Value of y_x with the help of forward differences :

$$f(x) = y_x = E^x y_0 = (1+\Delta)^x y_0$$

$$= \left(1 + {}^xC_1 \Delta + {}^xC_2 \Delta^2 + {}^xC_3 \Delta^3 + \cdots\cdots\right) y_0$$

$$\Rightarrow \boxed{y_x = y_0 + {}^xC_1 \Delta y_0 + {}^xC_2 \Delta^2 y_0 + {}^xC_3 \Delta^3 y_0 + \cdots\cdots}$$

This formula enables us to know y_x provided we know the leading term and its differences

➡ eg evaluate 6 th term ie y_5 (y_0, y_1, y_2, y_3, y_4 being 1st 2nd 3rd 4th 5th terms)

$$y_5 = {}^5C_0 y_0 + {}^5C_1 \Delta y_0 + {}^5C_2 \Delta^2 y_0 + {}^5C_3 \Delta^3 y_0 + {}^5C_4 \Delta^4 y_0 + {}^5C_5 \Delta^5 y_0$$

➡ Note : finding y_0 given other entries i.e y_1, y_2, y_3, y_4

$$y_0 = E^{-1} y_1$$

$$= (1+\Delta)^{-1} y_1$$

$$= (1 - \Delta + \Delta^2 - \Delta^3 + \Delta^4 - \cdots\cdots) y_1$$

$$= y_1 - \Delta y_1 + \Delta^2 y_1 - \Delta^3 y_1 + \cdots\cdots$$

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Q Find the 7^{th} term of the sequence 2, 9, 28, 65, 126, 217 and also find the general term.

Sol.

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1^{st}	0	$\boxed{2}\ (y_0)$				
2^{nd}	1	9 (y_1)	$\boxed{7}\ (\Delta y_0)$			
3^{rd}	2	28 (y_2)	19	$\boxed{12}\ (\Delta^2 y_0)$		
4^{th}	3	65 (y_3)	37	18	$\boxed{6}\ (\Delta^3 y_0)$	
5^{th}	4	126 (y_4)	61	24	6	$\boxed{0}\ (\Delta^4 y_0)$
6^{th}	5	217 (y_5)	91	30	6	0

7^{th} term $= y_6 = E^6 y_0 = (1+\Delta)^6 y_0$

$= y_0 + {}^6C_1 \Delta y_0 + {}^6C_2 \Delta^2 y_0 + {}^6C_3 \Delta^3 y_0$
$+ {}^6C_4 \Delta^4 y_0 + {}^6C_5 \Delta^5 y_0 + {}^6C_6 \Delta^6 y_0$

$= 2 + 6(7) + \frac{6(5)}{2!}(12) + \frac{6(5)(4)}{3!}(6) + \frac{6(5)(4)(3)}{4!}(0)$

$= 2 + 42 + 15(12) + 20(6) + 15(0)$

$= 344$

General term

$y_n = y_0 + {}^nC_1 \Delta y_0 + {}^nC_2 \Delta^2 y_0 + {}^nC_3 \Delta^3 y_0 + {}^nC_4 \Delta^4 y_0 + \cdots$

$= 2 + n(7) + \frac{n(n-1)}{2!}(12) + \frac{n(n-1)(n-2)}{3!}(6) + 0$

$= n^3 + 3n^2 + 3n + 2$

Verification : $y_6 = (6)^3 + 3(6)^2 + 3(6) + 2 = 344$

Q Find the first term of the series whose second and subsequent terms are 8, 3, 0, −1, 0

Sol.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	~~$\Delta^4 y$~~
0	y_0	....			
1	$\boxed{8}$ (y_1)		....		
		$\boxed{-5}$ Δy_1		....	
2	3 (y_2)		$\boxed{2}$ $\Delta^2 y_1$		
				$\boxed{0}$ $\Delta^3 y_1$	
3	0 (y_3)	−3			
			2		
4	−1 (y_4)	−1		0	
			2		
		1			
5	0 (y_5)				

Now $\quad y_0 = E^{-1} y_1$

$= (1+\Delta)^{-1} y_1$

$= (1 - \Delta + \Delta^2 - \Delta^3 + \Delta^4 - \cdots\cdots) y_1$

$= y_1 - \Delta y_1 + \Delta^2 y_1 - \Delta^3 y_1$

$= 8 - (-5) + 2 - 0 = 15$

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$$\nabla y_n = y_n - y_{n-1}$$

$$\Rightarrow \quad y_{n-1} = y_n - \nabla y_n$$

$$= (1-\nabla) y_n$$

Similarly $\quad y_{n-2} = y_{n-1} - \nabla y_{n-1}$

$$= (1-\nabla) y_{n-1}$$

$$= (1-\nabla)(1-\nabla) y_n$$

$$= (1-\nabla)^2 y_n$$

& $\quad y_{n-3} = (1-\nabla)^3 y_n$

Generalising this concept

$$y_{n-k} = (1-\nabla)^x y_n$$

$$= (1 - {}^xC_1 \nabla + {}^xC_2 \nabla^2 + \cdots + (-1)^x \nabla^x) y_n$$

$$= y_n - {}^xC_1 \nabla y_n + {}^xC_2 \nabla^2 y_n + \cdots + (-1)^x \nabla^x y_n$$

eg: find y_{-1} & y_0 (given 6 data points
$y_0, y_1, y_2, y_3, y_4, y_5$)

$y_{-1} = y_{5-6}$. Here $n=5$, $x=6$

$y_0 = y_{5-5}$ Here $n=5$, $x=5$