Q1 according To Taylor Series,

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a) + \cdots$$

as we know the rules of Shifting operator (E)

$E(y_2) = y_3$

$E^n(y_i) = y_{i+n}$ for $(i=1)$

$\Delta y_n = y_{n+1} - y_n$

$= E(y_n) - y_n$

$\Delta y_n = (E-1)y$

On comparing we get $\Delta = E-1$

$E = \Delta + 1$ —①

So, we can define the relation

$$E(x) = f(x+h) = f(x) + hf(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \cdots$$

Let $D = \frac{d}{dx}$ $= \underbrace{\left(1 + hD + \frac{(hD)^2}{2!} + \frac{(hD)^3}{3!}\right.}_{\text{series of } e^{hD}} \left. + \cdots\right) f(x) = e^{hD} \cdot f$

So $Ef(x) = e^{hD} f(x)$

$E = e^{hD}$

$\log E = hD$

$\frac{1}{h} \log E = D$ —② put ① in ②

$= \frac{1}{h} \log(\Delta + 1)$

Apply series of $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

$$D = \frac{1}{h}\left(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \cdots\right) \to H \cdot P$$

Scanned by CamScanner

Q2

85% of $60 = \frac{85}{100} \times 60 = 51$

we have,
to find the no. of students who have secured 51 marks and above

Marks obtained X	No of students	1st Diff	2nd Diff	3rd diff	4th Diff
Below 10	3				
		12			
Below 20	15		3		
		15		17	
Below 30	30		20		-47
		35		-30	
Below 40	65		-10		25
		25		-5	
Below 50	90		-15		
		10			
Below 60	100				

$x_0 = 60 \quad \therefore \; P = \frac{51-60}{10} = \frac{-9}{10} = -0.9$

$y(51) = y_0 + P\Delta y_0 + \frac{P(P+1)}{2!}\Delta^2 y_0$

$= 100 + \frac{(-0.9)}{10} \times 10 + \frac{(0.9(0.1)(15)}{2 \times 10 \times 10}$

$= 100 - 9 + 0.675$

$= 91 + 0.6\cancel{7}5$

$= 91.675 \approx 92$

Therefore, $100 . 92 = 8$
8 students got 85% and above

Scanned by CamScanner

Let, $k = f(x_0)$, $t = f(x_1)$ and $z = [x_0, x_1]$

$[x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0} \Rightarrow z = \frac{t-k}{0.4}$ — ①

$[x_1, x_2] = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow 10 = \frac{6-t}{0.7-0.4} \Rightarrow \frac{6-t}{0.3} = 10$

$\therefore t = 3$

$[x_0, x_1, x_2] = \frac{[x_1, x_2] - [x_0, x_1]}{x_2 - x_0} = \frac{10-2}{0.7} = \frac{50}{7}$

$\therefore z = 3$

Put values in eq^n — ①

$t - 0.4z = k$

$3 - 0.4 \times 5 = k$

$k = 1$

(b)

x	y	F.dd	S.dd	T.dd
0	13	−8	−2	2
1	10	−14	10	
3	−18	36		
6	90			

$y(x) = y_0 + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0 x_1 x_2] + (x - x_0)(x - x_1)(x_1 - x_2)[x_0 x, x_2 x_3]$

$y(x) = 18 + (x - 0)(-3) + x(x-1)(-2) + x(x-1)(x-3)2$

$\therefore y(x) = 18 - 10x^2 + 2x^3$

Slope at $y(x)$, when $x = 2$, $y'(x) = 6x^2 - 20x$

$y'(2) = -40 + 24 = \underline{\underline{-16}}$

Scanned by CamScanner

Q4

x	y	F dd	S.dd	T.dd
$x_0 = 0$	0	$2k$	$6.4k$	$\left[\frac{\frac{2K-7}{1.5} - 6.4K}{2}\right]$
$x_1 = 0.5$	K	$2(3-k)$	$\frac{2K-7}{1.5}$	
$x_2 = 1$	5	-1		
$x_3 = 2$	2			

$$Tdd = 4K - \frac{14}{3} - 18 + 12K \times \frac{1}{2} = \frac{16K-32}{3} \times \frac{1}{2} = \frac{8K-16}{3}$$

$$P_3(x) = y_0 + (x-x_0)[x_0, x_1] + (x-x_0)(x-x_1)[x_0 x_1, x_2] + [x-x_0)(x-x_1)(x-x_2)(x-x_3)[x_0 x_1, x_2 x_3]$$

Coefficient of x_3 : $(x-0)(x-0.5)(x-1)\left(\frac{8K-16}{3}\right)$

$$P_3(x) = x^3 \times \frac{(8K-16)}{3} + x^2(\ldots) + x(\ldots) + \ldots$$

Coefficent of $x^3 \Rightarrow \frac{8K-16}{3} = 6$

$$8K - 16 = 18$$

$$8K = 34$$

$$K = \frac{17}{4} = 4.25$$

Scanned by CamScanner

x) $\int_{-1}^{1} e^{-|x|}\,dx$

By analytical method

$\int_{-1}^{1} e^{-|x|}\,dx = 2\int_{0}^{1} e^{-x}\,dx = 2\left[\frac{1}{e}-1\right](-1) = 2\left[1-\frac{1}{e}\right]$

By Numerical differentiation,

By Simpson's Rule, $S_n = \frac{\Delta x}{3}\left[f(x_1)+f(x_3)+\dots\right]+\left[f(x_2)+f(x_4)+\dots\right]$ — ①

$\Delta x = \frac{b-a}{h} = \frac{1+1}{4} = \frac{2}{4} = 0.5$

$S_n = \frac{0.5}{3}\left[f(-1)+4f(0.5)+2(f(0))+4(f(0.5))+\frac{4\times1}{\sqrt{e}}+\frac{1}{e}\right]$

$= \frac{1}{3}\left(\frac{1}{e}+\frac{4}{\sqrt{e}}+1\right)$ — ②

As exact value of integral should be equal to the value from Simpson's rule,

so ① = ②

$2\left[1-\frac{1}{e}\right] = \frac{1}{3}\left[\frac{1}{e}+\frac{4}{e}+1\right]$

$\therefore 5e-7 = 4\sqrt{e}$ $\quad \sqrt{e}=t$

$5t^2-4t-7=0$

$t = \frac{4 \pm \sqrt{16 + 4\times7\times5}}{10}$

$t = \frac{4+\sqrt{156}}{10}$

$\sqrt{e} = 1.649$

$e = 2.719$

$\therefore e \approx 2.72$

Now Putting the value of e in ②

$S_n = \frac{1}{3}\left[\frac{1}{2.72}+\frac{4}{\sqrt{2.72}}+1\right]$

$\therefore \boxed{S_n = 1.2648}$

Scanned by CamScanner

(b) Let $f(x) = x^2$ and given

$2\left[\frac{1}{2}(1^2+9^2) + \alpha^2 + \beta^2 + 7^2\right)$ — ①

$h = \frac{9-1}{x} = \frac{8}{4} = 2$

By Trapezoidal Rule,

$T_n = \frac{h}{2}\left[f(x_0) + f(x_4) + 2(f(x_1) + f(x_2) + f(x_3))\right]$

$= \frac{2}{2}\left[1^2 + 9^2 + 2(3^2+5^2+7^2)\right)$

$T_n = 2\left[\frac{1^2+9^2}{2} + 3^2 + 5^2 + 7^2\right]$ — ②

On comparing ① & ②

$\boxed{\begin{matrix}\alpha = 3 \\ \beta = 5\end{matrix}}$

Q6 $h = 1.5 - 1 = 0.5$

$S = \frac{h}{3}\left(F_1 + F_6 + 4(f_1 + f_3 + f_5) + 2(f_2 + f_4)\right]$

$= \frac{1.5-1}{3}\left(2 + 2.1 + 4(2.4 + 2.1 + 2.6) + 2(2.715)\right)$

$\boxed{S = 7.78\cancel{3}3}$ → area bounded.

5

Scanned by CamScanner

(A)

1

$5x-y+z=10$

$2x+4y=12$ (x_0, y_0, z_0)

$x+y+5z=-1$ $(2, 3, 0)$

$x=\frac{1}{5}(10+y-z)$

$y=\frac{1}{2}(6-x)$

$z=-\frac{1}{5}(1+x+y)$

Gauss Jacobi, 1st Iteration, $x_1=\frac{1}{5}[10+3]=2.6$

$y_1=\frac{1}{2}[6+2]=2$

$z_1=-\frac{1}{5}[1+5]=-1.2$

$x_1=2.6,\ y_1=2,\ z_1=-1.2$

2nd Iteration, $x_2=\frac{1}{5}[10+2+1.5]=13.2/5=2.64$

$y_2=\frac{1}{2}[6-2.6]=1.7$

$z_2=-\frac{1}{5}[1+2.6+2]=-1.12$

3rd Iteration, $x_3=\frac{1}{5}[10+1.7+1.12]=2.564$

$y_3=\frac{1}{2}[6-2.64]=1.64$

$z_3=-\frac{1}{5}[1+2.64+1.7]=-1.068$

4th Iteration $x_4=\frac{1}{5}[10+1.68+1.068]=2.5496$

$y_4=\frac{1}{2}[6-2.564]=1.73$

$z_4=-\frac{1}{5}[1+2.64+1.7]=-1.068$

5th Iteration, $x_5=\frac{1}{5}[10+1.75+1.068]=2.5596$

$y_5=\frac{1}{2}[6-2.5496]=1.7252$

$z_5=-\frac{1}{5}[1+2.5496+1.73]=-1.05592$

Scanned by CamScanner

6 th Iteration, $x_6 = \frac{1}{5}[10 + 1.7252 + 1.05592] = 2.5$

$y_6 = \frac{1}{2}[6 - 2.5596] = 1.7202$

$z_6 = -\frac{1}{5}[1 + 2.5596 + 1.7252] = -1.05696$

(B) Rewriting the Eq,

$x = 1.5 - 2/3\, y$, $y = 5/3 - \frac{2x}{3} + z/3$, $z = 0.25 + 0.5y$

$(0.4, 1.6, 0.4) \to$ Initial Point

K = 1 / $x = 1.5 - 2/3 \times 1.6 = 0.433$

$y = 5/3 - 2/3 \times 0.4 + \frac{0.4}{3} = 1.511$

$z = -0.25 + 0.5 \times 1.6 = 0.506$

K = 2 / $x = 1.5 - 2/3 \times 1.511 = 0.493$

$y = 5/3 - 2/3 \times 0.433 + 0.506/3 = 1.507$

$z = 0.25 + 0.5 \times 1.511 = 0.504$

K = 3 / $x = 0.495 = 1.5 - 2/3 \times 1.507$

$y = 5/3 - 2/3 \times 0.498 + 0.504/3 = 1.505$

$z = -0.25 + 0.5 \times 1.507 = 0.5035$

K = 4 / $x = 1.5 - 2/3 \times 1.505 = 0.5035$

$y = 5/3 - \frac{2}{3} \times 0.495 + \frac{0.503}{3} = 1.504$

$z = -0.25 + 0.5 \times 1.505 = 0.502$

Scanned by CamScanner

Exact value of Differential Eqn

$$2y \cdot \frac{dy}{dx} = x^2$$

$$\int 2y \cdot dy = \int x^2 \cdot dx$$

$$y^2 = \frac{x^3}{3} + c$$

at $x=0$ & $y=2$

$$\boxed{c=4}$$

$$y^2 = \frac{x^3}{3} + 4$$

$y(1) \longrightarrow y^2 = 4.33$

$y = 2.080 = y(1)$ - (1)

Now, By Runga-Kutta Method

$h = 0.5$, $y_0 = 2$, $x_0 = 2$

$F(x, y) = x^2/2y$

$K_1 = h f(x_0, y_0) = 0$

$K_2 = h f(x_0 + h/2, y_0 + k_1/2)$

$= 0.5 * f\left(0 + \frac{0.5}{2}, 2+0\right.$

$= 0.5 * f(1/4, 2) = 0.00781$

$K_3 = hf(x_0 + h/2, y_0 + k_2/2)$

$= 0.5 f(1/4, 2 + 0.00781/2)$

$= 0.0078$

$K_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.5 f(0.5, 2.0078) = 0.03113$

$\Delta y = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) = \frac{1}{6}(0.06235) = 0.014$

$\Delta y = 0.0104$

Scanned by CamScanner

$x_1 = x_0 + h = 0.\cancel{7}\,0.5 = 0.5$

$y_1 = y_0 + \Delta y = 2 + 0.0104 = 2.0104$

To determine $y(1)$, $x_1 = 0.5$, $y_1 = 2.0104$, $h = 0.5$

$k_1 = hf(x_1, y_1)$

$k_1 = 0.5 f(0.5, 2.0104) = 0.5 \times 0.6217$

$\boxed{k_1 = 0.0311}$

$k_2 = hf(x_1 + h/2, y_1 + k_1/2)$

$= 0.0695$

$k_3 = hf(x_1 + h/2, y_1 + k_2/2)$

$= 0.069$

$k_4 = hf(x_1 + h, y_1 + k_3)$

$= 0.0675$

$\Delta y = \frac{1}{6}(k_1 + 2(k_2 + k_3) + k_4)$

$= \frac{1}{6}(0.3756) = 0.0626$

$x_2 = x_1 + h = 1$

$y_2 = y_1 + \Delta y = 2.0104 + 0.0626 = 2.073$

On comparing ① & ②

$y(1) = 2.080 \simeq \cancel{0}\ 2.076$

Scanned by CamScanner

$$F(x) = \frac{x^2}{y^2+1}$$

$$y_1 = y_0 + \int_{x_0}^{x} F(x_1, y_0)\,dx$$

$$= y_0 + \int_0^x \frac{x^2}{y^2+1}\,dx = 0 + \int_0^x \frac{x^2}{1}\,dx$$

$$\therefore y_1 = \frac{x^3}{3}$$

$$y_2 = y_0 + \int_{x_0}^{x} f(x,y)\cdot dx$$

$$y_2 = 0 + \int_0^x f(x,y)\,dx = \int_0^x \frac{x^2}{(x^3)^2+1}\,dx = \int_0^x \frac{9x^2\,dx}{(x^3)^2+9}$$

$$= 3 \times \frac{1}{3} \tan^{-1}\frac{x^3}{3}$$

$$y_2 = \tan^{-1}\frac{x^3}{3}$$

y_1, when $x = 0.25$

$$y_1 = \frac{(0.25)^3}{3} = 0.00520$$

y_2 when $x = 0.25$

$$y_2 = \tan^{-1}\frac{(0.25)^3}{3} = 0.2999^\circ = 0.00519 \text{ rad.}$$

$$\boxed{y = 0.005}$$

Scanned by CamScanner

Q10 As $f(x) = x^2 - 12$

when, $x = 3$, $f(x) = +ve$

$x = 4$, $f(x) = +ve$

it means root lies b/w (3, 4)

Initial guess, $x_0 = 3$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Iteration 1: $x_1 = 3 - (-3/6) = 3 + 1/2 = 3.5$

Iteration 2: $x_2 = 3.5 - \frac{(3.5)^2 - 12}{7} = 3.46428$

Iteration 3: $x_3 = 3.46428 - \left[\frac{(3.46428)^2 - 12}{2 \times 3.46428}\right] = 3.46410$

Iteration 4: $x_4 = 3.46410 - \left[\frac{-0.00001119}{2 \times 3.46410}\right]$

$= 3.46410$

Hence,

$$\boxed{x = 3.464}$$

Scanned by CamScanner