Ans 1: Given: $f(x) = 2x$ and $g(x) = x+2$

To prove: $f^{-1} \circ g^{-1} = (g \circ f)^{-1}$

$\Rightarrow f^{-1}(x) = x/2$, $g^{-1}(x) = x-2$

as, $(f^{-1} \circ g^{-1})(x) = f^{-1}((g^{-1})(x))$,

$(f^{-1} \circ g^{-1})(x) = f^{-1}(x-2)$

$\Rightarrow (f^{-1} \circ g^{-1})(x) = \frac{x-2}{x}$ — ①

as w.r.t $(g \circ f)(x) = g(f(x))$

$= g(2x)$

$= 2x+2$

Let $(g \circ f)^{-1}(x) = y$

$x = (g \circ f)(y)$

$x = 2y+2$

$y = \frac{x-2}{2} = (g \circ f)^{-1} x$ — ②

So, from ① & ②

$(f^{-1} \circ g^{-1})(x) = (g \circ f)^{-1}(x)$ Hence Proved

Ans 2: Given: $(a,b) \, R \, (c,d) \Rightarrow (ad = bc)$

So, to prove given relation is equivalence relation, it must be reflexive, symmetric and transitive

a) Reflexive → Since $(a,b) \, R \, (a,b)$ is there

if $[ab = ba]$ which is true, hence it is reflexive.

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b) Symmetric → Since $(a,b) R (c,d)$

If $ad = bc$, then $da = bc$

So, $cb = da$

$(c,d) R (a,b)$ which is true, hence given relation is symmetric

c) Transitive → Now let, $(a,b) R (c,d)$ and $(c,d) R (e,f)$

To prove → $((a,b),(e,f)) \in R \Rightarrow af = be$

$((a,b),(c,d)) \in R$ & $((c,d),(e,f)) \in R$

$ad = bc$ — ①

$cf = de$ - ②

① × ②

$ad \times cf = bc \times de$

$af = be$

$((a,b),(e,f)) \in R$

$\Rightarrow$ R is transitive

$\Rightarrow$ R is equivalence on $N \times N$ Hence Proved

Ans3 Given : $f : X \longrightarrow Y$ and also

$A \subseteq X, B \subseteq X$

To prove : (i) $[f(A \cap B) \subseteq f(A) \cap f(B)]$

$\Rightarrow$ let $y \in f(A \cap B)$ and

$x \in (A \cap B)$ which implies that

$y = f(x)$

$\because$ $x \in A$ So, $y = f(x) = f(A)$

$x \in B$ So, $y = f(x) = f(B)$ } given

therefore $y = f(x) \in f(A) \cap f(B)$

and $y \in f(A \cap B)$, So, $[f(A \cap B) \subseteq f(A) \cap f(B)]$. Hence Proved

(ii) $[f^{-1}(A \cap B) \subseteq f^{-1}(A) \cap f^{-1}(B)]$

$\Rightarrow$ let $x \in f^{-1}(A \cap B)$ then

$f(x) \in (A \cap B)$

Since, $f(x) \in A$,

$x \in f^{-1}(A)$.

Similarly, $f(x) \in B$

$x \in f^{-1}(B)$

$x \in f^{-1}(A) \cap f^{-1}(B)$

Hence $[f^{-1}(A \cap B) \subseteq f^{-1}(A) \cap f^{-1}(B)]$ Hence Proved

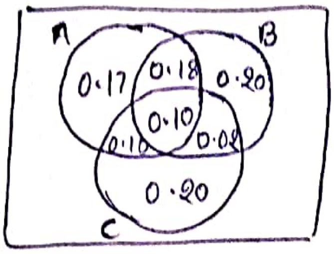
Ans 4. Given: Drink (A) = 0.55 $\quad$ $A \cap B = 0.28$

Drink (B) = 0.50 $\quad$ $B \cap C = 0.10$

Drink (C) = 0.42 $\quad$ $A \cap C = 0.10$

$A \cap B \cap C = 0.10$

Using Venn Diagram



a) People take energy drink = 0.17 + 0.20 + 0.20 + 0.10 + 0.10 + 0.02 + 0.18

= 0.97

People who don't drink = 1.00 − 0.97

= 0.03

% of people who don't drink = 3%

b) Who drink two brands exactly = 0.10 + 0.02 + 0.18

= 0.30

% of people = 30%

c) People who only drink A = 0.17

Percentage = 17%

Ans 5. Define $f: J \to Z$ by $\begin{cases} f(1) = 0 \\ f(n) = n/2, \text{ if } n \text{ is even} \\ f(n) = -\left(\frac{n-1}{2}\right), \text{ if } n \text{ is odd}, n > 1 \end{cases}$

We now show that f maps J onto Z. let $w \in Z$. If $w = 0$, then note that $f(1) = 0$. Suppose $w > 0$. Then $f(2w) = \frac{2w}{2} = w$. Suppose $w < 0$. Solving $w = -\left(\frac{n-1}{2}\right)$ for n, we get $n = -2w + 1$. Note that $-2w + 1$ is an odd positive number. So, $f(-2w+1) = -\left(\frac{2w+1-1}{2}\right) = w$. Hence, f maps J onto Z. The function f is one to one since $\frac{n}{2} = \frac{m}{2}$ implies $n = m$ and $-\left(\frac{n-1}{2}\right) = -\left(\frac{m-1}{2}\right) \Rightarrow n = m$.

Hence Proved

Ans 6. Given: $A = \{1, 2, 3, 4, 5, 6\}$

Check whether R is reflexive, symmetric, anti-symmetric & transitive.

(i) $R = \{(i, j) : |i - j| = 2\}$

a) For Reflexive :→ let $i \in A$, such that

$|i - i| = 0$ (which is even)

Therefore R is reflexive.

b) For Symmetric: Let $(i, j) \in R$

$\Rightarrow |i - j|$ is even $= 2$

$\Rightarrow |-(i-j)| = |j - i|$ is also even $= 2$

$\Rightarrow (j, i) \in R$ (which is true)

Therefor R is symmetric

c) For Anti-Symmetric: Since it is symmetric so it can't be anti-symmetric.

d) For Transitive: let $(i, j) \in R$ and $(j, k) \in R$

$|i - j| = 2$ and $|j - k| = 2$

$(i - j)$ is even and and $(j - k)$ is even $(i > j > k)$ (Assume)

$i - k = (i - j) + (j - k)$ {sum of two even no. is even}

$i - k = 2$ (even)

$\Rightarrow (i,k) \in R$ therefore R is transitive.

(ii) $R = \{(i,j) : |i-j| < 2\}$

a) Reflexive : let $i \in A$ such that

$|i-i| = 0$

i.e. $|i-j| < 2$, so given relation is reflexive.

b) Symmetric : let $(i,j) \in R$

$|i-j| < 2$

So, $|-(i-j)| < 2$

$|j-i| < 2$, $(j-i) \in R$ [which is true], so given relation is symmetric.

c) Transitive : $\because (i,j) \in R$ and $(j,i) \in R$.

let (j,k) is also there

$|i-j| < 2$, $|j-i| < 2$

$|j-k| < 2$

$(i-k) = (i-j) + (j-k) < 2$

So, $|i-k| < 2$ i.e. $(i,k) \in R$

Hence it is transitive.

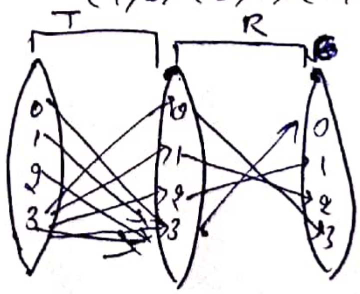
Ans 7: Given: $A = \{0,1,2,3\}$.

$R = \{(x,y) : x+y = 3\} \Rightarrow R = \{(0,3),(1,2),(2,1),(3,0)\}$.

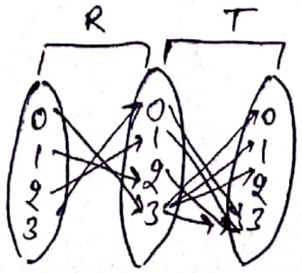
$S = \{(x,y) : \cancel{3x} (x+y) = 1\} \Rightarrow S = \{(0,3),(1,2),(2,1),(3,0)\}$

$T = \{(x,y) : \max(x,y) = 3\} \Rightarrow T = \{(0,3),(3,0)$
$(1,3)(3,1)(2,3)(3,2)(3,3)\}$.

1.) ROT = $ROT = \{(0,0)(3,3)(1,0)(2,0)$
$(3,2)(3,0)(3,1)\}$



2) TOR = $\{(0,0)(3,3)(0,1)$
$(0,2)(0,3)(2,3)\}$



3.) SOS = Using Matrix form:

$$M_{SOS} = M_S \times M_S = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \end{array} \times \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \end{array}$$

$$M_{SOS} = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \end{array}$$

So, $SOS = \{(0,0)(1,1)(2,2)(3,3)\}$

Ans 8 Suppose that $[0,1]$ is countable. Clearly $[0,1]$ is not a finite set, so we are assuming that $[0,1]$ is countably infinite.

Then there exists a bijection from N to $[0,1]$. In other words, we can create an infinite list which contains every real number. Write each number in their list in decimal notation. Such a list might look something like

1 0.0234242.....
2 0.03243423...
3 0.0500000...
4 0.2034203....

let N be the number ~~obtain~~ obtained as follows. For each $n \in N$; let the n^{th} decimal spot of N be equal to the N^{th} decimal spot of n^{th} number in the list. + 1 if that number is less than 9, and let it be 0 if that number is equal to 9. In the list above we would have $N = 0.1315$.

By construction, N is different from every number in our list and so our list is incomplete. But this contradicts the exists of a bijection from N to $[0,1]$. Hence $[0,1]$ must be uncountable.

Ans 9. Given function $f: R \to R$ such than $f(x) = x^3 + 1$

let A and B be two sets of real numbers.

let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$

$\Rightarrow 1 + (x_1)^3 = 1 + (x_2)^3$

$(x_1)^3 = (x_2)^3$

$x_1^3 - x_2^3 = 0$

$(x_1 - x_2)(x_1^2 + x_2^2 + x_1 x_2) = 0$

$\Rightarrow x_1 = x_2$

So it is ~~an~~ one-one function.

$y = 1 + x^3 \Rightarrow x^3 = y - 1$
$x = (y-1)^{1/3}$

So, y has a pre image in A
So, it is an onto function.

Ans10. (1) Given: $f : I \to I$, $f(x, y) = xy$

a) For one one function: let $F(x_1, y_1) = F(x_2, y_2)$
where $[x_1, y_1, x_2, y_2 \in I]$.

So, $(x_1 y_1 = x_2 y_2)$

So, these can be possible if may.
$[x_1 = x_2$ & $y_1 = y_2]$ So given function is one - one

b) For onto function: let $[y' = xy]$ $(y' \in I)$
So, $[x = y'/y]$.

Given: $f(x, y) = xy$ [let above value of x in it].
$f(y'/y \cdot y) = y'/y \times y$.
So $[f(x, y) = y'] \in I$
as [co-domain = Range $\in I$] function is onto.

Hence given $[f(x, y) = xy]$ is one - one and onto function.

(2) Given: $f : R \times R \to R \times R$, $[f(x, y) = (x+y, x-y)]$

a) For one one function: let $f(x_1, y_1) = f(x_2, y_2)$.
where $[x_1, y_1, x_2, y_2 \in R]$

So, $(x_1 + y_1, x_1 - y_1) = (x_2 + y_2, x_2 - y_2)$
two sets are equal if each element of one set equals to another set element.

So, $x_1 + y_1 = x_2 + y_2$, $x_1 - y_1 = x_2 - y_2$

$x_1 - x_2 + y_1 - y_2 = 0$ — (A)
$x_1 - x_2 - y_1 + y_2 = 0$ — (B)
on solving (A) & (B)

$$\cancel{x_1} - \cancel{x_2} + y_1 - y_2 = 0$$
$$-\cancel{x_1} \mp \cancel{x_2} - y_1 + y_2 = 0$$
$$2y_1 - 2y_2 = 0$$

$[y_1 = y_2]$ (Put in ①)

$[x_1 = x_2]$

as $[x_1 = x_2$ & $y_1 = y_2]$ So, given $f(x,y)$ is one-one

b) For onto function: let $y' = x+y$, $x = y' - y$

$y'' = x - y$, $x = y'' + y$ where $(y', y'' \in R)$

WRT $f(x,y) = (x+y, x-y)$

So, $f(x,y) = (y' - y + y, y'' + y - y)$

$f(x,y) = (y', y'') \in R$.

as [co-domain = Range $\in R$], Hence given $f(x,y)$ is onto function.

Thus $f(x,y) = \{(x+y), (x-y)$ is one-one & onto function.

3) Given: $f: N \to N$, $[f(j) = j^2 + 2]$

a) for one one function: let $f(x_1) = f(x_2)$, $(x_1, x_2 \in N)$

$x_1^2 + 2 = x_2^2 + 2$

$[x_1 = x_2]$, Hence given function is one-one.

b) for onto function: let $[y = j^2 + 2]$, $y \in N$

$[j = \sqrt{y-2}]$.

WRT $f(j) = j^2 + 2$

$f(j) = (\sqrt{y-2})^2 + 2 = y - 2 + 2 = y \in N$

So [Co-domain = Range $\in N$], Hence given $f(j)$ is onto function.

Thus $[f(j) = j^2 + 2]$ is ~~oneand~~ one-one and onto function.

4.) Given: $f: N \times N \to N$, $f(x,y) = (2x+1)2^y - 1$

a) for one-one function: let $f(x_1, y_1) = f(x_2, y_2)$, $x_1, y_1 \in N$

$x_2, y_2 \in N$

$$(2x_1+1)2^{y_1} - \cancel{1} = (2x+1)2^{y_2} - \cancel{1}$$

$$2x_1+1 = 2x_2+1 \quad , \quad 2^{y_1} = 2^{y_2}$$

$(x_1 = x_2)$ take log on both the sides

$(y_1 = y_2)$

as $(x_1 = x_2)$ & $(y_1 = y_2)$ hence given $f(x)$ is one - one.

b) <u>For onto function</u> :- let $[y = (2x+1)2^y - 1]$ where $y \in N$

$$y = (2x+1)2^y - 1$$

$$(2x+1)2^y = y+1$$

$$2x+1 = \frac{y+1}{2^y}$$

<u>WRT</u>

$f(x,y) = (2x+1)2^y - 1$

Put value of x in above $f(x,y)$ we, get.

$$f\left(\frac{\frac{y+1}{2^y} - 1}{2}, y\right) = \left[\cancel{2}\left(\frac{\frac{y+1}{2^y} - 1}{\cancel{2}}\right) + 1\right]2^y - 1$$

$$= \left[\left(\frac{y+1-2^y}{2^y}\right) + 1\right]2^y - 1$$

$$= \left[\frac{y+\cancel{1}-\cancel{2^y}+\cancel{2^y}}{\cancel{2^y}}\right)\cancel{2^y} - \cancel{1}$$

$$[f(x,y) \Rightarrow y \in N]$$

So, [co-domain = Range $\in N$], so given $f(x,y)$ is onto function.

Thus $f(x,y)$ is one one and onto function.

Ans 11: $f: R \to R$, $f(x) = ax+b$

<u>To prove</u> $\to$ $f(x)$ is inverctible

$\Rightarrow$ which means we have to show that $f(x)$ is one - one and onto function.
Because if any function is one-one & onto it is invertible.

a) for one one function: let $f(x_1) = f(x_2)$, $[x_1, x_2 \in R]$

$ax_1 + b = ax_2 + b$

$[x_1 = x_2]$ so given $f(x)$ is one one function.

b) for onto function: let $(y = ax + b)$, $y \in R$

$$\left[\frac{y-b}{a} = x\right].$$

WKT, $f(x) = ax + b$

$$f\left(\frac{y-b}{a}\right) = a\left(\frac{y-b}{a}\right) + b = y \in R$$

So, [co domain = Range $\in R$] so given $f(x)$ is onto function.

So from above, $f(x)$ is invertible.

Determining Inverse

Inverse of $x = f^{-1}(y)$

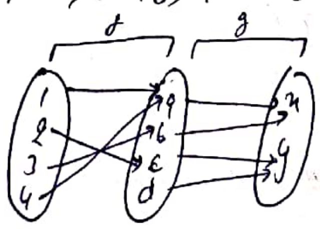
$$\left[f^{-1}(x) = \frac{y-b}{a}\right].$$

Ans 12: Given: $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$ & $C = \{x, y, z\}$

$f = \{(1,a), (2,c), (3,b), (4,a)\}$

$g = \{(a,x), (b,x), (c,y), (d,y)\}$

$g \circ f =$



$g \circ f = \{(1,x), (2,y), (3,x), (4,x)\}$

Ans 13: Show = $[1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1]$ — ①

By using mathematical Induction

Step 1 → Verification: Putting $n = 1, 2$ in eq ① (RHS)

$P(1) = 3$ (True)

$P(2) = 7$ (True)

Hence both are true

Step 2 Inductive Property : Assume for $[n=k]$ given $P(n)$ is true and to prove that for $[n=k+1]$ given relation is true.

So, $1+2+2^2+\_\_\_\_+2^n = 2^{n+1}-1$

$[\text{Put } n=k]$

$1+2+2^2+\_\_\_\_+2^k = 2^{k+1}-1$ — (A)

To prove

$1+2+2^2+\_\_\_\_+2^{k+1} = 2^{k+2}-1$ — (B)

So, using eq^n (A), Add both side 2^{k+1}, we get

$1+2+2^2+\_\_\_\_+2^k+2^{k+1} = 2^{k+1}-1+2^{k+1}$

$= 2\times 2^{k+1}-1$

$= 2^{k+2}-1$ (RHS)

Hence proved for $(n=k+1)$, $P(n)$ is true.

Step 3 Conclusion : From Step ① & step ②, it has been proved that the relation ~~___~~ for given $P(n)$ is always true for all positive integer n.

H.P

Ans 4. i) $a_n - 5a_{n-1} + 6a_{n-2} = 1$ — ① $(a_0=1, a_1=1)$.

Substitute $(n=n+2)$ in eq ①, we get

$[a_{n+2} - 5a_{n+1} + 6a_n = 1]$ — ②

Using concept of generating function, WRT

$a_{n+2} = \dfrac{G(x)-a_0-a_1x}{x^2}$, $a_{n+1} = \dfrac{G(x)-a_0}{x}$, $a_n = G(x)$

Put these values in above eq^n ②, we get

$$\frac{G(x)-a_0-a_1x}{x^2} - \left(\frac{5G(x)-5a_0}{x}\right) + 6G(x) = 1$$

So, $G(x) - 1 - x - 5xG(x) + 5x + 6x^2G(x) = x^2$

So, $(6x^2-5x+1)G(x) + 4x - 1 = x^2$

$(6x^2-5x+1)G(x) = x^2 - 4x + 1$

So, $G(x) = \frac{x^2}{6x^2-5x+1} - \frac{4x}{6x^2-5x+1} + \frac{1}{6x^2-5x+1}$

let $[G(x) = A + B + C]$ — ③

where, $A = \frac{x^2}{6x^2-5x+1} = \frac{1}{6} - \frac{\left(\frac{1}{6} - \frac{5x}{6}\right)}{6x^2-5x+1}$

So, using concept of partial function

we get $\left[\frac{x^2}{6x^2-5x+1} = \frac{1}{2} - \frac{1}{3(3x-1)} + \frac{1}{2(2x-1)}\right]$

B) $\frac{4x}{6x^2-5x+1} = \frac{4x}{(3x-1)(2x-1)}$ Using partial function

let $\frac{4x}{(3x-1)(2x-1)} = \frac{A}{(3x-1)} + \frac{B}{(2x-1)}$

$2A + 3B = 4$
$A + B = 0.$

So, $A = -4$ & $B = 4$

$\Rightarrow \frac{4x}{6x^2-5x+1} = \frac{-4}{(3x-1)} + \frac{4}{(2x-1)}$

C) $\frac{1}{6x^2-5x+1} = \frac{1}{(3x-1)(2x-1)}$

$= \frac{-3}{(3x-1)} + \frac{2}{(2x-1)}$

Put values of A, B, C in ③

$G(x) = \frac{1}{2} - \frac{1}{3(3x-1)} + \frac{1}{2(2x-1)} + \frac{4}{(3x-1)} - \frac{4}{(2x-1)} - \frac{3}{(3x-1)} + \frac{2}{(2x-1)}$

$G(x) = \frac{1}{2} - \frac{2}{3} \times \frac{1}{(3x-1)} - \frac{3}{2} \times \frac{1}{(2x-1)}$

$G(x) = \frac{1}{2} - \frac{2}{3} \times \frac{1}{(1-3x)} + \frac{3}{2} \times \frac{1}{(1-2x)}$

$\left\{ \begin{array}{l} 1 - 3x = 3^n \\ 1 - 2x = 2^n \end{array} \right\}$

$\left[G(x) = \frac{1}{2} - \frac{2}{3} \times (3)^n + \frac{3}{2} \times (2)^n\right]$

(2.) $[a_n + a_{n-1} = 3n.(2)^n]$, $a_0 = 1$ & $a_1 = 1$.

$$a_n + a_{n-1} = 3n.(2)^n \quad -(1)$$

Put $(n = n+1)$ in eq^n (1), we get

$$a_{n+1} + a_n = 3(n+1).(2)^{n+1}$$

$$a_{n+1} + a_n = 6[n.2^n + 2^n] \quad -(2)$$

Using Generating function Concept

$$a_{n+1} = \frac{G(x) - a_0}{x}, \quad a_n = G(x), \quad 6n.2^n = \frac{6x}{1-2x}, \quad 2^n = \frac{1}{1-2x}$$

Substitute above value in eq^n (2)

$$\frac{G(x) - 1}{x} + G(x) = \frac{6x}{1-2x} + \frac{6}{1-2x}$$

$$(1+x)\,G(x) = 1 + \frac{6x^2}{1-2x} + \frac{6x}{1-2x}$$

So, $G(x) = \dfrac{1}{1+x} + \dfrac{6x^2}{(1-2x)(1+x)} + \dfrac{6x}{(1-2x)(1+x)}$

let $[G(x) = A + B + C] \quad -(3)$

$$B = \frac{6x^2}{(1-2x)(1+x)} = \frac{6x^2}{(2x^2+x-1)} = 3 + \frac{(3-3x)}{(1-2x)(1+x)}$$

$$\frac{6x^2}{(1+x)(1-2x)} = 3 + \frac{1}{(2x-1)} + \frac{2}{(1+x)}$$

$$C = \frac{6x}{(1-2x)(1+x)} = \frac{A}{1-2x} + \frac{B}{1+x}$$

$$\left\{\begin{array}{l} A - 2B = 6 \\ A + B = 0 \\ A = 2 \\ B = -2 \end{array}\right\}$$

$$\frac{6x}{(1-2x)(1+x)} = \frac{2}{1-2x} - \frac{2}{(1+x)}$$

Substitute value of B & C in eq^n (3), we get

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$$G(x) = \frac{1}{(1+x)} + \frac{1}{(2x-1)} + \frac{\cancel{2}}{(1\cancel{+}x)} + 3 + \frac{2}{(1-2x)} - \frac{\cancel{2}}{\cancel{(}1+x)}$$

$$G(x) = \frac{1}{1+x} + \frac{1}{1-2x} + 3$$

$$[G(n) = (-1)^n + 2^n + 3].$$

(3) $a_n - 4a_{n-1} + 4a_{n-2} = (n+1)^2$ — (1) , $a_0 = 1, a_1 = 1$

Using operator method

Substitute $(n = n+2)$ in eq^n (1) we get

$$a_{n+2} - 4a_{n+1} + 4a_n = (n+3)^2 \quad \text{— (2)}$$

from eq^n (2), it is second order recurrence relation.

So, total sol^n = $[a_n = (a_n)^h + (a_n)^p]$

i.e. $(a_n = CF + PS$

For CF

$$a_{n+2} - 4a_{n+1} + 4a_n = 0$$

let $(a_n = r^n)$

$$r^{n+2} - 4r^{n+1} + 4r^n = 0$$

Divide by r^n both sides

$$r^2 - 4r + 4 = 0 \Rightarrow (r-2)^2 = 0$$

i.e. $(r_1 = r_2 = 2)$ as roots of eq^n are real and equal

So, $[(a_n)^h = C_1(2)^n + C_2 n(2)^n]$.

For PS From eq^n (2)

$$a_{n+2} - 4a_{n+1} + 4a_n = n^2 + 6n + 9$$

Convert it in terms of E, we get

$$(E^2 - 4E + 4)a_n = n^2 + 6n + 9$$

$$a_n = \frac{1}{(E^2-4E+4)} (n^2+6n+9)$$

WRT, $(E = 1+\Delta)$

$$a_n = \frac{1}{1+\Delta^2+2D-4-4\Delta+4}$$

$$a_n = \frac{1}{\Delta^2-2\Delta+1} (n^2+n-1)$$

$$a_n = \frac{1}{1}\left[\left(1+\frac{\Delta^2-2\Delta}{1}\right)^{-1}\right] n^2+6n+9$$

$$\Delta(n^2+6n+9) = (n+1)^2+6(n+1)+9-(n^2+6n+9)$$

$$= 2n+7.$$

$$\Delta^2(n^2+6n+9) = 2n+9-2n-7 = 2$$

So, $(a_n)^P = n^2+6n+9+4n+20 = n^2+10n+29$

So, $a_n = (c_1+c_2)2^n + n^2+10n+29$

$\because$ AT (a) $(a_0 = 1, a_1 = 1)$

$\therefore$ $c_1 = -28$, $c_2 = \frac{17}{2}$

(4) $a_{n+2} - 3a_{n+1} + 2a_n = 4n.(3)^n$ — (1)

using operator method

eq^n (1) is second order recurrence relation

$$a_n = (a_n)^n + (a_n)^P$$

a) for CF

aux $eq^n \Rightarrow a_{n+2} - 3a_{n+1} + 2a_n = 0$

let $(a_n = x^n)$

$$x^{n+2} - 3x^{n+1} + 2^r = 0$$

Divide above eq^n by x^n

$$x^2 - 3x + 2 = 0$$

$(x-1)(x-2) = 0$ So, $(x_1 = 1$ & $x_2 = 2)$ roots are equal and real

$CF = (a_r)^n = C_1(1)^n + C_2(2)^n$

B) For PS

using eq^n ①

$a_{n+2} - 3a_{n+1} + 2a_n = 4n \cdot 3^n$

convert LHS of eq^n ① in terms of E

$(E^2 - 3E + 2)a_n = 3^n \cdot 4n$

$$a_n = \frac{1}{(E^2 - 3E + 2)} \cdot 3^n \cdot 4n$$

$$a_n = 3^n \left[\frac{1}{(9E^2 - 9E + 2)}\right] \cdot 4n$$

WRT $(E = 1 + \Delta)$

$$a_n = 3^n \left[\frac{1}{9(1+\Delta)^2 - 9(1+\Delta) + 2}\right] \cdot 4n$$

$$a_n = 3^n \left[\frac{1}{\cancel{9} + 9\Delta^2 + 18\Delta - \cancel{9} - 9\Delta + 2}\right] 4n$$

$$a_n = 3^n \left[\frac{1}{9\Delta^2 + 9\Delta + 2}\right] 4n$$

$$a_n = \frac{3^n}{2}\left[\left(1 + \frac{9\Delta^2 + 3\Delta}{2}\right)^{-1}\right] 4n$$

$$a_n = \frac{3^n}{2}\left[1 - \frac{9\Delta}{2}\right] 4n$$

$\Delta(4n) = 4n + 4 - 4n = 4$

$$(a_n)^P = \frac{3^n}{2}\left[4n - \frac{9}{2} \times 4\right] = \frac{3^n}{2}\left[4n - 18\right]$$

$$= 3^n[2n - 9]$$

$a_n = C_1 + C_2(2)^n + 3^n(2n - 9)$

Ans 15 Given → $a_0 = 100$, $a_1 = 110$

ATQ

$$a_n - a_{n-1} = 2(a_{n-1} - a_{n-2})$$

$$a_n - a_{n-1} = 2a_{n-1} - 2a_{n-2}$$

$$2a_{n-2} - 3a_{n-1} + a_n = 0$$

let ($a_n = x^n$)

$$2x^{n-2} - 3x^{n-1} + x^n = 0$$

Divide above eq^n by x^{n-2}

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0 \quad (x_1 = 1, x_2 = 2)$$

roots are real and unequal

$$a_n = c_1 + c_2(2)^n$$

ATQ ($a_0 = 100$, $a_1 = 110$)

$$a_0 = c_1 + c_2(2)^0 = 100$$

$$c_1 + c_2 = 100$$

$$a_1 = c_1 + 2c_2 = 110$$

$$c_1 + 2c_2 = 110$$

$$c_1 + c_2 = 100$$

$$c_2 = 10$$

$$c_1 = 90$$

So, $a_n = 90 + 10(2)^n$ (Explicit Formula)

Ans 16 If we expand it along first row, Dn will be equal to 6 times the determinant of matrix given by deleting the first row & first columns minus 6 times the det of matrix

So, $D_n = 6 \cdot D_{n-1} - 1 \cdot \det$ (matrix left after deleting) ($6 > 0$)

Again delete the first row of first column of this matrix, we left with a copy of A_{n-2} (det of which is D_{n-2})

$$A_{50} = \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$$

Ans 18 By using mathematical Induction

Base Step: let $n=2$ (for), we know two sets S_1 & S_2 in Universal set U by De morgan's law.

$$\left[\overline{S_1 \cup S_2} = \bar{S_1} \cap \bar{S_2}\right]$$

Inductive Step: let assume that $B_1, B_2, B_3 \cdots B_n$ are the n subsets of Universal set U.

So, our goal is to show that given any $n+1$ subsets S_1, S_2, S_3. ---- S_{n+1} of $U = \overline{S_1 \cup S_2 \cup -- \cup S_{n+1}} = \bar{S_1} \cap \bar{S_2} \cap -- \cap \bar{S}_{n+1}$

So, let $S_1, S_2, S_3 -- S_n$ be any set of Universal set.

$$\text{So, } \overline{S_1 \cup S_2 \cup ---S_n \cup S_{n+1}} = \overline{\overline{S_1 \cup S_2 \cup --- S_n} \cup S_{n+1}}$$

$$= \overline{S_1 \cup S_2 \cup S_3 - - \cup S_n}$$

$$= \bar{S_1} \cap \bar{S_2} \cap -- \cap \bar{S_n} \cap \overline{S_{n+1}}$$

which is true.

$$\text{Hence } \left[\overline{\cup_{i=1}^{n} = S_i}\right] = \bar{S_1} \cap \bar{S_2} \cap \bar{S_3} \cap --- \cap \bar{S_n}$$

Hence Prove

Ans 19 $f: X \to Y$, let (A, B) be subset of X

Prove $\rightarrow$ $A \subset B \Rightarrow f(A) \subset f(B)$

Proof $\rightarrow$ Take an arbitary element $x \in f(A)$, then there exists a $y \in A$ so that

$$f(y) = x.$$

Now take an arbitary element $x \in f(B)$, then there exists a $y \in B$ So that

$$f(y) = x$$

So, for every $y \in A$ and $y \in B$, $f(y) \in f(A)$ and $f(y) \in f(B)$
also for any $x \in f(A)$, $x \in f(B)$
Therefore, $[f(A) \subset f(B)]$
Hence, $[A \subset B \Rightarrow f(A) \subset f(B)]$ H.P

The converse is not possible because inverse of a function is not their so, it is not possible that first if given function is subject of another function then its respective elements are subset of each other.

Ans20 Given : $(a,b) R (c,d)$ if ~~ad~~ $a+d = b+c$.
Show given relation is equivalence relation. Hence we have to show that Given Relation is Reflexive, Symmetric and Transitive

a) Reflexive : Here $(a,b) R (c,d) \Leftrightarrow$ ~~ad~~ $a+d =$ ~~bc~~ $b+c$
so for $(a,b) R (a,b)$
$a+b = b+a$.

R is reflexive.

b) Symmetric : For $(a,b) R (c,d) \in N \times N$
$\Rightarrow a+d = b+c$
$\Rightarrow b+c = a+d$
$\Rightarrow c+b = d+a$
$\Rightarrow (c,d) \in (a,b)$

R is symmetric

c) Transitive : for $(a,b) R (c,d)$ and $(c,d) R (e,f)$
$\Rightarrow a-b = c-d$ and $c-d = e-f$
$\Rightarrow a-b = e-f$
$\Rightarrow a+f = e+b$
$\Rightarrow (a,b) R (e,f)$
R is transitive   So R is equivalence relation