Maths Part-B
(End-Sem Assignment)

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Q.2) (A.) Checking if the Solⁿ is Exact :-

Condition: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Given Eqⁿ

$$\left(1+e^{x/y}\right)dx + \left(1-\frac{x}{y}\right)e^{x/y}\,dy = 0$$

$$M = 1+e^{x/y}$$

$$\frac{\partial M}{\partial y} = 0 + \left(\frac{-x}{y^2}\right)e^{x/y}$$

$$\frac{\partial M}{\partial y} = -\frac{x}{y^2}e^{x/y}$$

$$N = e^{x/y} - e^{x/y}\frac{x}{y}$$

$$\frac{\partial N}{\partial x} = \frac{1}{y}\cdot e^{x/y} - \frac{1}{y}e^{x/y} - \frac{x}{y^2}e^{x/y}$$

$$\frac{\partial N}{\partial x} = -\frac{x}{y^2}e^{x/y}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$ given Eqⁿ is Exact Differential Eqⁿ

Solution is :-

$$\underset{(y\text{ constant})}{\int M\,dx} + \underset{\left(\substack{\text{terms of }N\\ \text{independent of }x}\right)}{\int N\,dy} = C$$

$$\Rightarrow \int\left(1+e^{x/y}\right)dx + \int 0\,dy = C$$

$$\therefore \boxed{x + ye^{x/y} = C}$$

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Q.2) (B.) Given Eq^n :- $y'' - 4xy' + (4x^2-3)y = 0$ —(i)

Given That :- $y = e^{x^2}$ is a sol^n to eq^n

$\therefore$ Let part of C.F $= u = e^{x^2}$

Comparing Eq(i) with $y'' + Py' + Qy = R$

$P = -4x$, $Q = 4x^2 - 3$, $R = 0$

The Transformed Eq^n is;

$$\frac{d^2V}{dx^2} + \left(P + \frac{2}{u}\frac{du}{dx}\right)\frac{dV}{dx} = \frac{R}{V}$$

$$\frac{d^2V}{dx^2} + \left(-4x + \frac{2}{e^{x^2}}e^{x^2}\cdot 2x\right)\frac{dV}{dx} = 0$$

$$\therefore \frac{d^2V}{dx^2} = 0 \quad , \quad \frac{dV}{dx} = C_1$$

$$\therefore V = C_1x + C_2$$

So Second Independent Sol^n of Eq(i) is

$$y = V = C_1x + C_2$$

So Complete Sol^n of Eq(i) is

$$\therefore \boxed{y = uV = e^{x^2}(C_1x + C_2)}$$

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Q.3 (A.) Probability of Boy $= p = \frac{1}{2}$

Probability of girl $= q = \frac{1}{2}$

$n = 5$, $N = 320$

(i) Prob. of 2 Boys and 3 girls $= P(X=2)$

$^5C_2 p^2 q^3 \Rightarrow {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 \Rightarrow \frac{5 \times 4 \times 3!}{2! \times 3!} \times \frac{1}{32} = \frac{10}{32}$

No. of families $= N \times \frac{10}{32} = 320 \times \frac{10}{32} = 100$

% of families having 2 boys & 3 girls $= \frac{10}{32} \times 100 = \boxed{31.25\%}$

(ii) Probability of atleast one boy $= 1 - P(X=0)$

$= 1 - {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$

$= 1 - \frac{5!}{5!} \times \frac{1}{32} \Rightarrow 1 - \frac{1}{32}$

$= \frac{31}{32}$

No. of families with atleast 1 boy $= 320 \times \frac{31}{32} = 310$

% of families with atleast 1 boy $= \frac{31}{32} \times 100$

$\boxed{\Rightarrow 96.875\%}$

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Q.3) (B) Given funcⁿ :- $f(x) = \cos x - 3x + 1 = 0$

Given interval $[0.60, 0.61]$

$f(0.60) = 0.025$

$f(0.61) = -0.010$

$\therefore$ Roots lie b/w 0.60 & 0.61

Since, $f(0.60) \cdot f(0.61) < 0$

Let $a = 0.60$, $b = 0.61$

Bisection Method :-

1st Iterations :-

$$x_1 = \frac{a+b}{2} = 0.605$$

$$f(0.605) = 0.007502$$

$\therefore$ Roots ~~lie~~ lie b/w $x_1 (0.605)$ & $b (0.61)$

2nd Iteration :-

$$x_2 = \frac{x_1 + b}{2} = 0.6075$$

$$f(0.6075) = -0.001422$$

So, now Roots will lie b/w $[0.605, 0.6075]$

Hence, the real root after 2 iterations will be $\boxed{x = 0.6075}$

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Q.4| (A.) Central difference operator $= \delta$
Average difference operator $= \mu$

we know,

$$\Rightarrow \frac{1}{2}\left[\Delta f(x) + \nabla f(x)\right] = \frac{1}{2}\left[f(x+h) - f(x) + f(x) - f(x-h)\right]$$

$$= \frac{1}{2}\left[f(x+h) - f(x-h)\right]$$

$$= \frac{1}{2}\left[E - E^{-1}\right] f(x)$$

$$= \mu\delta f(x)$$

$$\therefore \mu\delta f(x) = \frac{1}{2}\left[E - E^{-1}\right] f(x) \quad \& \quad \delta = E^{1/2} - E^{-1/2}$$

Therefore;

$$(1 + \delta^2\mu^2) f(x) = \left[1 + \frac{1}{4}(E - E^{-1})^2\right] f(x)$$

$$= \left[1 + \frac{1}{4}(E^2 - 2 + E^{-2})\right] f(x)$$

$$= \frac{1}{4}\left[E + E^{-1}\right]^2 f(x)$$

$$= \left[1 + \frac{1}{2}(E^{1/2} - E^{-1/2})^2\right]^2 f(x)$$

$$= \left[1 + \frac{\delta^2}{2}\right]^2 f(x)$$

Hence,

$$\boxed{1 + \delta^2\mu^2 = \left[1 + \frac{\delta^2}{2}\right]^2}$$

Hence Proved

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Q.4 (B.) $2x + y - z = 4$ —(i)

$x - y + 2z = -2$ —(ii)

$-x + 2y - z = 2$ —(iii)

Rearranging the given eqn & writting in diagonal dominance method :-

$2x + y - z = 4$ [Replace eq. (2) & (3)]

$-x + 2y - z = 2$

$x - y + 2z = -2$

$x_n = \frac{1}{2}[4 - y + z]$, $y_n = \frac{1}{2}[2 + x + z]$

$z_n = \frac{1}{2}[-2 + y - x]$

Initial guess (0.75, 0.75, -0.75)

Applying guess Siedal Rule :-

I^st Iteration :-

$x_1 = \frac{1}{2}[4 - 0.75 - 0.75] = 1.250$

$y_1 = \frac{1}{2}[2 + 1.25 - 0.75] = 1.250$

$z_1 = \frac{1}{2}[-2 - 1.250 + 1.250] = -1.00$

II^nd Iteration

$x_2 = \frac{1}{2}[4 - 1.25 + (-1)] = 0.875$

$y_2 = \frac{1}{2}[2 + 0.875 - 1] = 0.937$

$z_2 = \frac{1}{2}[-2 + 0.9375 - 0.875] = -0.9687$

So, from the two ~~Itter~~ iterations soln are :-

$$\boxed{\begin{aligned} x &= 0.875 \\ y &= 0.937 \\ z &= -0.968 \end{aligned}}$$

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Q.5 (A) Given value of integration:-

$$I = 2\left[\frac{1}{2}(1+9^2) + \alpha^2 + \beta^2 + \gamma^2\right] \text{ --- (i)}$$

forming table:-

$n = 4,\ a = 1,\ b = 9$; so $h = \frac{b-a}{n} = 2$

x :	1	3	5	7	9
y :	1	9	25	49	81
	y_0	y_1	y_2	y_3	y_4

By Trapezoidal Rule:-

$$\int_1^9 x^2\,dx = \frac{h}{2}\left[(y_0 + y_4) + 2(y_1 + y_2 + y_3)\right]$$

$$= 2\left[\frac{(1+9^2)}{2} + (3^2 + 5^2 + 7^2)\right]$$

$$= 2\left[\frac{1}{2}(1+9^2) + (3^2 + 5^2 + 7^2)\right] \text{ --- (ii)}$$

On Comparing Eq (i) & (ii) we get;

$$\boxed{\begin{array}{c}\text{If } \alpha = 3, \text{ then } \beta = 5 \\ \text{or} \\ \text{If } \alpha = 5, \text{ then } \beta = 3\end{array}}$$

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Q.5 (B.) $\frac{dy}{dx} = x + y^2$; $y(0) = 1$, $h = 0.1$

$f(x, y) = x + y^2$; $x_0 = 0$, $y_0 = 1$

Using Runge - Kutta Method

$K_1 = hf(x_0, y_0) = 0.1(0 + 1^2) = 0.1$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right) = hf(0.05, 1.05)$

$= 0.11525$

$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right) = hf(0.05, 1.05762)$

$= 0.11686$

$K_4 = hf(x_0 + h, y_0 + K_3) = hf(0.1, 1.11686)$

$= 0.13474$

$y_1 = y_0 + \Delta y$

$\Delta y = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[(0.1) + 2(0.11525) + 2(0.11686) + (0.13474)]$

$\Delta y = 0.11649$

$y_1 = y_0 + \Delta y = 1.11649$

So, $\boxed{y(0.1) = 1.11649}$

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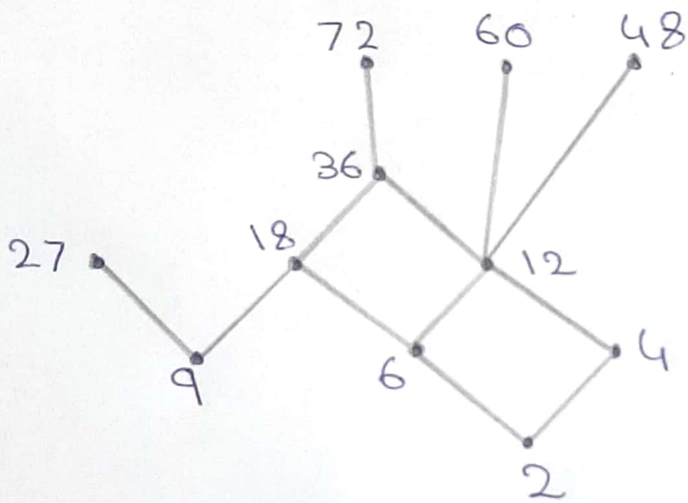
Q.6) Poset $P = (\{2,4,6,9,12,18,27,36,48,60,72\}, |)$
where "a|b" means "a divides b"



Hasse Diagram

1.) Maximal Elements = $\{27, 48, 60, 72\}$

2.) Minimal Elements = $\{2, 9\}$

3.) GLB(2,9) = ϕ (does not exist)

4.) LUB(2,9) = $\{18\}$