# Mid Sem Assignment

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Q.1 $(2xe^y + 3y^2)dy + (3x^2 + \lambda e^y)dx = 0$ — (i)

Rearrange eqn (i), we get

$(3x^2 + \lambda e^y)dx + (2xe^y + 3y^2)dy = 0$ — (ii)

This eqn is of the form

$$Mdx + Ndy = 0$$

where, $M = 3x^2 + \lambda e^y$

and $N = 2xe^y + 3y^2$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial(3x^2 + \lambda e^y)}{\partial y} \Rightarrow \frac{\partial M}{\partial y} = \lambda e^y$

$\therefore \frac{\partial N}{\partial x} = \frac{\partial(2xe^y + 3y^2)}{\partial x} \Rightarrow \frac{\partial N}{\partial x} = 2e^y$

$\because$ The given differential eqn (ii) is an exact differential eqn

$\therefore$ It must follow $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\therefore \lambda e^y = 2e^y$

$\therefore \boxed{\lambda = 2}$

$\therefore$ The value of constant $\lambda$ must be 2 for the given diff. eqn to be exact diff. eqn

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Q.2) Determine the P.I. of $(D-1)^2 y = e^x \sec^2 x \tan x$

$(D-1)^2 y = e^x \sec^2 x \tan x$ —— (i)

The Auxiliary eqⁿ is

$(m-1)^2 = 0$

$\therefore m = 1, 1$ (Real & equal Roots)

$\therefore$ C.F. of eq. (i) $= (c_1 + c_2 x) e^x$

$\therefore$ C.F. $= c_1 e^x + c_2 x e^x$

Let ~~C.F. = $Ay_1 + By_2$~~ $y = Ay_1 + By_2$

$\therefore$ By variation of Parameters, we get

$y_1 = e^x$, $y_2 = xe^x$

$$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^x & xe^x \\ e^x & xe^x + e^x \end{vmatrix}$$

$= e^{2x}(x+1) - e^{2x}(x)$

$= e^{2x}$

$\therefore A = -\int \frac{R(x) y_2}{W} dx + d_1$

$= -\int \frac{e^x . \sec^2 x \tan x . xe^x}{e^{2x}} dx + d_1$

$= -\int x . \sec^2 x \tan x \, dx + d_1$

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Applying Integration by parts

$u = x$ , $v = \sec^2 x \tan x$ by ILATE

$$\therefore = -\left[x.\int \sec^2 x \tan x \, dx - \int\left[\frac{d.x}{dx} \cdot \int \sec^2 x \tan x \, dx\right]\right] + d_1$$

$$\left[\int \sec^2 x \tan x \, dx = \int \frac{\sin x}{\cos^3 x} dx\right.$$

Let $\cos x = t$

$-\sin x \, dx = dt$

$\therefore dx = -\frac{dt}{\sin x}$

$$= -\int \frac{dt}{t^3} = -\left[\frac{t^{-2}}{-2}\right] = \frac{t^{-2}}{2}$$

$$\left.= \frac{1}{2\cos^2 x} = \frac{\sec^2 x}{2}\right]$$

$$\therefore A = -\left[x.\frac{\sec^2 x}{2} - \int \frac{\sec^2 x}{2} dx\right] + d_1$$

$$= -\left[\frac{x\sec^2 x}{2} - \frac{\tan x}{2}\right] + d_1$$

$$\therefore A = -\left(\frac{x\sec^2 x - \tan x}{2}\right) + d_1$$

$$\therefore B = \int \frac{R(x)y_1}{W} dx + d_2 = \int \frac{e^x . \sec^2 x \tan x . e^x}{e^{2x}} dx + d_2$$

$$= \int \sec^2 x \tan x \, dx + d_2$$

$$= \frac{\sec^2 x}{2} + d_2 \quad \text{[solved previously]}$$

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$\therefore y = Ay_1 + By_2$

$$= \left[\left(\frac{x\sec^2 x - \tan x}{2}\right) + d_1\right]e^x + \left[\frac{\sec^2 x}{2} + d_2\right]x.e^x$$

$$= \frac{e^x \tan x}{2} - \frac{e^x . x\sec^2 x}{2} + d_1 e^x$$

$$+ \frac{xe^x \sec^2 x}{2} + d_2 xe^x$$

$$y = \underbrace{d_1 e^x + d_2 x.e^x}_{C.F.} + \underbrace{\frac{e^x \tan x}{2}}_{P.I.}$$

$$\therefore P.I. = \frac{e^x \tan x}{2}$$

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Q.3) A Random variable has the distribution function

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $f(X=x)$ | $K$ | $2K$ | $3K$ | $K^2$ | $K^2+K$ | $2K^2$ | $4K^2$ |

Determine (i) K (ii) $P(X<5)$

Solⁿ (i) $\because \sum P(X=x)=1$

$\therefore K+2K+3K+K^2+K^2+K+2K^2+4K^2=1$

$8K^2+7K-1=0$

$8K^2+8K-K-1=0$

$8K(K+1)-1(K+1)=0$

$(8K-1)(K+1)=0$

$\therefore K=-1, \frac{1}{8}$

$\because$ Probability cannot be negative

$\therefore \boxed{K=\frac{1}{8}}$

(ii) $P(X<5)=P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$=K+2K+3K+K^2$

$=K^2+6K$

$=\left(\frac{1}{8}\right)^2+6\left(\frac{1}{8}\right)$

$=\frac{1+48}{64}=\frac{49}{64}$

$\therefore \boxed{P(X<5)=\frac{49}{64}}$

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Q.4| A target is to be destroyed in a bombing exercise. There is 75% chance that any one bomb will strike the target. Assume that atleast 2 hits are required to destroy the target completly. How many bombs must be dropped in order that the chance of destroying the target is equal or more than 99%.

Solⁿ| $p$ = Probability of hitting the target = $\frac{3}{4}$
$q$ = Probability of not hitting the target = $\frac{1}{4}$

$\because$ Atleast 2 hits are required & chances are $\geq 99\%$

$\therefore$ $P(X \geq 2) \geq \frac{99}{100}$

$1 - P(X=0) - P(X=1) \geq \frac{99}{100}$

$\therefore$ $P(X=0) + P(X=1) \leq \frac{1}{100}$ ; ${}^nC_0 \left(\frac{3}{4}\right)^0 \left(\frac{1}{4}\right)^n + {}^nC_1 \left(\frac{3}{4}\right)^1 \left(\frac{1}{4}\right)^{n-1} \leq \frac{1}{100}$

$\left(\frac{1}{4}\right)^n + n\frac{3}{4}\left(\frac{1}{4}\right)^{n-1} \leq \frac{1}{100}$

$1 + 3n \leq \frac{4^n}{100}$

$100 + 300n \leq 4^n$

$\because$ at $n=6$, we get $1900 \leq 4096$ ~~cond~~ and putting $n=7$, violates it.

$\therefore$ $\boxed{n=6}$ are the no. of bombs which must be dropped.

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Q.5)

Solⁿ) (i) Bisection Method:

$$x = \sqrt{12}$$
$$x^2 = 12$$
$$\therefore f(x) = x^2 - 12$$

Interval [3, 4] here $a = 3$, $b = 4$

$f(3) = -3$ and $f(4) = 4$

$\therefore$ Roots lie b/w 3 and 4

1st approximation

$$x_1 = \frac{a+b}{2} = 3.5$$
$$f(3.5) = 0.25 \ (+ve)$$

Interval (3, 3.5)

2nd Approximation $x_2 = \frac{a + x_1}{2} = 3.25$

$$f(3.25) = -1.4375 \ (-ve)$$

Interval [3.25, 3.5]

3rd Approxⁿ $x_3 = \frac{x_1 + x_2}{2} = 3.375$

$$f(3.375) = -0.6093 \ (-ve)$$

Interval [3.375, 3.5]

4th Approxⁿ $x_4 = \frac{x_3 + x_1}{2} = 3.4375$

$$f(3.4375) = -0.1835 \ (-ve)$$

Interval [3.4375, 3.5]

5th Approxⁿ $x_5 = \frac{x_4 + x_1}{2} = 3.4687$

$$f(3.4687) = 0.0318 \ (+ve)$$

Interval [3.4375, 3.4687]

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<u>6th Approxm:</u> $x_6 = \frac{x_4 + x_5}{2} = 3.4531$

$f(3.4531) = -0.0761$ (-ve)

Interval $[3.4531, 3.4687]$

<u>7th Approxm:</u> $x_7 = \frac{x_6 + x_5}{2} = 3.4609$

$f(3.4609) = -0.0221$ (-ve)

Interval $[3.4609, 3.4687]$

<u>8th Approxm:</u> $x_8 = \frac{x_5 + x_7}{2} = 3.4648$

$f(3.4648) = 0.0048$ (+ve)

Interval $[3.4609, 3.4648]$

<u>9th Approxm:</u> $x_9 = \frac{x_8 + x_7}{2} = 3.4628$

$f(3.4628) = -0.0090$ (-ve)

Interval $[3.4628, 3.4648]$

<u>10th Approxm:</u> $x_{10} = \frac{x_9 + x_8}{2} = 3.4638$

$f(3.4638) = -0.0020$ (-ve)

Interval $[3.4638, 3.4648]$

<u>11th Approxm:</u> $x_{11} = \frac{x_{10} + x_8}{2} = 3.4643$

$f(3.4643) = 0.0013$ (+ve)

Interval $[3.4638, 3.4643]$

<u>12th Approxm:</u> $x_{12} = \frac{x_{10} + x_{11}}{2} = 3.4640$

$f(3.4640) = -0.0007$ (-ve)

Interval $[3.4640, 3.4643]$

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13th Approxm $x_{13} = \frac{x_{12}+x_{11}}{2} = 3.4641$

$f(3.4641) = -0.00001$ (-ve)

$\because$ Approximation 12 & 13 have same value of $x$ ~~at~~ upto 3 decimal places

$\therefore \boxed{x = 3.464}$

(ii) Regula Falsi Method

$\because f(x) = x^2 - 12$

Interval $[\underset{x_0}{3}, \underset{x_1}{4}]$

Iteration 1 : $x_2 = x_0 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} \cdot f(x_0)$

$= 3 - \frac{1}{4-(-3)} \cdot (-3)$

$= 3 + \frac{3}{7} \Rightarrow \frac{24}{7}$

$x_2 = 3.4285$ , $f(3.4285) = -0.2453$ (-ve)

Interval $[3.4285, 4]$

Iteration 2 : $x_3 = 3.4615$, $f(x_3) = -0.0180$ (-ve)

Interval $[3.4615, 4]$

Iteration 3 : $x_4 = 3.4639$, $f(x_4) = -0.0013$ (-ve)

Interval $[3.4639, 4]$

Iteration 4 : $x_5 = 3.4640$, $f(x_5) = -0.0007$ (-ve)

Interval $[3.4640, 4]$

Iteration 5 : $x_6 = 3.4640$ , $f(x_6) = -0.0007$ (-ve)

$\because$ Iteration 4 & 5 are having same values

$\therefore \boxed{x = 3.464}$

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(iii) Newton-Raphson's Method

$$f(x) = x^2 - 12, \quad f'(x) = 2x$$

$\because$ Interval [3, 4]

$\therefore$ Initial guess $x_0 = 3$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Iteration 1: $x_1 = 3 - \frac{(-3)}{6} = 3 + \frac{1}{2} = 3.5$

Iteration 2: $x_2 = 3.46428$

Iteration 3: $x_3 = 3.46410$

Iteration 4: $x_4 = 3.46410$

$\because x_3 = x_4$

$\therefore \boxed{x = 3.464}$

(iv) Iteration Method

$$f(x) = x^2 - 12$$

$\because$ Interval [3, 4]

$$x^2 - 12 = 0$$
$$x^2 = 12$$
$$x = \sqrt{12} = \phi(x)$$
$$\phi'(x) = 0 < 1$$

$\therefore$ Initial guess $= 3 = x_0$, $\phi(x) = \sqrt{12}$

Iteration 1: $x_1 = \phi(x_0) = 3.4641$

$x_2 = \phi(x_1) = 3.4641$

$\because x_1 = x_2$

$\therefore \boxed{x = 3.464}$

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Q.6)

Solⁿ) Let P(x) represent the probability of batteries with 'x' lifetime (in hrs)

Given $\mu = 400$ hrs, $\sigma = 45$ hrs

(i) To find: $P(X \geq 470)$

converting to Z score

$P(Z \geq 1.55)$ or $P\left(\frac{x-\mu}{\sigma} \geq \frac{470-400}{45}\right)$

from given table $P(Z=1.55) = 0.4394$

∴ by property of normal distribution, we get

$P(Z \geq 1.55) = 0.5 - P(Z < 1.55)$

$= 0.5 - 0.4394$

$= 0.0606$

∴ Hence % of batteries with a lifetime atleast 470 hrs

$= 0.0606 \times 100$

$= 6.06\%$

(ii) To find: $P(385 < x < 415)$

Converting to Z score

$P\left(\frac{385-400}{45} < \frac{x-\mu}{\sigma} < \frac{415-400}{45}\right)$

$P(-0.33 < Z < 0.33)$

is Equivalent to $2 \times P(0 < Z < 0.33)$

from table $P(0 < Z < 0.33) = 0.1293$

so $2\,P(0 < Z < 0.33) = 2 \times 0.1293$

$\boxed{P(385 < x < 415) = 0.2586}$

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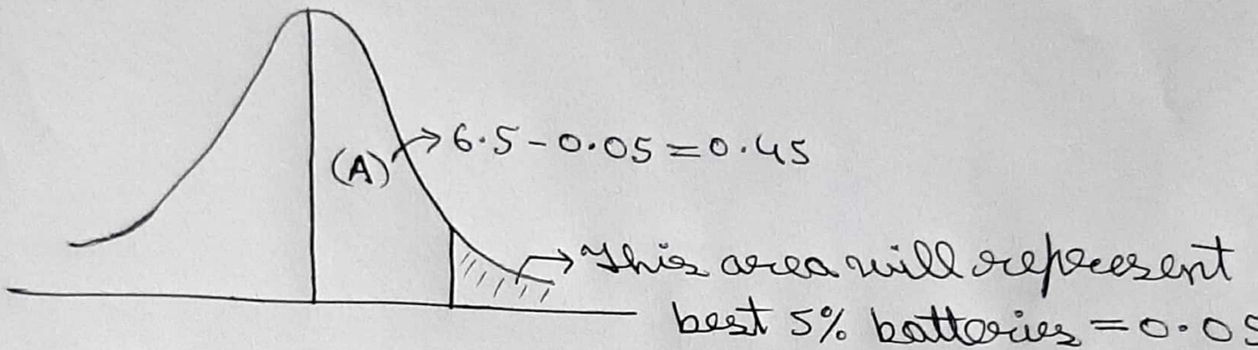
(iii) Minimum life of best 5% of batteries:




Value of z corresponding to this area = 1.65
(from table)

$$z = \frac{x-\mu}{\sigma} \quad \text{or} \quad 1.65 = \frac{X-400}{45}$$

$$X = 1.65 \times 45 + 400$$

$$= 74.25 + 400$$

$$\boxed{X = 474.25 \text{ Hours}}$$

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Q.7| Solve the differential equation

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \sin x \quad \text{—(i)}$$

Sol$^n$| In operator form, eq.(i) can be written as

$$(D^3 - 3D^2 + 4D - 2)y = e^x + \sin x$$

Auxiliary eq.$^n$ can be written as

$$m^3 - 3m^2 + 4m - 2 = 0$$

Let $m = 1$

$\therefore 1 - 3 + 4 - 2 = 0$

$\therefore 0 = 0$

now, divide $m^3 - 3m^2 + 4m - 2$ by $m - 1$

$$\begin{array}{r|l} m-1 & m^3 - 3m^2 + 4m - 2 \;\big(\, m^2 - 2m + 2 \\ & m^3 - m^2 \\ \hline & -2m^2 + 4m - 2 \\ & -2m^2 + 2m \\ \hline & 2m - 2 \\ & 2m - 2 \\ \hline & 0 \end{array}$$

$\therefore (m-1)(m^2 - 2m + 2) = 0$

$\therefore m = 1 \;;\; m^2 - 2m + 2 = 0$

$\therefore m = 1 \pm i$

$\therefore$ Roots are $m = 1, 1+i, 1-i$

$\therefore$ C.F. $= c_1 e^x + e^x(c_2 \cos x + c_3 \sin x)$

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$$P.I. = \frac{1}{f(D)} X = \frac{1}{(D^3-3D^2+4D-2)}(e^x + \sin x)$$

$$= \frac{1}{(D^3-3D^2+4D-2)} e^x + \frac{1}{(D^3-3D^2+4D-2)} \sin x$$

Rule-I
Replace $D \to a$
$\therefore D \to 1$
$\because f(1) = 0$
$\therefore$ case fails

Rule-III
Replace $D^2 \to -a^2$
$D^2 \to -1$

$$\therefore P.I. = x \cdot \frac{1}{(3D^2-6D+4)} e^x + \frac{1}{-D+3+4D-2} \sin x$$

Rule-I
Replace $D \to a$
$D \to 1$

here, multiply numerator & Denominator by $(3D-1)$

$$= x \cdot \frac{1}{3-6+4} e^x + \left(\frac{1}{3D+1}\right)(\sin x) \times \left(\frac{3D-1}{3D-1}\right)$$

$$= xe^x + \frac{(3D-1)\sin x}{9D^2-1}$$

Rule III Replace $D^2 \to -a^2$
$D^2 \to -1$

$$\therefore P.I. = xe^x - \frac{1}{10}(3\cos x - \sin x)$$

$\therefore$ C.S. of Eq.(i) $= y = C.F. + P.I.$

$$\therefore C.S. = c_1 e^x + e^x(c_2 \cos x + c_3 \sin x) + xe^x - \frac{1}{10}(3\cos x - \sin x)$$

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Q.8) Solve the differential equation

$$\frac{d^2y}{dx^2} + (1-\cot x)\frac{dy}{dx} - y\cot x = \sin^2 x \quad \text{—(i)}$$

Sol$^n$) Let $\frac{d^2y}{dx^2} + (1-\cot x)\frac{dy}{dx} - y\cot x = 0$ —(ii)

$\therefore$ $\boxed{\text{C.S. of eq (ii)} = \text{C.F. of eq (i)}}$

$\therefore$ we first solve for eq (ii), by using change of Dependent variables method

$\therefore$ $P = 1-\cot x$, $Q = -\cot x$, $R = 0$

$1 - P + Q = 1 - 1 + \cot x - \cot x = 0$

$\therefore$ Part of C.F. $\boxed{u = e^{-x}}$

$\therefore$ Let $y = uv$ be the sol$^n$ of eq (ii)

$\therefore$ The transformed eq (ii) is

$$\frac{d^2v}{dx^2} + \left(P + \frac{2}{u}\frac{du}{dz}\right)\frac{dv}{dx} = \frac{R}{u}$$

$$\frac{d^2v}{dx^2} + \left[(1-\cot x) + \frac{2}{e^{-x}}(-e^{-x})\right]\frac{dv}{dx} = 0$$

$$\frac{d^2v}{dx^2} + (-1-\cot x)\frac{dv}{dx} = 0$$

putting $\frac{dv}{dx} = z$, we get

$$\frac{dz}{dx} + (-1-\cot x)z = 0 \quad \text{—(iii)}$$

which is $1^{st}$ order $1^{st}$ Degree L.D.E.

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$$I.F. = e^{\int(-1-\cot x)dx} = e^{-(x+\log \sin x)} = e^{-x} \cdot e^{-\log \sin x}$$

$$I.F. = \frac{e^{-x}}{\sin x}$$

$\therefore$ The Sol$^n$ of Eq.$^n$ (iii) is

$$z \cdot \frac{e^{-x}}{\sin x} = \int 0 \cdot dx + c_1$$

$$\therefore z = \frac{dv}{dx} = c_1 e^{x} \sin x$$

By Integrating, we get

$$v = c_1 \int e^{x} \sin x \, dx + c_2$$

$$v = c_1 \left[ \frac{e^{x}}{1^2+1^2} (\sin x - \cos x) \right] + c_2$$

$$v = \frac{c_1}{2} e^{x} (\sin x - \cos x) + c_2$$

$\therefore$ The C.S. of Eq (ii) is

$$y = uv = e^{-x} \left[ \frac{c_1}{2} e^{x} (\sin x - \cos x) + c_2 \right]$$

$$\therefore C.S. = y = \frac{c_1}{2} (\sin x - \cos x) + c_2 e^{-x}$$

$\therefore$ C.F. of Eq. (i) is

$$C.F. = \frac{c_1}{2} (\sin x - \cos x) + c_2 e^{-x}$$

$\therefore$ Let $\frac{c_1}{2} = c_3$ (another constant)

$$y_1 = \sin x - \cos x$$

$$y_2 = e^{-x}$$

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$\therefore$ By using Variation of Parameters Method;

$$W = y_1 y_2' - y_1' y_2$$
$$= (\sin x - \cos x)(-e^{-x}) - (\cos x + \sin x)(e^{-x})$$
$$= -2e^{-x} \sin x$$

Let $P.I. = u y_1 + v y_2$

where, $u = -\int \frac{y_2 R(x)}{W} dx = -\int \frac{e^{-x} \sin^2 x}{(-2e^{-x} \sin x)} dx$

$$= \frac{1}{2} \int \sin x \, dx$$

$$u = -\frac{\cos x}{2}$$

$$v = \int \frac{y_1 R(x)}{W} dx = \int \frac{(\sin x - \cos x)(\sin^2 x)}{(-2e^{-x} \sin x)} dx$$

$$= -\frac{1}{2} \int e^{x} (\sin^2 x - \sin x \cos x) dx$$

$$= -\frac{1}{2} \int e^{x} \left( \frac{1 - \cos 2x}{2} - \frac{\sin 2x}{2} \right) dx$$

$$v = \int \left[ \frac{-e^{x}}{4} + \frac{1}{4} e^{x} \cos 2x + \frac{1}{4} e^{x} \sin 2x \right] dx$$

$$v = -\frac{1}{4} e^{x} + \frac{1}{4} \left[ \frac{e^{x}}{(1^2 + 2^2)} (\cos 2x + 2 \sin 2x) \right]$$
$$+ \frac{1}{4} \left[ \frac{e^{x}}{(1^2 + 2^2)} (\sin 2x - 2 \cos 2x) \right]$$

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$$V = -\frac{1}{4}e^{x} + \frac{e^{x}}{20}(\cos 2x + 2\sin 2x)$$
$$+ \frac{e^{x}}{20}(\sin 2x - 2\cos 2x)$$
$$= -\frac{1}{4}e^{x} + \frac{e^{x}}{20}(3\sin 2x - \cos 2x)$$

Let $P.I. = u y_1 + v y_2$

$$= -\frac{1}{2}\cos x(\sin x - \cos x)$$
$$+ \left(-\frac{1}{4}e^{x} + \frac{e^{x}}{20}(3\sin 2x - \cos 2x)\right)e^{-x}$$

Finally, the C.S. of eq.(i) is

$C.S. = y = C.F. + P.I.$

$$y = \left(-\frac{\cos x}{2} + c_3\right)(\sin x - \cos x)$$
$$+ \frac{e^{-x}}{20}\left[(3\sin 2x - \cos 2x - 5)e^{x} + 20c_2\right]$$

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Q.9] Solve the differential equation

$$y'' - 4xy' + (4x^2-1)y = -3e^{x^2}\sin 2x \quad \text{---(i)}$$

Sol^n] We can solve eq.(i), by using Reduction to Normal Form Method

$P = -4x$, $Q = 4x^2 - 1$, $R = -3e^{x^2}\sin 2x$

Let $y = uv$ be the sol^n of eq.(i)

Now, $v = e^{-\frac{1}{2}\int P.dx} = e^{-\frac{1}{2}\int(-4x)dx} = e^{x^2}$ --- (ii)

∴ The Reduced eq. is

$$\frac{d^2u}{dx^2} + Q_1 u = R_1 \text{ --- (iii) (Normal form)}$$

when, $Q_1 = Q - \frac{1}{2}\frac{dP}{dx} - \frac{P^2}{4}$

$$= 4x^2 - 1 - \frac{1}{2}(-4) - \frac{1}{4}(16x^2)$$

$Q_1 = 1$ (Constant)

$$R_1 = \frac{R}{V} = \frac{-3e^{x^2}\sin 2x}{e^{x^2}} = -3\sin 2x$$

Putting the values of $Q_1$ & $R_1$ in eq.(iii) we get

$$\frac{d^2u}{dx^2} + u = -3\sin 2x \quad \text{---(iv)}$$

which is L.D.E. with constant coefficients

∴ In operation form eq.(iv) is written as

$$(D^2+1)u = -3\sin 2x$$

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$\therefore$ The Auxiliary Eq$^n$ of Eq.(iv) is

$$m^2 + 1 = 0$$

$$\therefore m = \pm i$$

$\therefore$ C.F. of Eq.(iv) $= C_1 \cos x + C_2 \sin x$

$$\therefore P.I. = \frac{1}{f(D)} X = \frac{1}{(D^2+1)}(-3\sin 2x)$$

$$= (-3)\frac{1}{(D^2+1)}(\sin 2x)$$

Apply, Rule-III

Replace $D^2 \to -a^2$

$D^2 \to -4$

$$= (-3)\frac{1}{(-4+1)} \sin 2x$$

$$\therefore P.I. = \sin 2x$$

$\therefore$ The C.S. of Eq$^n$ (i) is

$$u = C.F. + P.I.$$

$$= C_1 \cos x + C_2 \sin x + \sin 2x$$

$\therefore$ ~~C.F.~~ C.S. $= y = uv$

$$\boxed{y = e^{x^2}(C_1 \cos x + C_2 \sin x + \sin 2x)}$$

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Q.10)

Solⁿ) Let P be the probability of a condensor being defective

$$P = \frac{1}{100}$$

$$n = \text{no. of boxes} = 100$$

$$\therefore \lambda(\text{mean}) = nP = 100 \times \frac{1}{100} = 1$$

$$P(X \geq 3) = 1 - P(0) - P(1) - P(2)$$

According to Poisson Distribution

$$\left[ P(X=r) = \frac{e^{-\lambda} \lambda^{r}}{r!} \quad , r = 0, 1, 2, \ldots, n \right]$$

$$\therefore \quad P(0) = \frac{e^{-1} \cdot 1^{0}}{0!} = e^{-1}$$

$$P(1) = \frac{e^{-1} \cdot 1^{1}}{1!} = e^{-1}$$

$$P(2) = \frac{e^{-1} \cdot 1^{2}}{2!} = \frac{e^{-1}}{2}$$

$$P(X \geq 3) = 1 - e^{-1} - e^{-1} - \frac{e^{-1}}{2} = 1 - \left(2 + \tfrac{1}{2}\right) e^{-1}$$

$$= 1 - \frac{2.5}{e} = 0.08630$$

$$\boxed{P(X \geq 3) = 0.08}$$

∴ Probability of getting 3 or more faulty condensors = 0.08