**UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN**

| Program | B.Tech (All SoCSBranches) | Semester | III |
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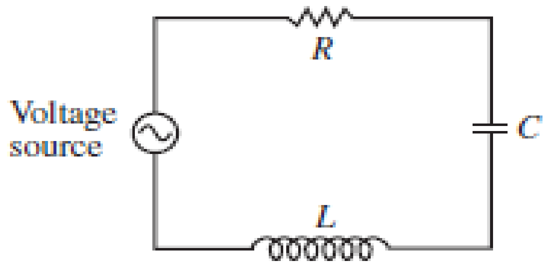
| Course | Discrete Mathematical Structures | Course Code | CSEG2006 |

1. Show that the set of all real valued continuous functions defined in the open interval (0, 1) is a vector space over the field of real numbers, with respect to the operations of vector addition and scalar multiplication defined as
$(f+g)(x) = f(x) + g(x)$
$(\alpha f)(x) = \alpha\ f(x), \quad \alpha \in R$ and $0 < x < 1$.
2. Suppose $V = \{x | x \in \mathbb{R}, x > 0\}$. Define vector addition and scalar multiplication as follows:
Vector addition: for $x \in V, y \in V$, define $x \oplus y = xy$
Scalar multiplication: for $r \in \mathbb{R}, x \in V$, define $r \odot x = x^r$
Show that $V(\oplus, \odot)$ is a vector space over the field of real numbers.
3. Let $W$ be the union of the first and third quadrants in the $xy$-plane. That is, let
$$W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \geq 0 \right\}.$$
   - **i.** If $u$ is in $W$ and $c$ is any scalar, is $cu$ in $W$? Why ?
   - **ii.** Find specific vectors $u$ and $v$ in $W$ such that $u + v$ is not in $W$. (This is enough to show that $W$ is not a vector space.)
4. Verify whether the set of all points in $\mathbb{R}^2$ of the form $(3s, 2+5s)$ forms a vector space where $s \in \mathbb{R}$.
5. If a mass $m$ is placed at the end of the spring and if the mass is pulled downward and released, the mass-spring system will begin oscillate. The displacement $y$ of the mass from its resting position is given by a function of the form $y(t) = c_1\ cos\omega t + c_2\ sin\omega t$, where $\omega$ is a constant that depends on the spring and the mass. Show that the set of all functions described by $y(t)$(with $\omega$ fixed and$c_1, c_2$arbitary) is a vector space.
6. Under what condition of $\beta$ the set $\{(\beta, 1, 0),\ (1, \beta, 1),\ (0, 1, \beta)\}$ is L.I and L.D in $\mathbb{R}^3$.
7. Let $M_{2\times 2}$ be the vector space of all $2 \times 2$ matrices, and define $T: M_{2\times 2} \to M_{2\times 2}$ by $T(A) = A + A^T$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.
**i.** Show that $T$ is linear transformation.
**ii.** Describe the kernel of $T$.
8. The circuit in the figure consists of a resistor ($R\ ohms$), an inductor ($L\ henrys$), a capacitor ($C\ farads$) and an initial voltage source. Let $b = R/2L$ and suppose $R, L$ and $C$ have been

selected so that $b$ also equals $1/\sqrt{LC}$. Let $v(t)$ be the voltage (in volts) at time $t$, measured across the capacitor. It can be shown that $v$ is in the null space $H$ of the linear transformation that maps $v(t)$ into $Lv''(t) + R\,v'(t) + \left(\frac{1}{C}\right)v(t)$ and $H$ consists of all functions of the form $v(t) = e^{-bt}(c_1 + c_2 t)$. Find a basis for $H$.




**9.** Let $H = span\{u_1,\ u_2,\ u_3\}$ and $K = span\{v_1,\ v_2,\ v_3\}$ where

$$u_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, u_2 = \begin{bmatrix} 0 \\ 2 \\ -1 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 3 \\ 4 \\ 1 \\ -4 \end{bmatrix}, v_1 = \begin{bmatrix} -2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 3 \\ 2 \\ -6 \end{bmatrix}, v_3 = \begin{bmatrix} -1 \\ 4 \\ 6 \\ -2 \end{bmatrix}$$

Find bases for $H,\ K$ and $H + K$.

**10.** A linear transformation $T$ is defined on $V_3(C)$ by $T(a,b) = (a\alpha + b\beta,\ a\gamma + b\delta)$ where $\alpha, \beta, \gamma, \delta$ are fixed elements of $C$. Prove that $T$ is invertible if and only if $\alpha\delta - \beta\gamma \neq 0$.

**11.** Let $T: \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation defined by $T(a,b,c,d) = (2a, 0, 0, c+d)$ then find the range space, null space, rank and nullity of linear transformation.

**12.** Verify the Rank-Nullity theorem for the linear transformation $T: V_4 \to V_3$ defined by $T(e_1) = f_1 + f_2 + f_3,\ T(e_2) = f_1 - f_2 + f_3,\ T(e_3) = f_1,\ T(e_4) = f_1 + f_3$ when $\{e_1, e_2, e_3, e_4\}$ and $\{f_1,\ f_2,\ f_3\}$ are the standard basis $V_4$ and $V_3$ respectively.

**13.** Show that the linear transformation $T: R^3 \to R^3$, defined by $T(x,y,z) = (2x, 4x - y, 2x + 3y - z)$, is invertible and determine $T^{-1}$.

**14.** The linear transformation $T: R^3 \to R^2$ defined by $T(x,y,z) = (3x + 2y - 4z,\ x - 5y + 3z)$. Determine the matrix of $T$ relative to the bases $B_1 = \{(1,1,1), (1,1,0), (1,0,0)\}$, $B_2 = \{(1,3), (1,5)\}$.

**15.** If the matrix of a linear transformation $T$ on $V_3(R)$ with respect to the standard bases is $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$ what is the matrix of $T$ with respect to the bases $\{(0,1,-1), (1,-1,1), (-1,1,0)\}$.