PROBLEM-1 **ROHAN DEEPAK PASPALLU**
Topic: $B(x, y)$ **40093648**

Introduction

$B(x, y)$ denotes the beta function also known as the Euler's beta function. Beta function is a function which is defined for the values defined in a certain specific limits of a function. The formula of $B(x, y)$ is:

- $B(x, y) = \frac{(x-1)!(y-1)!}{(x+y-1)!}$ **For positive integers**[?]
- $B(x, y) = \int_0^1 \frac{t^{x-1}}{(1-t)^{y-1}} dt$ **For positive real numbers**

Properties of Beta Function

- Beta function is symmetric: $B(x, y) = B(y, x)$
- Beta function in terms of Gamma functions as: $B(x, y) = \frac{\Gamma x \Gamma y}{\Gamma(x+y)}$
- When x and y are postitive then it follows the form of gamma function.
- There can me multiple parameters in the beta function (i.e. not necessarily x and y).

Domain and Co-Domain

- The domain of the Beta function depends on the limits of the integral function, having a higher limit as well as a lower limit during which the required output of a given function can be obtained.
- The co-domain of a function depends on the domains. Here we have to manipulate the co-domains in the predefined form by solving the given problem and converting it into a beta function which can be executed only in some particular domain. Various examples of co-domains are:
  - $B(x, y) = \int_0^{\frac{\pi}{2}} (\sin\theta)^{2x-1} (\cos\theta)^{2x-1}$
  - $B(x, y) = \int_0^{\infty} \frac{t^{x-1}}{(1+t)^{x+y}} dt$

PROBLEM-2 **ROHAN DEEPAK PASPALLU**
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Requirements

1. **First Requirement**
   - **ID =** FR1
   - **Type =** Functional Requirements
   - **Version =** 1.0
   - **Difficulty =** Easy
   - **Description =** The function requires two variable inputs x and y to perform the functionality on them.
   - **Rationale =** x and y
2. **Second Requirement**
   - **ID =** FR2
   - **Type =** Functional Requirements
   - **Version =** 1.0
   - **Difficulty =** Easy
   - **Description =** The function requires the two variables to have a domain of R+ (i.e. positive real numbers).
   - **Rationale =** $x >= 0$ and $y >= 0$
3. **Third Requirement**
   - **ID =** FR3
   - **Type =** Functional Requirements
   - **Version =** 1.0
   - **Difficulty =** Easy
   - **Description =** The output of the requirement is also in R+
   - **Rationale =** $B(x, y) >= 0$

4. **Fourth Requirement**
   - **ID =** FR4
   - **Type =** Functional Requirements
   - **Version =** 1.0
   - **Difficulty =** Easy
   - **Description =** If the values are positive integers Z+ (including zero) the values do not have any issue. As, we can use gamma function to calculate B(x,y).
   - **Rationale =** $\{\forall x, y \in Z^+ \mid B(x,y){=}\frac{\Gamma x \Gamma y}{\Gamma(x+y)}\ \}$
5. **Fifth Requirement**
   - **ID =** FR5
   - **Type =** Functional Requirements
   - **Version =** 1.0
   - **Difficulty =** Difficult
   - **Description =** But, if the values are real numbers then we have to perform integration according to the specific function which needs to be mentioned to calculate that. Here, there is a compulsion for a function to be present.
   - **Rationale =** $\{\forall x, y \in R^+ \mid B(x,y){=}\int_0^1 \frac{t^{x-1}}{(1-t)^{y-1}} dt\}$
6. **Sixth Requirement**
   - **ID =** FR6
   - **Type =** Functional Requirements
   - **Version =** 1.0
   - **Difficulty =** Easy
   - **Description =** The range of positive real variables is declared between the range of 0 to 1.
   - **Rationale =** $\{\forall x, y \in [0,1] \mid B(x,y){=}\int_0^1 \frac{t^{x-1}}{(1-t)^{y-1}} dt\ \}$

7. **Seventh Requirement**
   - **ID =** FR7
   - **Type =** Functional Requirements
   - **Version =** 1.0
   - **Difficulty =** Difficult
   - **Description =** If we want to include negative values we can do that by including Image of a specific negative number.
   - **Rationale =** $x < 0$ and $y < 0$
8. **Eighth Requirement**
   - **ID =** FR8
   - **Type =** Functional Requirements
   - **Version =** 1.0
   - **Difficulty =** Easy
   - **Description =** There should be no other inputs other than the numeric values.
   - **Rationale =** $x, y \in R^+$
9. **Ninth Requirement**
   - **ID =** FR9
   - **Type =** Functional Requirements
   - **Version =** 1.0
   - **Difficulty =** Easy
   - **Description =** The two input variables can have similar as well as distinct values.
   - **Rationale =** $x = y$ or $x \neq y$

Assumptions

1. x and y are positive real numbers $x, y \in R^+$
2. For $x, y \in Z^+$ its easier to compute $B(x, y)$
3. There is no requirement for functions to calculate $B(x, y)$.

Algorithm-1

Stirling's approximation (or Stirling's formula) is an approximation for factorials. It is a good approximation, leading to accurate results even for small values of n.
The sterling's approximation equation is represented as:

$$B(x,y) = \frac{\Gamma x \Gamma y}{\Gamma(x+y)}$$

$$\Gamma x = \sqrt{\frac{2\pi}{x}}\left(\frac{x}{e}\right)^x$$

Advantages:

- The algorithm has a domain for all the positive real numbers.
- The algorithm can compute most of the values available.
- The algorithm acts as an approximation for the integration function.
- Reduces the complexity of an integration code.

Disadvantages:

- The algorithm cannot give accurate results.
- Though the complexity is reduced, but its much more complex compared to the later algorithm.
- Debugging the code is a bit difficult.
- For smaller values the difference between the actual answer and the required answer is quite different. But, as the size of numbers increase the difference becomes less.

Algorithm 1 Calculate Beta Function using Simpson's approximation

Require: value: $x > 0$ & $y > 0$ ▷ where $x, y \in \mathcal{R}^+$
Ensure: $result = Beta(x, y)$

procedure CalculateSquareRoot($value$)
    $squareroot \leftarrow math.sqrt(value)$
    return $squareroot$ ▷ It returns the squareroot
end procedure

procedure CalculatePower($value1$,$value2$)
    $power \leftarrow math.power(value1, value2)$
    return $power$ ▷ It returns the base to the power
end procedure

procedure CalculateGamma($value$)
    $value2 \leftarrow (\frac{value}{e})^{value}$
    $gamma \leftarrow$ CalculaeSquareRoot($value$)CalculatePower($value, value2$)
    return $gamma$ ▷ It returns the gamma value
end procedure

procedure CalculateBeta(x, y)
    $value1 \leftarrow$ CalculateGamma(x)
    $value2 \leftarrow$ CalculateGamma(y)
    $z \leftarrow x + y$
    $value3 \leftarrow$ CalculateGamma(z)
    $beta \leftarrow \frac{value1 * value2}{value3}$
    return $beta$ ▷ It returns the beta value
end procedure

$result \leftarrow$ CalculateBeta(x, y) ▷ Final result of $Beta(x, y)$

PROBLEM-3 **ROHAN DEEPAK PASPALLU**
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Algorithm-2

Beta function makes the use of gamma function to perform the computation of the positive integers
$B(x,y) = \frac{\Gamma x \Gamma y}{\Gamma(x+y)}$

$\Gamma x = (x-1)!$
Advantages:

- The algorithm gives the most accurate answers.
- The algorithm is easier to implement and debug.
- The execution takes place faster.
- Performs better functionality for the Beta function

Disadvantages:

- The algorithm can only be implemented for positive numbers.
- The values fed to the algorithm can only be positive integers.
- The algorithm doesn't consider integer values.

Algorithm 2 Calculate Beta Function using factorial

Require: value: $x > 0$ & $y > 0$ ▷ where $x, y \in \mathcal{Z}^+$

Ensure: $result = Beta(x, y)$

procedure CALCULATEFACTORIAL($value$)
    $value2 \leftarrow value - 1$
    if $value2 \neq 1$ **then**
        $fact \leftarrow$ CALCULATEFACTORIAL($value2$)
        return $fact$ ▷ It returns the factorial
    else
        return 1
    end if
end procedure

procedure CALCULATEGAMMA($value$)
    $value2 \leftarrow value - 1$
    $gamma \leftarrow$ CALCULATEFACTORIAL($value2$)
    return $gamma$ ▷ It returns the gamma value
end procedure

procedure CALCULATEBETA(x, y)
    $value1 \leftarrow$ CALCULATEGAMMA(x)
    $value2 \leftarrow$ CALCULATEGAMMA(y)
    $z \leftarrow x + y$
    $value3 \leftarrow$ CALCULATEGAMMA(z)
    $beta \leftarrow \frac{value1 * value2}{value3}$
    return $beta$ ▷ It returns the beta value
end procedure

$result \leftarrow$ CALCULATEBETA(x, y) ▷ Final result of $Beta(x, y)$

Bibliography

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