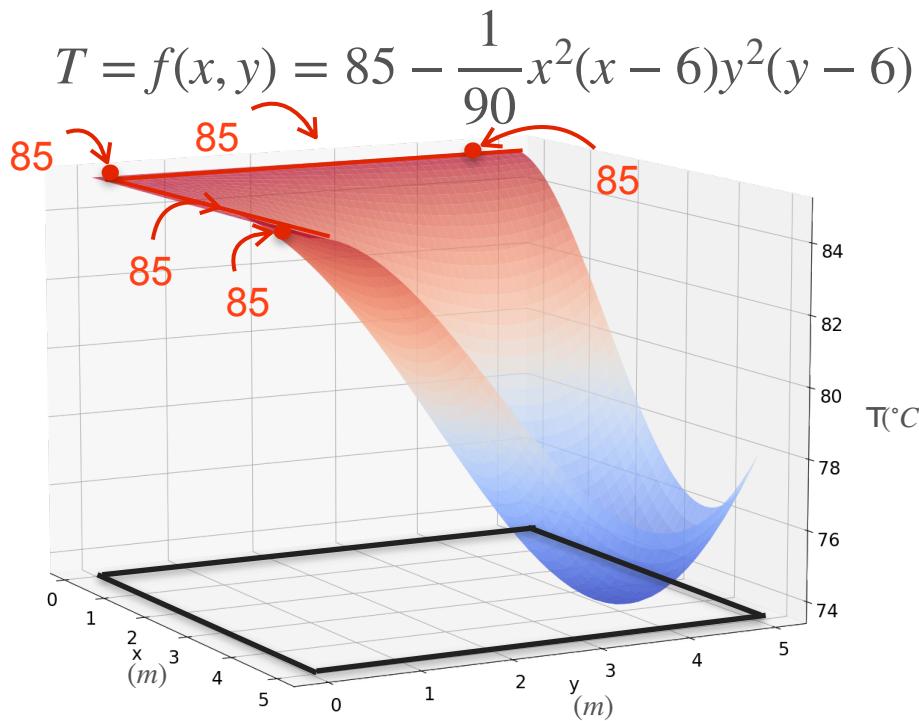
$$x = 4, y = 4$$

$$x = 6, y = 0$$

$$x = 6, y = 6$$

Outside

Motivation for Optimization in Two Variables



Candidate points for the minima

$$\begin{aligned}x &= 0 \\y &= 0\end{aligned}$$

Maxima

$$\begin{aligned}x &= 0, y = 0 \\x &= 0, y = 4\end{aligned}$$

Maxima

$$\begin{aligned}x &= 0, y = 6\end{aligned}$$

Outside

$$\begin{aligned}x &= 4, y = 0\end{aligned}$$

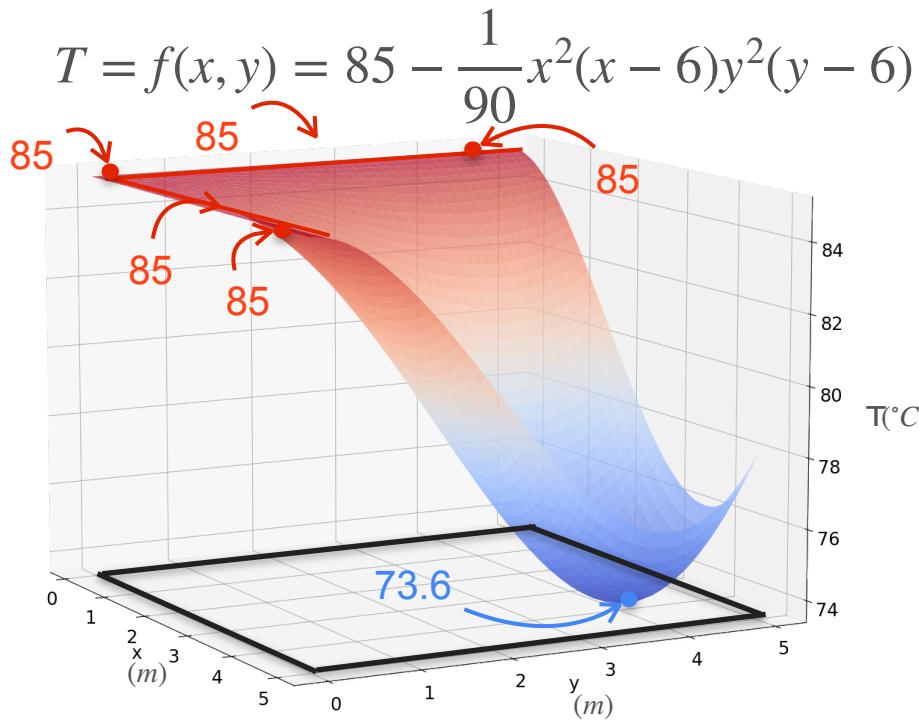
Maxima

$$\begin{aligned}x &= 4, y = 4\end{aligned}$$

$$\begin{aligned}x &= 6, y = 0 \\x &= 6, y = 6\end{aligned}$$

Outside

Motivation for Optimization in Two Variables



Candidate points for the minima

$$\begin{aligned}x &= 0 \\y &= 0\end{aligned}$$

Maxima

$$\begin{aligned}x &= 0, y = 0 \\x &= 0, y = 4\end{aligned}$$

Maxima

$$\begin{aligned}x &= 0, y = 6\end{aligned}$$

Outside

$$\begin{aligned}x &= 4, y = 0\end{aligned}$$

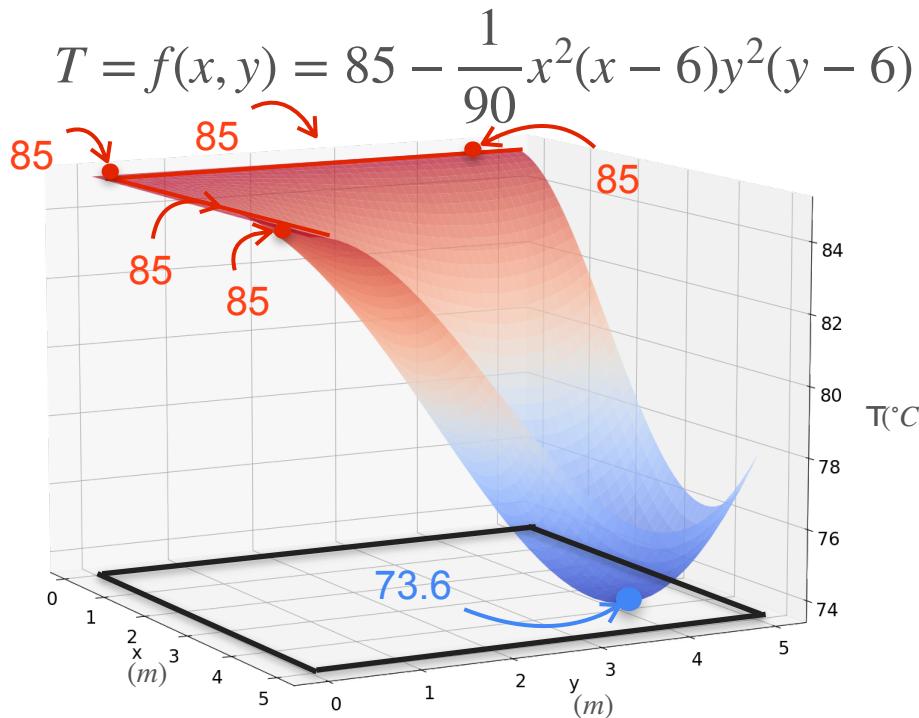
Maxima

$$\begin{aligned}x &= 4, y = 4\end{aligned}$$

$$\begin{aligned}x &= 6, y = 0 \\x &= 6, y = 6\end{aligned}$$

Outside

Motivation for Optimization in Two Variables



Candidate points for the minima

$$\begin{aligned}x &= 0 \\y &= 0\end{aligned}$$

Maxima

$$\begin{aligned}x &= 0, y = 0 \\x &= 0, y = 4\end{aligned}$$

Maxima

$$\begin{aligned}x &= 0, y = 6\end{aligned}$$

Outside

$$\begin{aligned}x &= 4, y = 0\end{aligned}$$

Maxima

$$\begin{aligned}x &= 4, y = 4\end{aligned}$$

Minimum

$$\begin{aligned}x &= 6, y = 0 \\x &= 6, y = 6\end{aligned}$$

Outside



DeepLearning.AI

Gradients and Gradient Descent

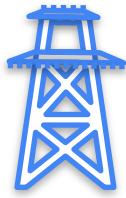
Optimization using gradients
- Analytical method

Linear Regression: Analytical Approach

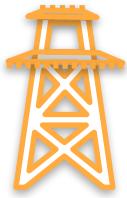
Linear Regression: Analytical Approach



Linear Regression: Analytical Approach



(1, 2)

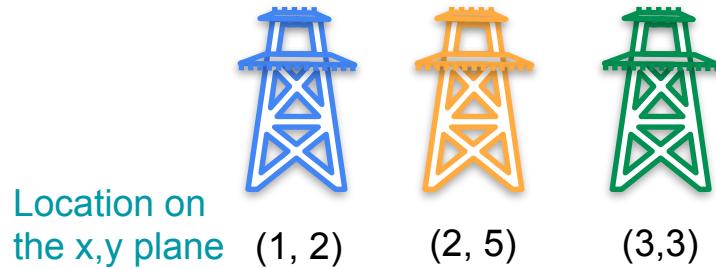


(2, 5)

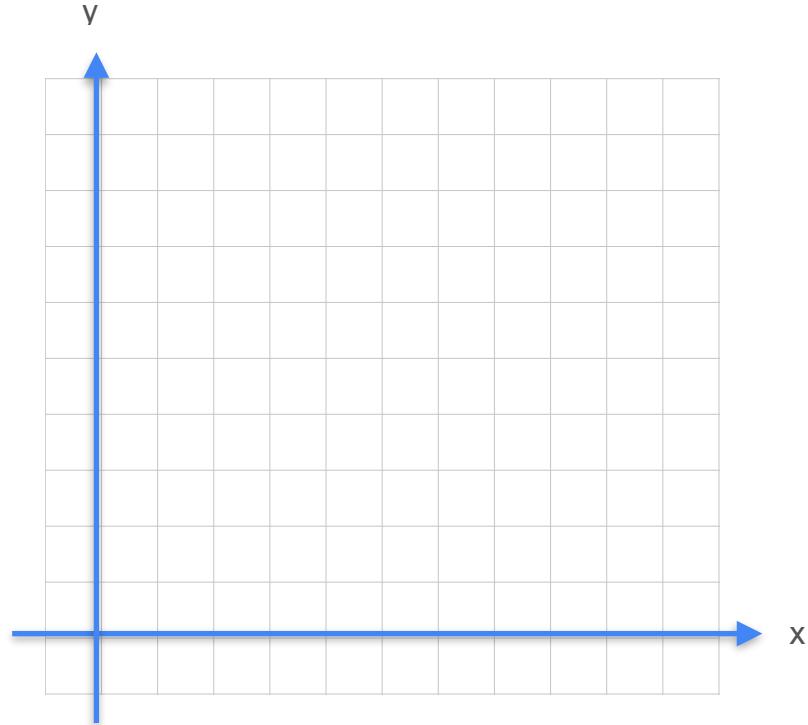
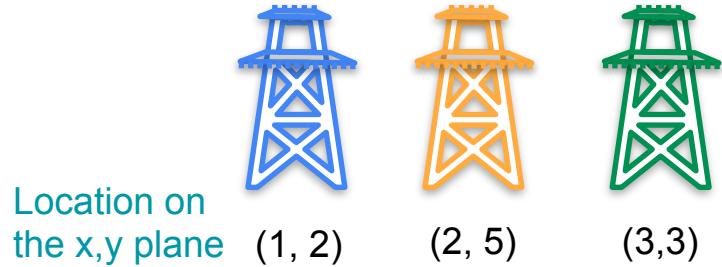


(3,3)

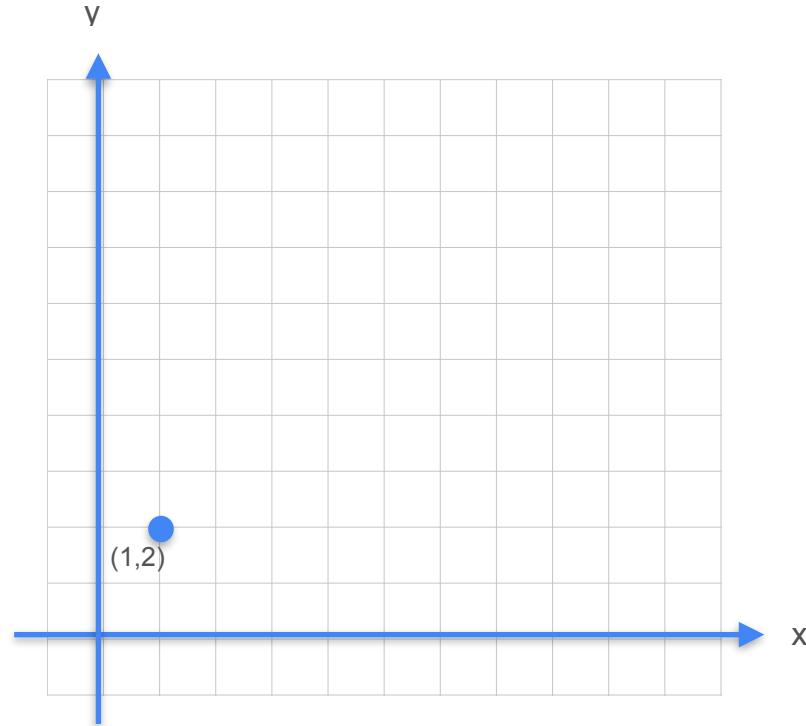
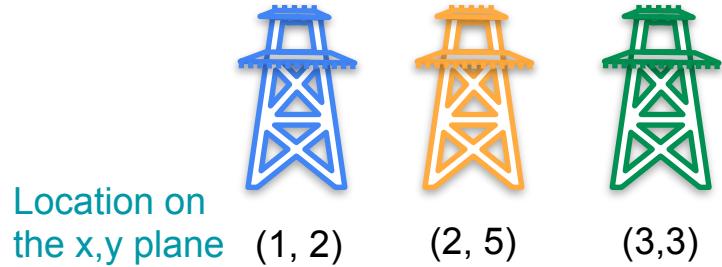
Linear Regression: Analytical Approach



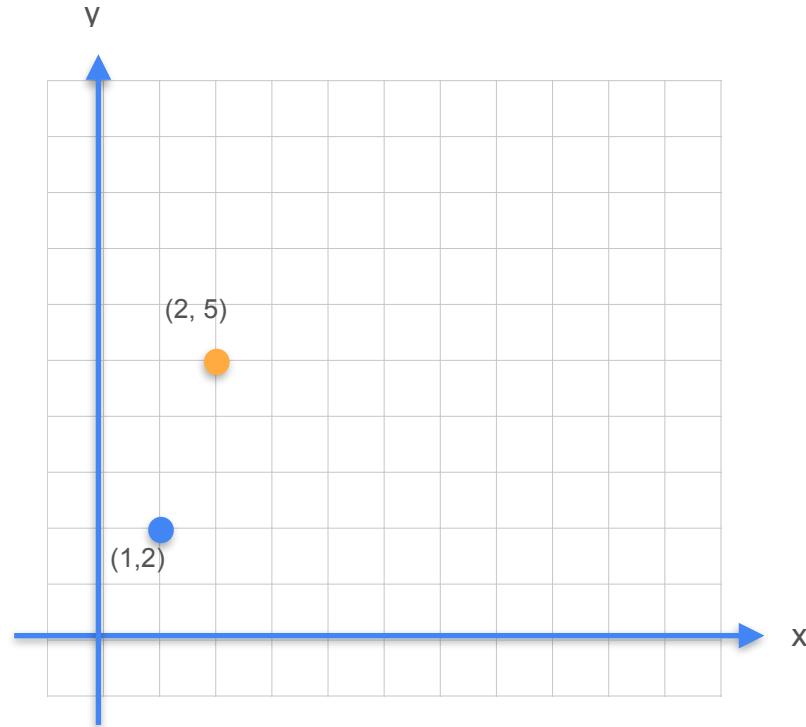
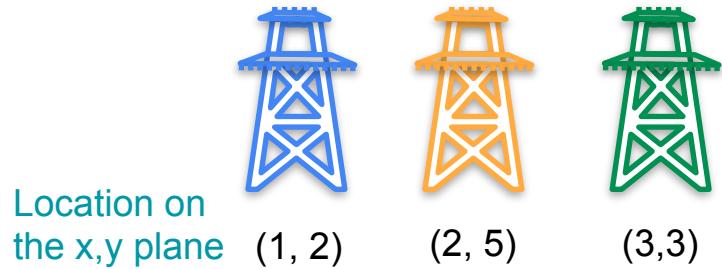
Linear Regression: Analytical Approach



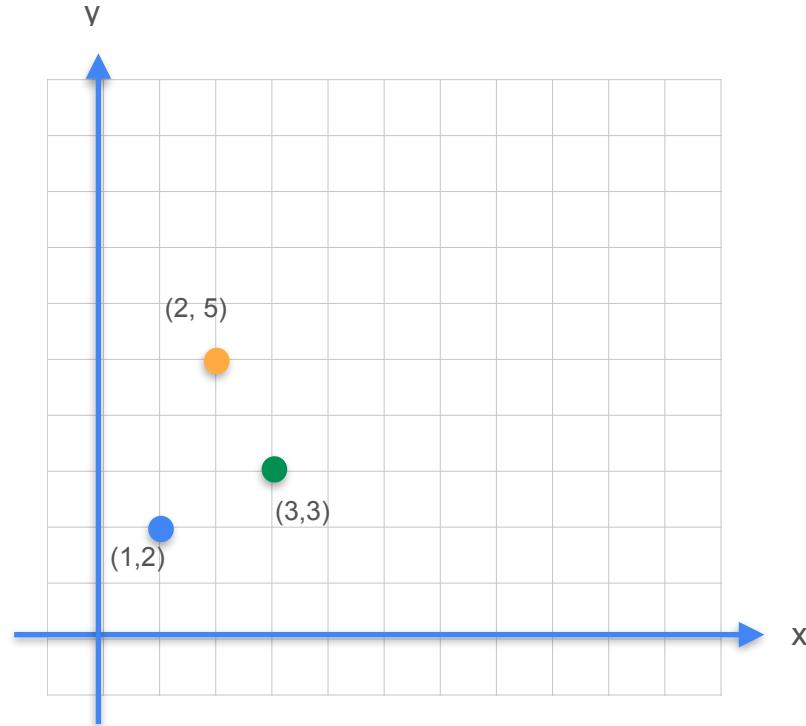
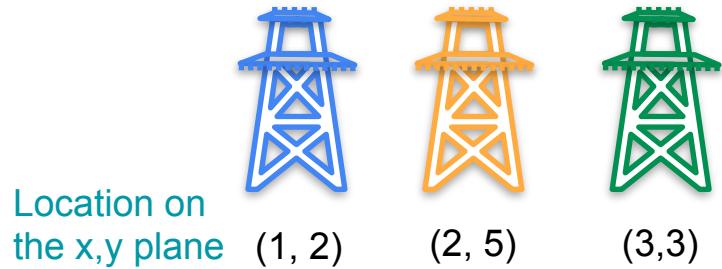
Linear Regression: Analytical Approach



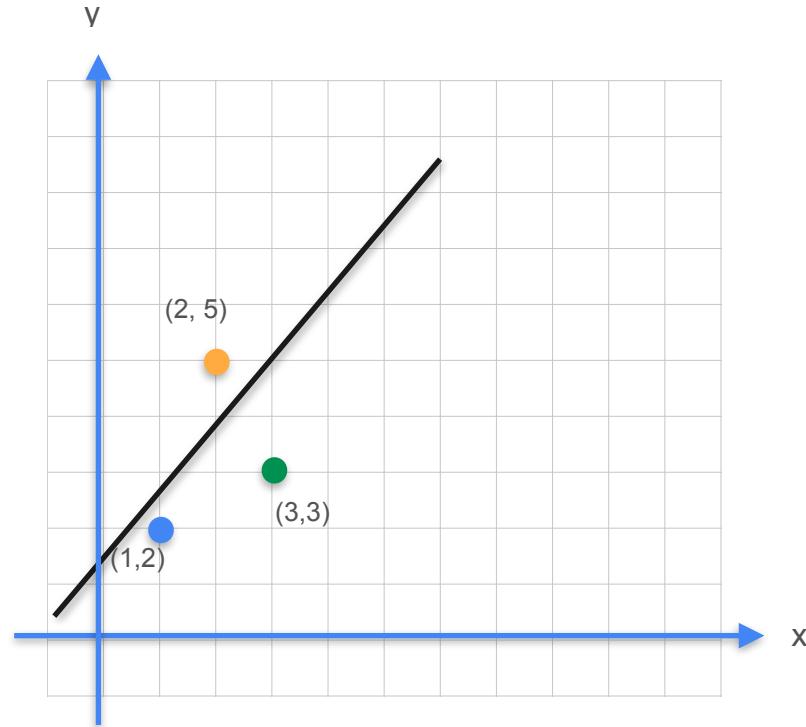
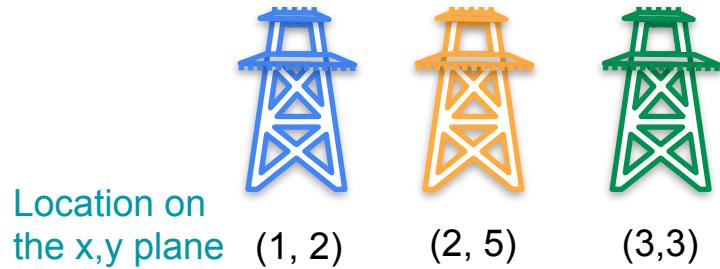
Linear Regression: Analytical Approach



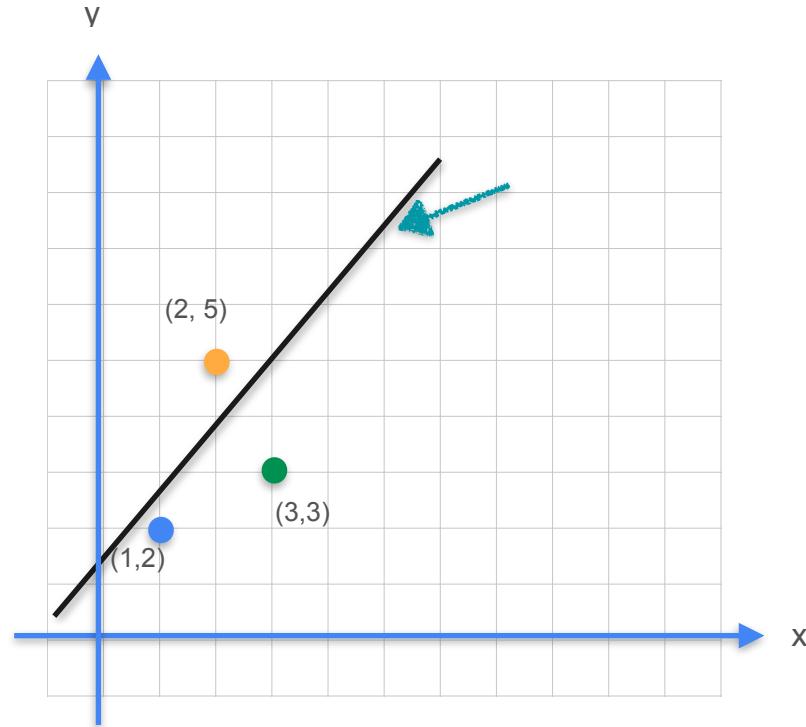
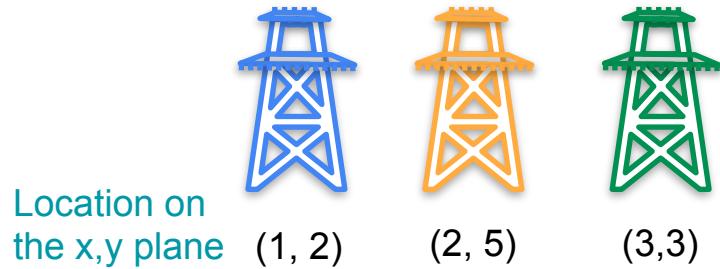
Linear Regression: Analytical Approach



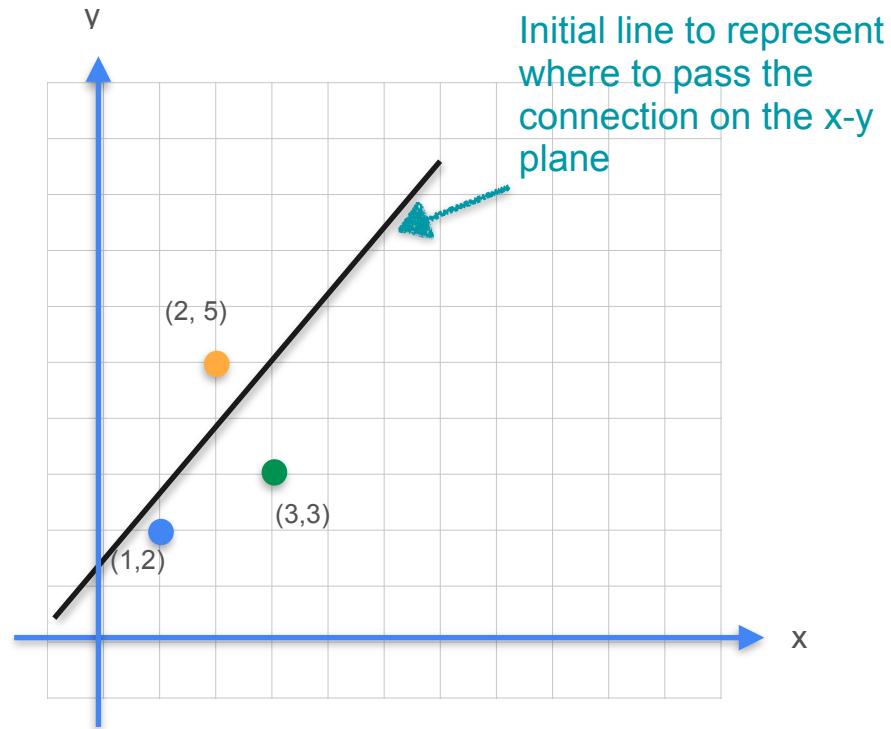
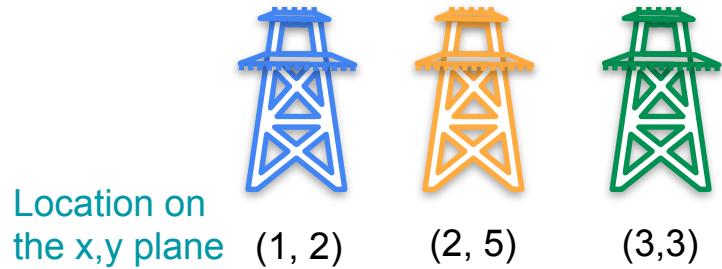
Linear Regression: Analytical Approach



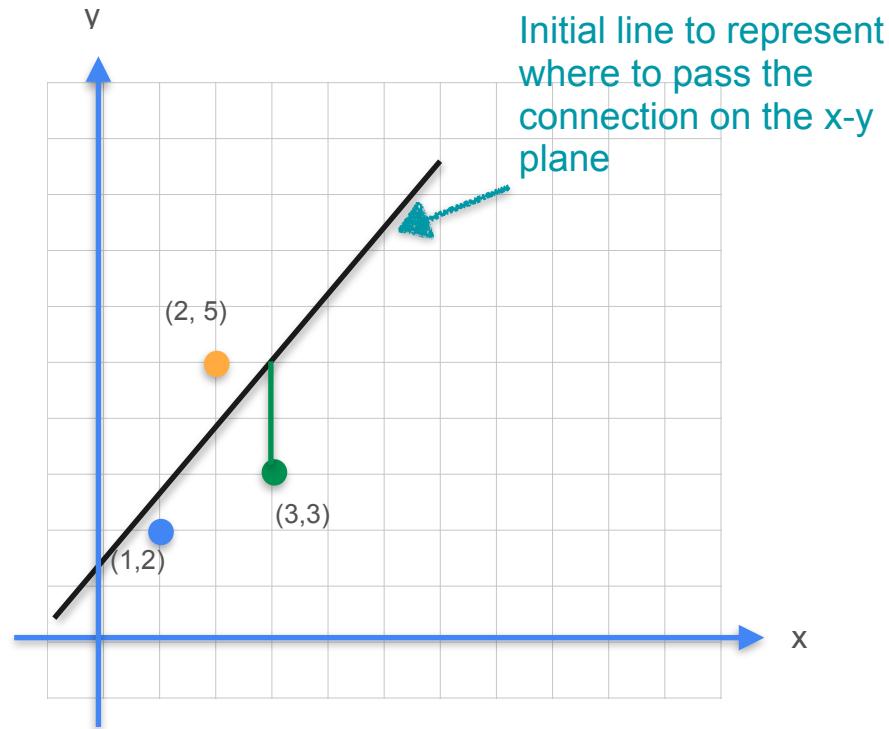
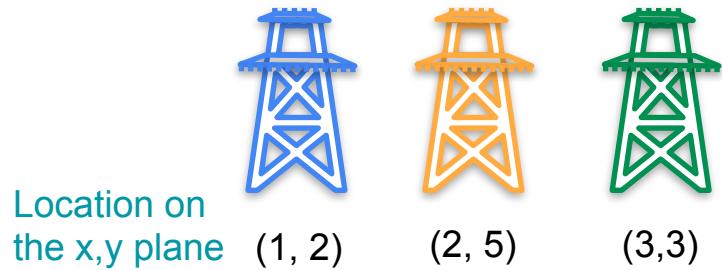
Linear Regression: Analytical Approach



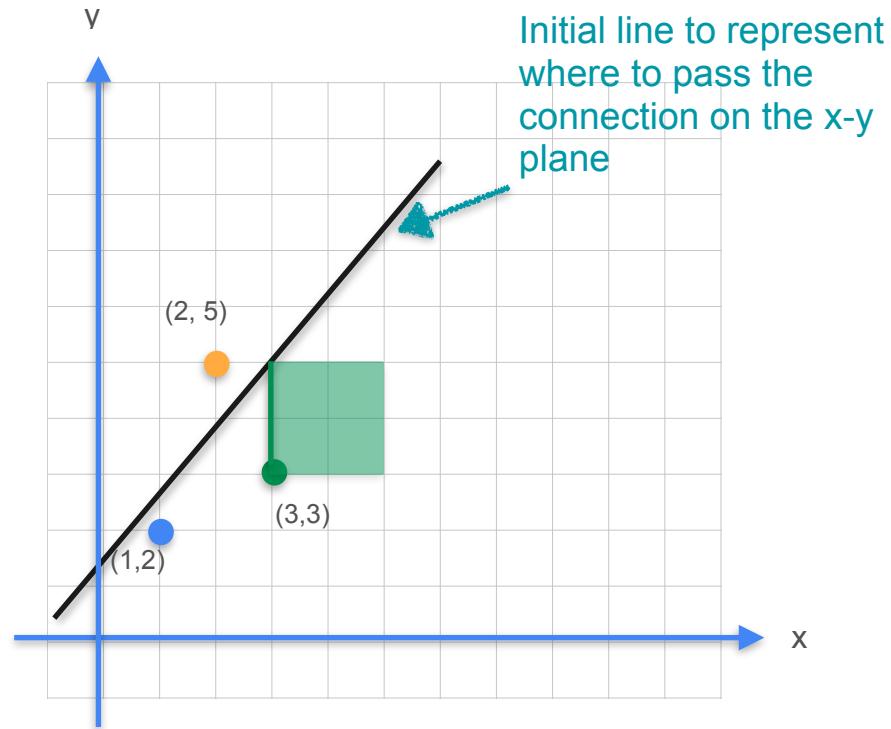
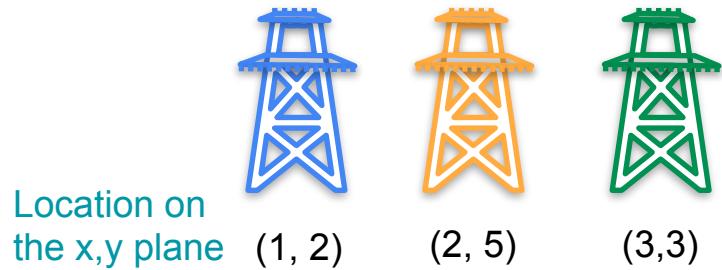
Linear Regression: Analytical Approach



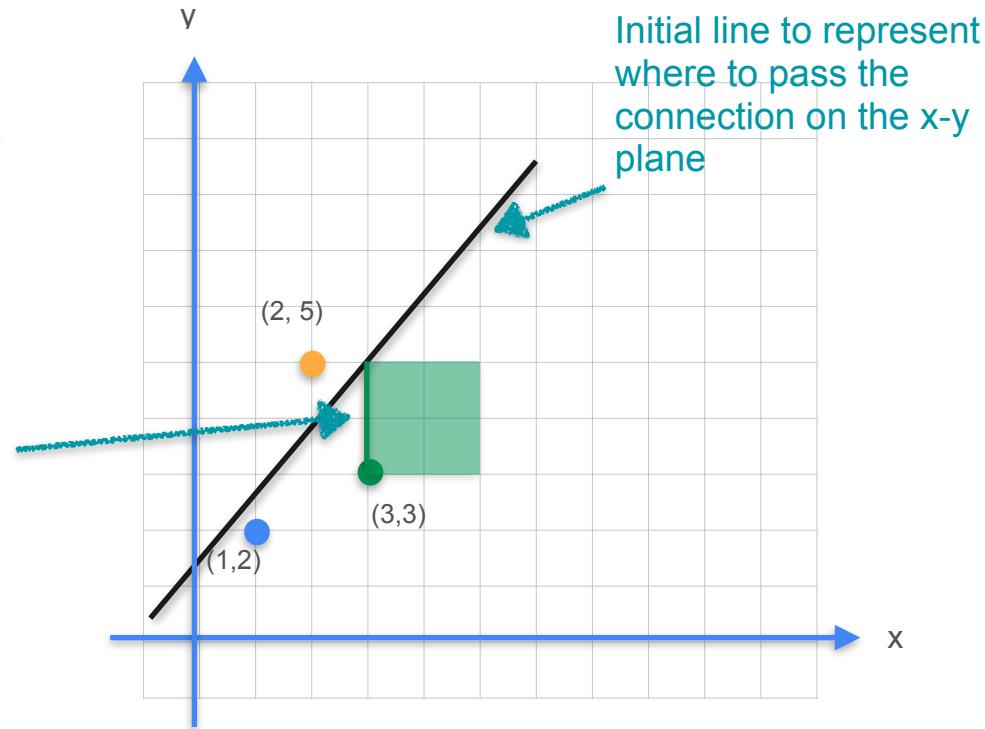
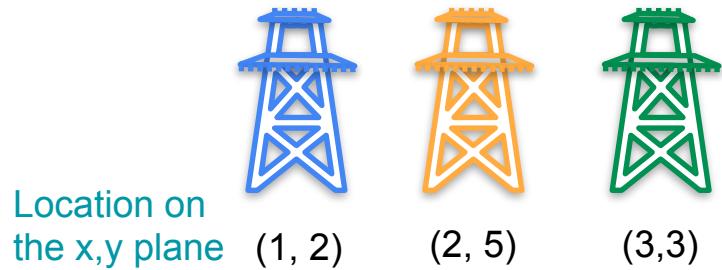
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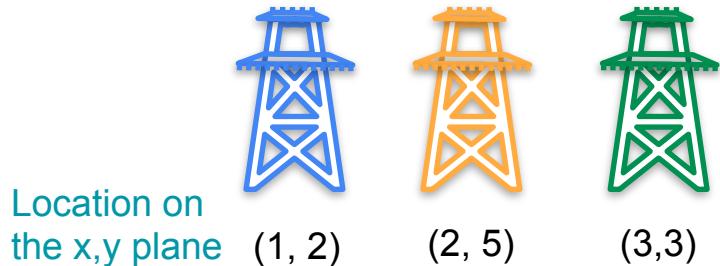
Linear Regression: Analytical Approach



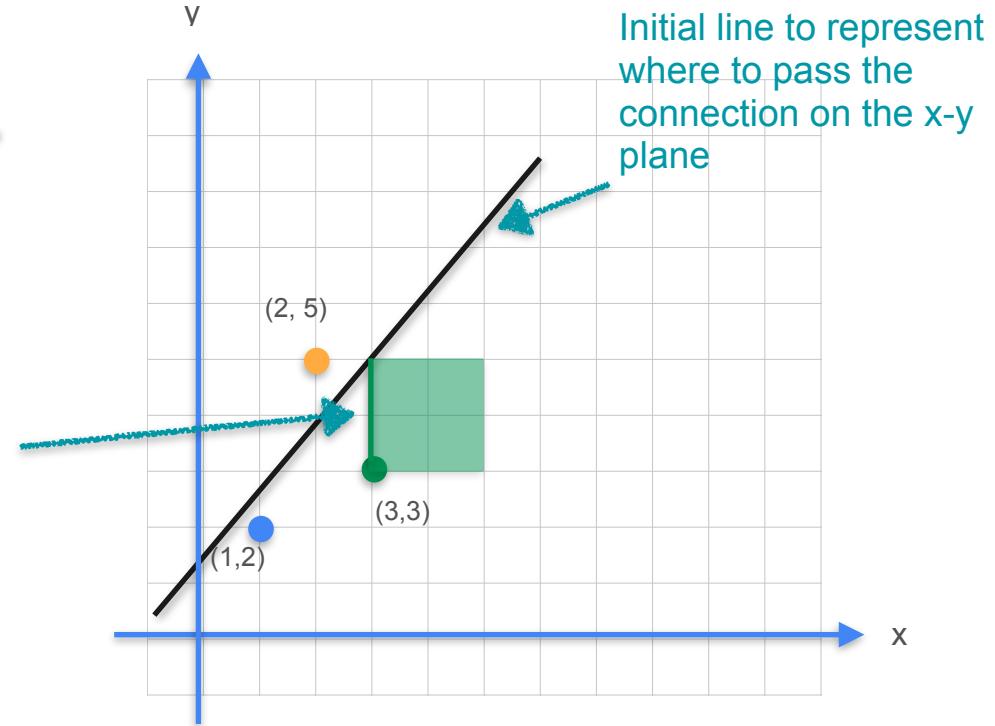
Linear Regression: Analytical Approach



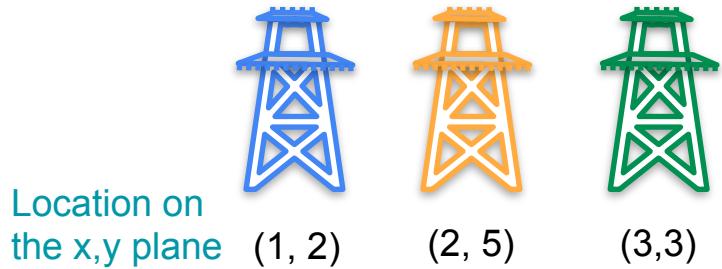
Linear Regression: Analytical Approach



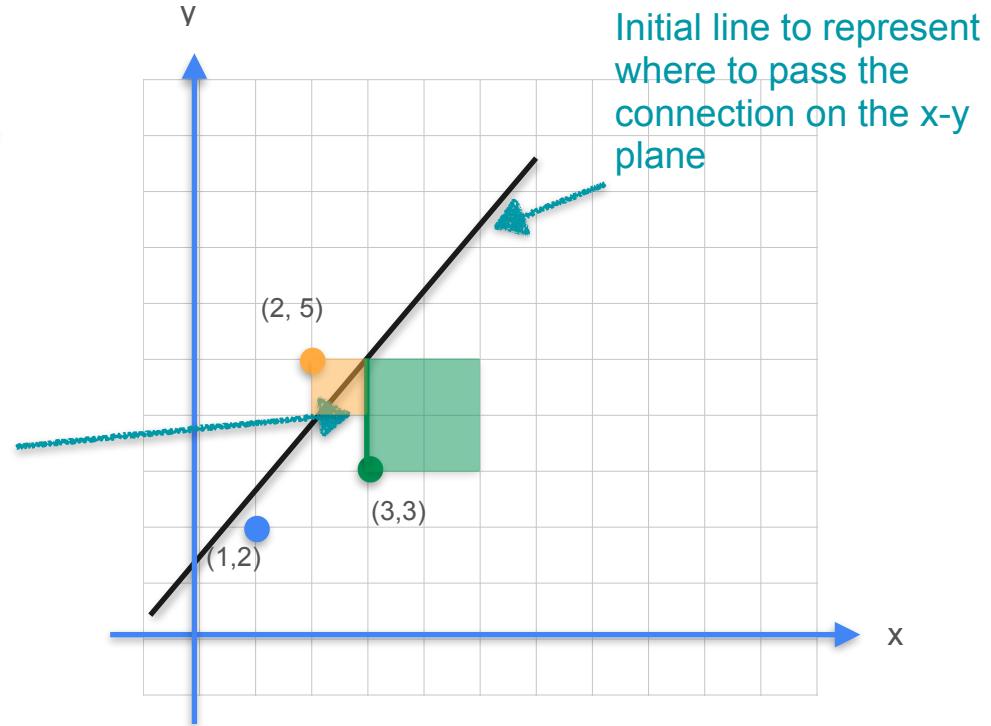
The cost of connecting connection to the powerline



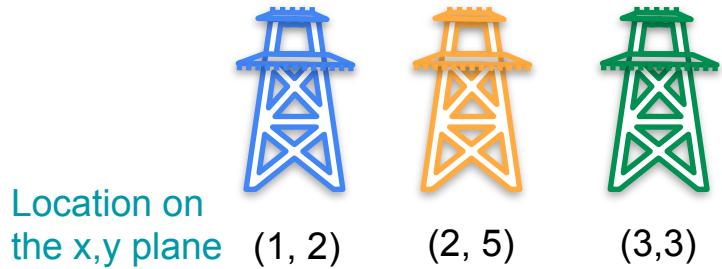
Linear Regression: Analytical Approach



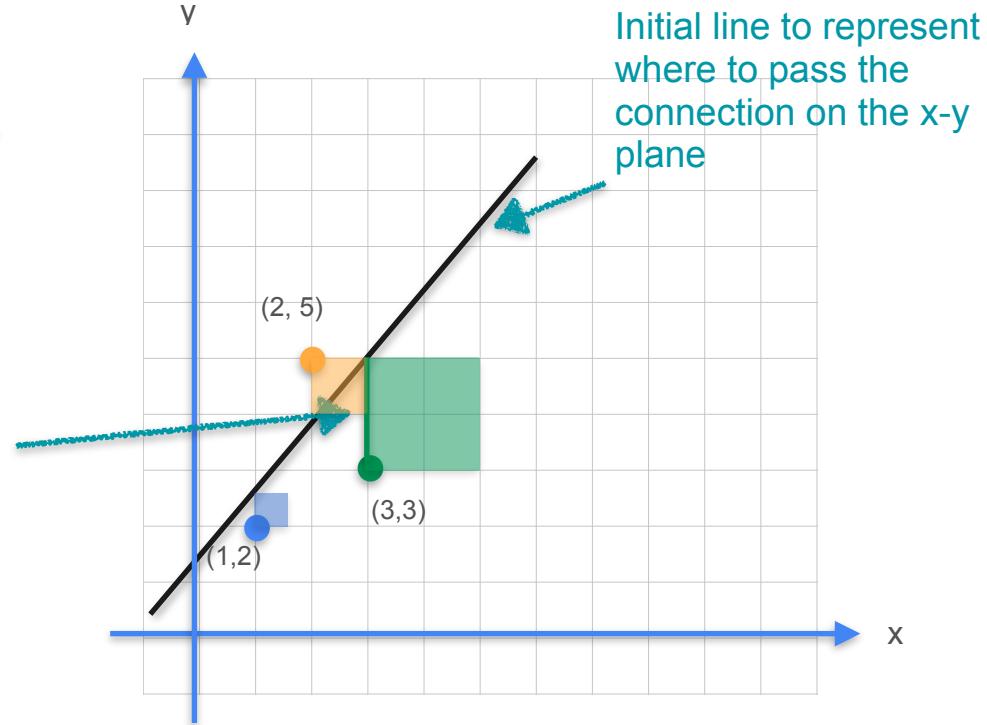
The cost of connecting connection to the powerline



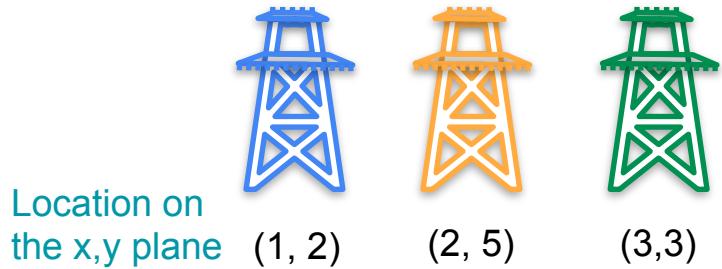
Linear Regression: Analytical Approach



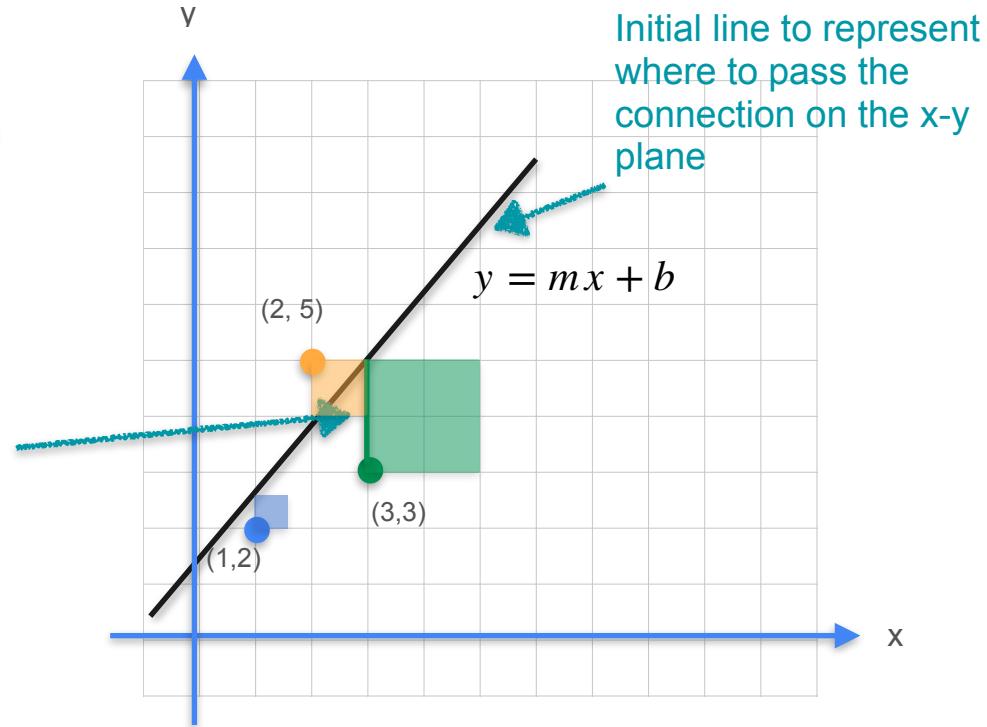
The cost of connecting connection to the powerline



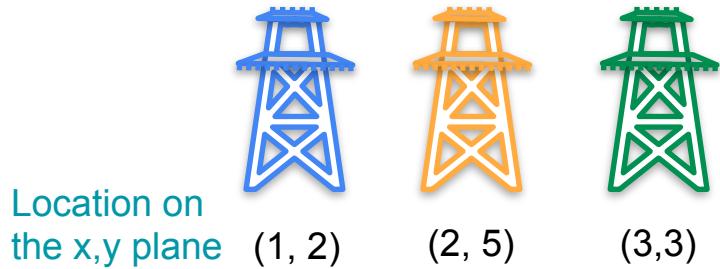
Linear Regression: Analytical Approach



The cost of connecting connection to the powerline

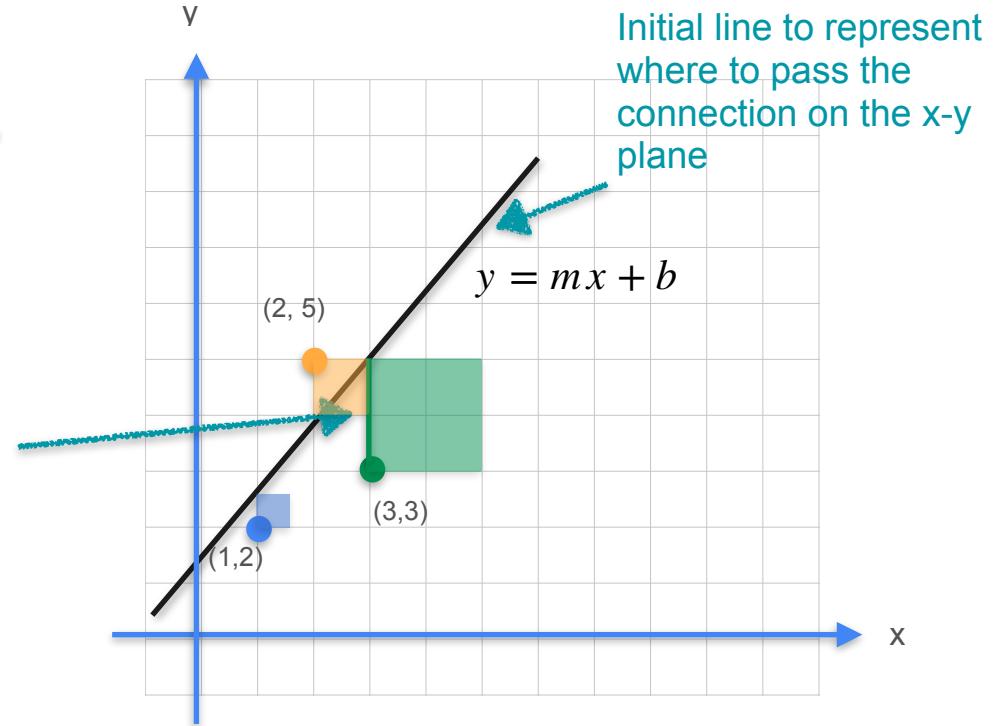


Linear Regression: Analytical Approach

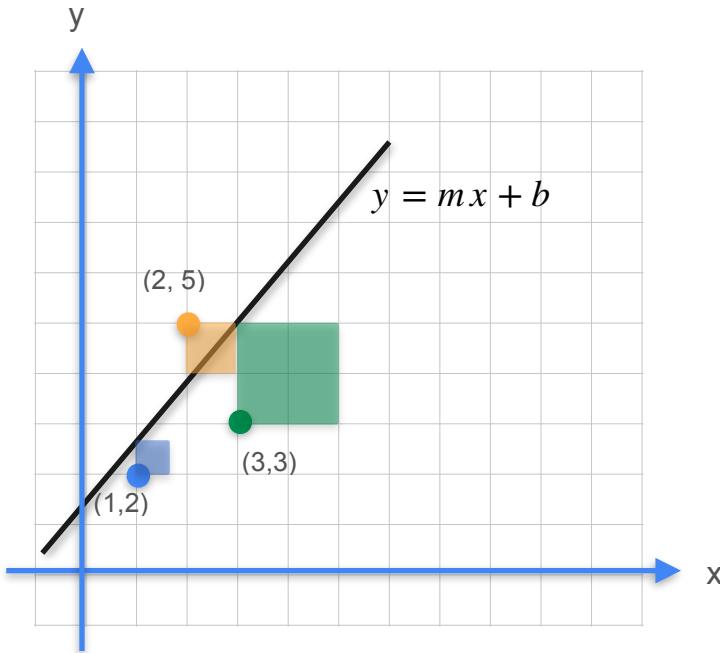


The cost of connecting connection to the powerline

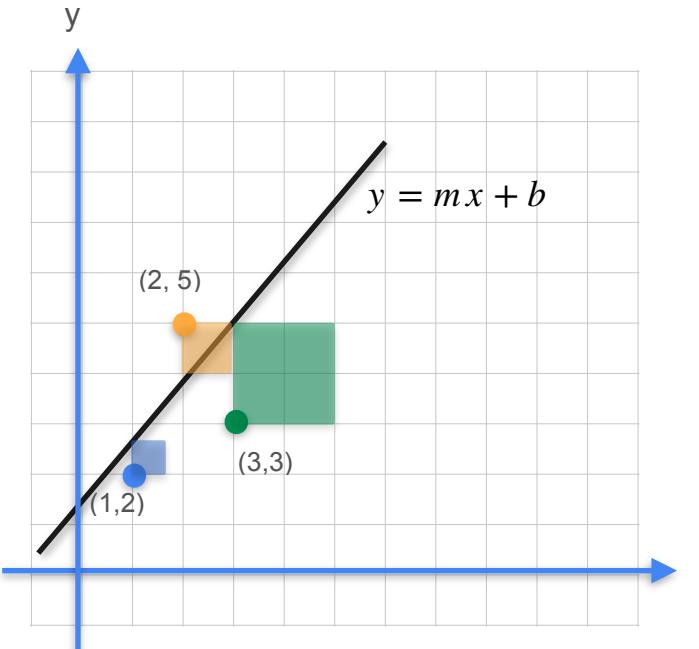
Goal: Find m, b such that you minimize sum of squares cost



Linear Regression: Analytical Approach

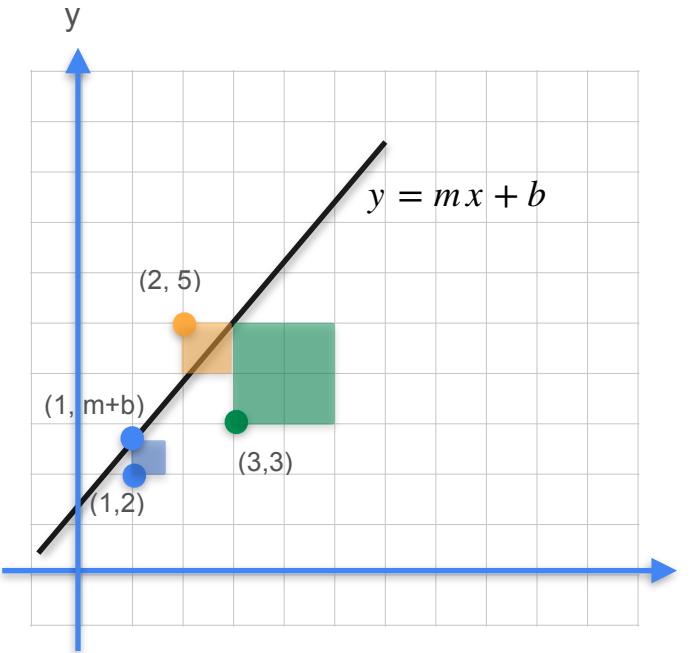


Linear Regression: Analytical Approach



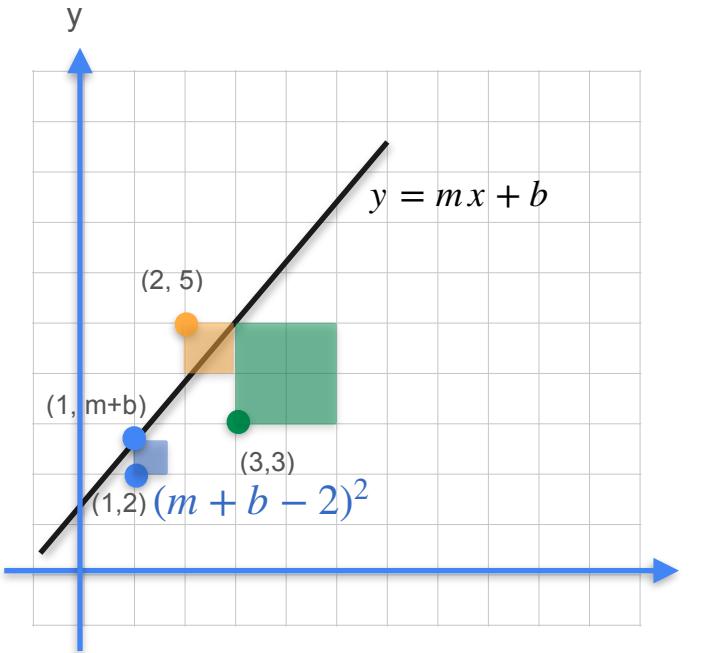
Goal: Minimize sum of squares cost

Linear Regression: Analytical Approach



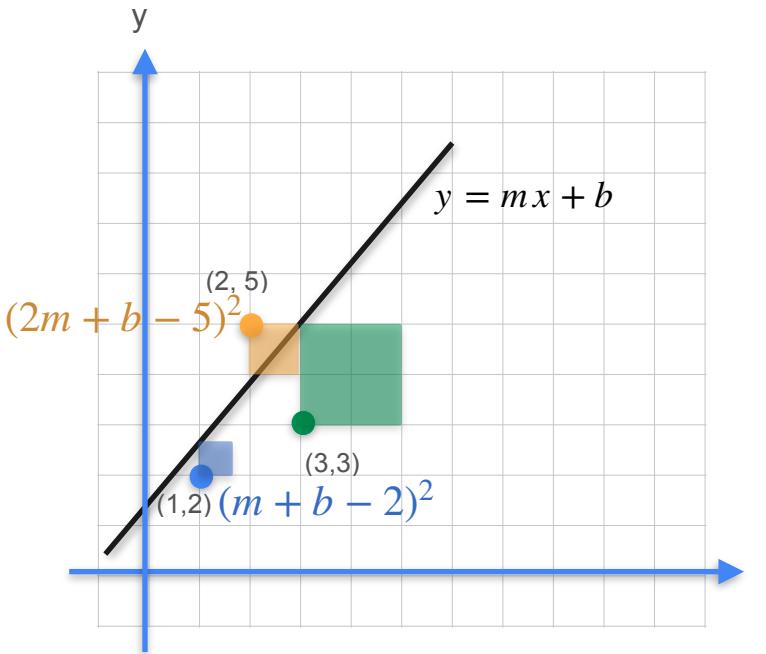
Goal: Minimize sum of squares cost

Linear Regression: Analytical Approach



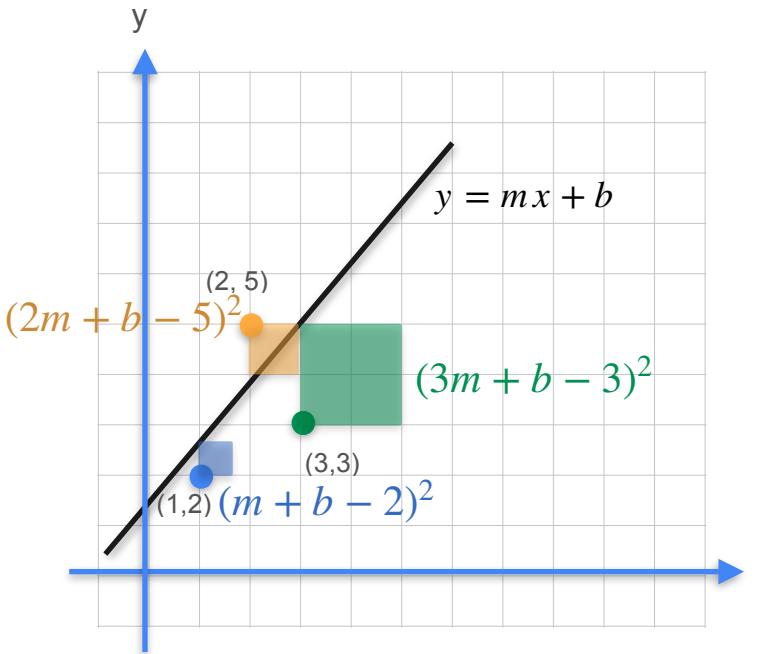
Goal: Minimize sum of squares cost

Linear Regression: Analytical Approach



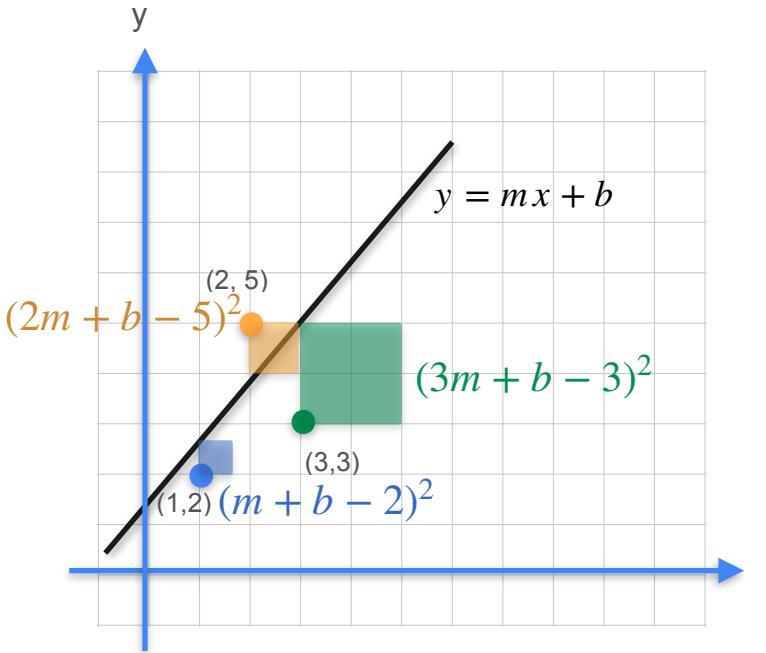
Goal: Minimize sum of squares cost

Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

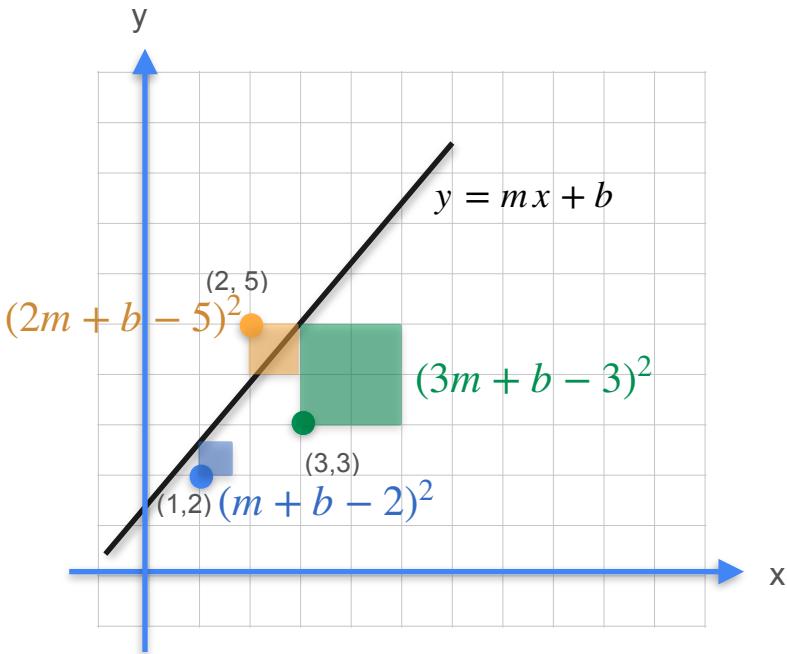
Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$

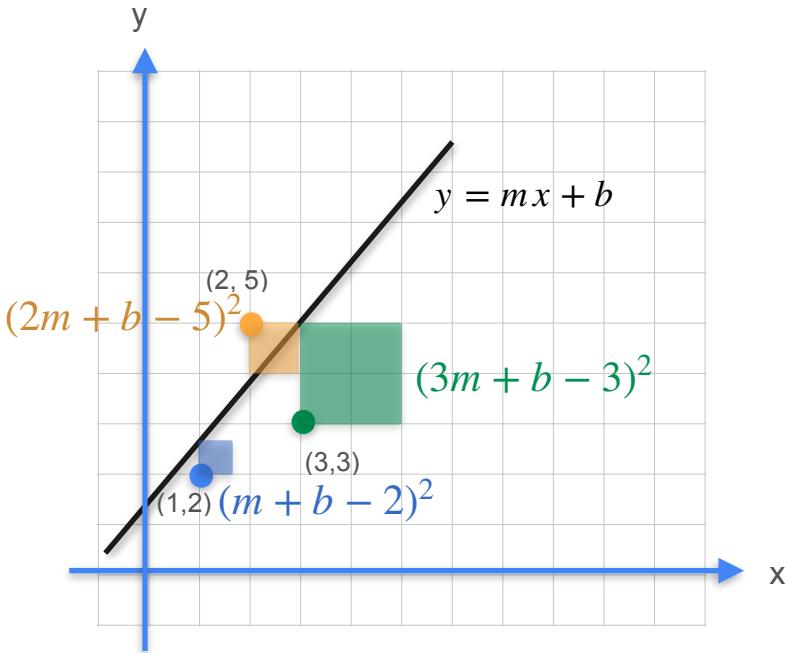
Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$
$$m^2 + b^2 + 4 + 2mb - 4m - 4b$$

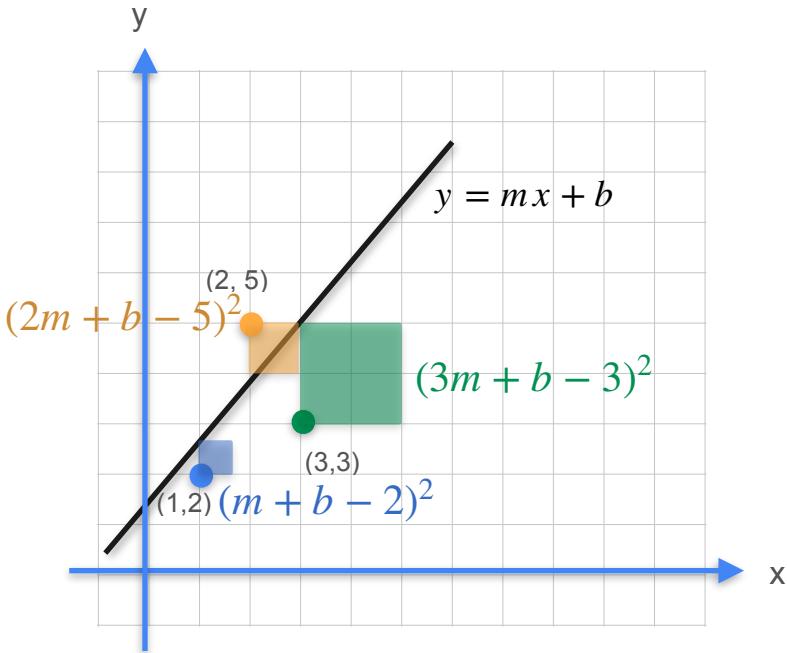
Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$\begin{aligned}(m + b - 2)^2 &+ (2m + b - 5)^2 + (3m + b - 3)^2 \\ m^2 &+ b^2 + 4 + 2mb - 4m - 4b \\ +4m^2 &+ b^2 + 25 + 4mb - 20m - 10b\end{aligned}$$

Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

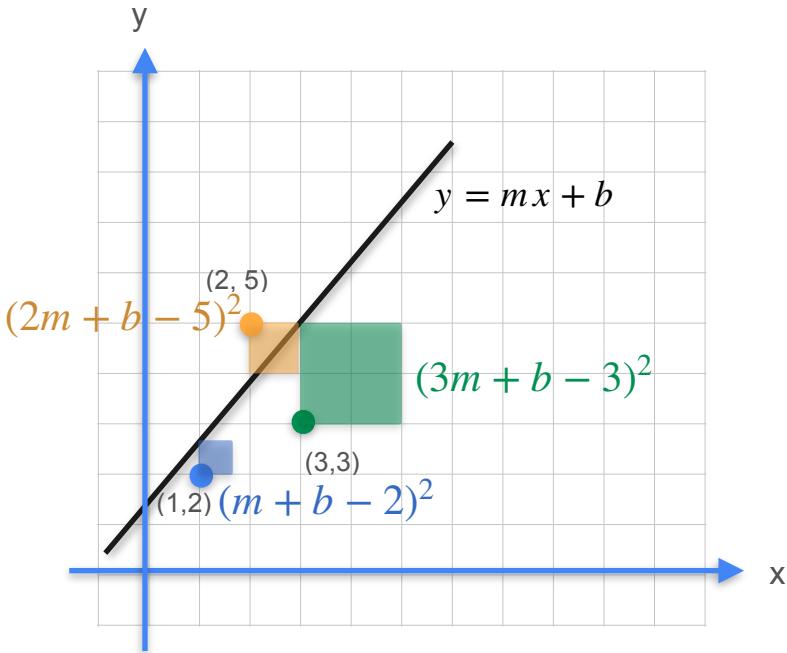
$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$

$$m^2 + b^2 + 4 + 2mb - 4m - 4b$$

$$+4m^2 + b^2 + 25 + 4mb - 20m - 10b$$

$$+9m^2 + b^2 + 9 + 6mb - 18m - 6b$$

Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

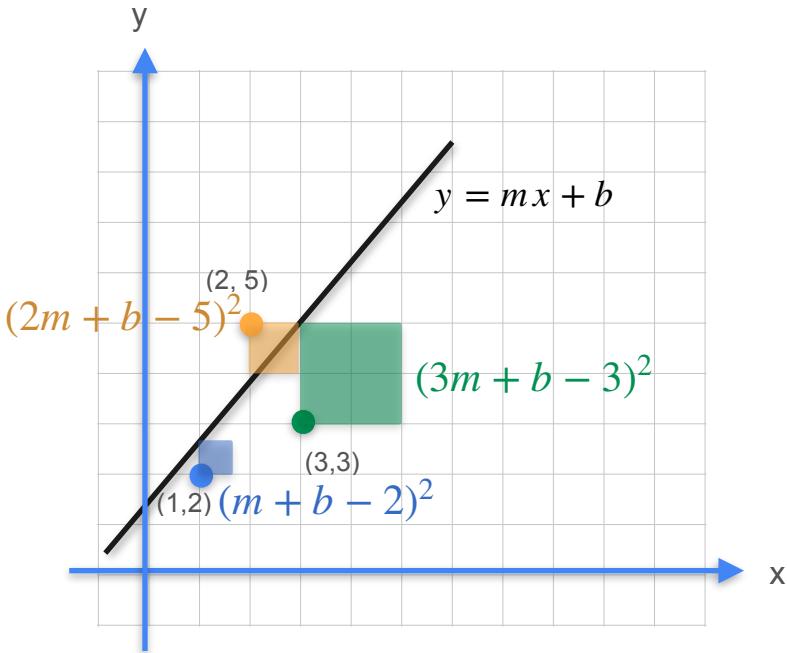
$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$

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$$+4m^2 + b^2 + 25 + 4mb - 20m - 10b$$

$$+9m^2 + b^2 + 9 + 6mb - 18m - 6b$$

Linear Regression: Analytical Approach

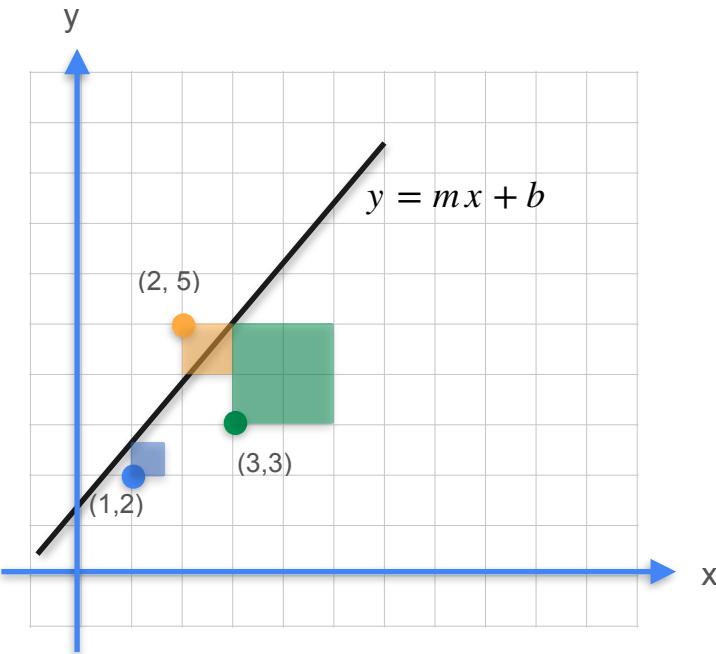


Goal: Minimize sum of squares cost

$$\begin{aligned}(m + b - 2)^2 &+ (2m + b - 5)^2 + (3m + b - 3)^2 \\ m^2 &+ b^2 + 4 + 2mb - 4m - 4b \\ &+ 4m^2 + b^2 + 25 + 4mb - 20m - 10b \\ &+ 9m^2 + b^2 + 9 + 6mb - 18m - 6b\end{aligned}$$

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

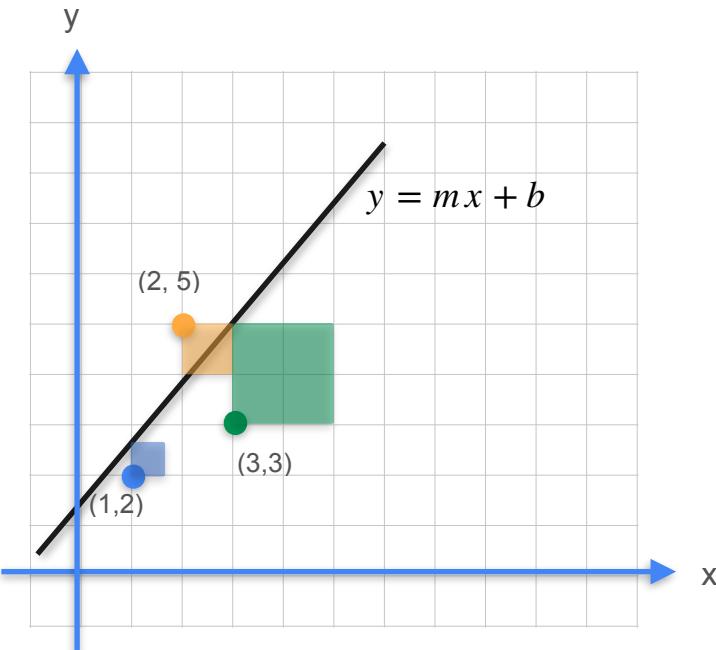
Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

Linear Regression: Analytical Approach

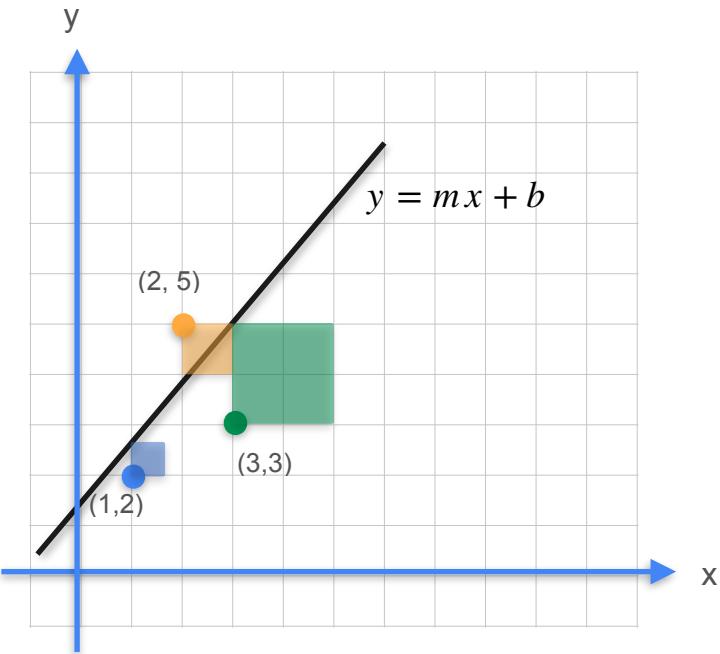


Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 0$$

Linear Regression: Analytical Approach



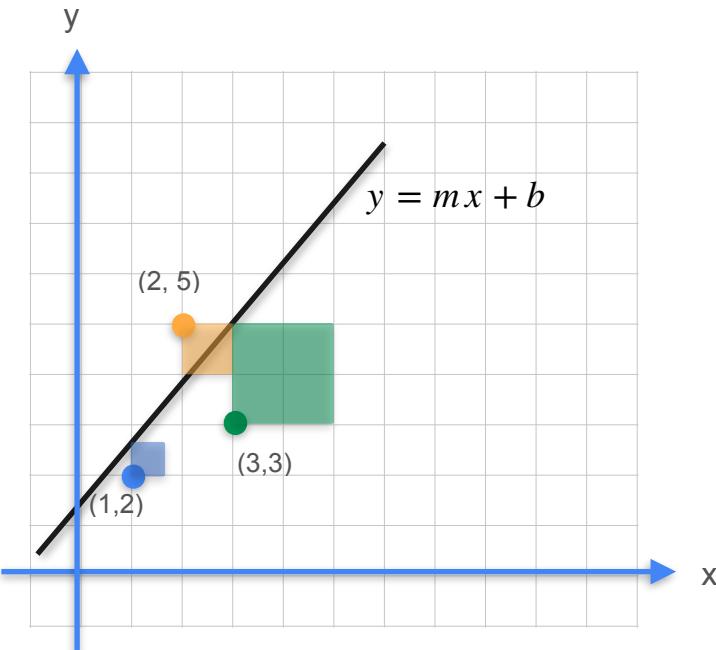
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$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 0$$

$$\frac{\partial E}{\partial b} = 0$$

Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

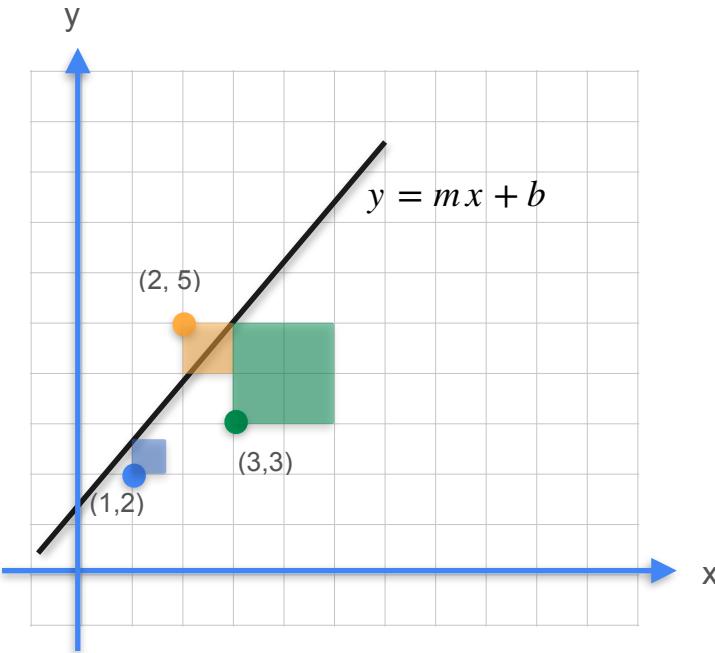
$$\frac{\partial E}{\partial m} = 0$$

Quiz:

$$\frac{\partial E}{\partial b} = 0$$

Find the partial derivative of E with respect to m

Linear Regression: Analytical Approach



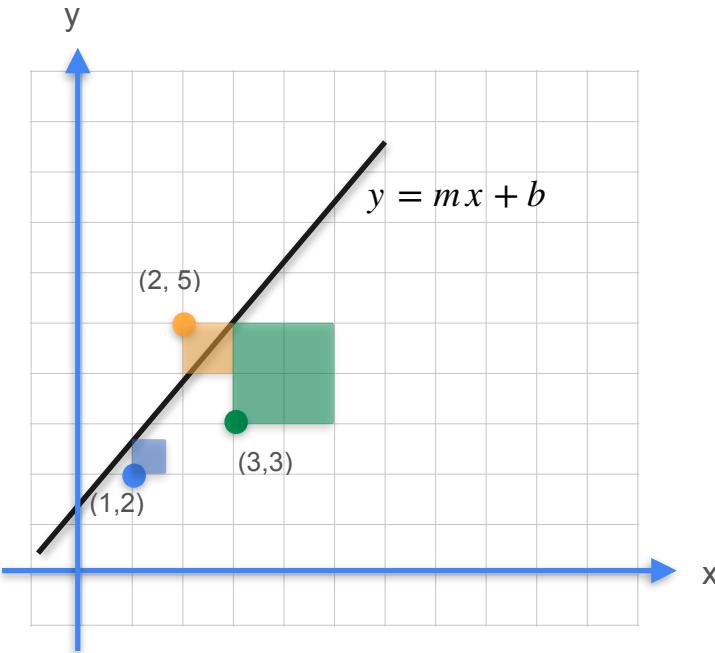
Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42$$

$$\frac{\partial E}{\partial b} =$$

Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

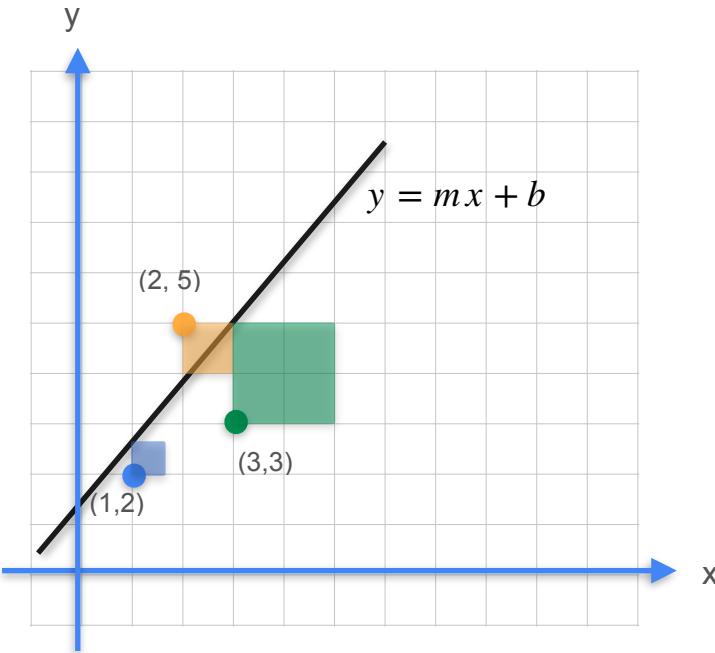
$$\frac{\partial E}{\partial m} = 28m + 12b - 42$$

$$\frac{\partial E}{\partial b} =$$

Quiz:

Find the partial derivative of E with respect to b

Linear Regression: Analytical Approach



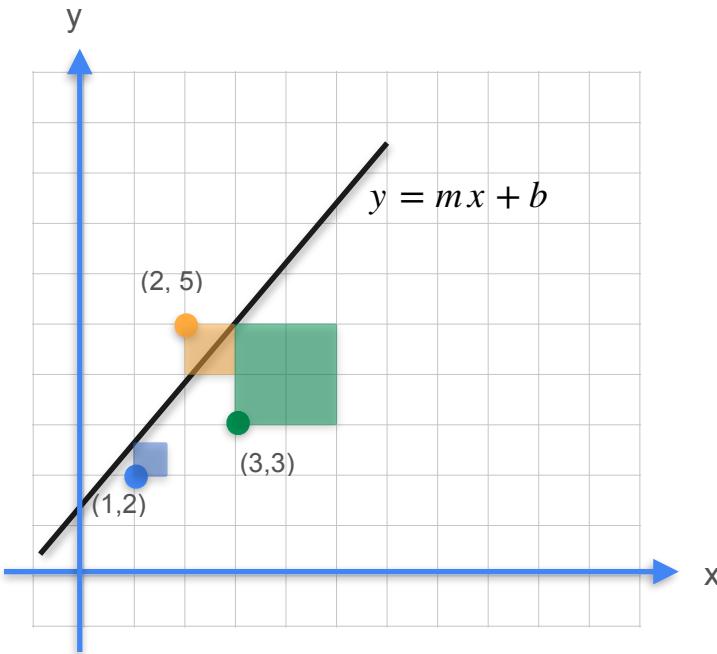
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$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20$$

Linear Regression: Analytical Approach



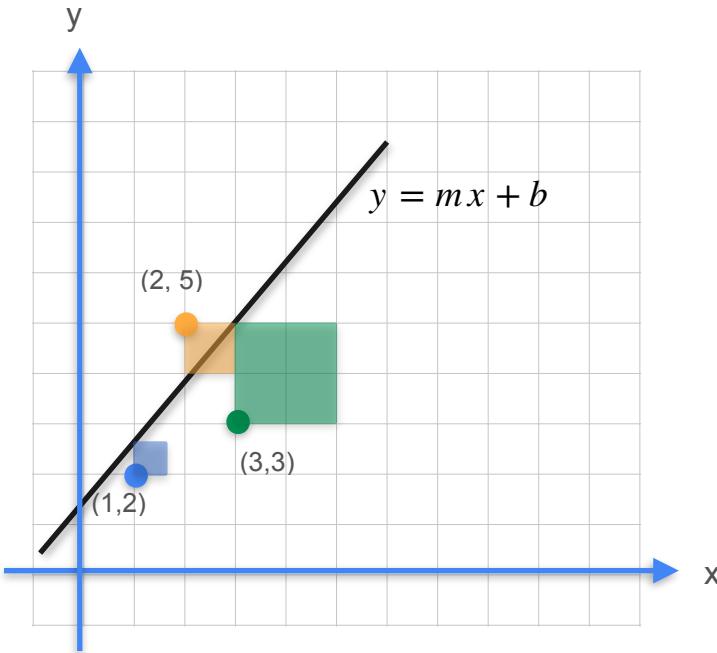
Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20$$

Linear Regression: Analytical Approach



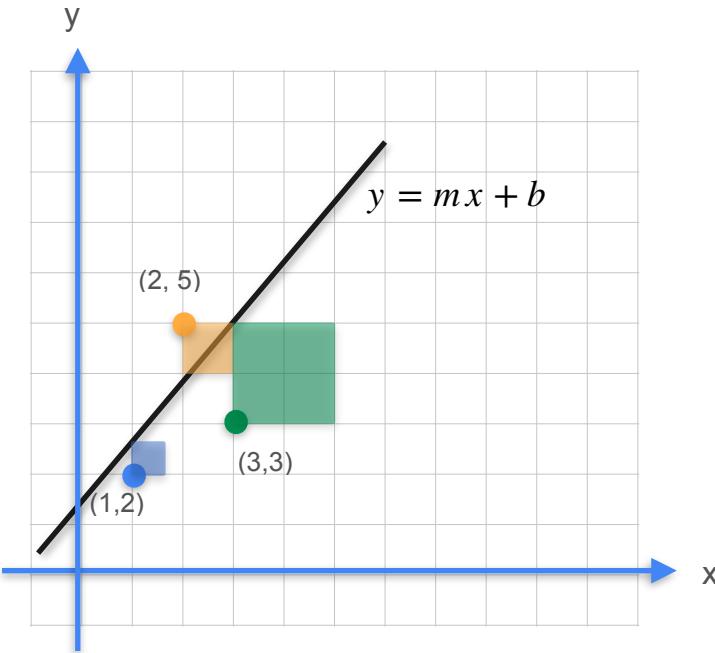
Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20$$

Linear Regression: Analytical Approach



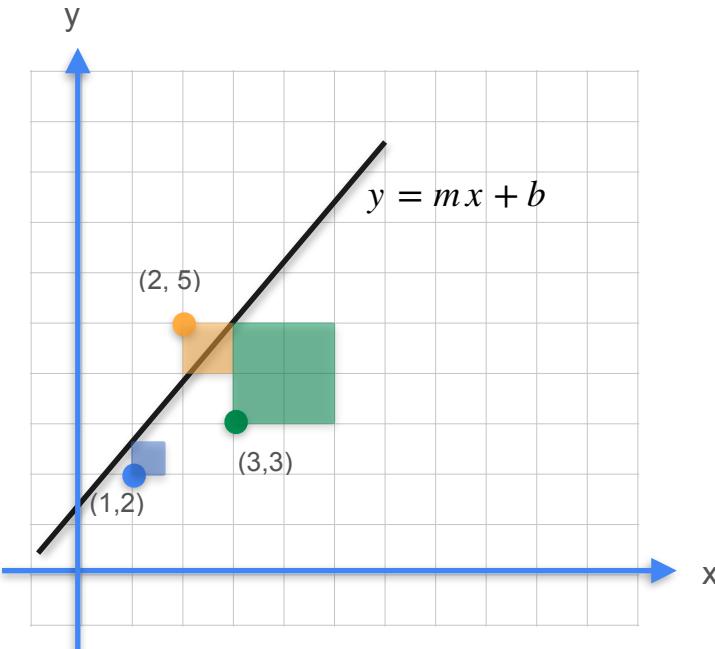
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$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

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Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

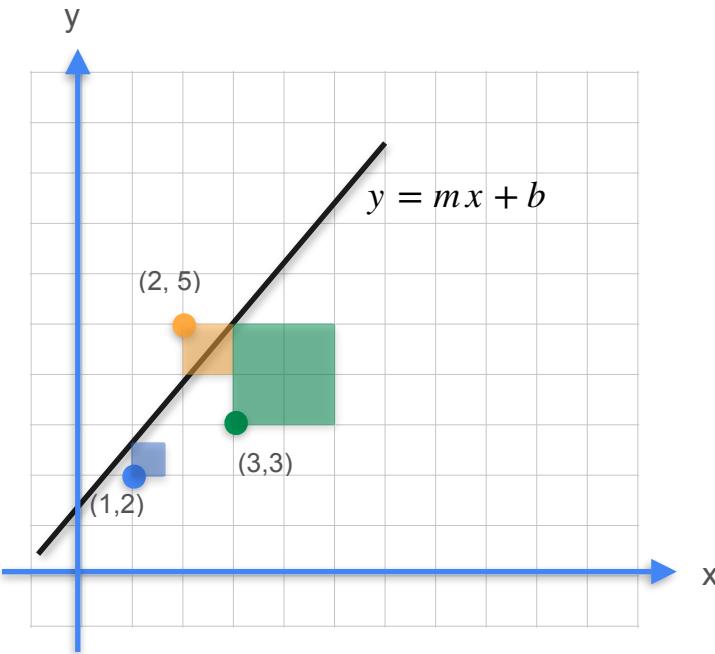
$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$m =$$

Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

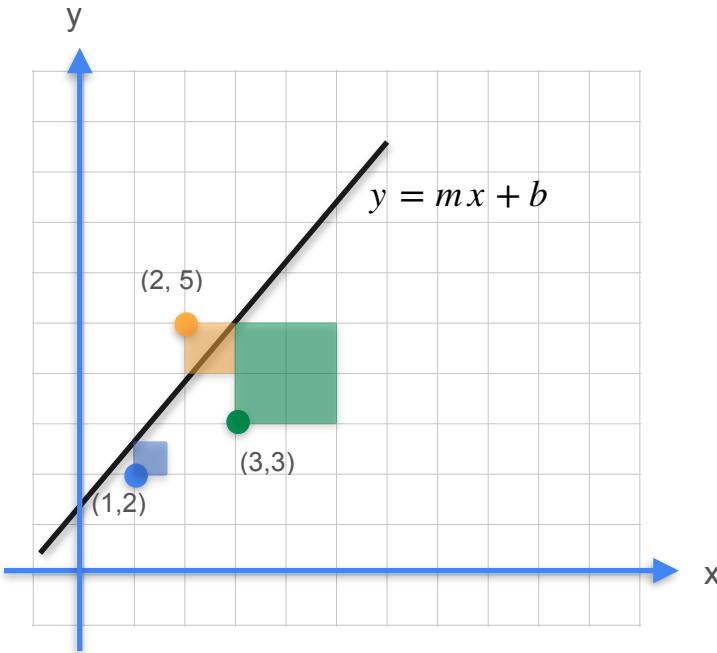
$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$m =$$

$$b =$$

Linear Regression: Analytical Approach



Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

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$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$



Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

$$12b + 24m - 40 = 0$$



Linear Regression: Analytical Approach

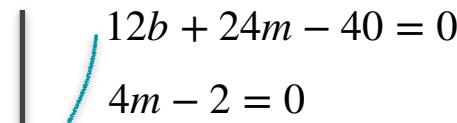
Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$


$$\begin{aligned} 12b + 24m - 40 &= 0 \\ 4m - 2 &= 0 \end{aligned}$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$


$$\begin{aligned} 12b + 24m - 40 &= 0 \\ 4m - 2 &= 0 \\ m &= \frac{2}{4} = 0.5 \end{aligned}$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$


$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

$$6b + 12(0.5) - 20 = 0$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$


$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

$$6b + 12(0.5) - 20 = 0$$

$$6b + 6 - 20 = 0$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

$$6b + 12(0.5) - 20 = 0$$

$$6b + 6 - 20 = 0$$

$$6b - 14 = 0$$

Linear Regression: Analytical Approach

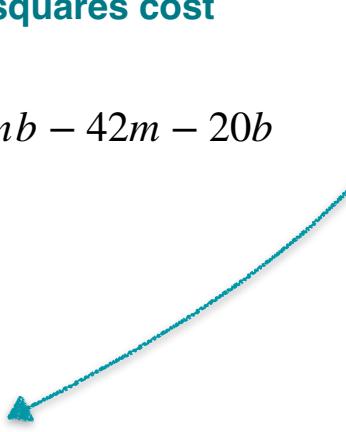
Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$



$$12b + 24m - 40 = 0$$

$$4m - 2 = 0$$

$$m = \frac{2}{4} = 0.5$$

$$6b + 12(0.5) - 20 = 0$$

$$6b + 6 - 20 = 0$$

$$6b - 14 = 0$$

$$b = \frac{14}{6} = \frac{7}{3}$$

Linear Regression: Analytical Approach

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

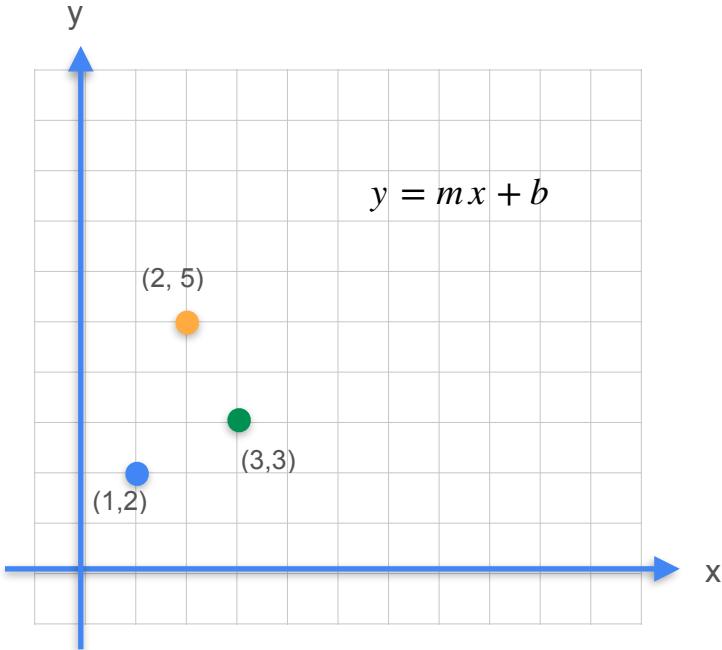
$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

$$m = \frac{1}{2}$$

$$b = \frac{7}{3}$$

$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

Linear Regression: Optimal Solution

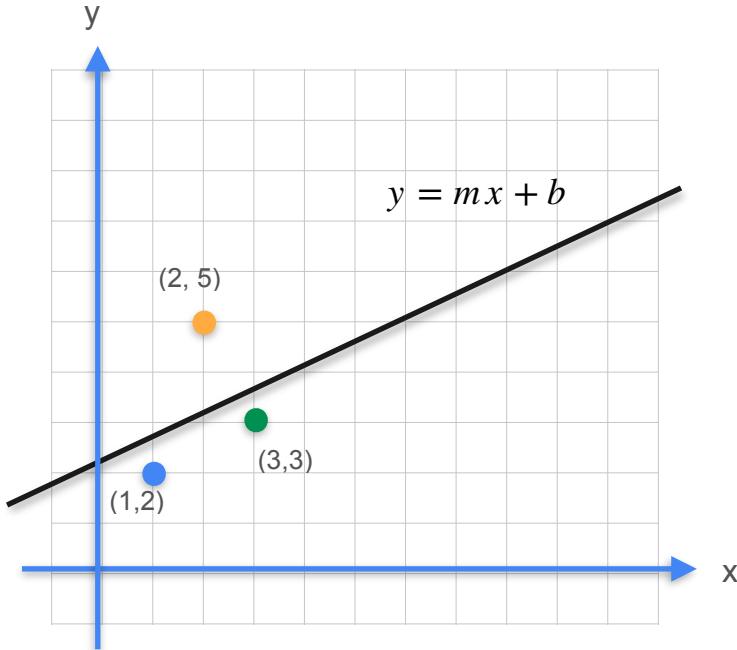


$$m = \frac{1}{2}$$

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$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

Linear Regression: Optimal Solution

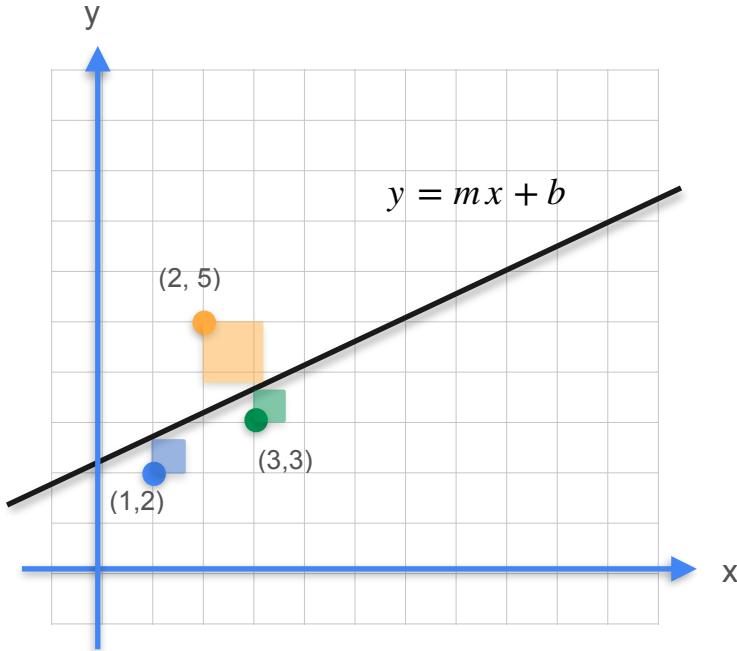


$$m = \frac{1}{2}$$

$$b = \frac{7}{3}$$

$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

Linear Regression: Optimal Solution



$$m = \frac{1}{2}$$

$$b = \frac{7}{3}$$

$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$

Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

$$\frac{\partial E}{\partial m} = 28m + 12b - 42 = 0$$

$$\frac{\partial E}{\partial b} = 6b + 12m - 20 = 0$$

Gradient Descent to the rescue

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$



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Gradients and Gradient Descent

**Optimization using Gradient
Descent in one variable -
Part 1**

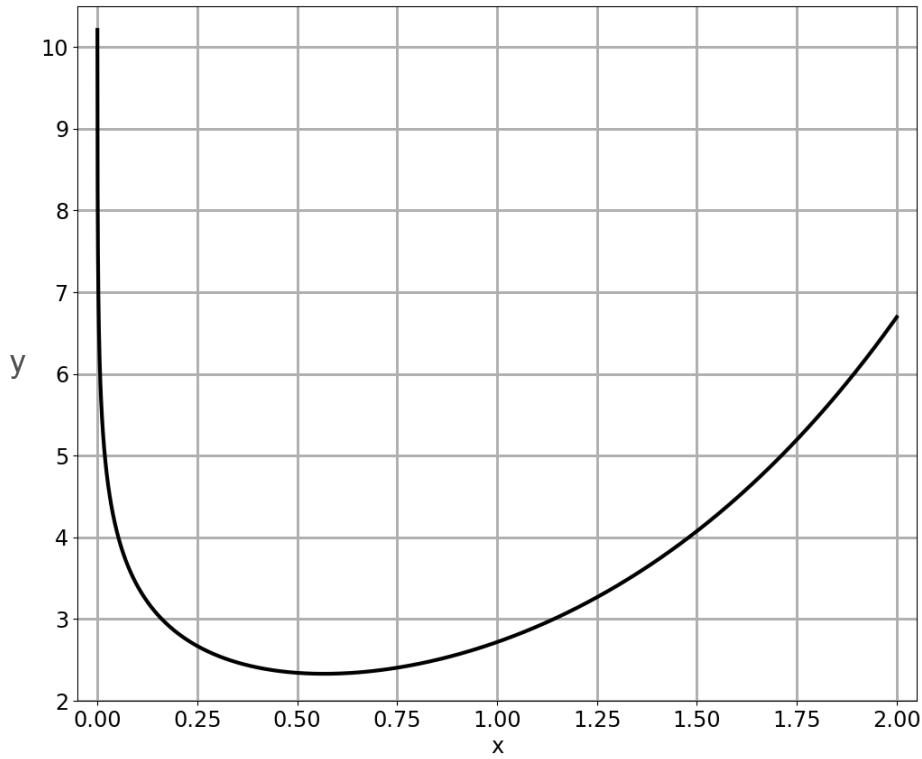
Hard To Optimize Functions

Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Hard To Optimize Functions

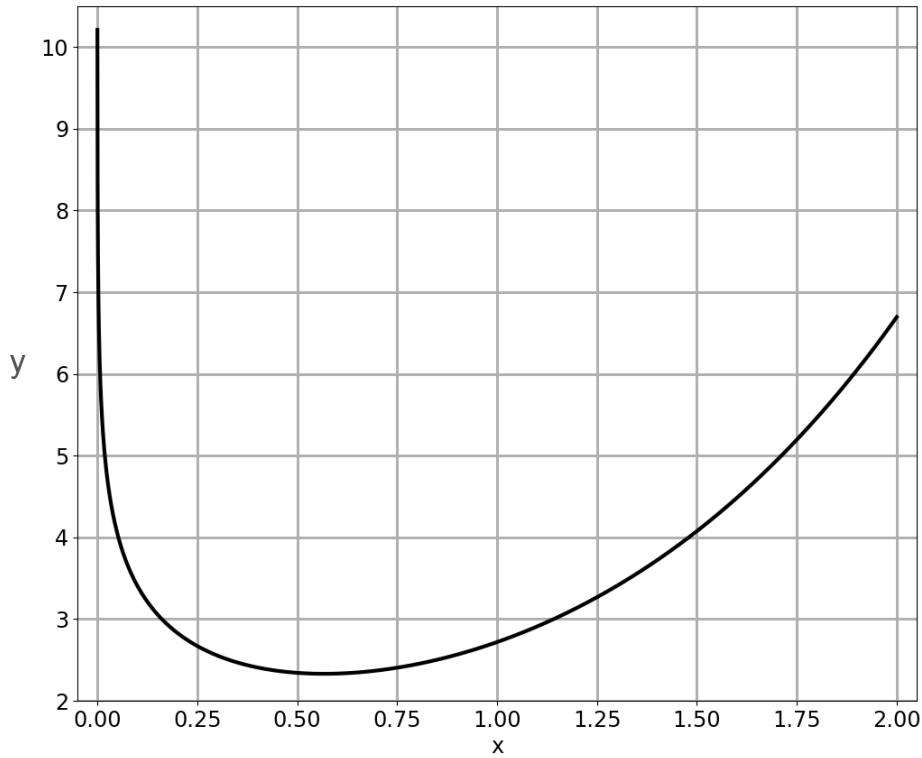
$$f(x) = e^x - \log(x)$$



Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

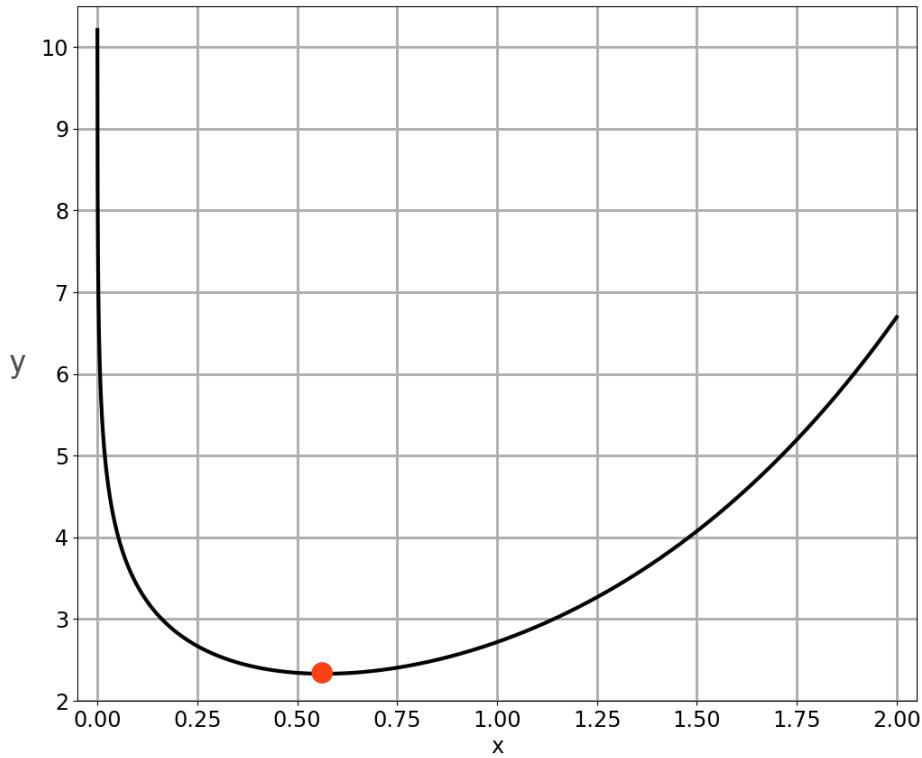
Minimum?



Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

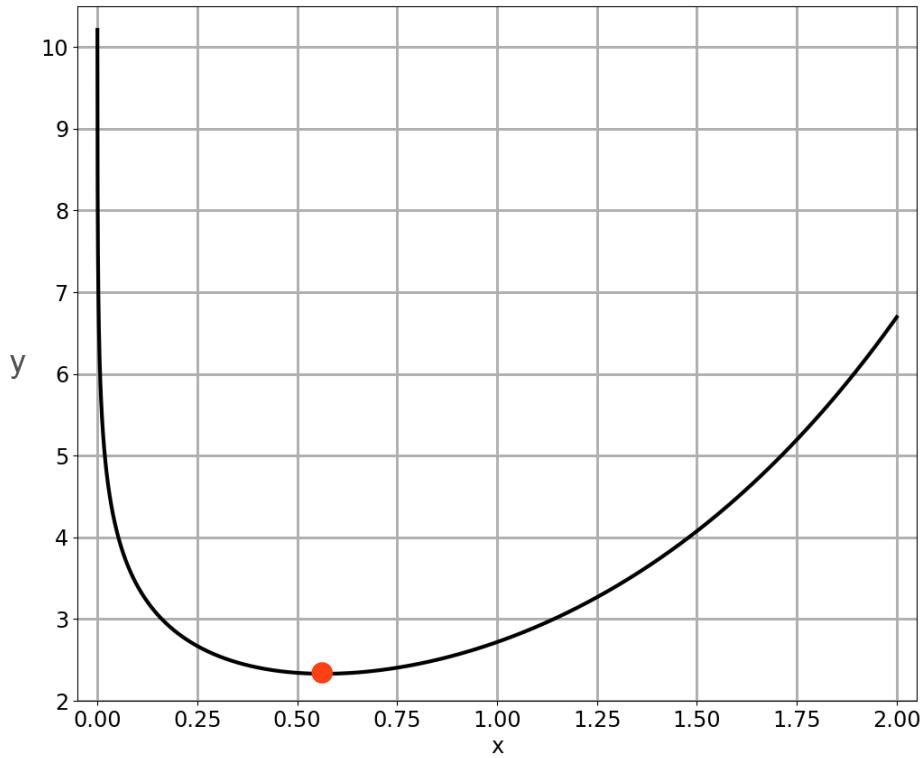


Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

$$f'(x)=0$$

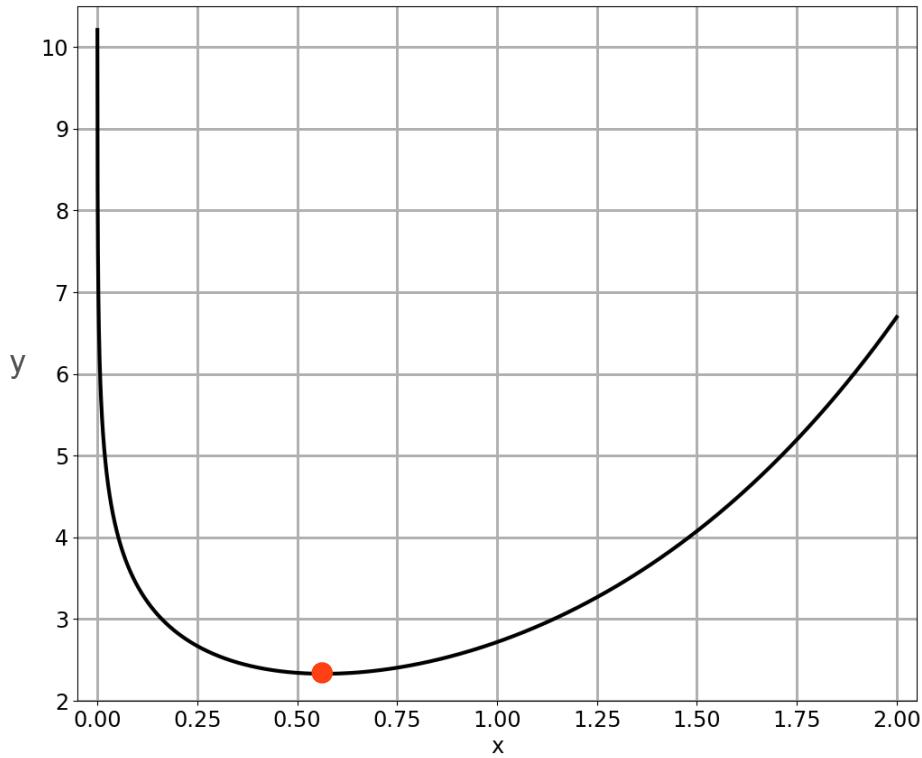


Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

$$f'(x) = e^x - \frac{1}{x} = 0$$

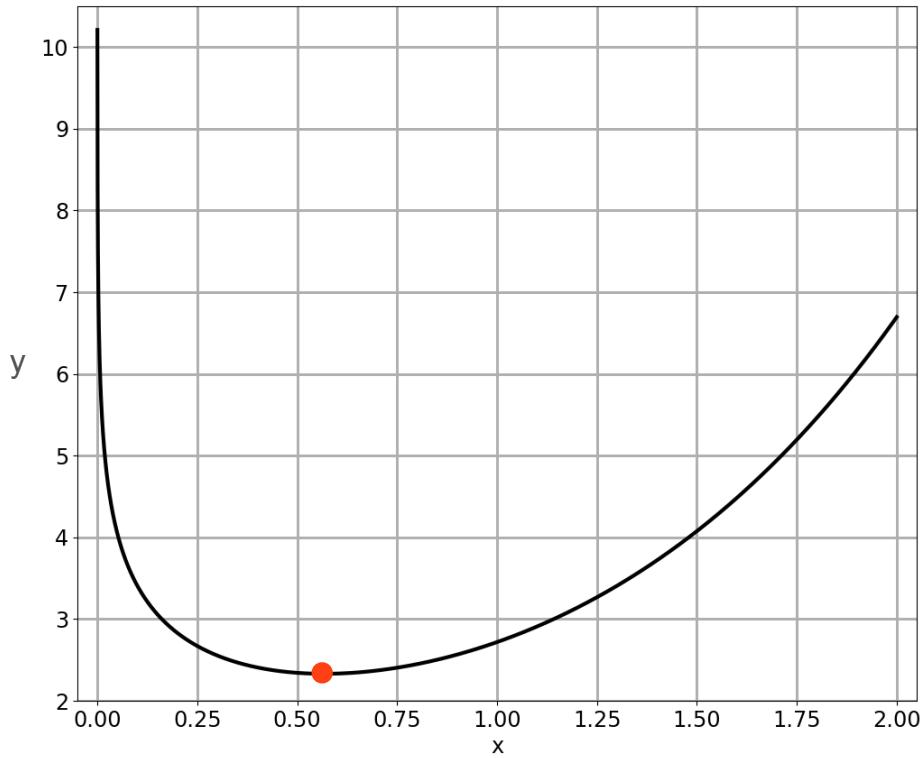


Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

$$f'(x) = e^x - \frac{1}{x} = 0$$

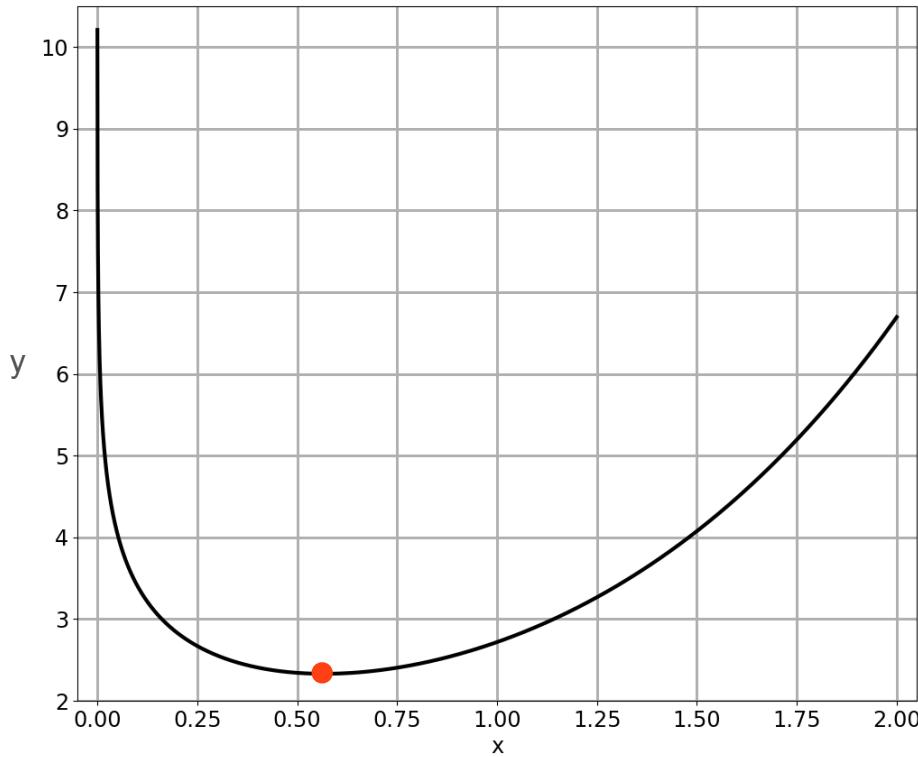


Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

$$f'(x) = e^x - \frac{1}{x} = 0$$

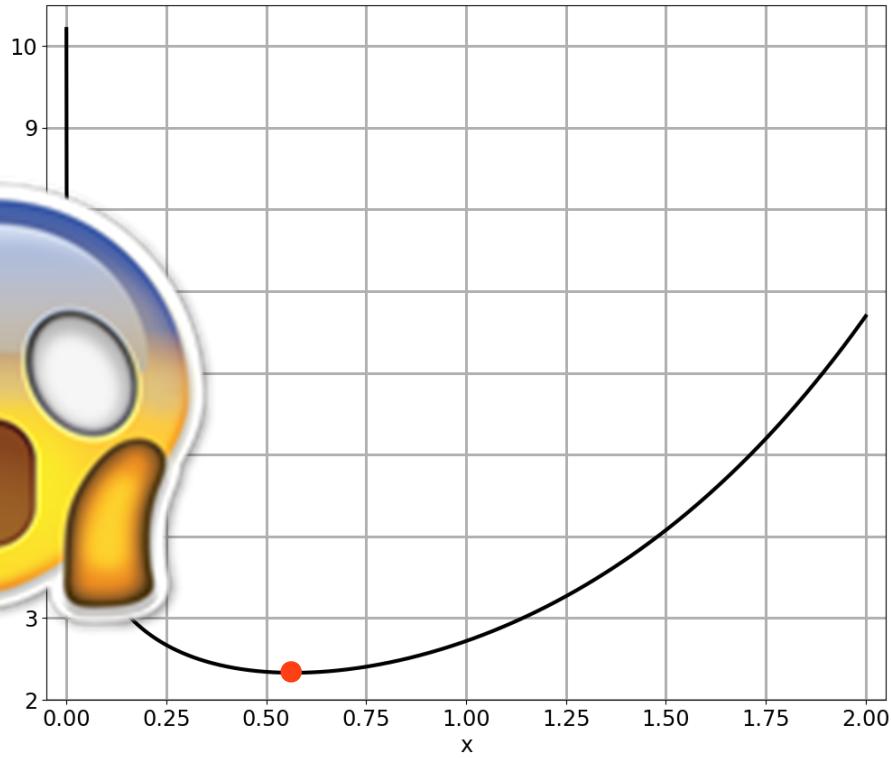


Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

Minimum?

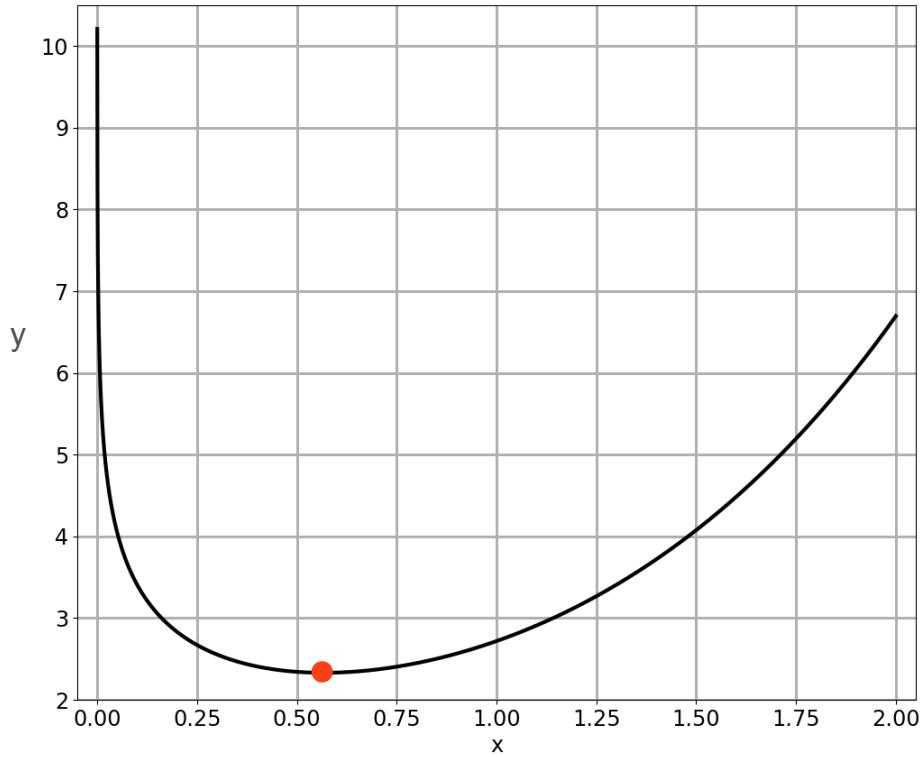
$$f'(x) = e^x - \frac{1}{x} = 0$$



Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

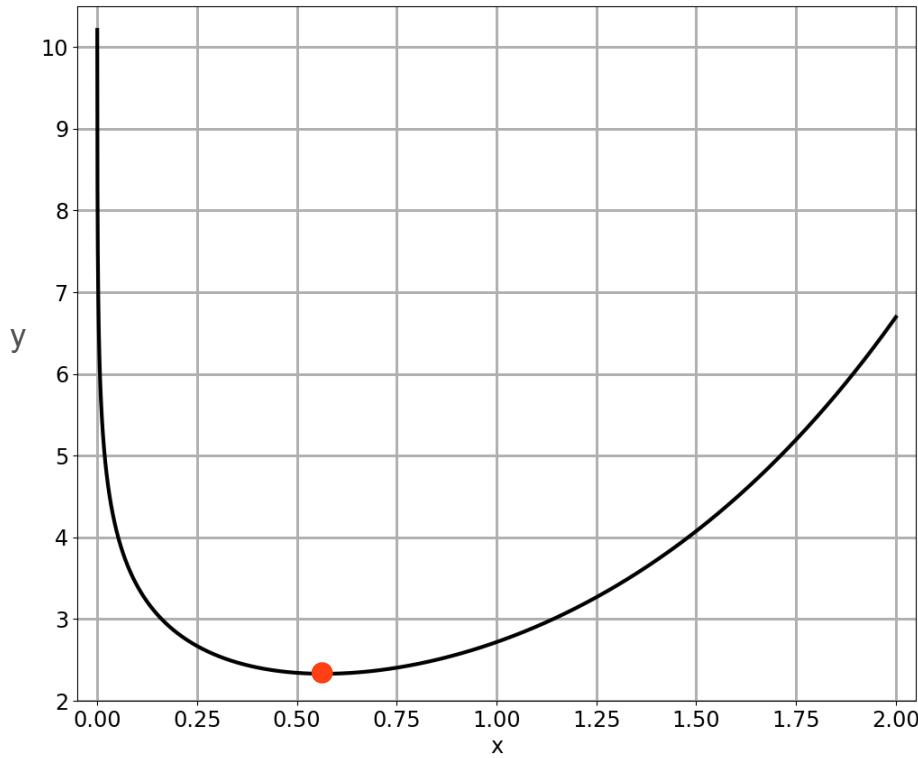
$$f'(x) = e^x - \frac{1}{x}$$



Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x} = 0$$



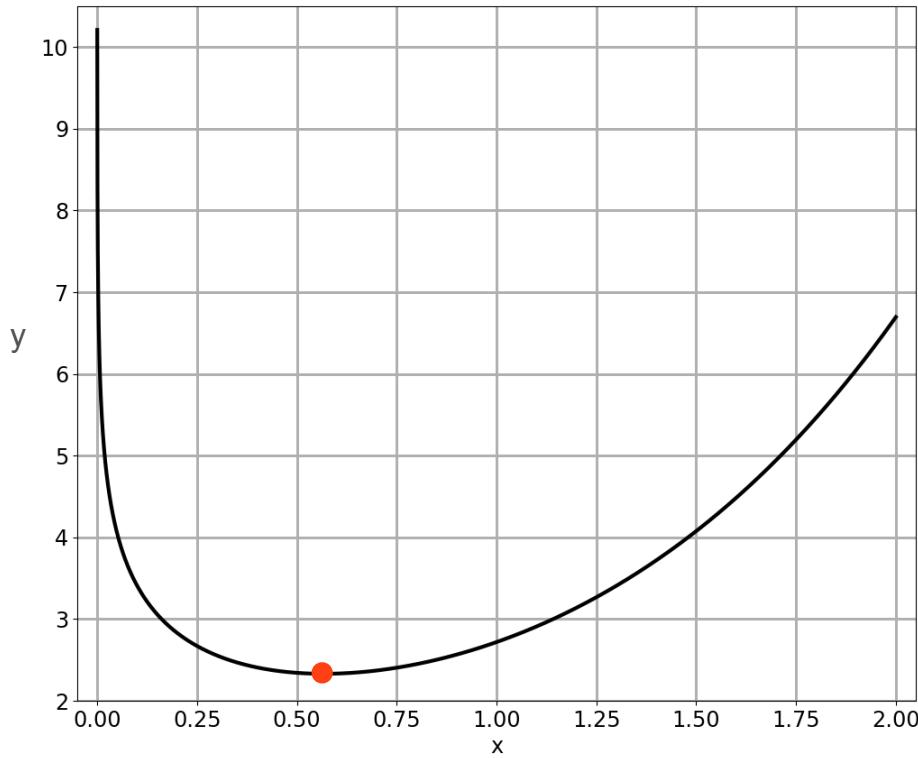
Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x} = 0$$



$$e^x = \frac{1}{x}$$



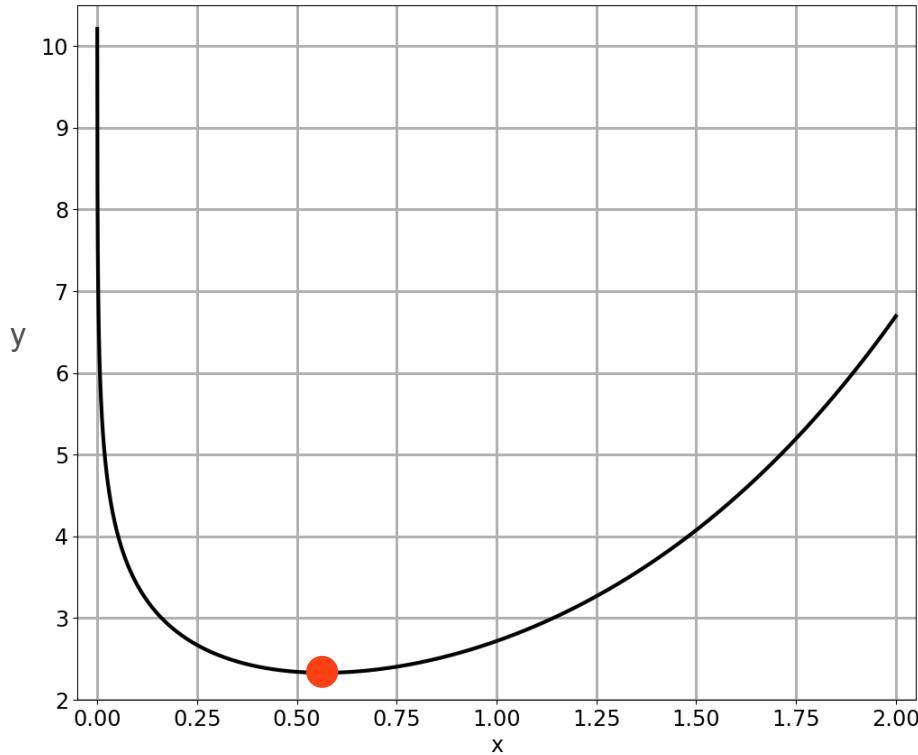
Hard To Optimize Functions

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x} = 0$$

→ $e^x = \frac{1}{x}$

Solution: $x = 0.5671\dots$



Hard To Optimize Functions

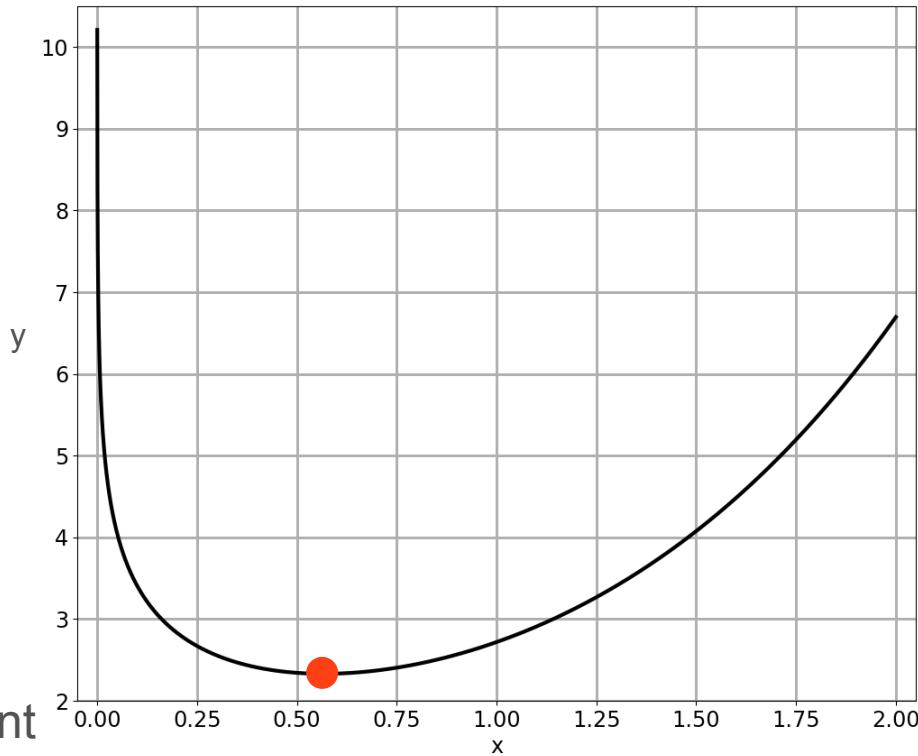
$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x} = 0$$

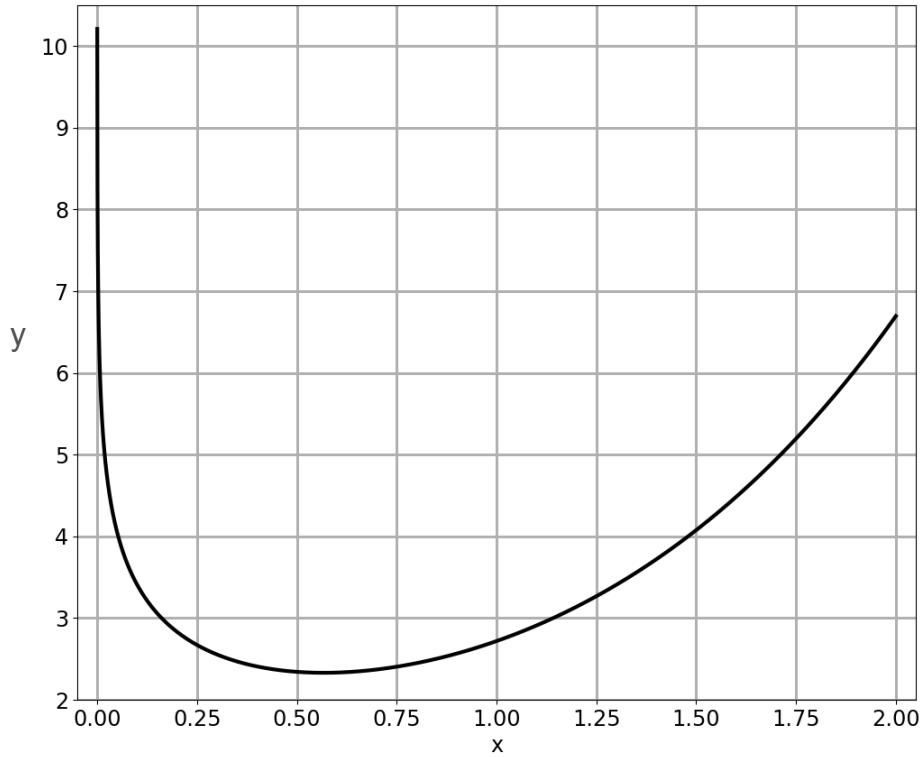
→ $e^x = \frac{1}{x}$

Solution: $x = 0.5671\dots$

Also known as the Omega constant

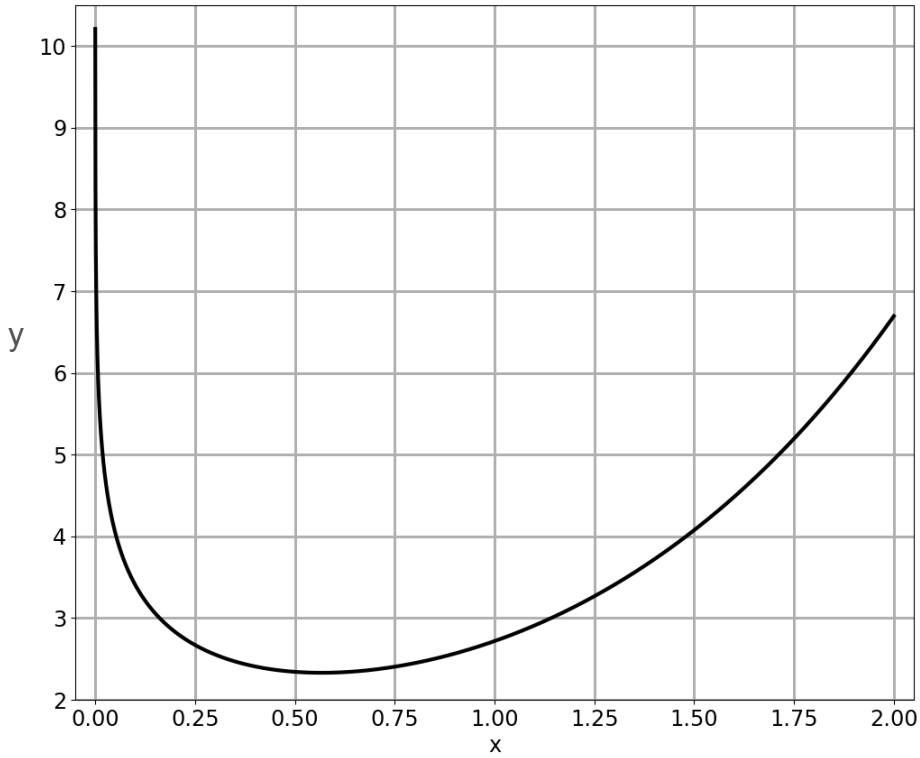


Method 1: Try Both Directions



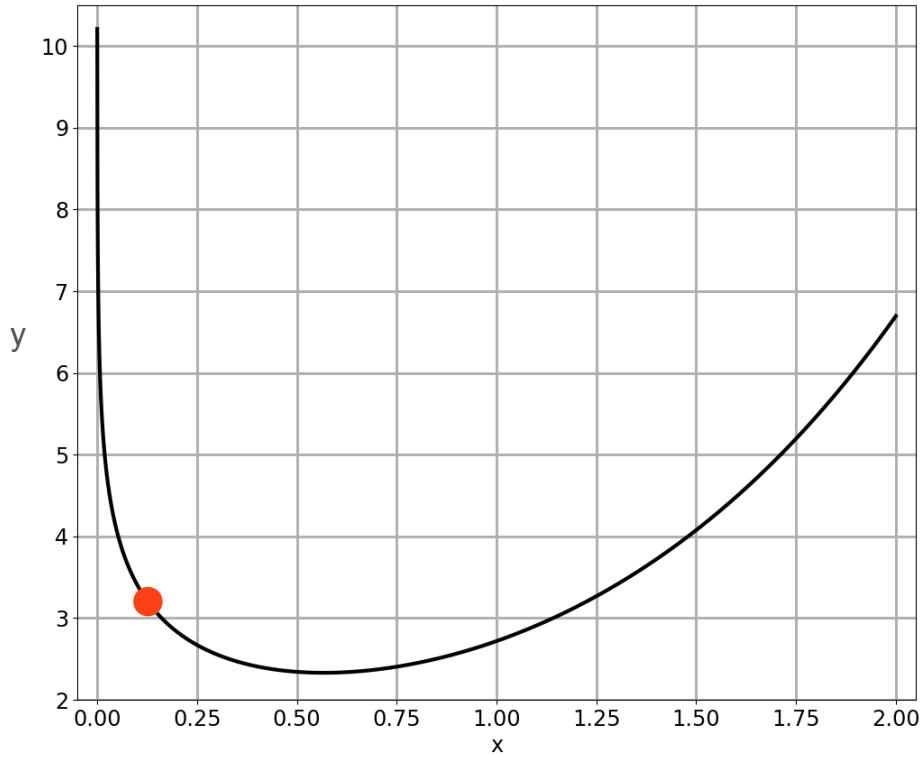
Method 1: Try Both Directions

Is there any
other way?



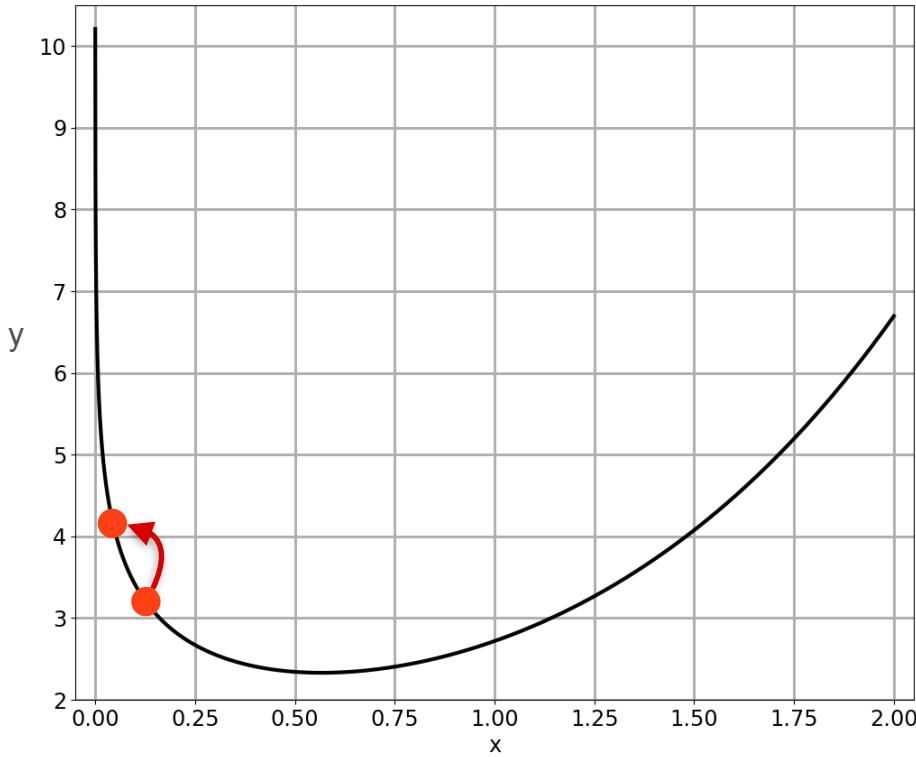
Method 1: Try Both Directions

Is there any
other way?



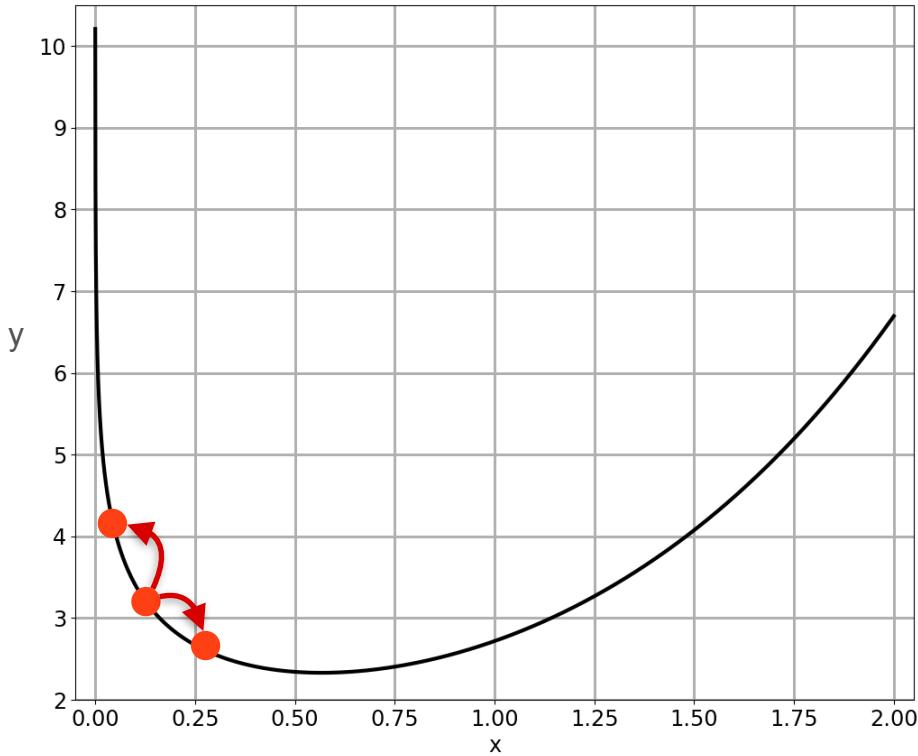
Method 1: Try Both Directions

Is there any
other way?



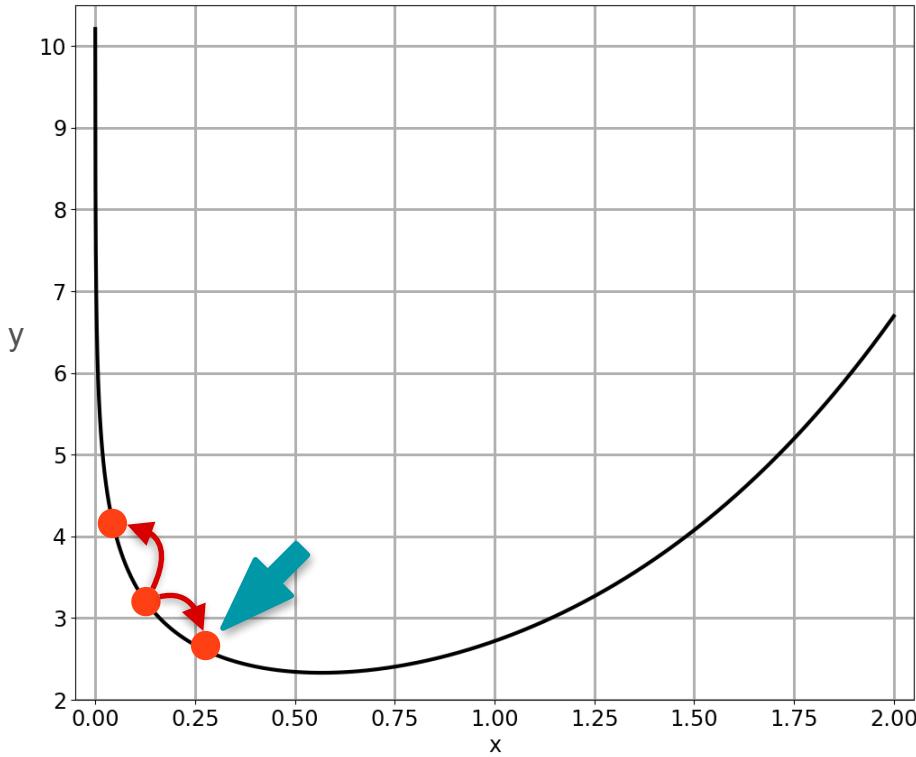
Method 1: Try Both Directions

Is there any
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Method 1: Try Both Directions

Is there any
other way?

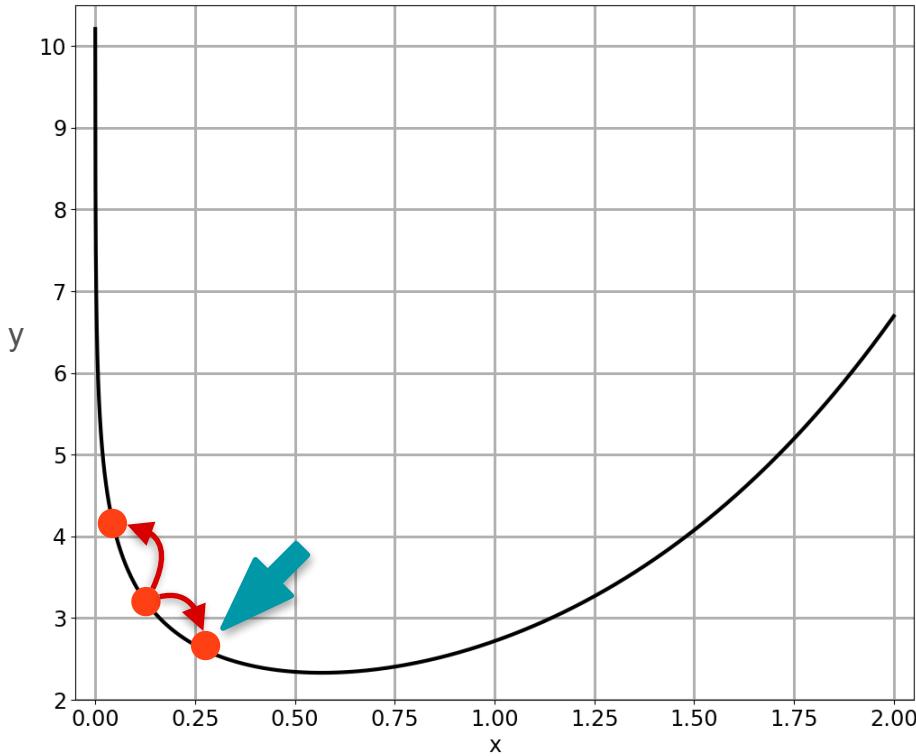


Method 1: Try Both Directions

Is there any
other way?



Repeat!

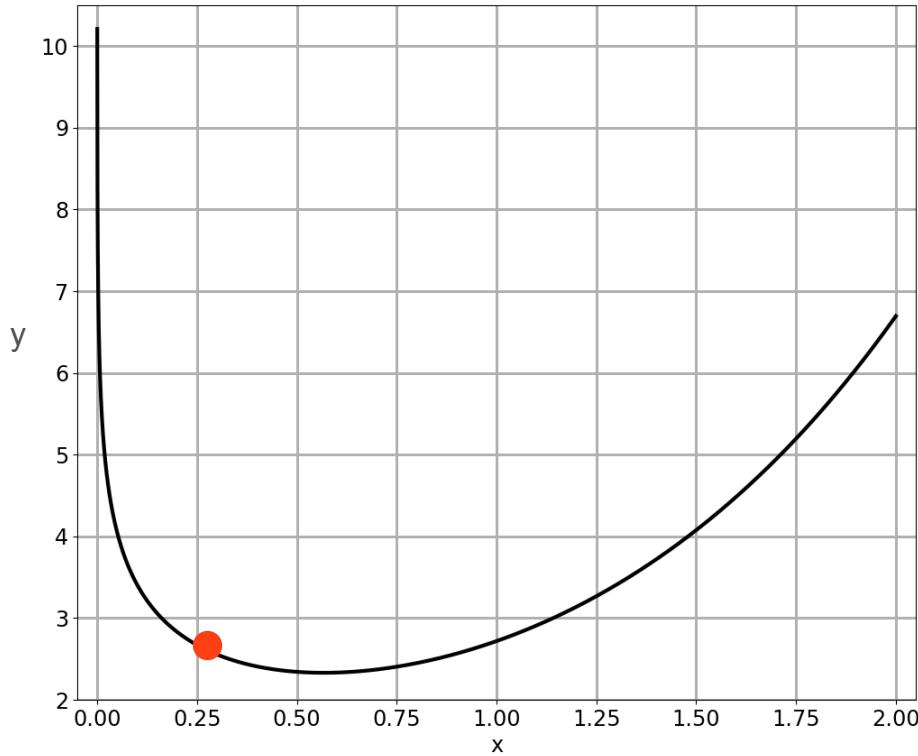


Method 1: Try Both Directions

Is there any
other way?



Repeat!

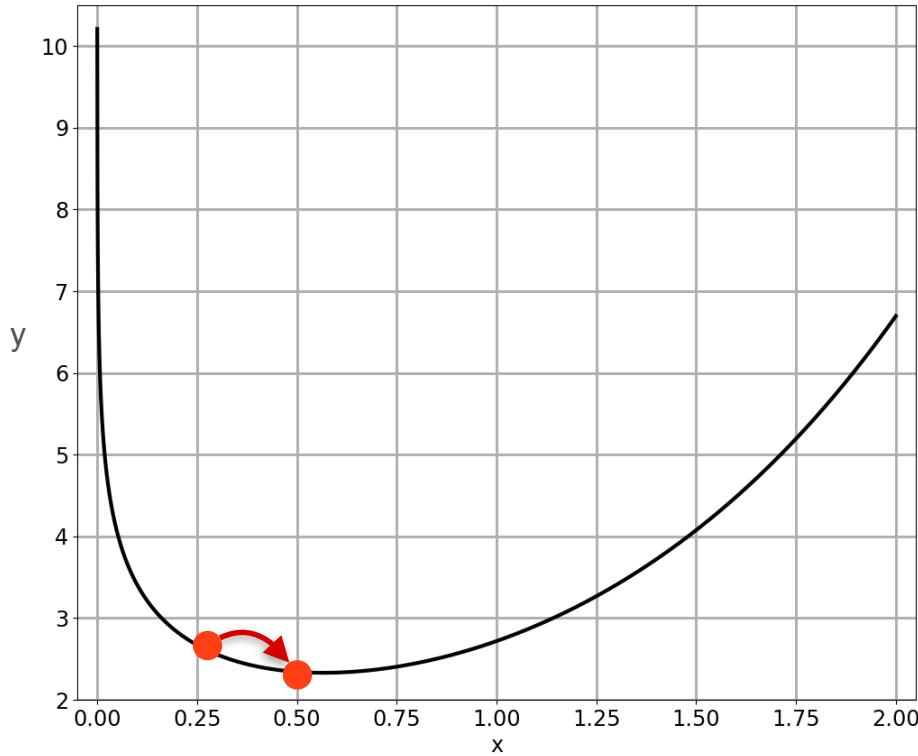


Method 1: Try Both Directions

Is there any
other way?



Repeat!

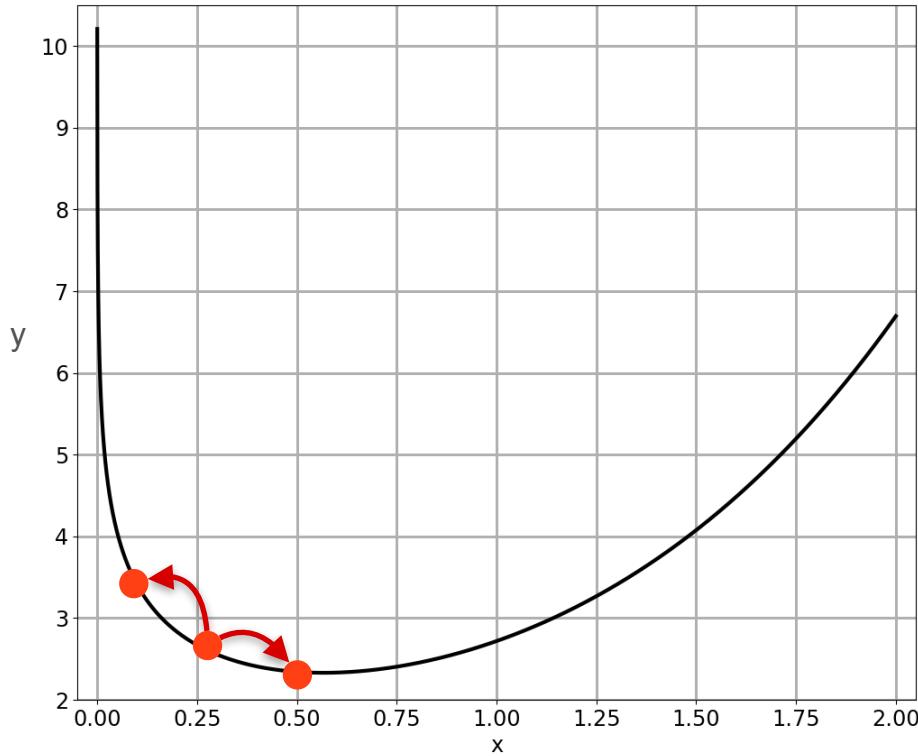


Method 1: Try Both Directions

Is there any
other way?



Repeat!

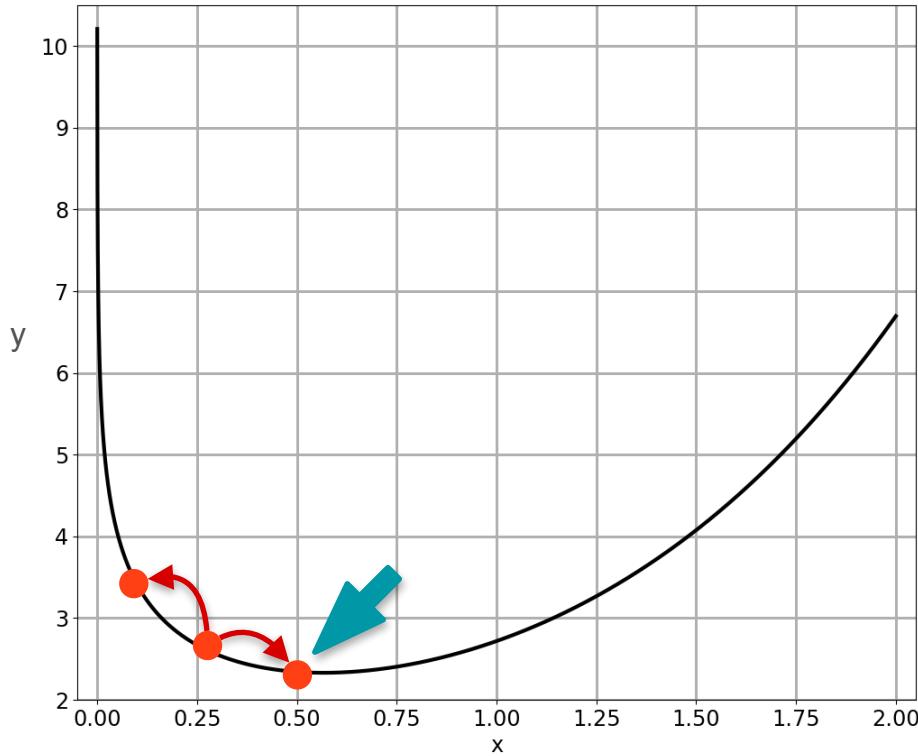


Method 1: Try Both Directions

Is there any
other way?



Repeat!

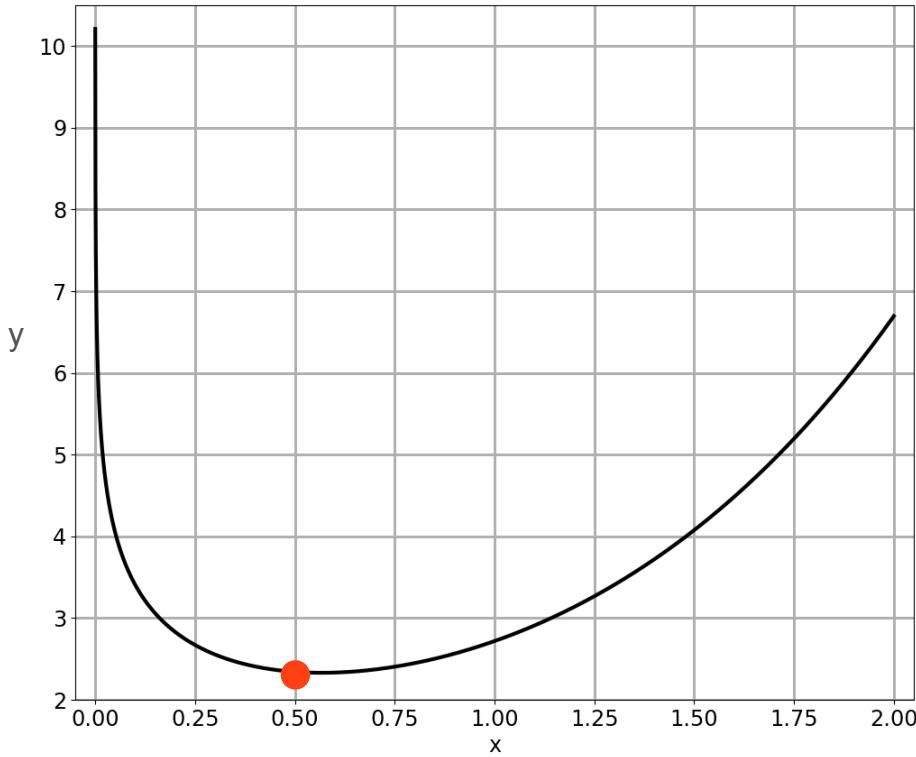


Method 1: Try Both Directions

Is there any
other way?



Repeat!

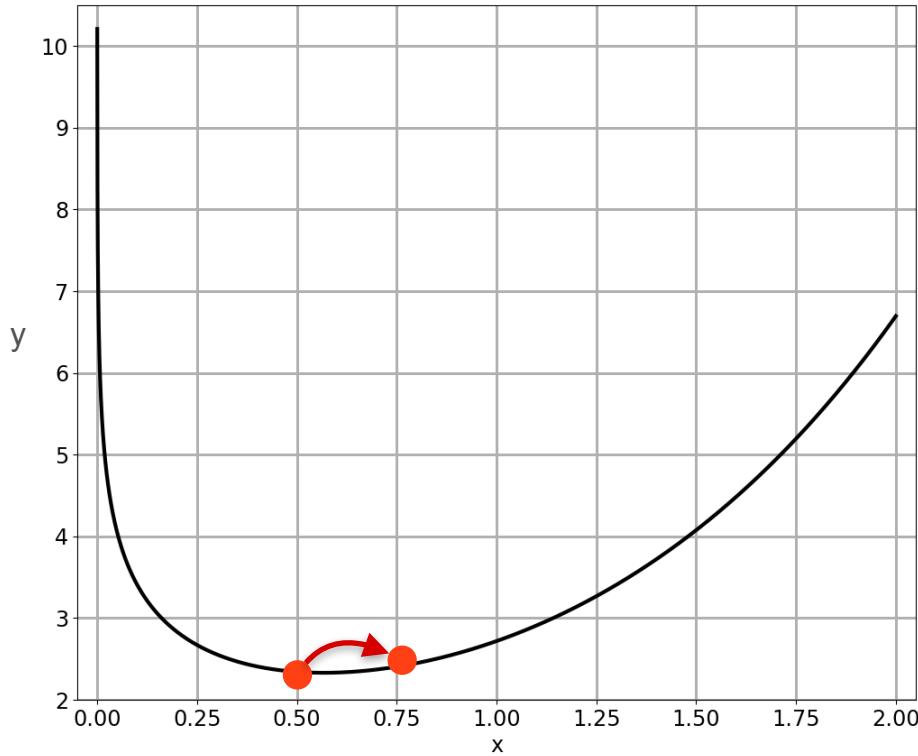


Method 1: Try Both Directions

Is there any
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Repeat!

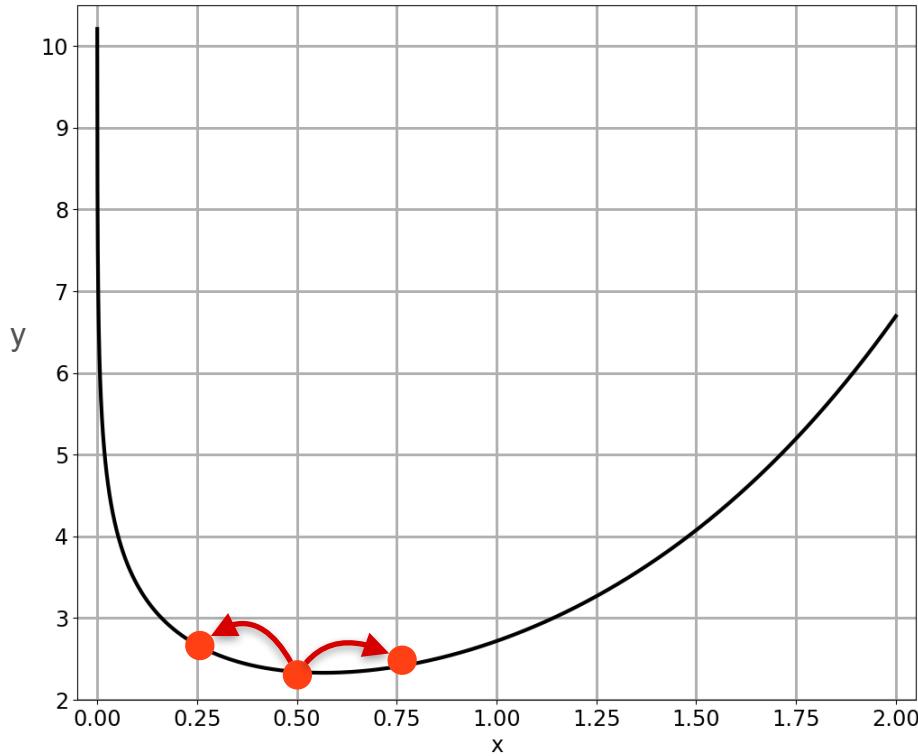


Method 1: Try Both Directions

Is there any
other way?



Repeat!

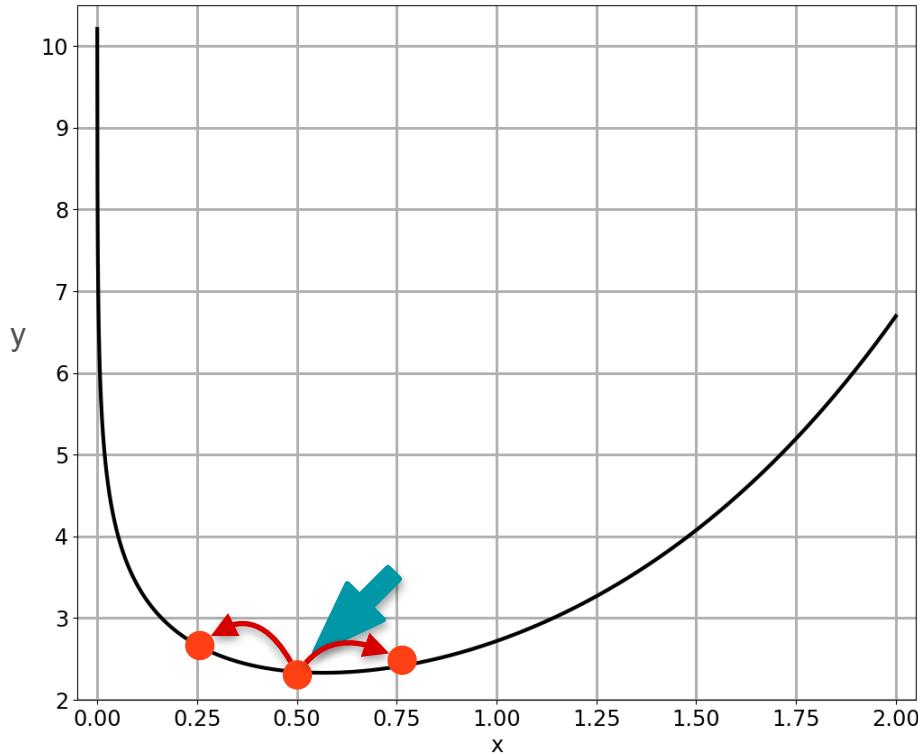


Method 1: Try Both Directions

Is there any
other way?



Repeat!

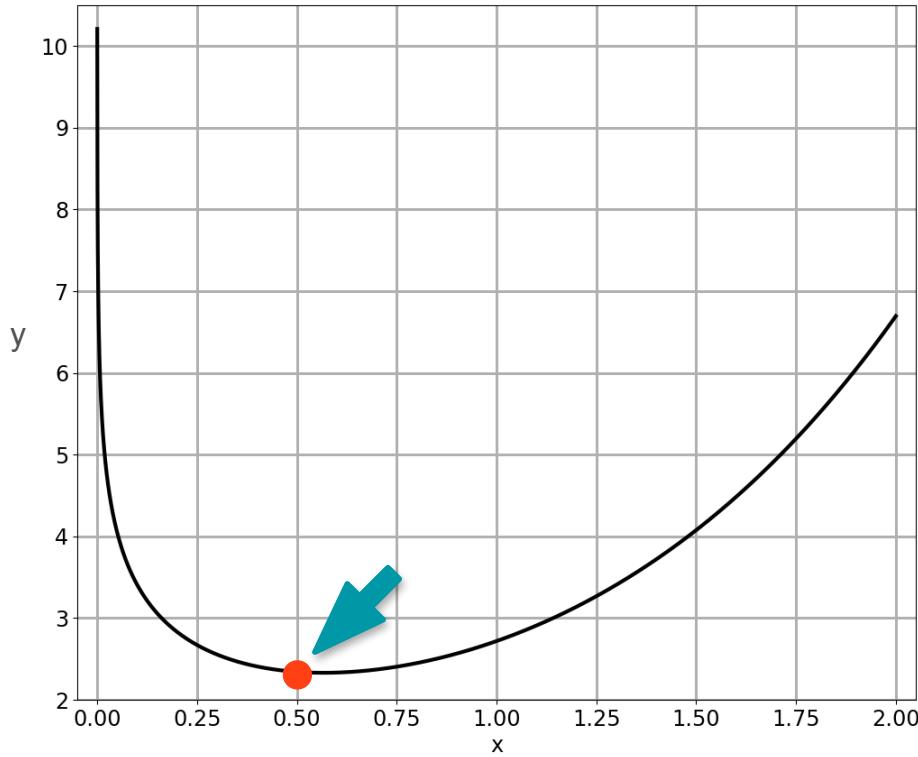


Method 1: Try Both Directions

Is there any
other way?



Repeat!



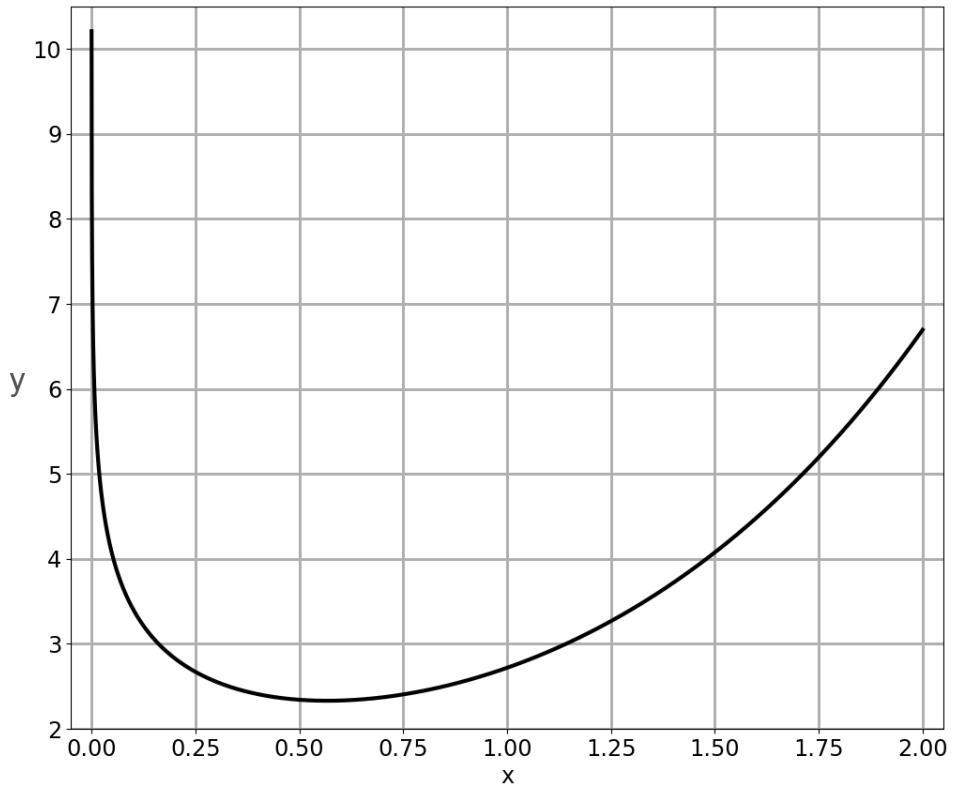


DeepLearning.AI

Gradients and Gradient Descent

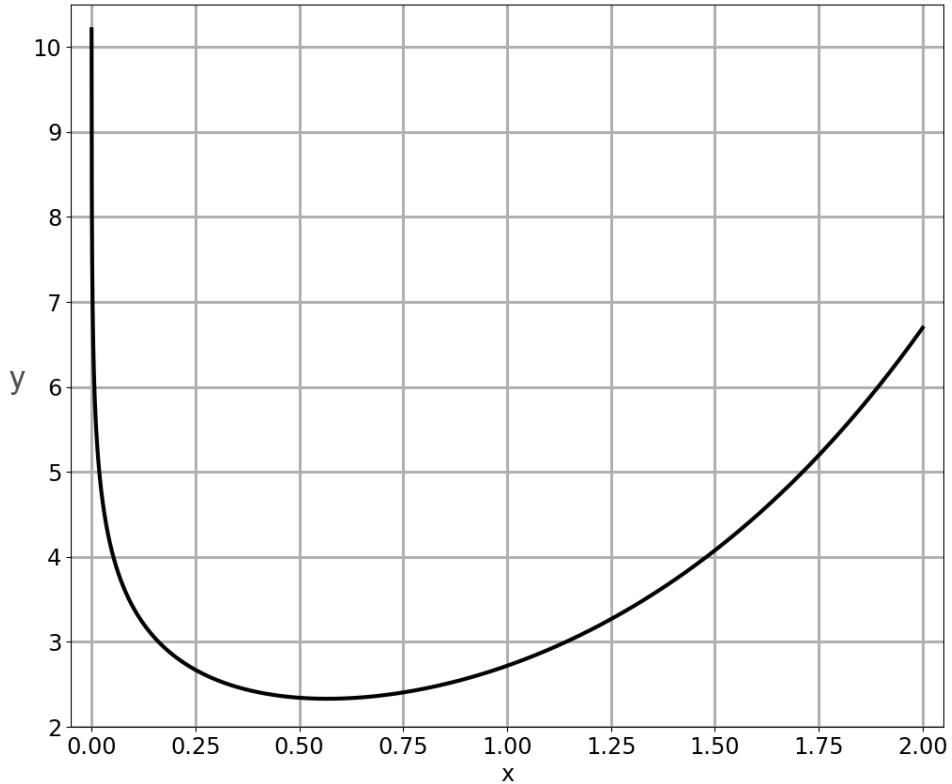
**Optimization using Gradient
Descent in one variable -
Part 2**

Method 2: Be Clever



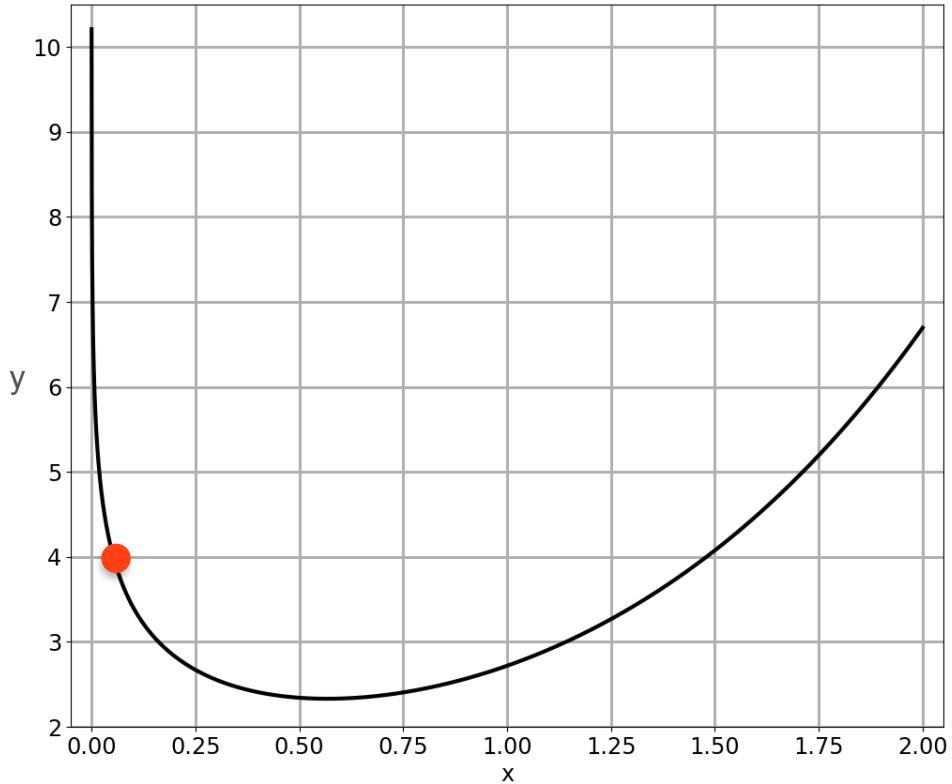
Method 2: Be Clever

Try something
smarter...



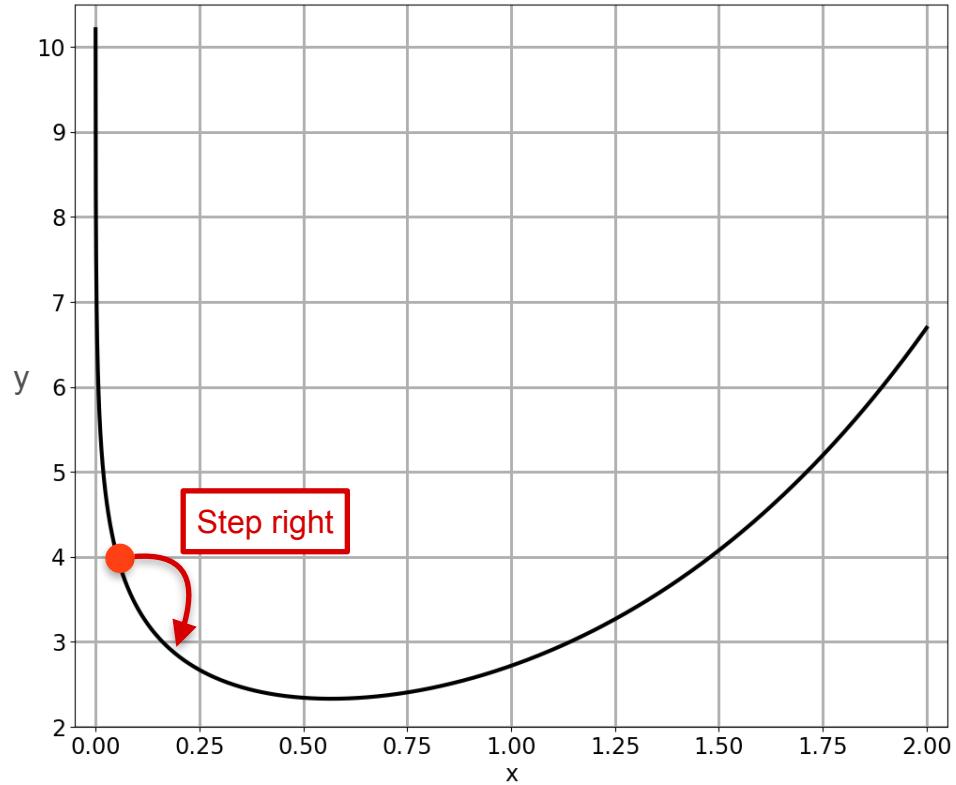
Method 2: Be Clever

Try something
smarter...



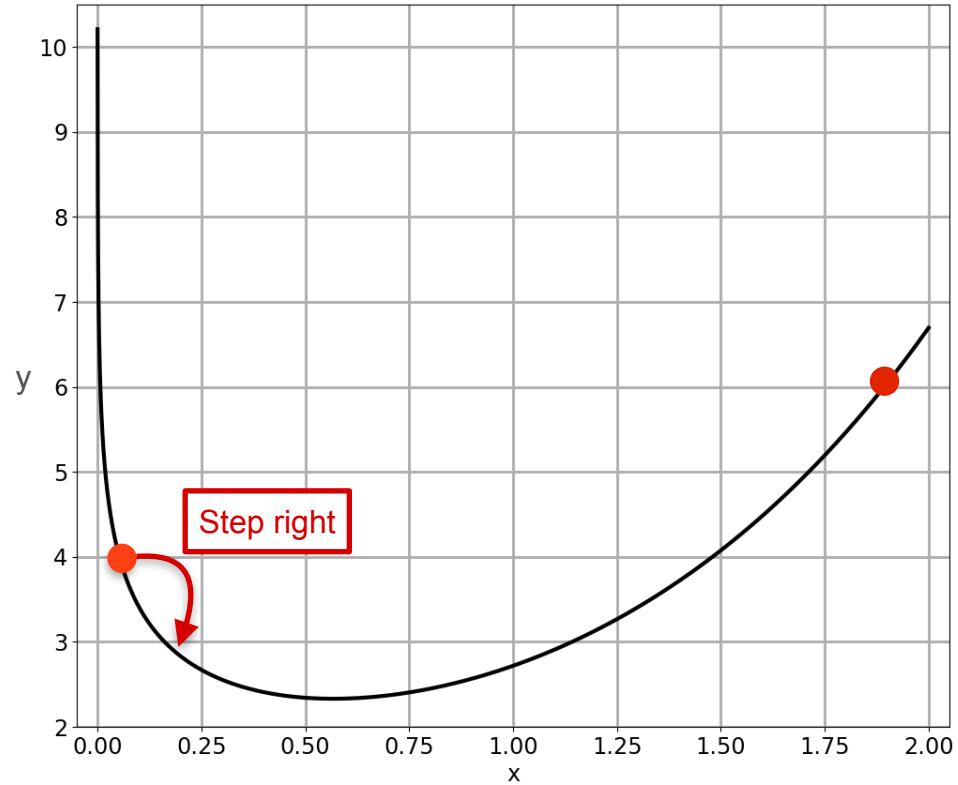
Method 2: Be Clever

Try something
smarter...



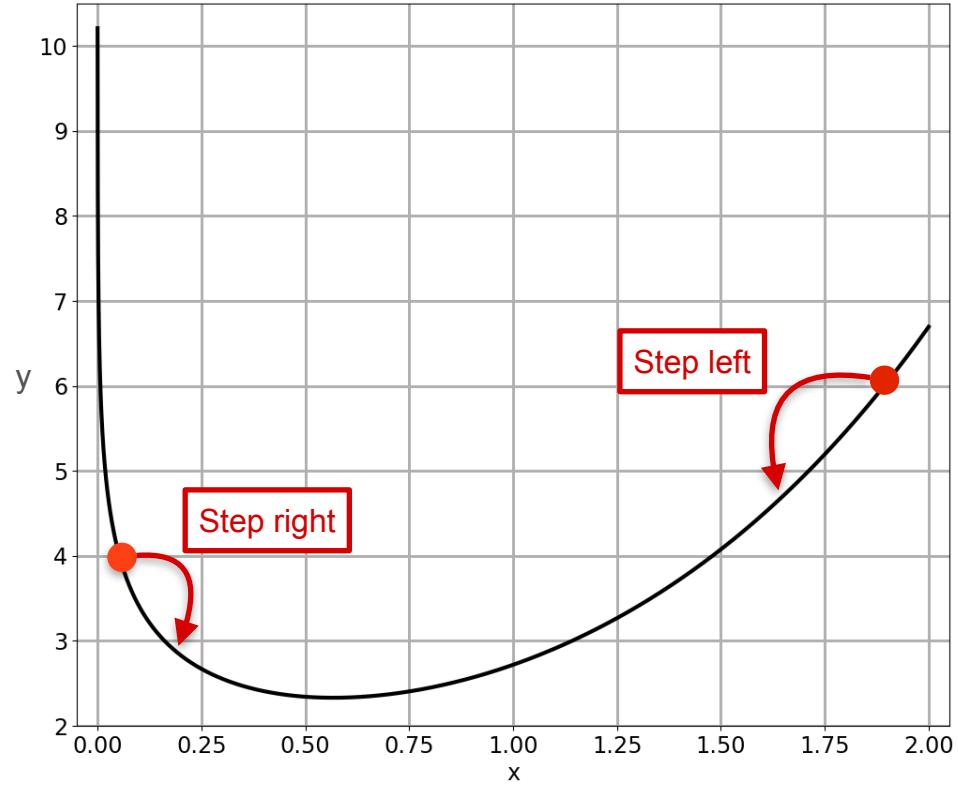
Method 2: Be Clever

Try something
smarter...



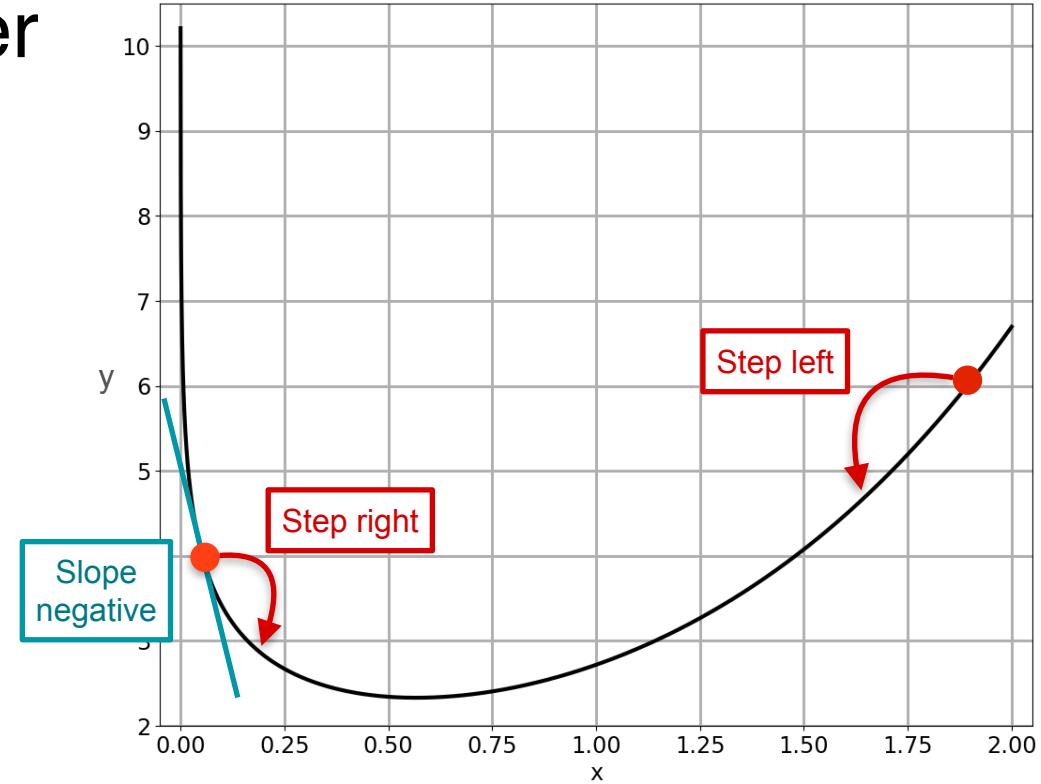
Method 2: Be Clever

Try something
smarter...



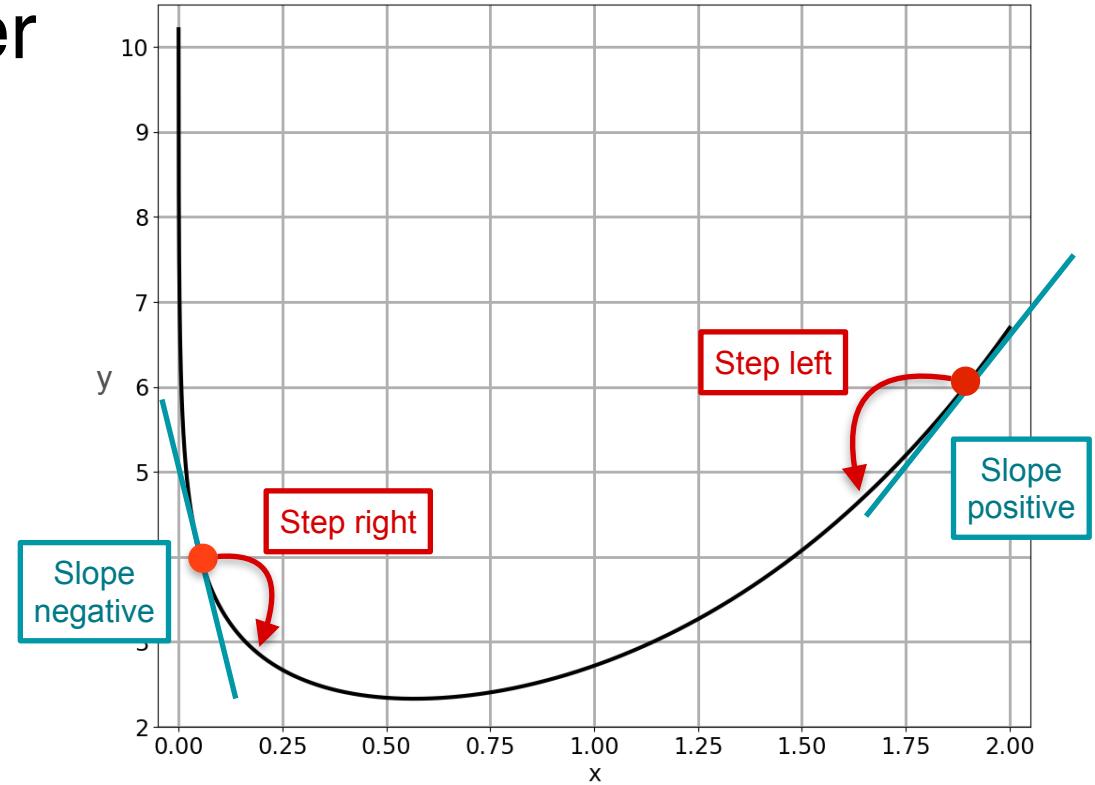
Method 2: Be Clever

Try something
smarter...



Method 2: Be Clever

Try something
smarter...

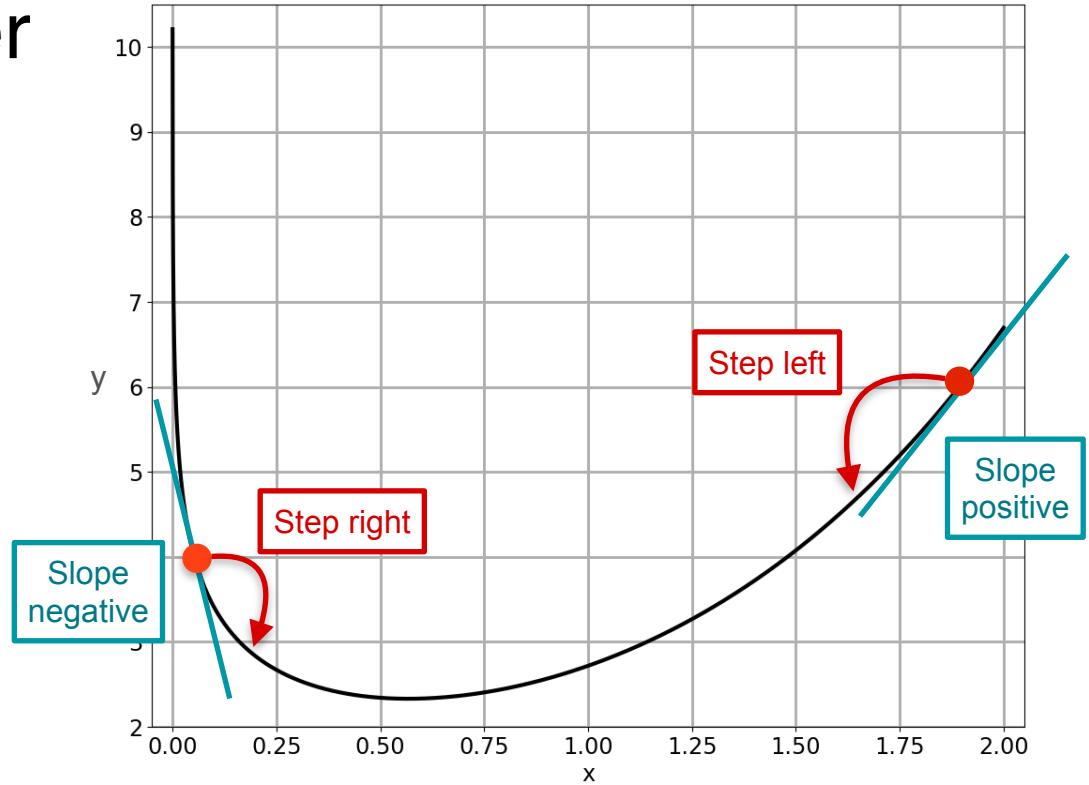


Method 2: Be Clever

Try something
smarter...



new point

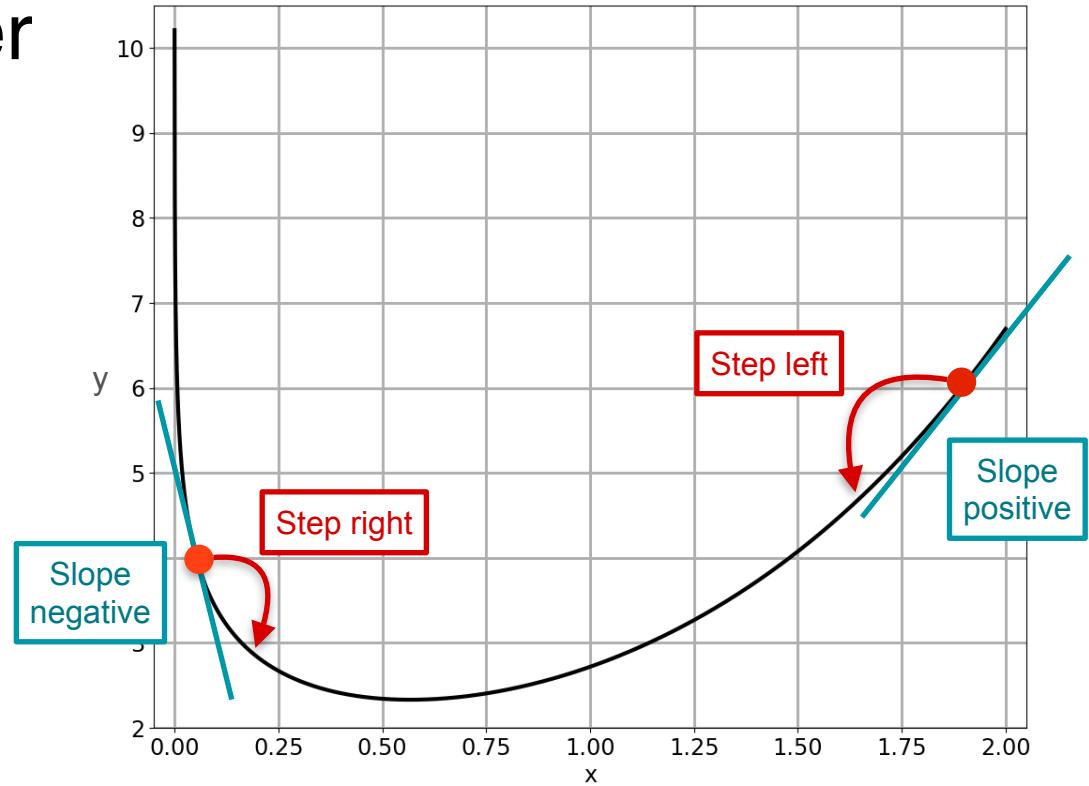


Method 2: Be Clever

Try something
smarter...



new point = old point

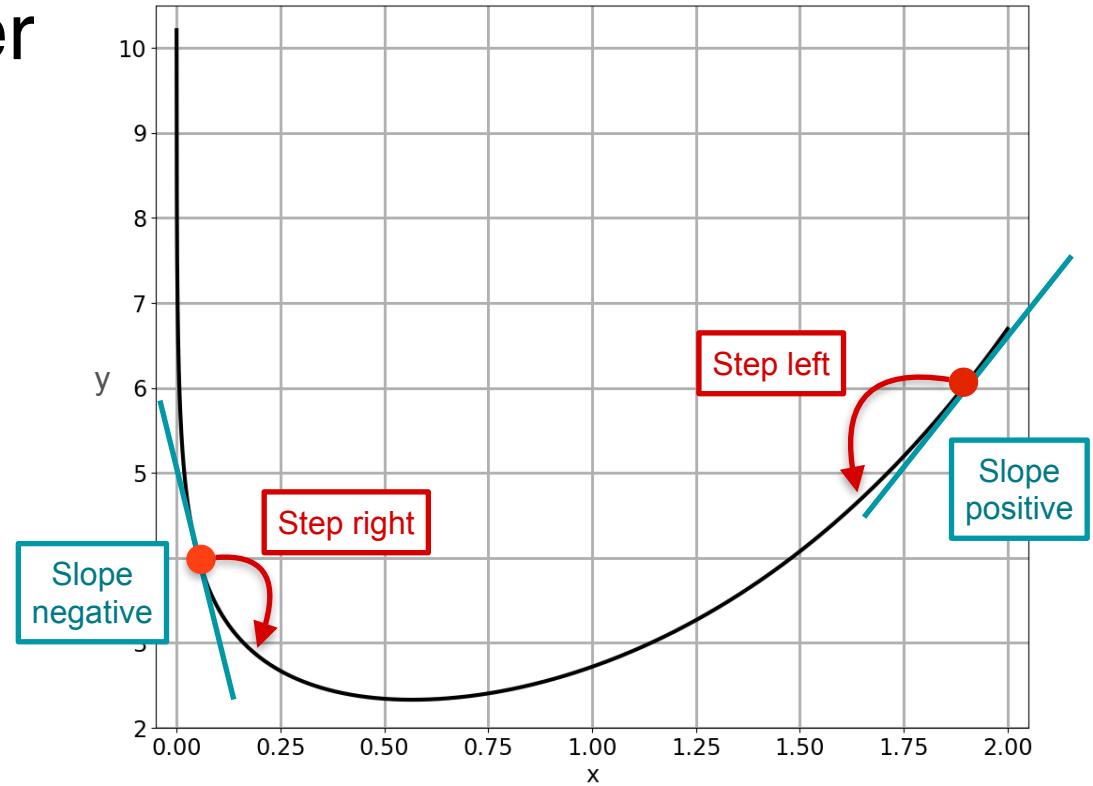


Method 2: Be Clever

Try something
smarter...



new point = old point - slope



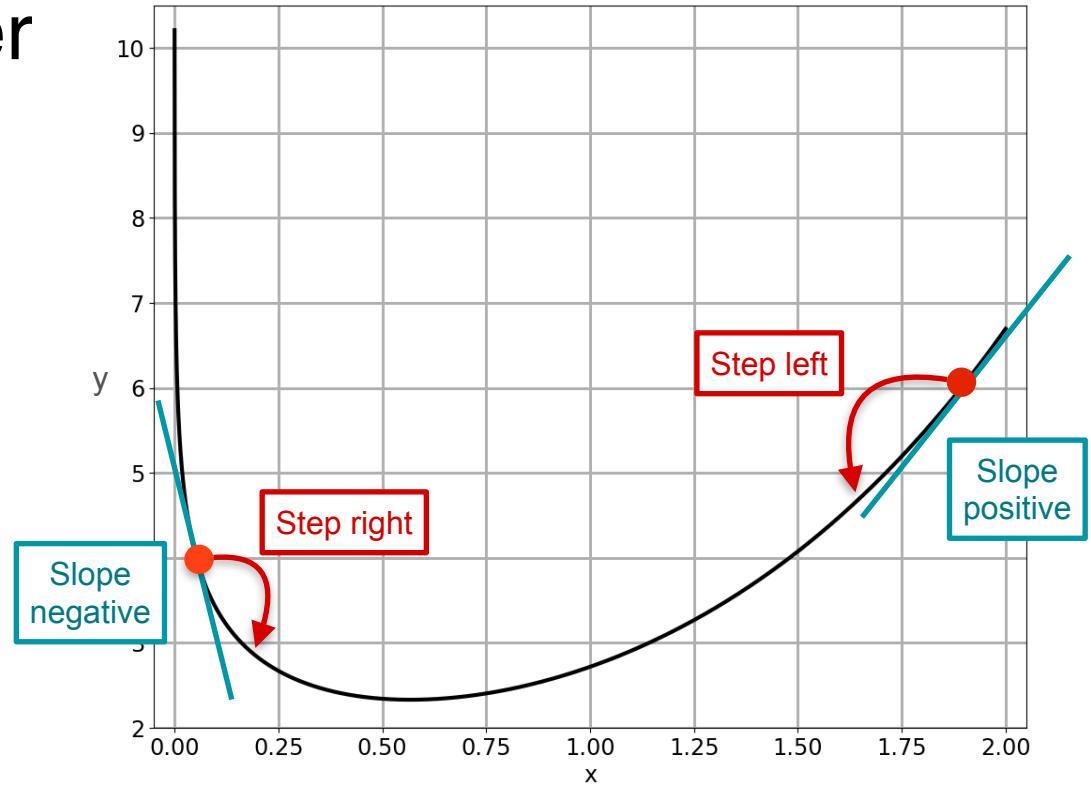
Method 2: Be Clever

Try something
smarter...



new point = old point - slope

x_1



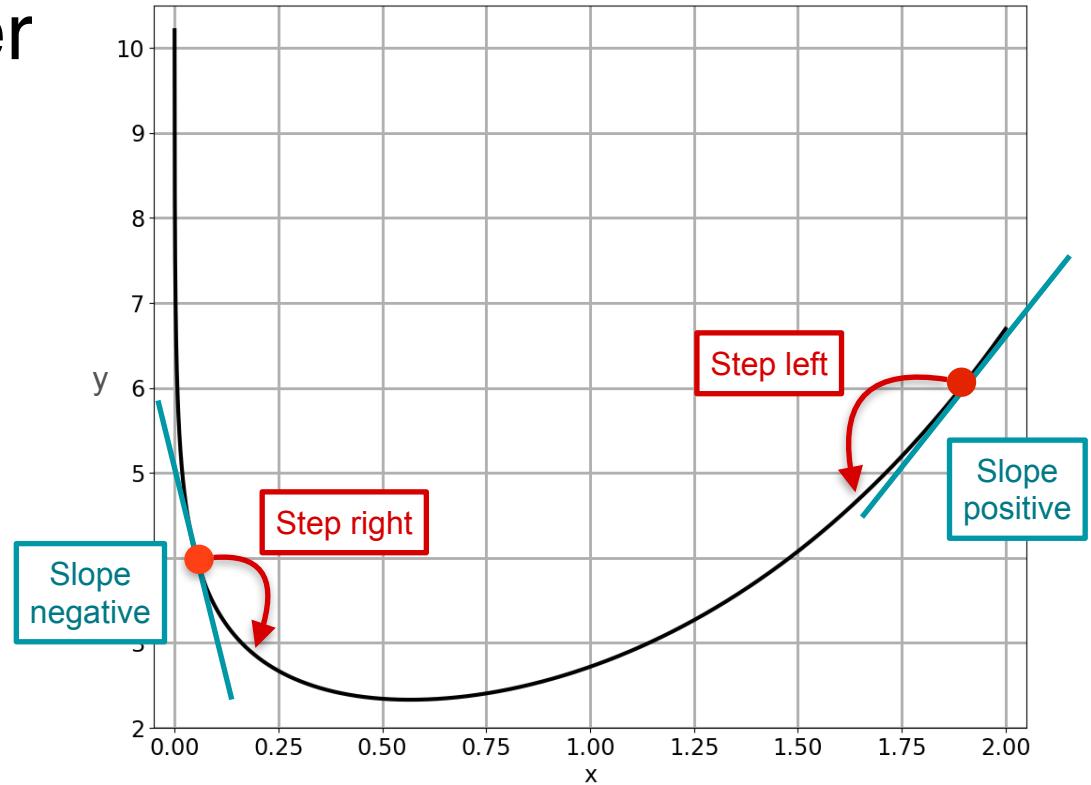
Method 2: Be Clever

Try something
smarter...



new point = old point - slope

$$x_1 = x_0$$



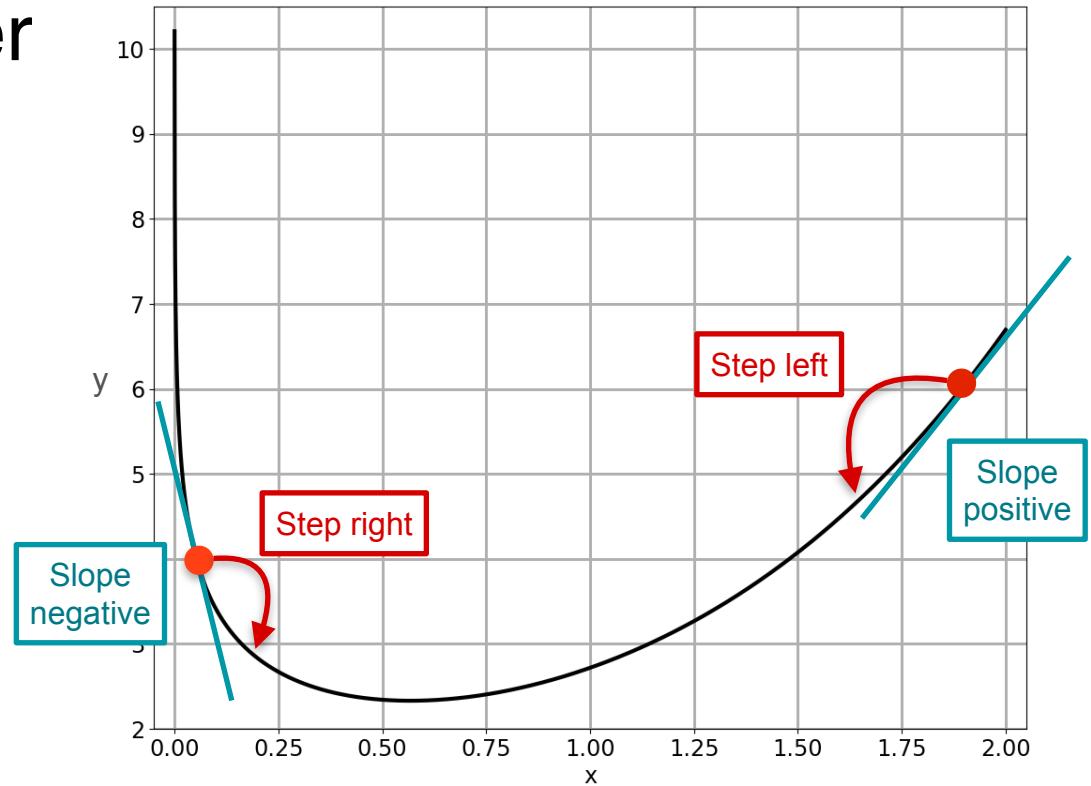
Method 2: Be Clever

Try something
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$



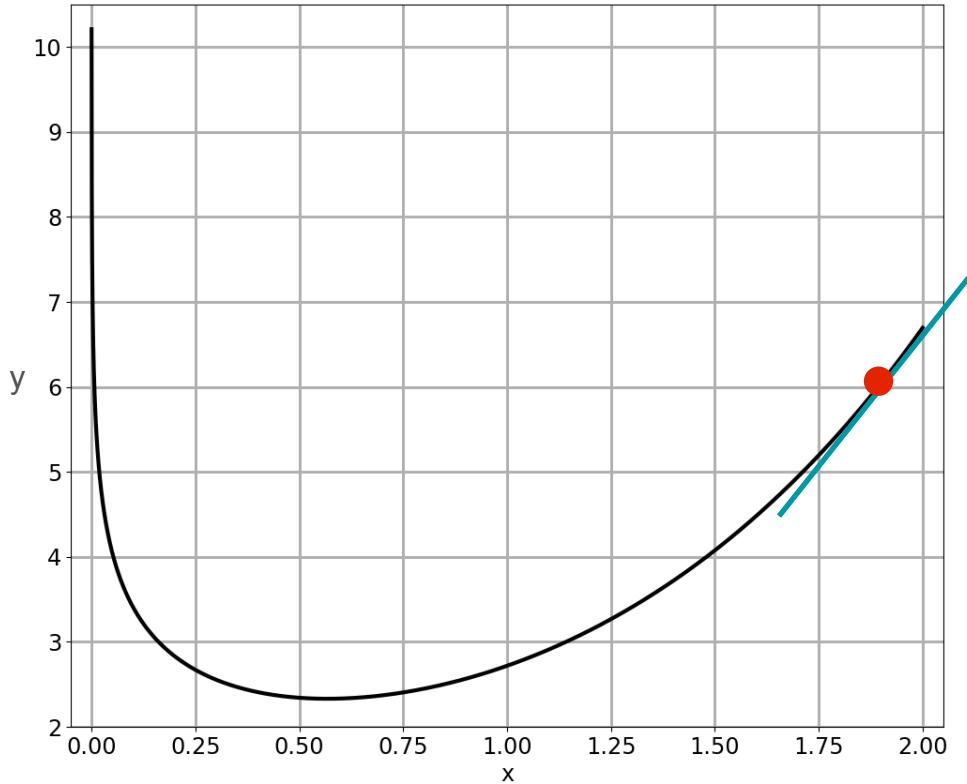
Method 2: Be Clever

Try something
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$



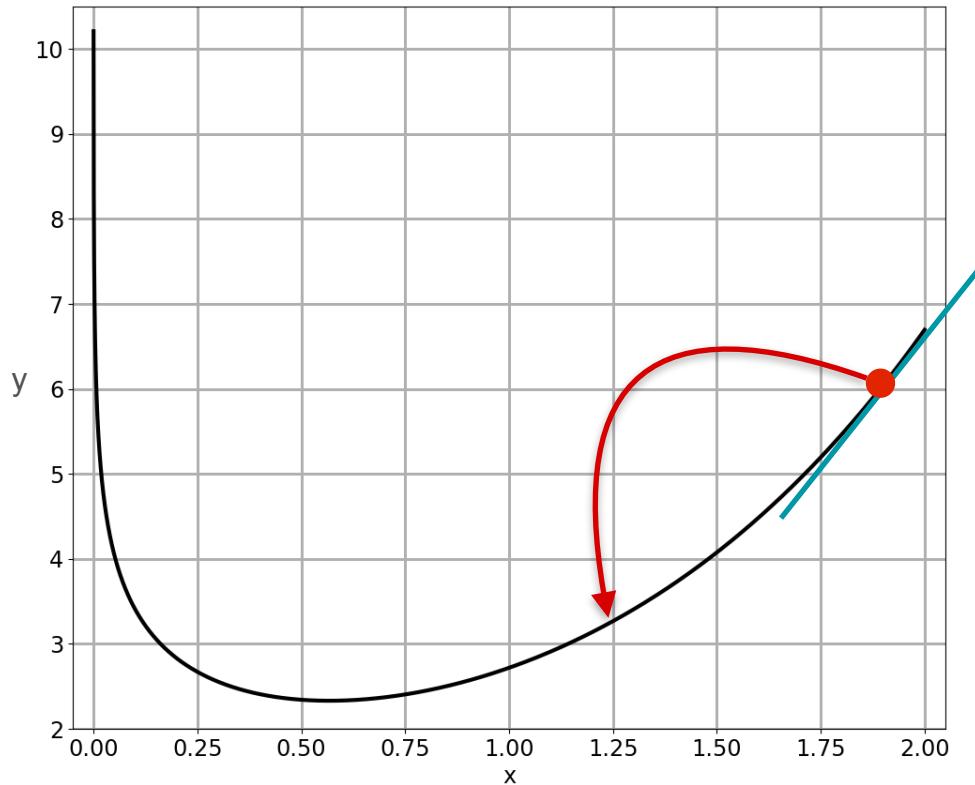
Method 2: Be Clever

Try something
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$



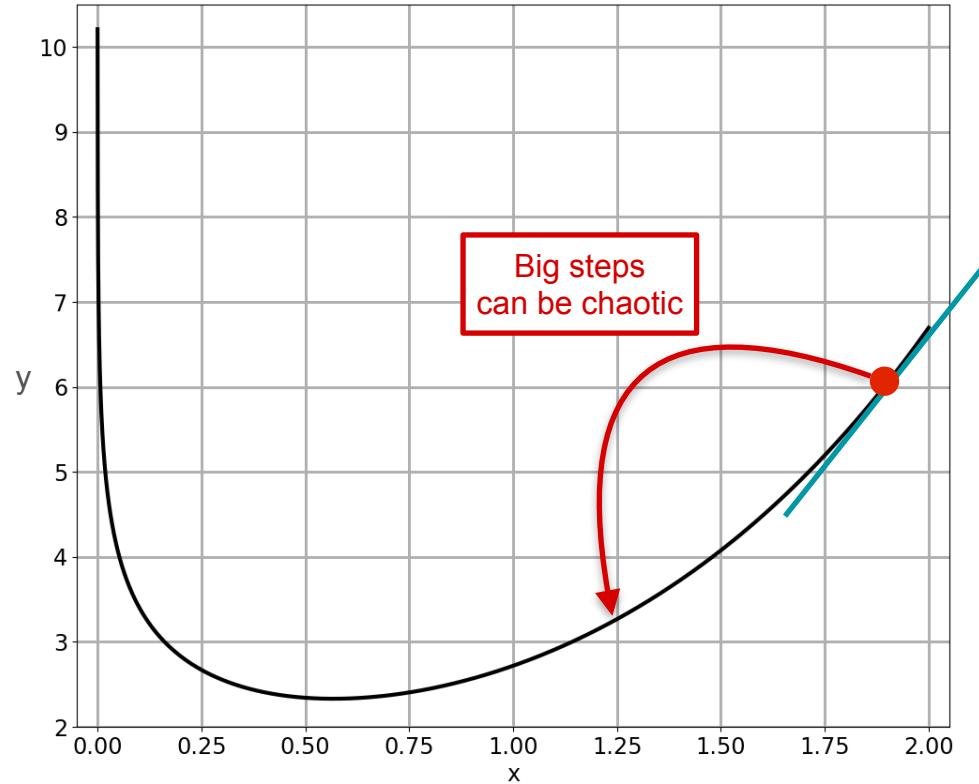
Method 2: Be Clever

Try something
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$



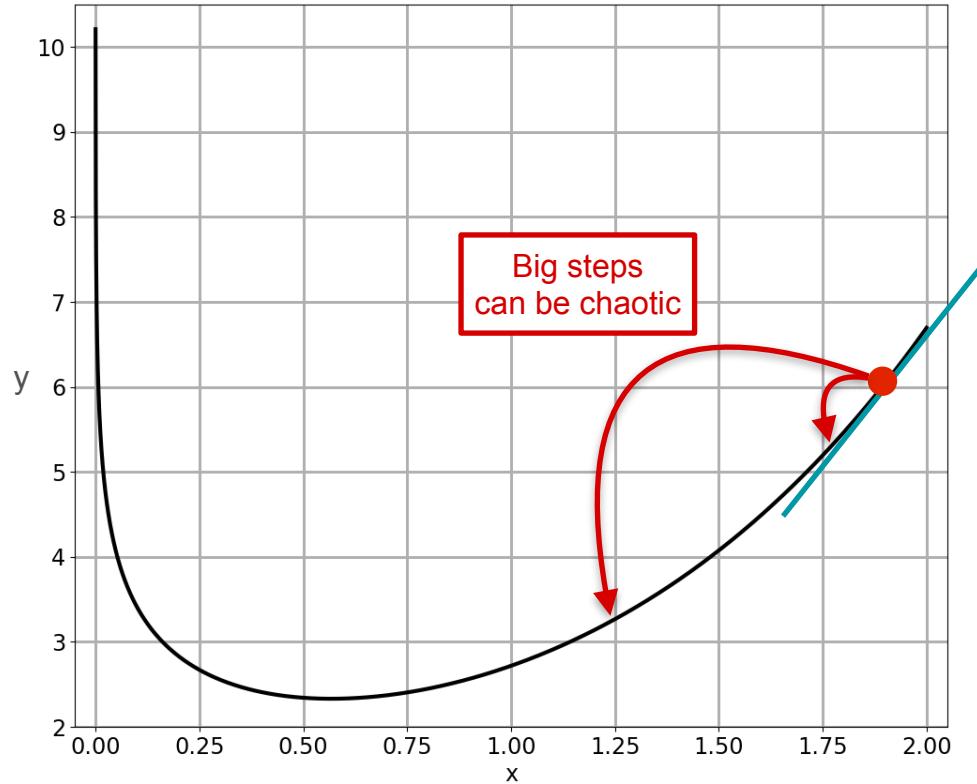
Method 2: Be Clever

Try something
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$



Method 2: Be Clever

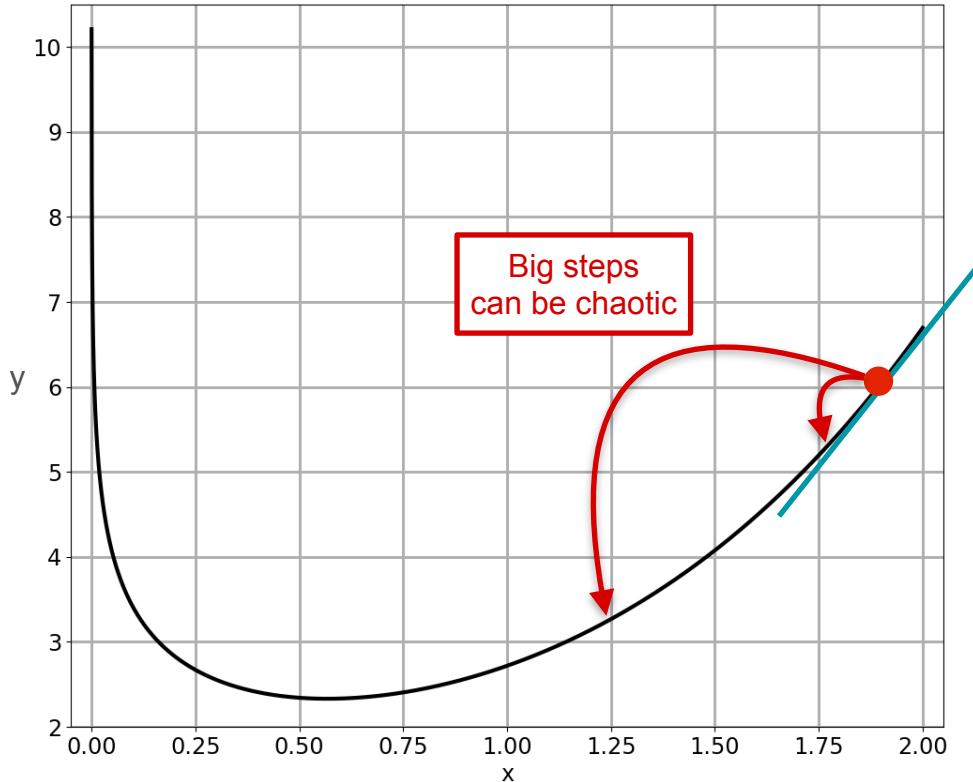
Try something
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

$$x_1 = x_0 - 0.01 f'(x_0)$$



Method 2: Be Clever

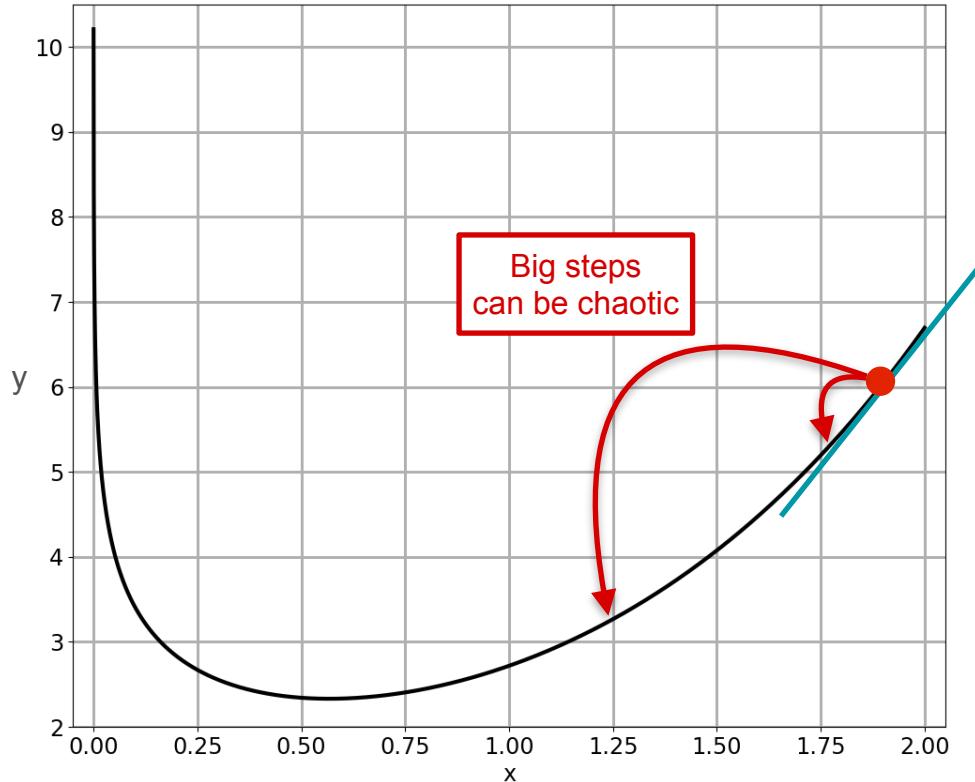
Try something
smarter...



new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

$$x_1 = x_0 - \alpha f'(x_0)$$



Method 2: Be Clever

Try something
smarter...



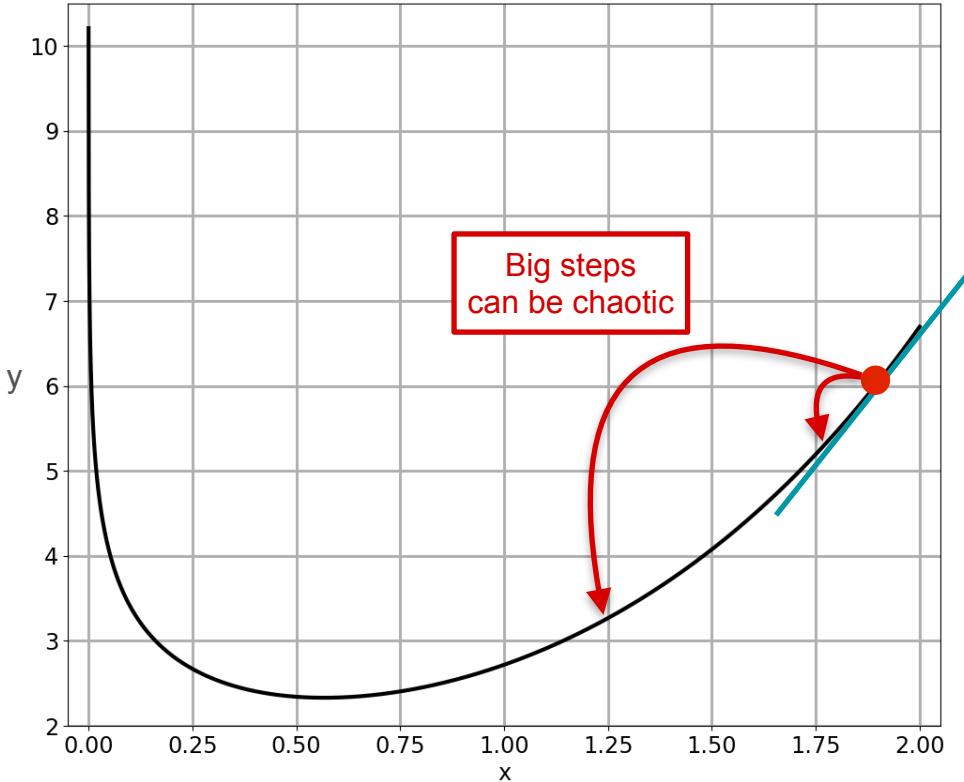
new point = old point - slope

$$x_1 = x_0 - f'(x_0)$$

$$x_1 = x_0 - \alpha f'(x_0)$$



Learning rate

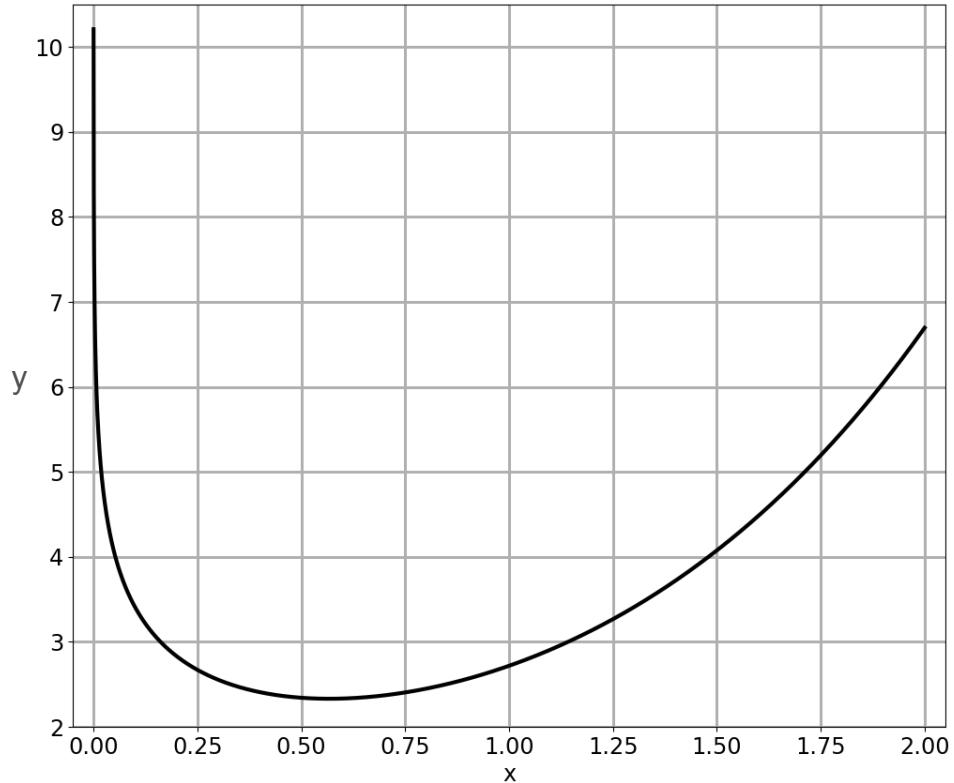


Method 2: Be Clever

Try something
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

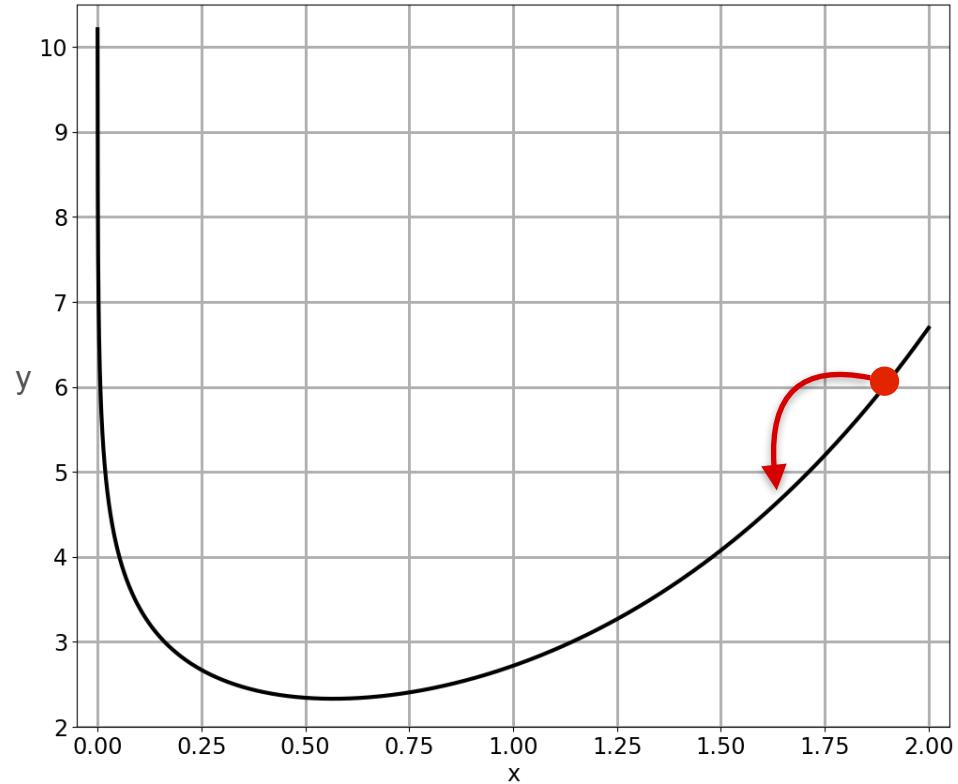


Method 2: Be Clever

Try something
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

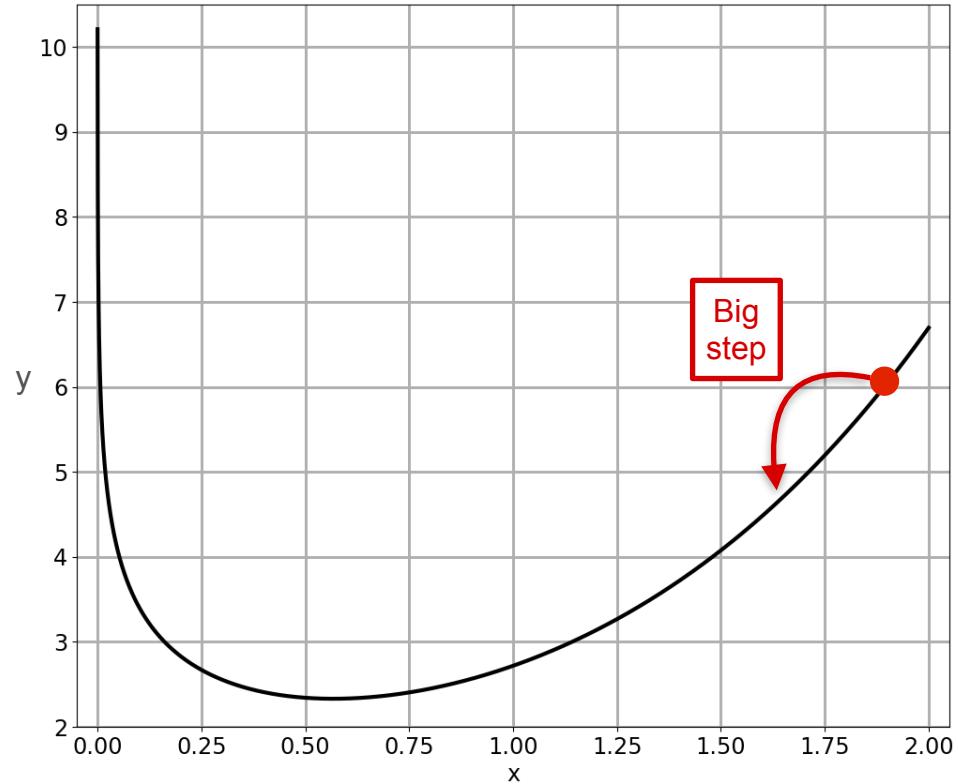


Method 2: Be Clever

Try something
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

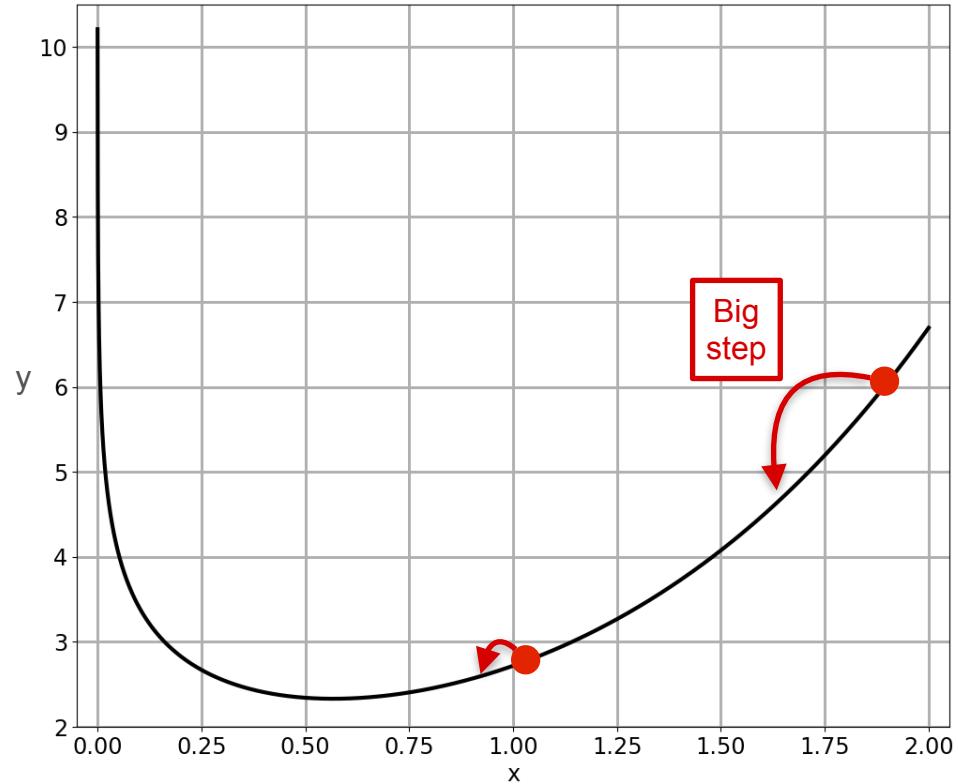


Method 2: Be Clever

Try something
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

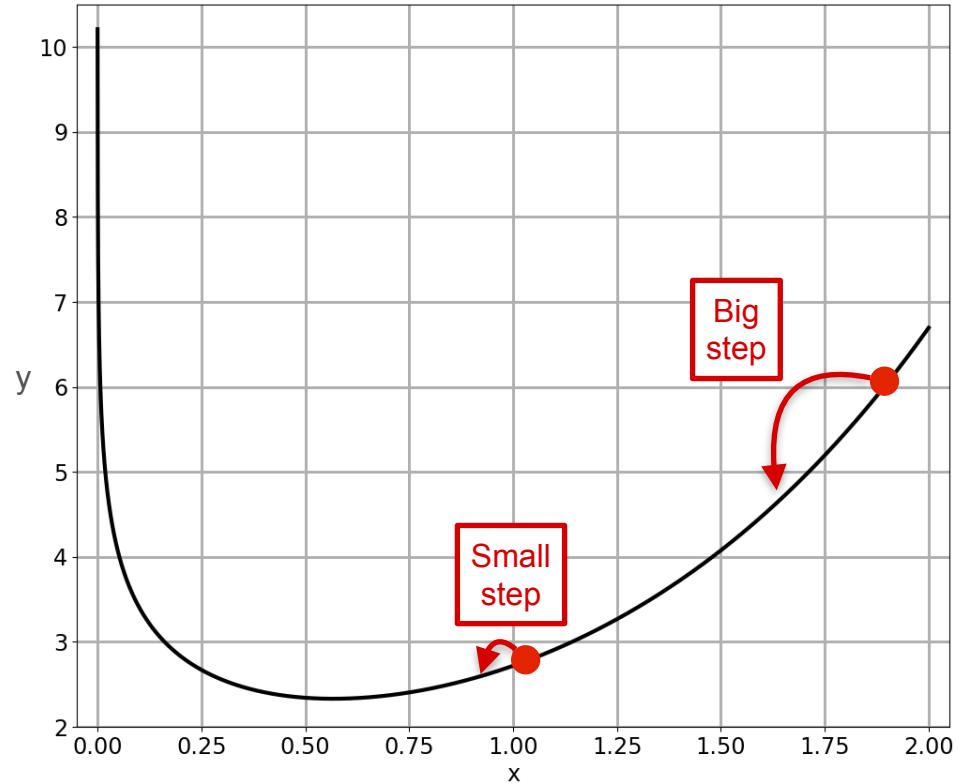


Method 2: Be Clever

Try something
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

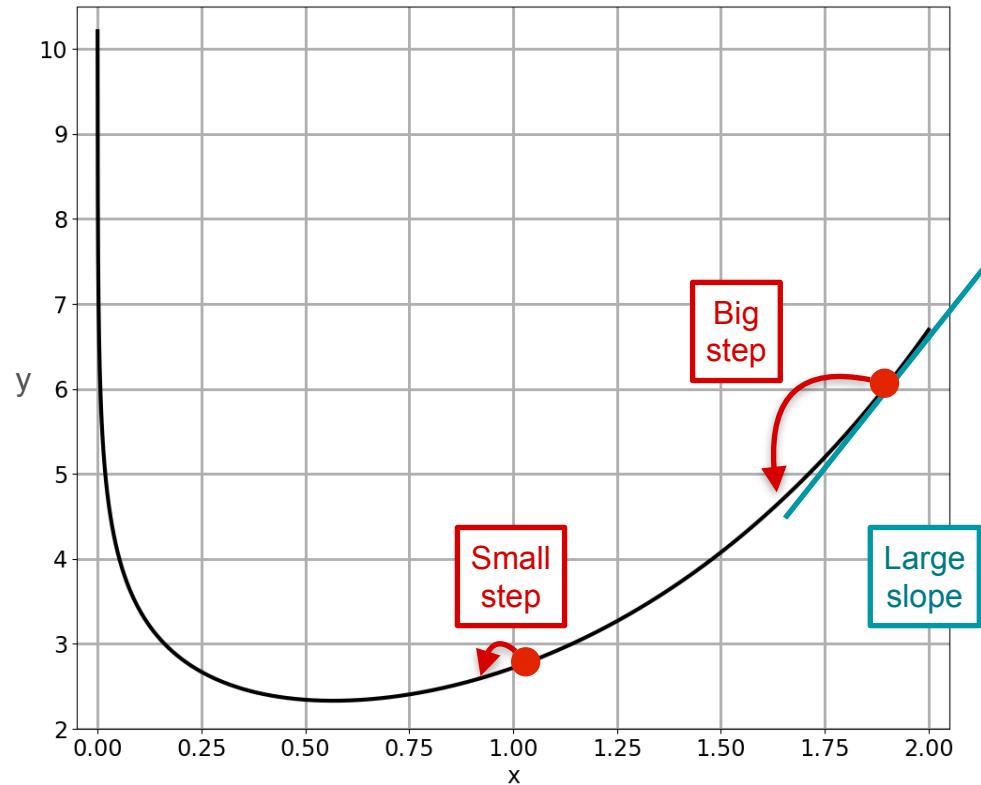


Method 2: Be Clever

Try something
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

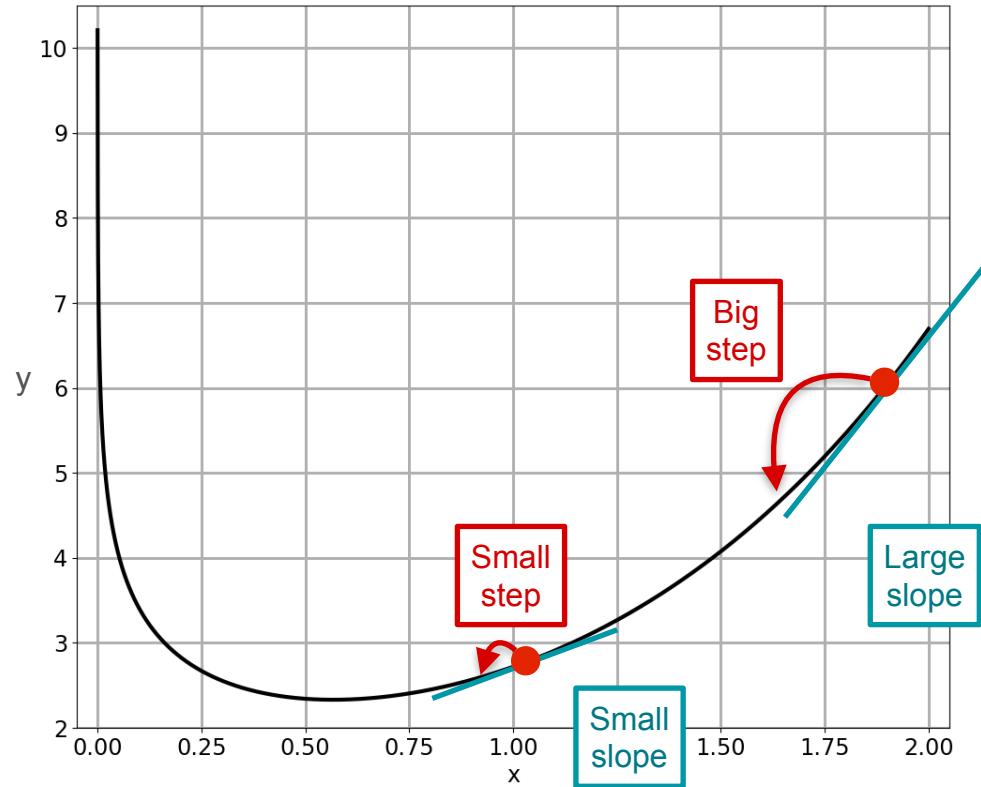


Method 2: Be Clever

Try something
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$



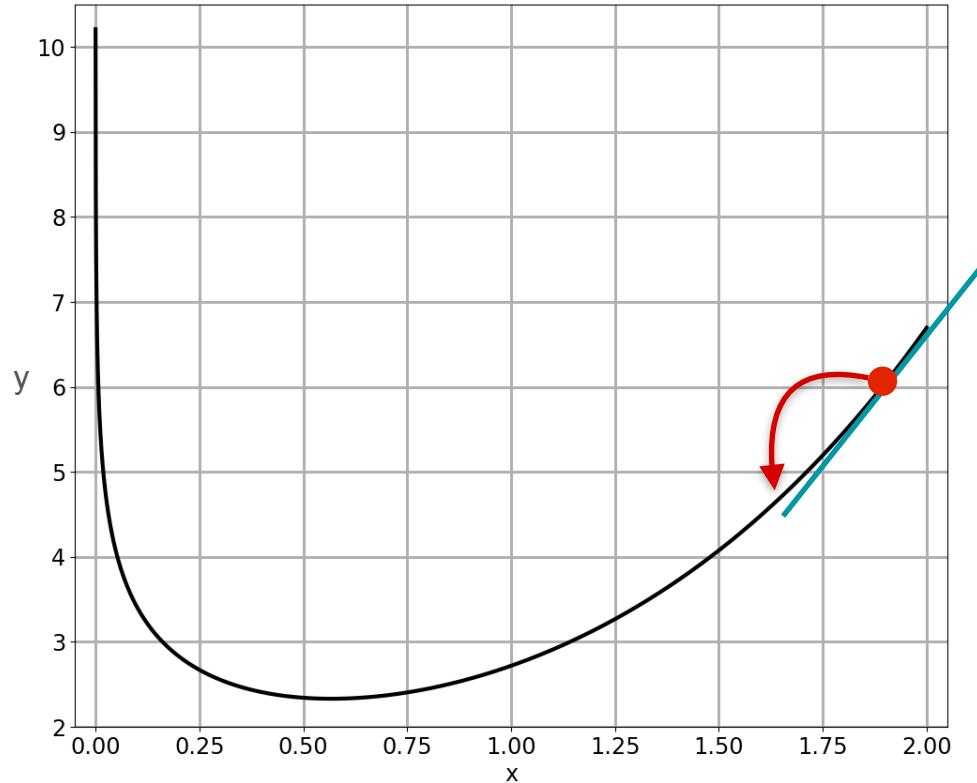
Method 2: Be Clever

Try something
smarter...



$$x_1 = x_0 - \alpha f'(x_0)$$

Gradient descent



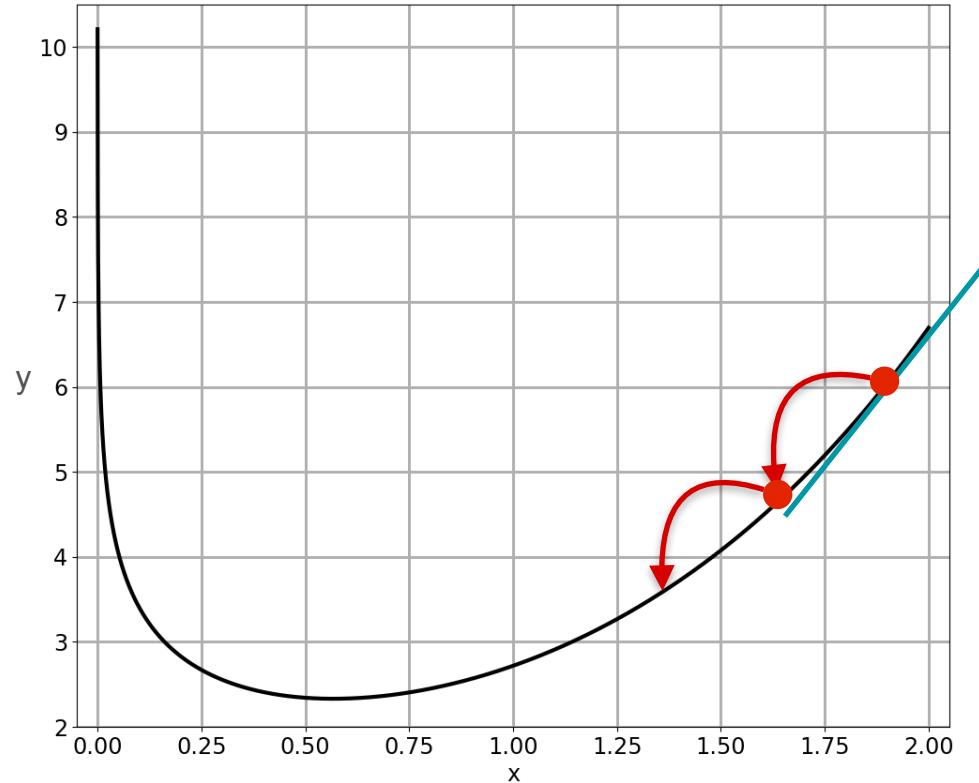
Method 2: Be Clever

Try something
smarter...



$$x_2 = x_1 - \alpha f'(x_1)$$

Gradient descent



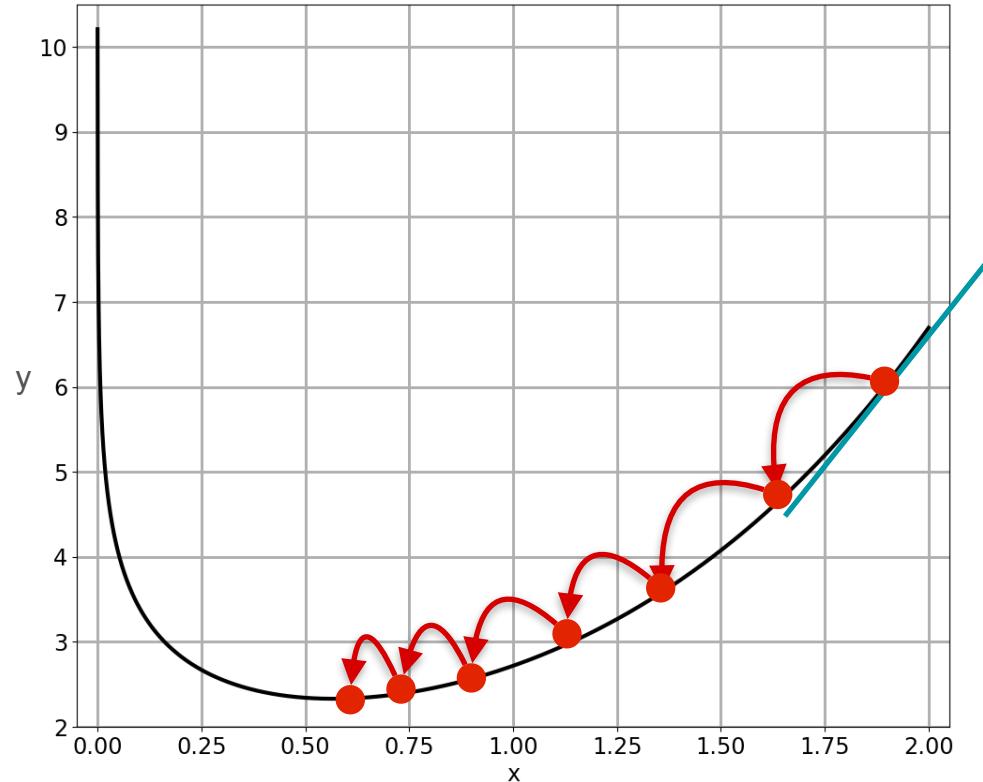
Method 2: Be Clever

Try something
smarter...



$$x_{20} = x_{19} - \alpha f'(x_{19})$$

Gradient descent



Gradient Descent

Function: $f(x)$

Goal: find minimum of $f(x)$

Step 1:

Define a learning rate α

Choose a starting point x_0

Step 2:

Update: $x_k = x_{k-1} - \alpha f'(x_{k-1})$

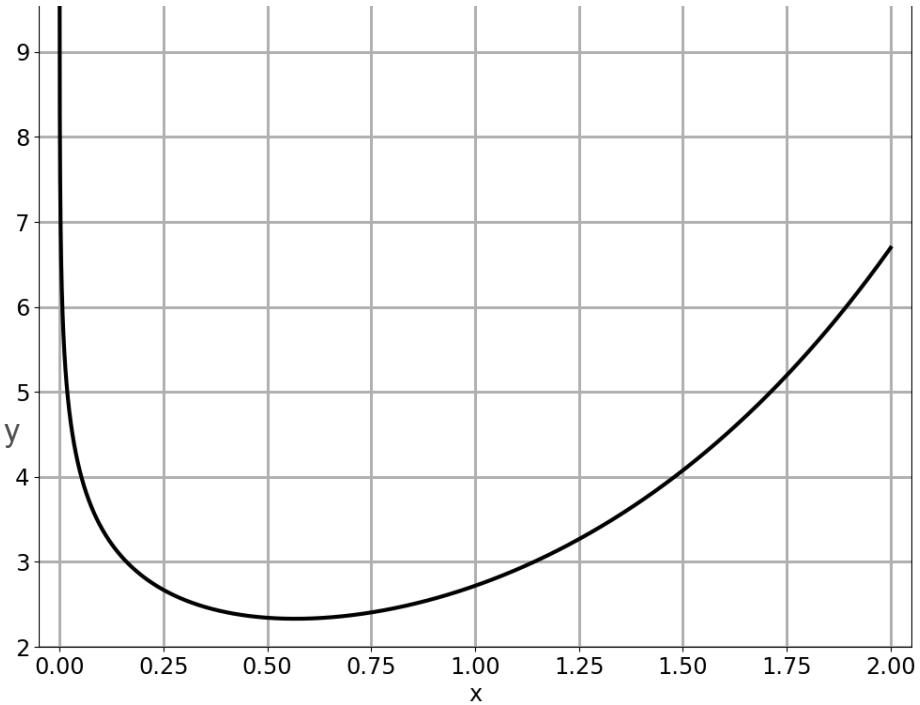
Step 3:

Repeat Step 2 until you are close enough to
the true minimum x^*

Gradient Descent

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x}$$



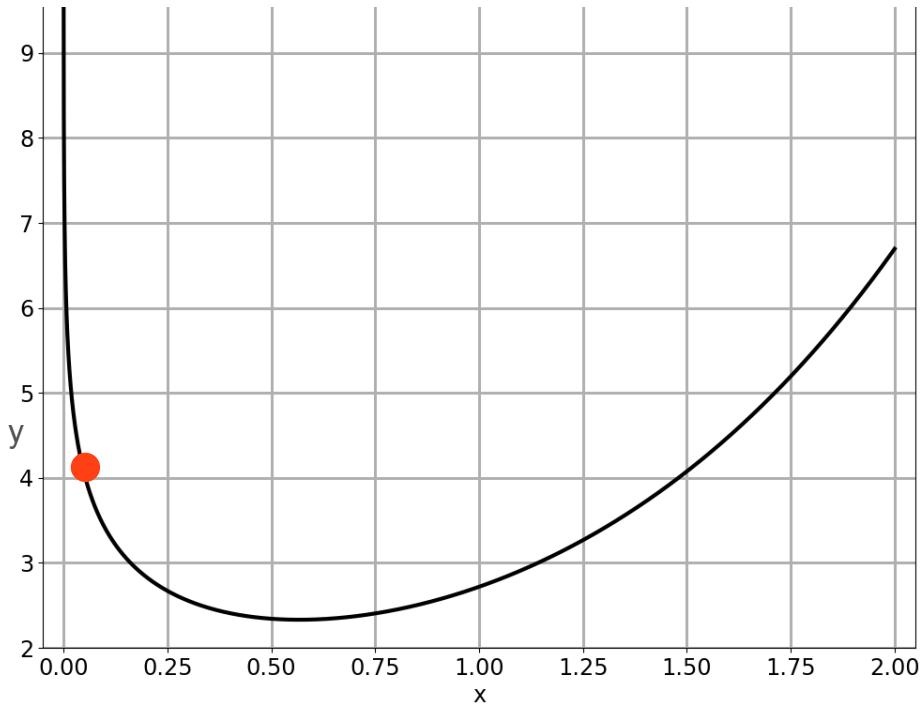
Gradient Descent

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x}$$

Start: $x = 0.05$

Rate: $\alpha = 0.005$



Gradient Descent

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x}$$

Start: $x = 0.05$

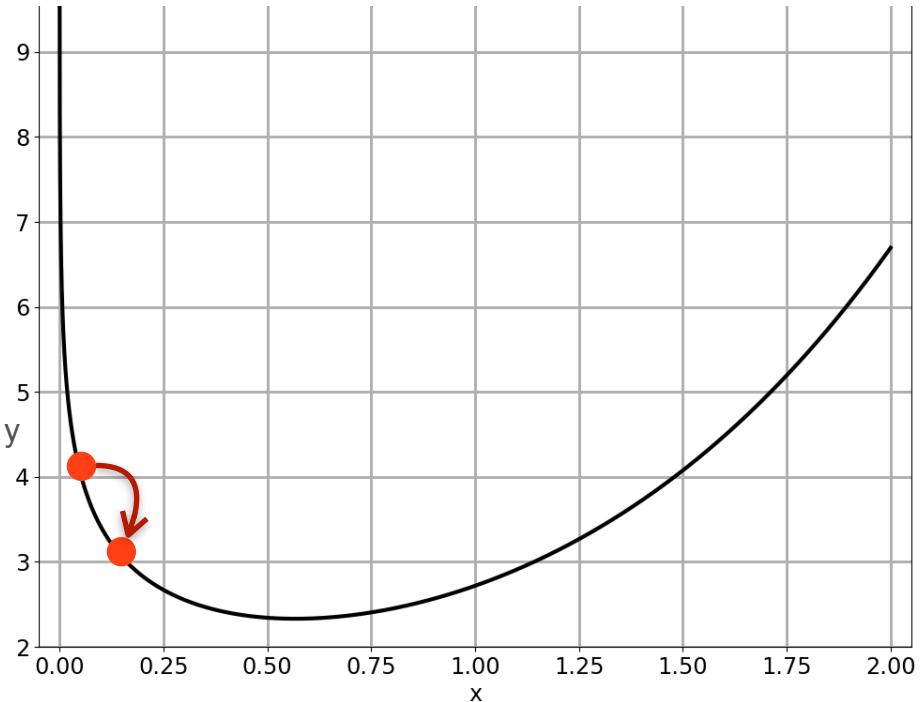
Rate: $\alpha = 0.005$

Find:

$$f'(0.05) = -18.9$$

Move by $-0.005f'(0.05)$

$$x \mapsto 0.1447$$



Gradient Descent

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x}$$

Start: $x = 0.05$

Rate: $\alpha = 0.005$

Find:

$$f'(0.05) = -18.9$$

Move by $-0.005f'(0.05)$

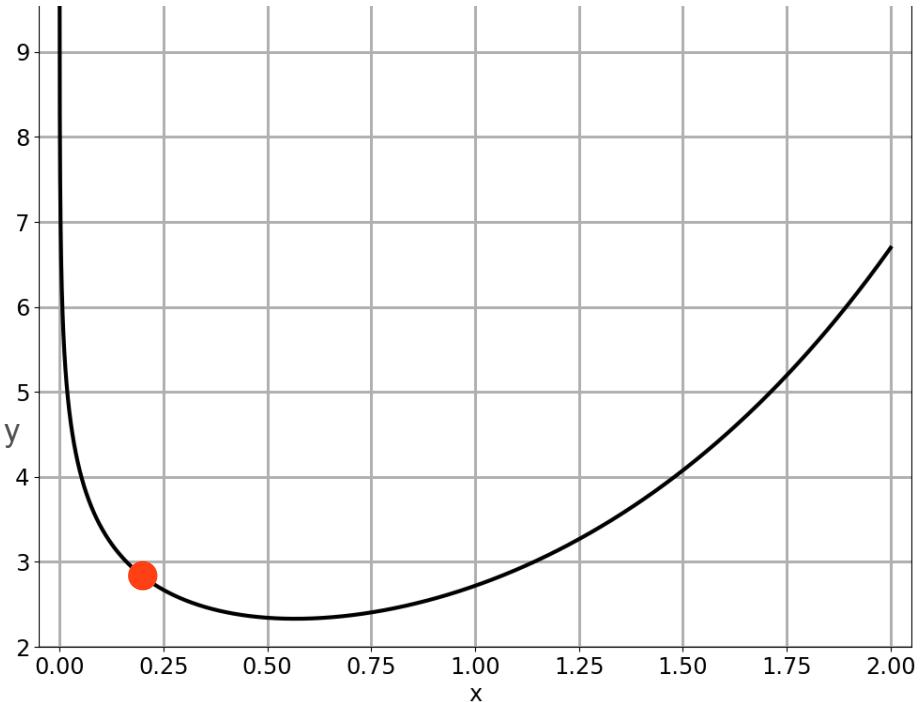
$$x \mapsto 0.1447$$

Find:

$$f'(0.1447) = -5.7552$$

Move by $-0.005f'(0.05)$

$$x \mapsto 0.1735$$



Gradient Descent

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x}$$

Start: $x = 0.05$

Rate: $\alpha = 0.005$

Find:

$$f'(0.05) = -18.9$$

Move by $-0.005f'(0.05)$

$$x \mapsto 0.1447$$

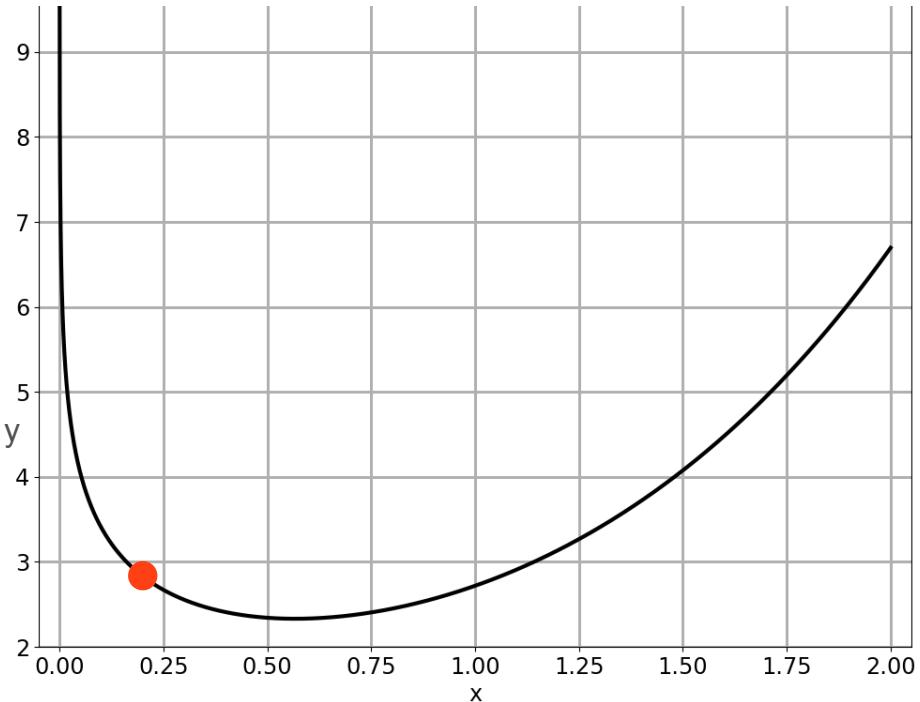
Find:

$$f'(0.1447) = -5.7552$$

Move by $-0.005f'(0.05)$

$$x \mapsto 0.1735$$

Repeat!



Gradient Descent

$$f(x) = e^x - \log(x)$$

$$f'(x) = e^x - \frac{1}{x}$$

Start: $x = 0.05$

Rate: $\alpha = 0.005$

Find:

$$f'(0.05) = -18.9$$

Move by $-0.005f'(0.05)$

$$x \mapsto 0.1447$$

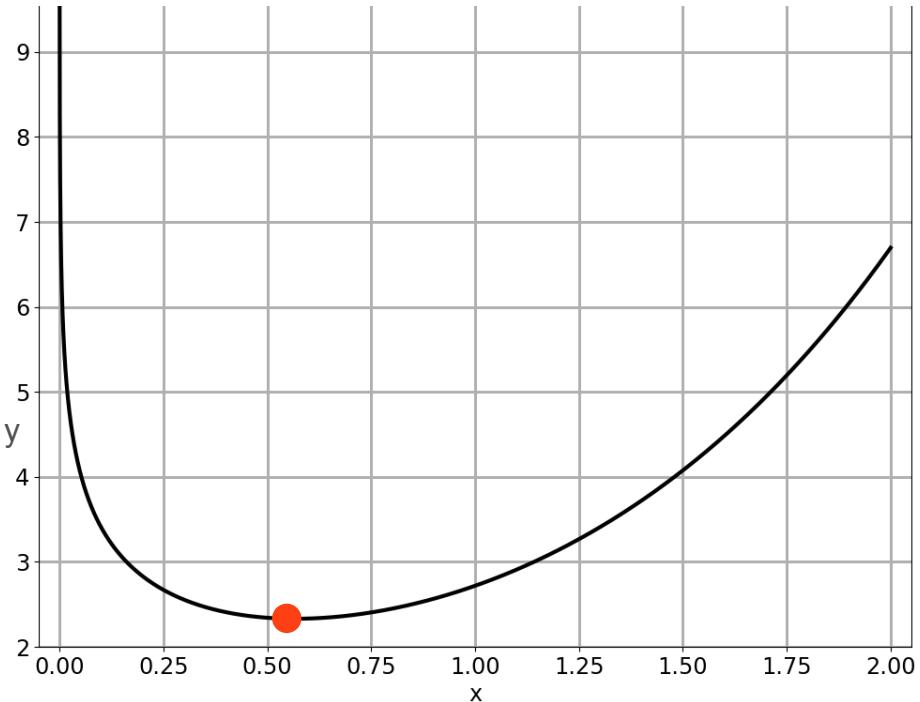
Find:

$$f'(0.1447) = -5.7552$$

Move by $-0.005f'(0.05)$

$$x \mapsto 0.1735$$

Repeat!





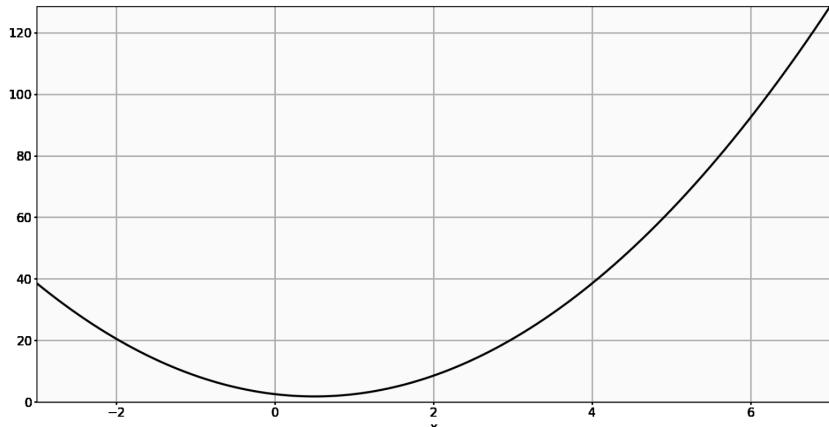
DeepLearning.AI

Gradients and Gradient Descent

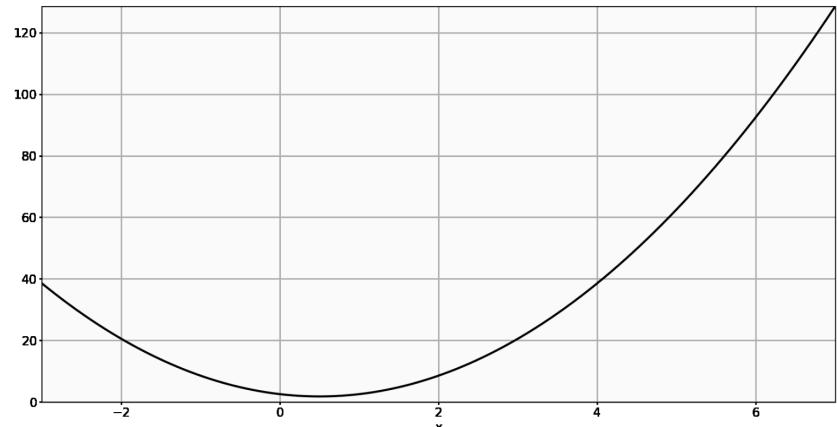
**Optimization using Gradient
Descent in one variable -
Part 3**

What Is a Good Learning Rate?

Too large

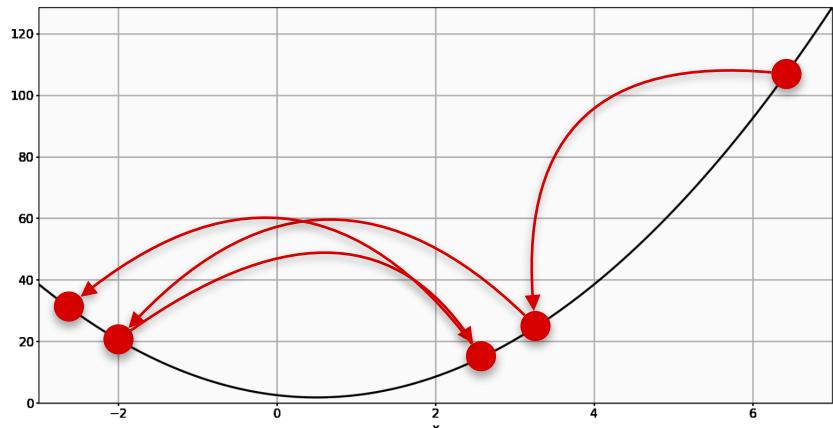


Too small

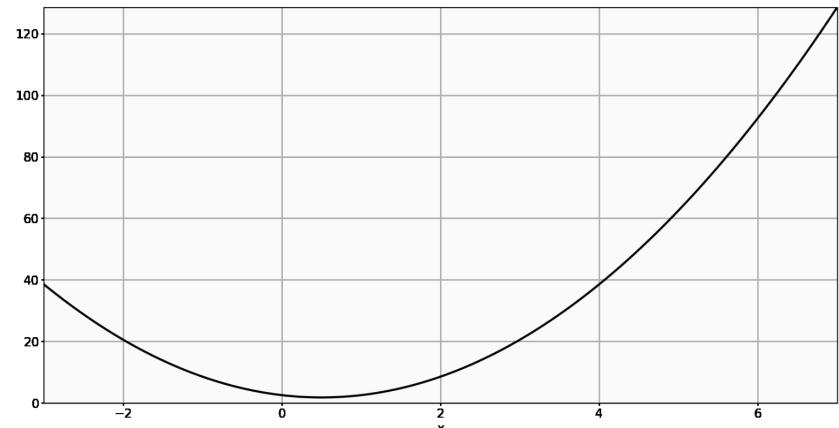


What Is a Good Learning Rate?

Too large

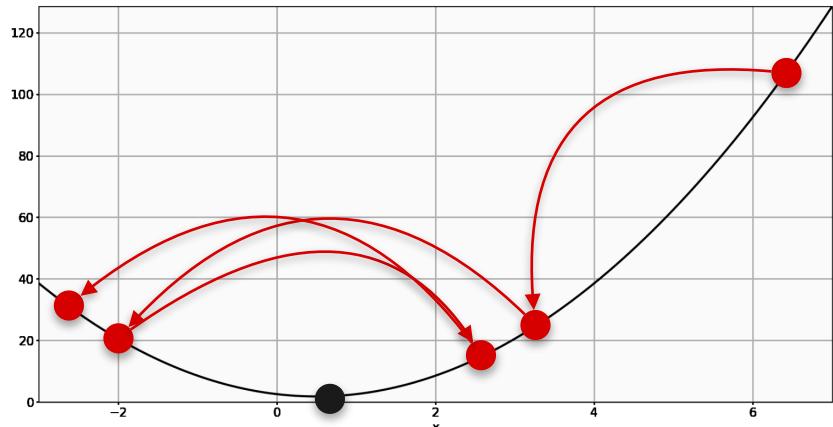


Too small

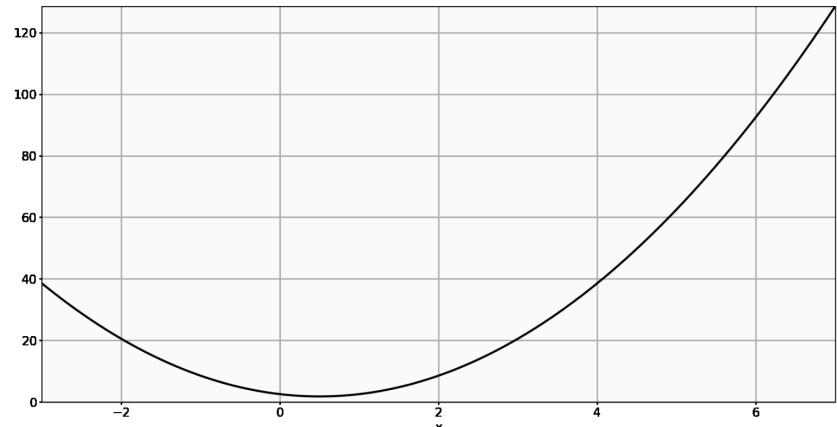


What Is a Good Learning Rate?

Too large

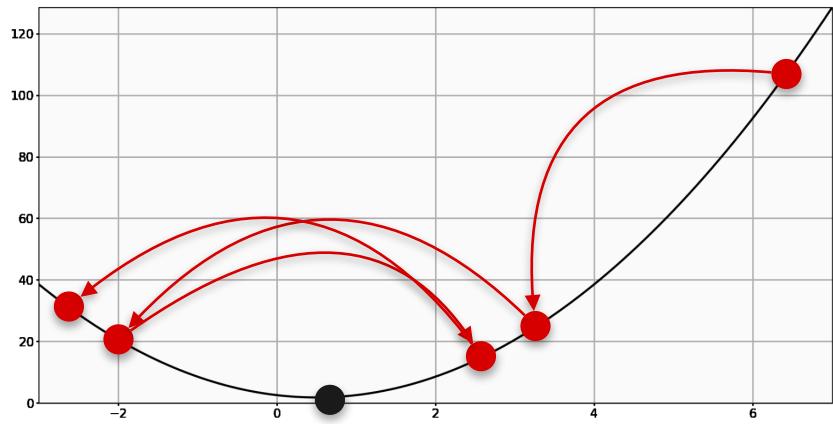


Too small

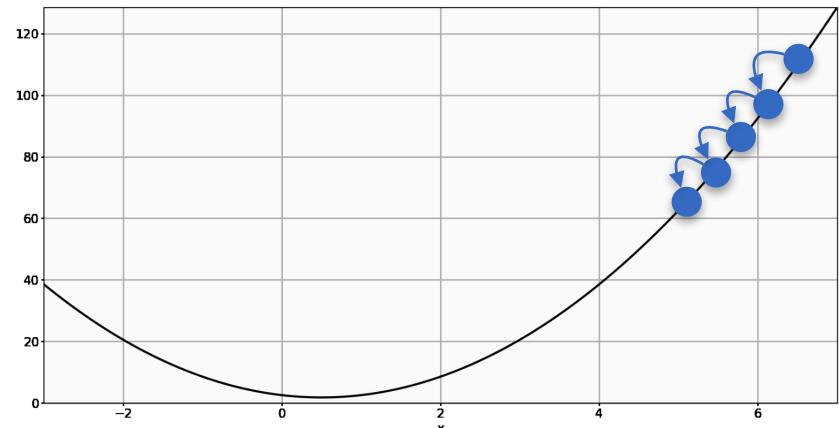


What Is a Good Learning Rate?

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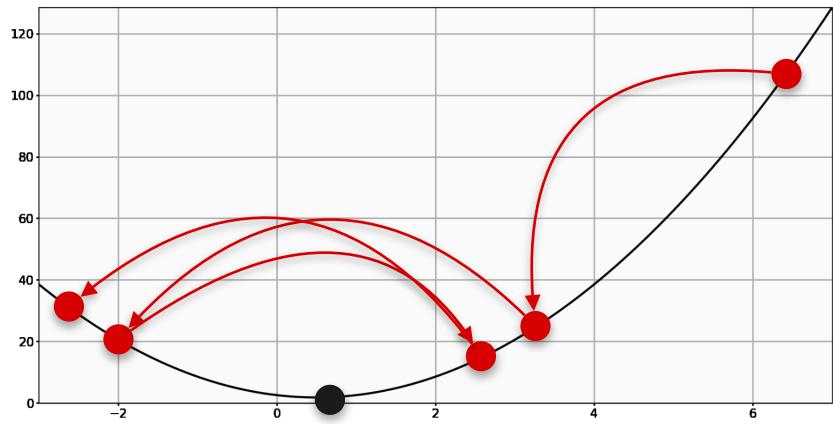


Too small

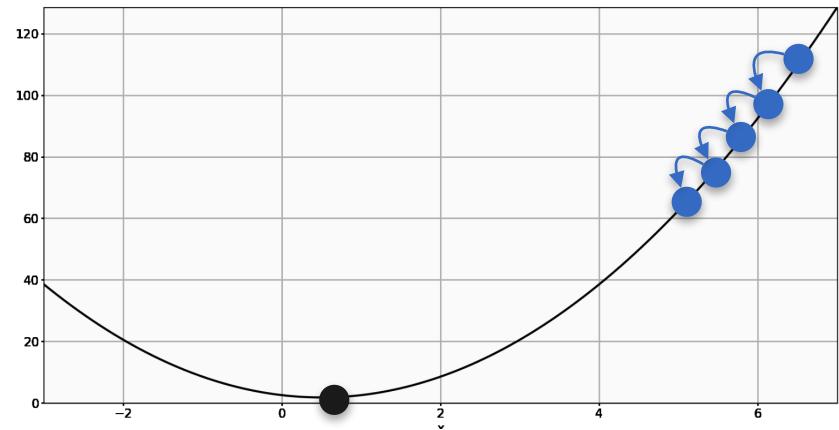


What Is a Good Learning Rate?

Too large

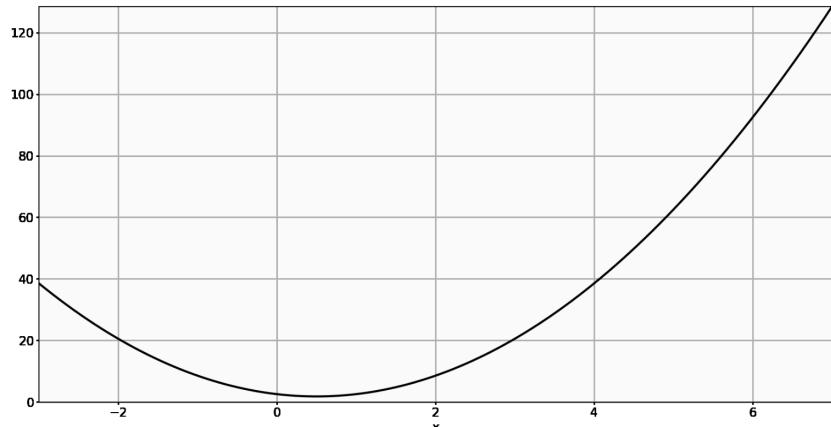


Too small



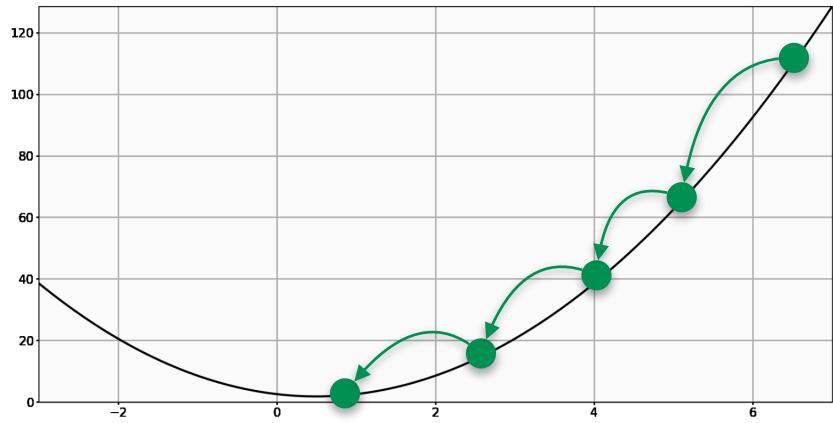
What Is a Good Learning Rate?

Just right



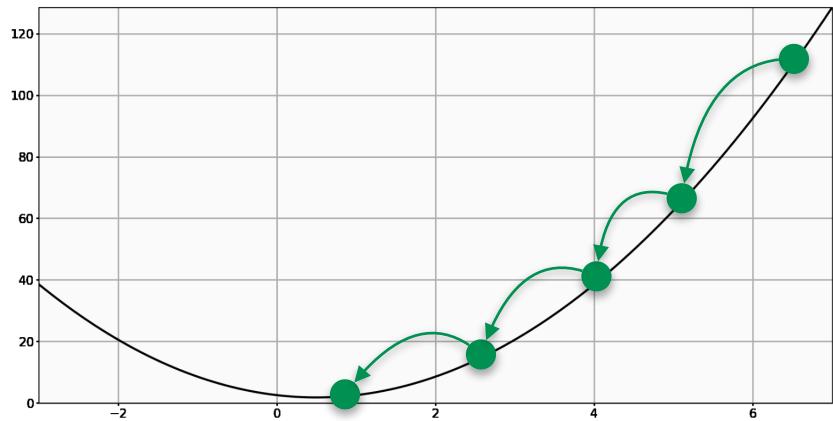
What Is a Good Learning Rate?

Just right

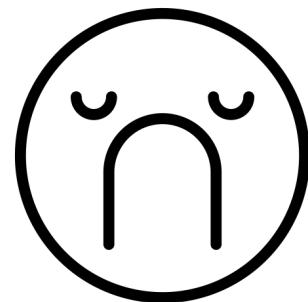


What Is a Good Learning Rate?

Just right

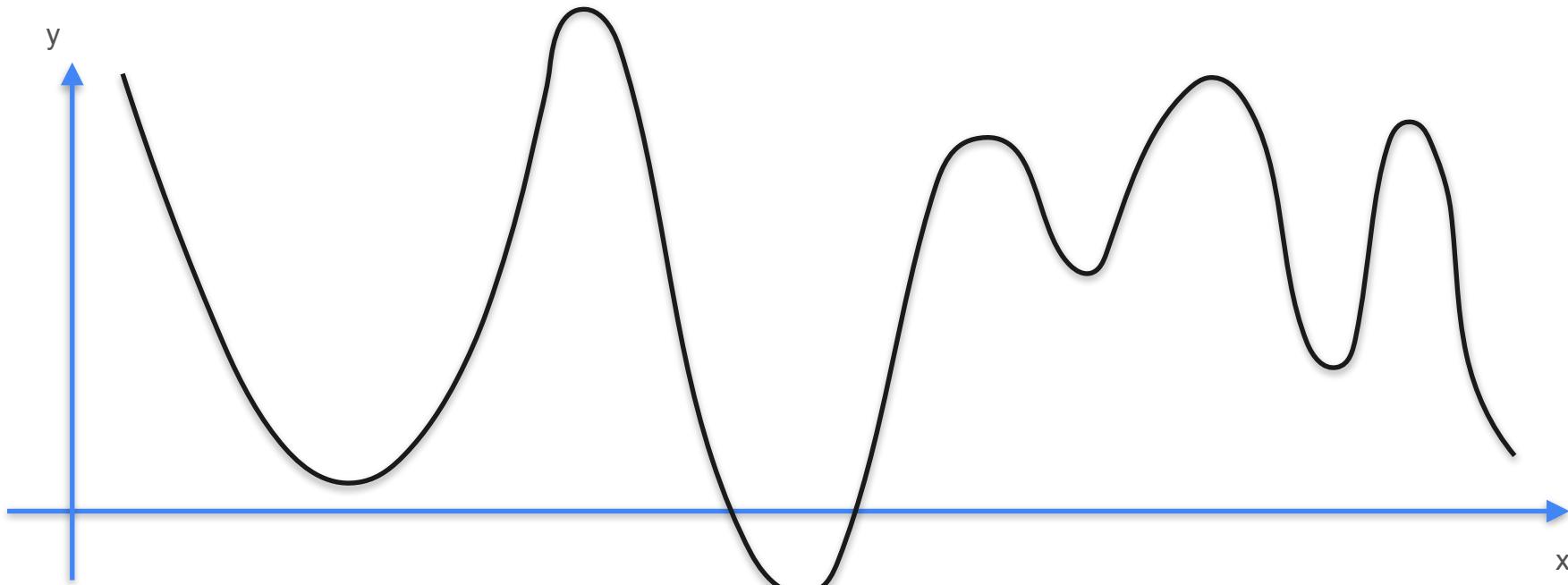


Unfortunately, there is no rule to give the best learning rate α

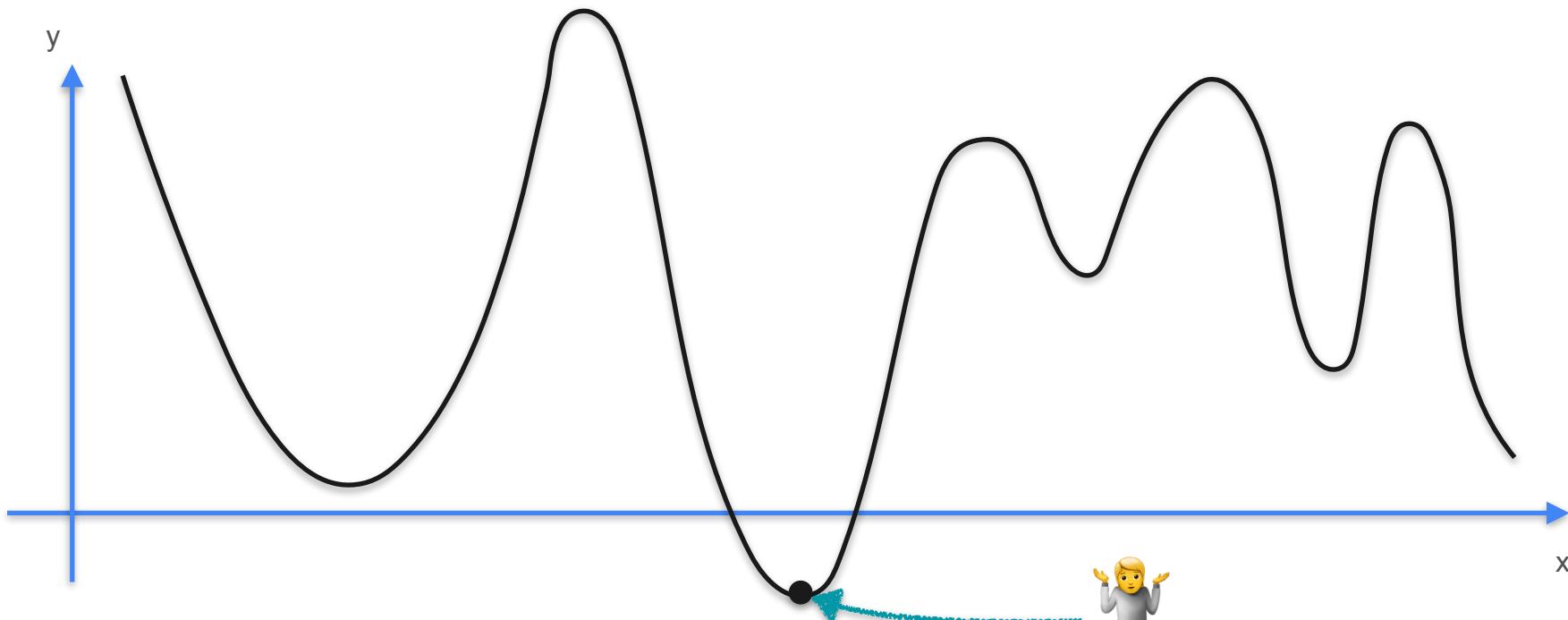


Drawbacks of Gradient Descent

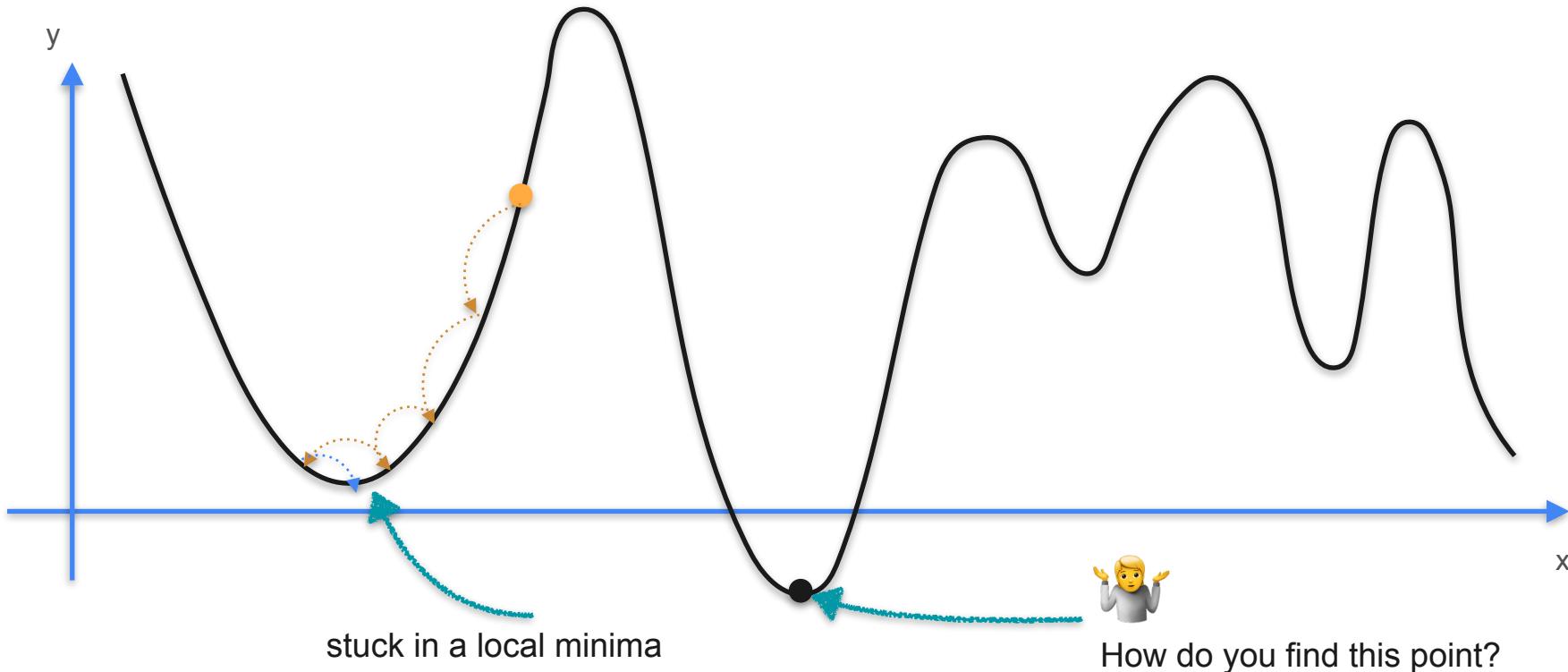
Drawbacks of Gradient Descent



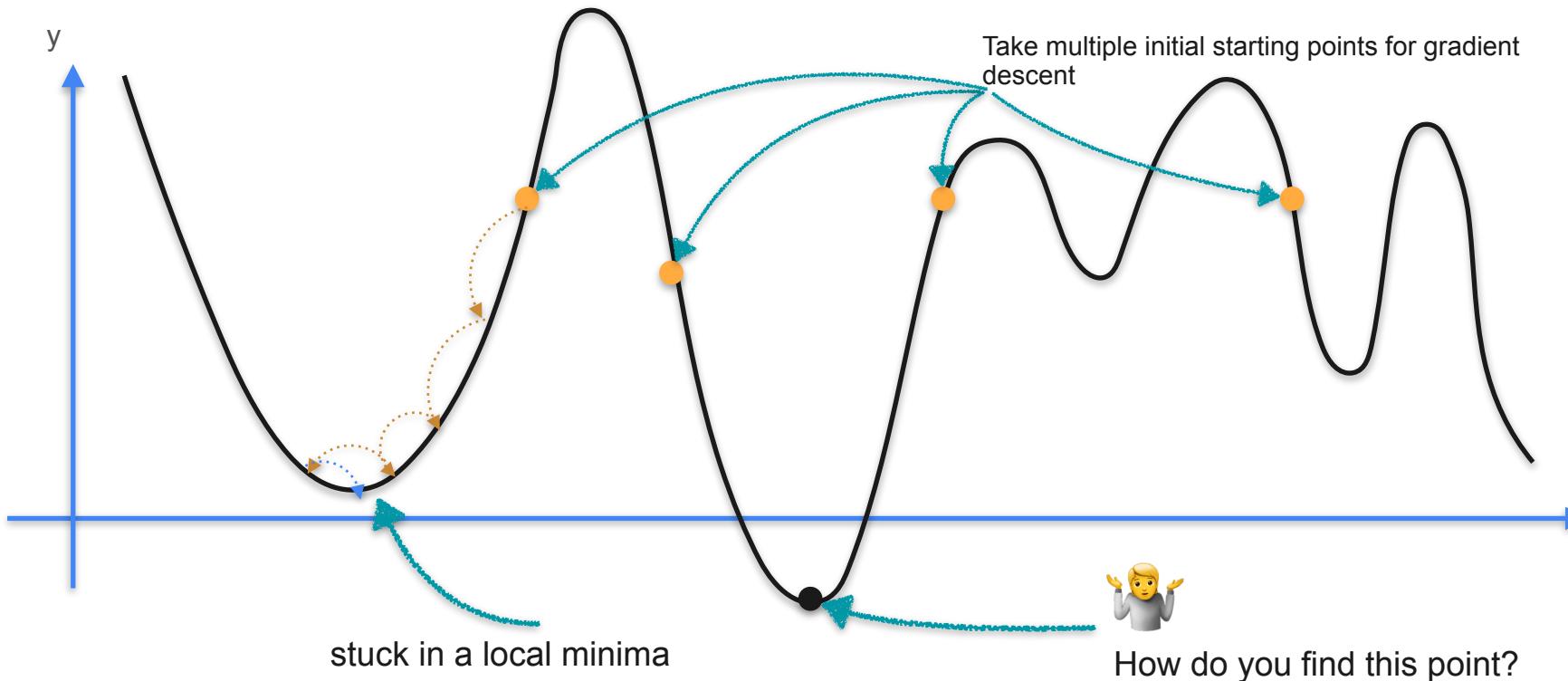
Drawbacks of Gradient Descent



Drawbacks of Gradient Descent



Drawbacks of Gradient Descent



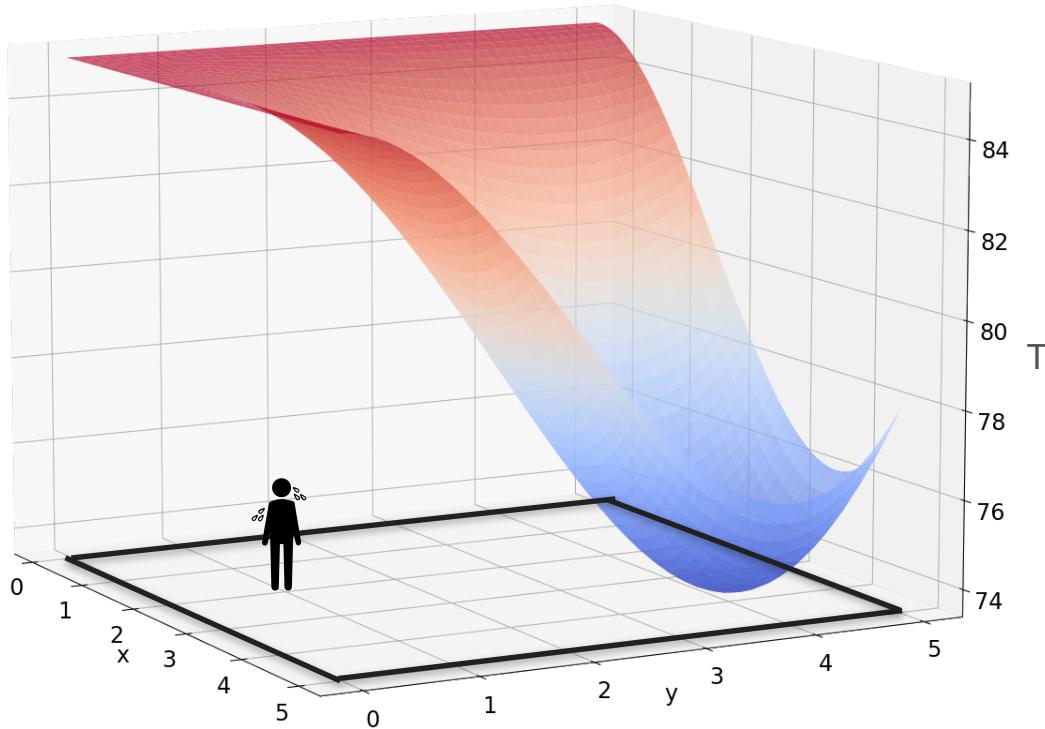


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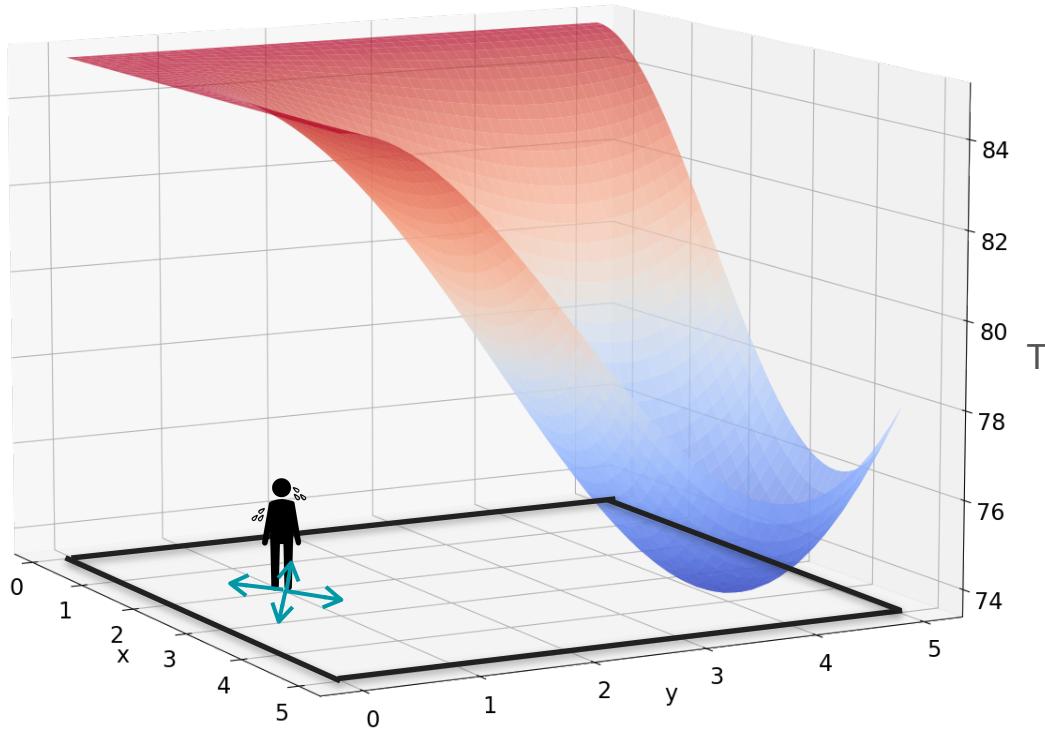
Gradients and Gradient Descent

**Optimization using Gradient
Descent in two variables -
Part 1**

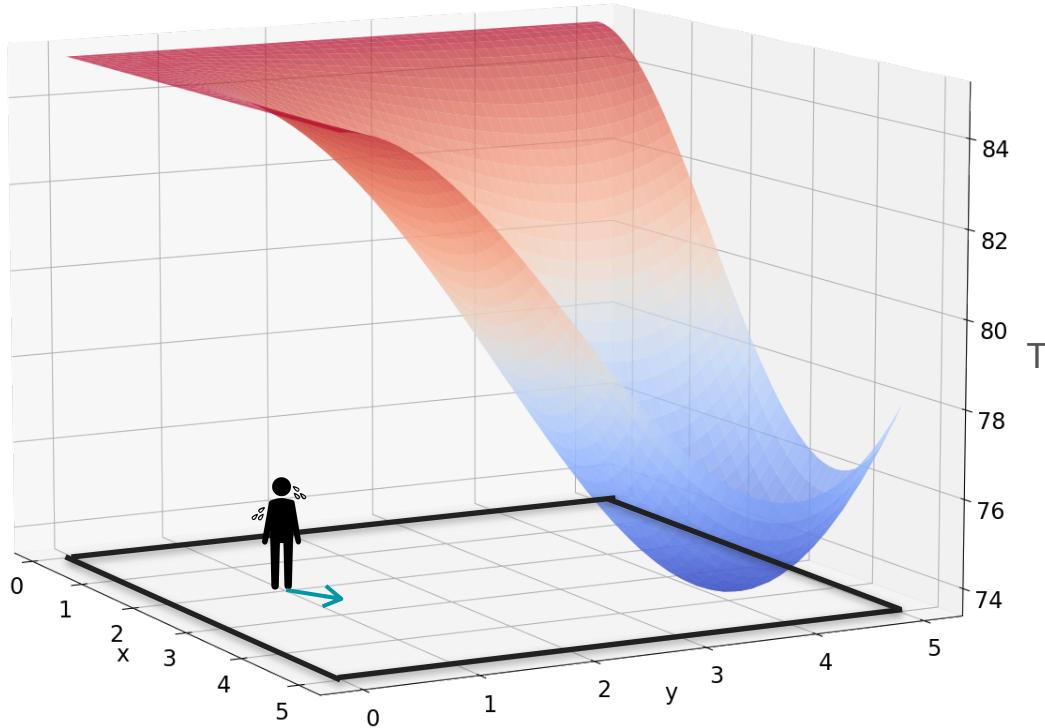
Gradient Descent With Heat Example



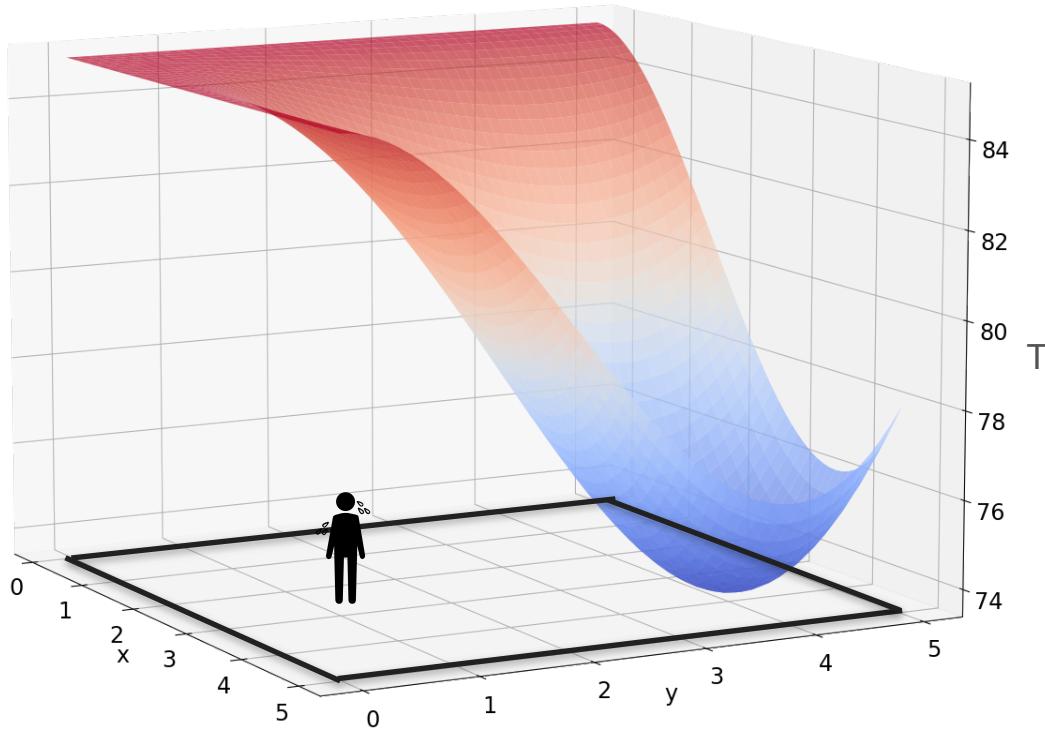
Gradient Descent With Heat Example



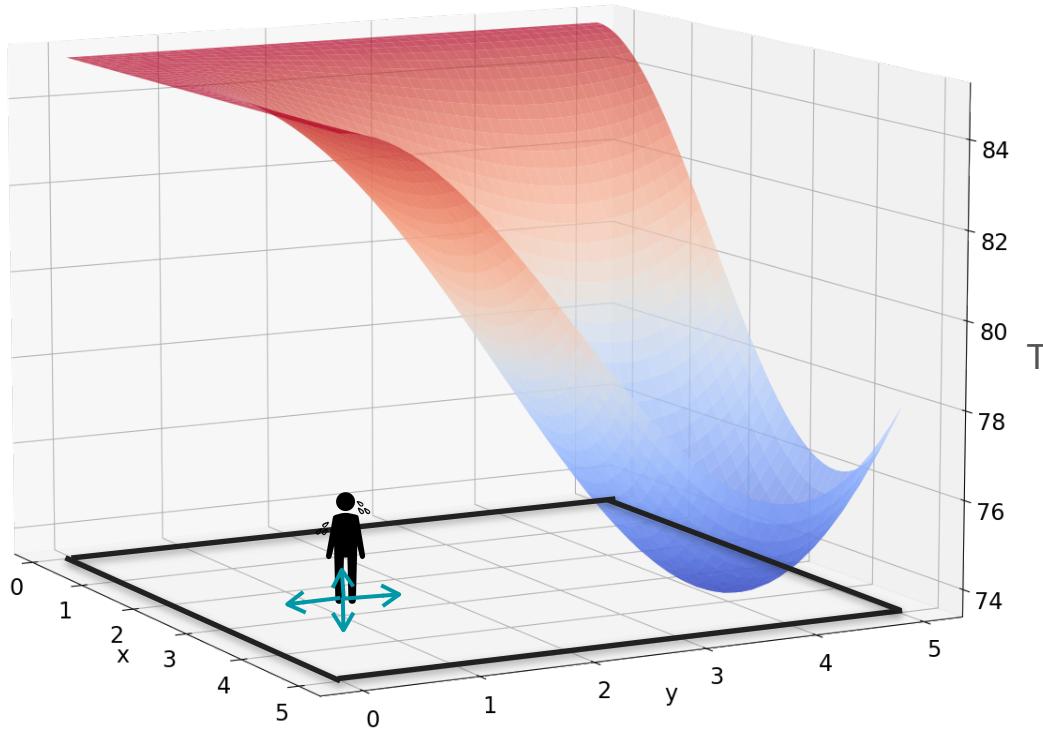
Gradient Descent With Heat Example



Gradient Descent With Heat Example

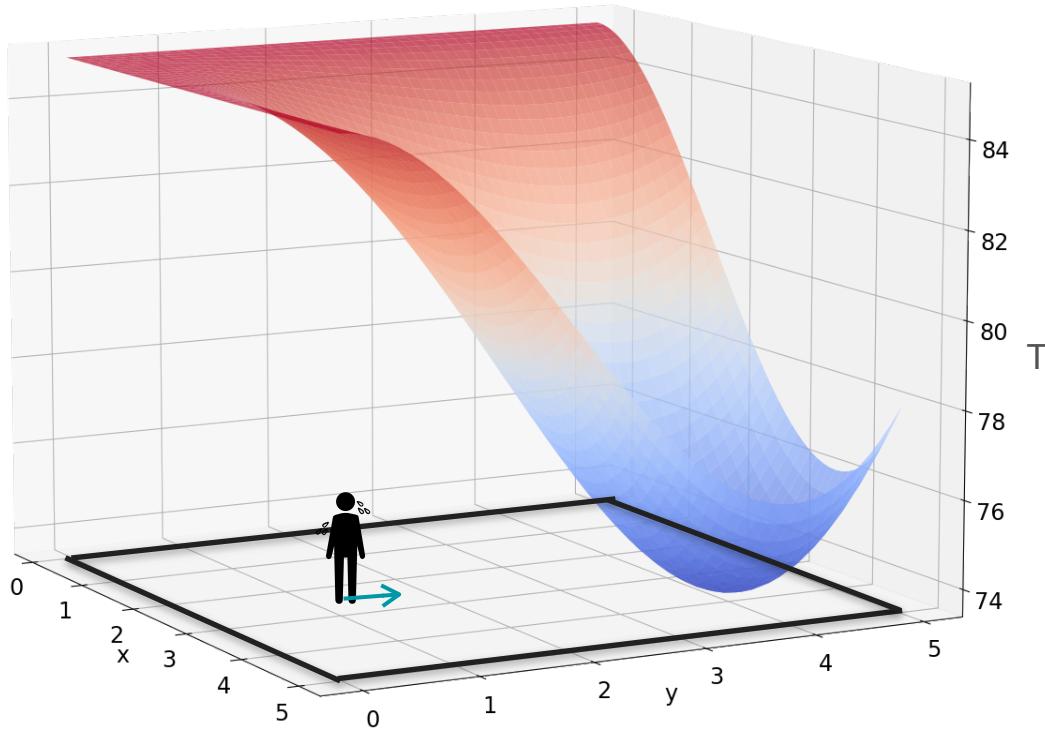


Gradient Descent With Heat Example

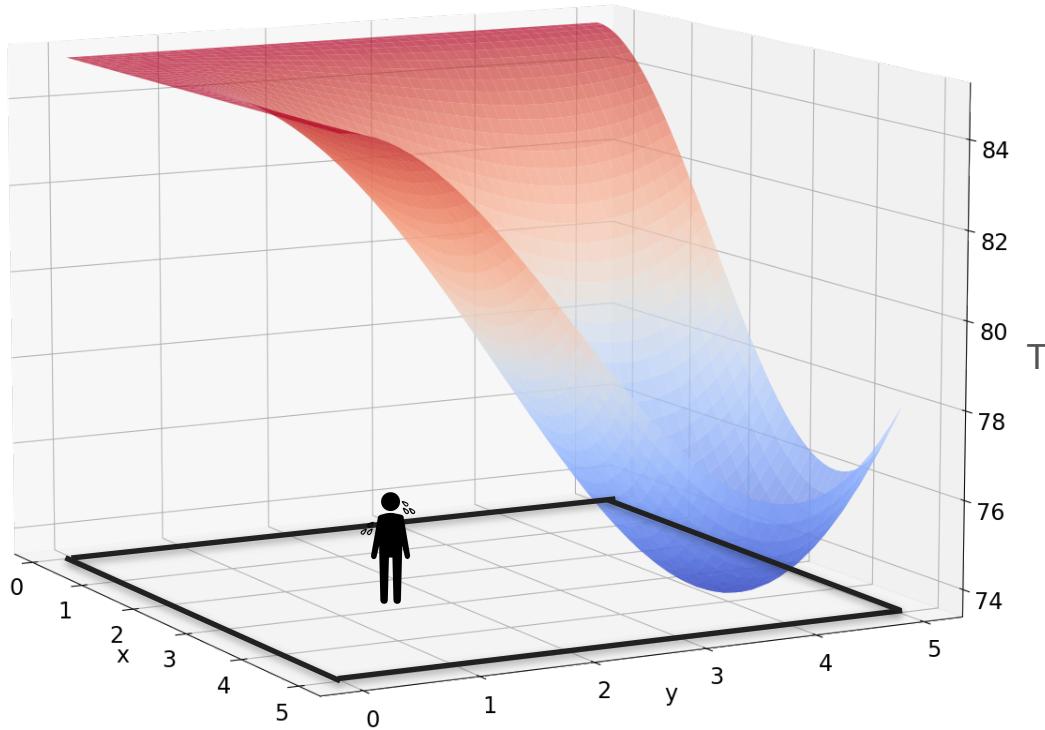


Gradient Descent With Heat Example

Repeat!

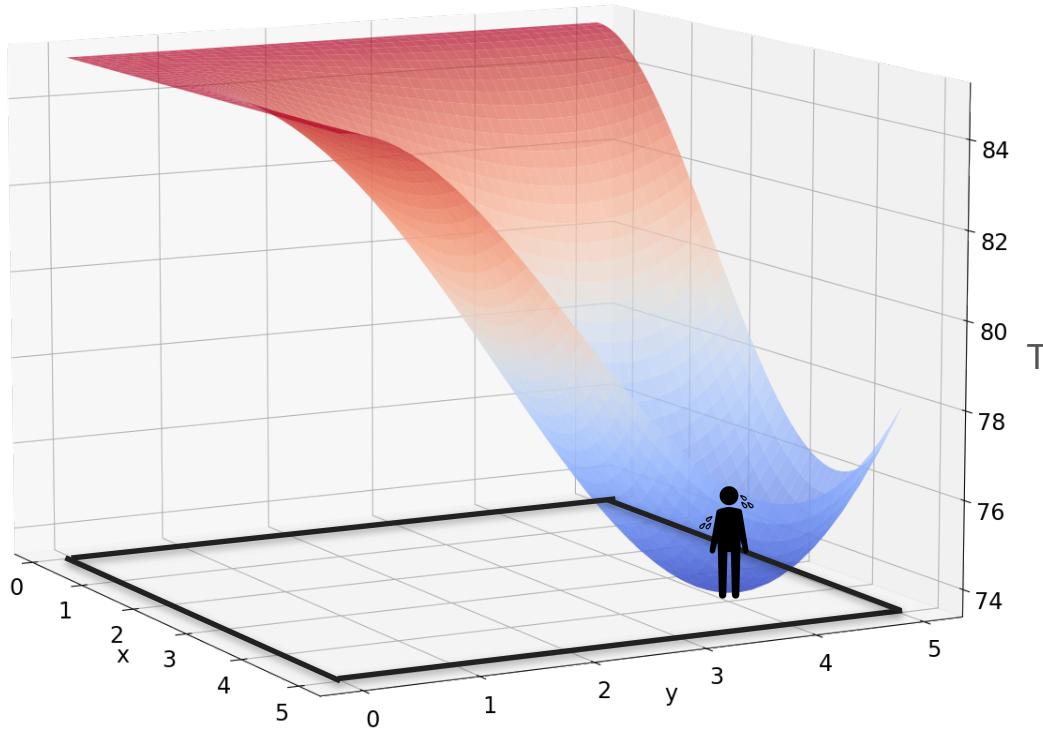


Gradient Descent With Heat Example



Gradient Descent With Heat Example

Repeat!



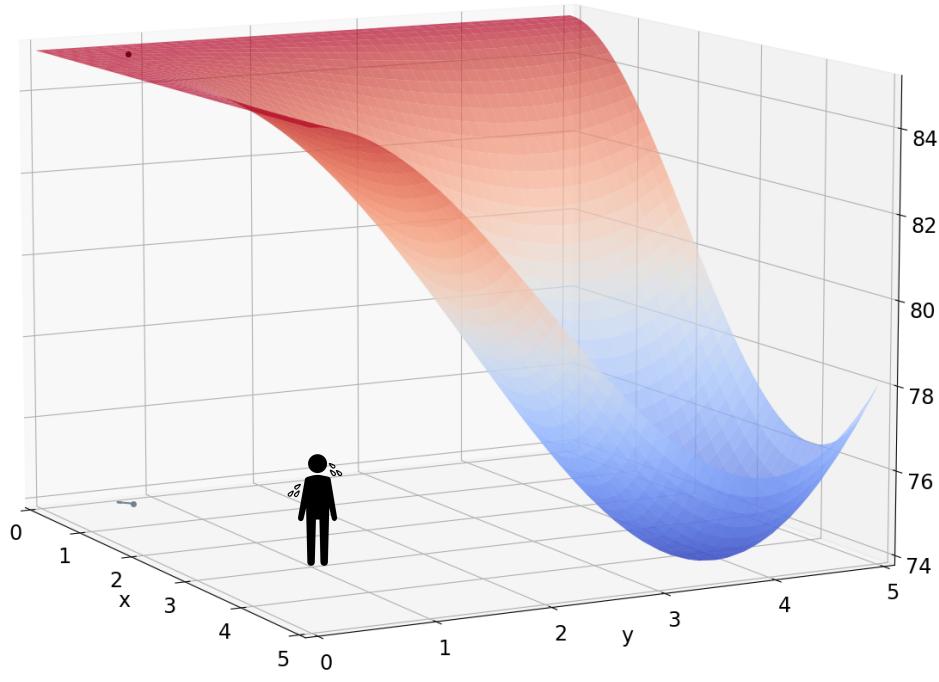


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Gradients and Gradient Descent

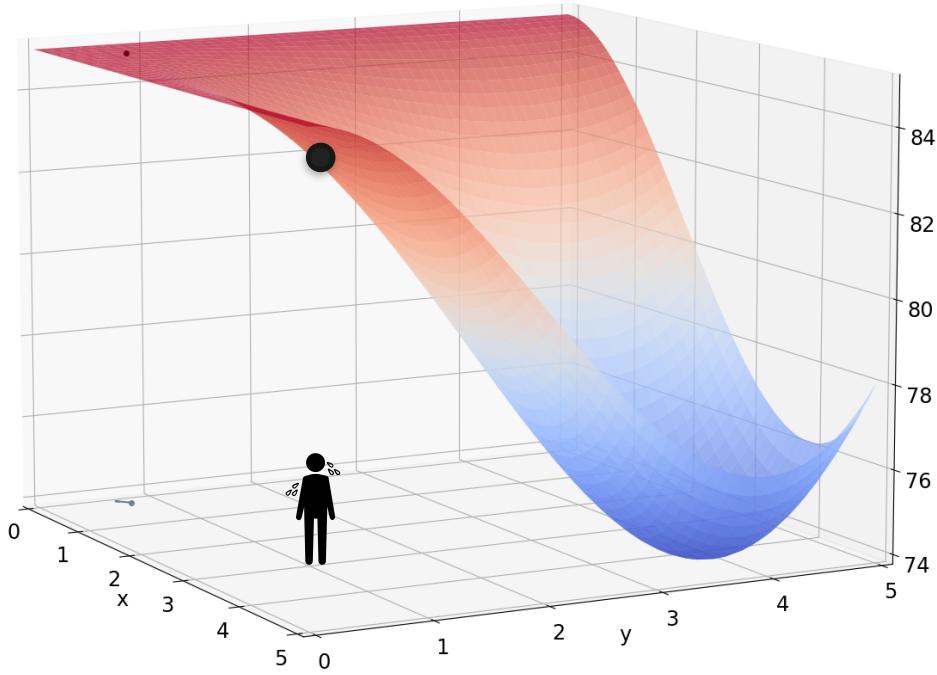
**Optimization using Gradient
Descent in two variables -
Part 2**

Idea for Gradient Descent



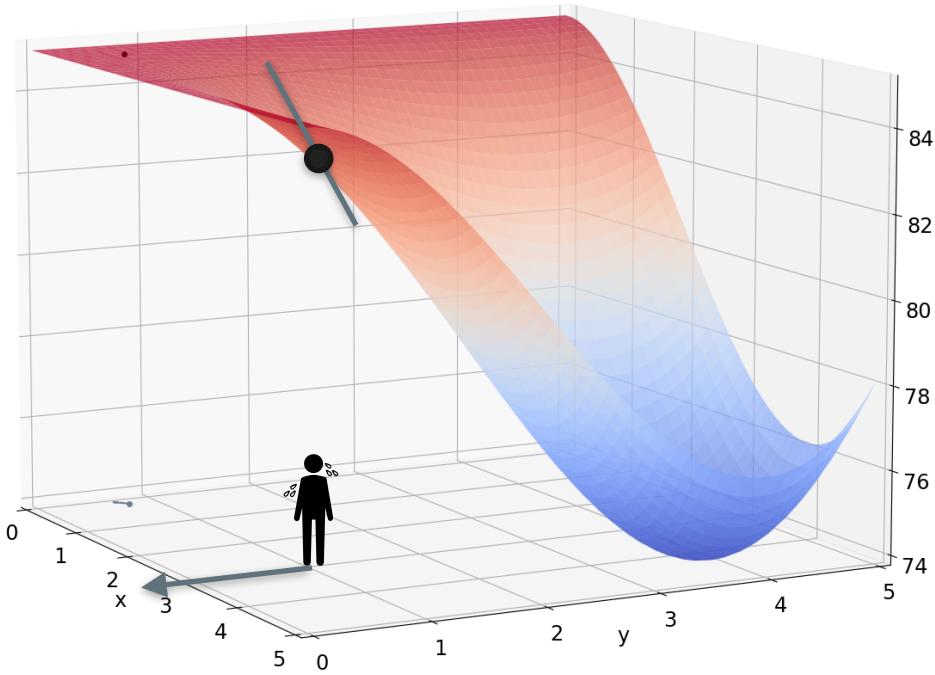
Idea for Gradient Descent

Initial position: (x_0, y_0)



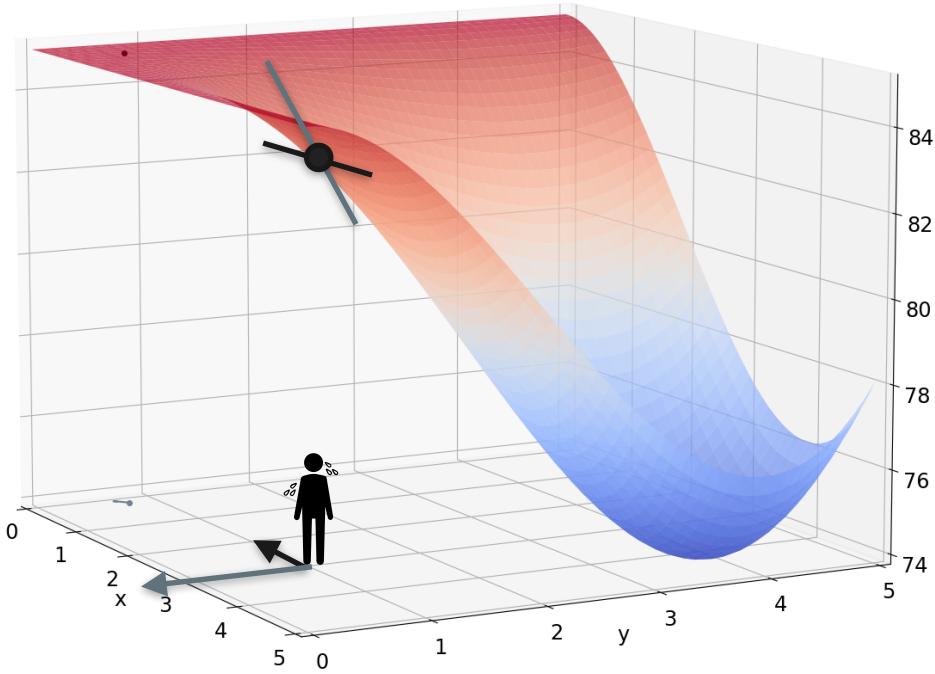
Idea for Gradient Descent

Initial position: (x_0, y_0)



Idea for Gradient Descent

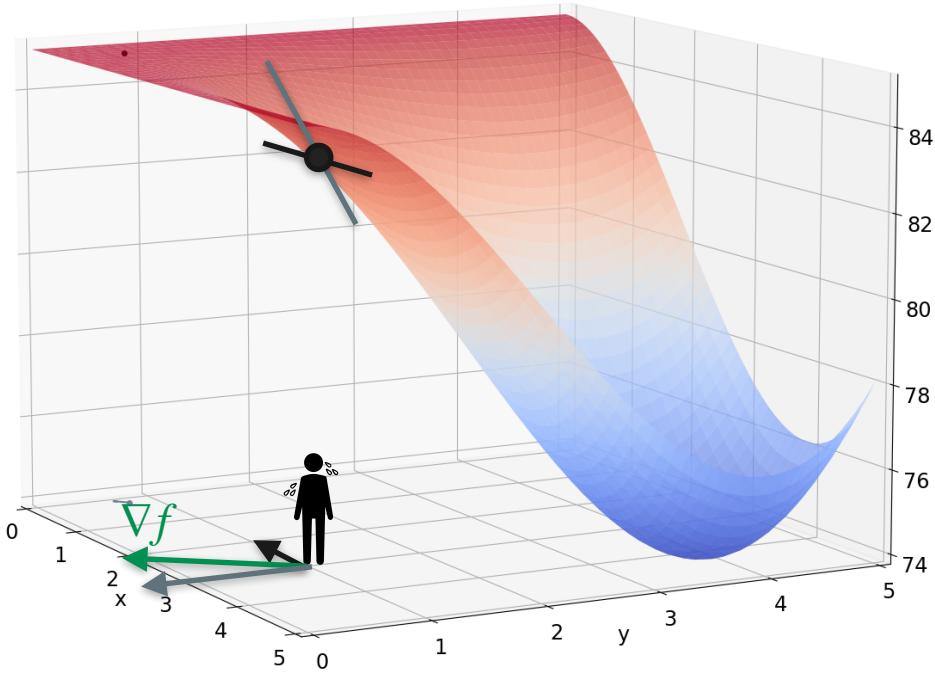
Initial position: (x_0, y_0)



Idea for Gradient Descent

Initial position: (x_0, y_0)

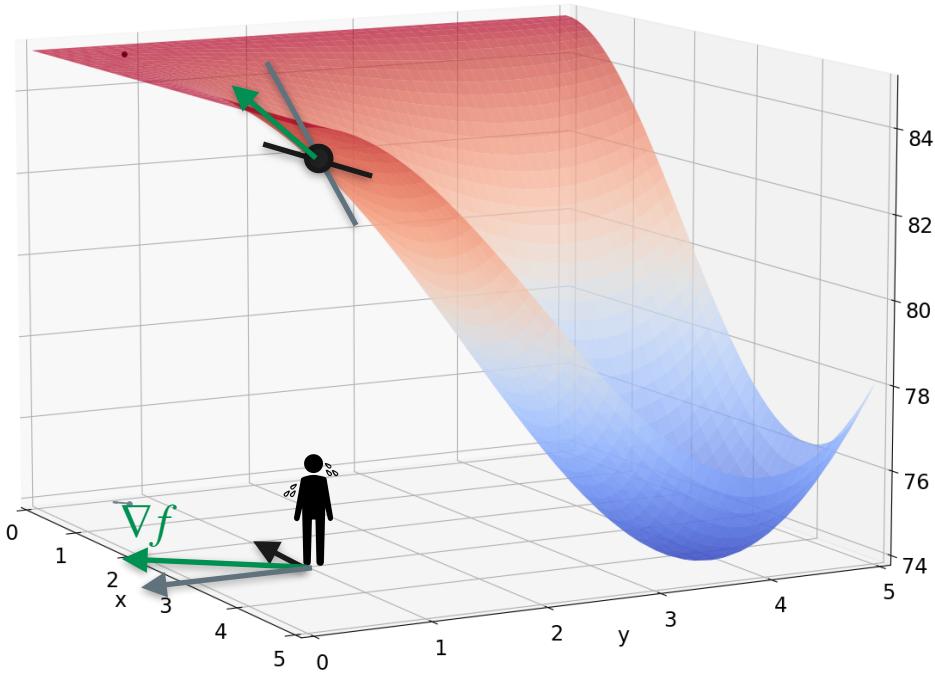
Direction of greatest ascent: ∇f



Idea for Gradient Descent

Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

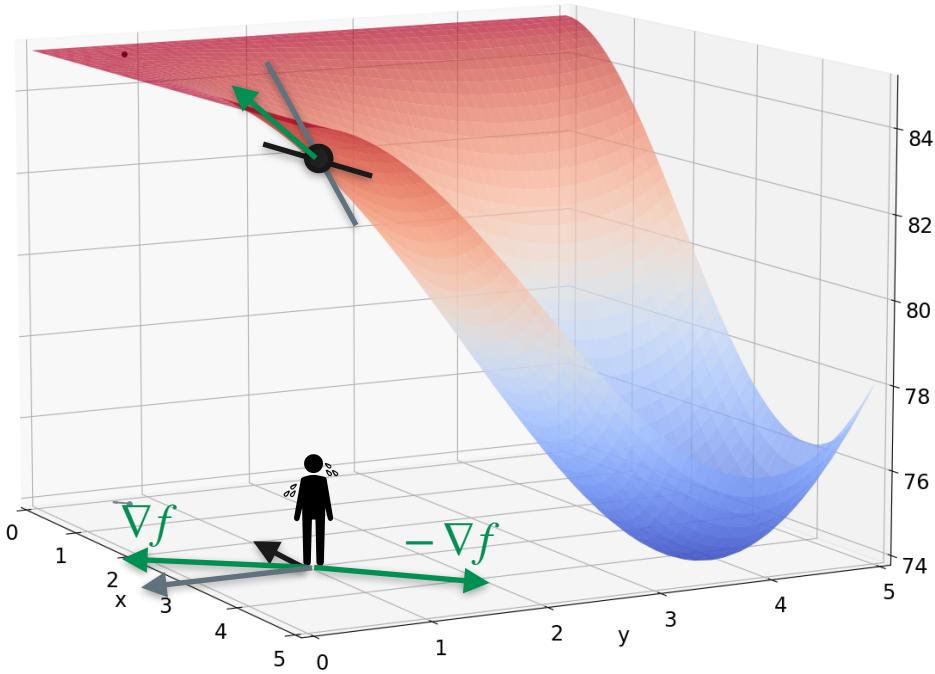


Idea for Gradient Descent

Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

Direction of greatest descent: $-\nabla f$

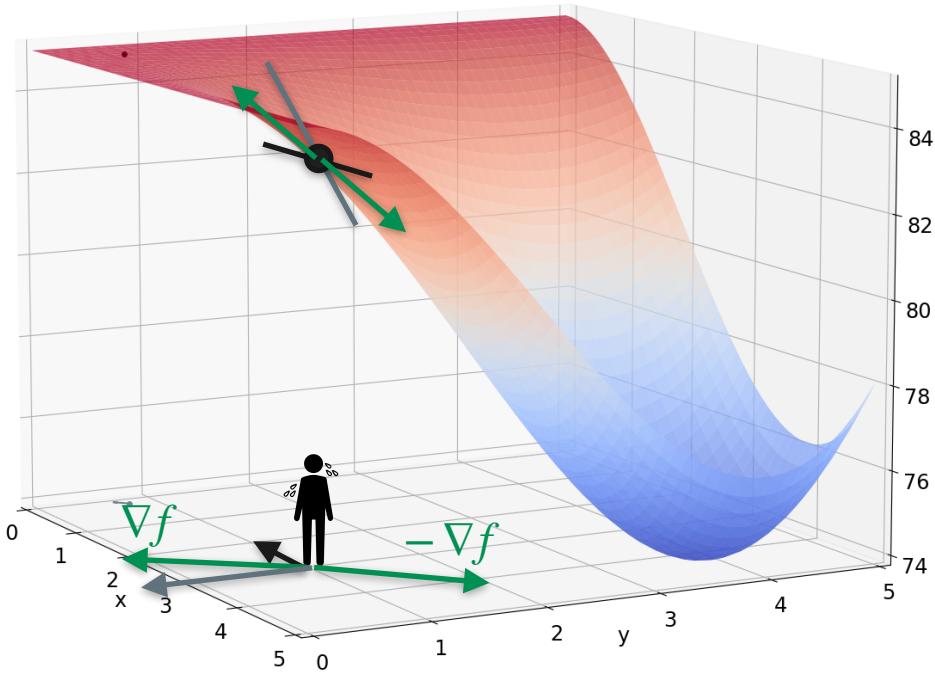


Idea for Gradient Descent

Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

Direction of greatest descent: $-\nabla f$

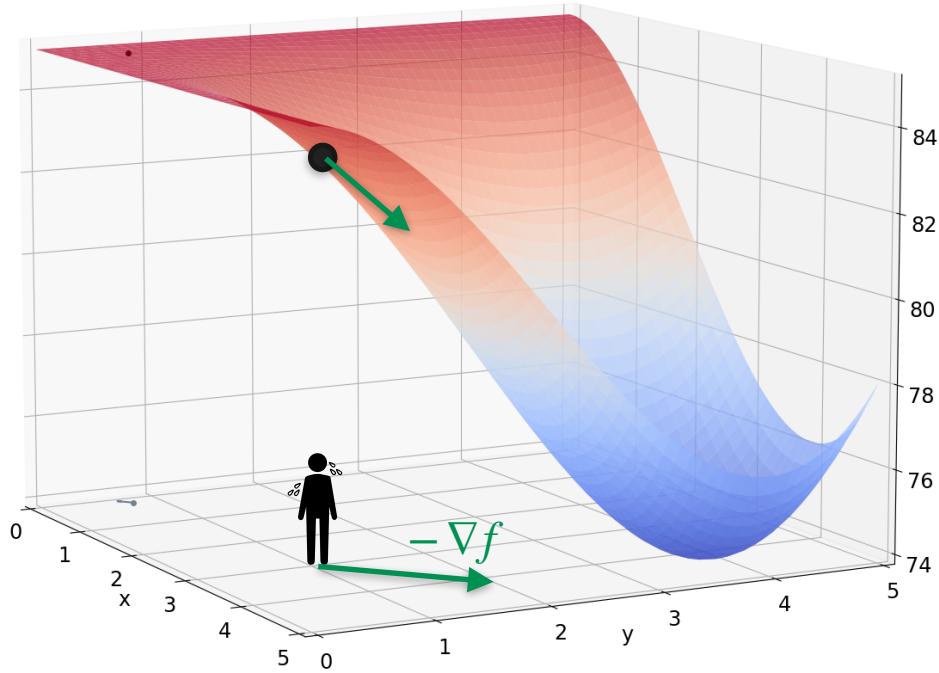


Idea for Gradient Descent

Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

Direction of greatest descent: $-\nabla f$



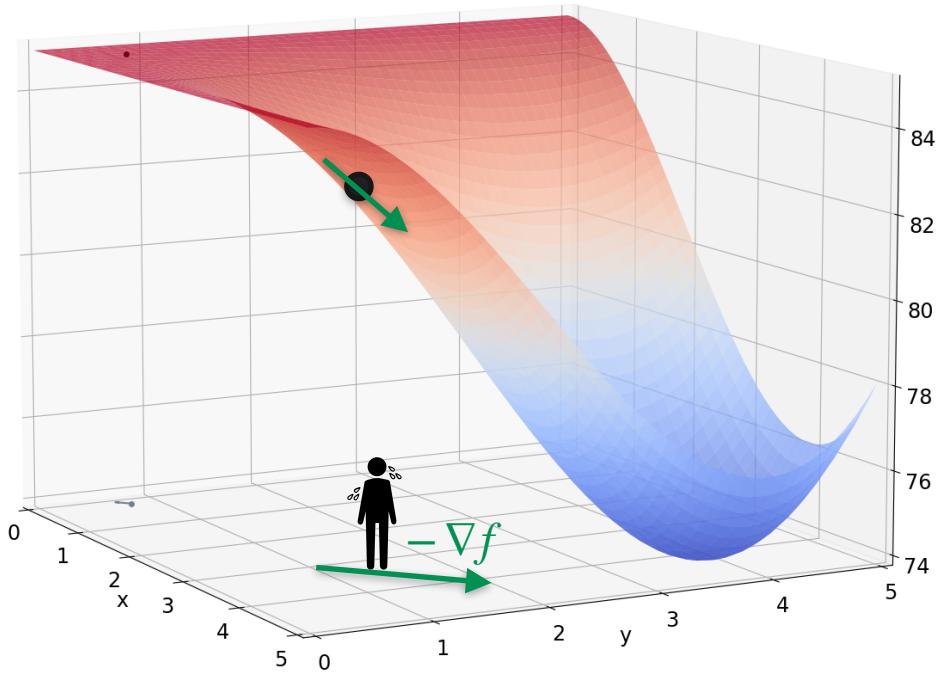
Idea for Gradient Descent

Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

Direction of greatest descent: $-\nabla f$

Updated position: $(x_0, y_0) - \alpha \nabla f$



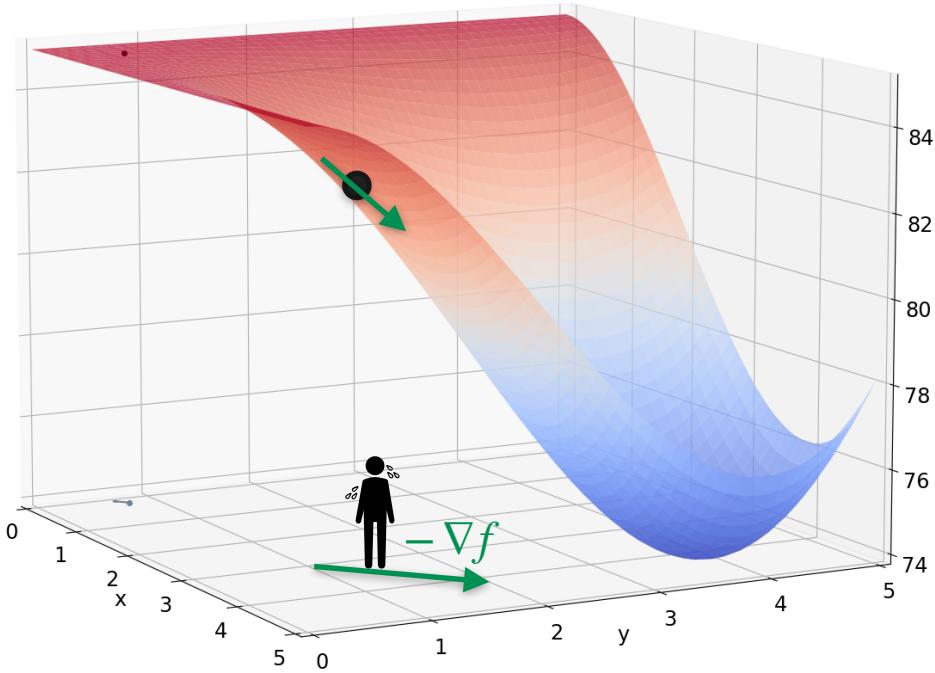
Idea for Gradient Descent

Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

Direction of greatest descent: $-\nabla f$

Updated position: $\underbrace{(x_0, y_0) - \alpha \nabla f}_{(x_1, y_1)}$



Idea for Gradient Descent

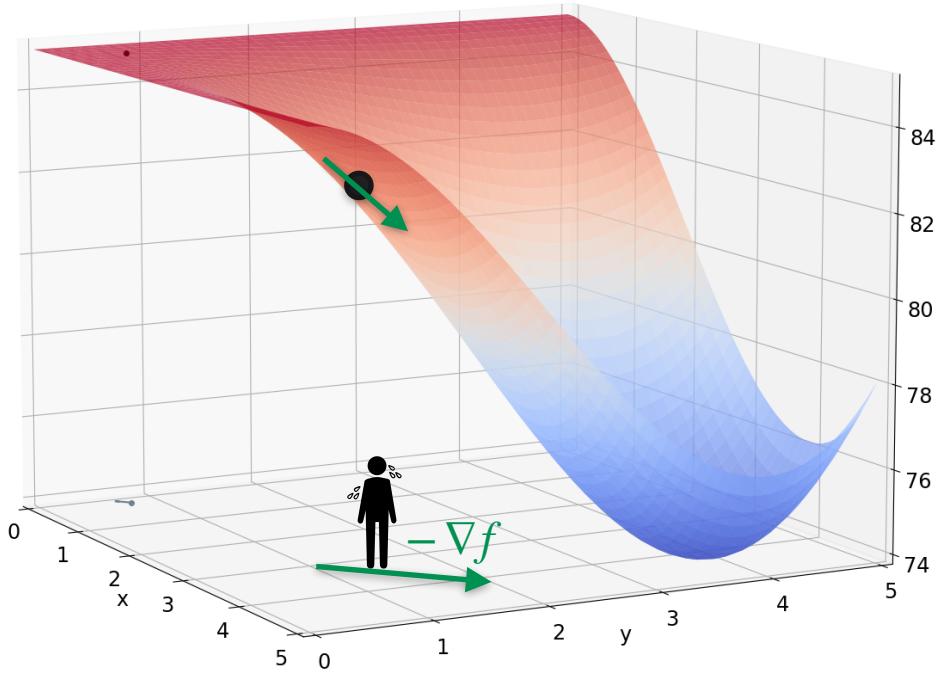
Initial position: (x_0, y_0)

Direction of greatest ascent: ∇f

Direction of greatest descent: $-\nabla f$

Updated position: $\underbrace{(x_0, y_0) - \alpha \nabla f}_{(x_1, y_1)}$

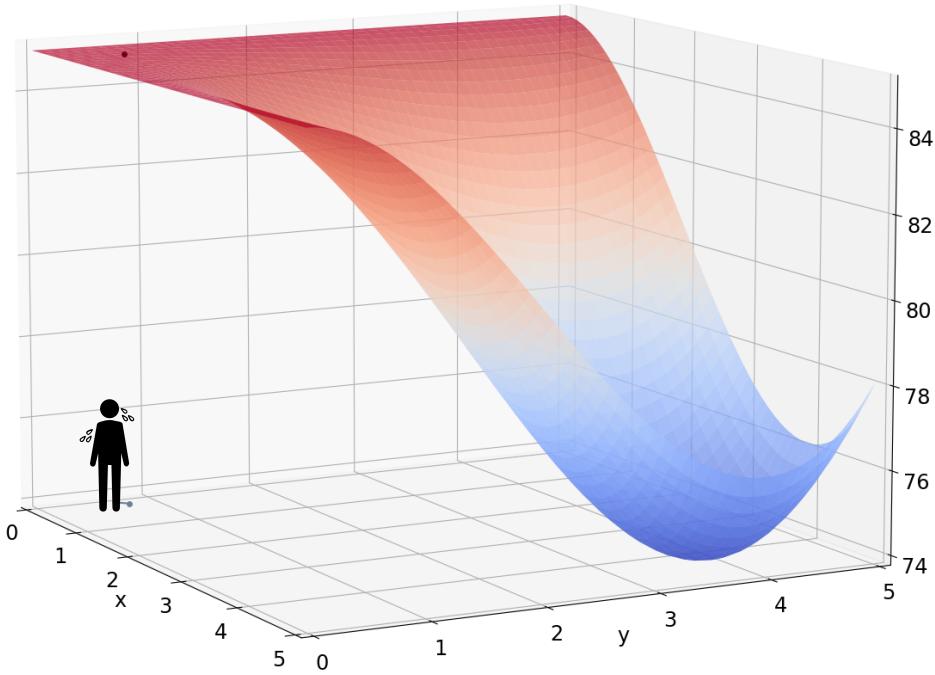
Better point!



Method 2: Gradient Descent

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

Start: $x = 0.5, y = 0.6$

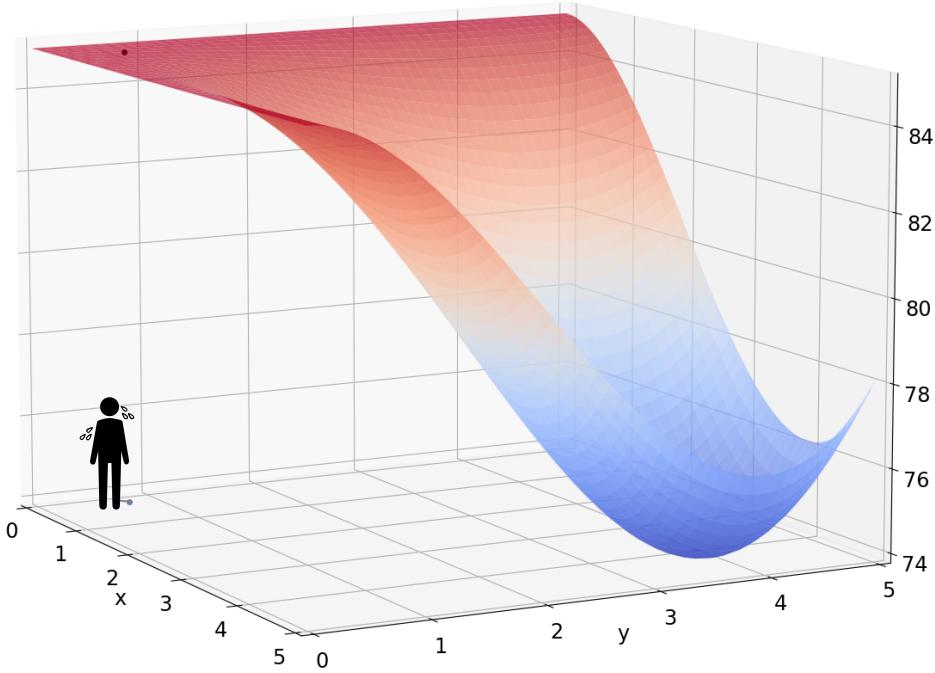


Method 2: Gradient Descent

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

Start: $x = 0.5, y = 0.6$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$



Method 2: Gradient Descent

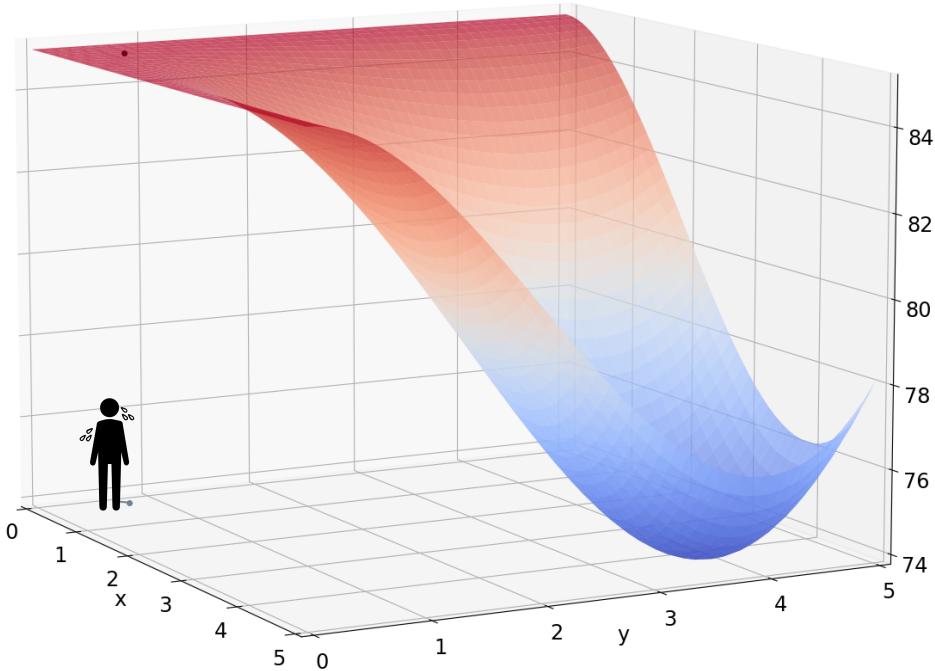
$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

Start: $x = 0.5, y = 0.6$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = -\frac{1}{90}x(3x - 12)y^2(y - 6)$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}x^2(x - 6)y(3y - 12)$$



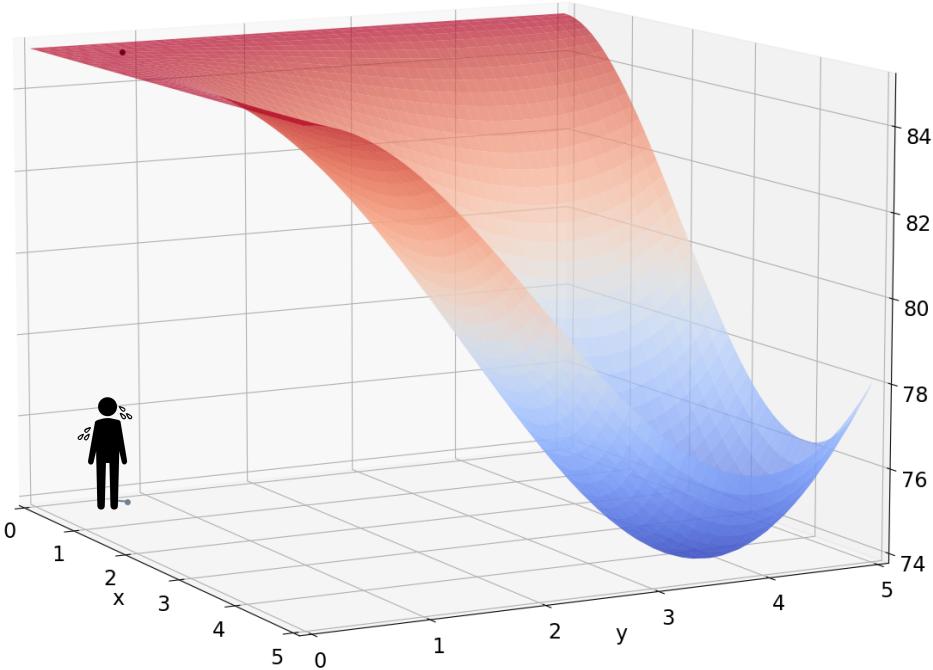
Method 2: Gradient Descent

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

Start: $x = 0.5, y = 0.6$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla f = \begin{bmatrix} -\frac{1}{90}x(3x - 12)y^2(y - 6) \\ -\frac{1}{90}x^2(x - 6)y(3y - 12) \end{bmatrix}$$



Method 2: Gradient Descent

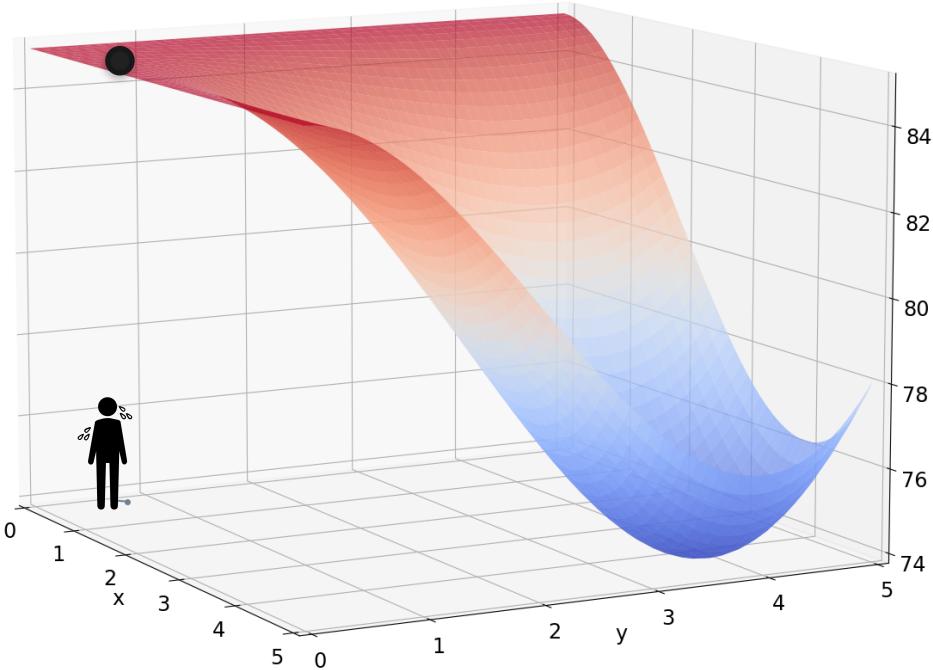
$$T = f(x, y) = 85 - \frac{1}{90}x^2(x - 6)y^2(y - 6)$$

Start: $x = 0.5, y = 0.6$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla f = \begin{bmatrix} -\frac{1}{90}x(3x - 12)y^2(y - 6) \\ -\frac{1}{90}x^2(x - 6)y(3y - 12) \end{bmatrix}$$

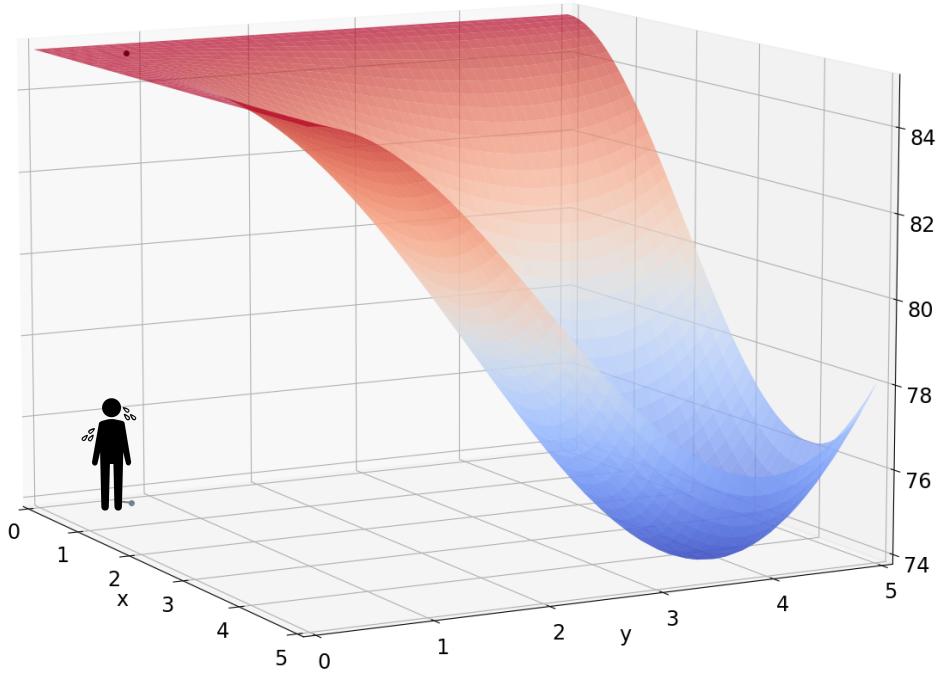
$$\nabla f(0.5, 0.6) = \begin{bmatrix} 0.1134 \\ -0.0935 \end{bmatrix}$$



Method 2: Gradient Descent

Start: $x = 0.5, y = 0.6$

$$\nabla f(0.5, 0.6) = \begin{bmatrix} 0.1134 \\ -0.0935 \end{bmatrix}$$

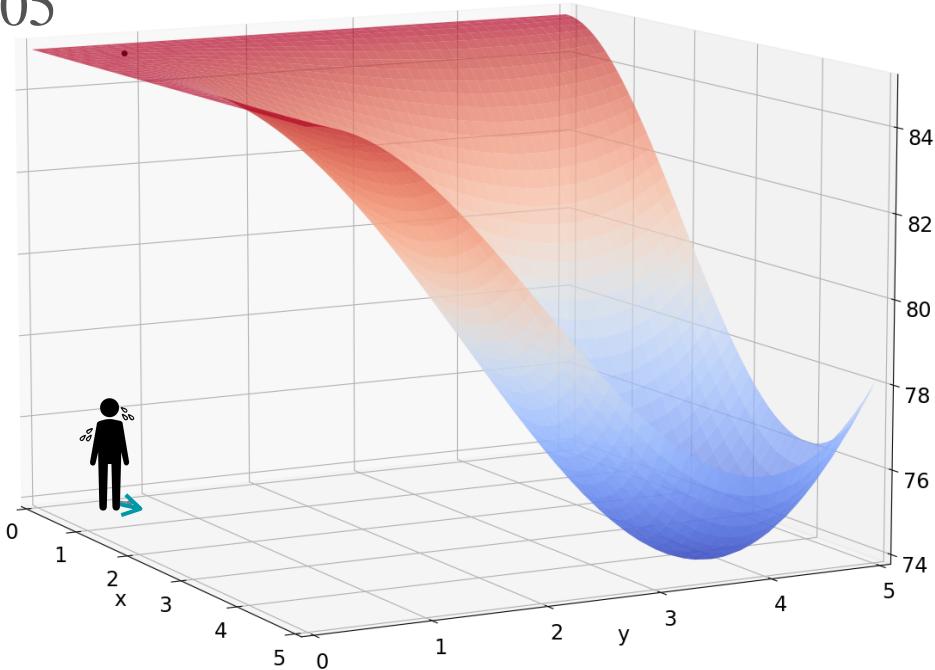


Method 2: Gradient Descent

Start: $x = 0.5, y = 0.6$ Rate: $\alpha = 0.05$

$$\nabla f(0.5, 0.6) = \begin{bmatrix} 0.1134 \\ -0.0935 \end{bmatrix}$$

Move by
 $-0.05 \nabla f(0.5, 0.6)$



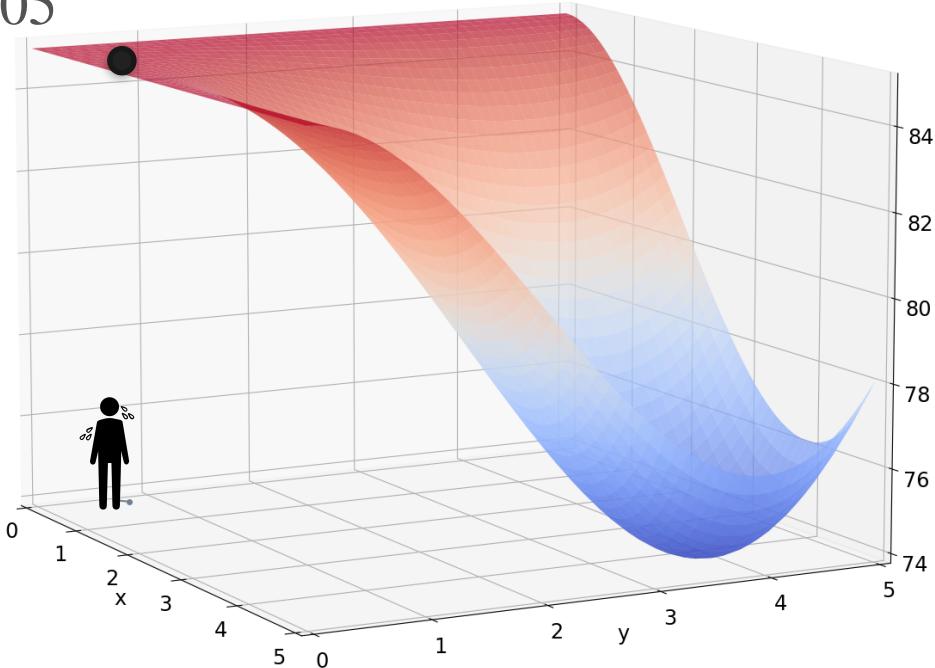
Method 2: Gradient Descent

Start: $x = 0.5$, $y = 0.6$ Rate: $\alpha = 0.05$

$$\nabla f(0.5, 0.6) = \begin{bmatrix} 0.1134 \\ -0.0935 \end{bmatrix}$$

Move by
 $-0.05 \nabla f(0.5, 0.6)$

$$\begin{aligned} x &\mapsto 0.5057 \\ y &\mapsto 0.6047 \end{aligned}$$



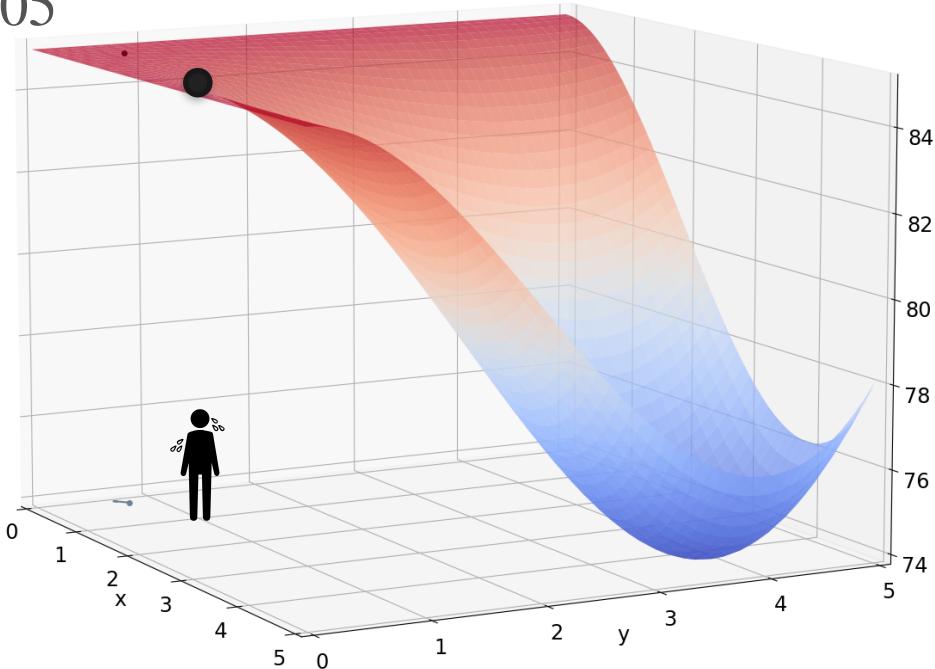
Method 2: Gradient Descent

Start: $x = 0.5$, $y = 0.6$ Rate: $\alpha = 0.05$

$$\nabla f(0.5, 0.6) = \begin{bmatrix} 0.1134 \\ -0.0935 \end{bmatrix}$$

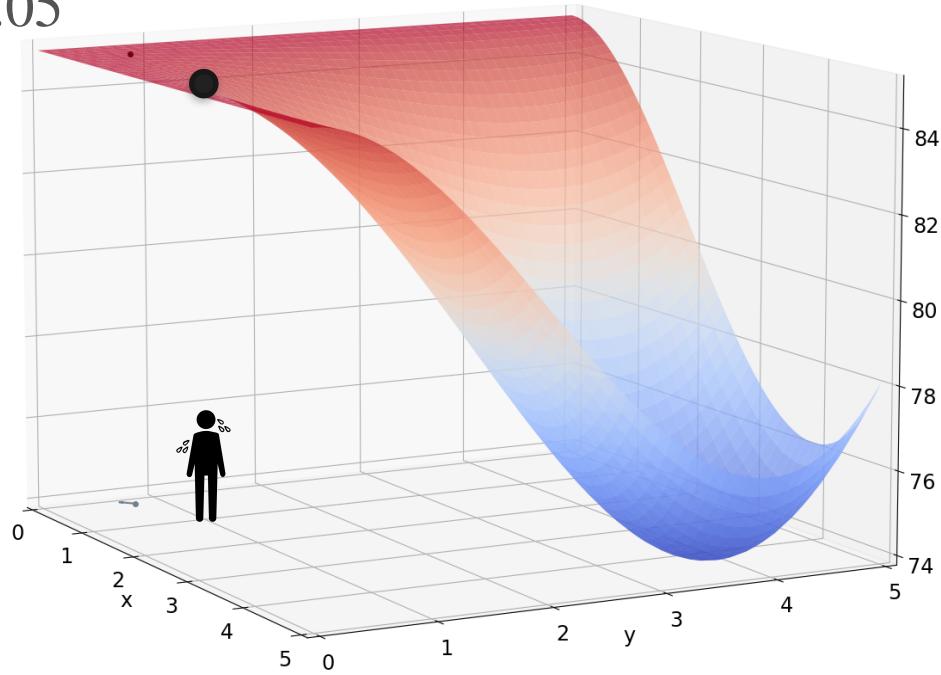
Move by
 $-0.05 \nabla f(0.5, 0.6)$

$$\begin{aligned} x &\mapsto 0.5057 \\ y &\mapsto 0.6047 \end{aligned}$$



Method 2

Start: $x = 0.5, y = 0.6$ Rate: $\alpha = 0.05$



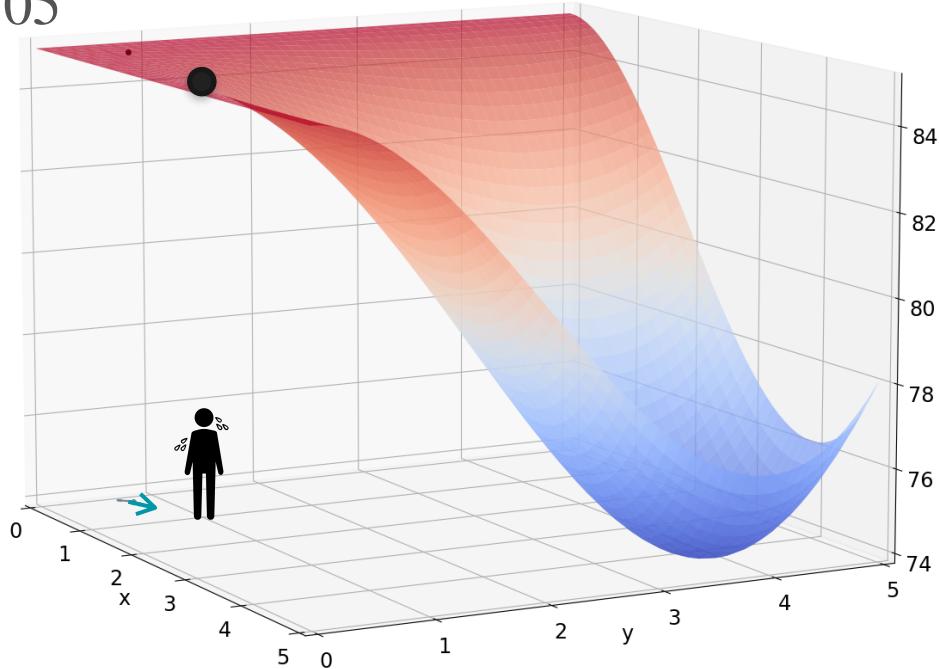
Method 2

Start: $x = 0.5, y = 0.6$ Rate: $\alpha = 0.05$

Find:

$$\nabla f(0.5057, 0.6047) = \begin{bmatrix} -0.1162 \\ -0.0961 \end{bmatrix}$$

Move by
 $-0.05 \nabla f(0.5057, 0.6047)$



Method 2

Start: $x = 0.5$, $y = 0.6$ Rate: $\alpha = 0.05$

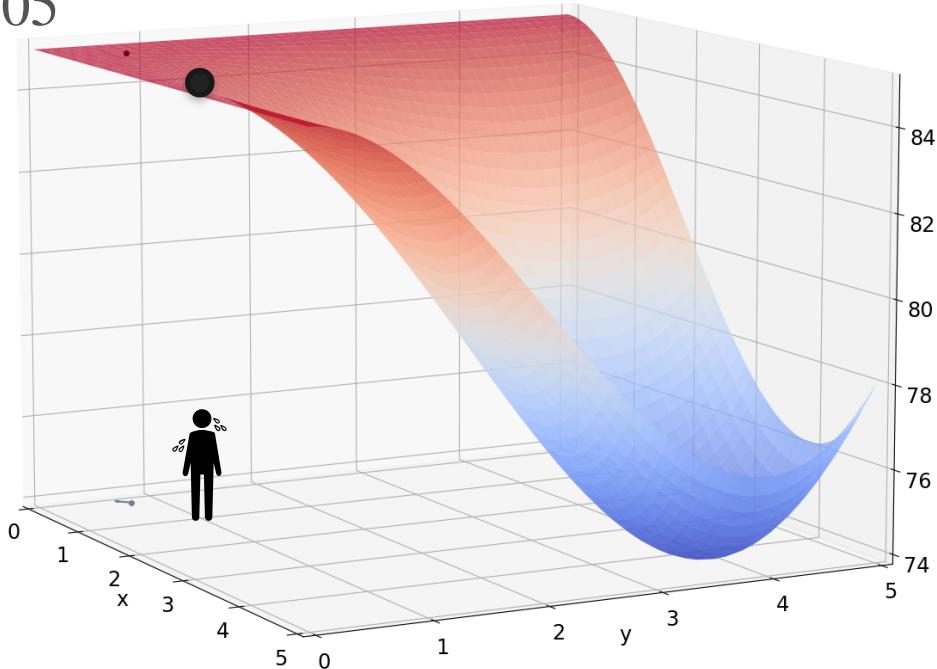
Find:

$$\nabla f(0.5057, 0.6047) = \begin{bmatrix} -0.1162 \\ -0.0961 \end{bmatrix}$$

Move by
 $-0.05 \nabla f(0.5057, 0.6047)$

$$\begin{aligned} x &\mapsto 0.5115 \\ y &\mapsto 0.6095 \end{aligned}$$

Repeat!



Gradient Descent

Start: $x = 0.5$, $y = 0.6$ Rate: $\alpha = 0.05$

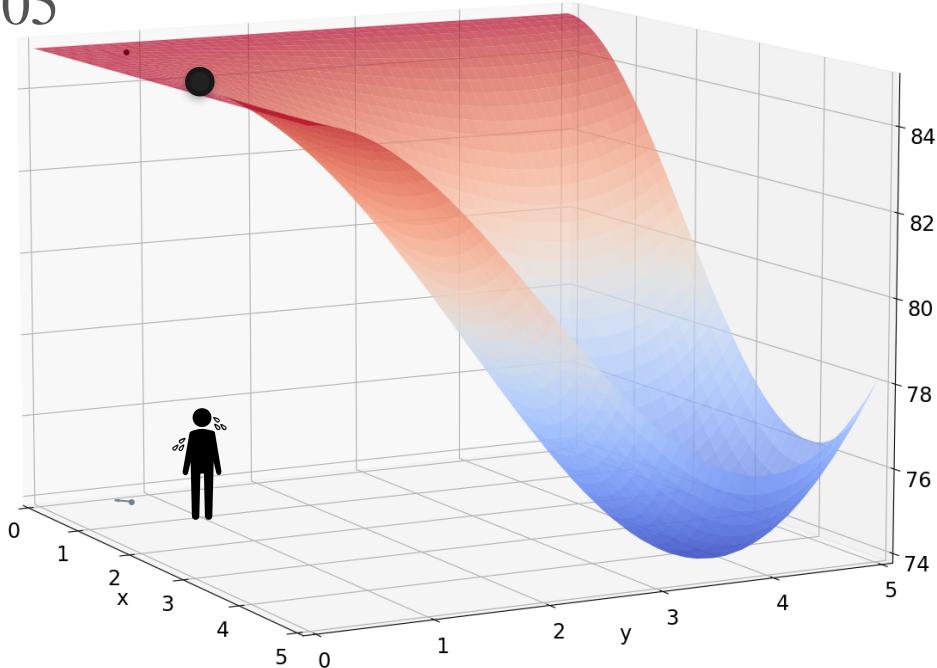
Find:

$$\nabla f(0.5057, 0.6047) = \begin{bmatrix} -0.1162 \\ -0.0961 \end{bmatrix}$$

Move by
 $-0.05 \nabla f(0.5057, 0.6047)$

$$\begin{aligned} x &\mapsto 0.5115 \\ y &\mapsto 0.6095 \end{aligned}$$

Repeat!



Gradient Descent

Start: $x = 0.5$, $y = 0.6$ Rate: $\alpha = 0.05$

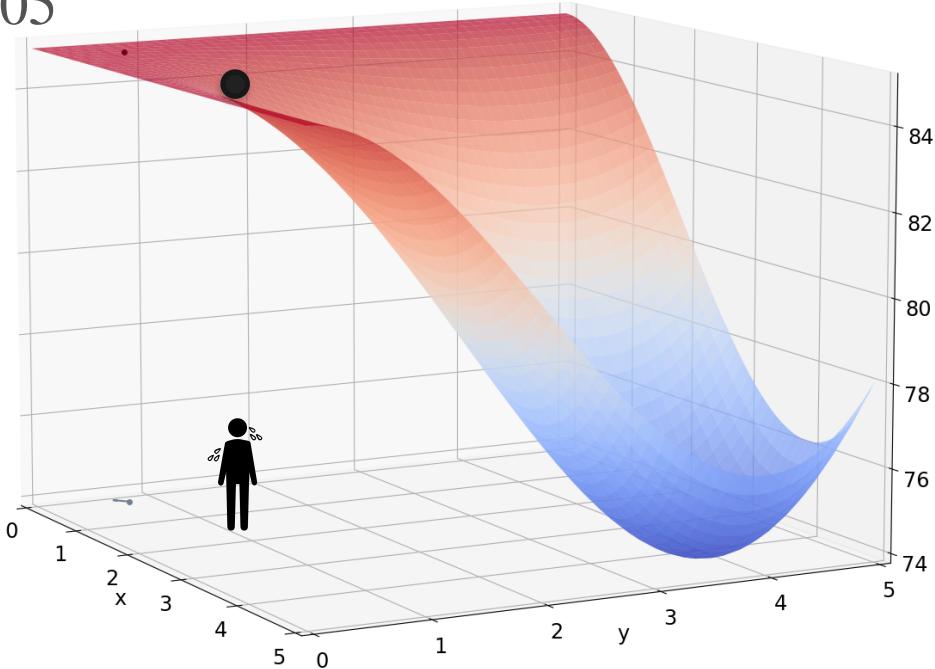
Find:

$$\nabla f(0.5057, 0.6047) = \begin{bmatrix} -0.1162 \\ -0.0961 \end{bmatrix}$$

Move by
 $-0.05 \nabla f(0.5057, 0.6047)$

$$\begin{aligned} x &\mapsto 0.5115 \\ y &\mapsto 0.6095 \end{aligned}$$

Repeat!



Gradient Descent

Start: $x = 0.5$, $y = 0.6$ Rate: $\alpha = 0.05$

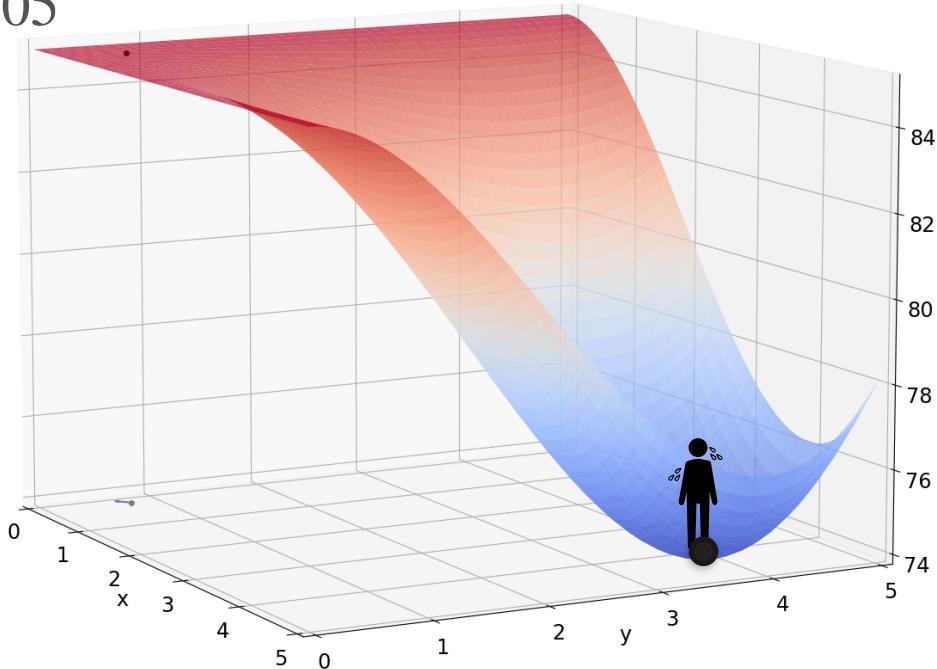
Find:

$$\nabla f(0.5057, 0.6047) = \begin{bmatrix} -0.1162 \\ -0.0961 \end{bmatrix}$$

Move by
 $-0.05 \nabla f(0.5057, 0.6047)$

$$\begin{aligned} x &\mapsto 0.5115 \\ y &\mapsto 0.6095 \end{aligned}$$

Repeat!



Gradient Descent

Gradient Descent

Function: $f(x, y)$

Gradient Descent

Function: $f(x, y)$

Goal: find minimum of $f(x, y)$

Gradient Descent

Function: $f(x, y)$

Goal: find minimum of $f(x, y)$

Step 1:

Define a learning rate α

Choose a starting point (x_0, y_0)

Gradient Descent

Function: $f(x, y)$

Goal: find minimum of $f(x, y)$

Step 1:

Define a learning rate α

Choose a starting point (x_0, y_0)

Step 2:

Update:
$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} x_{k-1} \\ y_{k-1} \end{bmatrix} - \alpha \nabla f(x_{k-1}, y_{k-1})$$

Gradient Descent

Function: $f(x, y)$

Goal: find minimum of $f(x, y)$

Step 1:

Define a learning rate α

Choose a starting point (x_0, y_0)

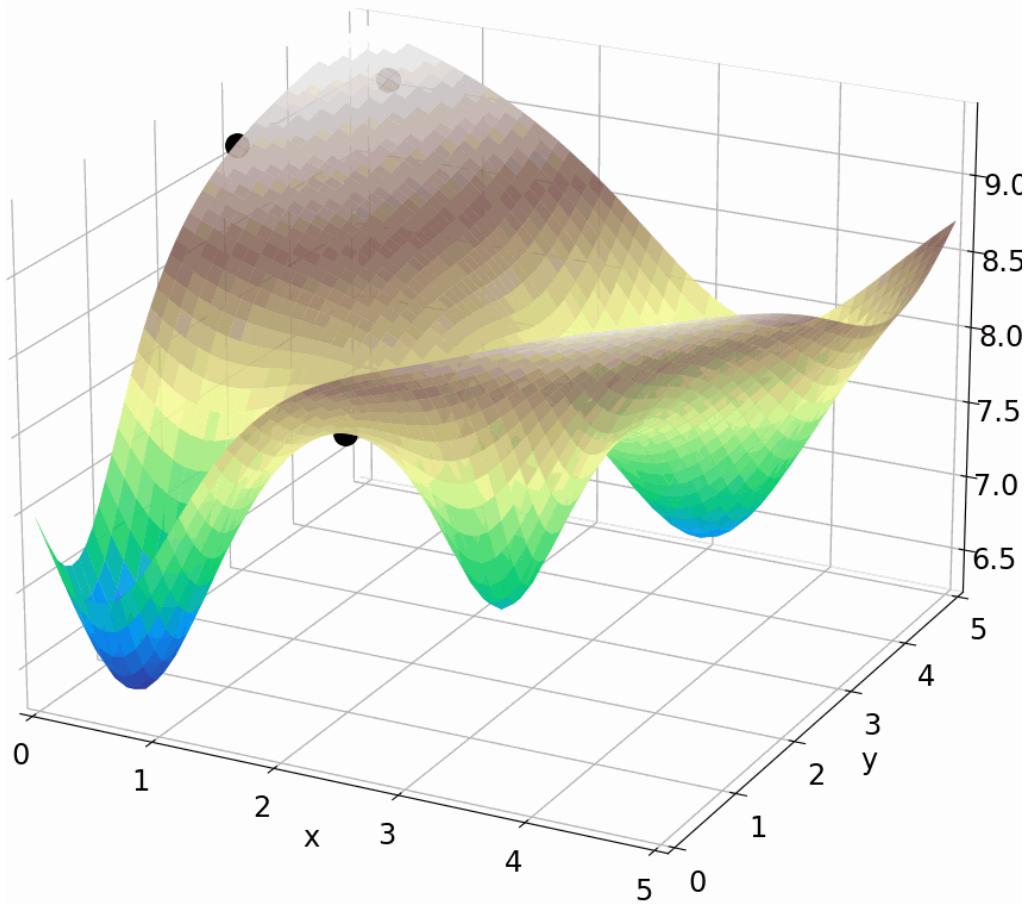
Step 2:

Update: $\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} x_{k-1} \\ y_{k-1} \end{bmatrix} - \alpha \nabla f(x_{k-1}, y_{k-1})$

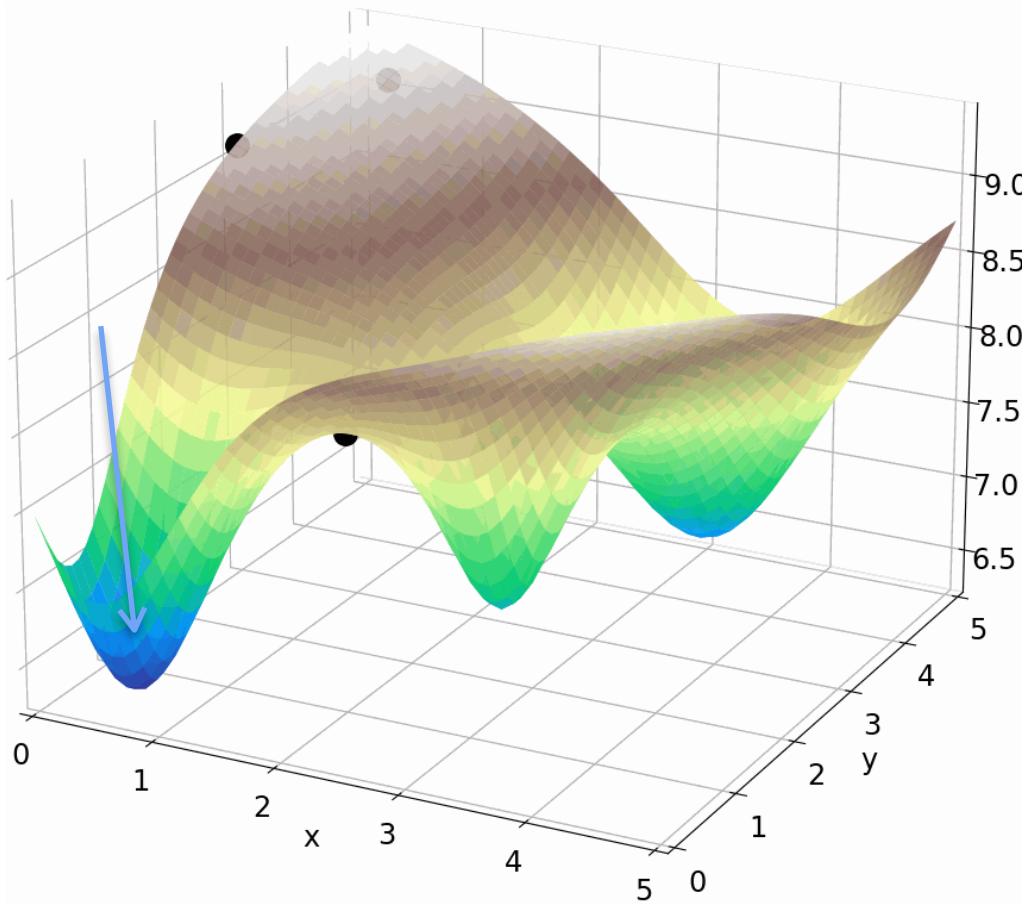
Step 3:

Repeat Step 2 until you are close enough to
the true minimum (x^*, y^*)

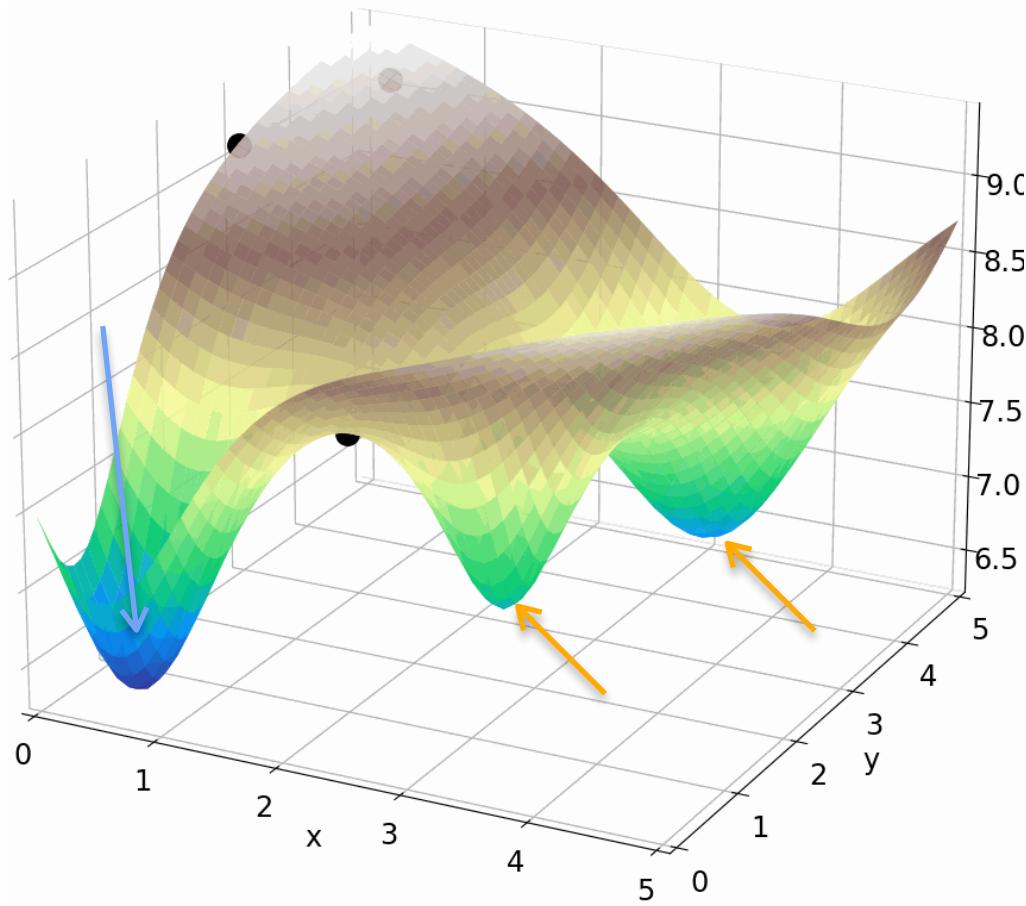
Gradient Descent With Local Minima



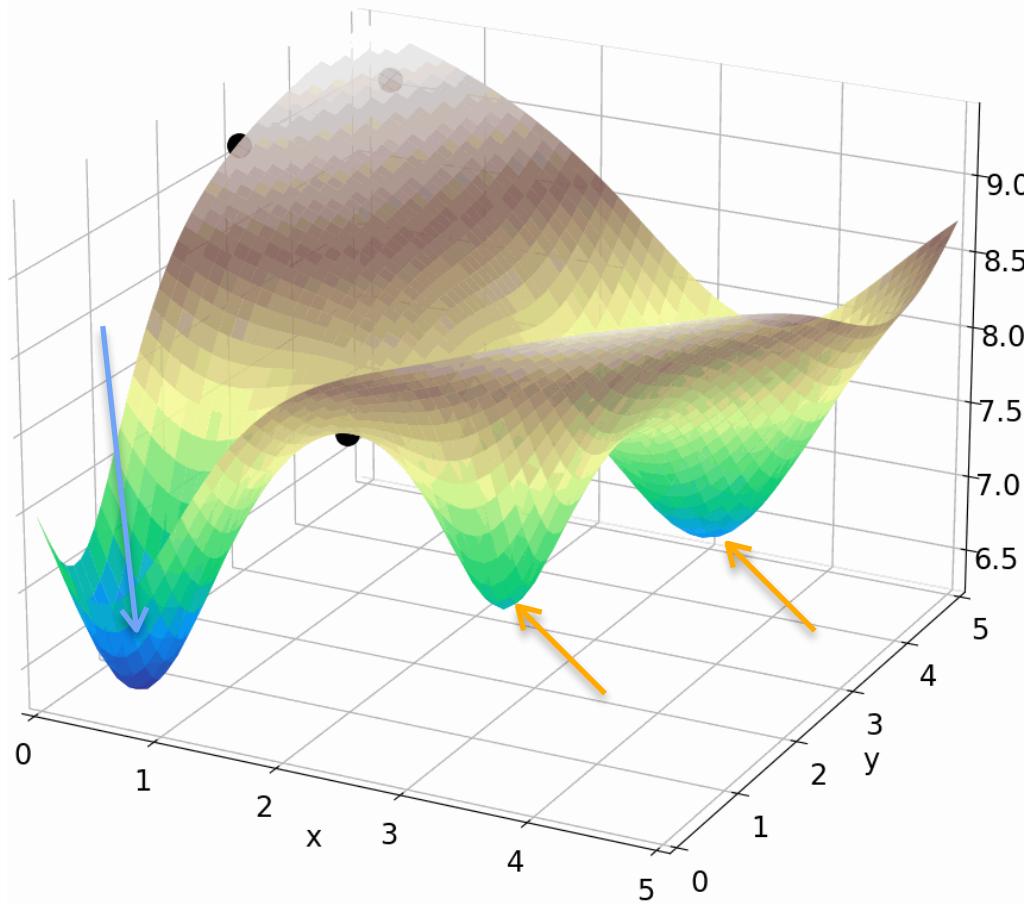
Gradient Descent With Local Minima



Gradient Descent With Local Minima



Gradient Descent With Local Minima



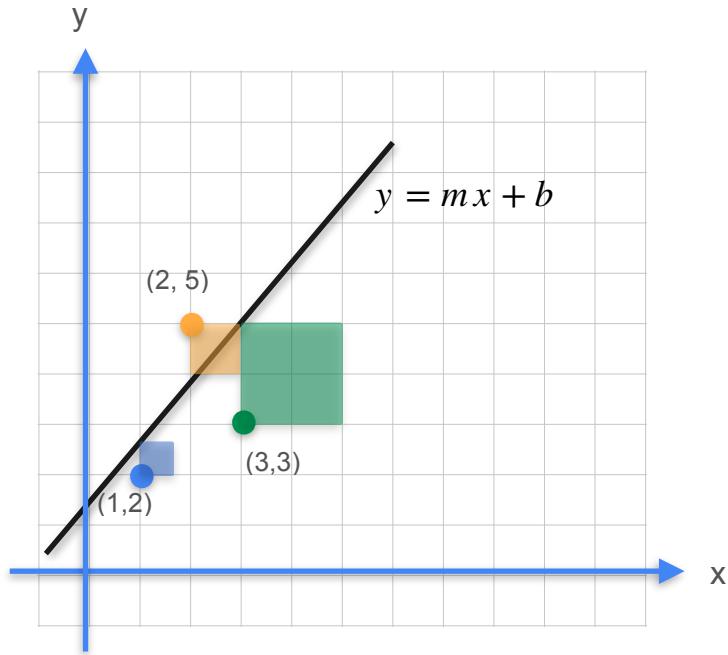


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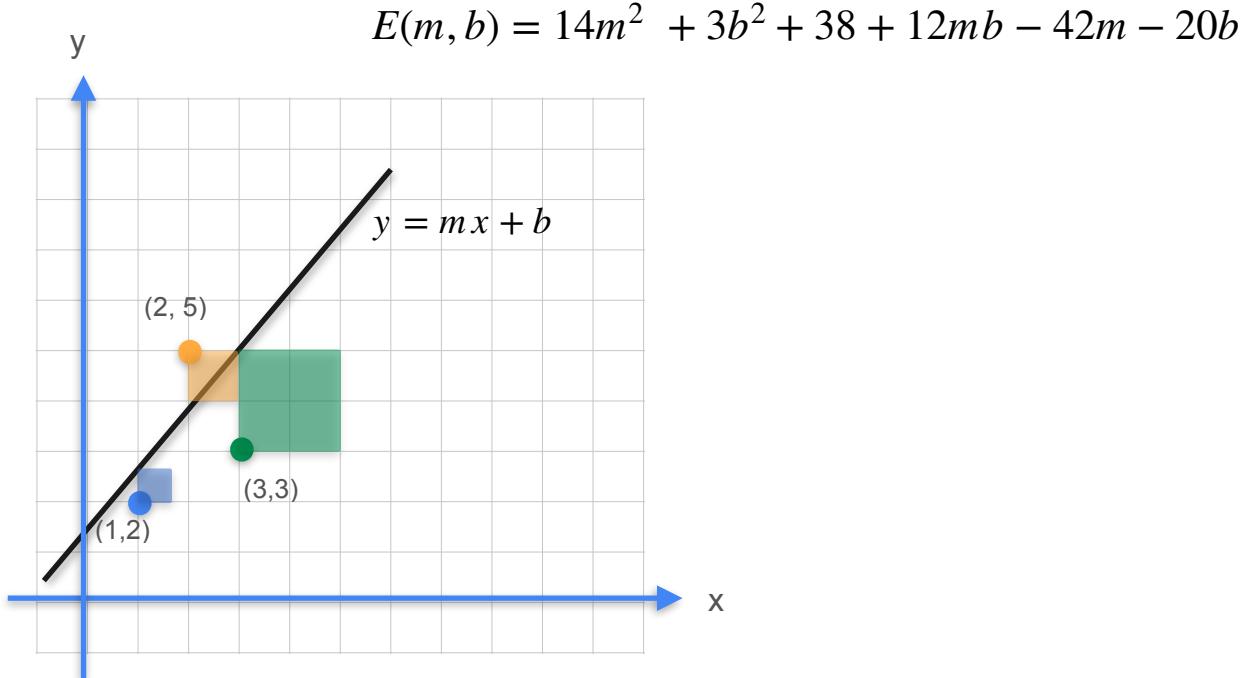
Gradients and Gradient Descent

**Optimization using Gradient
Descent - Least squares**

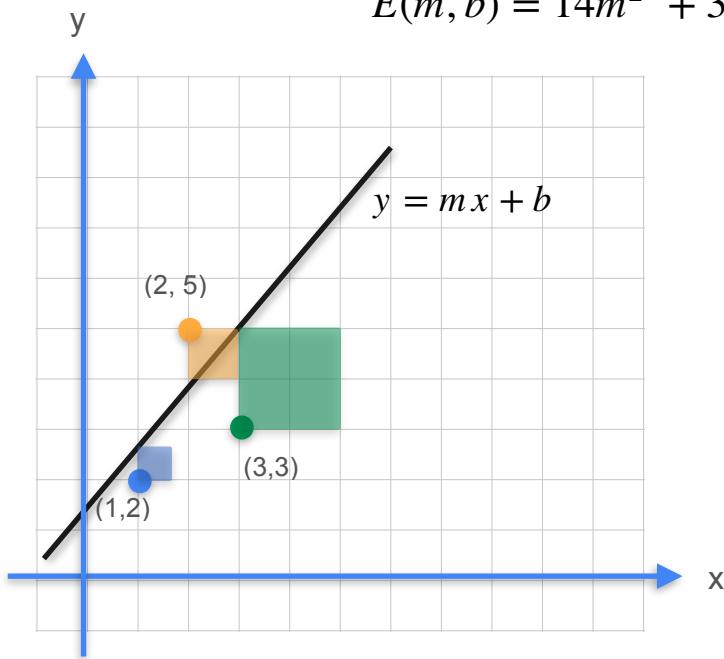
Gradient Descent With Power Lines Example



Gradient Descent With Power Lines Example

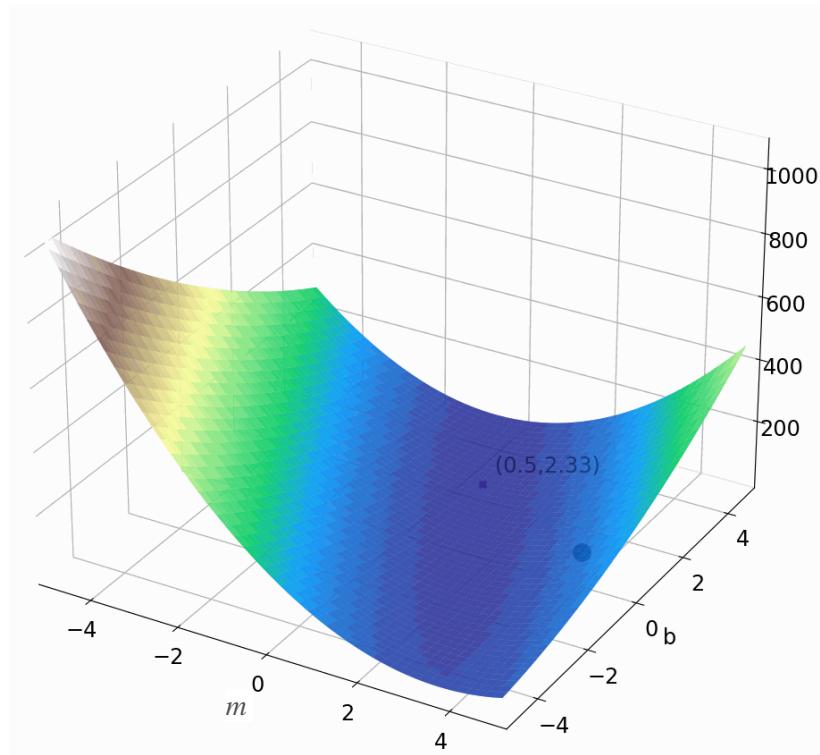


Gradient Descent With Power Lines Example



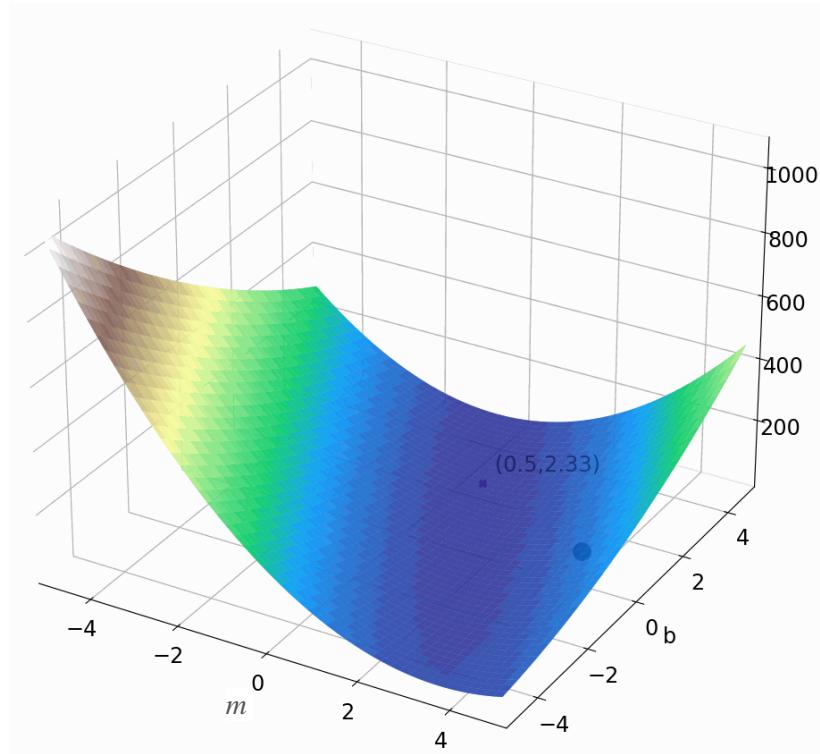
$$E\left(m = \frac{1}{2}, b = \frac{7}{3}\right) \approx 4.167$$

Linear Regression: Gradient Descent



Linear Regression: Gradient Descent

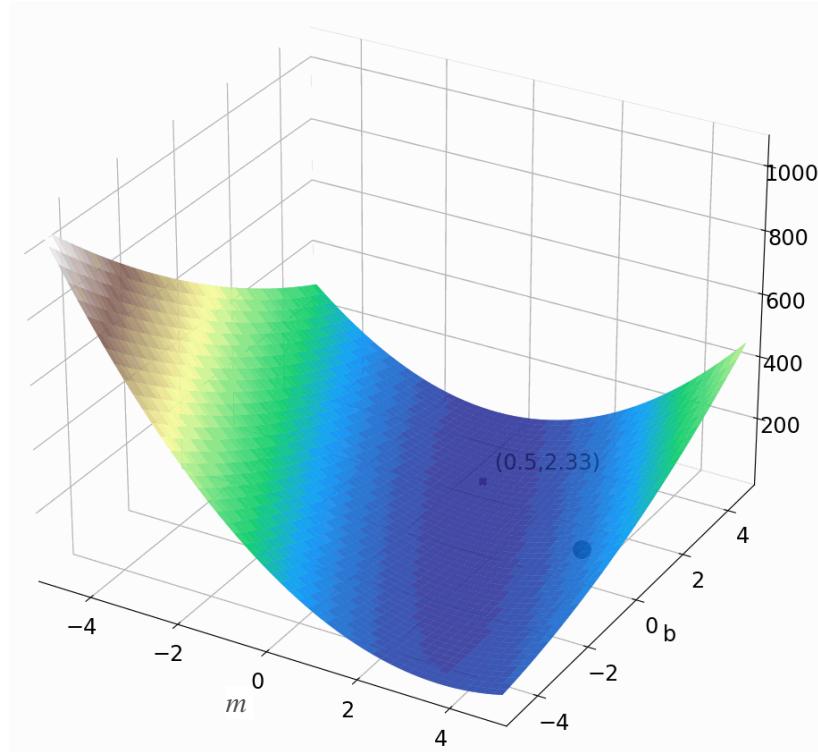
Goal: Minimize sum of squares cost



Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$



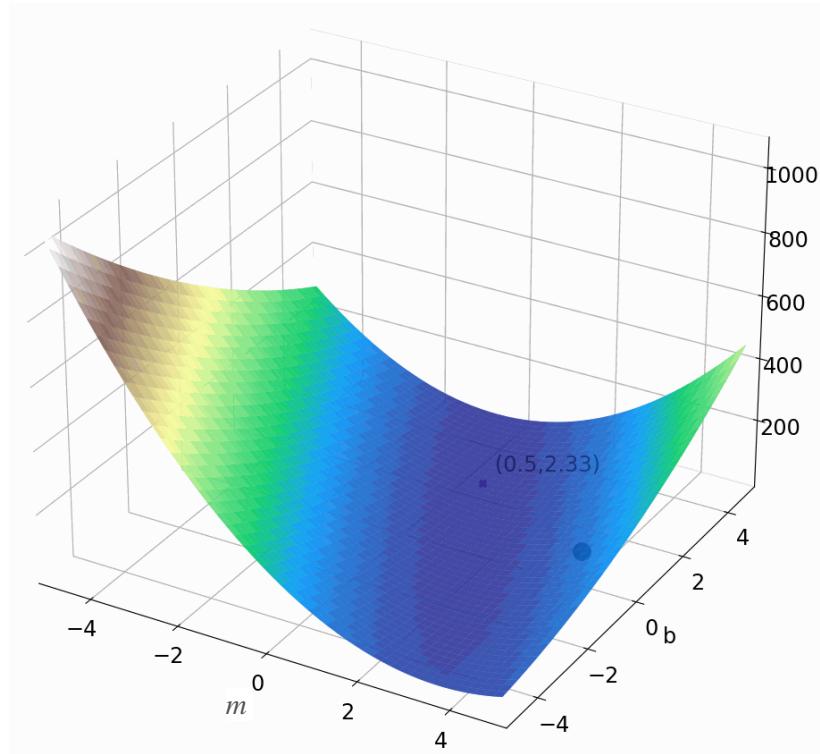
Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$m =$$

$$b =$$

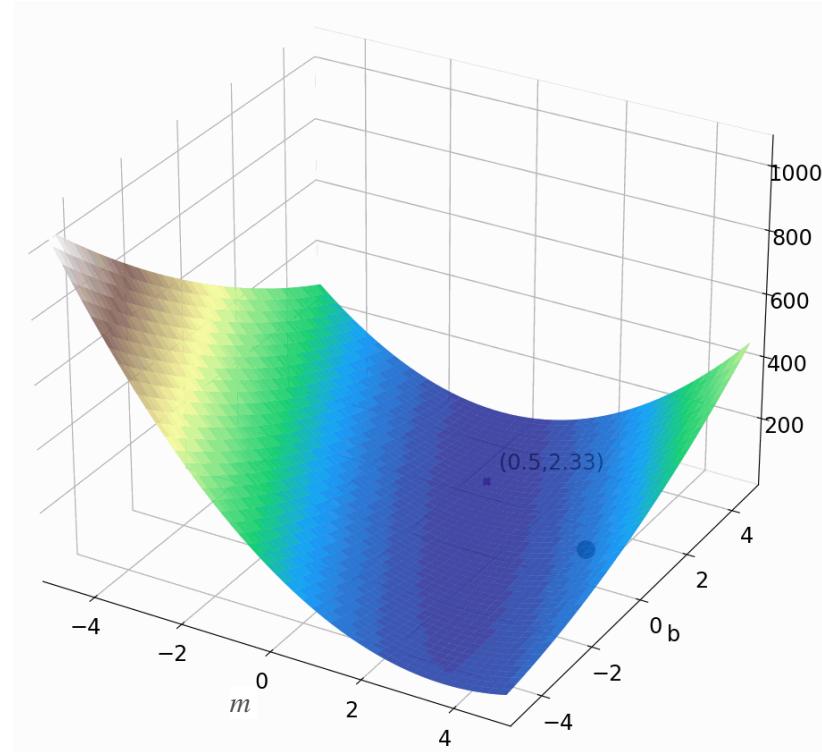


Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

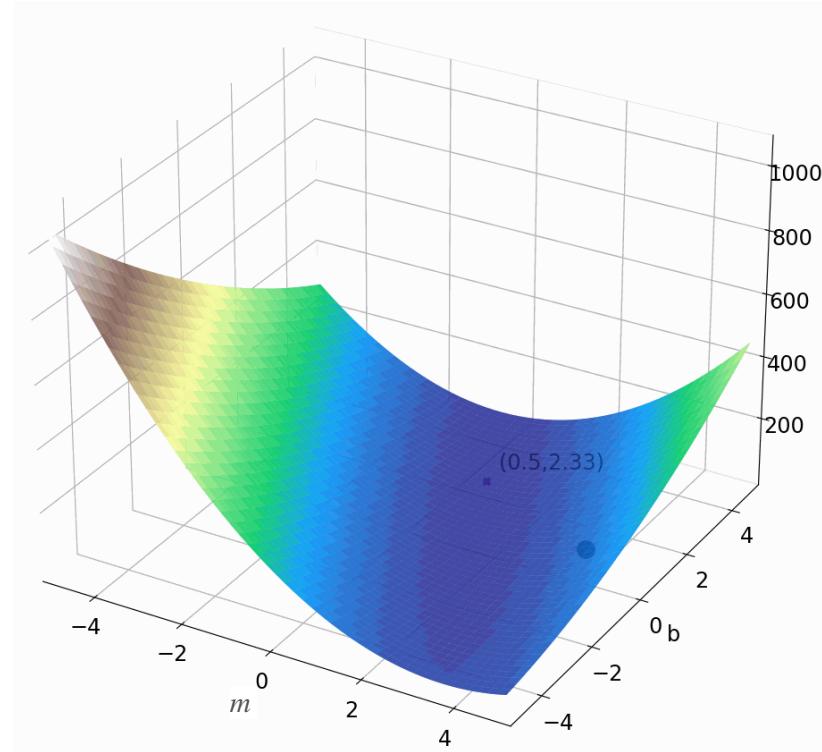


Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

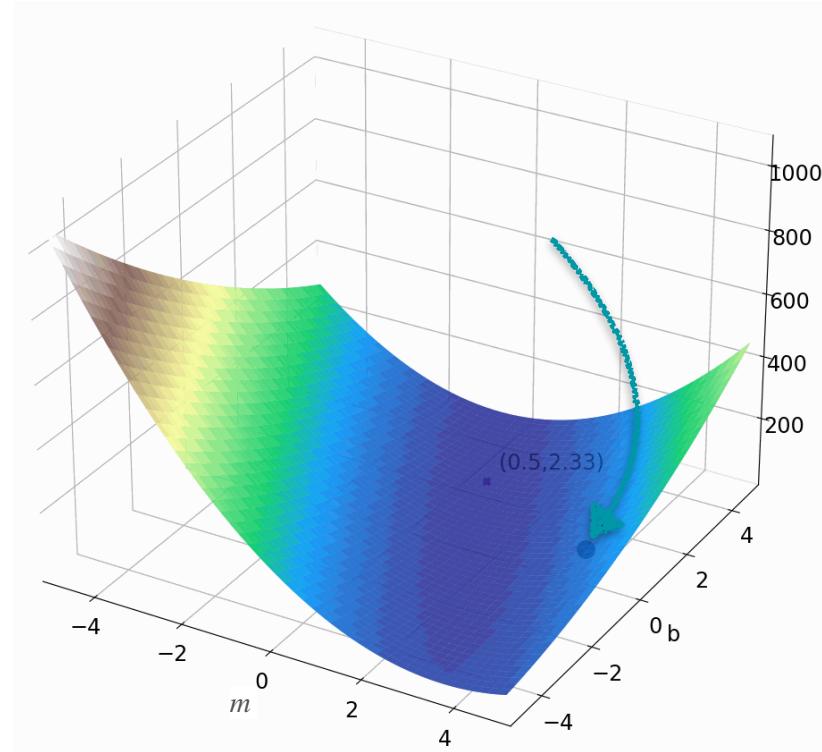


Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

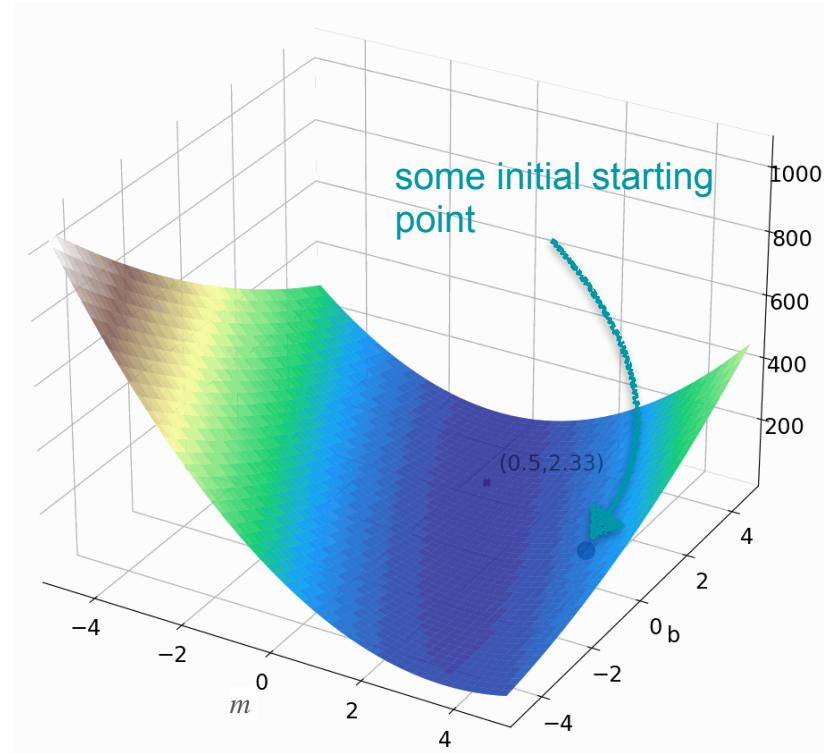


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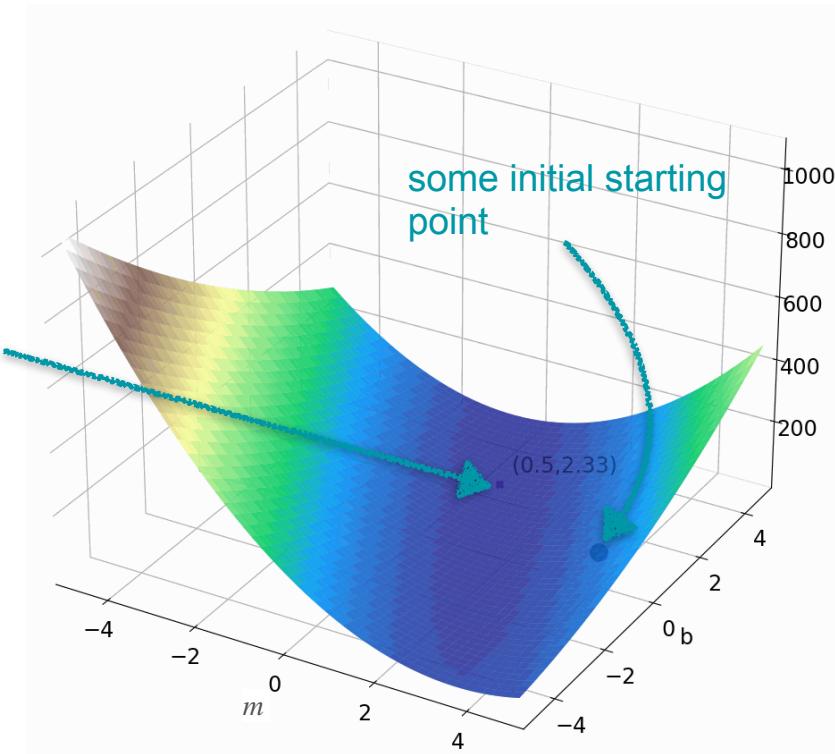


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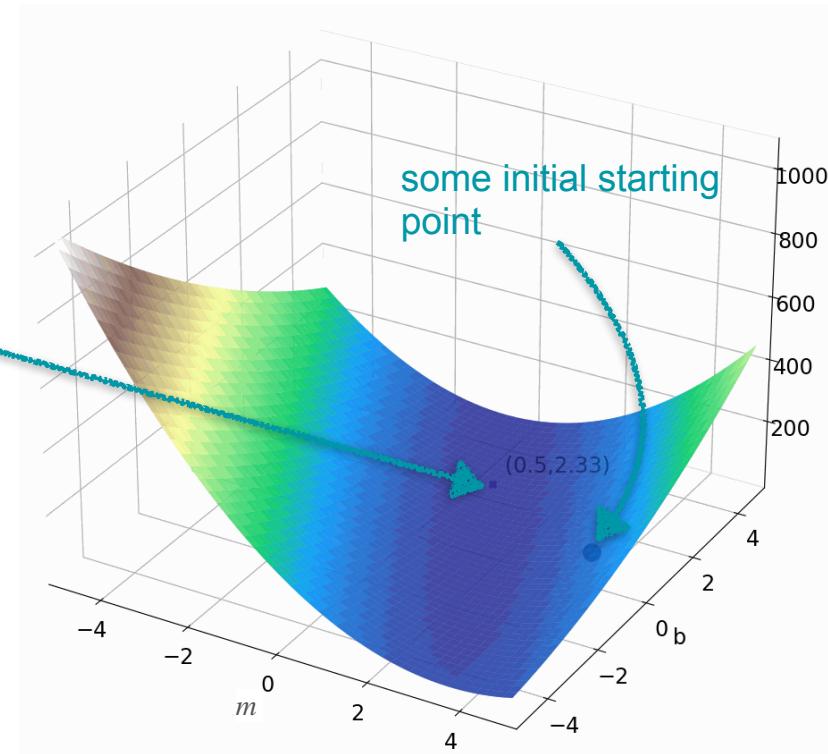
Linear Regression: Gradient Descent

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The points m, b such that
the cost is minimum



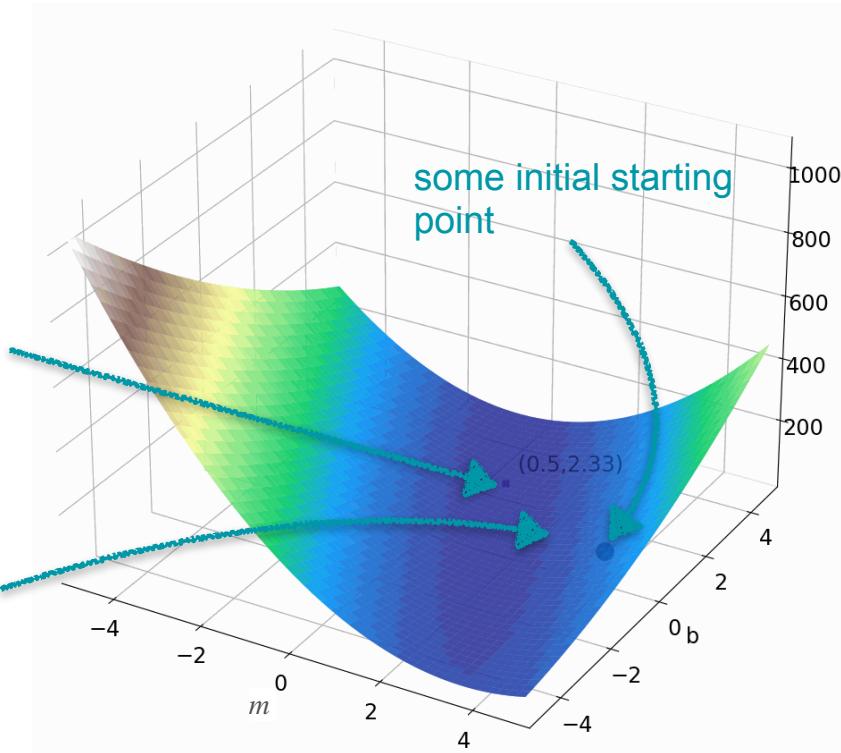
Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

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$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

The points m, b such that
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Linear Regression: Gradient Descent

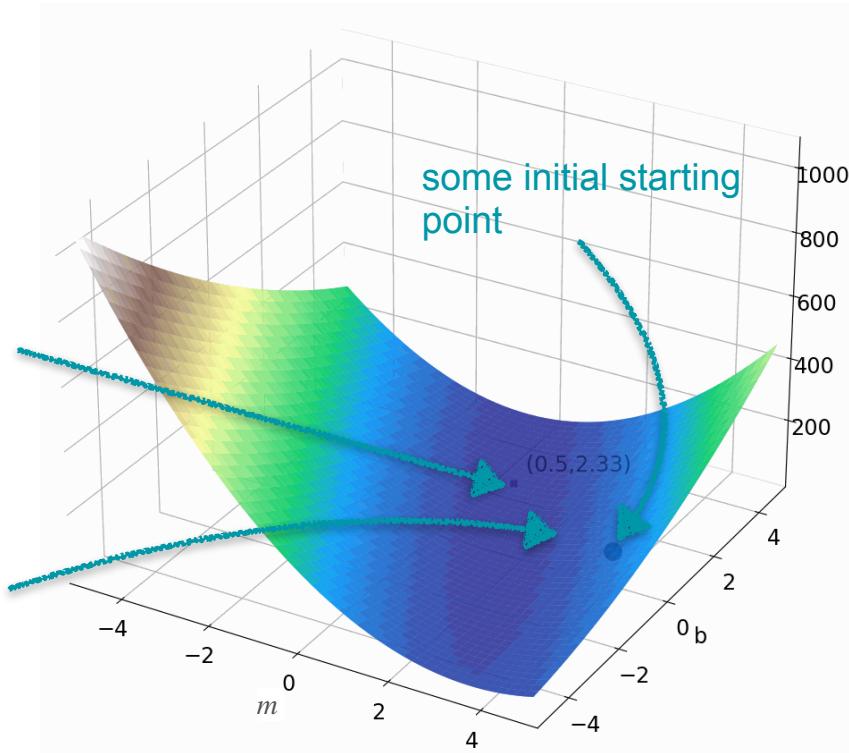
Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

The points m, b such that
the cost is minimum

descend until you
find the minimum



Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

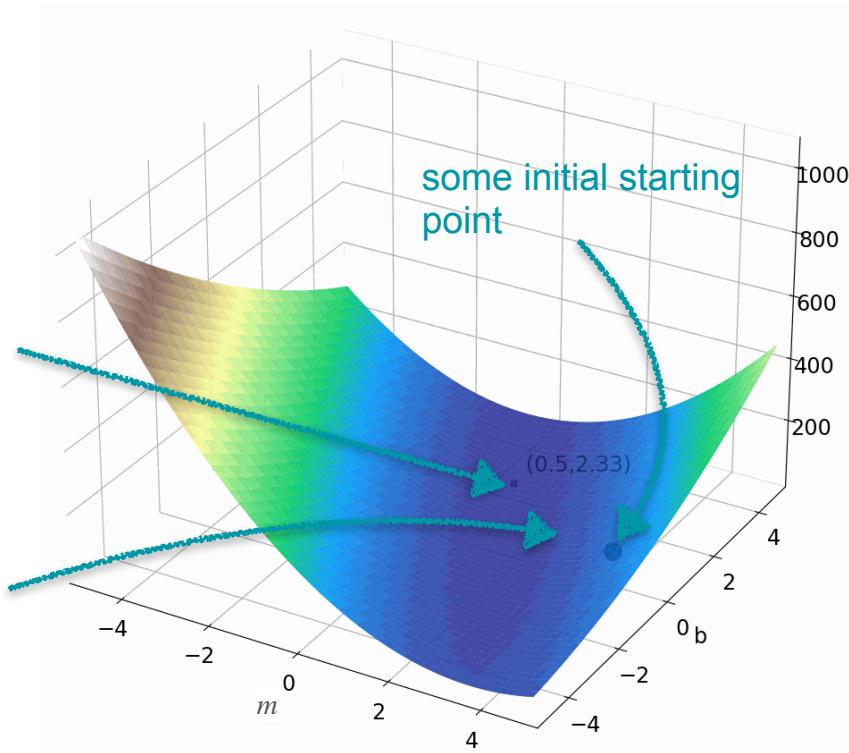
$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

Steps:

descend until you
find the minimum

The points m, b such that
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Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

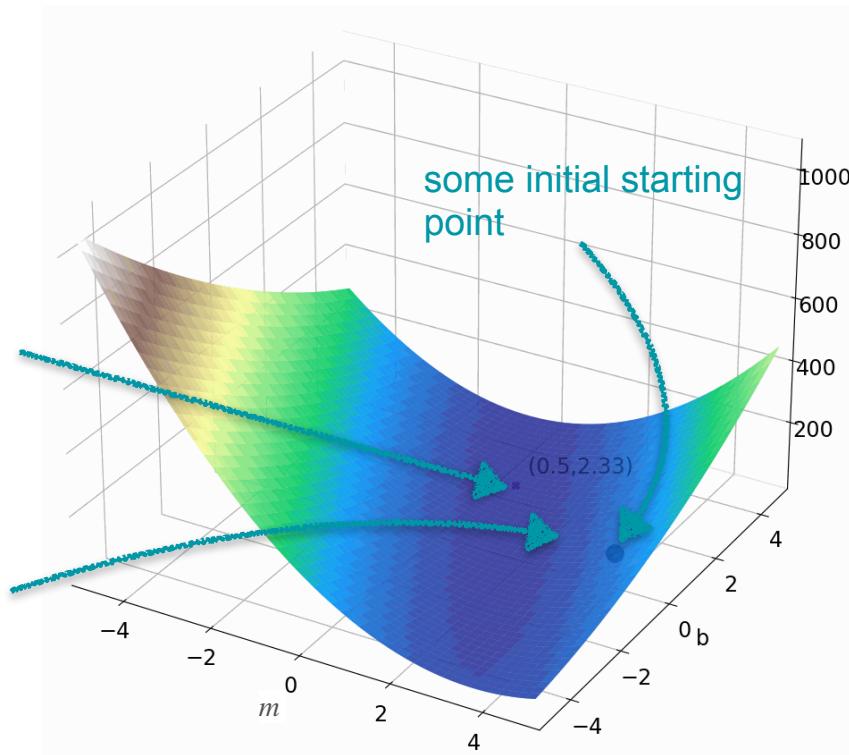
$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

Steps:

Start with (m_0, b_0)

descend until you
find the minimum

The points m, b such that
the cost is minimum



Linear Regression: Gradient Descent

Goal: Minimize sum of squares cost

$$\nabla E = [28m + 12b - 42, 6b + 12m - 20]$$

$$\begin{aligned} m &= ? \\ b &= ? \end{aligned}$$

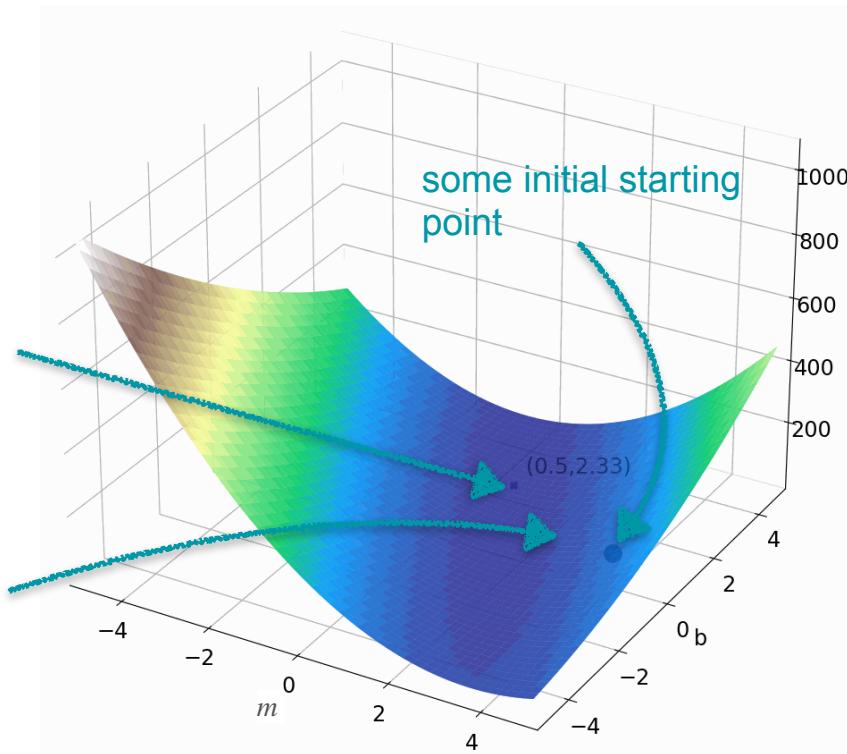
Steps:

Start with (m_0, b_0)

Iterate

$$(m_{k+1}, b_{k+1}) = (m_k, b_k) - \alpha \nabla E(m_k, b_k)$$

descend until you
find the minimum



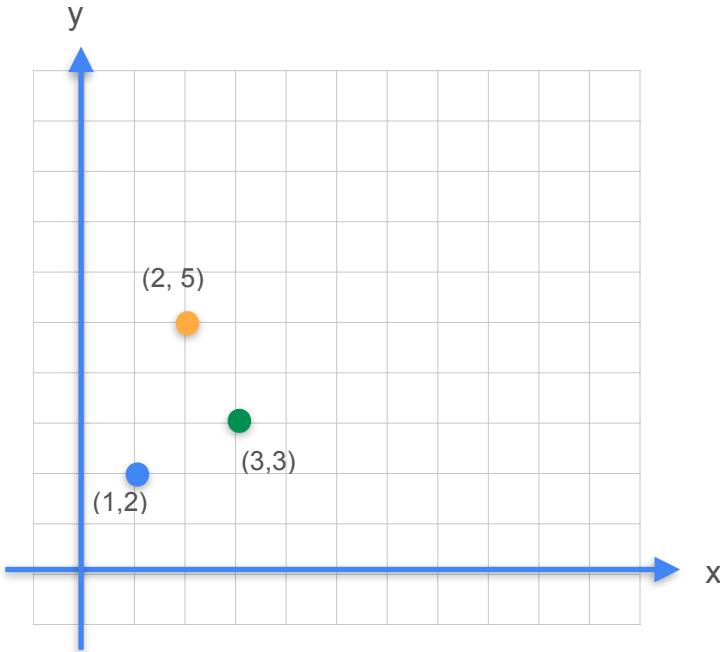


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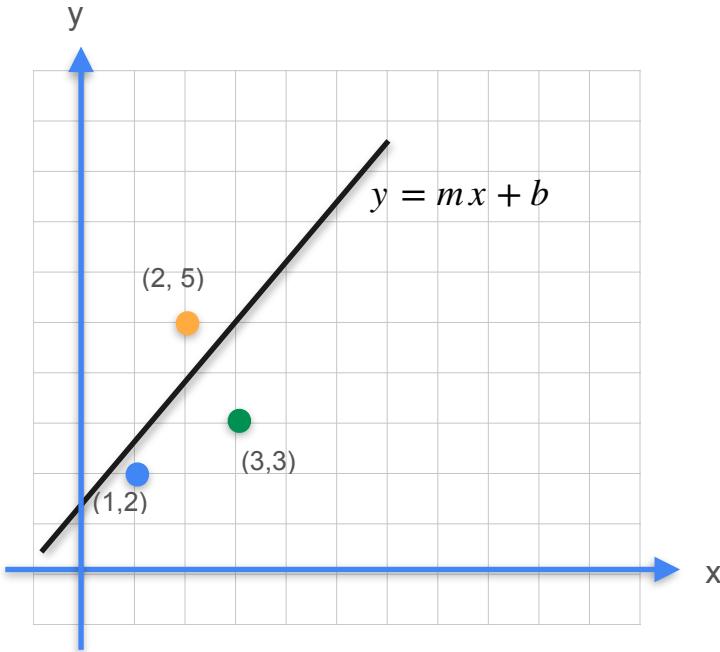
Gradients and Gradient Descent

**Optimization using Gradient
Descent - Least squares
with multiple observations**

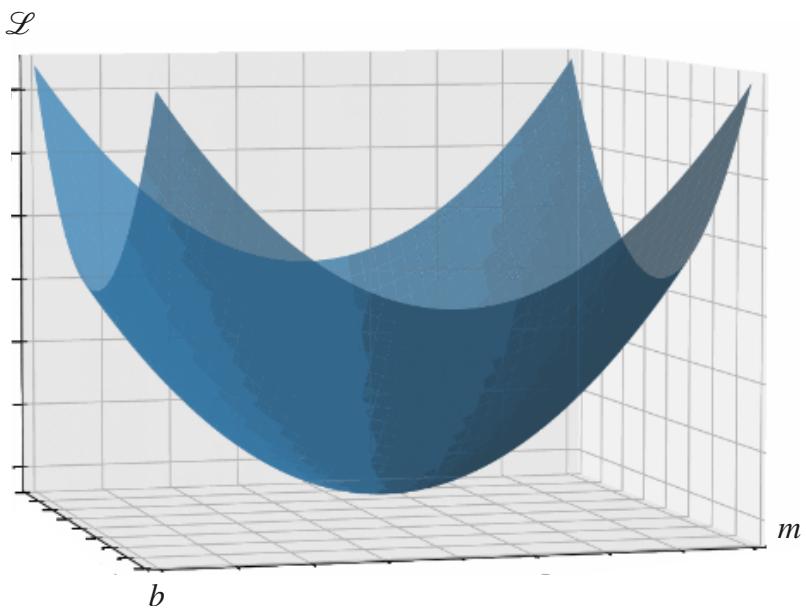
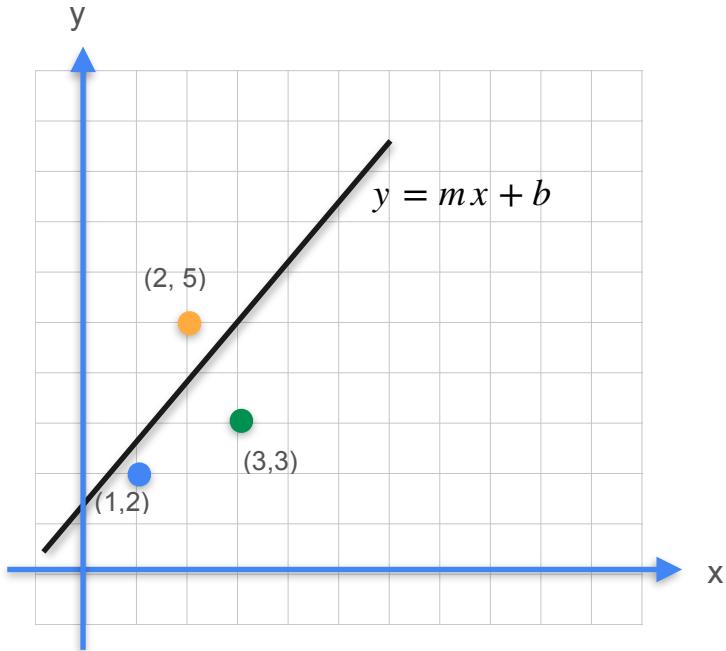
Gradient Descent



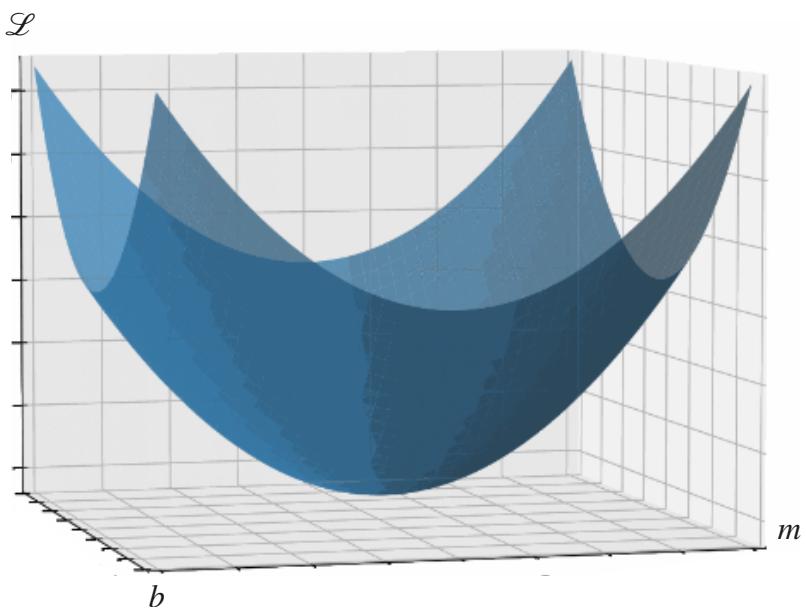
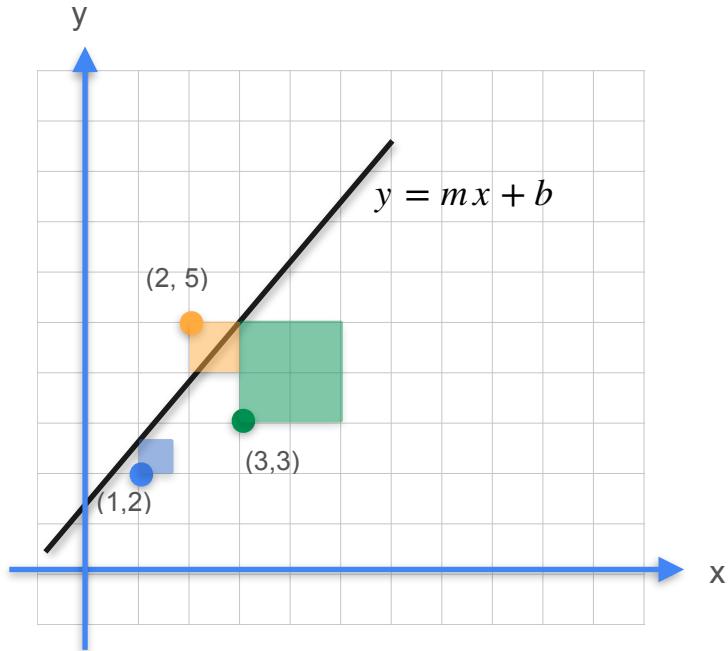
Gradient Descent



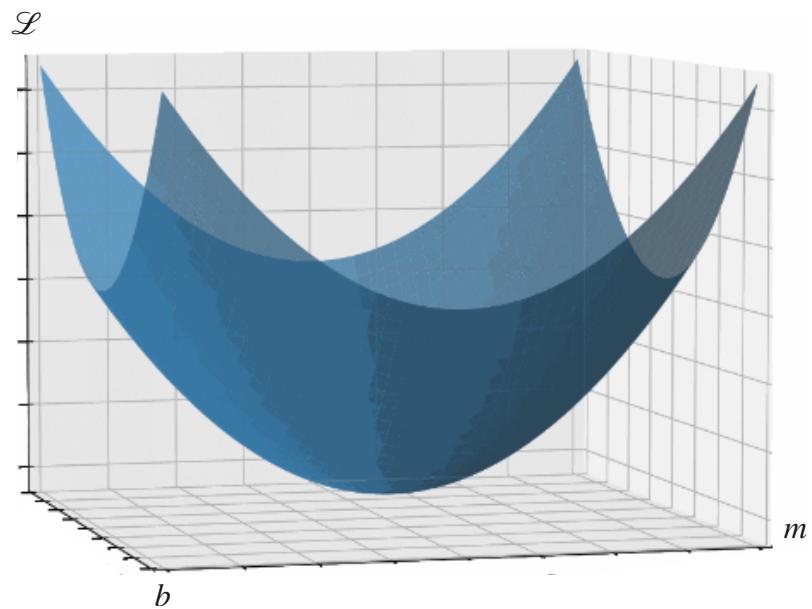
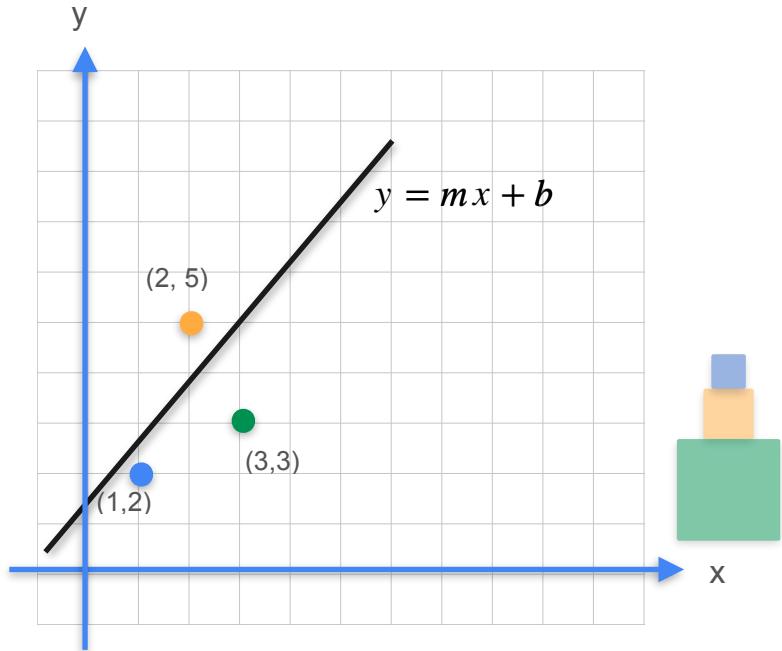
Gradient Descent



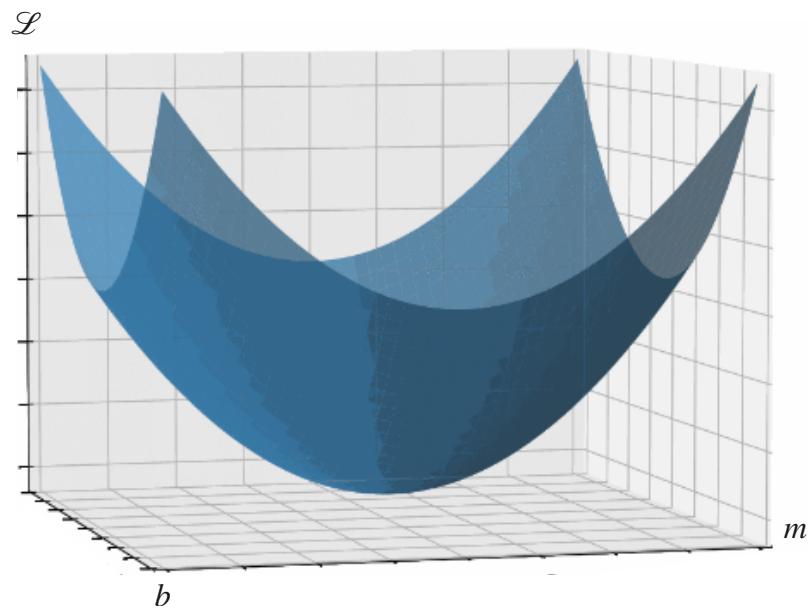
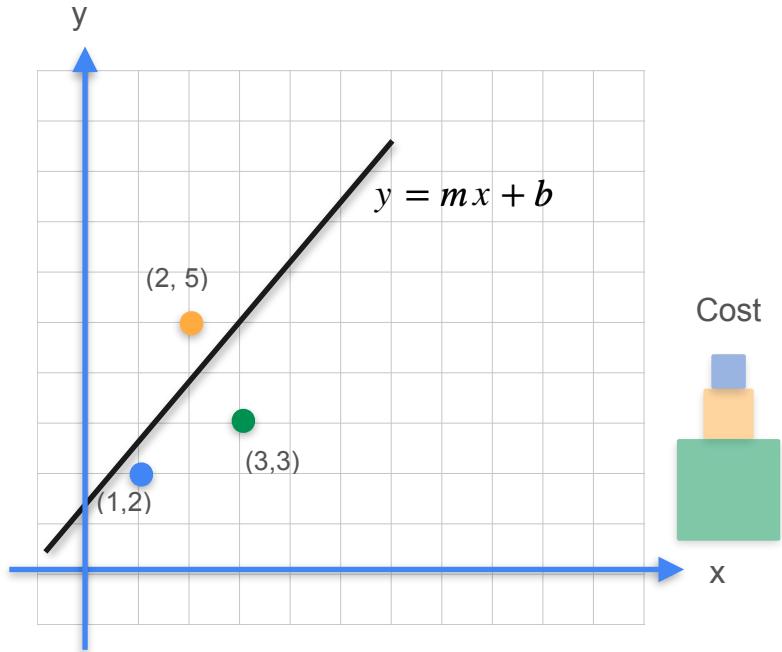
Gradient Descent



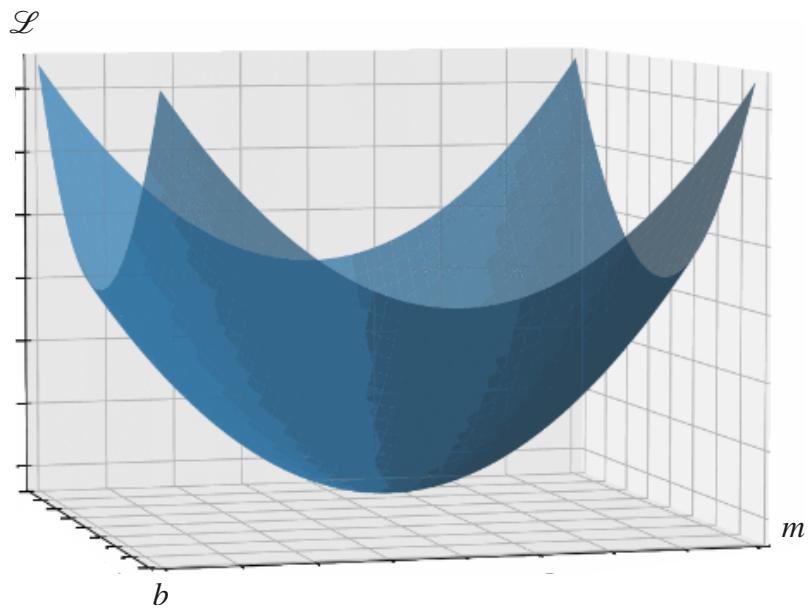
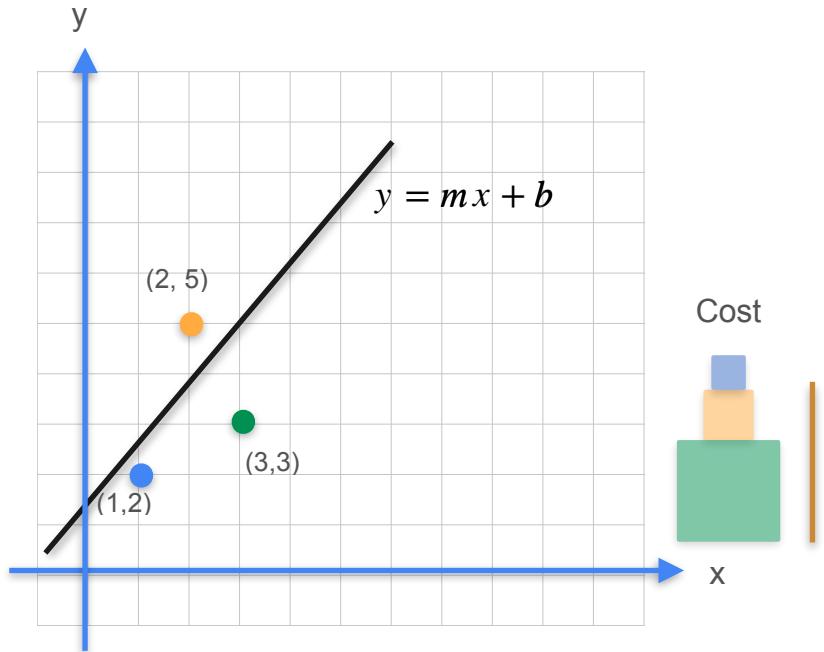
Gradient Descent



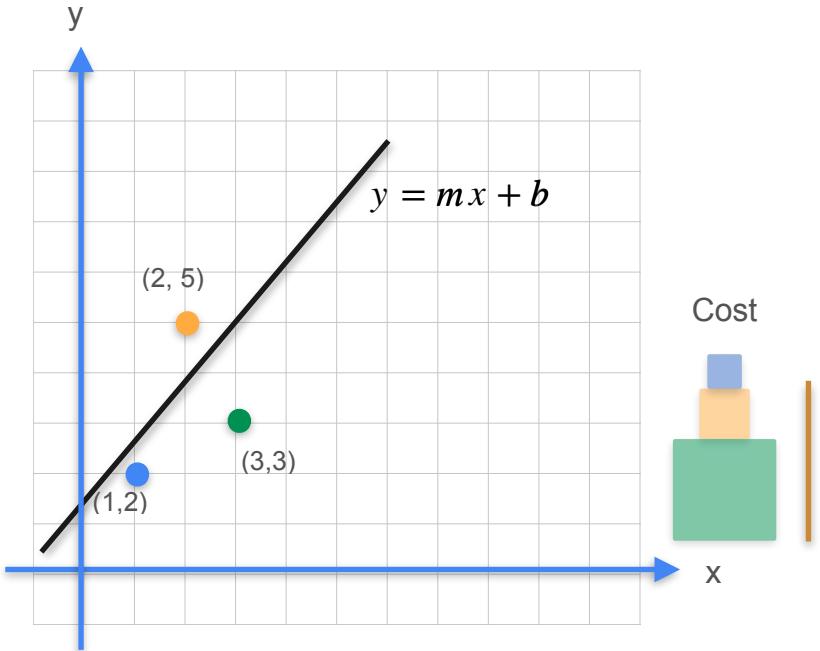
Gradient Descent



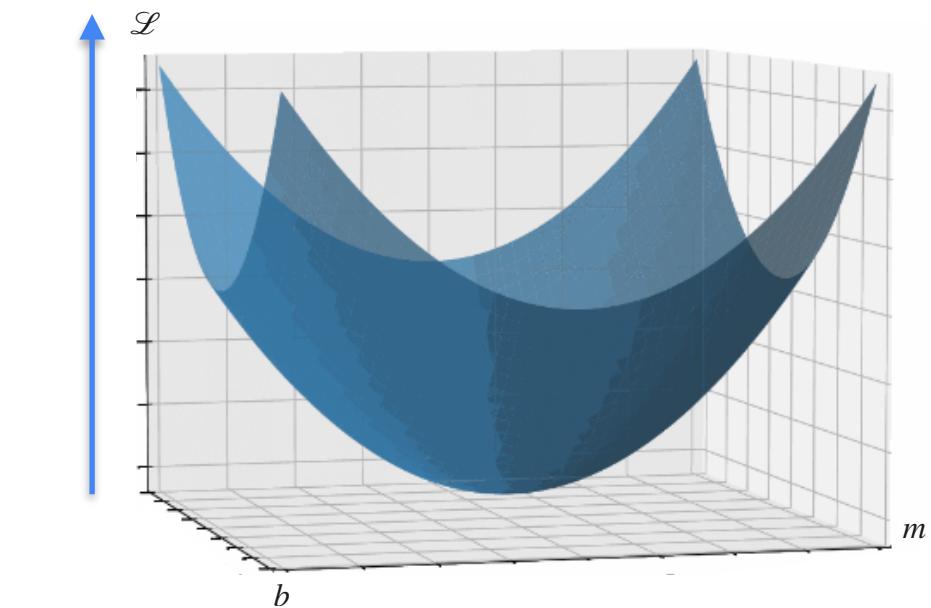
Gradient Descent



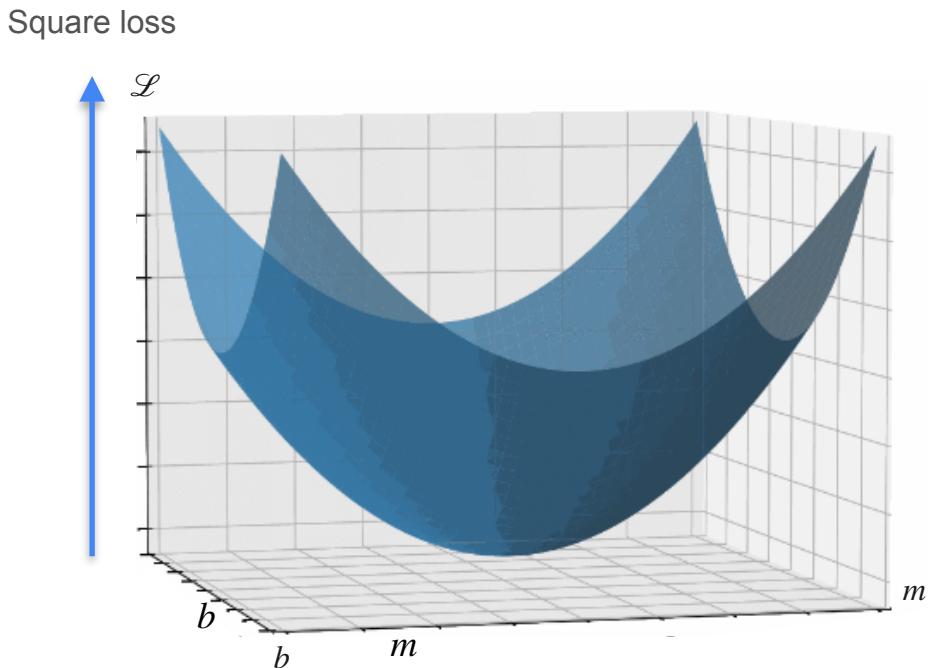
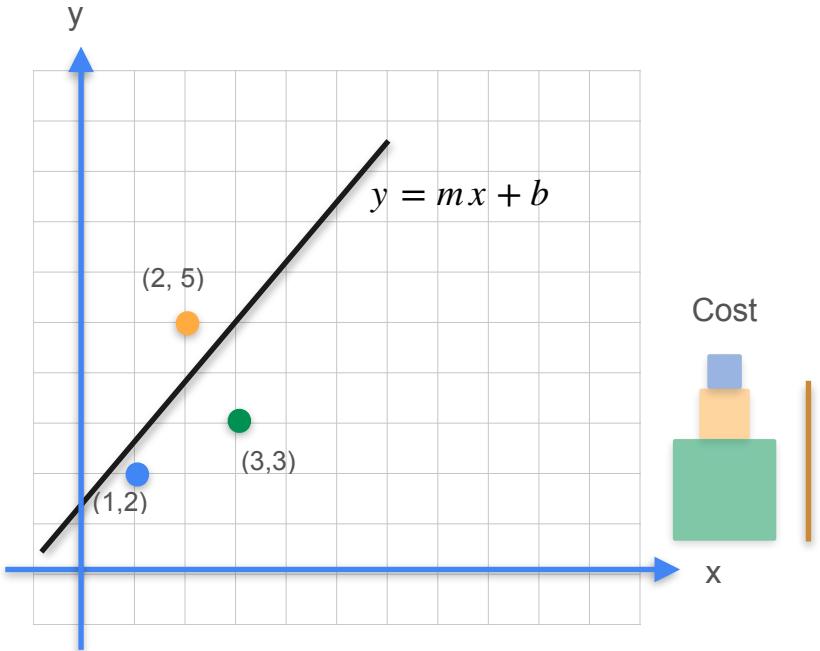
Gradient Descent



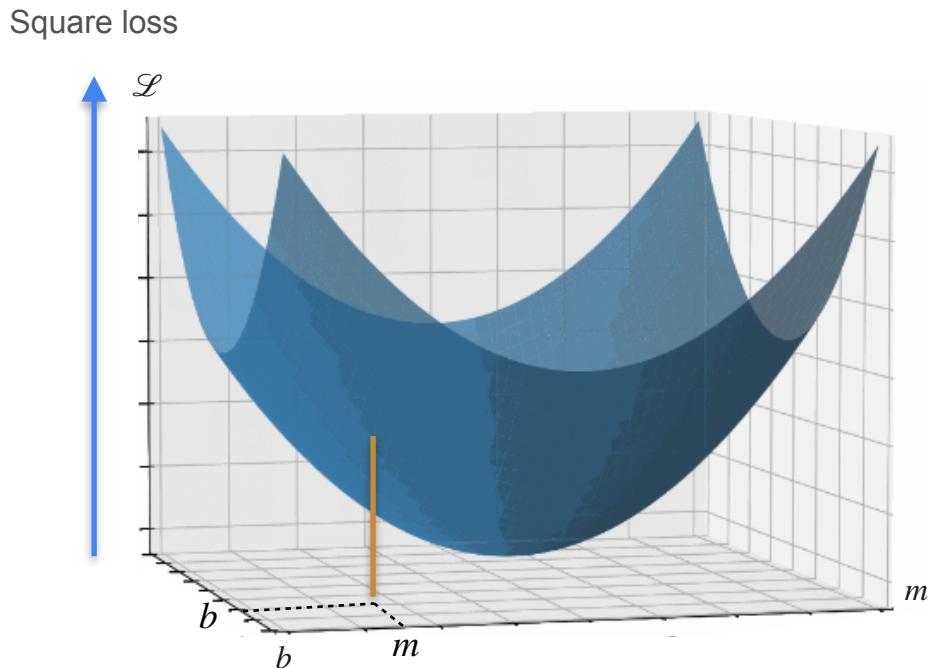
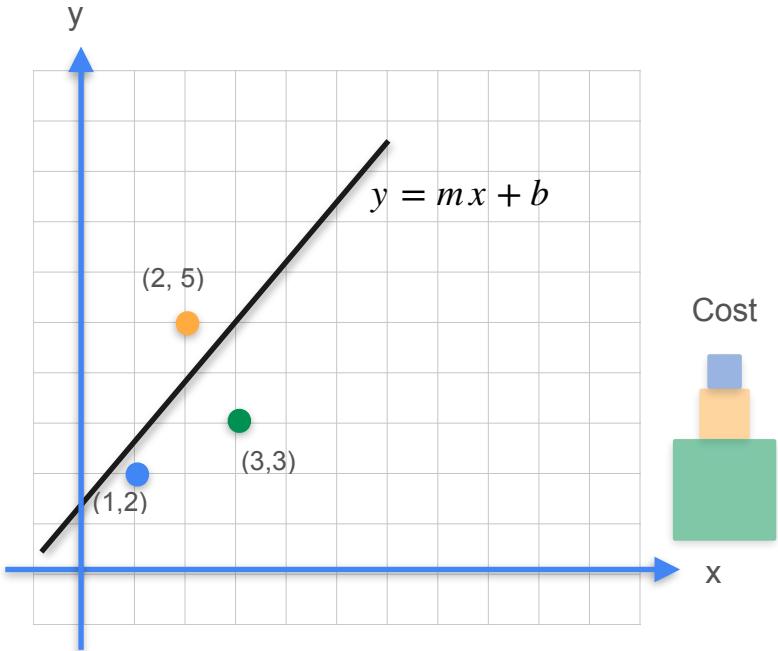
Square loss



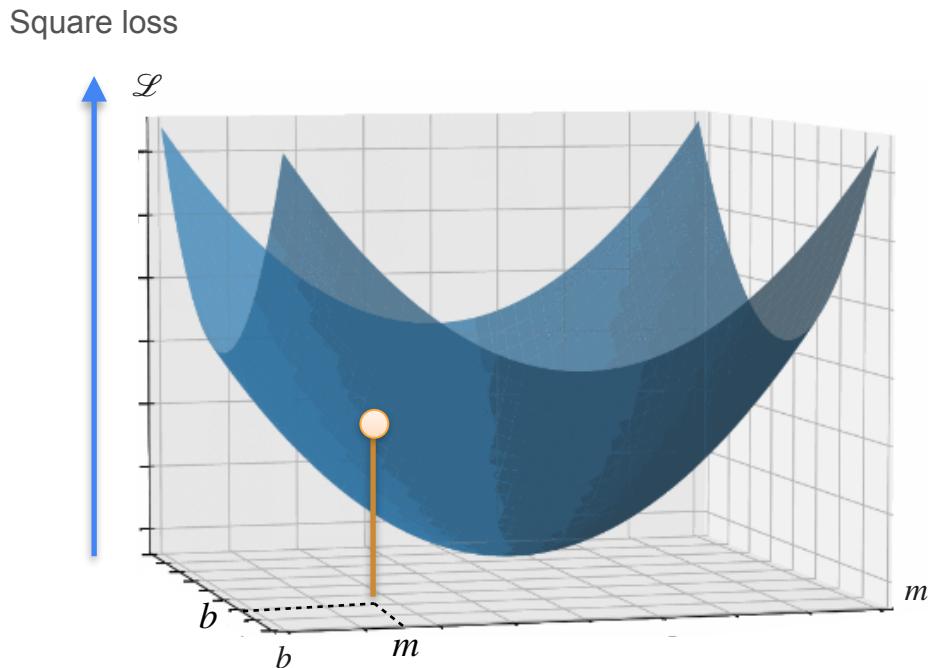
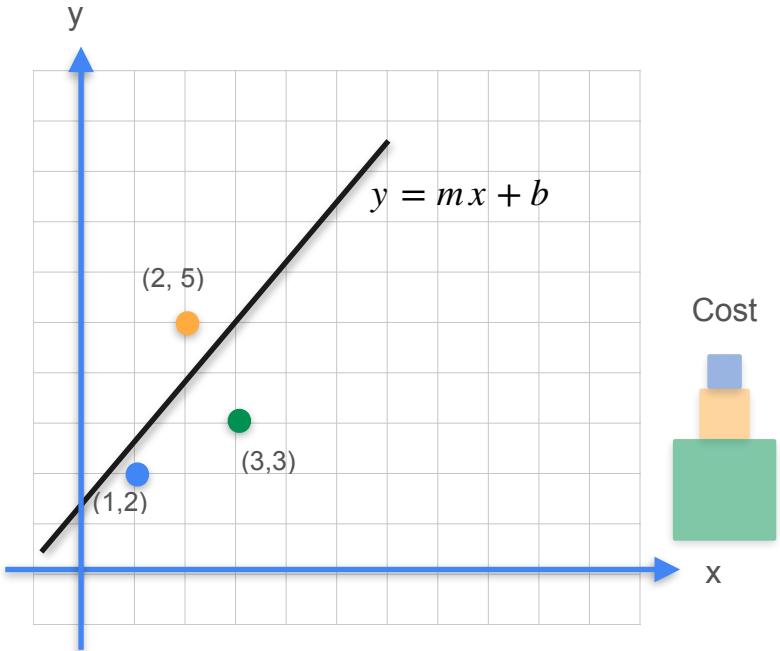
Gradient Descent



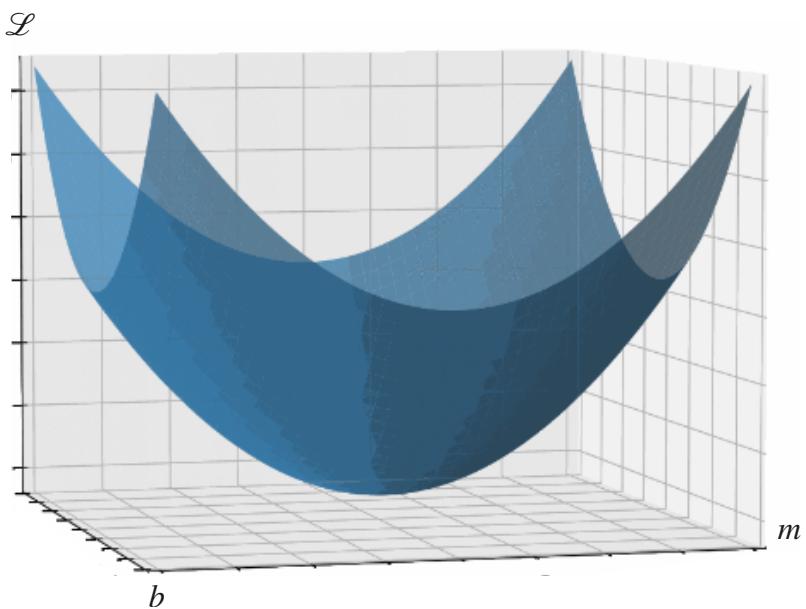
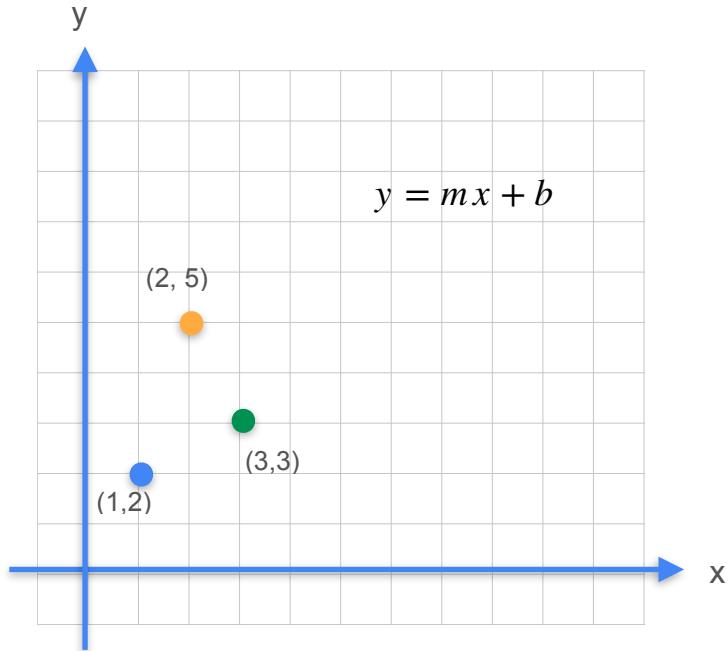
Gradient Descent



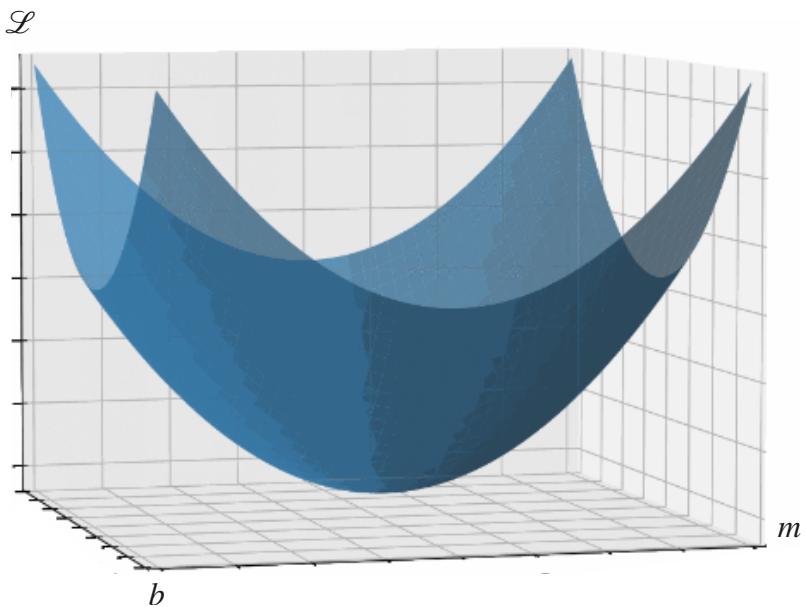
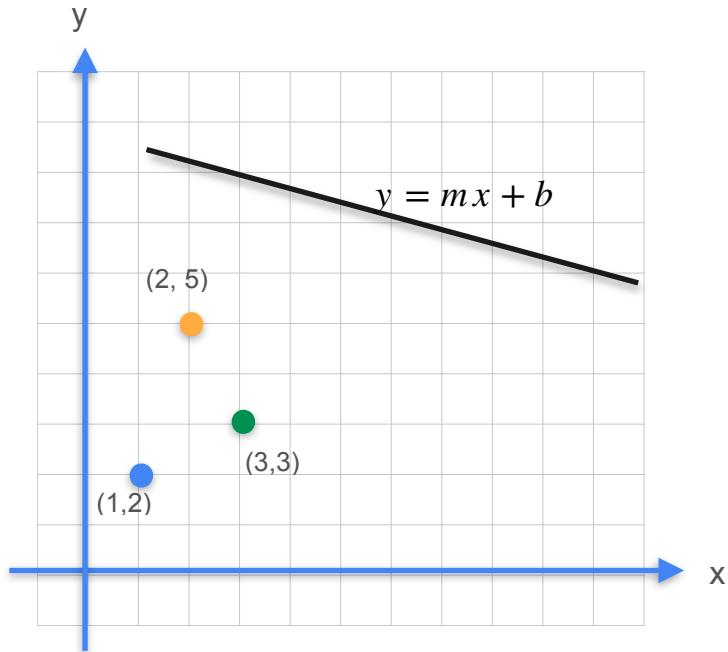
Gradient Descent



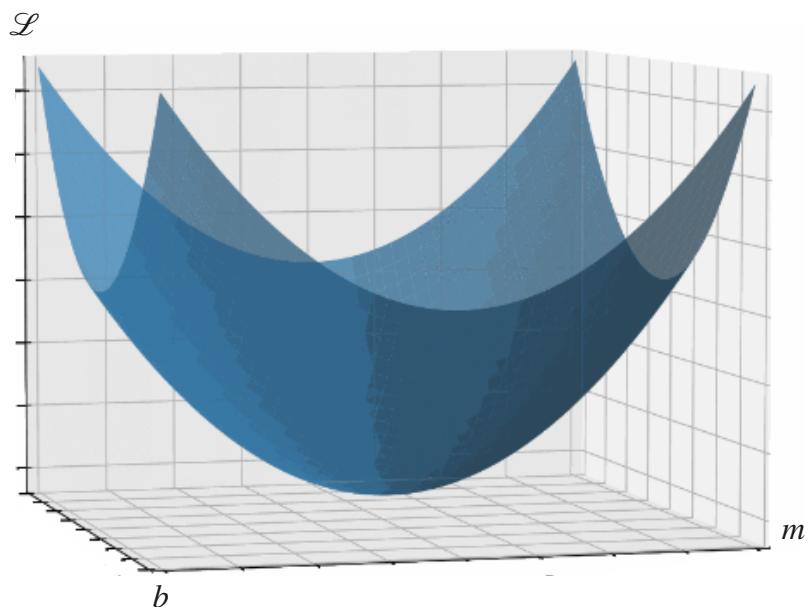
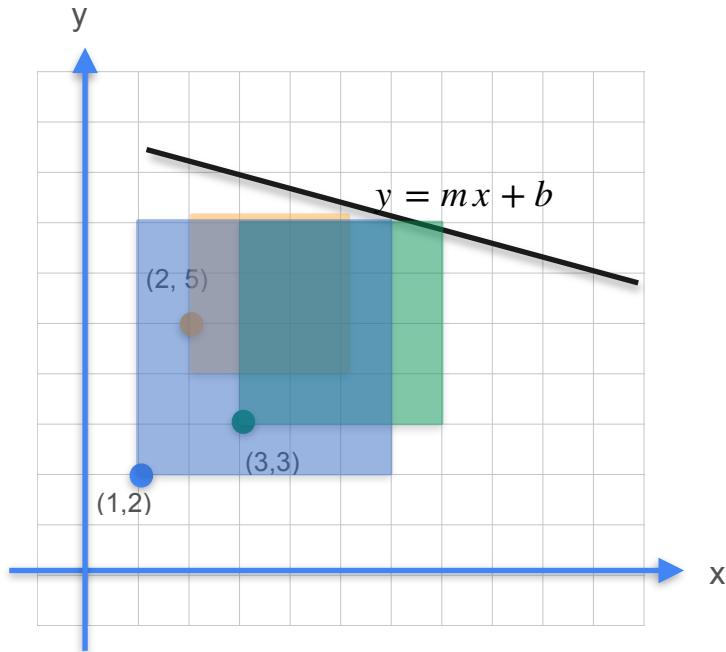
Gradient Descent



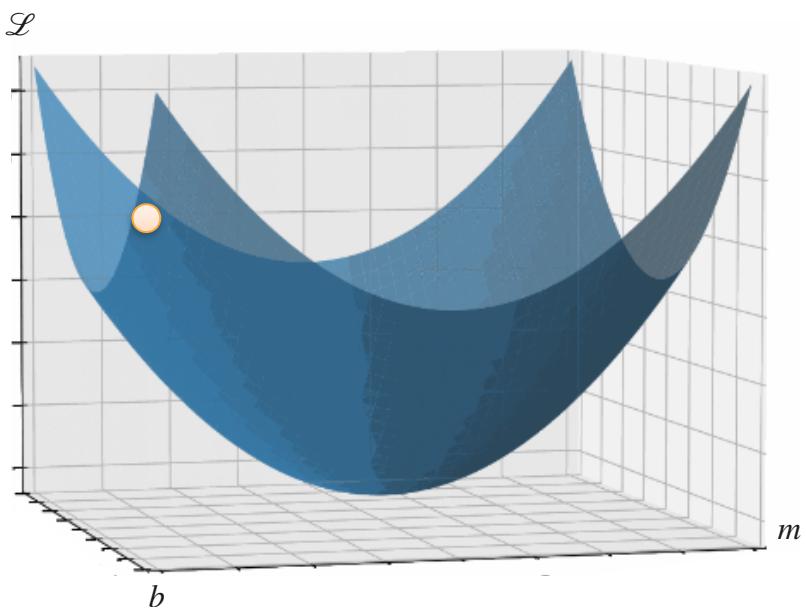
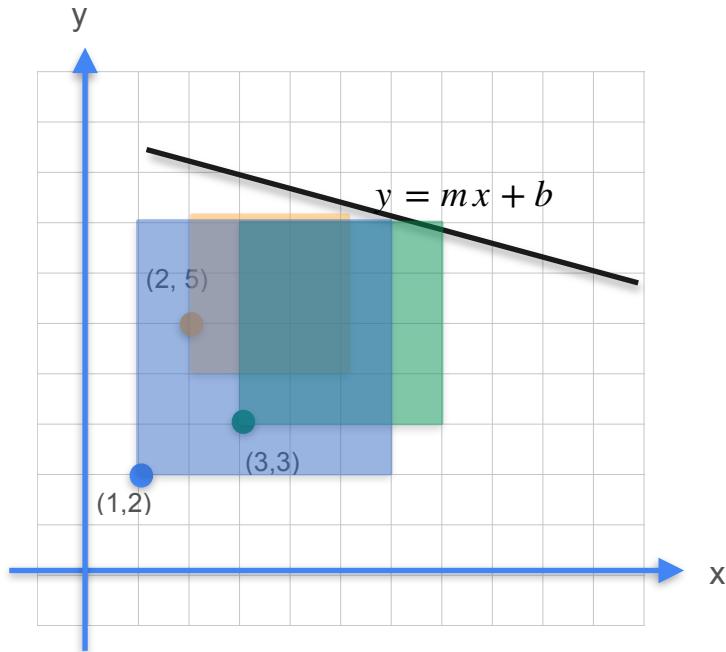
Gradient Descent



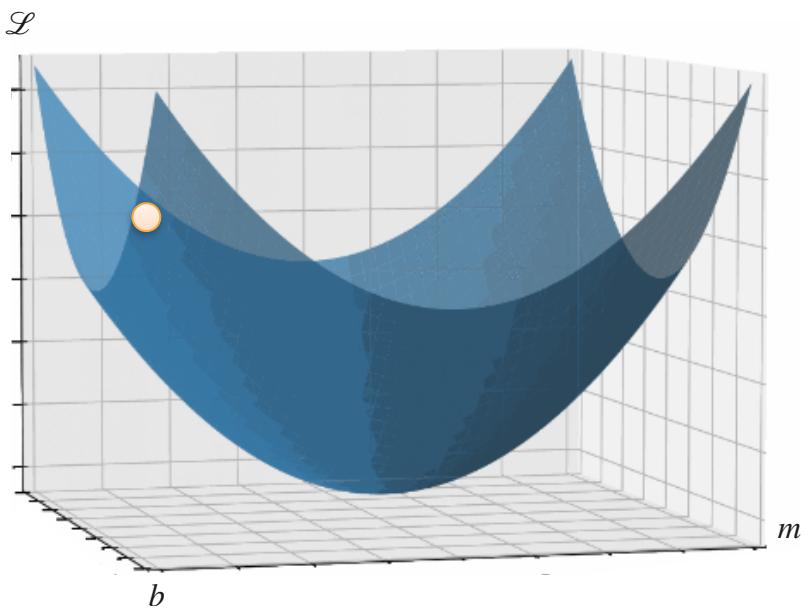
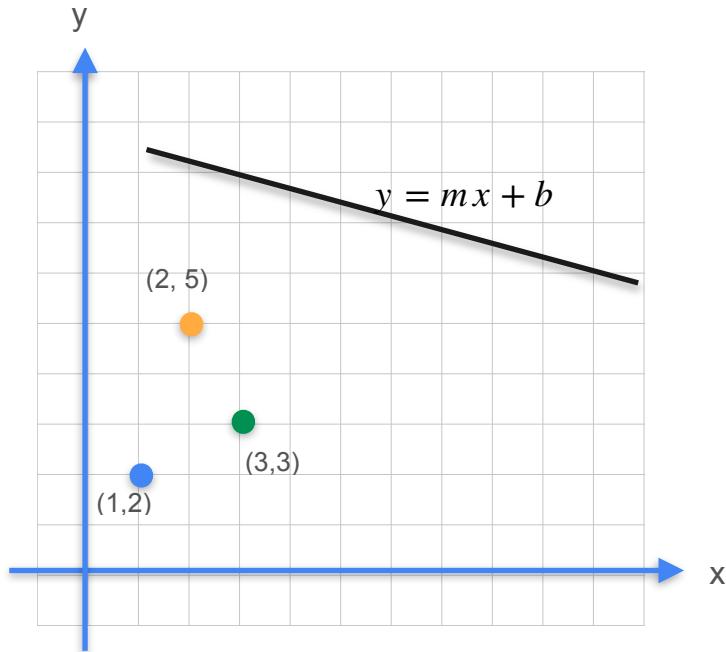
Gradient Descent



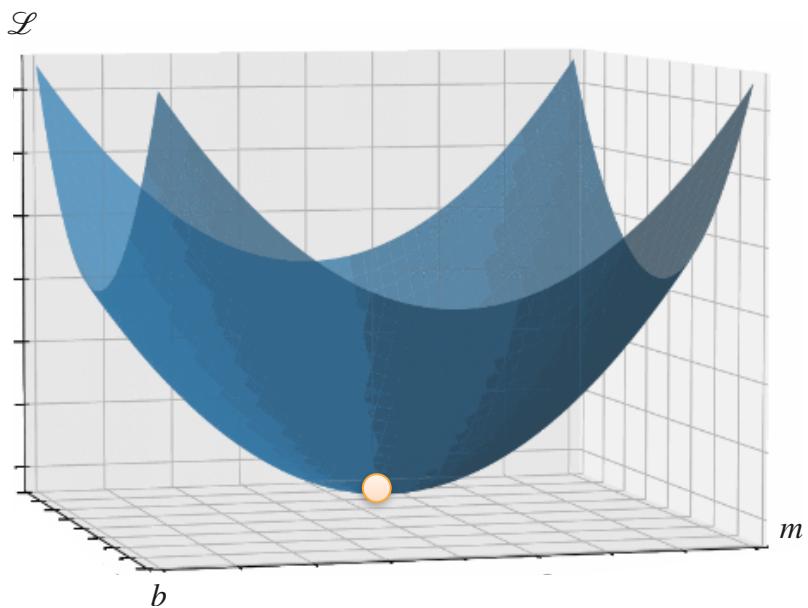
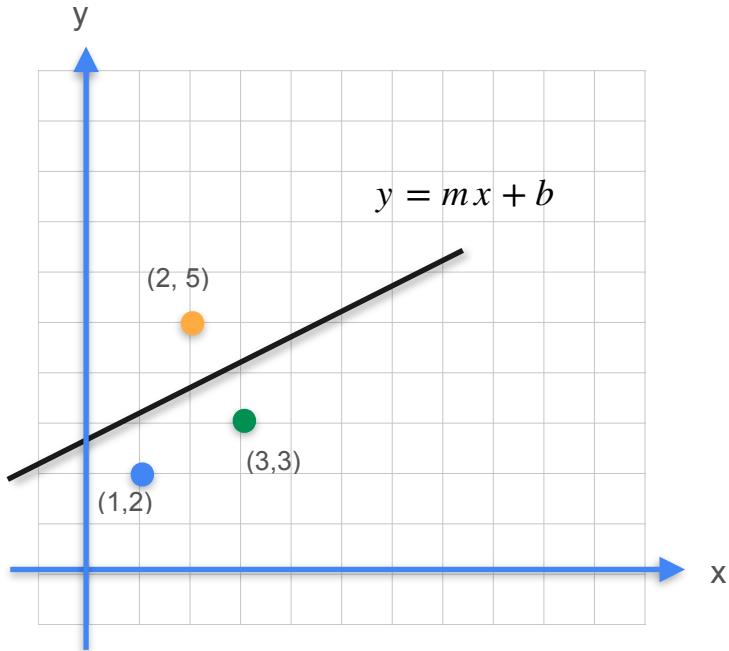
Gradient Descent



Gradient Descent



Gradient Descent



Another Example

Another Example



Another Example



TV advertisement
budget

Another Example



TV advertisement
budget



Another Example



TV advertisement
budget



Number of sales

Another Example

Another Example

TV budget

Sales

Another Example

TV budget	Sales
230.1	22.1

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

Goal: Predict sales in terms of TV budget

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
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Goal: Predict sales in terms of TV budget

Tool: Linear regression

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
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Goal: Predict sales in terms of TV budget

Tool: Linear regression

$$y = mx + b$$

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3

Goal: Predict sales in terms of TV budget

Tool: Linear regression

$$y = mx + b$$

Another Example

TV budget	Sales
230.1	22.1
44.5	10.4
17.2	9.3



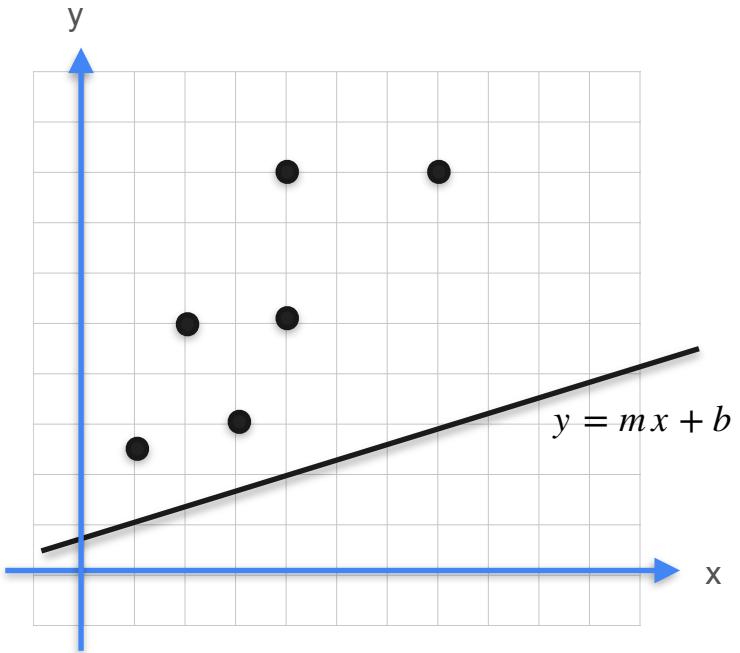
Multiple observations

Goal: Predict sales in terms of TV budget

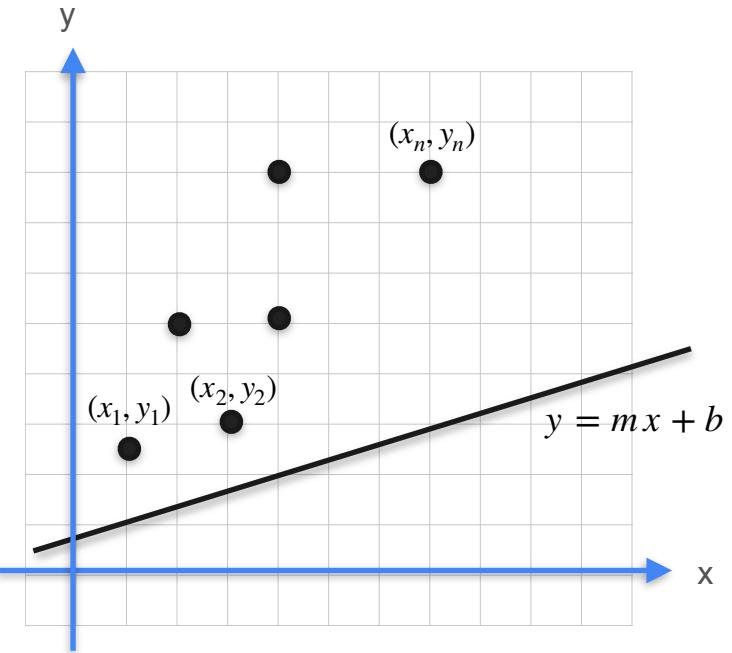
Tool: Linear regression

$$y = mx + b$$

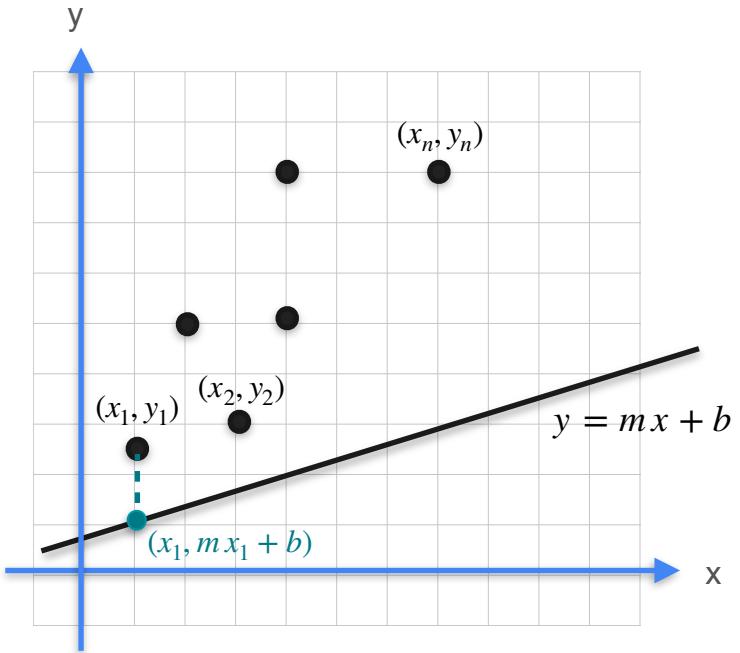
Gradient Descent



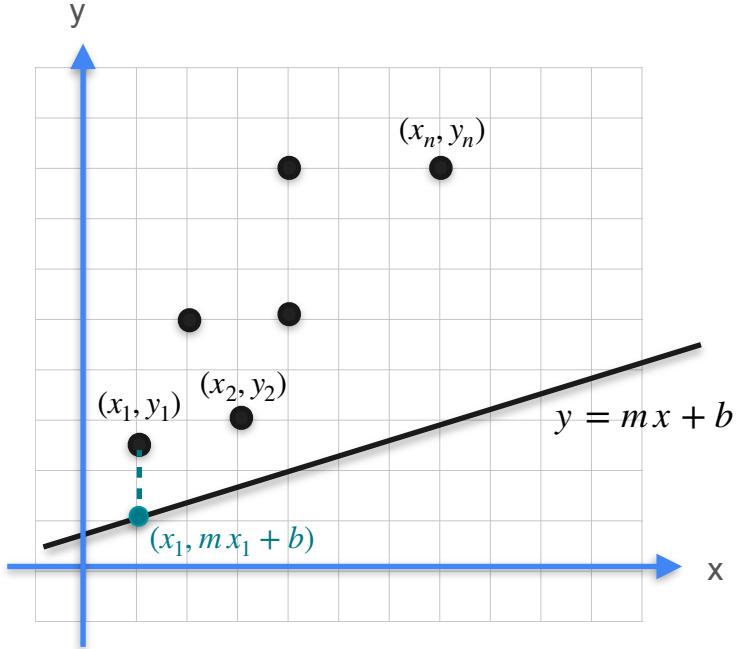
Gradient Descent



Gradient Descent

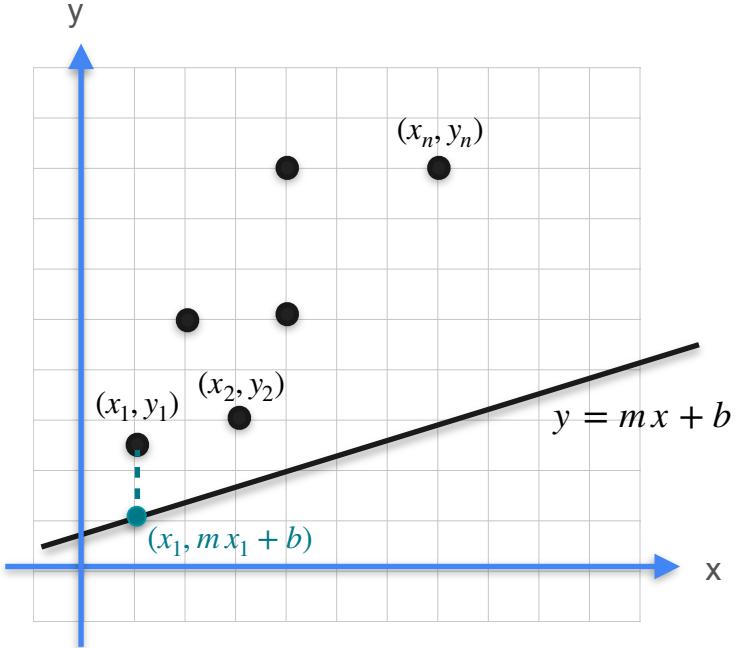


Gradient Descent



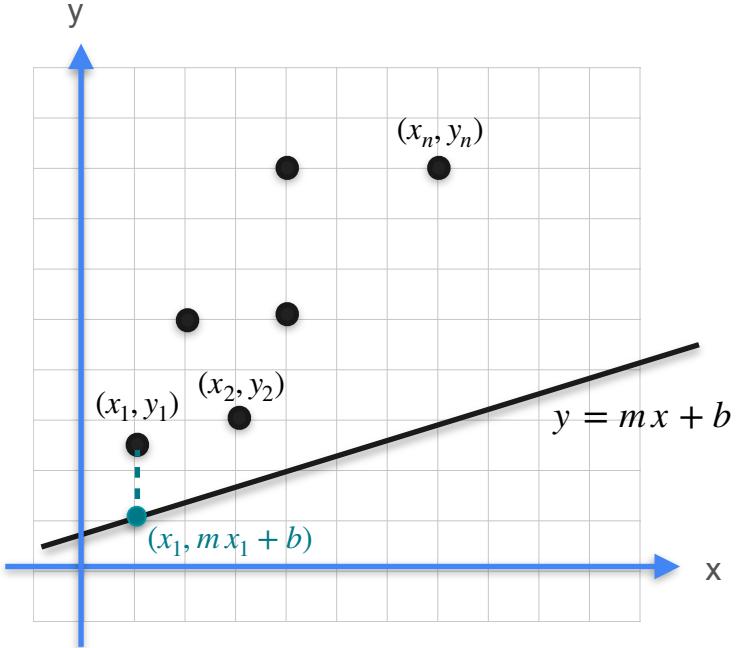
Loss
↓
 $mx_1 + b - y_1$

Gradient Descent



Loss
↓
 $(mx_1 + b - y_1)^2$

Gradient Descent



Cost

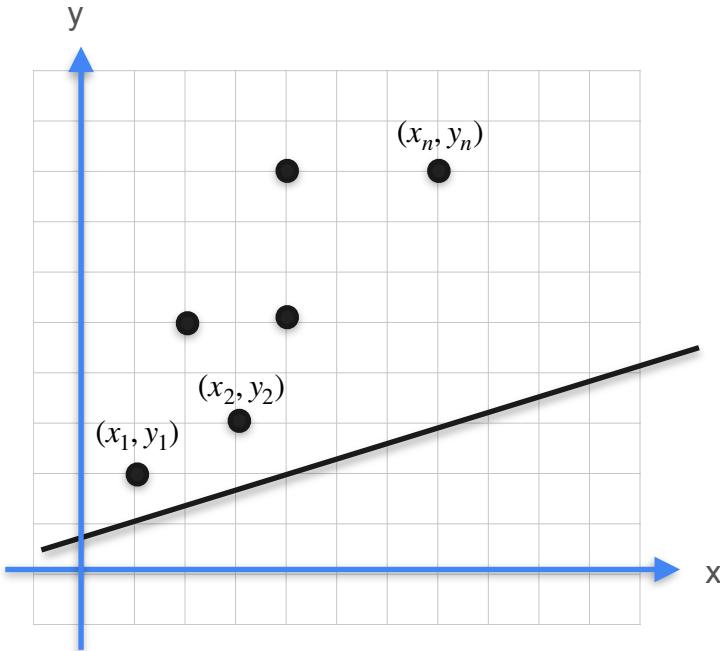


$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

Loss

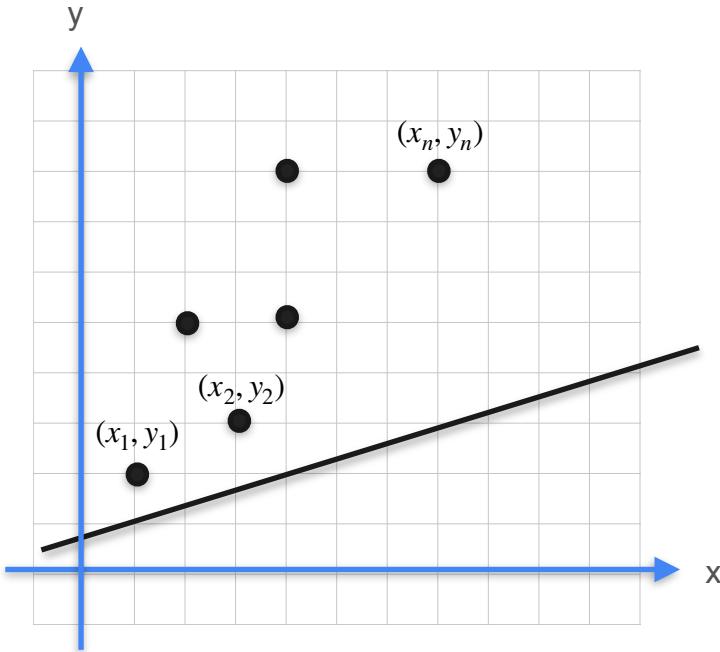


Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} \left[(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2 \right]$$

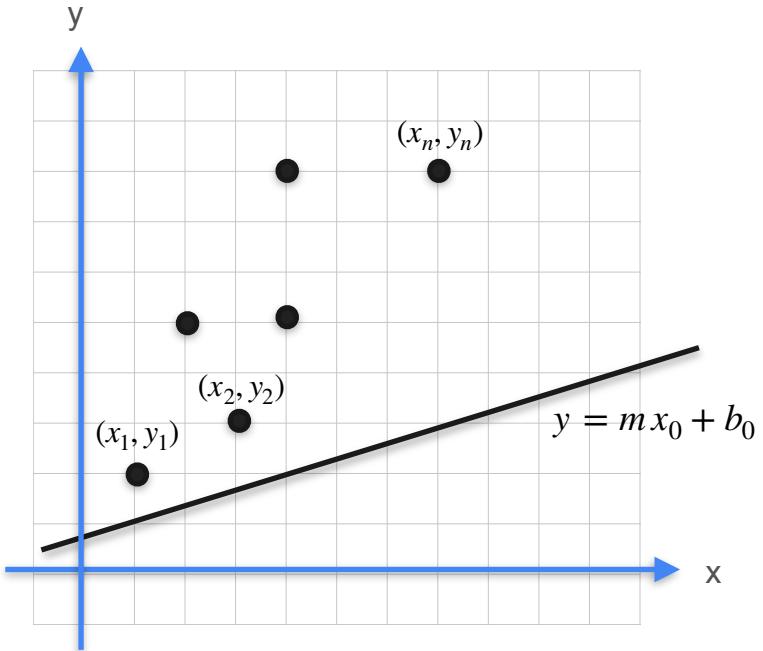
Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix}$$

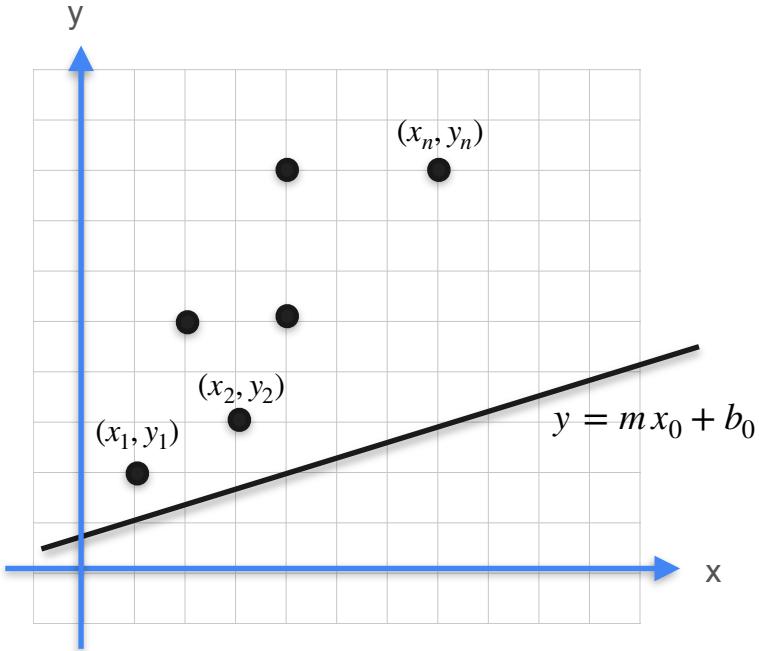
Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix}$$

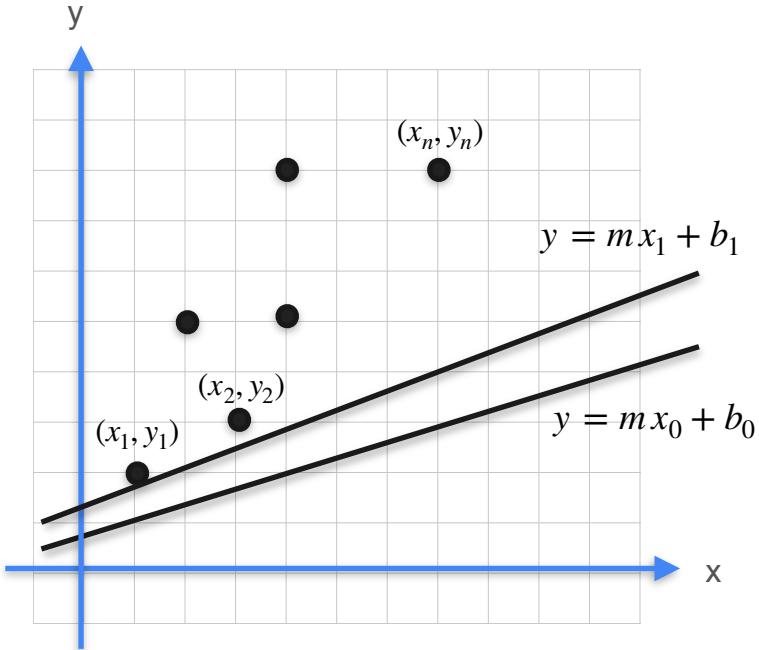
Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix} \xrightarrow{\text{ }} \begin{bmatrix} m_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} m_0 \\ b_0 \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_0, b_0)$$

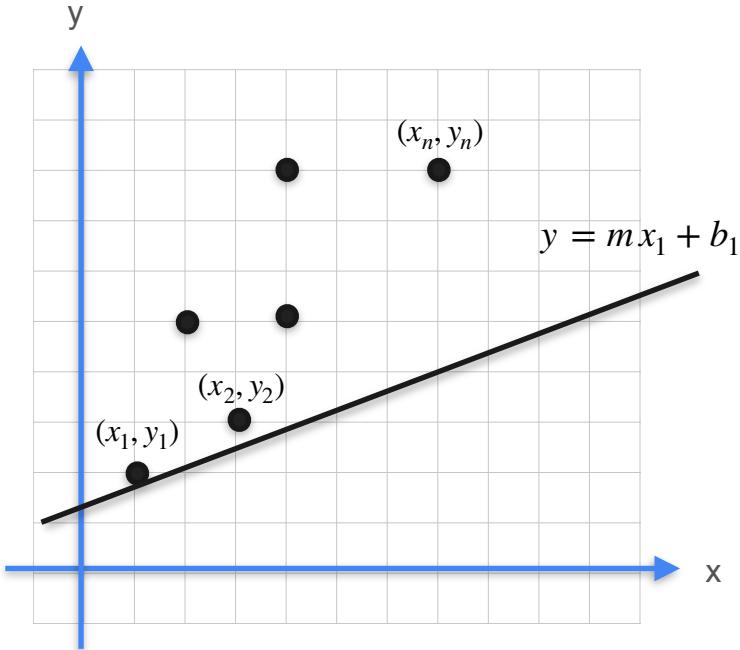
Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix} \xrightarrow{\text{ }} \begin{bmatrix} m_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} m_0 \\ b_0 \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_0, b_0)$$

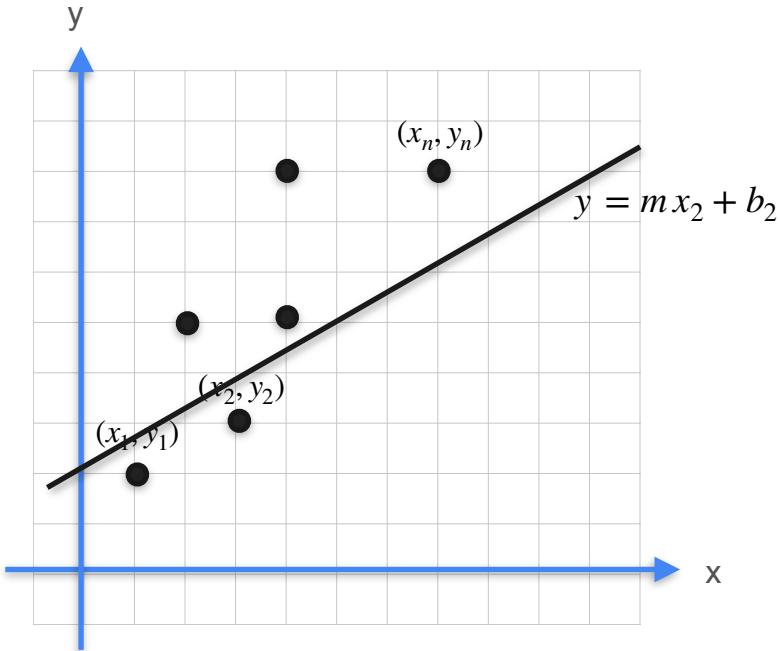
Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_0 \\ b_0 \end{bmatrix} \xrightarrow{\text{ }} \begin{bmatrix} m_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} m_0 \\ b_0 \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_0, b_0)$$

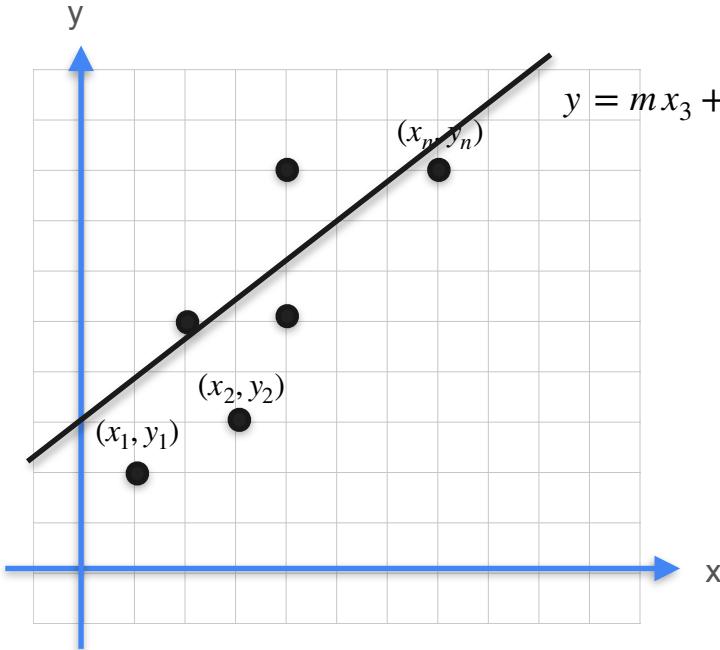
Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_1 \\ b_1 \end{bmatrix} \rightarrow \begin{bmatrix} m_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} m_1 \\ b_1 \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_1, b_1)$$

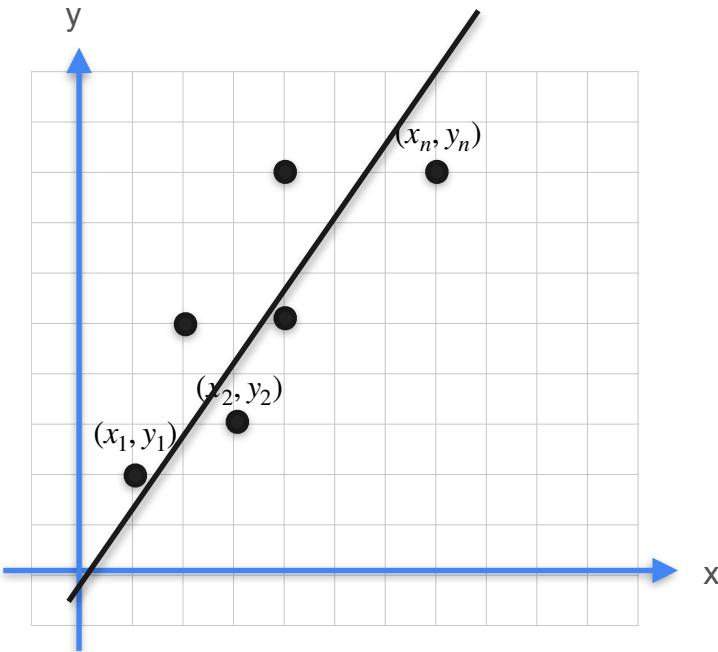
Gradient Descent



$$y = mx_3 + b_3 \quad \mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_2 \\ b_2 \end{bmatrix} \rightarrow \begin{bmatrix} m_3 \\ b_3 \end{bmatrix} = \begin{bmatrix} m_2 \\ b_2 \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_2, b_2)$$

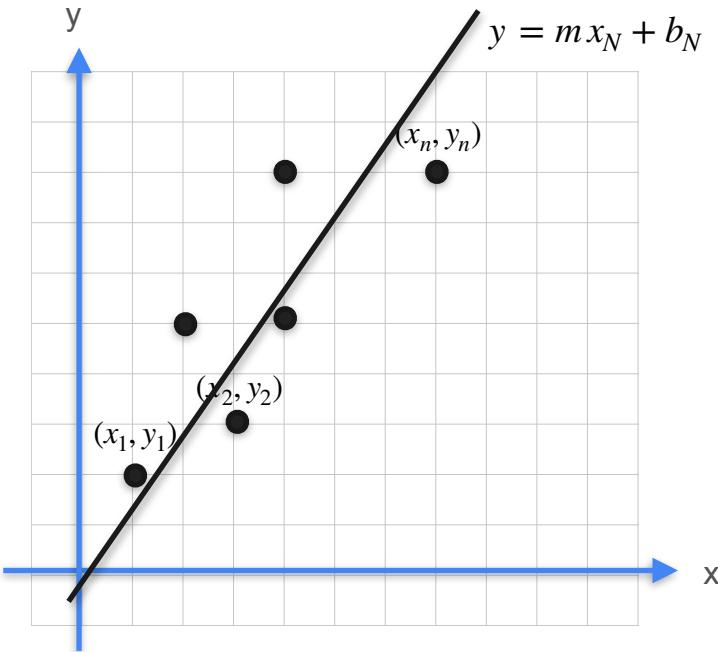
Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_N \\ b_N \end{bmatrix} \rightarrow \begin{bmatrix} m_N \\ b_N \end{bmatrix} = \begin{bmatrix} m_{N-1} \\ b_{N-1} \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_{N-1}, b_{N-1})$$

Gradient Descent



$$\mathcal{L}(m, b) = \frac{1}{2m} [(mx_1 + b - y_1)^2 + \dots + (mx_n - y_n)^2]$$

$$\begin{bmatrix} m_N \\ b_N \end{bmatrix} \xrightarrow{\text{ }} \begin{bmatrix} m_N \\ b_N \end{bmatrix} = \begin{bmatrix} m_{N-1} \\ b_{N-1} \end{bmatrix} - \alpha \nabla \mathcal{L}_1(m_{N-1}, b_{N-1})$$



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Gradients and Gradient Descent

Conclusion