Introduction to Quantum Mechanics

Classical Mechanics explained about the motions of
the objects or particles characterised by mass and velocity.
(Newton's Laws of Motion). These particles are either directly obser-
vable or can be observed under microscope. When these classical
concepts were applied to electrons (atomic dimensions), the
actual behaviour was not precisely explained. Hence classical
Mechanics cannot be applied to atomic dimensions.

Classically speaking, when light negatively charged electrons go round
heavy positively charged nucleus in circular orbits, the
electrons experiences an attractive electrostatic force due to
which the distance between them decreases. Hence the
orbiting electron experiences centripetal acceleration. Therefore
the energy of the electron decreases continuously (because
an accelerated charged particle radiates energy continuously)
comes closer and closer to the nucleus and finally falls
into the nucleus collapsing the structure and the stability
of atom. Hence classical Mechanics failed to explain the
stability of the atom.

Classical Mechanics failed to explain the spectral
series of Hydrogen atom. The Hydrogen spectrum (experimentally)
consists of discrete set of lines represented by wavenumber

$$\bar{\nu} = R\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right] \quad (n_1 > n_2) \text{ (R - Rydberg Constant)}$$

This shows that electromagnetic radiations of certain
definite wavelengths will be emitted (but not continuously)

Similarly classical theory was not capable of explaining Spectral radiancy (distribution) of a black body.

Max Planck in 1900 (Dec. 14) at a meeting of German Physical Society read his paper "On the ~~way~~ ~~The~~ Theory of the Energy Distribution Law of the Normal Spectrum". This was the Start of revolution of Physics "Quantum Mechanics" (the date of presentation). Later on, Modern Quantum Mechanics was developed by Schrodinger and others. Many paths converged depicting the failure of classical Mechanics. (1st paper - 1900 - On an improvement of Wien's Equation for the Spectrum)

*** Quantum Physics is a generalisation of classical Physics that includes classical laws as special cases.

*** Quantum Physics extends that range to the region of small dimensions.

*** Just as 'c', the vel of light signifies universal constant, the Planck's constant characterizes Quantum Physics. This was introduced when he was explaining the properties of Thermal Radiation in his paper in 1900.

($h = 6.65 \times 10^{-27}$ erg sec

$h = 6.625 \times 10^{-34}$ Joule sec)

Max Planck was awarded Nobel prize in 1918 for his discovery of energy quanta. His Nobel lecture was on "The Genesis and present state of development of Quantum Theory".

Quantum Mechanics was much ~~successful~~ successful in giving both the qualitative and quantitative explaination of

(i) Photo Electric Effect (1905)
(ii) Black body Radiation (1901)
(iii) Emission of line spectra (1913)
(iv) Compton Effect (1924)

Along with explaination of the phenomena of Interference, diffraction and polarisation on electro-magnetic Theory. Later on confirmed the idea of wave particle duality.

The most outstanding development in modern science was the conception of Quantum Mechanics in 1925 (began in 1900) This new approach was highly successful in explaining about ~~ato~~ the behaviour ~~of~~ of atoms ~~molecules~~ molecules and nuclei.

The basic idea of Quantum Theory was first introduced by Max Planck and further developments and interpretations were made by Einstein, Bohr, Schrodinger, de-Broglie, Heisenberg, Born and Dirac.

CHRONOLOGICAL DEVELOPMENT OF QUANTUM THEORY

1900 paper - "On an Improvement of Wien's Equation for the Spectrum"
Planck guessed black body radiation formula.

paper. "On the theory of the Energy distribution law of Normal Spectrum", Planck derived the radiation formula.

1905 paper "On a heuristic point of view about the creation and conversation of light" - Einstein introduced light quanta" Explains about black body radiation, production of electrons by ultraviolet light (photoelectric Effect)
"when a light ray starting from a point is propagated, the energy is not continuously distributed over an ever increasing volume, but it consists of a finite number of ~~eq~~ energy quanta, localised in space, which move without being divided and which can be absorbed or emitted as a whole".
Einstein received Nobel prize in 1921 for his discovery of Law of Photo Electric Effect but not for his (Special) Theory of Relativity.

1913 - paper "On the Constitution of Atoms and Molecules - Bohr gave the first successful interpretation of atomic Spectra. The Quantisation of angular Momenta was given in the subsequent paper in the same year. Bohr was awarded the 1922 Nobel prize in physics for his investigation in Structure of atoms and radiation emanating from them.

1914 - paper - "On the excitation of 2536 A° mercury resonance line by electron collisions" - Frank & Hertz showed that by varying the kinetic energies of the electrons hitting an atom it is possible to produce controlled excitation of atoms and Molecules.
(Discrete energy states of atoms).
They received Nobel prize for the year 1925 for the discovery of the laws ~~of~~ governing the impact of an electron upon an atom.

1917 - Paper "On the Quantum Theory of Radiation"
Einstein put forward the existence of stimulated emission and introduced Einstein's ~~coffe~~ A & B coefficients.

1922 - paper. Stern & Gerlach performed the experiment, in the presence of strong inhomogeneous magnetic field, a beam of ~~sti~~ silver atoms split into two beams, (Proof of Space Quantisation)

1923 - Compton reported his studies on scattering of X-rays by solid materials (compton effect) for which he received Nobel prize in 1927.

1924 - paper - "Investigations on Quantum Theory" - de-Broglie suggested the idea of waves associated with electrons.
"In order to have a stable state, the length of the channel must be in resonance with the wave.
He was awarded 1929 Nobel prize in physics for his 'discovery of the wave nature of Electrons".

1. Newton's Cospusculas Theory of Light : Interference, Diffraction & Polarisation - Not explained

2. Huygen's Wave Theory of Light : Light - hypothetical Ether Medium Explained above three.

3. Maxwell's Electromagnetic Theory :

4. Some expts couldn't be explained by above three. : PhotoElectric, compton Effect, emission, Absorption of Light.

4. This gave birth to Quantum Theory of Light. (Max Planck).

Matter
↓
many no. of Oscillating Particles → Vibrate with any Frequency (classically)
↓ ↓
Vibrate with different frequencies (Quantum) radiate energy continuously (classically)
↓
Energy is Quantised
$E = n(h\nu)$
↓
Vibrating particle do not radiate energy continuously (but discontinuously) - moves from one quantised State to other.
↓
No radiation when it is in a Quantized State.
↓
Exchange of energy between light & matter is not continuous.

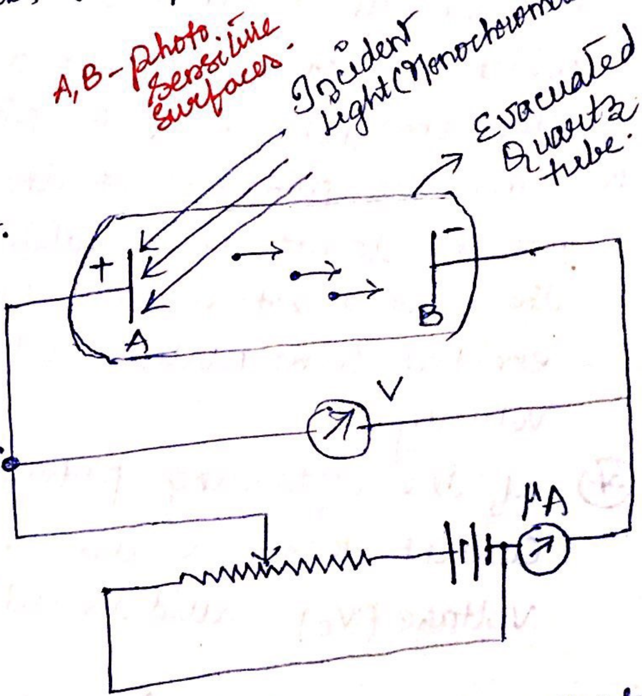
1

Photo Electric Effect.

The emission of electrons from a metal plate when illuminated by light or any other radiation of any wavelength (suitable) or frequency [ultraviolet light on zinc plate and ordinary light on alkali metals like lithium, sodium, potassium...] is called "Photoelectric Effect". The emitted electrons are called as "Photo Electrons".

This was discovered by Hertz when ultraviolet light was incident on zinc plates, sparks were produced in the air gap of his transmitter.

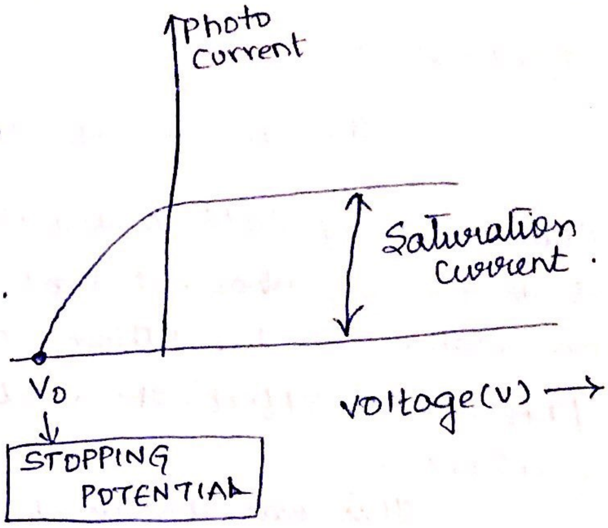
'A' and 'B' are two photosensitive surfaces enclosed in an evacuated Quartz chamber. These two acts as the electrodes as they are connected to variable supply. The metal plate on which the light strikes is the anode (A). The photoelectrons that are released from 'A' have sufficient energy to reach the cathode (B) though there is repulsion between cathode and (slower) photoelectrons. This constitutes the Photocurrent or Photoelectric current.



The number of photoelectrons emitted and their kinetic energy depends on the following factors.

(i) Potential Difference between two Electrodes A & B.
(ii) Intensity of Incident Radiation
(iii) The Frequency of Incident Radiation
(iv) The photometal used.

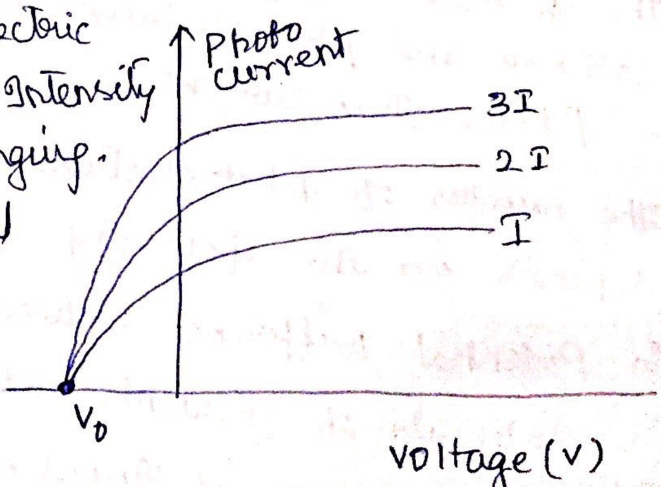
(i) Effect of Potential Difference



- when the positive potential is increased, the photoelectric current increases and reaches to a saturation value (Maximum current).
- Further increase in potential do not produce any increase in current.
- Though the potential difference is zero, there is some current, due to incident radiation which not only provides a conducting path but also an electromagnetic force to photo electrons.
- when the potential of the plate is made negative, the photo current do not immediately drop down to zero but flows in the same direction. This shows that photo electrons are emitted from plate (on which light is incident) with finite velocity.
- If the retarding potential is further increased, the photo current decreases and becomes zero at a particular value for voltage (V_0), called as cut-off potential or stopping potential.

(ii) Effect of Intensity of Incident Radiation

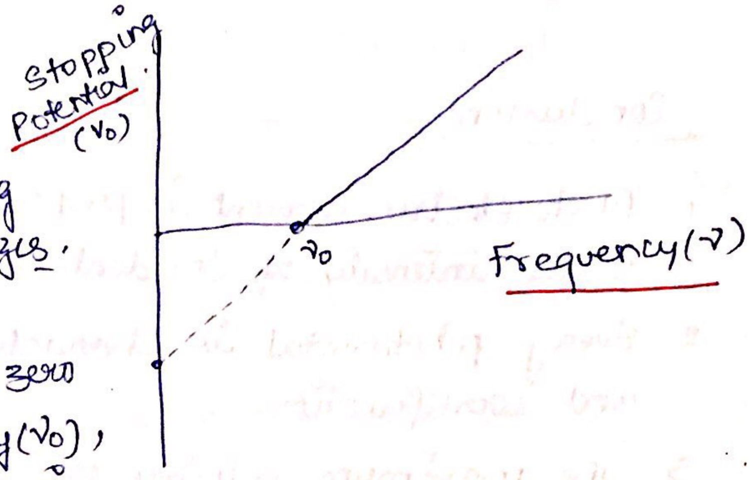
This is the variation of photoelectric current with voltage when the Intensity of Incident ~~r~~ Intensity is changing. The photo current is proportional to Intensity ie the saturation current increases. But the stopping potential is same for all intensities of light of the same frequency 'ν'.



This confirms that the stopping potential is independent of the intensity of incident light.

(iii) The effect of Frequency of incident Radiation

For the same photo metal and same intensity of incident radiation, when the frequency of incident radiation is changing how the stopping potential changes.



The frequency (ν_0) where the Stopping potential (V_0) becomes zero is called the Threshold frequency (ν_0), and the ~~Threshold~~ corresponding wavelength (λ_0) is called Threshold wavelength.

The frequency above which photo electric effect happens and stops below this frequency is called "Threshold Frequency":

Hence ~~This~~ Threshold freq is defined as the minimum frequency (ν_0) of the incident radiation that can produce Photo Electric emission (or) the frequency capable of liberating photoelectrons without giving any additional energy.

Basing on this Threshold or critical Frequency, the minimum energy ϕ for an electron to escape from a metal surface is called "Work Function", given by $\phi = h\nu_0$.

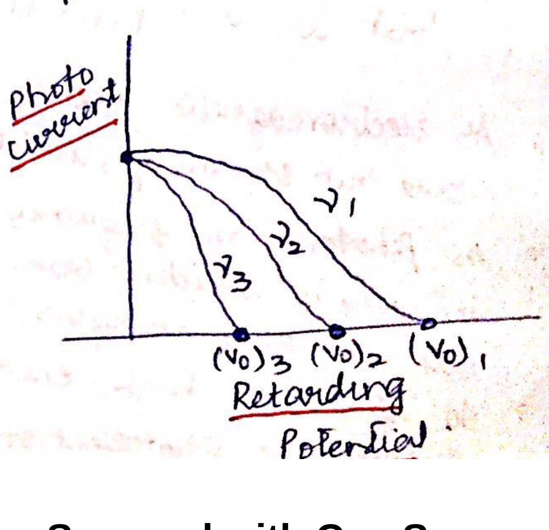
Greater is the workfunction of a metal, more energy is required for an electron to leave the metal surface and hence higher is the Threshold Frequency for photoelectric emission.

$\nu_1 > \nu_2 > \nu_3$

Intensity of light is constant.

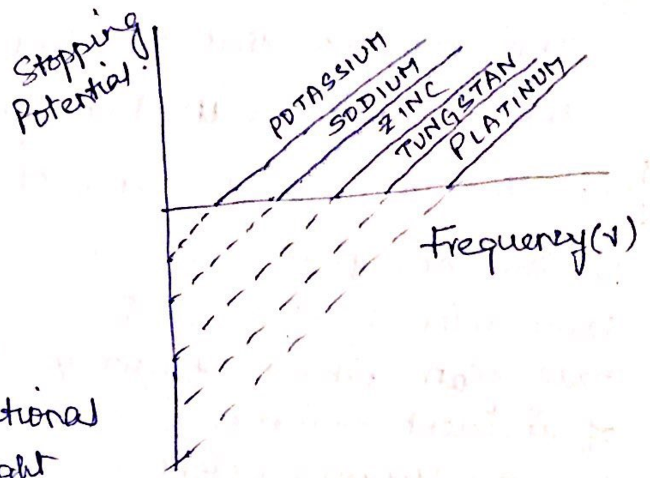
Photo current = Saturation (constant)

$(V_0)_1 > (V_0)_2 > (V_0)_3$.



(iv) Effect of Photometal

The graphical variation shows that the Threshold Frequency is a function of photometal.



Conclusions

1. Photo Electric current is proportional to the intensity of Incident light
2. Every photometal is characterised by Threshold Frequency and workfunction.
3. The maximum velocity or the kinetic energy of the photo electrons depends on the frequency of the radiation but not on the intensity of incident radiation.
4. The rate at which photoelectrons are emitted from photo metal is independent of Temperature. This shows That Photo Electric Effect & Thermoionic emission are two different Phenomena.
5. Photoelectron emission from a photosensitive surface is instantaneous and emission continues as long as the frequency of the incident light is greater than that of the Threshold Frequency. The time lag between incidence and emission is less than 10^{-8} seconds.
6. ~~Threshold~~ Stopping Potential is proportional to Frequency but is independent of intensity of incident light.

* As Electromagnetic Energy is concentrated in photons (not spread out as a wave but stream of particles), There is no delay in emission of Photoelectrons

* As photons of frequency 'ν' have an energy of $h\nu$ (constant), hence if ~~intens~~ intensity of incident beam is changed, The number of photoelectrons changes but not their energies.
So if ν is high, energy of photons will be high and hence max energy the Photoelectrons have.

EINSTEIN'S PHOTO ELECTRIC EQUATION.

From Planck's idea that light comprises of Photons, Einstein explained Photoelectric effect, where one photon is completely absorbed by one electron which gains quantum of energy and be emitted from the metal.

The photon's energy is utilised in two ways.

(i) A part of energy is used to free the electron from the atom and away from the metal surface, known as Photo Electric Work Function (W_0)

(ii) Other part is utilised in increasing the kinetic energy of the emitted electron. ($\frac{1}{2} mv^2$).

ie $$\boxed{h\nu = W_0 + \frac{1}{2} m v^2.}$$

This is known as EINSTEIN'S PHOTOELECTRIC EQUATION.

(or) $$h\nu = W_0 + KE_{Max}.$$
$$= h\nu_0 + KE_{Max}.$$

(or) $$\boxed{KE_{Max} = h(\nu - \nu_0)}$$

If $\nu < \nu_0$ No photo Electric Effect.

Here $W_0 = h\nu_0 = \frac{hc}{\lambda_0}$

(or) $$\lambda_0 = \frac{hc}{W_0} = \frac{(3 \times 10^8) \times (6.625 \times 10^{-34})}{W_0}$$

$$\boxed{\lambda_0 = \frac{19.875 \times 10^{-26}}{W_0} \text{ metre}}$$

(W_0 in Joules)

(ii)

$$\lambda_0 = \frac{19.875 \times 10^{-26}}{1.602 \times 10^{-19}\, W_0}$$

$$= \frac{12.4 \times 10^{-7}}{W_0} \text{ metre}$$ (if W_0 is in electronvolts)

$$\boxed{\lambda_0 = \frac{12{,}400}{W_0} \mathring{A}}$$

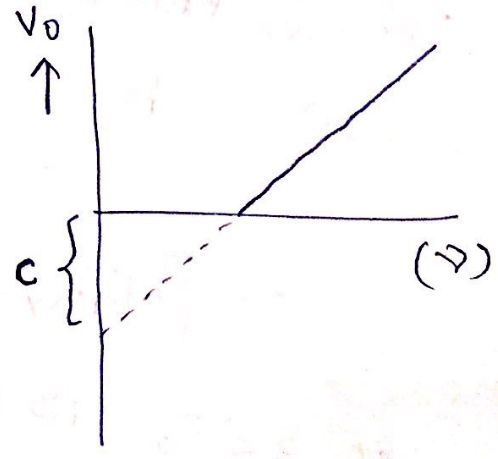
If V_0 is the stopping potential, Then

$$eV_0 = h\nu - h\nu_0 \qquad [\because KE_{max} = h\nu - h\nu_0]$$
$$[\because KE = eV]$$

$$\boxed{V_0 = \left(\frac{h}{e}\right)\nu - \left(\frac{h}{e}\right)\nu_0}$$
$$y = mx - c$$

This is in the form of ($y = mx - c$)

If a graph is plotted between stopping potential (V_0) on Y-axis and Frequency (ν) on X-axis, The plot would be a straight line of slope (h/e).



* The term photon was coined by chemist GILBERT LEWIS in 1926.

PHOTON.

* Invisible entity (like electron) proved experimentally
* 'h' ε, 'ν' for photon just like 'e' ε, 'm' for electron
* Energy in terms of Quantum ($h\nu$) $E = n(h\nu)$.

→ Quantum Theory $E = n(h\nu)$
→ Quantum Mechanics $E = (n+1/2)\, h\nu$.
⇒ Limiting energy of photon is not zero but is $\frac{1}{2}(h\nu)$ (when $n=0$)

$$\text{Energy},\ E = h\nu = \left(\frac{hc}{\lambda}\right)$$

→ As photon travels with vel of light, $E = mc^2$.

$$(\text{or})\ m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{hc}{\lambda c^2} = \frac{h}{c\lambda}.$$

$$\text{Momentum},\ \boxed{P = mc} = \left(\frac{h\nu}{c^2}\right)c = \frac{h\nu}{c} = \frac{h}{\lambda}.\ (\text{or})\ P = \left(\frac{E}{c}\right) \Rightarrow \boxed{E = PC}$$

→ No Rest Mass but behaves as though they have gravitational Mass.
→ Photons are Electrically Neutral
→ Cannot be deflected by electric or Magnetic fields.
→ Photons cannot be ionised.
→ Massless particle:

$$\boxed{E^2 = m^2c^4 + p^2c^2}$$

Proof. (Relativistically).

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-v^2/c^2}}$$

$$E^2 = m^2c^4/[1-v^2/c^2]$$

$$P = \gamma mv = \frac{mv}{\sqrt{1-v^2/c^2}}$$

$$p^2 = m^2v^2/[1-v^2/c^2]$$

$$p^2c^2 = m^2v^2c^2/[1-v^2/c^2].$$

$$\therefore E^2 - p^2c^2 = \frac{m^2c^4}{[1-(v^2/c^2)]} - \frac{m^2v^2c^2}{[1-(v^2/c^2)]}$$

$$= \frac{m^2c^2}{[1-(v^2/c^2)]}[c^2 - v^2]$$

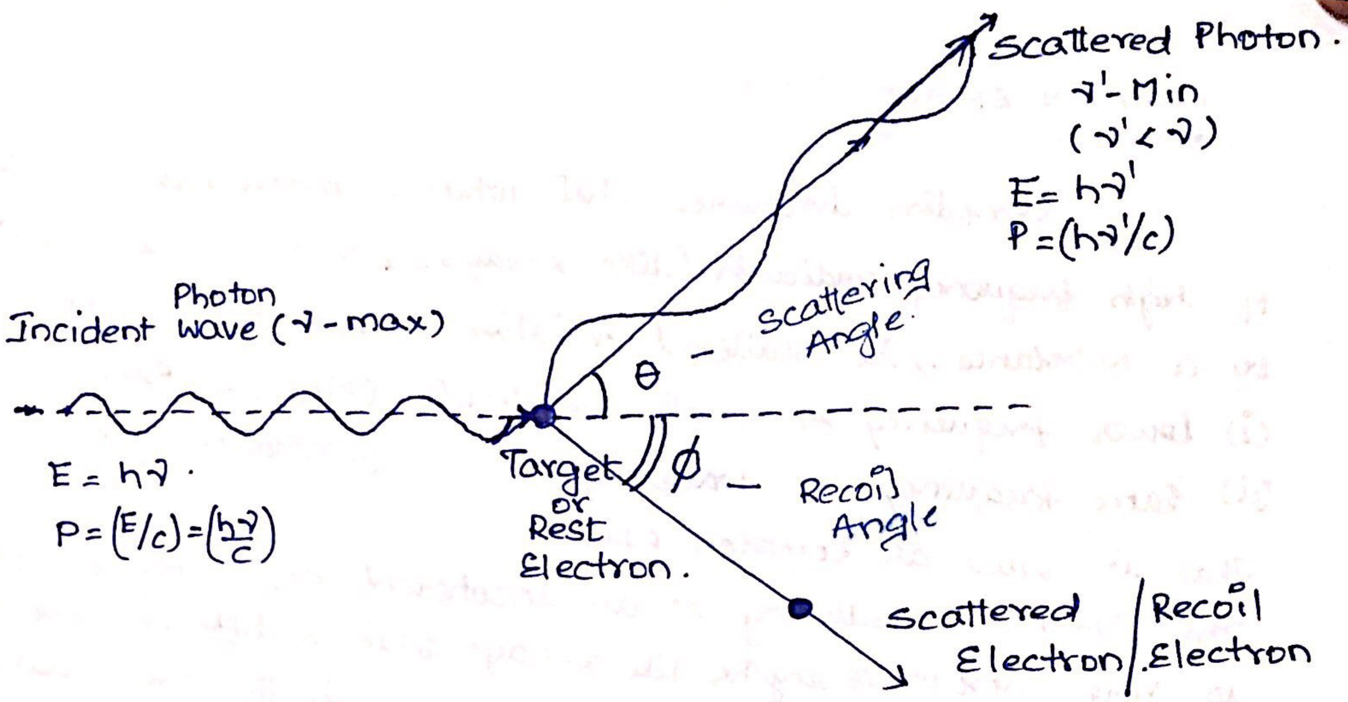
$$\therefore E^2 - p^2c^2 = m^2c^4. \Rightarrow \boxed{E^2 = p^2c^2 + m^2c^4}$$

|| 7 ||

COMPTON EFFECT

Compton discovered that when a monochromatic beam of high frequency radiation (like x-rays or γ-rays) is scattered by a substance., The scattered radiation consists of two components -
(i) lower frequency or greater wavelength (Modified Radiation)
(ii) Same frequency or wavelength. (Unmodified Radiation)
This is called as Compton Effect.
This compton scattering is an incoherent scattering which is is very short wavelengths like X-rays. This is different from that of Coherent Scattering which is observed in the visible range and at longer wavelengths. In coherent scattering there will not be any change in frequency or wavelength. [em radiation of freq 'ν' incidents on Free charges, Absorption takes place and start oscillating with freq 'ν'. Then oscillating charges radiates em radiation of same freq 'ν'].

In This incoherent (COMPTON) scattering, we have modified Frequencies, Unmodified Frequencies and Recoil Electrons.
The phenomenon of scattering is due to an elastic collision between an X-ray photon, The incident radiation and the electron of scatterer which is assumed to be initially at rest.
when the photon of energy ($h\nu$) collides with the rest electron, photon looses (transfers) some of its energy to electron. Due to this, the Scattered photon will have a smaller energy ($h\nu'$) where as the Electron gains the Kinetic Energy. Here ν' is less than ν ($\nu' < \nu$) or $\lambda' > \lambda$ (longer wavelength).
As this is an elastic collision, the law of conservation of energy and momentum holds good.
To derive an expression for change in frequency (or wavelength), Compton considered the relativistic Mass of the electron after collision. This is acceptable because only a small fraction of photon's (highly energetic) energy imparts sufficient energy to recoiled electron.



When a photon of energy '$h\nu$' collides with the electron at rest, a small fraction of $h\nu$ is sufficient to make the electron free (though it is bound to nucleus). Hence work function is negligibly small when compared with $h\nu$. Here electron gets accelerated in collision as it gains kinetic energy and recoils. Hence in ~~the~~ after the collision, there will be recoiling of electron and scattering of photon with ϕ and θ as recoil angle and scattering angle respectively.

Before Collision

Energy of incident photon, $E = h\nu$.

Momentum of incident Photon, $P = (E/c) = (h\nu/c)$

Rest energy of electron $= m_0c^2$ (where m_0 - rest mass of Electron)

Momentum of the rest electron $= 0$.

After Collision.

Energy of scattered photon, $E' = h\nu'$.

Momentum of scattered photon, $P' = (E'/c) = (h\nu'/c)$

Energy of the recoil electron $= mc^2$ (where m is the mass of the electron moving with a velocity of v)

Momentum of the recoil electron $= mv$.

Here 'm' and 'm₀' are related relativistically as

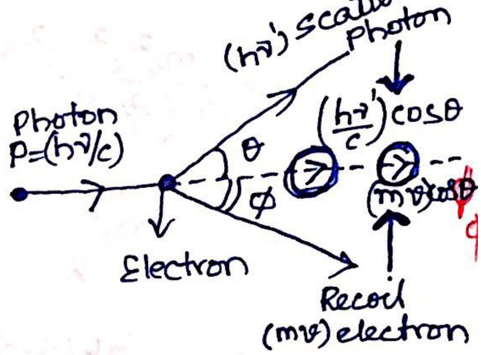
$$\boxed{m = \frac{m_0}{\sqrt{1-(v^2/c^2)}}}$$

Applying the law of conservation of energy for the Photon-Electron System before & after collision, we get

$$h\nu + m_0c^2 = h\nu' + mc^2.$$

$$\Rightarrow mc^2 = h(\nu-\nu') + m_0c^2 \longrightarrow (A)$$

As momentum is a vector quantity (unlike energy) directions should be taken into consideration when principle of conservation of momentum is analysed.



Applying the principle of conservation of Momentum along the direction of incident photon before & after collision, we have (Photon-Electron System)

$$\frac{h\nu}{c} + 0 = \left(\frac{h\nu'}{c}\right)\cos\theta + (mv)\cos\phi$$

(considering horizontal components of momentum vectors of photon and electron along the direction of incident photon)

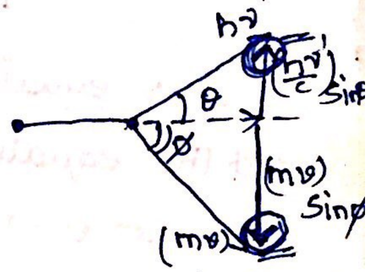
SEE THE ROUNDED ARROW HEADS

$$\Rightarrow mvc(\cos\phi) = h\nu - h\nu'(\cos\theta) \longrightarrow (1)$$

Similarly applying the principle of conservation of momentum in a direction perpendicular (vertical component) to incident photon before & after collision, we get

$$0 + 0 = \left(\frac{h\nu'}{c}\right)\sin\theta - (mv)\sin\phi$$

$$\Rightarrow (mvc)\sin\phi = h\nu'\sin\theta. \longrightarrow (2)$$



(vector diagram)

Squaring and Adding (1) & (2), we get

$$m^2v^2c^2 = [h^2\nu^2 + h^2\nu'^2\cos^2\theta - 2h^2\nu\nu'\cos\theta] + h^2\nu'^2\sin^2\theta.$$

$$= h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta.$$

$$m^2v^2c^2 = h^2[\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta] \longrightarrow (3)$$

Squaring eq (A), we get

$$m^2c^4 = h^2[\nu^2+\nu'^2-2\nu\nu'] + m_0^2c^4 + 2h(\nu-\nu')m_0c^2 \longrightarrow (4)$$

Subtracting (3) from eq (4), we get

$$(4)-(3) \Rightarrow m^2c^4 - m^2c^2v^2 = \{h^2(\nu^2+\nu'^2-2\nu\nu') + m_0^2c^4 + 2h(\nu-\nu')m_0c^2\}$$
$$-\{h^2(\nu^2+\nu'^2-2\nu\nu'\cos\theta)\}.$$

$$\Rightarrow m^2c^2[c^2-v^2] = -2h^2\nu\nu' + m_0^2c^4 + 2h(\nu-\nu')m_0c^2 + (2\nu\nu'h^2\cos\theta)$$
$$= -2h^2\nu\nu'[1-\cos\theta] + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$\Rightarrow \frac{m_0^2c^2(c^2-v^2)}{[1-(v^2/c^2)]} = -2h^2\nu\nu'[1-\cos\theta] + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$\Rightarrow \cancel{m_0^2c^4} = -2h^2\nu\nu'[1-\cos\theta] + 2h(\nu-\nu')m_0c^2 + \cancel{m_0^2c^4}$$

$$\Rightarrow \cancel{2}h(\nu-\nu')m_0c^2 = 2h^2\nu\nu'[1-\cos\theta].$$

$$\Rightarrow \left[\frac{\nu-\nu'}{\nu\nu'}\right] = \frac{h}{m_0c^2}[1-\cos\theta].$$

$$\Rightarrow \boxed{\left[\frac{1}{\nu'}-\frac{1}{\nu}\right] = \frac{h}{m_0c^2}[1-\cos\theta].}$$

This equation shows that ν' will be less than ν ($\nu'<\nu$) as such the equation ~~is~~ becomes positive because the maximum value of $\cos\theta = 1$ and h, m_0 and c are constants.

The above equation can also be written as

$$\left[\frac{c}{\nu'}-\frac{c}{\nu}\right] = \frac{h}{m_0c}[1-\cos\theta]$$

$$\Delta\lambda = \lambda'-\lambda = \frac{h}{m_0c}[1-\cos\theta]. \quad = \lambda_c[1-\cos\theta]$$

$$= \frac{h}{m_0c}[1-\{1+2\sin^2(\theta/2)\}]$$

where $\lambda_c = \frac{h}{m_0c}$ is called Compton wavelength

$$\boxed{\Delta\lambda = \lambda'-\lambda = \frac{2h}{m_0c}\sin^2(\theta/2)}$$

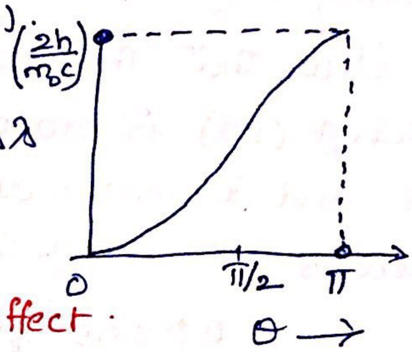
$\lambda_c = 0.0243\,Å$

If we substitute the constant values for h, m_0 and c, we get

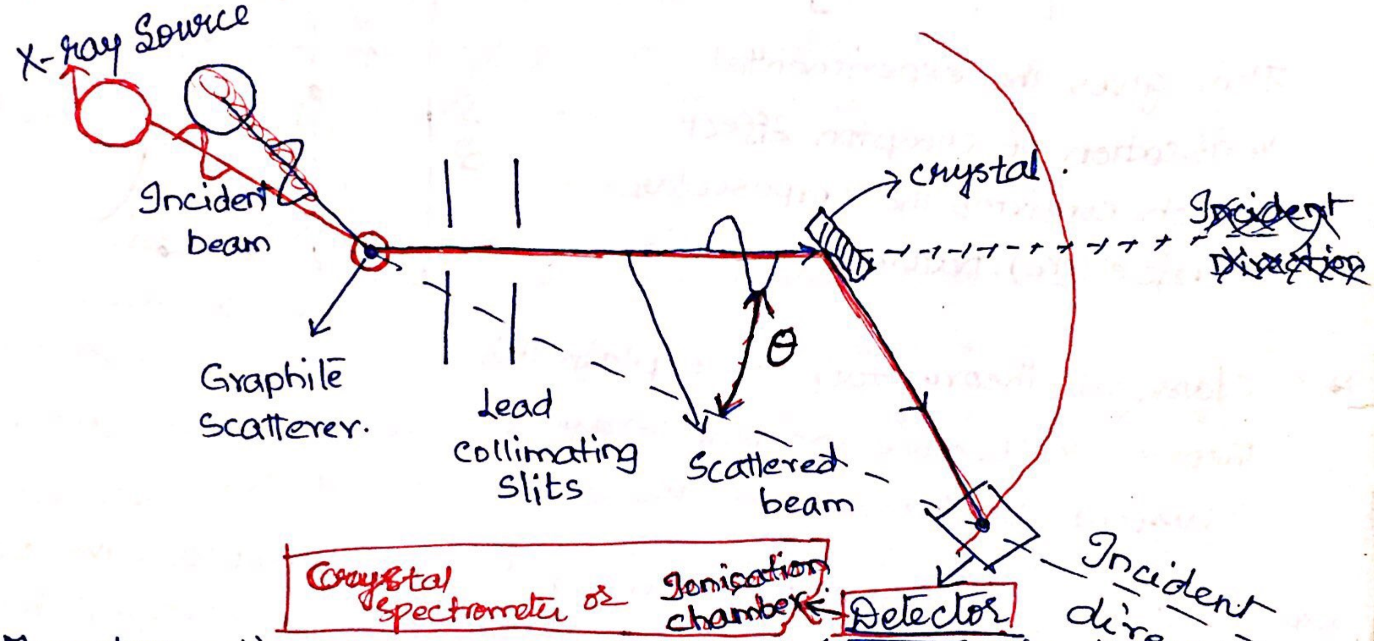
$$\Delta\lambda = \lambda' - \lambda = 0.0485 \sin^2(\theta/2)$$

From this, it is clear that the maximum shift in wavelength is about $0.05\,A^{\circ}$ (when $\theta = 180^{\circ}$, for head-on collision) & smaller wavelength radiation should be used for such a shift (X-Rays or γ-rays).

The variation of $\Delta\lambda$ and 'θ' is as shown



Experimental Verification of Compton Effect:

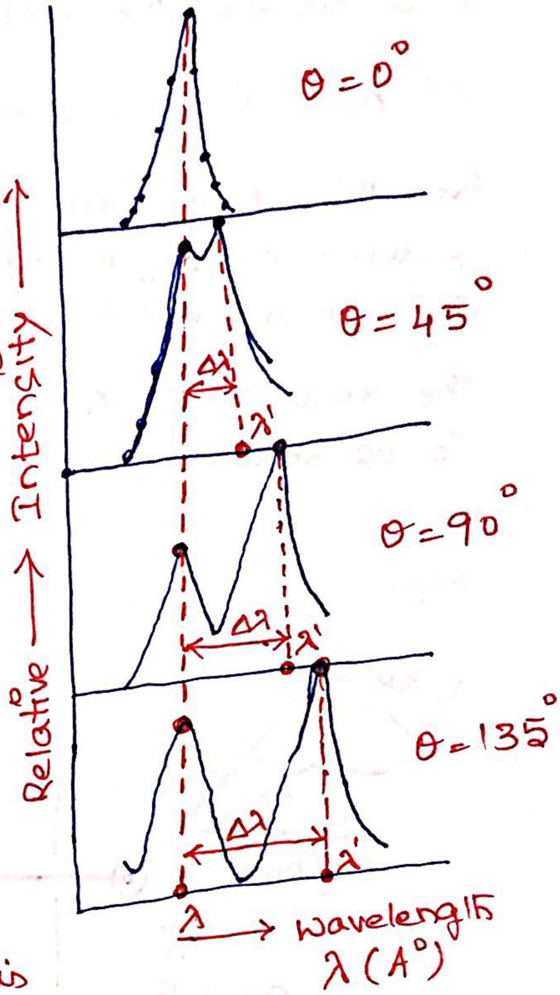


Monochromatic X-rays of wavelength 'λ' are allowed to incident on a Graphite (carbon) scatterer, which scatters the X-rays in different directions. The distribution of intensity with wavelength is measured for any scattering angle 'θ'. The scattered wavelengths are measured by considering Bragg's Reflections from a crystal. The intensities are measured by a Detector ~~like~~ like an Ionisation chamber.

Here the Incident beam consists of a single wavelength but the scattered beam comprises of two intensity peaks at two wavelengths λ (same as incident wave and λ', such that $\Delta\lambda = \lambda' - \lambda$.

Squaring eq (A), we get

Dependence of Intensity of x-rays at different angle 'θ'.



λ' is found to be function of the scattering substance and scattering Angle.

It is clear that the change in wavelength (Δλ) is proportional to 'θ' and increases as 'θ' increases. $\left[\because \Delta\lambda = \frac{2h}{m_0c} \cdot \sin^2(\theta/2)\right]$.

$[\theta\uparrow \sin\theta\uparrow]$

This gives the experimental verification of compton effect which confirms the corpuscular (Particle like) nature of radiation.

** Classical theory fails to explain this theory ** (because incident x-ray is classical Electromagnetic wave).

** According to Compton, incoming x-ray is not a wave of frequency 'ν' but collection of photons (each of Energy E = hν).

* Compton shift depends on scattering angle but not the initial wavelength.

* $\Delta\lambda = 0$ for $\theta = 0$ corresponding to "GRAZING" collision
(Minimum change) where incident photon is scarcely deflected

$\Delta\lambda = \frac{2h}{m_0c}$. for $\theta = 180°$ corresponding to "head-on" collision
(Maximum change) where the incident photon is reversed in direction

* $\Delta\lambda \; (\lambda_c) = \frac{h}{m_0c} = 0.0243 \, A°$ compton wavelength at $\theta = 90°$.

Pair Production & Pair Annihilation.

Interaction of Radiation with Matter.

Any type of radiation can travel indefinitely through a perfect vaccum. when the radiation travels through a gas, a liquid or a solid, it looses energy. The kind of interaction through which energy is lost depends on the type of radiation.

They are: ~~(i) the~~

(i) The interaction of heavy charged particles with matter.

(ii) The interaction of electrons with matter.

(iii) The interaction of γ-rays with matter.

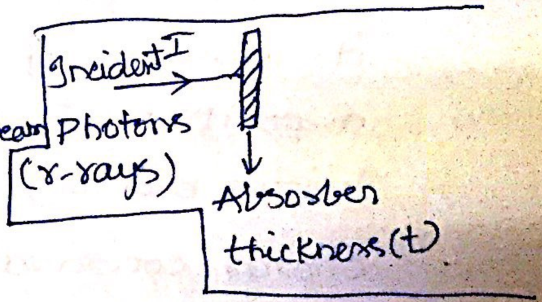
(iv) The interaction of neutrons with matter.

Interaction of γ-rays with Matter.

The γ-(Gamma) rays are highly penetrating and uncharged quantas of energy. Due to This, the interaction of γ-rays with matter is different from the interaction of charged particles with Matter.

When a beam of photons incident on a thin absorber of thickness 't', the matter absorbs the γ-rays where the photon disappears and its energy is converted into kinetic energy of absorbed particles, governed by the equation

$$\boxed{I = I_0 \exp(-\mu t)}$$



where I is the Intensity of Incident beam

linear Attenuation constant ← μ is Absorption Coefficient

t is thickness of Absorber

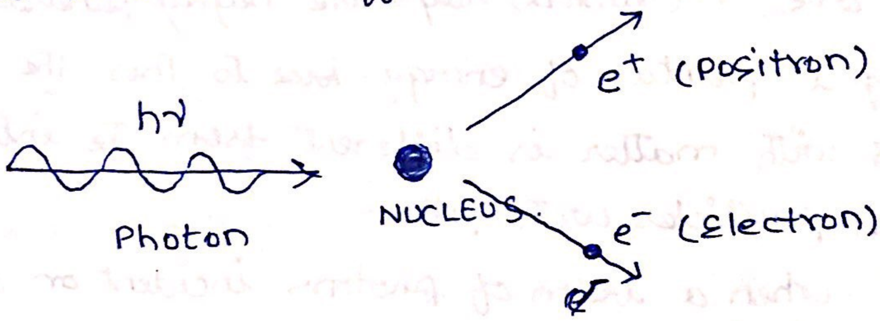
I_0 is Initial Intensity when $t=0$

There are ~~t~~
Three reasons are mainly responsible for the absorption of γ-rays

(i) Photo-Electric Absorption
where photon can give its complete energy to electron ($E = h\nu - \phi$)
Observed for low photon Energies.

(ii) Compton Scattering
where part of photon's energy is transferred to Electron.)
observed when Photon's have high energy (medium)

(iii) Pair Production
where the electromagnetic energy is converted into Matter. Here the photon with very high energy disappears in the coulomb's field of atomic nucleus to produce an electron-positron pair. This process is the best example for the conversion of radiant energy into rest mass energy ~~and~~ as well as into Kinetic energy.



$$h\nu \longrightarrow e^- + e^+$$

When a high energy photon ($h\nu$) encounters with a nucleus, it looses all of its energy creating an electron (e^-) and a positron (e^+ – similar to electron but with positive charge, Positive Magnetic Moment Positive electron) having kinetic energies also.

charge conservation holds good as charge of photon is zero and sum of charges of Electron & Positron is zero.
Similarly, conservation of Relativistic energy and Momentum holds good.

In this process, the energy absorbed by recoil of nucleus is negligible because of enoromous mass of nucleus.
Hence the total relativistic energy conservation is given by

$$h\nu = (E_-) + (E_+)$$

$$= [m_0c^2 + K_-] + [m_0c^2 + K_+]$$

$$= (K_-) + (K_+) + 2m_0c^2. \qquad [K_+ > K_-]$$

Here E_-, E_+ are the total Relativistic Energies of Elec & Positron respec

K_-, K_+ are the Kinetic energies of Electron & Positron respectively

The positron is produced with slightly larger kinetic energy than that of electron because the coulomb interaction of the pair with the positively charged nucleus leads to acceleration of Positron and deceleration of electron.

The minimum or Threshold energy for a photon to create Pair production is "$\underline{\underline{2m_0c^2}}$" $[\because E = h\nu - 2m_0c^2]$.

As rest mass energies of electron and positron is 0.51 MeV each, the total rest mass Energy is 1.02 MeV; which corresponds to a wavelength of $\underline{\underline{0.012A^\circ}}$ or 1.2pm $[1.2\times10^{-12}m = (1.2\times10^{-2})\times10^{-10}m = 0.012A^\circ]$. Electromagnetic radiations of this wavelengths are called "Gamma(γ)" rays that are the result of emissions from radioactive nuclei and also available in nature from cosmic rays.

The Pair Production is predominant when the energy of Photon is greater than or equal to $2m_0c^2$ (or) the photon is in the ~~ver~~ short wavelength region in the Electromagnetic spectrum.

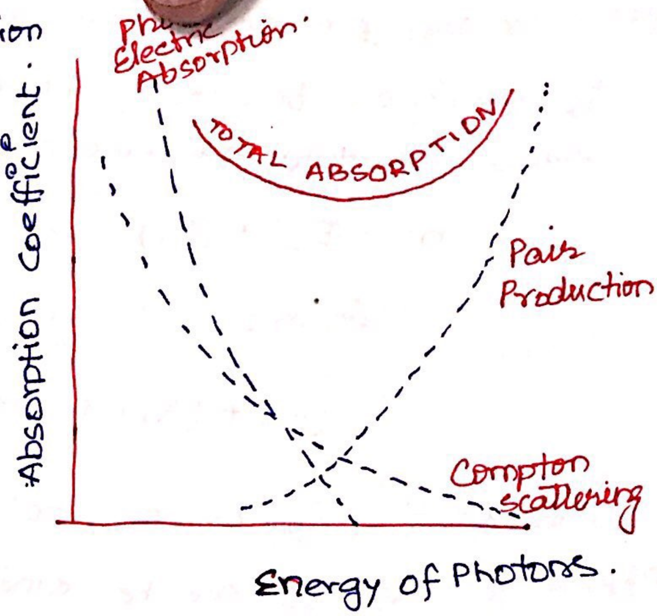
Pair production is produced in the laboratory by Bremsttrahlung Photons from Particle Accelerators.

Equation 99 (A). We

(< 1 MeV)
Very low Energies - Photo Electric Absorption (decreases with increase of Energy)

Medium energies (At 1 MeV) - Compton Scattering is observed

Very High Energies ($> 2m_0c^2$) - Pair Production is predominant. (represented by absorption at high energies).



Pair Annihilation.

Closely related inverse process of pair production is Pair Annihilation.

An electron and a positron which are (essentially) at rest near one another, unite and are annihilated. ie Matter disappears and we get ~~radiat~~ radiant energy. ie when they combine, Their mass is transformed into energy in the form of γ-rays.

$$e^+ + e^- \rightarrow 2\gamma.$$

In this process, the rest masses of electron and positron ($m_0c^2 + m_0c^2$) are converted into two 0.51 MeV photons.

$$m_0c^2 = [9.11 \times 10^{-31}\ \text{kg}][3 \times 10^8\ \text{m/s}]^2$$

$$= [9.11 \times 10^{-31}][9 \times 10^{16}]\ \text{Joule}$$

$$= \frac{[9.11 \times 10^{-31}][9 \times 10^{16}]}{1.602 \times 10^{-19}}\ \text{eV}$$

$$= \underline{0.51\ \text{MeV}}.$$

This is according to Mass-Energy Equivalence.

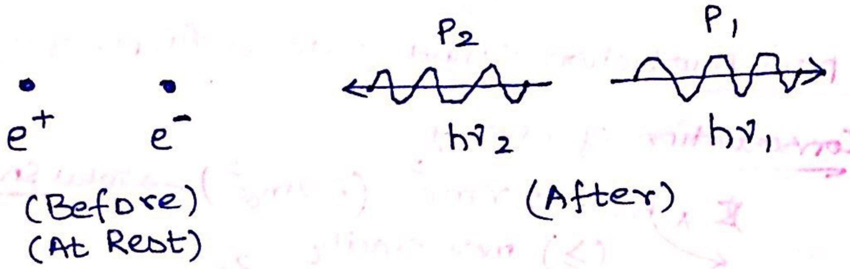
$\therefore$ The wavelength of Annihilation radiation is given by

$$E = h\nu = \frac{hc}{\lambda} = m_0 c^2$$

$$\therefore \underline{\underline{\lambda = 0.024\,A^\circ}}$$

In Annihilation, the initial momentum of system is zero (as at rest). As the momentum is to be conserved and if only one photon is emitted, it cannot have zero momentum. Hence two photons moving with equal momenta in opposite directions will be created.

$[P_1 + P_2 = 0$ or $P_1 = -P_2]$.



As they are equal in magnitude

ie $\quad P_1 = P_2$

$$\frac{h\nu_1}{c} = \frac{h\nu_2}{c}$$

$$\Rightarrow \nu_1 = \nu_2 = \nu.$$

Total Relativistic Energy conservation is given by

$$m_0c^2 + m_0c^2 = h\nu + h\nu.$$

$$\Rightarrow h\nu = m_0c^2 = 0.51\,MeV.$$

As electron & positron are at rest, they have no kinetic Energies. Hence the energy of photon equals rest mass energy which is $\underline{0.51MeV}$

* Energy & Linear Momentum could not be both conserved if pair production were to occur in empty space, so it does not occur in empty space.
* Pair production always involves atomic nucleus, that carries away initial photon momentum
* No Nucleus or other particle is needed for Pair Annihilation.

Rest mass Energy of electron = 0.51 MeV. (m_0c^2)
Positron = 0.51 MeV (m_0c^2)
Total = 1.02 MeV. ($2m_0c^2$)

* Hence Pair Production requires a photon whose energy is equal to or greater than 1.02 MeV. ($2m_0c^2$)
* Any extra energy of Photon becomes K.E for electron & Positron.

$$\boxed{E = h\nu - 2m_0c^2 \quad (h\nu \geq 2m_0c^2)}$$

K.E of Elec or position. ($E = K_- + K_+$)

Solution

* **Show that pair production cannot occur in Empty Space** ***.

Sol, from Conservation of Energy.

$h\nu = 2\gamma mc^2$ ($= 2m_0c^2$) → Total Energy.

Photon Energy.

($\geq$) more exactly

$= (\gamma mc^2) + (\gamma mc^2)$

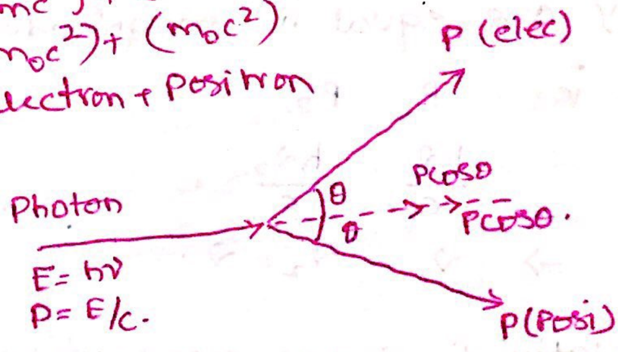
$= (m_0c^2) + (m_0c^2)$

electron + positron

Initial Momentum of Photon, $P = \frac{E}{c} = \left(\frac{h\nu}{c}\right)$

Momentum of Elec = $p\cos\theta$
(final) Posi = $p\cos\theta$.

Total = $2p\cos\theta$.



Conservation of Momentum

$$\frac{h\nu}{c} = 2p\cos\theta$$

$$h\nu = 2pc(\cos\theta)$$

$$= 2(\gamma mv)c(\cos\theta) \quad (p = \gamma mv)(= m_0 v)$$

$$= 2\gamma mc^2\left(\frac{v}{c}\right)\cos\theta \quad \text{[multiply \& divide by } c\text{]}$$

$$< 2\gamma mc^2 \quad \left[\text{As } \left(\frac{v}{c}\right) < 1 \text{ \& } \cos\theta \leq 1\right]$$

$$< 2m_0c^2$$

- But condition to be satisfied for Pair production is $h\nu \geq 2m_0c^2$.

But as $h\nu < 2m_0c^2$, pair production cannot happen in Empty space

De-Broglie's wave Hypothesis - Matter waves-
(Nature loves Symmetry)

Newton - Matter & Radiation consisted of particles. (Photon Theory)

20th Century - Interference, Diffraction & polarisation - ~~Radi~~ light is a wave.

- Photo/Compton/ - Corpuscular Theory explained on basis of
  - Quantum Theory

For a photon / Material particles $E = mc^2$ or $m = E/c^2 = (h\nu/c^2)$

$P = E/c$ or $P = (h\nu/c) = (h/\lambda)$

De-Broglie - wave particle parallelism of optics to all Fundamental entities of physics (Electrons/protons/neutrons/Atoms and Molecules).

* Correspondence between wave & particle is not confined only to Electromagnetic Radiation but also valid for Matter particles.

* A Moving particle has a wave associated with it and the particle is controlled by the wave.

* ~~Come~~ from Bohr's Theory of Hydrogen Atom.
* Led to further concepts of Quantum Mechanics by Schrodinger.
* Moving particle is associated with a wave (de-Broglie's wave) that has a wavelength given by

$$\lambda = \frac{h}{mv} = \frac{h}{P}.$$

More precisely $\lambda = \dfrac{h}{\gamma mv}$ $[\because P = \gamma mv]$

where γ = Relativistic Factor

$$= \frac{1}{\sqrt{1-(v^2/c^2)}}$$

* ~~When~~ Greater is particle's Momentum, shorter is the wavelength.

*** Particle's characteristics (E & P) and wave characteristics (ν & λ) are related to each other by Planck's Constant 'h'.

De-Broglie's wave Hypothesis.

wave particle Duality.

Particle.

* Easy to understand
* Mass
* located at definite point
* Velocity (v)
* Momentum (p)
* Energy (E)

wave.

* Difficult to grasp.
* Disturbance.
* spread out - relatively large region
* Frequency
* wavelength
* phase or wave velocity.
* Amplitude/Intensity.
* k

* Radiation is a wave spread out in the space and also a particle localised at a point in space.
* Radiation sometimes behaves like a particle at sometimes and as a wave at other time.

Radiation - wave - Visible/UV/IR/X-Rays.
- Expts based on Interference/Diffraction - (wave theory of Light)
- Presence of two waves simultaneously at the same position and same time.

Radiation - particle - Planck's Quantum Theory
(corpuscles) - Black body Radiation } can't be explained by wave theory.
- photo Electric Effect }
- Compton Effect
- Interaction of Radiation with Matter(Photons).

Radiation cannot exhibit its wave and particle properties simultaneously.

De-Broglie's concept of matter waves.

Newton proposed the famous corpuscular theory that light (radiation) consists of particles. It was established that light is a kind of wave motion basing on Interference, Diffraction and polarisation. Thus light is said to have dual nature.

Louis de-Broglie extended the wave-particle parallelism of optics to all the fundamental entities of physics (viz electrons, protons, atoms, neutrons etc. He proposed that Matter also has dual charecter (particle & wave) similar to that of radiation. ie There is a connection between waves and corpuscules not only in radiation but also in matter. Just as the photon is controlled by waves the moving particle is associated with a wave ~~contro~~ that controls the particle. This is totally based on "NATURE LOVES SYMMETRY".

According to de-Broglie's hypothesis, ~~the~~ a moving particle is associated with a wave that has a wavelength given by

$$\lambda = \left(\frac{h}{mv}\right) = \frac{h}{p}$$

where $p = mv$ is the momentum.

$$\left[\because E = h\nu = \frac{hc}{\lambda}.\quad E = mc^2 \quad \therefore mc^2 = \frac{hc}{\lambda} \quad \lambda = \frac{hc}{mc^2} = \frac{h}{mc}\right]$$

From Planck's Quantum theory of radiation $\lambda = \left(\frac{h}{mc}\right)$

Momentum and energy are related by $E = \frac{p^2}{2m}$

$$\text{or } p = \sqrt{2mE}.$$

$$\therefore \lambda = \frac{h}{\sqrt{2mE}}.$$

De-Broglie's wavelength for an electron

$$\lambda = \frac{h}{\sqrt{2eVm_0}}.$$

$$\left[\because \frac{1}{2}m_0v^2 = eV. \quad v = \sqrt{\frac{2eV}{m_0}}. \quad \therefore \lambda = \frac{h\sqrt{m_0}}{m_0\sqrt{2eV}} = \frac{h}{\sqrt{2eVm_0}}\right]$$

The wave velocity 'u' is given by $u = \nu\lambda$ where 'ν' is the frequency of matter waves.

From Planck's theory, $E = h\nu$.

From Einstein's equivalence, $E = mc^2$.

From ~~at~~ the two equations, we have

$$mc^2 = h\nu$$

$$\Rightarrow \nu = \left(\frac{mc^2}{h}\right)$$

$\therefore$ The de-Broglie's wavelength is $\lambda = (h/mv)$

Hence the wave velocity is now given by

$$u = \nu \cdot \lambda$$

$$= \left(\frac{mc^2}{h}\right)\left(\frac{h}{mv}\right)$$

$$= \left(\frac{c^2}{v}\right)$$ [v - speed of ~~matter~~ material particle]

As $v < c$, $u > c$.

i.e wave velocity is greater than ~~that of~~ the velocity of light.

(or) $u = \frac{h}{2m\lambda}$

wave velocity in terms of wave length.

[$\nu = E/h = \frac{\frac{1}{2}mv^2}{h} = \frac{eV}{h}$

$= \frac{h}{2m}\left(\frac{eV2m}{h^2}\right)$

$= \frac{h}{2m}\left(\frac{1}{\lambda^2}\right)$ $\left[\because \lambda = \frac{h}{\sqrt{2eVm}}\right]$

$\therefore u = \nu \cdot \lambda$

$= \frac{h}{2m} \cdot \left(\frac{1}{\lambda^2}\right)\lambda = \frac{h}{2m\lambda}$].

Properties of Matter waves.

(*) The de-Broglie wavelength is given by $\lambda = \left(\frac{h}{mv}\right)$.

(1) When mass of the particle is less, greater will be the wavelength of wave $(m\downarrow, \lambda\uparrow)$

(2) Smaller is the velocity of the particle, greater is the wavelength of the wave $(v\downarrow \lambda\uparrow)$

(3) When $v = 0$, $\lambda = \infty$ or when $v = \infty$, $\lambda = 0$

All this waves are generated by motion of particles. These wave are produced irrespective of presence of charge (as λ is not dependent on q). This shows that the matter waves are not Electromagnetic waves.

(4) The velocity of matter waves depends on the velocity of material particle which is not constant as the velocity of electromagnetic wave.

(5) The velocity of matter waves is greater than that of the velocity of an electromagnetic wave (Light).

(6) The Matter wave representation is a symbolic representation. ie waves have particle like properties and particles have wave like properties.

(7) The wave nature of matter ~~it~~ results in an uncertainity in the location of position of particle because the wave spreads in outer region but not confined to a point.
If wave spreads out in large region, the probability of finding the particle will be maxium, ~~and it~~ and is minimum if the wave is small.

The experimental proof for matter waves was given by DAVISSON & GERMER'S electron diffraction experiment. When there was the reflection of electrons from Nickel Target, they observed (accidentally) that at the target was heated due to the relections that become anomalous.
The Intensity of reflected beam has maximas and minima that proved ~~the~~ ~~a~~ diffraction of electrons. (wave) (Matter wave).

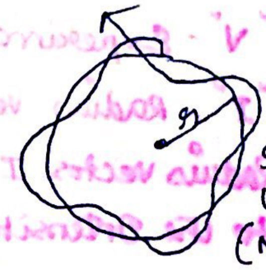
G.P. THOMSON'S experiment also proved the existence of matter waves.

De-Broglie Hypothesis.

From the concept of wave particle duality, the material particle of mass 'm' ~~beha~~ at some point in the space specified by mass, velocity, momentum and energy, also behaves like a wave spread out in space over a relatively large region of space, charecterised by frequency, wavelength, phase, amplitude, intensity etc.

According to Bohr's theory, the electron will be moving around the ~~orbit~~ nucleus in its orbit

According to De-Broglie, the electrons are material particles which has wave motion. If a wave moves along with it, when it goes round and round the ~~wave~~ nucleus, we can define the formation of standing wave, whose stability is taken to be the whole number of wavelengths in complete round that can ~~so~~ interfere constructively



The circumeference of the circular path of the electron is equal to an integral multiple of the wavelengths associated with the wave.

ie $$2\pi r = n\lambda$$

This is ~~de~~ taken from Bohr's condition for the quantisation of orbital Angular momentum.

ie $$mvr = \frac{nh}{2\pi}$$

$$\Rightarrow 2\pi r = \left(\frac{nh}{mv}\right)$$

from these two equations, we can say that

$$n\lambda = \frac{nh}{mv}$$

or $$\boxed{\lambda = h/mv}$$

where λ is called as the de-Broglie wavelength

$E = h\nu \qquad E = mc^2$ (From Planck's Theory of Radiation)

(λ associated with Material particle)

$\Rightarrow mc^2 = h\nu$

$\Rightarrow mc^2 = \frac{hc}{\lambda}$

$\Rightarrow \lambda = \frac{h}{mc}$

$\lambda = \frac{h}{p}$

$$\boxed{\lambda = \frac{h}{p}}$$ (where p - Momentum associated with photon moving with Velocity of light).

$$\boxed{\lambda = \frac{h}{mv} = \frac{h}{p}}$$ (where p - Momentum of the particle of mass 'm' moving with velocity 'v')

Relativistically $\boxed{\lambda = \frac{h}{\gamma m v}}$

If E is the Energy (Kinetic) of material particle, then

$$E = \frac{1}{2} m v^2.$$

$$= \frac{1}{2m} (m^2 v^2)$$

$$= \frac{p^2}{2m} \qquad \text{where } p = mv.$$

or $p = \sqrt{2mE}$.

$$\therefore \boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

For an Electron of rest mass 'm_0', charge 'e' and is accelerated by a Potential of 'V' where it acquires velocity 'v'. Then

$$eV = \frac{1}{2} m_0 v^2.$$

$$\text{or } v = \sqrt{\frac{2eV}{m_0}}.$$

$$\therefore \lambda = \frac{h}{m_0 v}$$

$$= \frac{h \cdot \sqrt{m_0}}{m_0 \sqrt{2eV}}$$

$$\Rightarrow \lambda = \frac{h}{(\sqrt{2eVm_0})}$$

$$\lambda = \frac{h}{\sqrt{2eVm_0}}.$$

$h = 6.625 \times 10^{-34}$ Joule-sec

$e = 1.602 \times 10^{-19}$ Coul.

$m_0 = 9.11 \times 10^{-31}$ Kg.

$$\sqrt{\frac{150}{V}} = \boxed{\lambda = \frac{12.26}{\sqrt{V}} \text{ A}^\circ}$$

$\lambda = 2d \sin\theta$ (from GP Thompson's Expt on Matter waves)

$E = h\nu$ $\qquad E = mc^2.$ (from Planck's theory of Radiation) (λ associated with Material particle)

$$\Rightarrow mc^2 = h\nu.$$

$$\Rightarrow mc^2 = \frac{hc}{\lambda}$$

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$$\therefore \lambda = \frac{h}{m_0 v}$$

$$= \frac{h\sqrt{m_0}}{m_0\sqrt{2eV}}$$

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$$\sqrt{\frac{150}{V}} = \boxed{\lambda = \frac{12.26}{\sqrt{V}} A^\circ}$$

$\lambda = 2d \sin\theta$ (from GP Thompson's Expt on Matter waves)

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Properties of Matter waves.

1. Lighter is the particle, greater is the wavelength associated.
2. Smaller velocity

2. when $v=0$, $\lambda=\infty$ $\Rightarrow$ wave becomes indeterminate
$v=\infty$, $\lambda=0$ $\Rightarrow$ no wave.

$\Rightarrow$ Matter waves are generated by motion of particles.
(whether particles may be charged or uncharged).

3. These waves are not Electromagnetic waves.
4. Velocity of Matter wave depends on velocity of particle (not constant like EM wave)
5. Velocity of Matter wave is greater than velocity of light ('u' or

6.

Particle associated with Matter' wave.
↓
Two different Velocities

↓ Mechanical Motion of particle (v)
↓

↓ Propagation of the wave (u) (u_p) [Phase (wave) Velocity].

Phase vel: velocity of the planes of constant phase that advances through medium

Proof 1. (u)

$E=h\nu$, $E=mc^2$.

$\Rightarrow h\nu = mc^2$.

$\Rightarrow \nu = \dfrac{mc^2}{h}$

Wave(Phase) Velocity $\}$ $u(u_p) = \nu\lambda$

$= \left(\dfrac{mc^2}{h}\right)\lambda$

$= \left(\dfrac{mc^2}{h}\right)\left(\dfrac{h}{mv}\right)$

$$\boxed{u = \left(\frac{c^2}{v}\right)}$$

Proof (2) (u)

wave Equation travelling along "+x" direction $y=a[\sin(\omega t-kx)]$

where $\omega = 2\pi\nu$ Angular Freq, propagation Const, $k=(2\pi/\lambda)$

By def. phase Velocity, $\boxed{u(u_p) = \left(\dfrac{\omega}{k}\right)}$

Here $\omega t - kx = \phi$ (constant) (wavefronts) [For any wave, $\omega t-kx$ - Phase of wave Motion]

$\dfrac{d}{dt}(\omega t - kx) = 0$

$\omega - k\left(\dfrac{dx}{dt}\right) = 0$.

$$\Rightarrow \boxed{\left(\frac{dx}{dt}\right) = u = \left(\frac{\omega}{k}\right)}$$

$\therefore \boxed{u = \dfrac{c^2}{v}}$

$u = \nu\lambda$

$= \left(\dfrac{E}{h}\right)\left(\dfrac{h}{mv}\right) = \dfrac{E}{mv} = \dfrac{mc^2}{mv} = \dfrac{c^2}{v}$.

From both, as $c > v$, $u > c$.

experiment also proved the existence of matter waves.

Heisenberg's Uncertainity Principle.

In classical physics, some variables like position co-ordinates, momentum and its components, angular momentum etc called as the dynamical variables can be measured at any instant of time. In the case of the micro-physical system, there is a limit to accuracy in measuring these kind of values which was established by Heisenberg. W. This principle that ~~at~~ gives the relation between these limits is known as Uncertainity Principle. This uncertainity Principle is a direct consequence of wave-Particle duality.

According to this principle,

It is impossible to specify precisely and simultaneously ~~both~~ the values of both members of particular pairs of physical variables that describes the behaviour of an atomic system.

(OR) ~~The~~ order of ~~ma~~

(OR) The order of magnitude of the product of the uncertainities in the knowledge of two variables (pair) must be ~~equal to~~ atleast (equal to) planck's constant.

(1) $\Delta p \cdot \Delta x = h$ — ~~Δp~~ uncertainity in momentum & position.

(2) $\Delta E \cdot \Delta t = h$ — uncertainity in Energy & time.

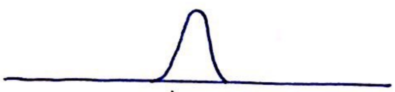
(3) $\Delta J \cdot \Delta \theta = h$. — uncertainity in Angular Momentum & Angle.

In classical Mechanics, basing on the initial fixed position, momentum can be defined. But in Quantum Mechanics, the particle is defined in terms of wave packet, in which the particle may be found. when the wave packet is small, the position of particle will be fixed, but the wave spreads out rapidly due to which the velocity becomes indeterminate when the wave packet is large, The velocity can be fixed but the position becomes indefinite. This is the crux of the principle.

In this way, ~~There~~ the certainity in position involves the uncertainity in momentum (or velocity) and vice-versa.

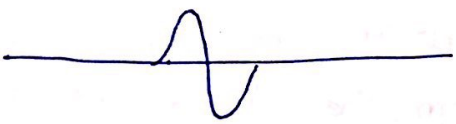
we cannot exactly ~~define~~ or (it is impossible) to define where ~~the~~ within the wavepacket the particle is and what is its exact momentum

①



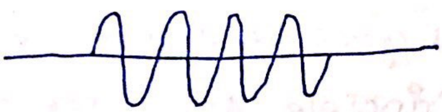
Position - Very well defined
Momentum - Very poorly defined

②



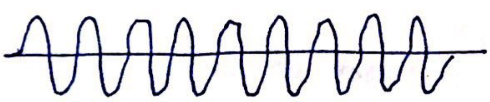
Position - well defined
Momentum - poorly defined.

③



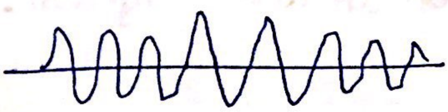
Position - less well defined
Momentum - Better defined.

④



Position - Poorly defined
Momentum - well defined.

⑤



Position - poorly defined
Momentum - poorly defined.

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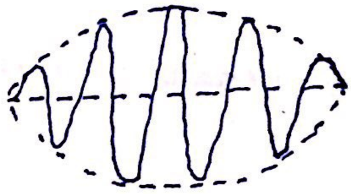
Heisenberg's Uncertainity Principle.

Heisenberg's Uncertainity principle is the direct consequence of dual nature of matter.

In classical Mechanics, a moving particle has a fixed position in space and a definite momentum at any instant basing on the initial values.

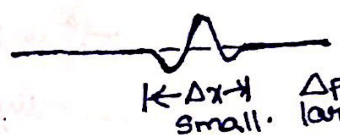
In wave Mechanics, the particle is described in Terms of a wave packet. Here it is necessary to define a guiding wave, the equation ~~for~~ which is defined ~~by~~ (derived) by Schroedinger. The physical significance of which is that the amplitude of the guiding wave defines the probability [$\psi\psi^*$] ~~of~~ finding the material particle at a point. If amplitude is zero, probability is infinitesimal where the mechanical process is accompanied by the wave. If the amplitude of the guiding wave is maximum, then the probability of finding the particle at that point is maximum, which is equivalent to particle in a wavepacket moving with velocity 'v'.



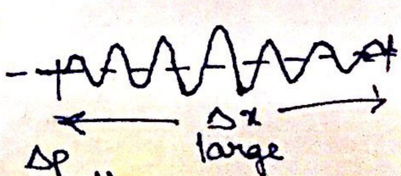
Let us consider a wavepacket.
The particle that corresponds to this wavepacket may be anywhere where the probability density $|\psi^2|$ is maximum (in the middle of the group) or anywhere where $|\psi^2|$ is of a non-zero value.

According to Bohr's probability interpretation, the particle may be found anywhere with in the wave packet.

(i) when the wave packet is small, the position of the particle may be fixed but the particle will be spread rapidly and hence the velocity becomes indeterminate. ie when the wave group (packet) is narrow, the particle's position can be defined but here the wavelength is not well defined as there are not enough waves to measure wavelength (λ). Hence the momentum (mv) from equation $\lambda = h/(mv)$ is not precisely determined.



(ii) when the wave packet is large, the velocity is fixed but there is large indefiniteness in position. ie in a clear wave the wavelength is definite and hence the momentum (mv) is precise. But here as the wave is spread out, the particle's position cannot be defined at a given time.



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This led to concept of uncertainity.

The uncertainity in position involves uncertainity in momentum (velocity)

i.e. the certainity in momentum involves uncertainity in position.

This shows that it is impossible to define where within the wave packet the particle is and what is its momentum.

According to Heisenberg's uncertainity Principle, it is impossible to specify (measure) and precisely and simultaneously the values of both exact position and exact momentum of an object at the same time.

Also, the order of magnitudes of the uncertainities in the two variables must be atleast planck's constant

ie $\boxed{\Delta p \cdot \Delta x \approx h}$. $\boxed{\Delta E \cdot \Delta t \approx h}$ $\boxed{\Delta J \cdot \Delta \theta \approx h}$

Exact Statement -

The product of uncertainities in determining the position and momentum of the particle can never be smaller than the order of $(h/4\pi)$

ie $$\Delta p \cdot \Delta x \geq \frac{h}{4\pi} \ (\text{or } \hbar/2) \quad \text{where } \hbar = h/2\pi.$$

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi} \ (\text{or } \hbar/2)$$

$$\Delta J \cdot \Delta \theta \geq \frac{h}{4\pi} \ (\text{or } \hbar/2).$$

Mathematical proof -

This can be proved basing on deBroglie's hypothesis of a wave associated with a particle or a wave packet that has many number of waves of slightly different frequencies that superimpose and has a Group velocity of 'G' = $(\Delta \omega/\Delta k)$.

Consider two simple harmonic waves of equal amplitudes and of nearly equal frequencies, given by

$$y_1 = a\left[\sin(\omega_1 t - k_1 x)\right]$$

$$y_2 = a\left[\sin(\omega_2 t - k_2 x)\right].$$

$k_1, k_2 = \frac{2\pi}{\lambda_1}$ & $\frac{2\pi}{\lambda_2}$ Prop. Constant

$\omega, \omega_2 = 2\pi\nu_1, 2\pi\nu_2$ Angular Freq.

According to principle of superposition,

$$y = y_1 + y_1$$

$$= 2a\sin(\omega t - kx)\left[\cos\left\{\left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta k}{2}\right)x\right\}\right]$$

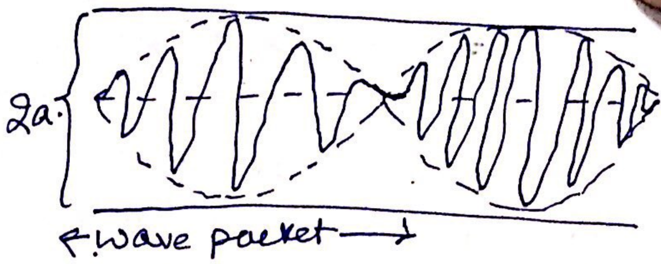
where $\omega = (\omega_1 + \omega_2)/2$

$\Delta\omega = \omega_1 - \omega_2$

$\Delta k = k_1 - k_2$

This wave packet of amplitude '2a' travels with Group velocity 'G', given by $G = \left(\frac{\Delta\omega}{\Delta K}\right)$

$$= \frac{(\omega_1 - \omega_2)}{(K_1 - K_2)}$$



Since the group velocity of the deBroglie wave packet associated with the moving particle is equal to velocity of particle, the loop so formed is equivalent to position of particle, where the particle can be anywhere. This happens when

$$\cos\left[\left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta K}{2}\right)x\right] = 0$$

$$\Rightarrow \left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta K}{2}\right)x = (2n+1)\frac{\pi}{2}. \quad \text{where } n = 0,1,2\ldots$$

Let x_1 and x_2 are the positions at two points (nodes) at the same instants, then, the above can be written for x_1 and x_2 as

$$x_1 \to n \qquad \left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta K}{2}\right)x_1 = (2n+1)\frac{\pi}{2}$$

$$x_2 \to n+1 \text{ and } \left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta K}{2}\right)x_2 = [2(n+1)+1]\pi/2 = (2n+3)\pi/2$$

Subtracting ~~first~~ first from Second, we get

$$\left(\frac{\Delta K}{2}\right)(x_1 - x_2) = \pi.$$

$$(\text{or}) \quad \left(\frac{\Delta K}{2}\right)(\Delta x) = \pi$$

$$(\text{or}) \quad \Delta x = \left(\frac{2\pi}{\Delta K}\right),$$

$$= \frac{\cancel{2\pi}\cdot(h)}{\cancel{2\pi}(\Delta p)}$$

$$\Rightarrow \boxed{\Delta x \cdot \Delta p = h}$$

We know

$$K = \frac{2\pi}{\lambda}$$

$$= \frac{2\pi}{(h/p)}$$

$$= \left(\frac{2\pi p}{h}\right)$$

$$\Rightarrow \Delta K = \left(\frac{2\pi}{h}\right)\Delta p.$$

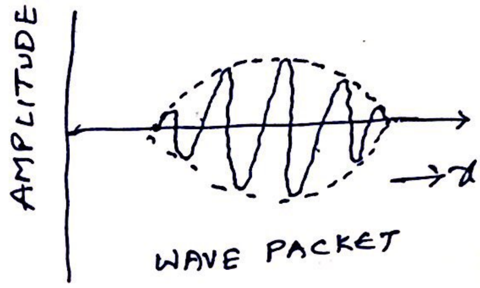
where Δp is an error or uncertainity in finding the momentum of a particle at a particular point.

More precisely $\Delta x \cdot \Delta p \geq \hbar$

$$(\text{or}) \quad \Delta x \cdot \Delta p \geq \frac{\hbar}{2}.$$

Group velocity.

~~As~~ The phase velocity of a wave associated with the particle is greater than the velocity of light [$u = (c^2/v)$]. This can be explained by assuming that each particle is associated with a group of waves [WAVE PACKET] instead of a single wave. Consider a pulse rather than a monochromatic wave, that consists of number of waves with slightly different frequencies. The superposition of these waves forms a wave packet or Group. Here. There may be variation slightly in the velocity & wavelength with phases & Amplitude. They interfere constructively over a small region of space where the particles can be located and outside this, they interfere destructively where the amplitude reduces to zero. This wave packet moves with its own velocity, known as GROUP VELOCITY or the observed velocity with which the Maximum amplitude of the group Advances or the velocity which the energy is transmitted in the group is called GROUP VELOCITY.



The individual waves forming a wave packet have an average velocity which is called as the PHASE (WAVE) VELOCITY.

$$y_1 = a_1 \sin(\omega_1 t - k_1 x)$$

$$y_2 = a_2 \sin(\omega_2 t - k_2 x)$$

$$y = y_1 + y_2$$

$$= a_1 \sin(\omega_1 t - k_1 x) + a_2 \sin(\omega_2 t - k_2 x)$$

$$= 2a \sin\left[\frac{(\omega_1+\omega_2)t}{2} - \frac{(k_1+k_2)t}{2}\right] . \cos\left[\frac{(\omega_1-\omega_2)t}{2} - \frac{(k_1-k_2)x}{2}\right]$$

$$= 2a \cos\left[\frac{(\omega_1-\omega_2)t}{2} - \frac{(k_1-k_2)x}{2}\right] . \sin[\omega t - kx]$$

where $\left(\frac{\omega_1+\omega_2}{2}\right) = \omega \quad \left(\frac{k_1+k_2}{2}\right) = k$

$\Rightarrow \quad y = 2a \cos\left[\frac{(\Delta\omega)t}{2} - \frac{(\Delta k)t}{2}\right] \cdot \sin[\omega t - kx].$

where $\Delta\omega = \omega_1 - \omega_2$ and $\Delta k = k_1 - k_2$.

A wave is said to have phase velocity of $u = (\omega/k)$

Another wave of frequency $(\Delta\omega/2)$ and propagation constant $(\Delta k/2)$ have the GROUP VELOCITY, $G = \left(\frac{\Delta\omega}{\Delta k}\right)$ (velocity of Envelope wave packet)

$= \left(\frac{\partial\omega}{\partial k}\right)$ [waves having small difference in their frequencies & wave numbers]

$$= \frac{\partial(2\pi\nu)}{\partial(2\pi/\lambda)}$$

$$= \frac{\partial\nu}{\partial(1/\lambda)}$$

$$= -\lambda^2\left(\frac{\partial\nu}{\partial\lambda}\right)$$

$$= -\lambda^2\left[\frac{\partial(\omega/2\pi)}{\partial\lambda}\right].$$

$$\boxed{G = \frac{-\lambda^2}{2\pi}\left[\frac{\partial\omega}{\partial\lambda}\right]}$$

Relation between u & G.

$$G = \left(\frac{d\omega}{dk}\right)$$

$$= \frac{d[uk]}{dk} \qquad \left[\because \text{Phase vel } u = (\omega/k)\right]$$

$$= u + k\left(\frac{du}{dk}\right)$$

$$= u + du(-\lambda/d\lambda)$$

$$\boxed{G = u - \lambda\left(\frac{du}{d\lambda}\right)}$$

when $k = \frac{2\pi}{\lambda}$

$$dk = -\frac{2\pi}{\lambda^2}(d\lambda)$$

$$\therefore \frac{k}{dk} = -\frac{\lambda}{d\lambda}.$$

In a dispersive medium (u is func of λ)

$G < u$.

In a Non-dispersive medium

$G = u. \qquad \left[\because \left(\frac{du}{d\lambda}\right) = 0\right]$

This is true for Electromagnetic waves in free space (vacuum) and Elastic waves in homogeneous medium.

Phase velocity has no physical significance because of motion of wave group, & not the motion of individual waves that make up the group, that corresponds to the motion of the body

$v_p > c$, $v_g < v_p$ or $v_g = v$.

This fact of $v_p > c$ for de Broglie waves do not violate the second postulate of Spl. Theory of Relativity.

* The amplitude of de Broglie waves that correspond to moving body reflects the probability of finding the particle at a particular place and particular time.
* De Broglie waves cannot be represented by an equation $y = A\cos[\omega t - kx]$ which describes an infinite series of waves of same amplitude.
* Instead, the moving body corresponds to a wave packet or wave group, whose waves have amplitudes depicting the existence of particle.

* $\boxed{v_g = u}$

$$v_g = \frac{d\omega}{dk} = \frac{[d\omega/dv]}{[dk/dv]}$$

$$\frac{1}{\sqrt{x}} \quad -\frac{1}{2}x^{-3/2} \quad -\frac{1}{2x^{3/2}}$$

$$\boxed{\omega} = 2\pi\nu = 2\pi\left[\frac{\gamma mc^2}{h}\right] = \frac{2\pi mc^2}{h\left[\sqrt{1-v^2/c^2}\right]}$$

$$\left(\frac{d\omega}{dv}\right) = \left[\frac{2\pi mc^2}{h}\right](-)\left[\frac{(-)2v/c^2}{2\left[1-v^2/c^2\right]^{3/2}}\right]$$

$$\left(\frac{u}{v}\right) \quad \left(\frac{uv' - vu'}{v^2}\right)$$

$$\boxed{k} = \frac{2\pi}{\lambda} = \frac{2\pi(\gamma mv)}{h} = \frac{2\pi mv}{h\sqrt{1-v^2/c^2}}$$

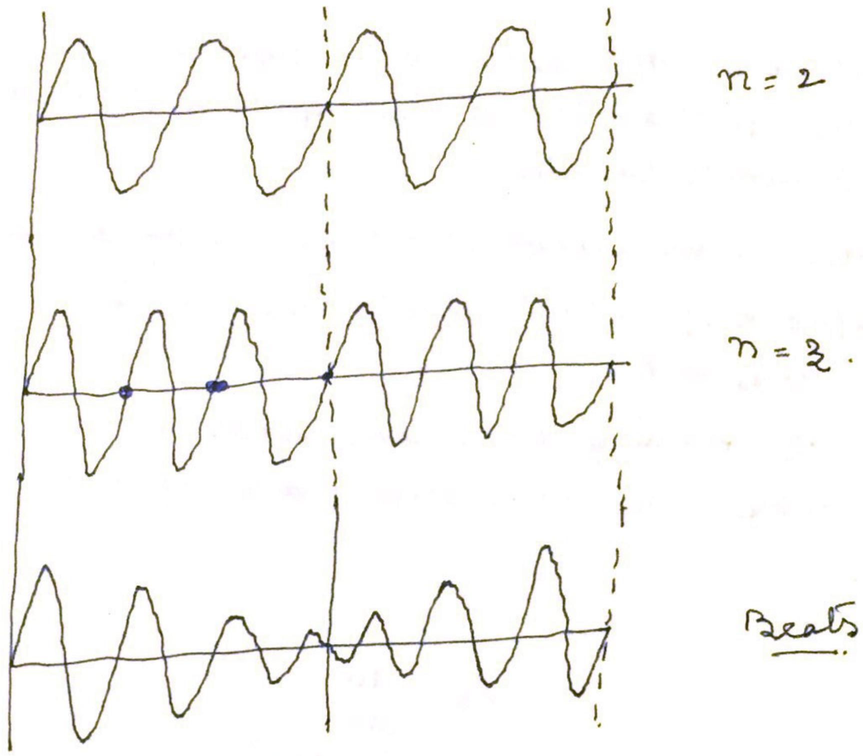
$$\left(\frac{dk}{dv}\right) = \frac{2\pi m}{h}\left[\frac{\cancel{\sqrt{1-v^2/c^2} - \frac{2v}{\sqrt{1-v^2/c^2}}}}{\sqrt{(1-v^2/c^2)}}\right]$$

$$= 2$$

$$= \frac{2\pi mc^2}{h}\left[\frac{v}{\sqrt{1-v^2/c^2}}\right].$$

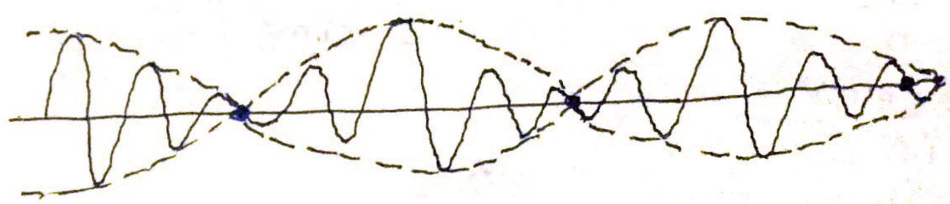
$$\frac{dk}{dv} =$$

The phase velocity of a wave associated with a particle is greater than the velocity of light, this drawback is explained by assuming that each moving particle is associated with a group of waves or a wave packet rather than a single wave.
i.e the de-Broglie waves are represented by a wave packet and hence the group velocity.



$$\boxed{y = 2a\left[\cos\left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta K}{2}\right)x\right]\sin(\omega t - Kx)}$$

Resultant displacement is another wave of freq $\left(\frac{\Delta\omega}{2}\right)$ and propagation constant $(\Delta k/2)$, modulated on a wave of freq ω and prop cons. k. (Modulation of two waves).



Resultant wave of Angular freq (ω) and wavenumber (K) that has superimposed upon it a modulation of ~~freq~~ angular freq $\left(\frac{\Delta\omega}{2}\right)$ and of wave number $(\Delta k/2)$.

6. Matter waves representation is only a Symbolic Representation.

7. This wave nature of matter introduces an uncertainity in the location of the position of the point because a wave a cannot be said exactly at this point or at that point.

But when the wave is large (Small), there is a maximum (minimum) probability of finding the particle.

$$y = A\cos(2\pi\nu t)$$

$$y = A\cos\left\{2\pi\nu\left[t - \frac{x}{v_p}\right]\right\},$$

$$= A\cos\left\{2\pi\left[\nu t - \frac{\nu x}{v_p}\right]\right\}.$$

$$= A\cos\left\{2\pi\left[\nu t - \frac{\nu x}{\nu\lambda}\right]\right\} \quad [\because v_p = \nu\lambda]$$

$$= A\cos\left\{2\pi\left[\nu t - \frac{x}{\lambda}\right]\right\}.$$

$$= A\cos\left\{2\pi\left[\left(\frac{t}{T}\right) - \left(\frac{x}{\lambda}\right)\right]\right\}.$$

$$= A\cos\left\{(2\pi\nu)t - \left(\frac{2\pi}{\lambda}\right)x\right\}$$

$c = \nu\lambda$
$v = \nu\lambda.$

$$y = A\cos\{\omega t - kx\}.$$

or

$$y = A\sin\{\omega t - kx\}.$$

$K = 2\pi/\lambda$
$= \frac{2\pi\nu}{v}$
$= (\omega/v)$
$\Rightarrow v = (\omega/k)$
$\propto v_p.$

$$\text{where } K = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{\cancel{2p}v_p} = \frac{\cancel{\omega}}{\cancel{2p}}\frac{\omega}{v_p}$$

$$\text{or } \boxed{v_p = \left(\frac{\omega}{K}\right)}$$

ω - radian per second

K - radian per metre (wave Number) - [No. of Radians corresponding to a wave train of 1 m long as there are 2π radians in one complete wave]

· Angular freq. is defined from uniform circular motion where a particle moves round a circle 'ν' times per second sweeps out $2\pi\nu$ radians per second.

Schrodinger Time Independent Wave Equation.

consider a particle of mass 'm' that moves with a velocity 'v' that can be represented as a wave of wavelength (λ) given by $\lambda = (h/mv)$ according to de-Broglie hypothesis. The particle that has the wave properties is defined by a wavefunction (ψ).

Let x, y, z be the coordinates of the particle that ~~it~~ is composed of system of stationary waves. ~~If~~ The function ψ that defines the displacement of the particle should be finite, single valued and periodic function. The ~~at~~ differential equation that defines the wave motion classically is given by

$$\left(\frac{\partial^2 \psi}{\partial t^2}\right) = v^2\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right)$$

$$= v^2 \nabla^2 \psi \longrightarrow (1)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is known as Laplacian Operator.

The Solution of the above Second order differtial equation is given by

$$\psi = \psi_0 \sin(\omega t) = \psi_0 \left[\sin(2\pi \nu t)\right].$$

Differentiating the equation w.r.t. time 't', we get

$$\left(\frac{\partial \psi}{\partial t}\right) = \psi_0 \left[\cos(2\pi \nu t)\right](2\pi \nu).$$

~~On~~ Differentiating once again, we get

$$\left(\frac{\partial^2 \psi}{\partial t^2}\right) = \psi_0 \left[-\sin(2\pi \nu t)\right](2\pi \nu)^2.$$

$$= -(4\pi^2 \nu^2)\left\{\psi_0 \sin(2\pi \nu t)\right\}.$$

$$= -(4\pi \nu^2)\psi.$$

Substituting the value of $\left(\frac{\partial^2 \psi}{\partial t^2}\right)$ from eq(1), (on LHS), we get.

$$v^2 \nabla^2 \psi = -(4\pi \nu^2)\psi.$$

$$v^2 \nabla^2 \psi = -4\pi^2 \nu^2 \psi.$$

$$= -\left(\frac{4\pi^2 v^2}{\lambda^2}\right)\psi \qquad [\because v = \nu\lambda \text{ or } c = \nu\lambda]$$

(or) $$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2}.\psi = 0.$$

From the de-Broglie's hypothesis, we have $\lambda = h/mv$

$$\therefore \quad \nabla^2 \psi + \frac{4\pi^2 m^2 v^2}{h^2}.\psi = 0.$$

$$\nabla^2 \psi + \left[\frac{m^2 v^2}{(h/2\pi)^2}\right]\psi = 0$$

$$\nabla^2 \psi + \frac{m^2 v^2}{\hbar^2}.\psi = 0 \qquad (\text{where } \hbar = h/2\pi).$$

Let E be the total energy of the particle and V be its potential energy.

Hence $(E - V)$ defines the Kinetic Energy given by $\frac{1}{2}mv^2$.

$$\Rightarrow \quad E - V = \frac{1}{2}mv^2.$$

$$\Rightarrow \quad 2(E-V) = mv^2$$

(or) $$m^2 v^2 = 2m(E-V)$$

Substituting this value in the second order differential equation, we get

$$\boxed{\nabla^2 \psi + \frac{2m}{\hbar^2}(E-V)\psi = 0} \quad \text{or} \quad \boxed{\nabla^2 \psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0}.$$

This equation is known as Schrodinger's time Independent Equation

The above equation can also be represented as.

$$\frac{\hbar^2}{2m}\nabla^2 \psi + (E-V)\psi = 0$$

$$\Rightarrow \quad \frac{\hbar^2}{2m}\nabla^2 \psi + E\psi - V\psi = 0$$

$$\Rightarrow \quad \frac{\hbar^2}{2m}\nabla^2 \psi - V\psi = -E\psi$$

$$\Rightarrow \left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = E\psi$$

$$\boxed{\hat{H}\psi = E\psi.}$$

Where $\hat{H} = \left[-\left(\frac{\hbar^2}{2m}\right)\nabla^2 + V\right]$ is Known as HAMILTONIAN OPERATOR.

In the case of a free particle, the potential energy. $V=0$.

Hence the Schrodinger wave equation for a free particle is given by

$$\boxed{\nabla^2\psi + \left(\frac{2m}{\hbar^2}\right)E\psi = 0.}$$

Schrodinger Time Dependent Equation.

To define this wave equation, Schrodinger Time Independent wave equation is to be taken and 'E' is to be eliminated. Here ψ is a variable associated the moving particle. So, this is a complex function of space coordinates of position & time.

The ~~d~~ second order differential equation that represents the wave motion in one dimension classically is given by

$$\left(\frac{\partial^2\psi}{\partial t^2}\right) = v^2\nabla^2\psi$$

The solution of this equation (ψ) can be defined as a function of space coordinates (as it is complex) by

$$\psi(x,y,z,t) = \psi_0(x,y,z)\cdot e^{-i\omega t}.$$

(or) $$\psi = \psi_0 e^{-i\omega t}.$$

Differentiating the equation w.r.t time 't', we get

$$\left(\frac{\partial\psi}{\partial t}\right) = \psi_0(-i\omega)e^{-i\omega t}$$

$$= \psi_0(-i\,2\pi\nu)\cdot e^{-i\omega t}$$

$$= -(2\pi\nu)\,i\cdot\psi.$$

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(or) $\left(\frac{\partial \psi}{\partial t}\right) = -i\left[2\pi\left(\frac{E}{h}\right)\right] \psi$ $\quad [\because E = h\nu]$

$$= -i\left[\frac{E}{(h/2\pi)}\right]\psi$$

$$= -\frac{iE}{\hbar}\psi .$$

$$\Rightarrow \quad E\psi = -\left(\frac{\hbar}{i}\right)\frac{\partial \psi}{\partial t}$$

$$= i^2\left(\frac{\hbar}{i}\right)\frac{\partial \psi}{\partial t} \qquad [\because i^2 = -1]$$

$$= i\hbar\left(\frac{\partial \psi}{\partial t}\right)$$

(or) $E\psi = i\hbar\left(\frac{\partial \psi}{\partial t}\right)$

Substituting this value in the Schrodinger Time Independent Eq,

$$\nabla^2\psi + \frac{2m}{\hbar^2}[E-V]\psi = 0,$$

we get

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left[i\hbar\frac{\partial \psi}{\partial t} - V\psi\right] = 0 .$$

$$\Rightarrow \nabla^2\psi = -\frac{2m}{\hbar^2}\left[i\hbar\frac{\partial \psi}{\partial t} - V\psi\right] .$$

$$\Rightarrow -\frac{\hbar^2}{2m}\nabla^2\psi = i\hbar\left(\frac{\partial \psi}{\partial t}\right) - V\psi$$

(or) $-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\left(\frac{\partial \psi}{\partial t}\right)$

The last equation describes the motion of a non-relativistic material particle.

(or) $\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = i\hbar\frac{\partial \psi}{\partial t}.$

$$\Rightarrow \boxed{\hat{H}\psi = \hat{E}\psi}$$

where $\hat{H}$ is the Hamiltonian operator & $\hat{E} = i\hbar\frac{\partial}{\partial t}$ is the Energy Operator.

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Wave function and its interpretation. (ψ)

When the concept of wave function (ψ) was introduced, it was thought that it is only an auxillary Mathematical quantity used for computations for experimental results. This is according to Millikan.

The first simple interpretation was given by Schrodinger, in terms of charge density.

If an electromagnetic wave has an amplitude of "A", then the energy per unit volume (energy density) is given by "A^2".

Therefore, Number of photons per unit volume $= \dfrac{\text{Energy per unit volume}}{\text{Energy of one(each) photon}}$

$\downarrow$

$$\text{photon Density} = \frac{A^2}{h\nu}.$$

$$\text{(or) photon density} \propto A^2.$$

Similarly, If ψ is the amplitude of the amplitude of the matter waves, then the particle density is proportional to "$\underline{\psi^2}$".

ie the square of the absolute value of ψ is the measure of particle density.

When this is multiplied by the charge of the particle, "ψ^2" now gives the measure of charge density.

If "ψ" refers to a single particle (in some cases only) where ψ is different from zero within that finite region, called as wave packet $|\psi|^2 (= \psi\psi^*)$ gives the probability of finding the particle in the state ψ. Here ψ^2 is the measure of Probability density. This is according to Max Born.

The probability of finding the particle in a volume of $dv = dx\,dy\,dz$ is given by

$$|\psi|^2 \, dx\,dy\,dz.$$

The total probability of finding the particle somewhere in the space (certainly) is unity.

ie $$\iiint |\psi|^2 \, dx\,dy\,dz = 1$$

This is called as the Normalisation condition.

Properties of wavefunctions

1. The wavefunction must be normalizable
2. The wavefunction must be single valued
3. The wavefunction must be finite every where
4. The wavefunction must be continuous and should have a continuous first derivative every where.
5.

$$K = \frac{2\pi}{\lambda} = \frac{2\pi P}{h} = \frac{P}{\hbar} = \frac{\sqrt{2mE}}{\hbar}$$

or $$K^2 = \frac{2mE}{\hbar^2}$$

Physical Significance of Zero point Energy

* Fundamental characteristic of ~~zero point energy~~ Quantum Mechanics
* which shows that in Quantum Mechanics, the minimum (Possible) energy of a particle confined in one dimensional box is not zero
* If energy (minimum) is zero, $(E=0, P=0)$ the particle will be located at a particular position $(\Delta x = 0)$ uncertainity in position is zero ie uncertainity in momentum, $\Delta p = 0$. with zero ~~with with~~ in Momentum
* $\Delta x = 0, \Delta P = 0$ violates uncertainity.
* The existence of ZPE preserves Heisenberg's Uncertainity Principle

$$E = 0 \Rightarrow P = 0 \Rightarrow mv = 0$$
$$\Rightarrow \frac{dx}{dt} = 0 \Rightarrow x = \text{Constant}$$

Physical Significance (Interpretation) of wavefunction ψ.

The quantity with which quantum Mechanics is concerned with is the wavefunction "ψ" of a body. The quantity that varies periodically is called a wave function, ψ, that has no direct physical significance and is also not an observable quantity. The square of its absolute magnitude $|\psi^2|$ defined at a particular position and particular time is proportional to the probability of finding the body there at that time. This probability can have any value between the limits zero and one. zero refers to the certainity of its absence where as one-corresponding to the certainity of presence. But the amplitude of any wave ~~at~~ may be positive or negative. But the probability being negative is a meaningless concept. Hence the wave function ψ, which is complex, & is the form of $(A+iB)$ is not an observable quantity and has no direct physical meaning.

If 'A' is the amplitude of an electromagnetic wave then A^2 (Intensity) defines Energy density ie Energy per unit volume. Similarly ~~where~~ $(A^2/h\nu)$ defines the photon density ie number of photons per unit volume. Similarly if ψ is the amplitude of a wave (rather ~~S~~ matter wave) at any instant then the particle density at any point is proportional to ψ^2. Hence ψ^2 is a measure of Particle density. If this is multiplied by charge, we get the charge Density.

$\therefore$ $A^2 \rightarrow$ photon Density

$\psi^2 \rightarrow$ Charge Density. (Particle Density)

$\psi^2 \rightarrow$ Probability of finding a particle (Probability Density)

According to Max Born, to explain how or where the particle is in relation to wave packet, he gave a new physical Significance as $\psi\psi^* = |\psi^2|$

where ψ^2 is measure of probability Density.

If ψ ~~to~~ is complex with both real & imaginary parts,

$\psi = A+iB$, then $\psi^* = A-iB$.

Here $\psi \times \psi^* = (A+iB)(A-iB)$

$= A^2+B^2$. (which is a positive quantity)

If dv is the volume element ($dv = dx\,dy\,dz$), then the total probability of finding the particle in the space must be unity, given by

$$\int_{-\infty}^{+\infty} P\,dV = 1 \quad \text{or} \quad \int_{-\infty}^{+\infty} |\psi(x,y,z)|^2 dV = 1 \quad \text{or} \quad \iiint |\psi|^2 dx\,dy\,dz = 1$$

where ψ is a function of (x,y,z), the space coordinates. The wave function that satisfies the above equation is said to be Normalised.

Every wavefunction can be normalised by multiplying with a constant.

Normalisation condition for motion of particle in one dimension of $\psi(x)$ is $L^{-1/2}$. Three dimension of $\psi(x,y,z)$ is $L^{-3/2}$.

Properties of ψ

1. Since Probability (P) can have only one value at a particular instant, ψ must be single-valued, ~~and~~ continuous and finite.
2. The partial derivatives $(\partial\psi/\partial x)$, $(\partial\psi/\partial y)$ and $(\partial\psi/\partial z)$ must be continuous and single-valued everywhere.
3. ψ must be normalisable.

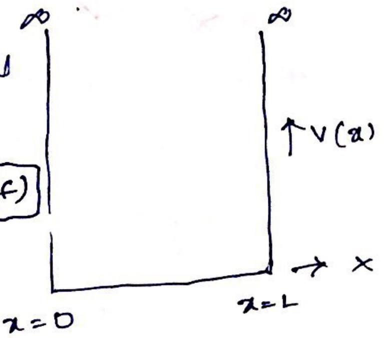
Postulates of Quantum Mechanics-

1. Wave functions to describe Physical System
   (i) $\psi(x)$ &, $[d\psi(x)/dt]$ must be finite for all values of x.
   If $\psi(x) = \infty$ $|\psi|^2 = \infty \Rightarrow$ particle is localised.
   This is a contradiction to with the wave property of system
   (ii) $\psi(x)$ &, $[d\psi(x)/dt]$ must be continuous for all values of x.
   (in the region where the potential $V(x,y,z)$ is infinite)
   (iii) $\psi(x)$ &, $[d\psi(x)/dt]$ must be single valued all x in the region
2. Operators for observable quantities.
3. ψ^2 that gives probability is essential to calculate the average or expectation values of the dynamical quantity defined by ψ. The expected or the average value of a dynamical quantity is the mathematical expectation for the result of single measurement.

One dimensional box.

consider an electron of mass 'm' confined to a one-dimensional box of length 'L' by an infinite barriers at two ends.

This line of length (L) is taken along x-axis and the electron is placed in an infinite deep potential well. The potential energy at any point on the line ~~and is constant and is zero.~~



$\boxed{\text{*** INTRODUCTORY EXPLAINATION(SEE BACK PAGE)}}$

Boundary conditions.

$V(x) = 0 \qquad (0 \leq x \leq L)$

$V(x) = \infty \qquad x \leq 0$ and $x \geq L$.

The movement of electron is restricted by the walls and the collision of electron with the wall is elastic (ie no loss of energy).

The wavefunction $\psi_n(x)$ of an electron in the region $x=0$ or at $x=L$ where $V(x)=0$ is defined by Schrodinger's equation as

$$\left(\frac{d^2\psi_n}{dx^2}\right) + \frac{2m}{\hbar^2} E_n \psi_n = 0. \qquad \left[\because V=0 \text{ in } \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0\right]$$

Here E_n is the total energy of the electron in the nth state ie (the kinetic energy).

The general solution for the above equation is given by

$$\cancel{\psi_n(x) = Ae^{ikx} + Be^{-ikx}}$$

*** $\psi_n(x) = [A\sin kx + B\cos kx]$ ✓ $\qquad \left[\because \frac{d^2\psi}{dx^2} + k^2\psi = 0\right]$

where $k^2 = \frac{2m}{\hbar^2}(E_n)$,

A & B are arbitrary constants.

By applying the boundary conditions ~~boundations~~, the constants A & B can be determined.

As the particle is enclosed between the rigid walls, the probability of finding the particle at any instant is given by ψ^2.

The value of the function vanishes ie $\psi_n(0) = 0$, $\psi_n(L) = 0$ $(x=0$ & $x=L)$, as the particle cannot penetrate through the walls.

This explains about the wave function to be continuous.

If this is not considered, $\psi_n(x)\, V(x)$ becomes infinite, then K.E should also be infinite which is not possible.

Hence, the above mentioned condition is called as the first Boundary Condition.

Applying the first boundary condition, $\psi = 0$ at $x = 0$, we get

$$0 = A(0) + B(1)$$

or $\cancel{A} = \cancel{B}$. or $B = 0$. ∵

Hence the value of wave function now is

$$\psi_n(x) = A \sin kx.$$

Applying the second boundary condition $\psi = 0$ at $x = L$, we get

$$\psi_n(L) = 0 = A \sin(kL).$$

The solution for above should be either $A = 0$ or $\sin kL = 0$.

As $A \neq 0$ (if $A = 0$ then $B = 0$ so ψ becomes zero – not possible)

$$\sin KL = 0.$$

$$\Rightarrow K_n L = n\pi$$

(or) $K_n = \left(\frac{n\pi}{L}\right)$ where $n = 1, 2, 3 \dots$

The wave equation is now written as

$$\psi_n(x) = A \sin\left[\left(\frac{n\pi}{L}\right)x\right]$$

From the equations of K, we come to conclusion that

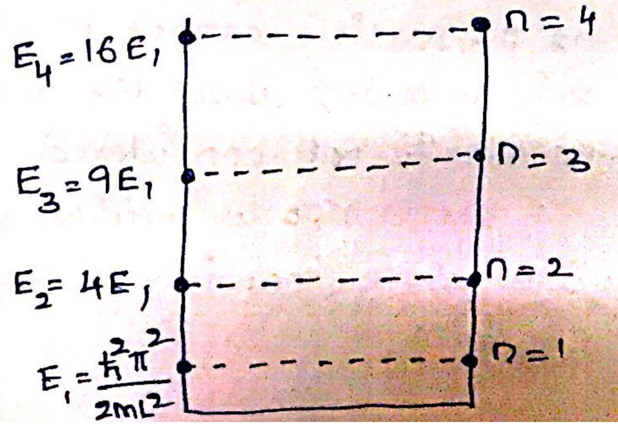
$$K^2 = \left(\frac{n^2\pi^2}{L^2}\right) \quad \text{and} \quad K^2 = \left(\frac{2m}{\hbar^2}\right) E_n \Rightarrow E_n = \frac{\hbar^2 k^2}{2m}$$

$$\Rightarrow \left(\frac{n^2\pi^2}{L^2}\right) = \left(\frac{2m}{\hbar^2}\right) \cdot E_n \quad E_1$$

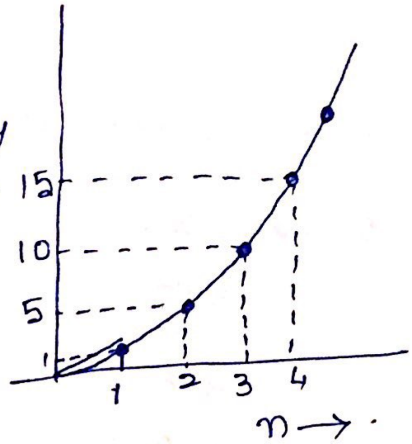
$$\text{(or)} \quad E_n = \frac{\hbar^2}{2m}\left[\frac{n\pi}{L}\right]^2 = n^2 \frac{\pi^2 h^2}{8\pi^2 m L^2} = \frac{h^2}{2m}\left[\frac{n}{2L}\right]^2.$$

This equation for E_n that defines the discrete set of energy values in an infinite deep potential well are called as "Eigen Energy States", that depends on 'n' called as the Principal Quantum number.

$n = 1$, $E_1 = \frac{h^2}{8mL^2}$ = zero point Energy.



The variation between the Principal Quantum number and the energy cand be drawn



The constant 'A' can be determined by using the normalization condition

$$\int_0^L \psi_n^*(x) \cdot \psi_n(x)\, dx = 1$$

$$\Rightarrow \int_0^L |\psi_n(x)|^2 dx = 1$$

$$\Rightarrow \text{(or)} \int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$\Rightarrow A^2 \int_0^L \frac{1}{2}\left[1 - \cos\left(\frac{2n\pi x}{L}\right)\right] dx = 1 \qquad \because \sin^2 x = \left[\frac{1-\cos 2\theta}{2}\right]$$

$$\Rightarrow \frac{A^2}{2}\left[x - \frac{L}{2\pi n}\sin\left(\frac{2n\pi x}{L}\right)\right]_0^L = 1$$

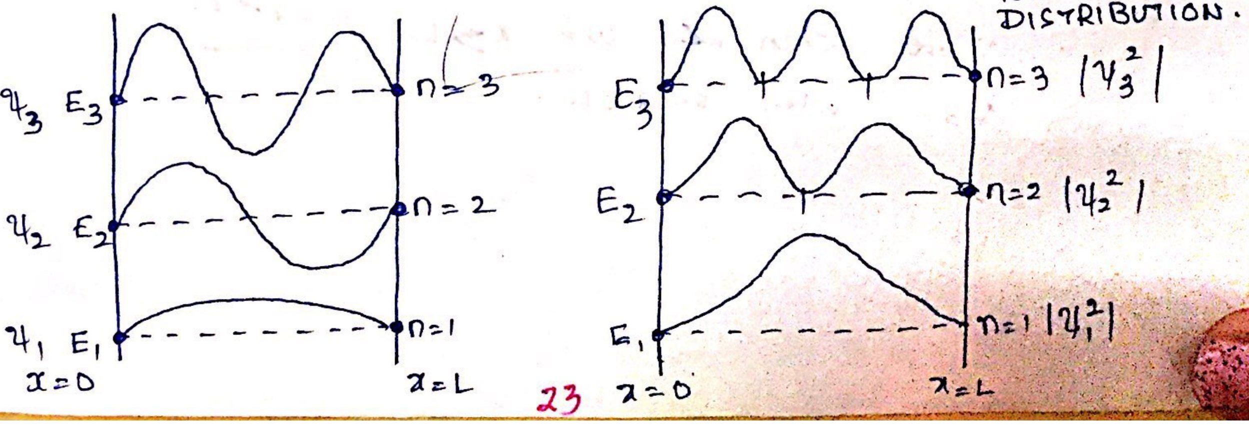
$$\Rightarrow \left(\frac{A^2 L}{2}\right) = 1$$

$$\text{(or)} \quad A = \sqrt{\frac{2}{L}}.$$

$$\therefore \boxed{\psi_n(x) = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{n\pi x}{L}\right)}.$$

This defines the ~~equati~~ wave function of an electron ~~in~~ enclosed in an infinite deep potential well.

QUANTUM MECHANICAL DISTRIBUTION.



$n = 0$ is not possible.

If $n=0$, $E = 0$

$\Rightarrow \frac{p^2}{2m} = 0$

$\Rightarrow p = 0$

or $\Delta p = 0$

Heisenberg $\Rightarrow$ $\Delta p \Delta x \geq \hbar/2$

$\Rightarrow \boxed{\Delta x = \infty.}$

But particle is fixed in between $x=0$ & $x=L$.

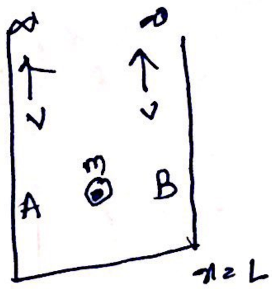
As this is contradiction to length of box.

*** **Introductory Explanation**

when the particle moves freely between the rigid walls A & B,

the potential Energy of the particle is ~~zero~~ constant (taken as zero) at $x=0$ or $x=L$ or $0<x<L$.

This is mainly because there are no forces acting on the particle.



when the particle collide with walls which are perfectly rigid, the particle reflects back. Here the collision is elastic and hence no loss of energy.

Hence the force (F) acting on the particle suddenly changes from zero to finite value within the length of the box.

As $F = \left(\frac{dV}{dx}\right)$ ~~$\Delta V \to \infty$ when $\Delta x = 0$~~.

*** $\therefore$ we should have $\Delta V \to \infty$ when $\Delta x = 0$ so that $F = (\partial V/\partial x)$ becomes finite. ***

$\therefore$ $V = \infty$ when $x \leq 0$ and $x \geq L$. $0 > x > L$.

$V = 0$ when $0 < x < L$.

Recoil Angle Direction.

Horizontal component of Momentum, $(mvc)\cos\phi = h\nu - h\nu'\cos\theta$.

Vertical component of Momentum, $(mvc)\sin\phi = h\nu'\sin\theta$.

$$\therefore \quad \tan\phi = \frac{h\nu'\sin\theta}{h\nu - h\nu'\cos\theta} = \frac{\nu'\sin\theta}{\nu-\nu'\cos\theta}$$

$$\nu' = \frac{\nu}{1+\alpha(1-\cos\theta)}$$ (See on reverse side) for proof. where $\alpha = \left(\frac{h\nu}{m_0c^2}\right)$

$$\tan\phi = \frac{\left[\frac{\nu}{1+\alpha(1-\cos\theta)}\right](\sin\theta)}{\nu - \frac{\nu(\cos\theta)}{1+\alpha(1-\cos\theta)}}$$

$$= \frac{\nu\sin\theta / \cancel{[1+\alpha(1-\cos\theta)]}}{\nu[1+\alpha(1-\cos\theta)] - \nu\cos\theta / \cancel{[1+\alpha(1-\cos\theta)]}}$$

$$= \frac{\nu\sin\theta}{\nu + \nu\alpha - \nu\alpha\cos\theta - \nu\cos\theta}$$

$$= \frac{\nu\sin\theta}{\cancel{\nu(1+\alpha)} \; \nu[1-\cos\theta] + \nu\alpha[1-\cos\theta]}$$

$$= \frac{\nu\sin\theta}{(\nu+\nu\alpha)[1-\cos\theta]}$$

$$= \frac{\cancel{\nu}\sin\theta}{\cancel{\nu}(1+\alpha)[1-\cos\theta]}$$

$$= \frac{2\sin(\theta/2)\cos(\theta/2)}{(1+\alpha)[\cancel{1}-\cancel{1}+2\sin^2(\theta/2)]}$$

$$= \frac{\cancel{2}\,\cancel{\sin(\theta/2)}\cos(\theta/2)}{\cancel{2}(1+\alpha)\sin^{\cancel{2}}\theta/2}$$

$$= \frac{\cot\theta/2}{1+\alpha}$$

$$\boxed{\tan\phi = \frac{\cot(\theta/2)}{1+\left(\frac{h\nu}{m_0c^2}\right)}}$$

6\. Method - I

$$(K.E)_{max} = \frac{[2h^2\nu^2/m_0c^2]}{1+(2h\nu/m_0c^2)}$$

Method - II.

Principle of conservation of Energy,

$$h\nu + m_0c^2 = h\nu' + mc^2.$$

$$\Rightarrow (m-m_0)c^2 = h\nu - h\nu'$$

$$\Rightarrow (K.E)_{max} = h\nu - h\nu'.$$

From eq $\lambda' - \lambda = \frac{h}{m_0c}[1-\cos\theta]$,

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}[1-\cos\theta].$$

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0c^2}[1-\cos\theta].$$

$$= \frac{1+\alpha(1-\cos\theta)}{\nu} \quad \text{where } \alpha = (h\nu/m_0c^2)$$

$$\therefore \nu' = \frac{\nu}{1+\alpha\cancel{\cos\theta}(1-\cos\theta)}$$

Substitute ν' in $(K.E)_{max}$ Eqn.

$$(K.E)_{max} = h\nu - \frac{h\nu}{1+\alpha\cancel{\cos\theta}(1-\cos\theta)}$$

$$= h\nu\left[1 - \frac{1}{1+\alpha\cancel{\cos\theta}(1-\cos\theta)}\right]$$

$$= \cancel{h\nu}\left[\frac{\cancel{(1+\alpha\cos\theta)-1}}{1+\cancel{\alpha\cos\theta}}\right] h\nu\left[\frac{\cancel{1}+\alpha(1-\cos\theta)-\cancel{1}}{1+\alpha(1-\cos\theta)}\right].$$

$$= \cancel{\frac{h\nu(\alpha\cos\theta)}{1+\alpha\cos\theta}} \quad \frac{h\nu\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}$$

When $\theta = 180°$ $\cos\theta = -1$

$$\therefore (K.E)_{max} = \frac{2h\nu\alpha}{1+2\alpha}.$$

$$(K.E)_{max} = \left[\frac{(2h^2\nu^2/m_0c^2)}{1+2(h\nu/m_0c^2)}\right] = \left[\frac{2E^2/E_0}{1+2(E/E_0)}\right].$$

$$K.E = \frac{2h^2\nu^2/m_0c^2}{1+2(h\nu/m_0c^2)}.$$

Method-1

Cons. of Energy.

$$h\nu + m_0c^2 = h\nu' + mc^2$$

$$\Rightarrow (m-m_0)c^2 = h\nu - h\nu'.$$

$$K.E\uparrow = h\nu - h\nu'.$$

From CE, $\Delta\lambda = \lambda_c[1-\cos\theta]$

$$\lambda' - \lambda = \frac{h}{m_0c}[1-\cos\theta]$$

$$\left[\frac{1}{\nu'} - \frac{1}{\nu}\right] = \frac{h}{m_0c^2}[1-\cos\theta]$$

$$\Rightarrow \frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0c^2}[1-\cos\theta]$$

$$= \frac{1+(h\nu/m_0c^2)[1-\cos\theta]}{\nu}$$

$$\left(\alpha = \frac{h\nu}{m_0c^2}\right) \quad \frac{1}{\nu'} = \frac{1+\alpha(1-\cos\theta)}{\nu}$$

$$\Rightarrow \nu' = \frac{\nu}{1+\alpha(1-\cos\theta)}.$$

$$\therefore \uparrow K.E = h\nu - \frac{h\nu}{1+\alpha(1-\cos\theta)}$$

$$= \frac{h\nu[\cancel{1}+\alpha(1-\cos\theta)-\cancel{1}]}{1+\alpha(1-\cos\theta)}$$

$$= \frac{h\nu[\alpha\{\cancel{1}-\cancel{1}+2\sin^2\theta/2\}]}{1+\alpha(\cancel{1}-\cancel{1}+2\sin^2\theta/2)}.$$

$$= \frac{h\nu\alpha(2\sin^2\theta/2)}{1+2\alpha\sin^2\theta/2}.$$

$$\theta = 180^\circ.$$

$$\uparrow K.E = \frac{2h\nu\alpha}{1+2\alpha}.$$

$$= \frac{2h^2\nu^2/m_0c^2}{1+2h\nu/m_0c^2}$$

$$= \frac{2E^2/E_0}{1+(2E/E_0)}$$

Method-2

$$\uparrow K.E = h\nu - h\nu'.$$

$$= h\nu\left[1-\left(\frac{\nu'}{\nu}\right)\right]$$

$$= h\nu\left[1-\left(\frac{\lambda}{\lambda'}\right)\right]$$

$$= h\nu\left[\frac{\lambda'-\lambda}{\lambda'}\right]$$

$$= \frac{h\nu}{\lambda'}[\Delta\lambda]$$

$$= \left(\frac{h\nu}{\lambda'}\right)\left[\frac{h}{m_0c}(1-\cos\theta)\right]$$

$$\left(\alpha = \frac{h}{m_0c}\right) \quad = \left(\frac{h\nu}{\lambda'}\right)[\alpha(1-\cos\theta)]$$

$$= \frac{h\nu[\alpha(1-\cos\theta)]}{\lambda+\alpha(1-\cos\theta)}$$

$$= \frac{h\nu[\alpha(1-\cos\theta)]}{\cancel{\alpha}[\lambda+\frac{\alpha}{\cancel{\alpha}}(1-\cos\theta)]}$$

$$= \frac{h\nu[\alpha(\cancel{1}-\cancel{1}+2\sin^2\theta/2)]}{\lambda+\alpha(\cancel{1}-\cancel{1}+2\sin^2\theta/2)}.$$

$$= \frac{2h\nu\alpha\sin^2\theta/2}{\lambda+2\alpha\sin^2\theta/2}.$$

$$= \frac{2h\nu\alpha\sin^2\theta/2}{\lambda[1+\frac{2\alpha}{\lambda}\sin^2\theta/2]}.$$

$$= \frac{\left(\frac{2h\nu^2}{c}\right)\alpha\sin^2(\theta/2)}{1+\frac{2\alpha\nu}{c}\sin^2\theta/2}.$$

$$= \frac{\left(\frac{2h\nu^2}{c}\right)\left(\frac{h}{m_0c}\right)\sin^2(\theta/2)}{1+\left(\frac{2\nu}{c}\right)\left(\frac{h}{m_0c}\right)\sin^2\theta/2}.$$

$$= \frac{2(h^2\nu^2/m_0c^2)\sin^2\theta/2}{1+\left(\frac{2h\nu}{m_0c^2}\right)\sin^2\theta/2}$$

$$\theta = 180^\circ, \quad E = \frac{2h^2\nu^2/m_0c^2}{1+(2h\nu/m_0c^2)\sin^2\theta/2}.$$

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6 Method-2 Energy of the Recoil Electron.

$$(K.E)_{Max} = \frac{[2h\nu^2/m_0c^2]}{1+(2h\nu/m_0c^2)}.$$

Method-2

$$E_K = h\nu - h\nu'$$

$$= h\nu\left[1-\left(\frac{\nu'}{\nu}\right)\right]$$

$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)$

$\therefore \lambda' = \lambda + \frac{h}{m_0c}(1-\cos\theta)$

$$= h\nu\left[1-\frac{\lambda}{\lambda'}\right]$$

$$= h\nu\left[\frac{\lambda'-\lambda}{\lambda'}\right]$$

$$= h\nu\left[\frac{\Delta\lambda}{\lambda'}\right].$$

$$= \frac{h\nu}{\lambda'}\left[\frac{h}{m_0c}[1-\cos\theta]\right]$$

$\alpha = (h/m_0c)$

$$= \frac{h\nu[\alpha(1-\cos\theta)]}{[\lambda+\alpha(1-\cos\theta)]}$$

$$= \frac{h\nu[\alpha\{\cancel{1}-\cancel{1}+2\sin^2(\theta/2)\}]}{\lambda+\alpha\{\cancel{1}-\cancel{1}+2\sin^2(\theta/2)\}}.$$

$E = h\nu$.

$= \frac{hc}{\lambda}$.

$c = \nu\lambda$

$\lambda = \left(\frac{c}{\nu}\right)$

$$= \frac{2h\nu\alpha\sin^2\theta/2}{\lambda+2\alpha\sin^2\theta/2}$$

$$= \frac{2h\nu\alpha\sin^2(\theta/2)}{\lambda\left[1+\left(\frac{2\alpha}{\lambda}\right)\sin^2(\theta/2)\right]}.$$

$$= \frac{2h\nu\alpha[\sin^2(\theta/2)]}{(c/\nu)\left[1+\frac{2\alpha}{\lambda}\sin^2(\theta/2)\right]}.$$

$$= \frac{2\left(\frac{h\nu^2}{c}\right)\alpha[\sin^2(\theta/2)]}{\left[1+\left(\frac{2\alpha}{\lambda}\right)\sin^2(\theta/2)\right]}.$$

$$= \frac{2\left(\frac{h\nu^2}{c}\right)\left(\frac{h}{m_0c}\right)[\sin^2(\theta/2)]}{1+2\left(\frac{h\nu}{m_0c^2}\right)\sin^2(\theta/2)}$$

$$(K.E)_{Max} = \frac{\left(\frac{2h^2\nu^2}{m_0c^2}\right)[\sin^2(\theta/2)]}{1+2(h\nu/m_0c^2)\sin^2(\theta/2)}$$

$\theta = 180^\circ$

$$(K.E)_{Max} = \frac{2h^2\nu^2/m_0c^2}{1+(2h\nu/m_0c^2)}$$

$$= \left[\frac{2E^2/E_0}{1+2E/E_0}\right]$$

7 $\lambda = 10.0\ pm = 10\times10^{-12}\ m$

$\lambda' = 10.5\ pm = 10.5\times10^{-12}\ m$

$(\text{Momentum})_{\text{recoil Electron}} = ?$

$\Delta\lambda = \lambda' - \lambda = 0.5\times10^{-12}\ m$

But $\lambda' - \lambda = \frac{h}{m_0 c}[1-\cos\theta]$

$\therefore \cos\theta = 1 - \Delta\lambda\left(\frac{m_0 c}{h}\right)$

$= 1 - 0.5\times10^{-12}\left[\frac{9.11\times10^{-31}\times3\times10^{8}}{6.625\times10^{-34}}\right]$

$= 1 - 0.5\times10^{-12}\left[\frac{9.11\times3}{6.625}\times10^{11}\right]$

$= 1 - \left[\frac{0.5\times9.11\times3}{6.625}\right]\times10^{-1}$

$= 1 - \frac{13.665}{6.625}\times10^{-1}$

$\cos\theta = 1 - 0.206 = 0.794$

$\theta = \cos^{-1}(0.794) = \cos^{-1}(0.8)$

$= \underline{\hspace{2cm}}$

$(mvc)\cos\phi = h\nu - h\nu'\cos\theta$

$(mvc)\sin\phi = h\nu'\sin\theta$

$m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta$

$= \frac{h^2c^2}{\lambda^2} + \frac{h^2c^2}{\lambda'^2} - \frac{2hc^2}{\lambda\lambda'}\cos\theta$

$m^2v^2 = h^2\left[\frac{1}{\lambda^2} + \frac{1}{\lambda'^2} - \frac{2\cos\theta}{\lambda\lambda'}\right]$

$p^2 = [6.625\times10^{-34}]^2$

$\left[10^{22} + \frac{10^{22}}{1.1025} - \frac{2\times0.794\times10^{22}}{1.05}\right]$

$p = 6.625\times10^{-34}\times10^{11}$

$\left[\sqrt{1 + \frac{1}{1.1025} - \frac{1.588}{1.05}}\right]$

$= \underline{\hspace{2cm}}$

Direction.

① $\tan\phi = \frac{\cot(\theta/2)}{1+(h\nu/m_0c^2)}$

② ~~$(m-m_0)c^2 = h\nu - h\nu' = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda'}\right]$~~

$\lambda = 10\times10^{-12}\ m$

$\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{10^{-11}} = 3\times10^{19}\ Hz$

$h = 6.625\times10^{-34}\ J.Sec$

$m_0 = 9.11\times10^{-31}\ kg$

$c = 3\times10^{8}\ m/s$

$\theta = \underline{\hspace{2cm}}$

$\therefore \phi = \tan^{-1}\left[\frac{\cot(\theta/2)}{1+(h\nu/m_0c^2)}\right]$

$= \underline{\hspace{2cm}}$

① Radius - 10^{-14} m.

Maximum uncertainity $(\Delta x)_{max}$ = 2×10^{-14} m (diameter, d = 2r)

$\therefore \Delta p \cdot \Delta x = \hbar$. Electron.

$\Delta p = \frac{\hbar}{(\Delta x)_{max}} =$ ——— 5.275×10^{-21} kg.m/s.

Since minimum uncertainity in the momentum of electron should be equal to its momentum

$P = (\Delta P)_{min} = 5.275\times10^{-21}$ kg-m/s.

Energy, $E = \frac{P^2}{2m} = \frac{(5.275\times10^{-21})^2}{2\times9.1\times10^{-31}}$ J.

$= \frac{(5.275\times10^{-21})^2}{2\times9.1\times10^{-31}\times1.6\times10^{-19}}$ eV

$= 97$ MeV.

~~Bett~~ Should be 97 Mev for its existence but has only 4 MeV.

② - Proton

$\Delta x = 2\times10^{-14}$ m.

$\Delta p \Delta x = \hbar$.

$(\Delta P)_{min} = 5.275\times10^{-21}$ kg m/s.

$E = \frac{P^2}{2m} = \frac{(5.275\times10^{-21})^2}{2\times1.6\times10^{-27}}$ J

$= \frac{(5.275\times10^{-21})^2}{2\times1.6\times10^{-27}\times1.6\times10^{-19}}$ eV.

$E = 52$ keV.

$\simeq$ Same energy of proton.

* Show that the de Broglie wavelength for a material particle of rest mass 'm_0' and charge 'e', accelerated from rest through a potential difference of 'V' volts relativistically is given by

$$\lambda = \frac{h}{\sqrt{2m_0 eV\left\{1+\left(\frac{eV}{2m_0c^2}\right)\right\}}}.$$

Sol, The potential energy of a charged particle of charge 'e' when placed in a potential of 'V' is given by

$$P.E = (eV).$$

Relativistically, The Energy is given by

$$E^2 = p^2c^2 + m_0^2c^4.$$

But 'E' is the algebraic sum of PE & Rest mass energy, given by $E = eV + m_0c^2$.

Substituting 'E' in Relativistic energy, we get

$$\Rightarrow \left[eV + m_0c^2\right]^2 = p^2c^2 + m_0^2c^4.$$

$$\Rightarrow (p^2c^2) = \left[eV + m_0c^2\right]^2 - m_0^2c^4.$$

$$= (eV)^2 + (\cancel{m_0^2c^4}) + 2eVm_0c^2 - \cancel{m_0^2c^4}$$

$$\Rightarrow p^2 = \left(\frac{e^2V^2}{c^2}\right) + 2eVm_0.$$

$$= 2eVm_0\left[1+\left(\frac{eV}{2m_0c^2}\right)\right].$$

$$\Rightarrow p = \sqrt{2eVm_0\left[1+\left(\frac{eV}{2m_0c^2}\right)\right]}$$

$$\therefore \lambda = \frac{h}{p}$$

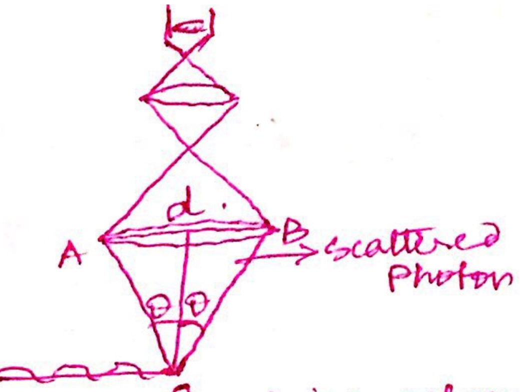
$$\boxed{\lambda = \frac{h}{\sqrt{2eVm_0\left[1+\frac{eV}{2m_0c^2}\right]}}}$$

γ-ray Microscope:

Atleast one photon enters microscope.

Limit of Resolution,

$$d = \frac{\lambda}{2\sin\theta}$$



d - distance between two points which can be resolved by microscope.

This is the range in which the electron would be visible when disturbed by the photon. ∴ The uncertainity in measurement of Position of electron is

$$\Delta x = d = \frac{\lambda}{2\sin\theta}.$$

Interaction between Photon & electron - compton effect.

To see this electron, the scattered photon should enter microscope within angle 2θ.

Momentum (Photon - electron during impact) → h/λ (= P)

component of Momentum (OA) = $-p\sin\theta = \left(\frac{h}{\lambda}\right)\sin\theta$

Momentum (OB) = $p\sin\theta = \left(\frac{h}{\lambda}\right)\sin\theta$.

∴ uncertainity in Mom., $\Delta P = 2p\sin\theta = 2\left(\frac{h}{\lambda}\right)\sin\theta$.

$$\therefore \Delta x \cdot \Delta P = \left[\frac{\lambda}{2\sin\theta}\right]\left[2\left(\frac{h}{\lambda}\right)\sin\theta\right]$$

$$= h.$$

Applications

1. Non Existence of electron in nucleus
2. Existence of proton in nucleus
3. Ground state energy of atomic electron in a Hydrogen Atom.

Scanned with CamScanner

$$\Delta t \cdot \Delta \omega = 2\pi$$

$$\Delta \omega = 2\pi(\Delta \nu)$$

$$= 2\pi\left(\frac{\Delta E}{h}\right)$$

$$\therefore \Delta t\left[2\pi\left(\frac{\Delta E}{h}\right)\right] = 2\pi$$

$$\therefore \Delta E \cdot \Delta t = \underline{h}$$

$$x = ct$$

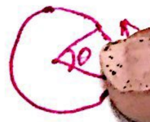
$$\Delta x = c(\Delta t)$$

$$\Delta p = \frac{\Delta E}{c} \quad \left(p = \frac{E}{c}\right)$$

$$\Delta p . \Delta x = \frac{\hbar}{2}$$

$$\frac{\Delta E}{c}(c\Delta t) = \frac{\hbar}{2}$$

$$\Delta E \cdot \Delta t \simeq \left(\hbar/2\right)$$



$$\theta = \frac{x}{r}$$

$$\Delta \theta = \frac{\Delta x}{r}$$

$$\Delta x = r(\Delta\theta)$$

$$L = mvr = pr$$

$$\Delta L = \Delta p(r)$$

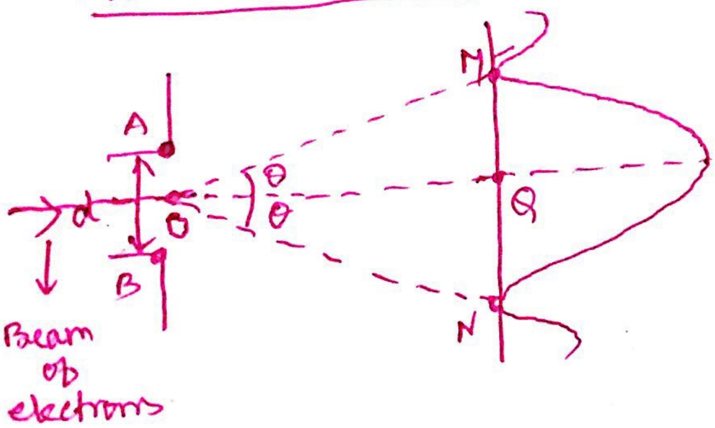
$$\Delta p = \left(\frac{\Delta L}{r}\right)$$

$$\Delta p . \Delta x = \hbar/2$$

$$\left(\frac{\Delta L}{r}\right)(r\Delta\theta) = \hbar/2$$

$$(\Delta L)(\Delta\theta) \simeq (\hbar/2)$$

Diffraction of electron by Narrow slit.



AB - Narrow slit of width - d

Beam - slit - diffraction.

$n = 1, \quad d\sin\theta = \lambda.$

θ - Angle of diffraction.

Difficult to locate at which position of slit, the beam enters the slit

∴ uncertainity in measurement of the Position of electron beam in the slit in a direction ⊥r to direction of incident beam is equal to width of the slit.

$$\Delta x = d = \frac{\lambda}{\sin\theta}.$$

$\vec{p}$ is momentum of electron. (along direction OM after diffraction).

Then momentum of electron in a direction ⊥r to the initial direction ie along QM is $(p\sin\theta)$. ~~Sly~~ along QN, Momentum $(-p\sin\theta)$.

Electron can be there anywhere from $-\theta$ to $+\theta$. $[p\sin\theta$ to $(-p\sin\theta)]$

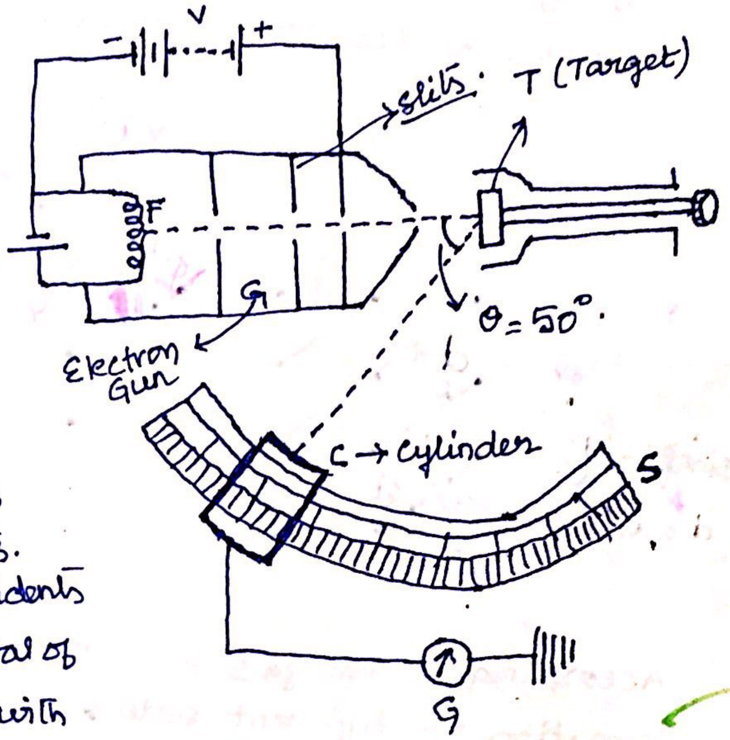
$$\Delta p = p\sin\theta - [-(p\sin\theta)] = 2p\sin\theta = 2\left(\frac{h}{\lambda}\right)\sin\theta.$$

$$\therefore \Delta p . \Delta x = \left[2\left(\frac{h}{\lambda}\right)\sin\theta\right]\left[\frac{\lambda}{\sin\theta}\right]$$

$$= \underline{2h}.$$

Davisson-Germer Experiment.
(Demonstration of Matter Waves - Experiment on Electron Diffraction)

The first experimental evidence of matter waves that confirmed the De-Broglie's hypothesis was given by DAVISSON and GERMER. They measured the de-Broglie wavelength associated with slow moving electrons by analysing about the reflections of electrons from Nickel Target. The reflected intensity has maxima and minima, which suspected that electrons are diffracted (i.e they behave like waves under certain conditions).

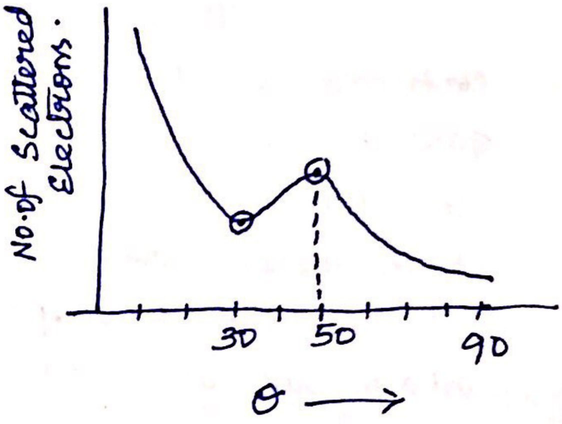


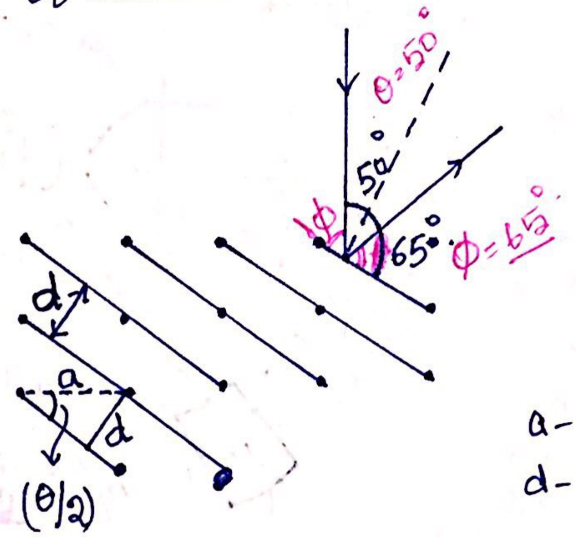
'G' is an electron gun that consists of a Tungsten Filament 'F'. Electrons are emitted due to Thermoionic emission when the filament becomes dull red. These electrons are accelerated by supplied potential difference (V). The electrons are collimated into fine electron pencil beam by slits. Then the beam of electrons incidents on a target, a large single crystal of Nickel. The ~~el~~ waves associated with the electrons (incident wave) gets diffracted in different directions. The angular distribution is measured by a Faraday Cylinder 'C', connected to a galvanometer, and can move on a circular graduated scale (S) between 29° to 90°.

The Faraday Cylinder consists of two insulated walls, which is used to receive the reflected electrons. The insulated walls are maintained at retarding potential so that fast moving electron enter into it. The slow moving electrons which are produced due to collision with Nickel atoms do not enter the Faraday cylinder instead they are reflected. Hence there is a deflection in the Galvanometer due to fast moving electrons reaching the Faraday cylinder.

Davisson and Germer measured the intensity of the scattered electrons as a function of angle 'θ', (given as polar diagram).

For a given accelerating voltage(V), there is a peak observed in the scattering curve. When the incident beam is exactly perpendicular to the crystal surface and at a potential of 54 volts, the peak was found be at 50° where there were maximum number of scattered electrons.



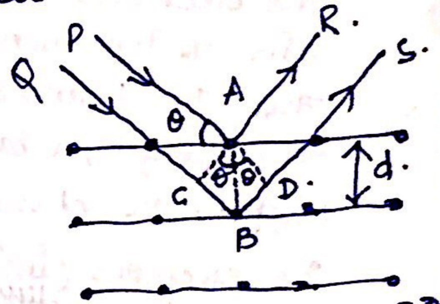


$\sin(\frac{\theta}{2}) = (\frac{d}{a})$

$d = a\sin(\theta/2)$

a - Inter Atomic Distance

d - Inter Planar Distance.



$\sin\theta = \frac{BC}{AB} = \frac{BD}{AB}$

P.D $\begin{cases} BC = d\sin\theta \\ BD = d\sin\theta. \end{cases}$

$\boxed{P.D = 2d\sin\theta = n\lambda}$

Bragg's law.

According to Bragg's law, there is the formation of different orders maximas when path difference in reinforcement is given by $\boxed{2d\sin\theta = n\lambda}$ Φ

For the first order maximum, n=1, we have

$$\lambda = 2d\sin\theta$$

for Nickel crystal, $a = 2.15\times10^{-10}$ m

$$d = a\sin(\theta/2)$$
$$= 2.15\times10^{-10}[\sin(25)^\circ]$$
$$= 2.15\times10^{-10}[0.4261]$$
$$= 0.9086\times10^{-10}\ m$$
$$= \underline{0.91\ A^\circ}.$$

Relation between Group velocity & particle velocity.

$$G = \lambda^2\left[\frac{u}{\lambda^2} - \frac{1}{\lambda}\left(\frac{du}{d\lambda}\right)\right]$$

$$= -\lambda^2\left[\frac{d}{d\lambda}\left(\frac{u}{\lambda}\right)\right]$$

$$G = -\lambda^2\left[\frac{d\nu}{d\lambda}\right] \quad [\because u = \nu\lambda] \Rightarrow \frac{1}{G} = -\frac{1}{\lambda^2}\left(\frac{d\lambda}{d\nu}\right) = \frac{d}{d\nu}\left[\left(\frac{1}{\lambda}\right)\right].$$

If $E \rightarrow$ Total Energy

$V \rightarrow$ potential Energy of particle,

$K \rightarrow$ Kinetic Energy, Then

$K = \frac{1}{2}mv^2 = E - V$. ($v$ is the velocity of the particle].

$$\Rightarrow v = \left[\frac{2(E-V)}{m}\right]^{1/2}.$$

$$G = u - \lambda\left(\frac{du}{d\lambda}\right) = \lambda^2\left[\frac{u}{\lambda^2} - \frac{1}{\lambda}\left(\frac{du}{d\lambda}\right)\right].$$

$$\frac{d}{d\lambda}\left(\frac{u}{\lambda}\right) = \left[u\left(-\frac{1}{\lambda^2}\right) + \frac{1}{\lambda}\left(\frac{du}{d\lambda}\right)\right]$$

$= -\lambda^2[$

From De-Broglie's Eq, $\lambda = \frac{h}{mv}$.

$$\Rightarrow \frac{1}{\lambda} = \frac{mv}{h}$$

$$\frac{1}{\lambda} = \frac{m}{h}\left[\frac{2(E-V)}{m}\right]^{1/2}.$$

Substituting this $(1/\lambda)$ value in $(1/G)$, we get.

$$\frac{1}{G} = \frac{d}{d\nu}\left[\frac{m}{h}\left\{\frac{2(E-V)}{m}\right\}^{1/2}\right]$$

$$= \frac{d}{d\nu}\left[\frac{m}{h}\left\{\frac{2(h\nu-V)}{m}\right\}^{1/2}\right] \quad [\because E = h\nu]$$

$$= \frac{1}{h}\frac{d}{d\nu}\left[\left\{2m(h\nu-V)\right\}^{1/2}\right].$$

$$= \frac{1}{\cancel{h}}\frac{1}{\cancel{2}}\left\{2m(h\nu-V)\right\}^{-1/2}\cdot(\cancel{2}m\cancel{h}).$$

$$= \frac{m}{\left\{2m(h\nu-V)\right\}^{1/2}} = \left[\frac{m}{2(E-V)}\right]^{1/2} = (1/G).$$

$$\Rightarrow \boxed{G = v}$$

Hence the material particle in motion is equivalent to Group of waves or wave packet.