ME 639 - Introduction to Robotics
Mini-project
IIT Gandhinagar

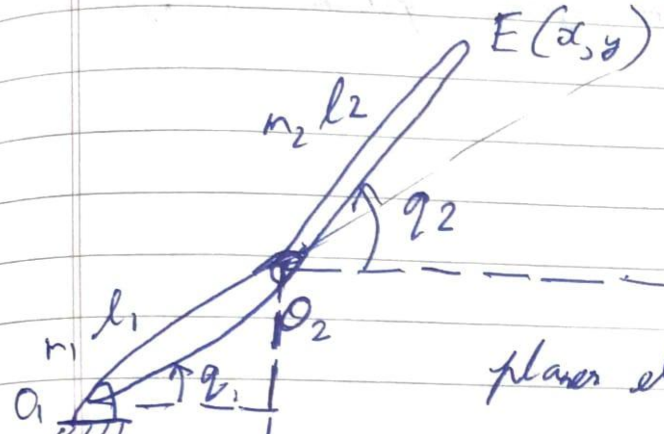
Rohan Naika
20110169

NOTE: Github link at the last page

Task 0:

The write up :

* 2R Manipulator :-



planar elbow manipulator

→ E - end effector
(x, y) - end effector position
(q_1, q_2) :- joint angles

* Assume a motor that are connected to each other links at O_1 & O_2

* Angles are sometimes θ_1, θ_2 or ρ_1, ρ_2 in various textbooks

* Task-1 -> Given arbitary trajectory:-
* of end effector (given (x,y) as a function of time
* make the robot follow this trajectory

* Task-2
-> the robot touch the wall and apply a prespected force at that location.
(constant)

* Task-3
-> Make the robo. behave like a virtual spring connected from the given point (x_0, y_0)

* <u>Mini Project</u>:-
<u>Short</u>:-
i) Trajectory following
ii) Apply a force on a wall.
iii) Act like a spring.

Now, here, for the 2R Manipulator:-

$$x = l_1 \cos q_1 + l_2 \cos q_2$$

$$y = l_1 \sin q_1 + l_2 \sin q_2$$

or using simplified notation:-

$$\left.\begin{aligned} x &= l_1 c q_1 + l_2 c q_2 \\ y &= l_1 s q_1 + l_2 s q_2 \end{aligned}\right\} - 1$$

* Differentiating 1,

$$x = -l_1 \sin q_1\, q_1 - l_2 \sin q_2\, q_2$$

$$\& \; y = l_1 c q_1\, q_1 + l_2 c q_2\, q_2$$

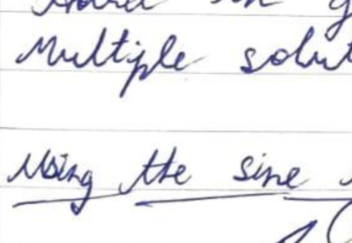
Here, end effector velocity :-

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & -l_2 s q_2 \\ l_1 c q_1 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

* For the task - i :-
We will need the reverse relationships. Given x, y we need to be able to find q_1, q_2.

i) Solve - numerically
ii) Derive a closed form expression :-
- Hard in general.
- Multiple solutions

* Using the sine rule :-



$$(x+y)^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos\theta$$

cosine rule + switching to acute angle :-

$$\therefore \theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2 l_1 l_2}\right)$$

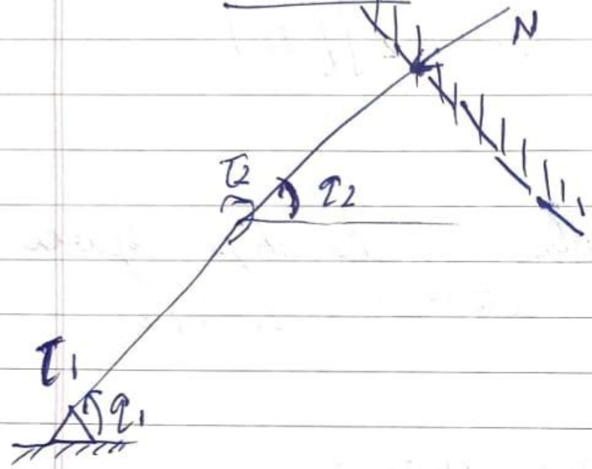
$$\underline{q_1 = \beta - \gamma} = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right)$$

$$\& \; q_2 = q_1 + \theta$$

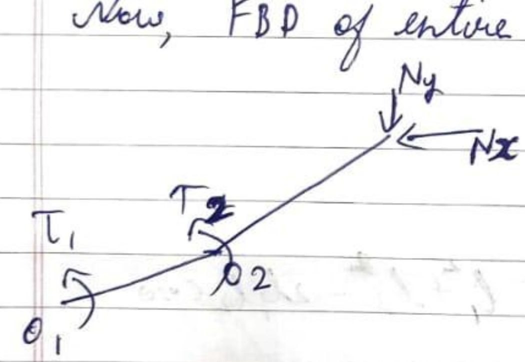
* That is the first level answer to τ_1

* Now, we will later start using the notation x_d and y_d (and q_{1d} & q_{2d}) here for actual values (they are not necessarily actual values

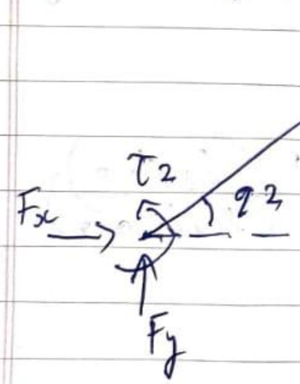
* For Task - ii)



Now, FBD of entire robot



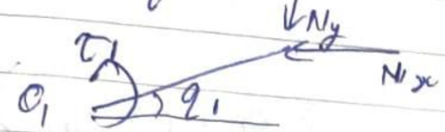
* F.B.D. for the individual hands:-



Here, $M_{O_2} = 0$

$\therefore + N_y l_2 \cos q_2 - N_x l_2 s q_2 = \tau_2$

• Now, FBD for link-1



$\Sigma M_{O_1} = 0$

$\Sigma M_O = 0 \quad N_y l_1 C q_2 - N_x l_1 S q_1 = \tau_1$

& $N_y l_2 C q_2 - N_x l_2 S q_2 = T_2$ — (11)

* In matrix form:-

$$\begin{bmatrix} \tau_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} -l_1 Sq_1 & l_1 Cq_2 \\ -l_2 Sq_2 & l_2 Cq_2 \end{bmatrix} \begin{bmatrix} N_x \\ N_y \end{bmatrix} \quad \text{— (4)}$$

• For T_3 (task-3) and next level answer to t,

× Understanding of dynamics:

→ Lagrange's equation:-

$\mathcal{L} = K - V$

$K \rightarrow$ kinetic & $V \rightarrow$ potential energy

$$\therefore \quad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i'$$ (Q_i' are generalized forces derived using principle of natural work)

Now, $K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2}_{\text{pure rotation of } L_1} + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2 + \frac{1}{2} m_2 v_{C_2}^2$

and here, $V_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2 \dot{q}_2}{2}\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2 \dot{q}_2}{2} \cos(q_2 - q_1)$

$\therefore \quad V = m_1 g \frac{l_1}{2} s q_1 + m_2 g \left(l_1 s q_1 + \frac{l_2}{2} s q_2\right)$

Here, $\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1)$

$\sin(q_2 - q_1) + m_1 g \frac{l_1}{2} c q_1 + m_2 g l_1 c_1$

Now, $\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) = \tau_1$

$- m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$

$+ m_2 g \frac{l_2}{2} s q_2 = \tau_2$

* Saving this, for some time later.

* $\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + \cdot (\dot{q}^2) \quad \dot{q}(x) \; cq = \tau_1$

$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + \ddot{q}_1 - \ldots \dot{q} \ldots sq = \tau_2$

Here, we note that ④ is valid for any forces F_x, F_y

and $F_x = kx$ $\left[\text{more generally } F_x = k_x(x - x_2)\right]$
$F_y = ky$ $\quad F_y = k_y(y - y_2)$

* From - ①

$F_x = k(l_1 c q_1 + l_2 c q_2), \quad F_y = k(l_1 s q_1 + l_2 s q_2)$

Now, from ④

$$k(l_1 sq_1 + l_2 sq_2) l_2 cq_2 - k(l_1 cq_1 + l_2 cq_2) l_2 sq_2 = \tau_2$$

$$k(l_1 sq_1 + l_2 sq_2) l_1 cq_1 - k(l_1 cq_1 + l_2 cq_2) l_1 sq_1 = \tau_{1s}$$

Now, set motor torques to be $\tau_1 + \tau_{1s}$ and $\tau_2 + \tau_{2s}$, respectively ! → Answer to T_3

* Either way to tackle:-
Solve that q_{1s} & q_{2d} from ③ → $\dot{q}_{1d}, \ddot{q}_{1d}$
↓
τ_1 & τ_2 from ③

* We'll still feedback control:-

→ What does go wrong or what happens wrong with no dynamics and statics?

→ What's wrong with getting force and position control sensation simultaneously.

* Motor control:-

→ What needs to be done to the motor level to control torque and achieve tracking?

- Mechanical & electrical dynamics:-

$$J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} = \tau_m - \tau l/r \quad \& \quad L\frac{di}{dt} + Ri = V - V_b = V - K_b \frac{d\theta_m}{dt}$$

r ↓ gear ratio

Task 1, 2, 3 and 4 :
Github repository link -
https://github.com/rohannaika/20110169_Mini-Project1_.git