Rohan Nyati
500075940
R177219148

Part - B

Q.2) If R be a relation in set of integers Z defined by

$R = \{(x,y) : x \in Z, y \in Z, (x-y) \text{ is divisible by } 6\}$.

Then Prove that R is an equivalence relation.

Solⁿ) Consider any $a, b, c \in Z$

since $a - a = 0 = 6 \cdot 0$

$\Rightarrow (a-a)$ is divisible by 6

$\Rightarrow (a, a) \in R$

$\therefore$ It is reflexive

Let $(a, b) \in R$

$\Rightarrow (a-b)$ is divisible by 6

$\Rightarrow a - b = 6q$ for some $q \in Z$

$b - a = 6(-q)$

$\therefore b - a$ is divisible by 6

$[\because q \in Z \Rightarrow -q \in Z \Rightarrow -6q \in Z]$

Thus $(a, b) \in R$

$\Rightarrow (b, a) \in R$

$\therefore$ R is symmetric.

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Let $(a,b) \in R$ and $(b,c) \in R$

$(a-b)$ is divisible by 6 and $(b-c)$ is divisible by 6

$(a-b) = 6q$ and $b-c = 6q'$ for some $q, q' \in Z$

$(b-c)+(a-b) = 6(q+q')$

$\Rightarrow (a-c) = 6(q+q')$

$[\because q, q' \in Z \Rightarrow q+q' \in Z]$

$\therefore (a,c) \in R$

Thus, $(a,b) \in R$ and $(b,c) \in R$

$\therefore (a,c) \in R$

$\therefore R$ is Transitive.

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500075940
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Q.3)

Sol^n) Injectivity of f :

Let x and y be two elements of domain(Q), such that

$f(x) = f(y)$

$2x = 2y$

$\therefore x = y$

$\therefore$ f is one - one

Surjectivity of f :

Let y be in the co-domain (Q), such that, $f(x) = y$

$2x = y$

$x = y/2 \in Q$ (Domain)

$\Rightarrow$ is onto

So f is one-one and onto, then it is bijective

$\therefore f^{-1}$ is invertible

find f^{-1} : Let $f^{-1}(x) = y$ — 1

$x = f(y)$

$x = 2y$

$\therefore y = x/2$

So $f^{-1}(x) = x/2$ from Eq. 1

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Injectivity of g:
Let (x) and (y) be 2 elements of domain(S), such that

$$g(x) = g(y)$$
$$x+2 = y+2$$
$$\therefore x = y$$

so, g is one-one

Surjectivity of g:
Let y be in the co-domain(S), such that $g(x) = y$

$$x+2 = y$$
$$x = y-2 \in S \text{(domain)}$$
$$\Rightarrow g \text{ is onto}$$

So g is bijective, and hence it is invertible

finding g^{-1}: Let $g^{-1}(x) = y$

$$x = g(y)$$
$$x = y+2$$
$$y = x-2$$
$$\therefore g^{-1}(x) = x-2$$

$\therefore$ for verification of $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

$f(x) = 2x$; $g(x) = x+2$

and $f^{-1}(x) = \frac{x}{2}$, $g^{-1}(x) = x-2$

Now, $(f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1})(x) = f^{-1}(x-2) = \frac{x-2}{2}$ (ii)

$(g \circ f)(x) = g(f(x)) = g(2x) = 2x+2$

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500075940
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Let $(g\circ f)^{-1} = y$

$x = (g\circ f)\, y$

$x = 2y + 2$

$\therefore y = \frac{x-2}{2}$

$\therefore (g\circ f)^{-1} = \frac{x-2}{2}$ —(iii)

$\therefore$ from Eq.(ii) and (iii), we get

$f^{-1}\circ g^{-1} = (g\circ f)^{-1}$

Hence Proved

Q.4)

Sol^n) $a_{n+2} - 2a_{n+1} + a_n = 2^n$ —(0) ; $a_0 = 2, a_1 = 1$

Let $G(x)$ be the generating func^n for the sequence $\{a_n\}$
Then applying general func^n on both sides of the given relation, we get

~~G(x)~~ from the property we know,

if $G(x)$ is the G.F for $\{a_n\}$ —(i)

then, $\frac{G(x) - a_0}{x}$ is the G.F. for (a_{n+1}) —(ii)

then, $\frac{G(x) - a_0 - a_1 x}{x^2}$ is the G.F. for (a_{n+2}) —(iii)

$\therefore$ G.F. for a^k is $\frac{1}{1-ax}$ —(iv)

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$\therefore$ Putting eq. (i) (ii) (iii) & (iv) in eq. (0)

$$\frac{G(x) - a_0 - a_1 x}{x^2} - \frac{2(G(x) - a_0)}{x} + G(x) = \frac{1}{1-2x}$$

put $a_0 = 2, a_1 = 1$

$\therefore$ after simplyfing; we get

$$G(x)[x^2 - 2x + 1] = 2 - 3x + \frac{x^2}{1-2x}$$

$$G(x) = \frac{2}{(1-x)^2} - \frac{3x}{(1-x)^2} + \frac{1}{(1-2x)} - \frac{1}{(1-2x)^2}$$

$$= \frac{1}{(1-x)^2} - \frac{3x}{(1-x)^2} + \frac{1}{1-2x}$$

G.F. for $\downarrow$ $\downarrow$ $\downarrow$

$\hookrightarrow = (n+1) \quad 3n \quad 2^n$

$\therefore a_n = (n+1) - 3n + 2^n$

$\therefore \boxed{a_n = 1 - 2n + 2^n}$

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Q. 5)

Sol^n) For a finite dimensional vector-space V(F), say of dimension n, if there is a subset B of V containing n elements which is a linearly independent set, then it is certainly a basis for the space. So, we have to check only the linear independence ness of the set in such cases. So, Here we have, $S = \{(1,2,1),(2,1,0),(1,-1,2)\}$

consider the determinent

$$A = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 2 \end{vmatrix} = 1\times(2) - 2(4) + 1(-3) \neq 0$$

Here, $\boxed{\det(A) \neq 0}$

∴ The given vectors are linearly independent

∴ They form a basis for $R^3(R)$.

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Q.6)

Solⁿ) $T : R^2(R) \to R^3(R)$

$T(x_1, x_3) = (x_1, x_1 + x_2, x_2)$

Let $u = (x, y) \in R^2$ then

$\therefore \boxed{R(T) = \{(x, x+y, y) : (x, y) \in R^2\}}$

$N(T) = \{u \in R^2 : T(u) = \bar{0} \in V\}$

$T(x, y) = (0, 0, 0)$

$(x, x+y, y) = (0, 0, 0)$

$\Rightarrow x = 0 ; \; x+y = 0 ; \; y = 0$

$\Rightarrow x = 0, \; y = 0$

$\therefore \boxed{N(T) = \{(0,0) \in U : T(u) = \bar{0} \in V\}}$

$\therefore$ N(T) = Required Null Space / Kernal Space

R(T) = Required Range Space

So Rank of T : $\rho(T) = \dim(R(T)) = 2$

Nullity of T : $\nu(T) = \dim(N(T)) = 0$

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Q.7| ~~(a)~~

Soln:| (a) The co-ordinate funcn. of T are given by homogenous polynomials of degree 1.

(b) The conditions for (a, b, c) to lie in kernel are

$$a = \frac{-2}{3}c; \quad b = \frac{4}{3}c$$

$\therefore$ The nullity of T is 1 by the dimension formula.