ASTRO 650 - Galactic Structure and Evolution: Problem Set 4
Due Thursday, April 28

1. **Stellar Population Models:** In this exercise you will become familiar with stellar population synthesis models and how they can be used to derive important galaxy properties such as stellar masses and star formation histories. We will be using a small subset of SSPs from Bruzual & Charlot (2003). The file bc03_models.txt contains 6 SSPs with ages of 1, 10, 100, 1000, 5000, 10000 Myr. The first column of the table contains the wavelength, and the subsequent columns contain the luminosity of the different SSPs. LUM10 corresponds to the 10 Myr SSP, LUM100 to the 100 Myr SSP, etc. The units of WAVE are Angstroms and the units of the LUMs are $L_\odot$ Å^{-1}.

   a) Make a nice color plot of all 6 SSPs. (Be sure that the spectra appear as continuous histograms rather than a bunch of points. Spectra are histograms, after all!) Your plot should be log-scaled on the Y-axis; it should have both axes labeled appropriately with units; and there should be a legend giving the different ages of the SSPs. Indicate the central wavelengths of the u,g,r,i,z filters (see Fukugita et al. 1996, AJ, 111, 1748). [10 pts]

   b) Next we will explore the impact of interstellar dust attenuation on galaxy spectra. We will use the dust attenuation law of Charlot & Fall (2000) which has the following form:

$$\tau(\lambda) = \tau_V \left(\frac{\lambda}{5000\ \text{Å}}\right)^{-0.7}$$

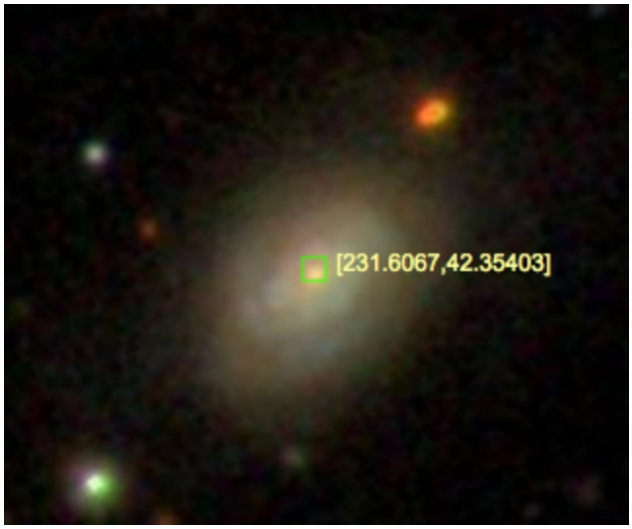
   where λ is the wavelength in Å and τ_V is V-band dust attenuation. Dust attenuation impacts the luminosity of a galaxy as follows:

$$L_{\text{dusty}}(\lambda) = L_0(\lambda)\ \exp(-\tau(\lambda))$$

   where $L_0(\lambda)$ is the galaxy's intrinsic luminosity, and $L_{\text{dusty}}(\lambda)$ is its attenuated luminosity.

   Your task is to make a plot showing the 1 Myr SSP attenuated by dust with the following values of τ_V: 0, 0.5, 1, 2. As in the previous problem, make sure your axes are labeled, you have a legend, etc. Use a linear scale for the Y-axis this time. You may find CFN 4.2.7 helpful for reference. [10 pts]

2. Now you are going to try your hand at comparing a real galaxy spectrum and models. The spatially integrated spectrum of an SDSS-IV MaNGA galaxy is given in the file: "manga_7443-6102.txt". The column labeled WAVE has units of Å and LUM has units of $10^6\ L_\odot$ Å^{-1}. An image of the galaxy is shown below for reference.



a) Generate a spectrum which is a linear combination of Bruzual & Charlot models with dust attenuation. For simplicity we'll use only 4 SSPs (1 Myr, 100 Myr, 1 Gyr, 10 Gyr). Mathematically you should combine the SSPs as follows:

$$L_{\rm model} = (c_1 L_1 + c_2 L_{100} + c_3 L_{1000} + c_4 L_{10\rm Gyr}) \times c_5 \exp\left(-\tau_V \ \left(\frac{\lambda}{5000}\right)^{-0.7}\right)$$

where $c_1 - c_5$ and τ_V are constants that you will determine by experimentation. Vary the constants until you get a good fit. In practice you would use a minimization routine to do this, but do it by hand to get a feel for the degeneracies. Note that the BC03 models do not include nebular emission lines. Your fit should be to the galaxy continuum only (mask out the emission lines as needed.) You will also notice some sky residuals in the red that are not well fit by the models. Once you are happy with your fit, make a plot of the galaxy spectrum scaled so that you can see the galaxy continuum well (the nebular emission lines might be cut off). Then overplot your best fit spectrum and each of the dust-reddened SSPs that make up the fit. Which spectral features did you find most helpful for constraining the relative contributions of the SSPs? Which features were best for constraining the level of dust attenuation? [30 pts]

b) Compute the galaxy's stellar mass from the constants you fit above. (Show the formula that you use.) Compare your answer to the stellar mass of the Milky Way. [10 pts]

3. **The Simple Closed Box Model of Chemical Evolution:**

Use the following definitions to derive the simple "closed box" model of chemical evolution.

Z = the mass fraction of metals in the ISM

R = fraction of stellar mass formed that is returned to the ISM by winds or supernovae

$\alpha = (1 - R)$ = the mass fraction locked up in long-lived remnants

q = the mass in metals returned to the ISM divided by the stellar mass formed in a given star formation event

$y = q/\alpha$ = the stellar yield: the mass in metals returned to the ISM divided by the mass locked up in long-lived remnants

$\Psi(t)$ = the star formation rate

S(t) = $\int_0^t dt' \Psi(t')$ = mass of stars formed at time t

$s(t) = \alpha S(t)$ = the mass remaining in stars at time t

$g(t)$ = the gas mass at time t

$M_b = g(t) + s(t)$ = the total baryonic mass of the system (a constant)

Assume that there are no inflows or outflows and thus, $dg/dt + ds/dt = 0$. Also assume that massive stars explode as supernovae immediately and low mass stars live forever.

a) Begin with the following equation:

$$\frac{d(gZ)}{dt} = \Psi(RZ + q) - Z\Psi$$

The left side of the equation, $d(gZ)/dt$), is the rate of change of the metal mass in the ISM. The right hand term, $\Psi(RZ + q)$, represents the metals returned by newly formed stars. (RZ = metals that existed in the gas the stars were formed from, and q = newly formed metals.) The last term, $-Z\Psi$, represents the metals removed from the ISM to make stars.

Show that this can be rewritten as:

$$\frac{d(gZ)}{dS} = -Z\alpha + q$$

[12 pts]

b) Show that the equation you derived in part a) can be rewritten as:

$$\frac{dZ}{dg} g = -y$$

(Hint, recall that $dg/dt + ds/dt = 0$). [12 pts]

c) Now integrate dZ from zero to $Z(t)$ and integrate the other size of the equation (containing dg) from M_b to $g(t)$. You should arrive at the solution to the closed box model. [6 pts]