Divided Difference Table

x	y_x	Δy_x	$\Delta^2 y_x$	$\Delta^3 y_x$	$\Delta^4 y_x$
x_0	y_0				
		$\Delta y_0 = \frac{y_1 - y_0}{x_1 - x_0}$			
x_1	y_1		$\Delta^2 y_0 = \frac{\Delta y_1 - \Delta y_0}{x_2 - x_0}$		
		$\Delta y_1 = \frac{y_2 - y_1}{x_2 - x_1}$		$\Delta^3 y_0 = \frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0}$	
x_2	y_2		$\Delta^2 y_1 = \frac{\Delta y_2 - \Delta y_1}{x_3 - x_1}$		$\Delta^4 y_0 = \frac{\Delta^3 y_1 - \Delta^3 y_0}{x_4 - x_0}$
		$\Delta y_2 = \frac{y_3 - y_2}{x_3 - x_2}$		$\Delta^3 y_1 = \frac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1}$	
x_3	y_3		$\Delta^2 y_2 = \frac{\Delta y_3 - \Delta y_2}{x_4 - x_2}$		$\Delta^4 y_1 = \frac{\Delta^3 y_2 - \Delta^3 y_1}{x_5 - x_0}$
		$\Delta y_3 = \frac{y_4 - y_3}{x_4 - x_3}$		$\Delta^3 y_2 = \frac{\Delta^2 y_3 - \Delta^2 y_2}{x_5 - x_2}$	
x_4	y_4		$\Delta^2 y_3 = \frac{\Delta y_4 - \Delta y_3}{x_5 - x_3}$		
		$\Delta y_4 = \frac{y_5 - y_4}{x_5 - x_4}$			
x_5	y_5				

$(x_0, y_0), (x_1, y_1), (x_2, y_2), \ldots\ldots$ be the given points.

$$\Delta y_{x_1}(y_0) = [x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0}$$

$$\Delta y_{x_2}(y_1) = [x_1, x_2] = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Delta y_{x_3}(y_2) = [x_2, x_3] = \frac{y_3 - y_2}{x_3 - x_2}$$

& so on

} First Divided Differences

SECOND DIVIDED DIFFERENCES

$$\Delta^2 y_0 = [x_0, x_1, x_2] = \frac{[x_1, x_2] - [x_0, x_1]}{x_2 - x_0} = \frac{\Delta y_1 - \Delta y_0}{x_2 - x_0}$$

$$\Delta^2 y_1 = [x_1, x_2, x_3] = \frac{[x_2, x_3] - [x_1, x_2]}{x_3 - x_1} = \frac{\Delta y_2 - \Delta y_1}{x_3 - x_1}$$

$$\Delta^2 y_2 = [x_2, x_3, x_4] = \frac{[x_4, x_3] - [x_2, x_3]}{x_4 - x_2} = \frac{\Delta y_3 - \Delta y_2}{x_4 - x_2}$$

& so on

THIRD DIVIDED DIFFERENCES

$$\Delta^3 y_0 = \frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0}$$

$$\Delta^3 y_1 = \frac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1}$$

$$\Delta^3 y_2 = \frac{\Delta^2 y_3 - \Delta^2 y_2}{x_5 - x_2}$$

& so on

Newton's Divided Difference Interpolation formula :

$$y_x = y_0 + (x - x_0)\Delta y_0 + (x - x_0)(x - x_1)\Delta^2 y_0 + (x - x_0)(x - x_1)(x - x_2)\Delta^3 y_0 + (x - x_0)(x - x_1)(x - x_2)(x - x_3)\Delta^4 y_0 + \cdots\cdots\cdots + (x - x_0)(x - x_1)\cdots\cdots(x - x_{n-1})\Delta^n y_0$$

Que : Using divided difference formula, find a polynomial satisfying the following data

	x	y_x	Δy_x	$\Delta^2 y_x$	$\Delta^3 y_x$	$\Delta^4 y_x$
x_0	-4	1245 y_0				
			-404 $\Delta y_0 = \frac{y_1 - y_0}{x_1 - x_0}$			
x_1	-1	33 y_1		94 $\Delta^2 y_0 = \frac{\Delta y_1 - \Delta y_0}{x_2 - x_0}$		
			-28 $\Delta y_1 = \frac{y_2 - y_1}{x_2 - x_1}$		-14 $\Delta^3 y_0$	
x_2	0	5 y_2		10 $\Delta^2 y_1 = \frac{\Delta y_2 - \Delta y_1}{x_3 - x_1}$		3 $\Delta^4 y_0$
			2 $\Delta y_2 = \frac{y_3 - y_2}{x_3 - x_2}$		13 $\Delta^3 y_1$	
x_3	2	9 y_3		88 $\Delta^2 y_2 = \frac{\Delta y_3 - \Delta y_2}{x_4 - x_2}$		
			442 $\Delta y_3 = \frac{y_4 - y_3}{x_4 - x_3}$			
x_4	5	1335 y_4				

Newton's divided difference formula is given by

$$y_x = y_0 + (x - x_0)\Delta y_0 + (x - x_0)(x - x_1)\Delta^2 y_0 + (x - x_0)(x - x_1)(x - x_2)\Delta^3 y_0 + (x - x_0)(x - x_1)(x - x_2)(x - x_3)\Delta^4 y_0$$

or $$y_x = 1245 + (x + 4)(-404) + (x + 4)(x + 1)(94) + (x + 4)(x + 1)(x - 0)(-14) + (x + 4)(x + 1)x(x - 2)(3)$$

$$= 3x^4 - 5x^3 + 6x^2 - 14x + 5$$

$$f(1) = y_1 = 3 - 5 + 6 - 14 + 5 = -5 \text{ Ans}$$