UPES

UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	**B. Tech SCS**	**Semester**	**II**
Course	**Mathematics II**	**Course Code**	**MATH 1005**
Session	**Jan-May 2020**	**Topic**	**Unit I, II, III and IV**

1. Solve the differential equation $(D^3 - 5D^2 + 7D - 3)y = e^{2x}cosh2x$.

2. Solve the differential equation $y'' + 4y' + 8y = cosx$, $y(0) = 1$ and $y'(0) = 0$.

3. Apply the method of variation of parameters to solve the differential equation $(D^2 + a^2)\, y = tanax$.

4. The probability mass function of a discrete random variable X is given by

$$p(x) = \begin{cases} k, & x = 0 \\ 2k, & x = 1 \\ 3k, & x = 2 \\ 0, otherwise. \end{cases}$$

(a) Find k. (b) Evaluate $P(X \geq 2), P(0 < X \leq 2)$ (c) find cumulative distribution function of X.

5. Out of 2000 families with 4 children each, how many would you expect to have (a) at least 1 boy (b) 2 boys (c) 1 or 2 girls and (d) no girls?

6. X is normally distributed and the mean of X is 12 and S.D. is 4
(a) find out the probability of the following:
(i) $X \geq 20$, (ii) $X \leq 20$ and (iii) $0 \leq x \leq 12$
(b) Find x' when $P(X > x') = 0.24$
(c) Find x_0' and x_1' when $P(x_0' < X < x_1') = 0.5$ and $P(X > x_1') = 0.25$.

7. Using Newton-Raphson method, find the real root of $f(x) = x\sin x + \cos x = 0$ which is near $x = \pi$ correct to three decimal places.

8. A slider in a machine moves along a fixed straight rod. Its distance x cm along the rod is given below for various values of time t seconds. Find the velocity of the slider and its acceleration when $t = 0.3$ seconds.

t	0.0	0.1	0.2	0.3	0.4	0.5	0.6
x	3.013	3.162	3.287	3.364	3.395	3.381	3.324

9. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using

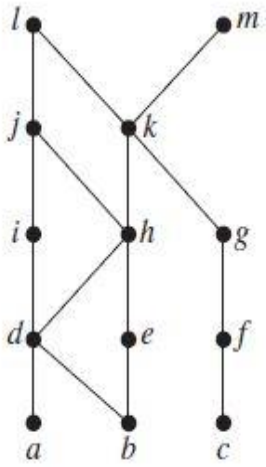
(i) Simpson's 1/3 rule taking h = 1/4

(ii) Simpson's 3/8 rule taking h = 1/6

10. Using Runge-Kutta method of fourth order, calculate $y(0.1)$, and $y(0.2)$ of the differential equation $\frac{dy}{dx} - \frac{2xy}{1+x^2} = 1$, $y(0) = 1$.

11. Draw the Hasse diagram for the "less than or equal to" relation on {0, 2, 5, 10, 11, 15}.

12. Answer these questions for the partial order represented by this Hasse diagram.



a) Find the maximal elements.

b) Find the minimal elements.

c) Is there a greatest element?

d) Is there a least element?

e) Find all upper bounds of $\{a, b, c\}$.

f) Find the least upper bound of $\{a, b, c\}$, if it exists.

g) Find all lower bounds of $\{f, g, h\}$.

h) Find the greatest lower bound of $\{f, g, h\}$, if it exists.

13. Determine whether these posets are lattices.

a) $(\{1, 3, 6, 9, 12\}, |)$ **b)** $(\{1, 5, 25, 125\}, |)$

c) $(\mathbf{Z}, \geq)$

d) $(P(S), \supseteq)$, where $P(S)$ is the power set of a set S