Program: B.Tech	B. Tech CS : IOT, Blockchain, OGI, OSS, CSF, BigData, AI+ML, Devops	Semester	I
Course	PHYSICS-I	Course Code	PHYS1008
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Key points and formula

- *Photo Electric Effect* explains the emission of electrons from a metal plate when illuminated by light or any other radiation of any wavelength or frequency (suitable). The emitted electrons are called 'photo electrons'. The emission of photo electrons are governed by the following equation (also known as Einstein's photoelectric equation):

  $$h\nu = W_0 + \frac{1}{2}mv^2$$

  where ν is the frequency of incident light, W_0 is the work function, m is the mass of photo electrons and v is the velocity of photo electrons.

- In *Compton Effect*, when a monochromatic beam of X-rays is scattered from a material (scattering is done by those electron which are almost free) then both the wavelength of primary radiation (λ, unmodified radiation) and the radiation of higher wavelength (λ', modified radiation) are found to be present in the scattered radiation. Presence of modified radiation in scattered X-rays is called Compton Effect. Compton Effect may be represented mathematically as:

  $$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

  where $\Delta\lambda$ is the Compton shift, θ is the scattering angle and h/m_0c is called the Compton wavelength of the electron and its value is 0.0243 Å.

  The direction of recoiled electron may be given as:

  $$\tan\emptyset = \frac{\cot\frac{\theta}{2}}{1 + \frac{h\nu}{m_0 c^2}}$$

  where $\emptyset$ is the angle between incident photon and recoiled electron.

- In *Pair production*, when a photon of sufficient energy (usually very high, like γ radiation), passes near the field of a nucleus, it materializes into an electron and a positron.

- In *Pair Annihilation*, when an electron and positron interact with each other due to their opposite charge (hence due to Coulombic attraction), both the particles annihilate resulting into the release of electromagnetic energy in the form of two γ- rays photons.

$$e^{-} + e^{+} \rightarrow \gamma + \gamma$$

- *De Broglie Wavelength*: $\lambda = \frac{h}{p}$

- *Phase Velocity*: When a monochromatic wave travels through a medium, its velocity of advancement in the medium is called the phase velocity V_p.

$$V_p = \frac{\omega}{k}$$

where $\omega = 2\pi\nu$ is the angular frequency and $k = 2\pi/\lambda$ is the wave number

Phase velocity of de-Broglie waves exceeds velocity of light $V_p = \frac{c^2}{v}$, where v is the speed of particle. This discrepancy is resolved by postulating that a moving particle is associated with a "wave packet" or a "wave group", rather than a single wave-train.

- *Group Velocity* (V_g) of a wave is the velocity with which the overall shape of the waves' amplitudes, known as the modulation or envelope of the wave, propagates through the space.

$$V_g = \frac{d\omega}{dk}$$

Relation between phase velocity and group velocity for a dispersive medium (in which wave velocity depends upon the wavelength) is:

$$V_g = V_p - \lambda \frac{dV_p}{d\lambda}$$

- *Heisenberg Uncertainty Principle* states that it is impossible to measure both the position and momentum simultaneously with unlimited accuracy. If Δx and Δp_x are uncertainties in measuring position and momentum, respectively, then

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$$

where $\hbar = \frac{h}{2\pi}$

Similarly, Heisenberg Uncertainty Principle may also be written for other conjugate quantities like E (energy) and t (time) as well as for L(angular momentum) and θ (angular position):

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

and

$$\Delta L \Delta\, \theta \geq \frac{\hbar}{2}$$

- *Wave function*,Ψ, is a quantity associated with a moving particle. It is a complex quantity and the solution of Schrodinger's equation.

  $|\Psi|^2$ is proportional to the probability of finding a particle at a particular point at a particular time. It is the *probability density*.

- *Properties of wave function*:
  (a) It must be finite everywhere.
  (b) It must be single valued.
  (c) It must be continuous and should have a continuous first order derivative everywhere.
  (d) It must be normalized.

- *Normalization* is done to make the solution of Schrodinger equation unique. It is done by taking the total probability of finding the particle within the space as one:

$$\int_{-\infty}^{\infty} |\Psi|^2\, dV = 1$$

  where dV is an infinitesimal volume element.

- *Schrodinger's wave equation* describes the motion of a particle w.r.t space and time. For time-independent (stationary) motion of the particle, Schrodinger equation may be written as:

$$\nabla^2\Psi + \frac{2m}{\hbar^2}(E - V)\Psi = 0$$

  For time dependent motion, the Schrodinger equation may be written as

$$i\hbar\frac{\partial\Psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi$$

  where ∇^2 is Laplacian operator and V is the potential energy; m is the mass of the moving particle.

- *Expectation value* of any quantity which is a function of 'x' say $f(x)$ is given by $< f(x) >= \int_{-\infty}^{\infty} f(x)|\psi|^2\, dx$

- An *operator* is a rule by means of which, from a given function we can find another function.

  Momentum operator: $\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$

  Energy operator: $\hat{E} = i\hbar\frac{\partial}{\partial t}$

- *Particle in a one dimensional box*

  I. Wave-function of the particle: $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{\pi n x}{L}$; where L= length of the box; n is quantum number

  II. Energy of the particle: $E = \frac{n^2h^2}{8m_oL^2}$; m_0 = rest mass of the particle, L = length of the box

- *Particle in a Three Dimensional Box:*

  I. Wave-function of the particle: $\psi = \left(\sqrt{\frac{2}{L}}\right)^3 \sin\frac{n_x\pi x}{L}\sin\frac{n_y\pi y}{L}\sin\frac{n_z\pi z}{L}$

  II. Energy of the particle: $E = (n_x^2 + n_y^2 + n_z^2)\frac{h^2}{8mL^2}$

2. MULTIPLE CHOICE QUESTIONS OF WAVE MECHANICS

CIT-PHYSICS (PHYS-105)_2016-17

1. Which of the following phenomena cannot be explained by wave theory of light?
   (a) Interference
   (b) Diffraction
   (c) Reflection
   (d) None of the above

2. In Compton effect, the change in wavelength of scattered photon depends on
   (a) Wavelength of incident photon
   (b) Intensity of Incident Radiation
   (c) Scattering Angle
   (d) All of the above

3. Calculate the de-Brogle wavelength (in Å), if an electron is accelerated from rest through a potential different $V = 50$V.
   (a) 2.4 Å (b) 9.2 Å
   (c) 1.73 Å (d) 5.7 Å

4. The operator for momentum is
   (a) $\frac{\hbar}{i}\nabla^2$ (b) $\frac{h}{i}\nabla^2$
   (c) $\frac{\hbar}{i}\nabla$ (d) $\frac{-\hbar}{i}\nabla$

5. If a proton and an alpha-particle are accelerated through the same potential difference, then the ratio of their de-Broglie wavelengths will be

   (a) $\sqrt{8}: 1$ (b) $1: \sqrt{8}$
   (c) $\sqrt{2}: 1$ (d) $1: \sqrt{2}$

6. A proton and an alpha-particle are confined in 1-D potential boxes of lengths L and L/2 respectively. If Ep and Eα are their ground state energies, then
   (a) $E_\alpha = E_p/2$ (b) $E_\alpha = 2E_p$
   (c) $E_\alpha = E_p$ (d) $E_\alpha = \sqrt{E_p}$

7. Which of the following phenomena shows the wave nature of an electron?

(a) Compton effect (b) Diffraction of electron by crystal
(c) Photoelectric effect (d) Thermionic emission

8. For a dispersive medium
   (a) Phase velocity is equal to the group velocity.
   (b) Phase velocity is not equal to the group velocity.
   (c) Both (a) and (b) are true
   (d) None of these are true

9. Schrodinger wave equation is applicable to
   (a) Relativistic motion only
   (b) Non-relativistic motion only
   (c) Neither for relativistic motion nor for non-relativistic motion
   (d) Both relativistic motion and non-relativistic motion

10. Choose the incorrect statement
    (a) The number of photoelectrons emitted is proportional to light intensity
    (b) The velocity of photoelectrons is proportional to frequency of light
    (c) Photoelectric effect is instantaneous process.
    (d) Stopping potential is independent of incident frequency.

11. Choose the incorrect statement. In the Compton scattering-
    (a) The increase in wavelength $\Delta\lambda$, is independent of the incident wavelength.
    (b) Compton wavelength is always a constant quantity.
    (c) The wavelength increase, $\Delta\lambda$, depends on scattering angle of photon
    (d) The wavelength increase is independent of scattering angle of photon.

12. The phase speed of matter waves for a relativistic free moving particle in vacuum is;
    (a) Equal to the speed of light in vacuum
    (b) Less than the speed of light in vacuum
    (c) More than the speed of light in vacuum
    (d) Equal to half the speed of light in vacuum

13. A moving particle is associated with a wave packet or group of waves. The group velocity is equal to;
    (a) Velocity of light
    (b) Velocity of sound
    (c) Velocity of particle
    (d) Square of the particle velocity

14. Uncertainty principle states that the error in measurement is due to;
    (a) Dual nature of light
    (b) Due to small size of particles
    (c) Due to large size of particles
    (d) Due to the error in measuring instruments

15. Uncertainty relation cannot hold for the following pairs;
    (a) Position and momentum
    (b) Energy and time
    (c) Linear momentum and angle
    (d) Angular momentum and angle

16. The duration of a radar pulse is 10^{-6} sec. The uncertainty in its energy will be;
    (a) 1.05×10^{-14} J
    (b) 1.05×10^{-21} J
    (c) 1.05×10^{-28} J
    (d) 1.05×10^{-31} J

17. Compton effect supports;
    (a) Wave nature of radiation
    (b) Particle nature of radiation
    (c) Both particle and wave nature of radiation
    (d) None of these

18. An X-ray photon is found to have its wavelength doubled on being scattered through 90^0. The wavelength is;
    (a) 0.024 m
    (b) 0.240 m
    (c) 0.042 m
    (d) 0.024 Å

19. X-ray of wavelength 1 Å are scattered at such an angle that the recoil electron has maximum kinetic energy. The wavelength of scattered rays is;
    (a) 0.048 Å
    (b) 0.480 Å
    (c) 2.048 Å
    (d) 1.048 Å

20. The equation of motion of matter waves are derived by;
    (a) Heisenberg
    (b) De-Broglie
    (c) Bohr
    (d) Schrodinger

21. $Sin\ 2x$ is an eigen function of the operator;
    (a) $-\frac{d}{dx}$
    (b) $+\frac{d}{dx}$
    (c) $-\frac{d^2}{dx^2}$

(d) $+\frac{d^2}{dx^2}$

22. The allowed eigen function must be;
    (a) Finite only
    (b) Continuous only
    (c) Single-valued only
    (d) All of the three

23. Schrodinger wave equation is
    (a) An equation of motion for an electron moving at non relativistic velocities
    (b) An equation of motion for proton accelerated at some potential difference V
    (c) An equation of motion of de-Broglie wave
    (d) None of these

24. Which of the following has more precise information about the position of a particle?
    (a) Energy
    (b) Wave-function
    (c) Probability
    (d) Expectation value

25. According to wave mechanics, a free particle can possess;
    (a) Discrete energies
    (b) Continuous energies
    (c) Only a single values energy
    (d) None of these

26. A particle moving in an infinite deep potential can have energies which are multiple of;
    (a) n^2
    (b) n
    (c) 1.5 n
    (d) n^2

3. TUTORIAL SHEET OF WAVE MECHANICS
CIT PHYSICS PHYS-105 (2016-17)

1. Find maximum wavelength that can liberate electron from potassium. Work function of potassium is 2.24 eV. **Ans: 5524Å**

2. The stopping Potential for a metal is 4.6 V when light of frequency 2×10^{15} Hz is incident on it. When light of frequency 2×10^{15} is used, the stopping potential is 12.9 V. Calculate the Planck's constant. (Given the charge of electron e = 1.6×10^{-19} C). **Ans:6.64×10^{-34} Js**

3. A metallic surface, when illuminated with light of wavelength λ_1, emits electrons with energies upto a maximum value E_1, and when illuminated with light of wavelength λ_2, where $\lambda_2 < \lambda_1$, it emits electrons with energies upto a maximum value E_2. Prove that plank's constant h and the work function φ of the metal are given by

$$h = \frac{(E_2 - E_1)\lambda_1\lambda_2}{C(\lambda_1 - \lambda_2)} \quad \text{and} \quad \varphi = \frac{E_2\lambda_2 - E_1\lambda_1}{(\lambda_1 - \lambda_2)}$$

4. Light of wavelength 800 nm is shone on a metal surface connected to a battery. The work function of the metal is 1.25 eV. Find the extinction voltage or retarding voltage at which the photoelectron current stops. Find the highest speed of the emitted photoelectrons at this incident frequency. **Ans: V_r = 0.3 volt, 3.2×10^5m/s**

5. Calculate the maximum percentage change in wavelength due to Compton scattering for incident photons of wavelength 1Å and 10Å. What information do you draw from the result?
Ans: 4.8%, 0.48%, the change is difficult to detect in sufficient larger wavelengths.

6. X-ray of photon of wavelength 0.3 Å is scattered through an angle of 60° by a free electron. Find the wavelength of scattered photon and the recoil energy of the electron.
Ans: 3.01212 Å, 2.57×10^{-16} J

7. A photon of energy E is scattered by an electron initially at rest (rest mass energy,E_0) (Compton scattering problem). Show that the maximum kinetic energy (KE_{max}) of the recoil electron can be calculated as $KE_{max} = \frac{2E^2/E_0}{\left(1 + 2E/E_0\right)}$

8. Show that the minimum energy of incident radiation should be ~ 256 KeV in order to transfer half of its energy to recoiled electron.

9. Calculate the de-Broglie wavelength of α-particle accelerated through a potential difference of 2000 volt. **Ans: 2.3×10^{-3}Å**

10. What will be the percentage error if an electron moves with a speed of equivalent KE of 100KeV if someone does not treat the electron relativistically? **Ans: $\lambda_R < \lambda_{non-R}$ by 8%**

11. Show that deBroglie wavelength associated with an electron accelerated from rest with a voltage V is approximately $\frac{12.27}{\sqrt{V}}$.

12. Calculate the ratio between the wavelength of an electron λ_e and a proton λ_p if the proton is moving with half the velocity of the electron. **Ans: 981**

13. Compute the ratio of Compton wavelength to de Broglie wavelength in the form of

$$\frac{\lambda_c}{\lambda_d} = \frac{\varphi_0}{(1-\varphi_0)^{1/2}} \text{ where } \varphi_0 = \frac{v}{c}$$

14. If an excited atom has maximum uncertainty of 10 ns in its life time, what would be the inherent fractional line broadening? Assume emitted wavelength = l (in meter unit).

Ans: λ/12π

15. Establish the relation $v_g \times v_p = c^2$ for a relativistically moving particle and its associated waves (consider particle velocity = group velocity). Find the phase and group velocity of the associated de Broglie wave of a moving electron (speed 0.9c).
Ans: (1st case) v_p=w/k = E/p = mc^2/mv = c^2/v = c^2/v_g and simplify. (2nd case), v_p = c/0.9, and v_g = 0.9c

16. An electron has a speed of 1.05×10^4 m/sec within the accuracy of 0.01%. Calculate the uncertainty in the position of the electron. **Ans: 5.5 ×10^{-5} m**

17. Which of the following wave functions cannot be solutions of Schrodinger's equation for any values of x? Why not? (a) $Y = A\sec x$, (b) $Y = A\tan x$, (c) $Y = A\exp(x^2)$, (d) $Y = \exp(-x^2)$?
Ans: (a-c) not solutions because Y is undefined for x= π/2, π/2, and ∞, respectively (d) is a solution as Y is finite in the range x = 0 to ∞

18. An electron is trapped in an infinitely deep well of width L. If the electron is in its ground state, what fraction of the time does it spend in central one-third of the well.

Ans: 0.61 or 61%

19. The formula$y = A\cos(wt - kx)$, where $k = \frac{w}{v}$, describes a wave that moves in the $+x$ direction along a stretched string. Show that this formula represents a solution of the wave equation

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2}$$

20. Find the value of the normalization constant N for the wave function
(a) $Y = Nx\exp(-x^2/2)$, (b) $Y = N\exp(-x^2/2a^2)\exp(-ikx)$.

Ans: N = [2/√π] $^{0.5}$, N = [1/a√ π] $^{0.5}$

21. According to the corresponding principle, quantum theory should give the same result as classical physics in the limits of large quantum numbers. Show that as $n \to \infty$, the probability of finding the trapped particle between x and $x + \Delta x$ is $\Delta x/L$. Also show that it is independent of x, which is the classical expectation.

22. Find the total number of maxima and minima (in probability) for a particle in a box having boundary between 0 and L for $n = 2$ state.
Ans: between 0 and L there are 2 maxima and 1 minimum (excluding boundary points).

23. (a) Calculate the lowest energy of an electron confined in a cubical box of each side 1Å.
(b) Find the temperature at which the average energy of the molecules of a perfect gas would be equal to the lowest energy of the electron, k_B = 1.38×10^{-23} J/K. **Ans: 6.02 ×10^{-18} J, 2.9×10^{5} K**

24. An electron is confined in a 1D infinite potential box of boundary between 0 and 2 nm. If the particle has 6 nodes find the particle energy in eV. **Ans: 4.57 eV**

4. SUBJECTIVE QUESTIONS OF WAVE MECHANICS
CIT PHYSICS (PHYS-105) 2016-17

Q1: Differentiate Quantum Mechanics from Classical Mechanics?

Q2: What is meant by wave-particle duality? What is its significance?

Q3: A frequency of 2.4 × 10^{15} Hz is used on magnesium with work function of 3.7 eV.

(a) What is energy transferred by each photon?
(b) Calculate the maximum KE of the ejected electrons.
(c) The maximum speed of the electrons.
(d) The stopping potential for the electrons.

Q4: Shows that in Compton Scattering, the recoil angle of an electron is given by $\phi = \tan^{-1}\left(\frac{\sin\theta}{\frac{\lambda'}{\lambda} - \cos\theta}\right)$, where θ and ϕ are scattering angle of scattered photon and electron respectively.

Q5: Show that Pair production can’t occur in empty space.

Q6: Establish the relation $v_g \times v_p = c^2$ for a relativistically moving microscopic particle, were v_g and v_p are group and phase velocity respectively.

Q7: Find the de-Broglie wavelength of an electron of energy V eV.

Q8: Show that a photon and electron of the same momentum have the same wavelength. Compare their wavelengths if the two have same energy.

Q9: How does the Bohr's orbit violate the uncertainty relation? Explain.

Q10: Find the expectation value $\langle p_x \rangle$ of the momentum of a particle in a 1-D box.

Q11: Find the expectation value $\langle x \rangle$ of the position of a particle trapped in a box L wide.

Q12: How the interaction of EM waves varies with the change in atomic size?

Q13: A beam of 50 keV electrons is directed at a crystal and diffracted electrons are found at an angle of 50^{o} relative to the original beam. What is the spacing of the atomic planes of the crystal (assume 1st order diffraction)? A relativistic calculation is needed for λ.

Q14: 1mW of light of wavelength 4560Å is incident on a Caesium (Cs) surface. Calculate the photo-electric current liberated, assuming quantum efficiency of 0.5%. Given: work function of Caesium (Cs) = 1.93 eV

Q15: A proton and a α-particle have the same kinetic energy.

(a) How do their speeds compare?
(b) How do their momenta compare?
(c) How do their de-Broglie wavelengths compare? Given $m_\alpha = 4m_p$

Q16: What is the significance of wave function? Derive Schrodinger time dependent and independent wave function.

Q17: At a certain time, the normalized wave function of a particle moving along the x-axis has the form

$$\Psi(x) = x + \beta \text{ for } -\beta < x < 0$$

$$= -x + \beta \text{ for } 0 < x < \beta 0$$

And zero elsewhere. Find the value of β and the probability that the particles position is between $x = {}^{\beta}/_{2}$ and $x = \beta$.