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Assignment

Q.1) Determine whether the set $M_{p\times n}$ of all $p\times n$ real matrices under matrix addition and scalar multiplication is a vector space.

Sol.ⁿ) Let $M_{p\times n}$ matrix be $= \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ u_{21} & u_{22} & \cdots & u_{2n} \\ u_{p1} & \cdots & \cdots & u_{pn} \end{bmatrix}$

1) $u+v = v+u$ (Commutativity of vector addition)

$$u+v = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ u_{21} & u_{22} & \cdots & u_{2n} \\ \vdots & & & \\ u_{p1} & u_{p2} & \cdots & u_{pn} \end{bmatrix} + \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & & & \\ v_{p1} & \cdots & \cdots & v_{pn} \end{bmatrix}$$

$$= \begin{bmatrix} u_{11}+v_{11} & u_{12}+v_{12} & \cdots & u_{1n}+v_{1n} \\ \vdots & & & \\ u_{p1}+v_{p1} & u_{p2}+v_{p2} & \cdots & u_{pn}+v_{pn} \end{bmatrix}$$

$$= \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ \vdots & & & \\ v_{p1} & v_{p2} & \cdots & v_{pn} \end{bmatrix} + \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ \vdots & & & \vdots \\ u_{p1} & u_{p2} & \cdots & u_{pn} \end{bmatrix}$$

$= v+u$

Hence Proved

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2) Since vector addition is commutative

$\therefore u+(v+w) = (u+v)+w$
(Associativity of vector addition)

3) The zero vector is $O = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix}$

$$u + O = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ \vdots & & & \\ u_{p1} & u_{p2} & \cdots & u_{pn} \end{bmatrix} + \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ \vdots & & & \\ u_{p1} & u_{p2} & \cdots & u_{pn} \end{bmatrix}$$

$\therefore u + O = u$ Hence Proved

4) For every $u \in V$, there exists a vector $-u$ such that
$u+(-u) = (-u)+u = 0$
(existence of an additive inverse)

$$u+(-u) = \begin{bmatrix} u_{11} & \cdots & u_{1n} \\ \vdots & & \\ u_{p1} & \cdots & u_{pn} \end{bmatrix} + \begin{bmatrix} (-u_{11}) & \cdots & (-u_{1n}) \\ \vdots & & \\ (-u_{p1}) & \cdots & (-u_{pn}) \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \\ 0 & \cdots & 0 \end{bmatrix}$$

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5.) $a(bu) = (ab)\,u$
(Associativity of Scalar Multi.)

$$a(bu) = a\begin{bmatrix} bu_{11} & \cdots & bu_{1n} \\ \vdots & & \\ bu_{pn} & \cdots & bu_{pn} \end{bmatrix} = ab\begin{bmatrix} u_{11} & \cdots & u_{1n} \\ \vdots & & \\ u_{p1} & \cdots & u_{pn} \end{bmatrix}$$

$\therefore\ a(bu) = (ab)u$ Hence Proved

6.) If $u, v \in V$ then $(u+v) \in V$ (Closure under addition)

$$u+v = \begin{bmatrix} u_{11}+v_{11} & u_{12}+v_{12} & \cdots & u_{1n}+v_{1n} \\ \vdots & & & \\ u_{p1}+v_{p1} & u_{p2}+v_{p2} & \cdots & u_{pn}+v_{pn} \end{bmatrix} \in V$$

7.) If a is any scalar ~~then~~ and $u \in V$, then $au \in V$

$$au = \begin{bmatrix} au_{11} & au_{12} & \cdots & au_{1n} \\ \vdots & & & \\ au_{p1} & au_{p2} & \cdots & au_{pn} \end{bmatrix} \in V$$

8.) $a(u+v) = au + av$ (Distributivity of Scalar multi over vector addition)

$$a(u+v) = \begin{bmatrix} au_{11}+av_{11} & \cdots & au_{1n}+av_{1n} \\ \vdots & & \\ au_{p1}+av_{p1} & \cdots & au_{pn}+av_{pn} \end{bmatrix} = au+av$$

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9) $(a+b)u = au + bu$ (Distributivity of Scalar multi over field addition)

$$(a+b)u = (a+b)\begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ \vdots \\ u_{p1} & u_{p2} & \cdots & u_{pn} \end{bmatrix} = \begin{bmatrix} au_{11}+bu_{11} & \cdots & au_{1n}+bu_{1n} \\ \vdots \\ au_{p1}+bu_{p1} & \cdots & au_{pn}+bu_{pn} \end{bmatrix}$$

$= au + bu$

H.P.

10) $1u = u$ (Existence of a Multiplicative Identity)

$$1u = \begin{bmatrix} 1.u_{11} & 1.u_{12} & \cdots & 1.u_{1n} \\ \vdots \\ 1.u_{p1} & 1.u_{p2} & \cdots & 1.u_{pn} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ \vdots \\ u_{p1} & u_{p2} & \cdots & u_{pn} \end{bmatrix} = u$$

H.P.

Therefore, we have verified that all 10 vector axioms hold for the set of $p \times n$ matrices in $M_{p \times n}$ under the defined operations of addition and scalar multiplication and so $M_{p \times n}$ is a Vector Space.

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Q.2 Let P_2 be the set of all polynomials of the form $P(x) = a_2 x^2 + a_1 x + a_0$ where a_0, a_1 and a_2 are real numbers. The sum of two polynomials $P(x) = a_2 x^2 + a_1 x + a_0$ and $q(x) = b_2 x^2 + b_1 x + b_0$ is defined in the usual way by

$$P(x) + q(x) = (a_2 + b_2) x^2 + (a_1 + b_1) x + (a_0 + b_0)$$

and the scalar multiple of $P(x)$ by the scalar c is defined by

$$c\,P(x) = c a_2 x^2 + c a_1 x + c a_0$$

Show that P_2 is a vector space.

Sol^n: 1) $u + v = v + u$

$$P(x) + q(x) = (a_2 + b_2) x^2 + (a_1 + b_1) x + (a_0 + b_0)$$
$$= (b_2 + a_2) x^2 + (b_1 + a_1) x + (b_0 + a_0)$$
$$= q(x) + P(x)$$

2) $u + (v + w) = (u + v) + w$

$$P(x) + [q(x) + r(x)] = (a_2 x^2 + a_1 x + a_0) + [(b_2 + c_2) x^2 + (b_1 + c_1) x + (b_0 + c_0)]$$

$$= [a_2 + (b_2 + c_2)] x^2 + [a_1 + (b_1 + c_1)] x + [a_0 + (b_0 + c_0)]$$

$$= [(a_2 + b_2) + c_2] x^2 + [(a_1 + b_1) + c_1] x + [(a_0 + b_0) + c_0]$$

$$= [P(x) + q(x)] + r(x)$$

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3) There exists a 0 vector such that $0+u=u+0=u$
The zero vector $O(x)=0x^2+0x+0$

$$P(x)+O(x) = (a_2+0)x^2+(a_1+0)x+(a_0+0) = P(x)$$

H.P.

4) For every $u \in V$, there exists $-u$ such that
$u+(-u)=(-u)+u=0$ (Existence of Additive Inverse)

$$P(x)+\{-P(x)\} = [a_2+(-a_2)]x^2+[a_1+(-a_1)]x+[a_0+(-a_0)]$$
$$=0$$

Hence Proved

5) $a(bu)=(ab)u$ (Associativity of Scalar Multiplication)

$$s(t\,P(x)) = s(ta_2x^2+ta_1x+ta_0)$$
$$= st(a_2x^2+a_1x+a_0)$$
$$=(st)P(x)$$

6) $a(u+v) = au+av$ (Distributivity of Scalar multi. over vector addition)

$$t[P(x)+q(x)] = (ta_2+tb_2)x^2+(ta_1+tb_1)x+(ta_0+tb_0)$$
$$= t\,P(x)+t\,q(x)$$

H.P.

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7) $(a+b)u = au + bu$ (Distributivity of Scalar multi over field addition)

$$(s+t)P(x) = (s+t)(a_2x^2+a_1x+a_0)$$
$$= s(a_2x^2+a_1x+a_0) + t(a_2x^2+a_1x+a_0)$$
$$= sP(x) + tP(x)$$

H.P.

8) If $u, v \in V$ then $u+v \in V$

$\because$ $P(x), q(x) \in V$, Then $P(x)+q(x) \in V$

9) If a is any scalar & $u \in V$, then $au \in V$

$P(x) \in V$ then $tP(x) \in V$
$$tP(x) = ta_2x^2 + ta_1x + ta_0 \in V$$

10) $1u = u$ (Existance of Multiplicative Identity)

$$1 \cdot P(x) = 1 \cdot a_2x^2 + 1 \cdot a_1x + 1 \cdot a_0$$
$$= a_2x^2 + a_1x + a_0$$

$\therefore$ $1 \cdot P(x) = P(x)$

Hence Proved