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(AI & ML)

Maths Assignment - 2

Q.1) X: Continous Random variable
Probability density funcⁿ. (P.D.f)

$$f(x) = \begin{cases} K(x-9)(10-x), & 9 \le x \le 10 \\ 0 & \text{otherwise} \end{cases}$$

by, property
$f(x) \geq 0$

$$\int_9^{10} f(x)\,dx = 1 \quad , 9 \le x \le 10$$

$$K\int_9^{10} (x-9)(10-x)\,dx = 1$$

$$K\int_9^{10} (-x^2+19x-90)\,dx = 1$$

$$K\left[-\frac{x^3}{3}+\frac{19x^2}{2}-90x\right]_9^{10} = 1$$

$$\therefore \boxed{K=6}$$

for K = 6,
distribution funcⁿ. of x;

$$f(x) = \begin{cases} 6(x-9)(10-x), & 9 \le x \le 10 \\ 0, & \text{otherwise} \end{cases}$$

Q.2) P (Success) = P(head) = p = 1/2
P (Fail) = P(tail) = q = 1/2
x = no. of toss = 4
r = prob. of 2 heads (r) = 2
by using binomial distribution,

$$P(X = r) = {}^{n}C_{r}\, p^{r} q^{(n-r)}$$

$$P(X=2) = {}^{4}C_{2}\left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{8}$$

so, probability of getting 2 heads in 4 toss $= \frac{3}{8}$.

Q.4) P(even) = p
P(odd) = q = 1 − p
P(even appears 5 times) = P(X = 5)

$$= {}^{10}C_5 \cdot p^5 \cdot q^5 \quad \text{— (1)}$$

P(even appear 4 times) = P(X=4)

$$= {}^{10}C_4\, p^4 q^6 \quad \text{— (2)}$$

According to ques.

$$ {}^{10}C_5\, p^5 q^5 = 2 \times {}^{10}C_4\, p^4 q^6$$

$$\frac{1}{5} p = \frac{2}{6} q$$

$$\frac{3}{5} p = q = 1 - p$$

$$\therefore \boxed{p = \frac{5}{8},\ q = \frac{3}{8}}$$

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Probability that even no. will not appear at all

$$P(X=0) = {}^{10}C_0\, p^0\, q^{10}$$

$$= {}^{10}C_0 \left(\frac{3}{8}\right)^{10}$$

$$= \left(\frac{3}{8}\right)^{10} \Rightarrow 0.000055$$

Q.5 λ = average = 15 years

using Poisson distribution;

$$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!} \quad ; \lambda = np$$

$$P(X=25) = \frac{e^{-15}(15)^{25}}{25!}$$

Probability that there will be no war in 25 years

$$= \frac{e^{-15}(15)^{24}(15)}{25!}$$

$$= 4.03 \times 10^{-8} \approx 0$$

Q.6 First 4 moments about $x = 2$

$\mu_1' = 1, \mu_2' = 3, \mu_3' = 15, \mu_4' = 40$

$$\text{Skewness } (\beta_1) = \frac{\mu_3}{\mu_2^{3/2}}$$

$$\text{Kurtosis} = \frac{\mu_4}{\mu_2} = \beta_2$$

$$\mu_2 = \mu_2' - \mu_1'^2 = 3 - 1 = 2$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3 = 15 - 9 + 2 = 8$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$
$$= 40 - 60 + 18 - 3$$
$$= -5$$

$$\beta_1 = \frac{\mu_3}{\mu_2^{3/2}} = \frac{8}{(2)^{3/2}} = \sqrt{8} = +ve$$

right tail is longer than left tail

$$\beta_2 = \frac{\mu_4}{\mu_3^2} = \frac{-5}{4} = -1.25 < 3$$

$\therefore$ The curve is flat topped / platykurtic.

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Q.7 X = no. of cars hired out per day
mean = 1.5 m

$$P(X=x) = \frac{e^{-m} m^{x}}{x!} = \frac{e^{-1.5}(1.5)^{x}}{x!}$$

(i) P (neither car is used)

$$P(X=0) = \frac{e^{-1.5}(1.5)^{0}}{0!} = e^{-1.5} = 0.2231$$

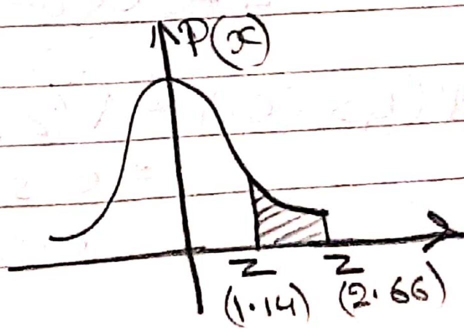
$= 22.31\%$

(2) P (some demand is refused)

$$\Rightarrow P(x>2) = 1 - P(x \leq 2)$$
$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$
$$= 1 - (e^{-1.5} \times 3.625) \approx 0.1912$$

$= 19.12\%$

Q.8 P (Correct ans) = 1/4
$E(x)$ or mean $= np = 80 \times 1/4 = 20$
variance $= npq = np(1-p) = 15.5$
standard deviation $= 3.94$
Causes work b/w 25 and 30
normal approx^n from 24.5 - 30.5 to include 25 & 30



$$\therefore z = \frac{24.5-20}{3.94} = 1.14$$

$$z = \frac{30.5-20}{3.94} = 2.66$$

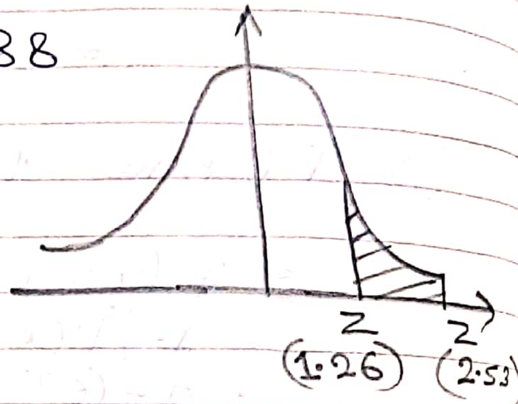
$Z(1.14) = 0.8728$
$Z(2.66) = 0.9960$

⇒ Area under ~~standard~~ shaded region that probability is $(0.9960 - 0.8728) = 0.1232$

⇒ Prob. without ~~corr~~ continuity correction factor

$$Z = \frac{25-20}{3.94} = 1.269$$

$$Z = \frac{30-20}{3.94} = 2.538$$

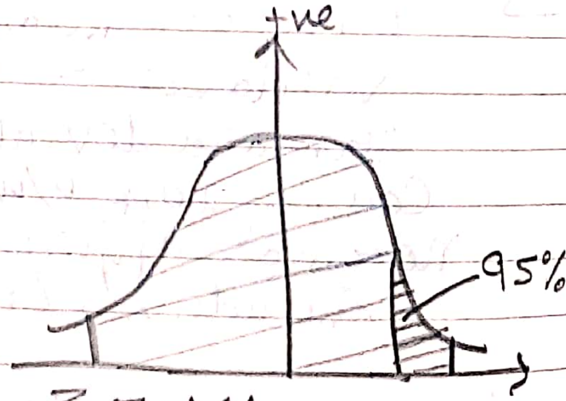


$Z(1.269) = 0.8961$
$Z(2.538) = 0.9943$

Area under shaded region that probability is
$0.9943 - 0.8961 = 0.0981$
⇒ Exact Probability

Q.9) mean $= \mu = 750$
standard deviation $= \sigma = 50$

$$Z = \frac{x-\mu}{\sigma}$$



1) $x = 668$

$$Z = \frac{668-750}{50} = -1.64$$

$P(x_1 > 668) = P(Z_1 < -1.64)$
$= 0.5 + P(-1.64 \le Z_0)$
$= 0.5 + 0.4495 \Rightarrow 0.9495$

$\therefore$ % Income exceeding Rs. 668 = 94.95
$\Rightarrow$ 95%

2.) If $x = 832$, then

$$z = \frac{832-750}{50} = 1.64$$

$$P(x_2 > 832) = P(z_2 > 1.64)$$
$$= 0.5 - 0.4495$$
$$= 0.0505$$

$\therefore$ % of income exceeding Rs. 832 = 5%

3.) Let x be the lowest income among the richest 100 persons

100 persons = 1% of 10,000

100 person represent 1% area of curve on right hand side

area b/w 0 and Z = 0.5 − 0.01 = 0.49

From Table Z;
$z(0.49) = 2.33$

$$z = \frac{x-\mu}{\sigma}$$

$$2.33 = \frac{x-750}{50}$$

$$x = 866.5$$

$\therefore$ Hence, the minimum income among 100 richest persons is Rs 866.5.