PROBLEM-1 **ROHAN DEEPAK PASPALLU**
Topic: $B(x, y)$ **40093648**

1 Introduction

$B(x, y)$ denotes the beta function also known as the Euler's beta function. Beta function is a function which is defined for the values defined in a certain specific limits of a function. The formula of $B(x, y)$ is:

- $B(x, y) = \frac{(x-1)!(y-1)!}{(x+y-1)!}$ **For positive integers**[1]
- $B(x, y) = \int_0^1 \frac{t^{x-1}}{(1-t)^{y-1}} dt$ **For positive real numbers**

2 Properties of Beta Function

- Beta function is symmetric: $B(x, y) = B(y, x)$
- Beta function in terms of Gamma functions as: $B(x, y) = \frac{\Gamma x \Gamma y}{\Gamma(x+y)}$
- When x and y are postitive then it follows the form of gamma function.
- There can me multiple parameters in the beta function (i.e. not necessarily x and y).

3 Domain and Co-Domain

- The domain of the Beta function depends on the limits of the integral function, having a higher limit as well as a lower limit during which the required output of a given function can be obtained.
- The co-domain of a function depends on the domains. Here we have to manipulate the co-domains in the predefined form by solving the given problem and converting it into a beta function which can be executed only in some particular domain. Various examples of co-domains are:
  - $B(x, y) = \int_0^{\frac{\pi}{2}} (\sin\theta)^{2x-1} (\cos\theta)^{2x-1}$
  - $B(x, y) = \int_0^{\infty} \frac{t^{x-1}}{(1+t)^{x+y}} dt$

References

[1] https://en.wikipedia.org/wiki/Beta_function