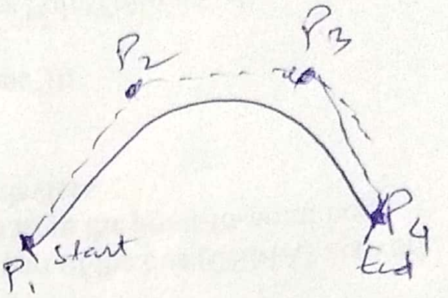
Bezier SPline Curve-

Degree of Polynomial $\xrightarrow{\text{depends}}$ on Control Points.

Control Points = 5 Then Polynomial order = 5-1=4 ⇒ x^4



$$Q(u) = \sum_{i=0}^{n} P_i * B_{i,n}$$

where P_i = Position Vector

n = Total Control Points − 1

$B_{i,n}$ = Barnstein Bezier Function also called Basis funⁿ.

$\Downarrow$

$B_{i,n}(u)$

For x-axis:-

$$X(u) = \sum_{i=0}^{n} x_i * B_{i,n}(u)$$

Now, we are defining Control Points = 4

$$\therefore Q(u) = P_0 * B_{0,3}(u) + P_1 * B_{1,3}(u) + P_2 * B_{2,3}(u) + P_3 * B_{3,3}(u)$$

$$\text{Now, } Q(u) = P_0 * (1-u)^3 + P_1 * 3 * u * (1-u)^2 + P_2 * 3 * u^2 * (1-u) + P_3 * u^3$$

We can convert this equⁿ into x, y, z form.

$$\text{So, } x(u) = (1-u)^3 * x_0 + 3 * u * (1-u)^2 * x_1 + 3 * u^2 * (1-u) * x_2 + u^3 * x_3$$

$y(u) = \ldots$

$z(u) = \ldots$

$$Bez_{i,n}(u) = {}^{n}C_i \cdot u^i \cdot (1-u)^{n-i}; \quad 0 \le u \le 1$$

where,

$${}^{n}C_i = \frac{\lfloor n}{\lfloor i \, \lfloor n-i} = \text{Binomial co-efficient}$$

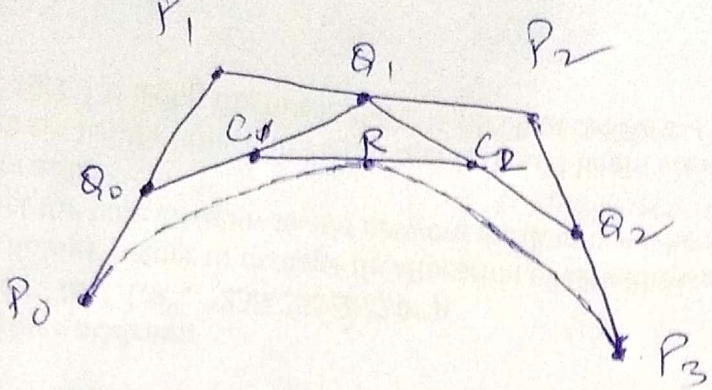
$$\text{Now, } B_{0,3} = \frac{\lfloor 3}{\lfloor 0 \, \lfloor 3} \cdot u^0 \cdot (1-u)^{3-0} = 1 \cdot 1 \cdot (1-u)^3 = (1-u)^3$$

$B_{1,3}(u) = 3 \cdot u \cdot (1-u)^2$

$B_{2,3}(u) = 3 \cdot u^2 \cdot (1-u)$

$B_{3,3}(u) = u^3$

$$\left.\begin{array}{l} B_{0,3}(u) \\ B_{1,3}(u) \\ B_{2,3}(u) \\ B_{3,3}(u) \end{array}\right\} \text{Blending functions.}$$



We can define ~~the~~ each line using parametric equation:

so,

$Q_0 = (1-u)P_0 + u.P_1$ — (1)

$Q_1 = (1-u).P_1 + u.P_2$ — (2)

$Q_2 = (1-u).P_2 + u.P_3$ — (3)

Now, in same way;

$C_1 = (1-u).Q_0 + u.Q_1$ — (4)

$C_2 = (1-u)Q_1 + u.Q_2$ — (5)

Now. $R = (1-u).C_1 + u.C_2$ — (6)

Now, put the value of C_1 & C_2 in equ^n (6)

$\therefore R = (1-u)[(1-u).Q_0 + u.Q_1] + u[(1-u).Q_1 + u.Q_2]$ — (7)

Now, put the value of Q_0, Q_1 & Q_2 in equ^n (7)

$$\therefore R = (1-u)\Big[\{(1-u)[(1-u)P_0 + u.P_1] + u[(1-u)P_1 + u.P_2]\}$$
$$+ \{u[(1-u)[(1-u)P_1 + u.P_2] + u[(1-u)P_2 + u.P_3]]\}\Big]$$