**UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN**

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| **Program** | **B.Tech (All SoCSBranches)** | **Semester** | **III** |
| **Course** | **Discrete Mathematical Structures** | **Course Code** | **CSEG2006** |

1. Prove that $f^{-1}og^{-1} = (gof)^{-1}$, where $f: Q \longrightarrow Q$ such that $f(x) = 2x$ and $g: Q \longrightarrow Q$ such that $g(x) = x + 2$.
2. Let $N = \{1,2,3,\dots\}$ and a Relation defined in $N \times N$ as follows: $(a, b)$ is related to $(c, d)$ iff $ad = bc$, then check whether R is an equivalence relation or not.
3. Let X and Y be two non-empty sets and let $f: X \longrightarrow Y$ is an into mapping and also $A \subseteq X, B \subseteq X$ then prove that i. $f(A \cap B) \subseteq f(A) \cap f(B)$ ii. $f^{-1}(A \cap B) \subseteq f^{-1}(A) \cap f^{-1}(B)$
4. In a survey concerning the energy drinking habits of people, it was found that 55 % take energy drink A, 50 % take energy drink B, 42 % take energy drink C, 28 % take energy drink A and B, 20 % take energy drink A and C, 12 % take energy drink B and C and 10 % take all the three energy drinks.
   **(i)** What percentage of people does not take energy drink?
   **(ii)** What percentage of people takes exactly two brands of energy drinks?
   **(iii)** What percentage of people takes the energy drink in A but not in B or C?
5. Show that the set of all integers $\mathbb{Z}$ is a countable set.
6. Consider the following relations on a set $A = \{1, 2, 3, 4, 5, 6\}$
   $(i)\ R = \{(i, j) : |i - j| = 2\}$, and $(ii)\ R = \{(i, j) : |i - j| < 2\}$
   Check whether $R$ is (i) reflexive, (ii) symmetric, (iii) anti-symmetric, and (iii) transitive.
7. If $A = \{0, 1, 2, 3\}$, $R = \{(x, y) : x + y = 3\}$, $S = \left\{(x, y) : \frac{3}{x+y} = 1\right\}$, $T = \{(x, y) : \max(x, y) = 3\}$.
   Compute $(i)\ RoT$, $(ii)\ ToR$, and $(iii)\ SoS$.
8. Check whether the set of real numbers is countable or not.
9. The function $f : \mathbb{R} \to \mathbb{R}$ is defined as $f(x) = x^3 + 1$, where $\mathbb{R}$ is the set of real numbers, then specify that $f$ is one-one and onto.
10. Specify the types (one-one or onto or both or neither) of the following function:
    **(i)** If I is set of non-negative integers and $f: I \times I \longrightarrow I$ such that $f(x, y) = xy$.
    **(ii)** If R is set of real numbers and $f: R \times R \longrightarrow R \times R$ and $f(x, y) = (x + y, x - y)$.
    **(iii)** If N is set of natural numbers including zero and $f: N \longrightarrow N$ such that $f(j) = j^2 + 2$.
    **(iv)** If N is set of natural numbers including zero and $f: N \times N \to N$ so that $f(x, y) = (2x + 1)\, 2^y - 1$.
11. Show that the mapping $f : \mathbb{R} \to \mathbb{R}$, which is defined as, $f(x) = ax + b$, where $a, b, x \in \mathbb{R}$, $a \neq 0$ is invertible. Determine its inverse also.
12. If A$= \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$ and $C = \{x, y, z\}$. Consider the function $f : A \to B$ and $g : B \to C$ defined by $f = \{(1, a), (2, c), (3, b), (4, a)\}$ and $g = \{(a, x), (b, x), (c, y), (d, y)\}$.
    Determine the composition function $(gof)$.

**13.** Show that $1+2+2^2+...+2^n = 2^{n+1}-1$ by mathematical induction for positive integers $n$.

**14.** Solve following recurrence relations by the method of generating function

**(i)** $a_n - 5a_{n-1} + 6a_{r-2} = 1$ **(iii)** $a_n - 4a_{n-1} + 4a_{r-2} = (n+1)^2$, given $a_0 = 1, a_1 = 1$.

**(ii)** $a_n + a_{n-1} = 3n\,2^n$ **(iv)** $a_{n+2} - 3a_{n+1} + 2a_n = 4n\,3^n$, with $a_0 = 1, a_1 = 1$.

**15.** Suppose that the population of a village is 100 at time $n = 0$ and 110 at time $n = 1$. The population increases from time $n-1$ to time $n$ is twice the increase from time $n-2$ to time $n-1$. Find a recurrence relation and initial conditions for the population at time $n$ and then find the explicit formula for it.

**16.** If $D_n$ is the value of the following determinant of order $n$

$$\begin{vmatrix} b & b & 0 & 0\dots\dots\dots & 0 & 0 & 0 \\ b & b & b & 0\dots\dots\dots & 0 & 0 & 0 \\ 0 & b & b & b\dots\dots\dots & 0 & 0 & 0 \\ .. & .. & .. & \dots\dots\dots. & .. & 0 & 0 \\ 0 & 0 & \dots\dots\dots & .. & b & b & b \\ 0 & 0 & \dots\dots.. & .. & 0 & b & b \end{vmatrix},$$

Find a recurrence relation for $D_n$. (Assume $b > 0$.)

**17.** If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, prove by principle of mathematical induction that for every integer $n \geq 3$,

$A^n = A^{n-2} + A^2 - I$. Hence find $A^{50}$. [$I$ is an identity matrix of order $3 \times 3$].

**18.** Let $U$ be a universal set and $S_1, S_2, S_3 \dots\dots \dots S_n$ be its any $n$ subsets. Use the principle of mathematical induction to show that $\overline{[\bigcup_{i=1}^{n} S_i]} = \overline{S_1} \cap \overline{S_2} \cap \overline{S_3} \cap \dots\dots\dots \cap \overline{S_n}$.

**19.** Let $f$ be a function with domain $X$ and range in $Y$ and let $A, B$ be subset of $X$. Then prove that $A \subset B \Rightarrow f(A) \subset f(B)$. Show that converse is not true.

**20.** A relation $R$ is defined on the set $N \times N$ as $(a,b)R(c,d)$ $iff$ $a + d = b + c$. Show that R is an equivalence relation on $N \times N$.