UNIT – 2,4 & 5

AC-DC & DC-DC Power Converters fed drives

Syllabus

- 2 – Pulse converter - performance parameters: harmonics, ripple, distortion, power factor
- 3 – Pulse converter - performance parameters: harmonics, ripple, distortion, power factor
- 6 – Pulse converter - performance parameters: harmonics, ripple, distortion, power factor
- Single phase and three phase power converter fed DC motor drive
- Effect of source impedance and overlap
- DC – DC chopper circuit using BJT and IGBT
- Chopper fed drives
- Braking and speed reversal of DC motor drives using choppers
- Zero Voltage and Zero Current switching concepts

References

1. Rashid. M. H., “ Power Electronics – circuits, Devices and Applications”, Prentice Hall India, New Delhi, 2013.

2. Gopal K. Dubey, “Fundamentals of Electrical Drives”, Narosa Publications, 2nd edition, 2002.

3. Simon Ang, “Power-Switching Converters”, CRC Press, 2nd edition, 2005.

Separately Excited DC Motor

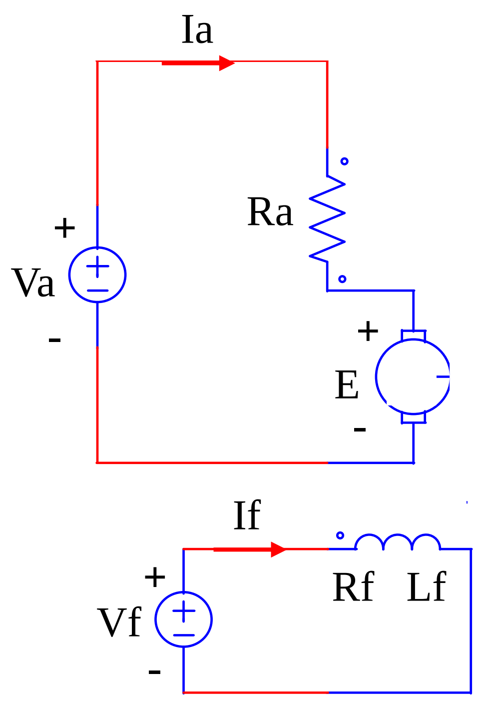
$$K_e\phi = K \text{ (constant)}$$

$$V = E + R_a I_a$$

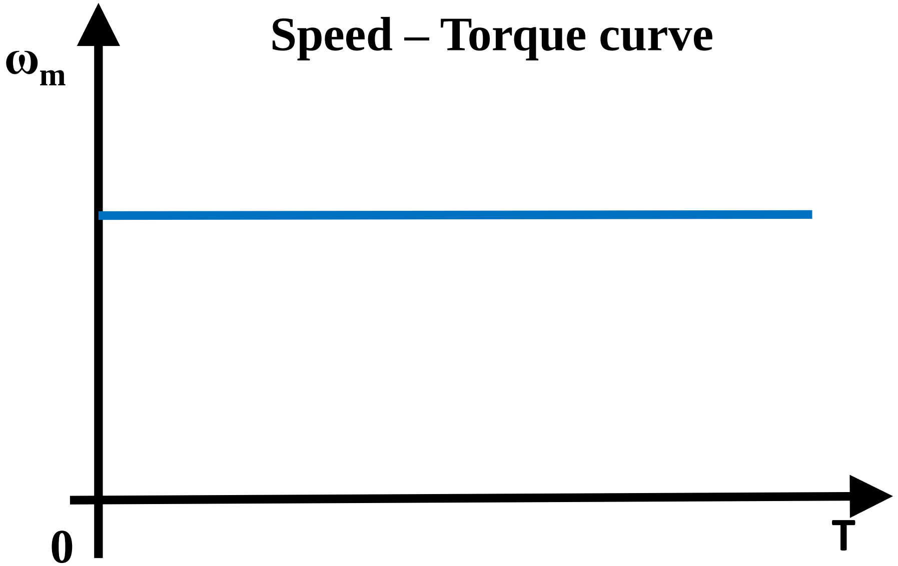
$$T = K_e \phi I_a = K I_a$$

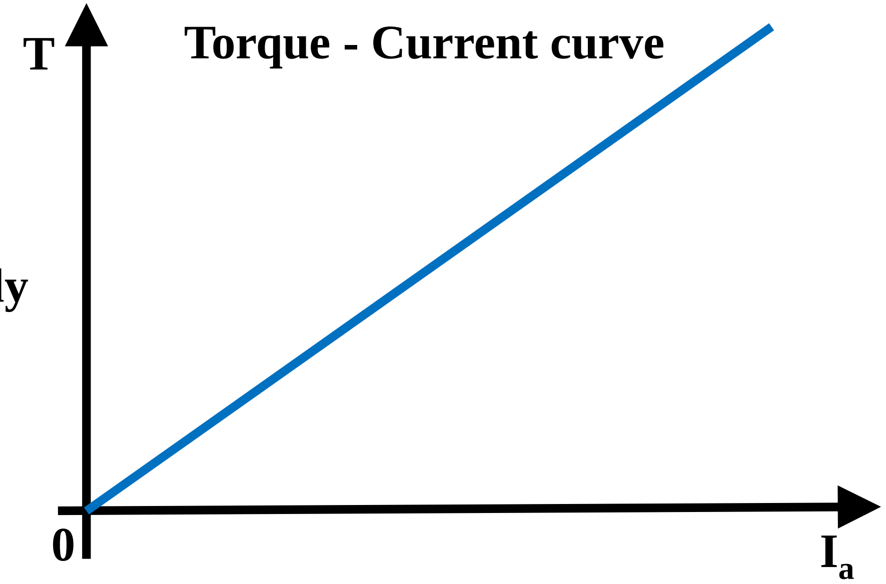
$$E = K\omega_m$$

$$\omega_m = \frac{V}{K} - \frac{R_a}{K} I_a = \frac{V}{K} - \frac{R_a}{K^2} T$$



Equivalent circuit of Separately Excited DC Motor

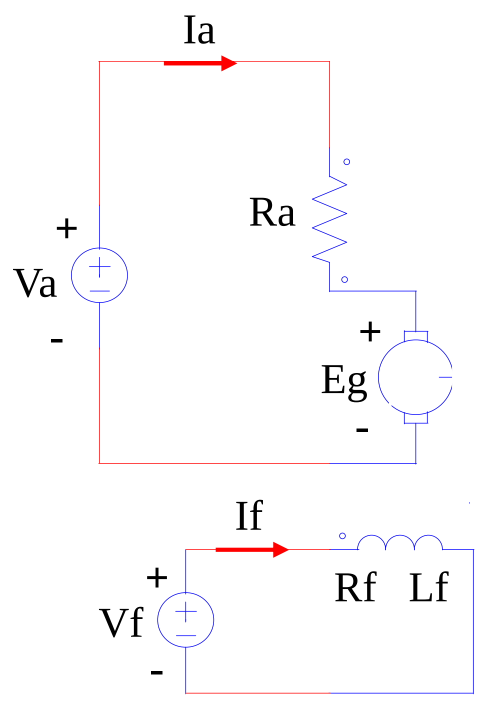




DC Drive Operating Modes

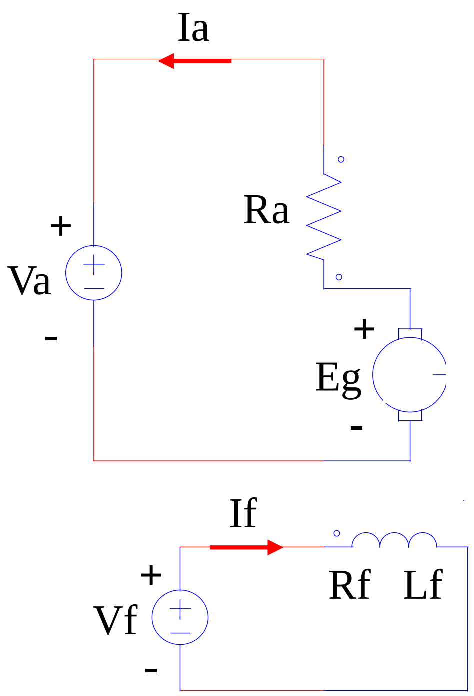
- In variable-speed applications, a dc motor may be operating in one or more modes:
  - motoring,
  - regenerative braking,
  - dynamic braking,
  - plugging, and
  - four quadrants.

Motoring: The arrangements for motoring are shown in Figure. Back emf Eg is less than supply voltage Va. Both armature and field currents are positive. The motor develops torque to meet the load demand.



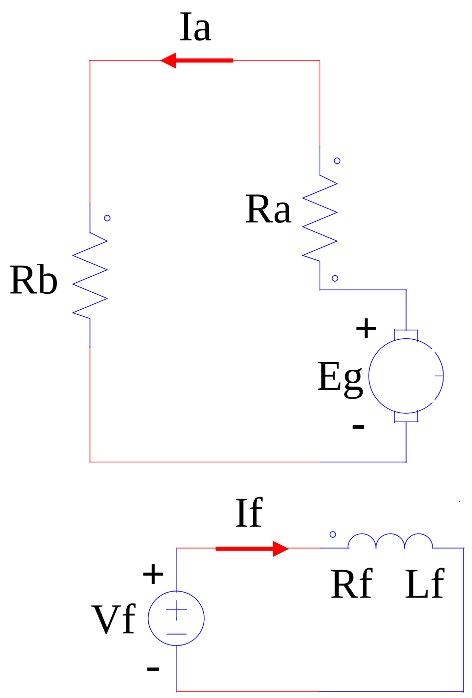
Regenerative braking:

- The arrangements for regenerative braking are shown in Figure.
- The motor acts as a generator and develops an induced voltage E_g. E_g must be greater than supply voltage V_a.
- The armature current is negative, but the field current is positive.
- The kinetic energy of the motor is returned to the supply.



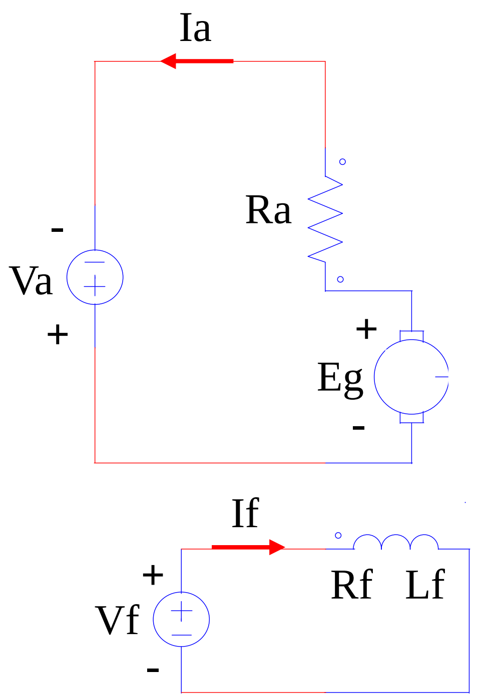
Dynamic braking:

- The arrangements shown in Figure are similar to those of regenerative braking, except the supply voltage V_a is replaced by a braking resistance R_b,.
- The kinetic energy of the motor is dissipated in R_b.



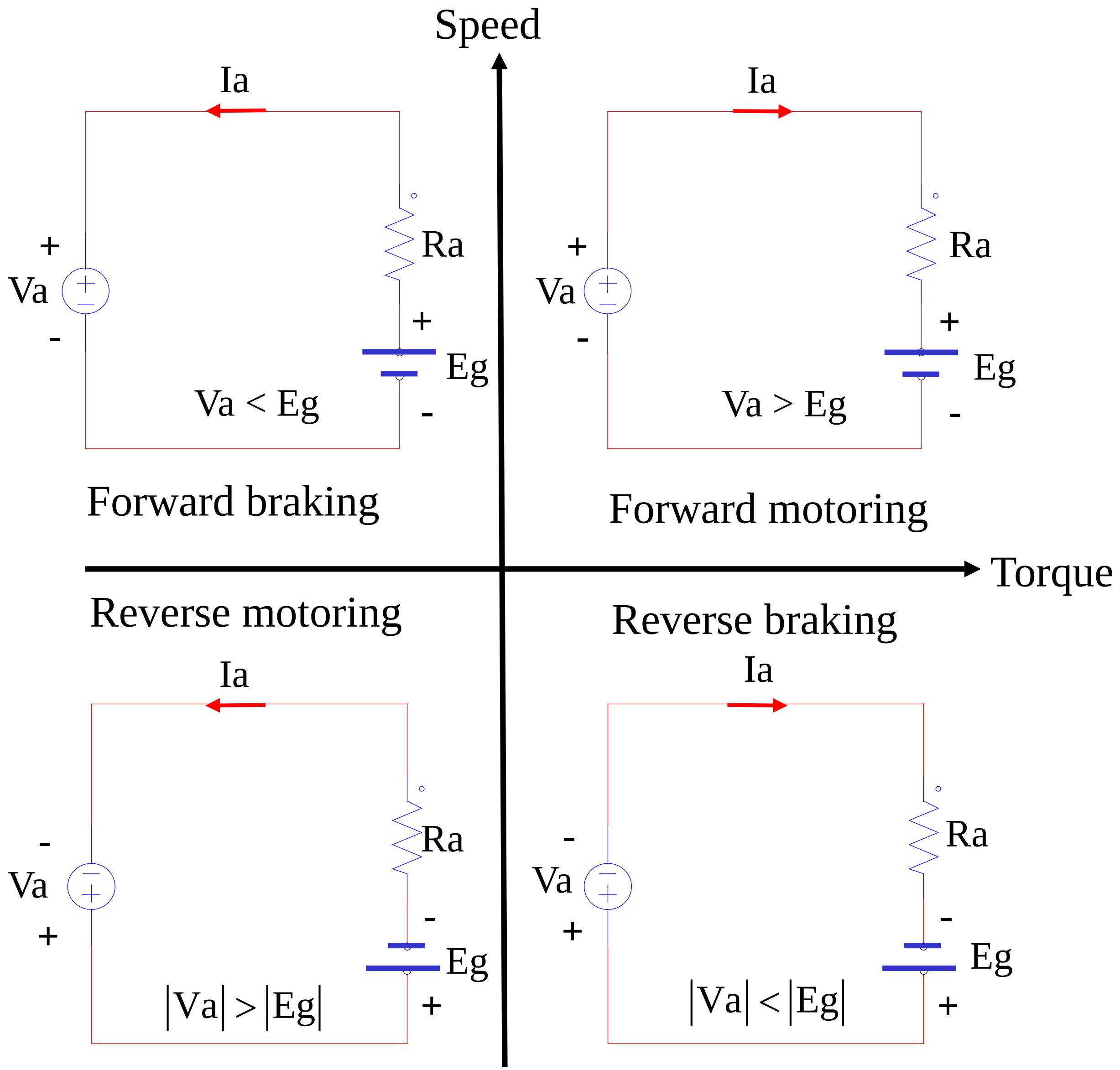
Plugging:

- Plugging is a type of braking. The connections for plugging are shown in Figure.
- The armature terminals are reversed while running. The supply voltage V_a and the induced voltage E_g act in the same direction.
- The armature current is reversed, thereby producing a braking torque. The field current is positive.
- For a series motor, either the armature terminals or field terminals should be reversed, but not both.



Four Quadrants:

- Figure shows the polarities of the supply voltage V_a, back emf E_g, and armature current I_a for a separately excited motor.



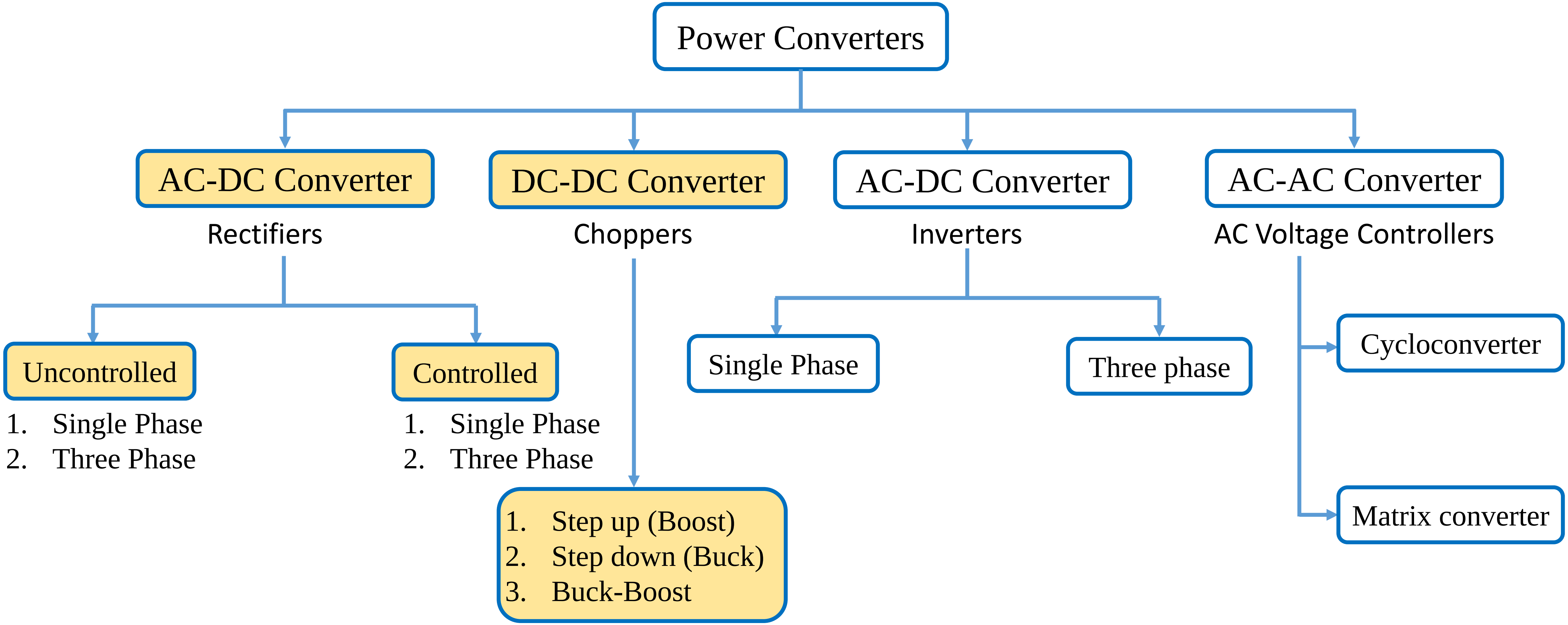
Four Quadrants:

- Figure shows the polarities of the supply voltage V_a, back emf E_g, and armature current I_a for a separately excited motor.
- In forward motoring (quadrant I), V_a, E_g, and I_a are all positive. The torque and speed are also positive in this quadrant.

- During forward braking (quadrant II), the motor runs in the forward direction and the induced emf E_g continues to be positive. For the torque to be negative and the direction of energy flow to reverse, the armature current must be negative. The supply voltage V_a should be kept less than E_g.

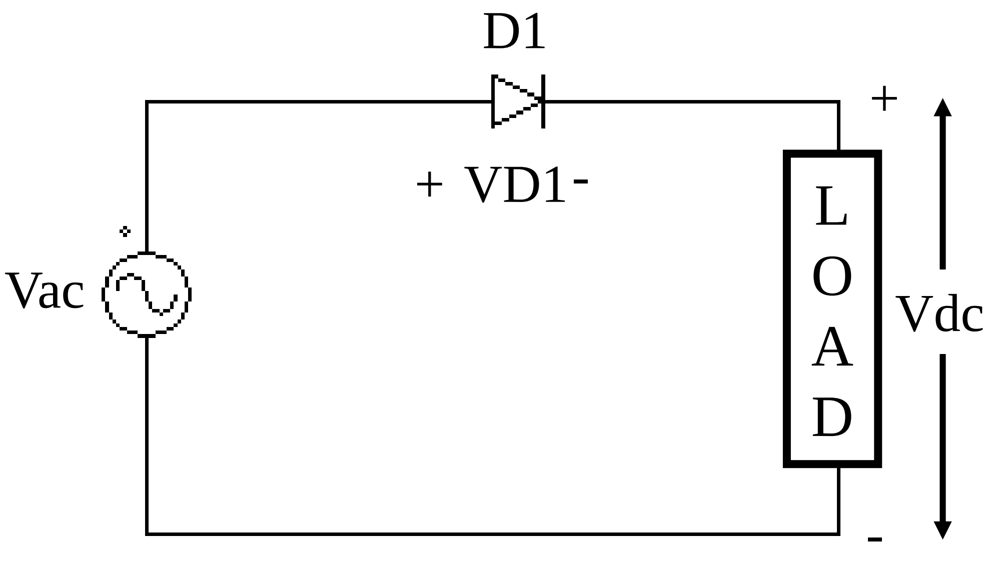
- In reverse motoring (quadrant III), V_a, E_g, and I_a are all negative. The torque and speed are also negative in this quadrant. To keep the torque negative and the energy flow from the source to the motor, the back emf E_g must satisfy the condition $| V_a | > | E_g |$. The polarity of E_g can be reversed by changing the direction of field current or by reversing the armature terminals.

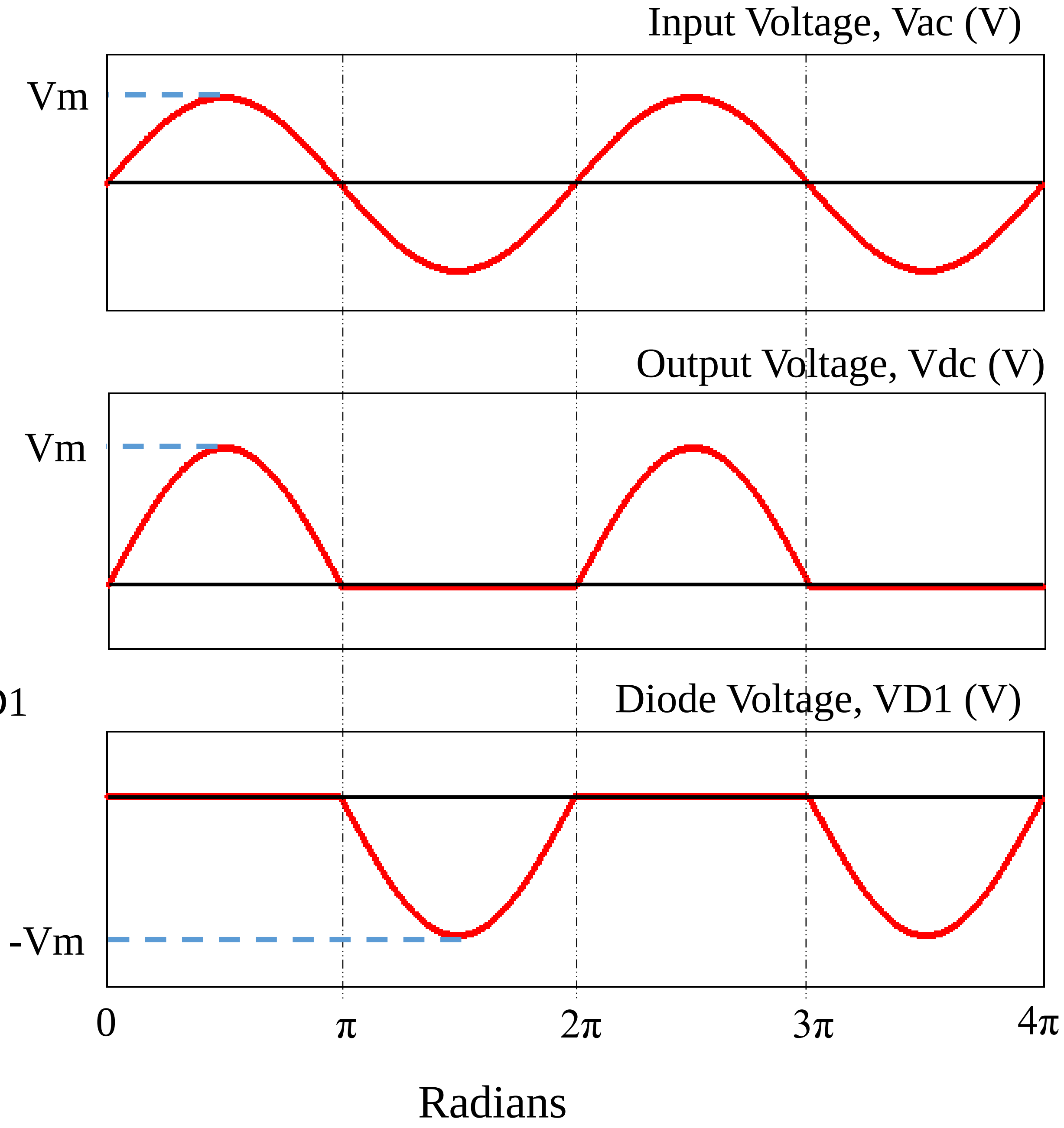
- During reverse braking (quadrant IV), the motor runs in the reverse direction. V_a, and E_g continue to be negative. For the torque to be positive and the energy to flow from the motor to the source, the armature current must be positive. The induced emf Eg must satisfy the condition $| V_a | < | E_g |$.


Power Converters
AC-DC Converter
Rectifiers
DC-DC Converter
Choppers
AC-DC Converter
Inverters
AC-AC Converter
AC Voltage Controllers
Uncontrolled
1. Single Phase
2. Three Phase
Controlled
1. Single Phase
2. Three Phase
1. Step up (Boost)
2. Step down (Buck)
3. Buck-Boost
Single Phase
Three phase
Cycloconverter
Matrix converter

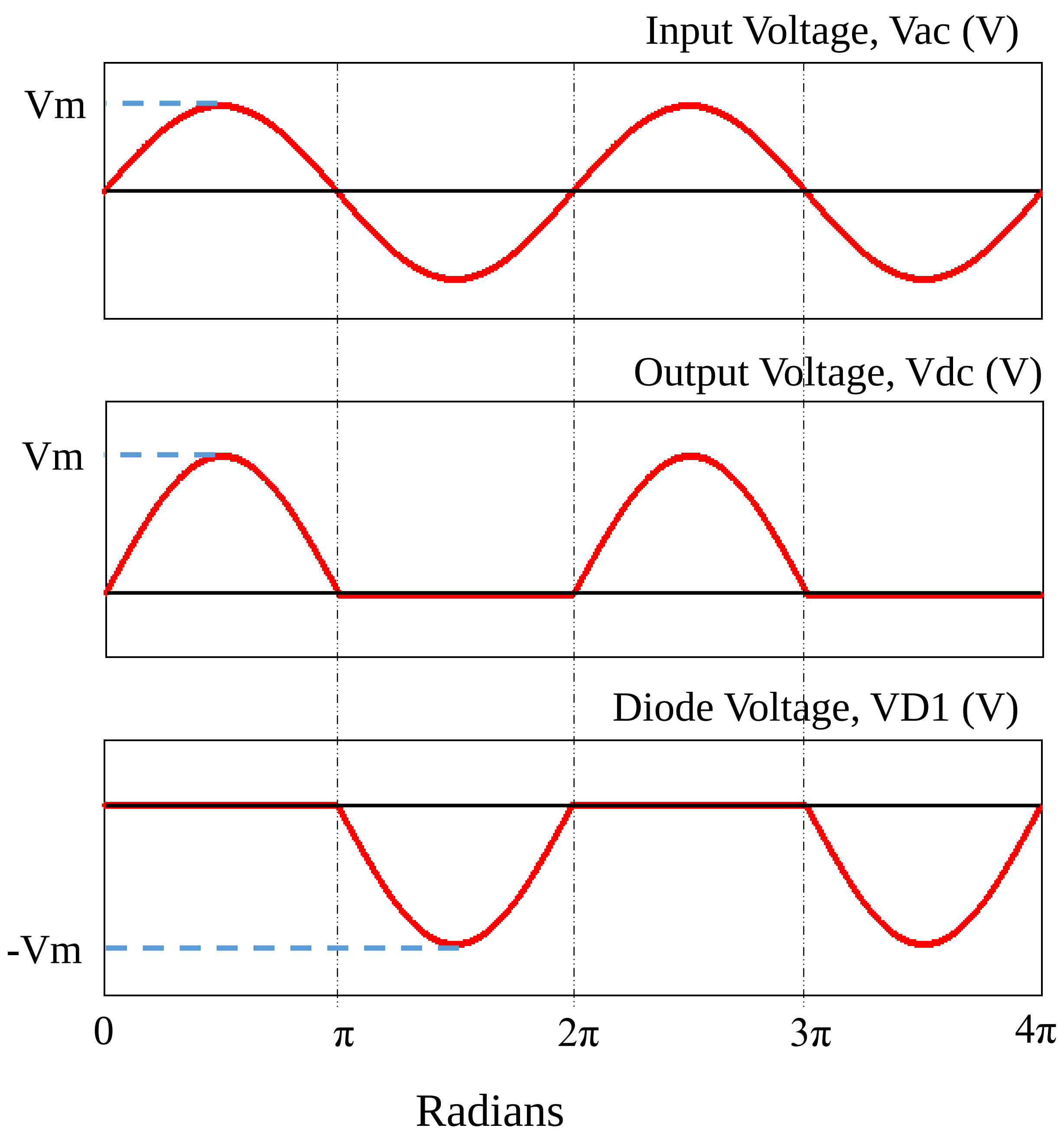
AC-DC Converters

Single phase half wave uncontrolled rectifier





- During positive cycle of the input voltage, Vac, diode,D1 conducts.
- During negative cycle of the input voltage, Vac, diode D1 is in blocking condition.



Average DC output voltage, Vdc = $\frac{1}{2\pi}\int_0^{\pi}\left(V_m \sin\omega t\right)d\omega t$

$$= \frac{V_m}{2\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$\boxed{\text{Vdc} = \frac{V_m}{\pi}}$$

RMS DC output voltage, Vrms = $\sqrt{\left[\frac{1}{2\pi}\int_0^{\pi}\left(V_m \sin\omega t\right)^2 d\omega t\right]}$

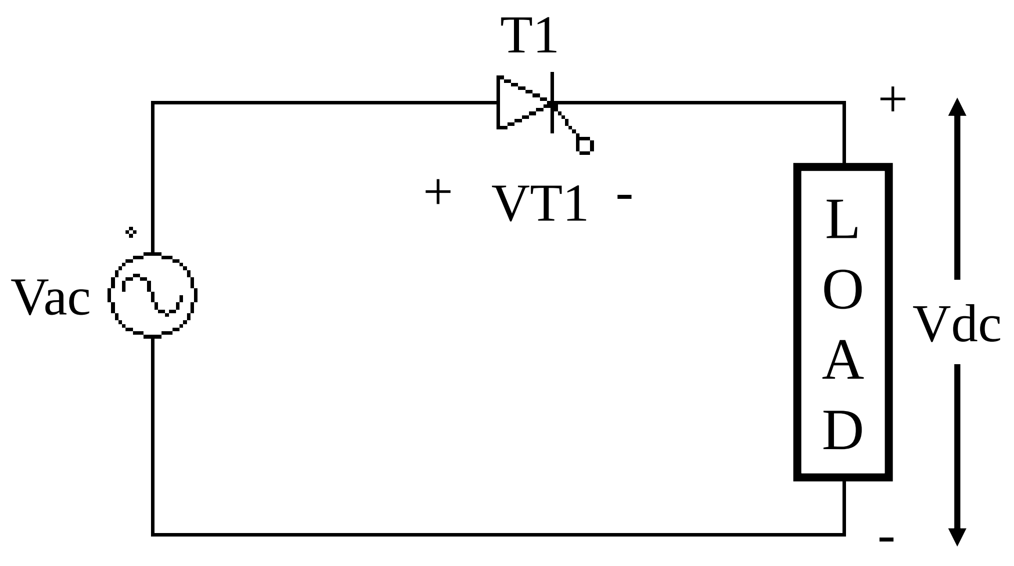
$$= \sqrt{\left[\frac{V_m^2}{2\pi}\int_0^{\pi}\sin^2\omega t\, d\omega t\right]}$$

$$= \sqrt{\left[\frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t\right]}$$

$$= \sqrt{\left[\frac{V_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}\right]}$$

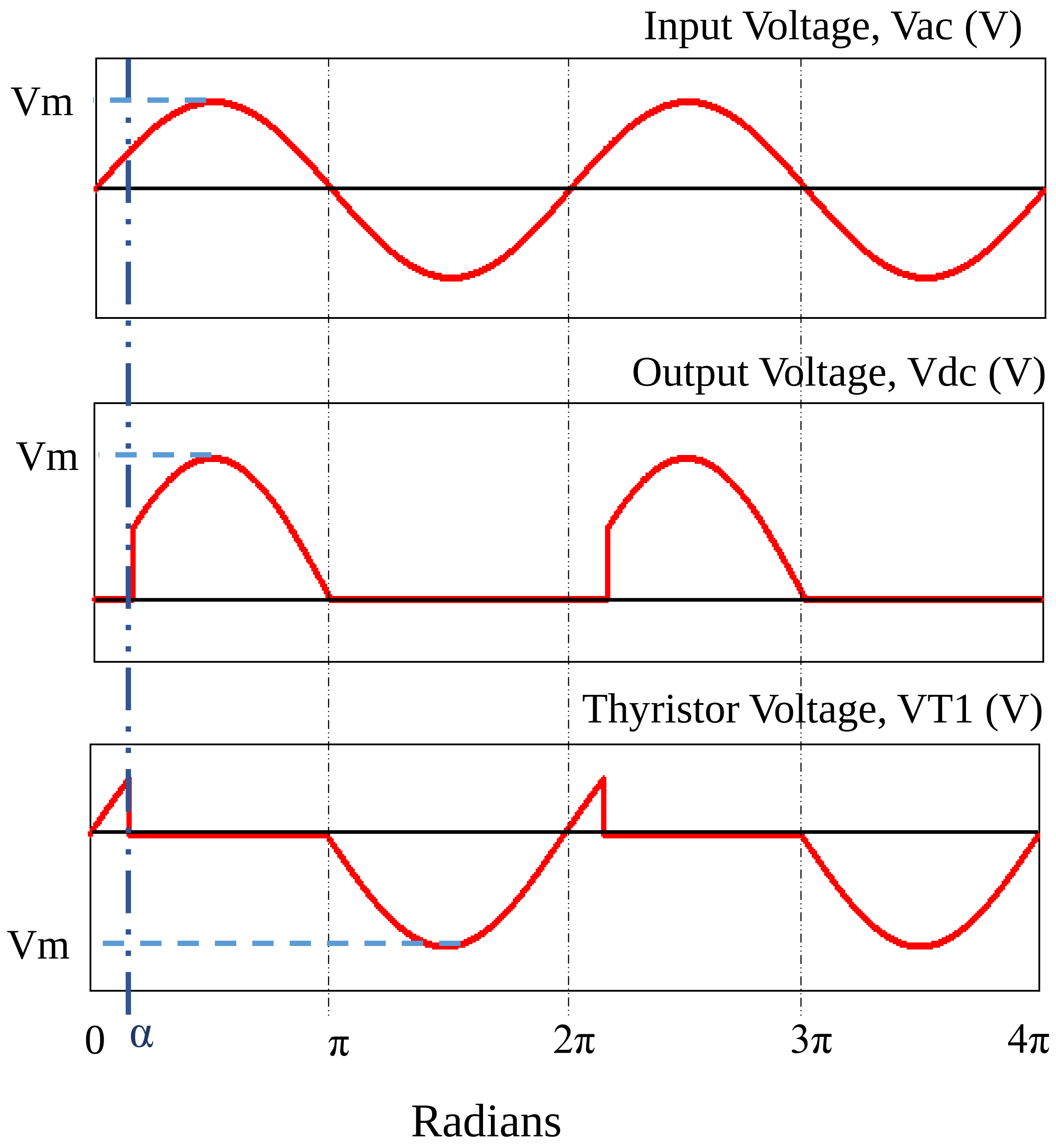
$$\boxed{\text{Vrms} = \sqrt{\left[\frac{V_m^2}{4\pi}[\pi]\right]} = \frac{V_m}{2}}$$

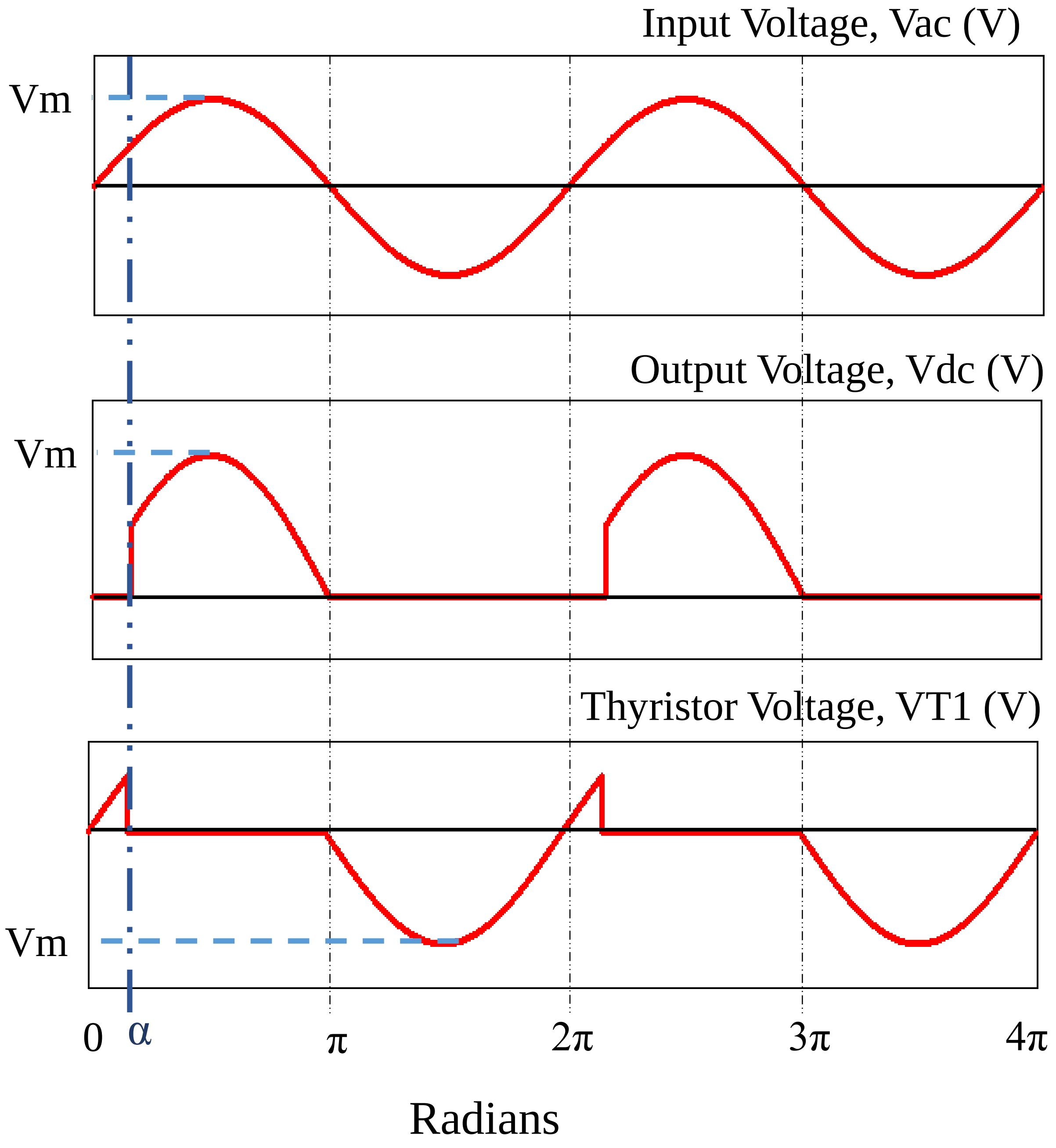
Single phase half wave controlled rectifier



- Until the thyristor,T1 is triggered at an angle α, T1 remains in blocking state in the positive cycle of the input voltage, Vac.

- Thyristor is unidirectional device. Thus, it block the negative cycle of the input voltage, Vac,.





Average DC output voltage, Vdc $= \dfrac{1}{2\pi}\int_{\alpha}^{\pi}(V_m \sin\omega t)\,d\omega t$

$$= \frac{V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$\boxed{\text{Vdc} = \frac{V_m}{2\pi}(1+\cos\alpha)}$$

RMS DC output voltage, Vrms $= \sqrt{\left[\dfrac{1}{2\pi}\int_{\alpha}^{\pi}(V_m \sin\omega t)^2\,d\omega t\right]}$

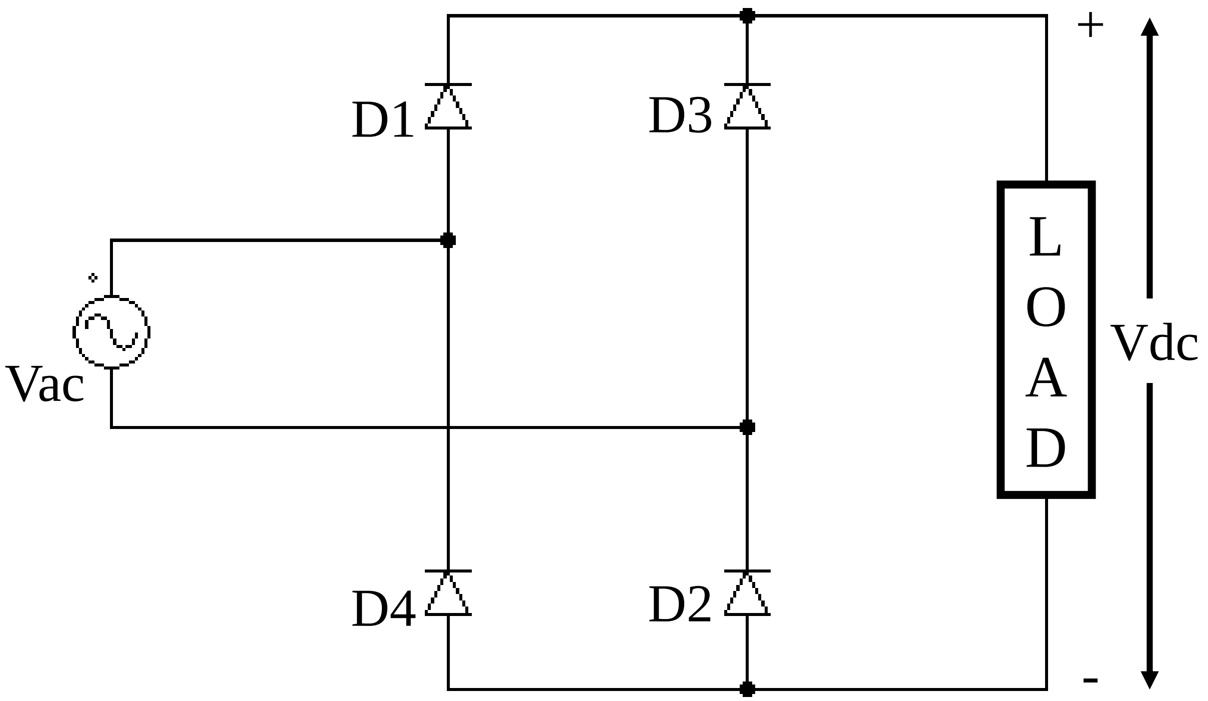
$$= \sqrt{\left[\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi}\sin^2\omega t\,d\omega t\right]}$$

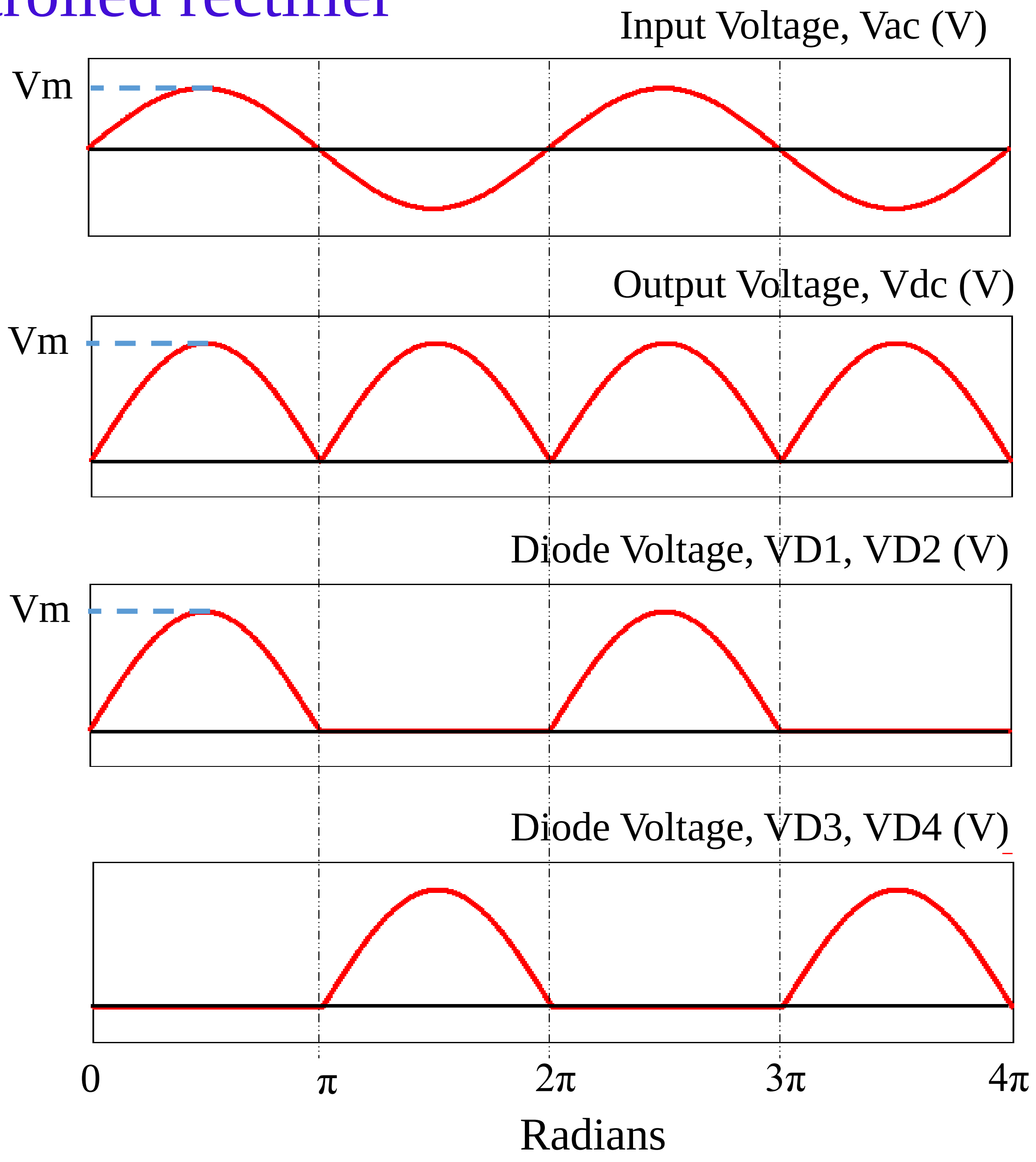
$$= \sqrt{\left[\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t\right]}$$

$$= \sqrt{\left[\frac{V_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi}\right]}$$

$$\boxed{\text{Vrms} = \frac{V_m^2}{2\sqrt{\pi}}\sqrt{\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}}$$

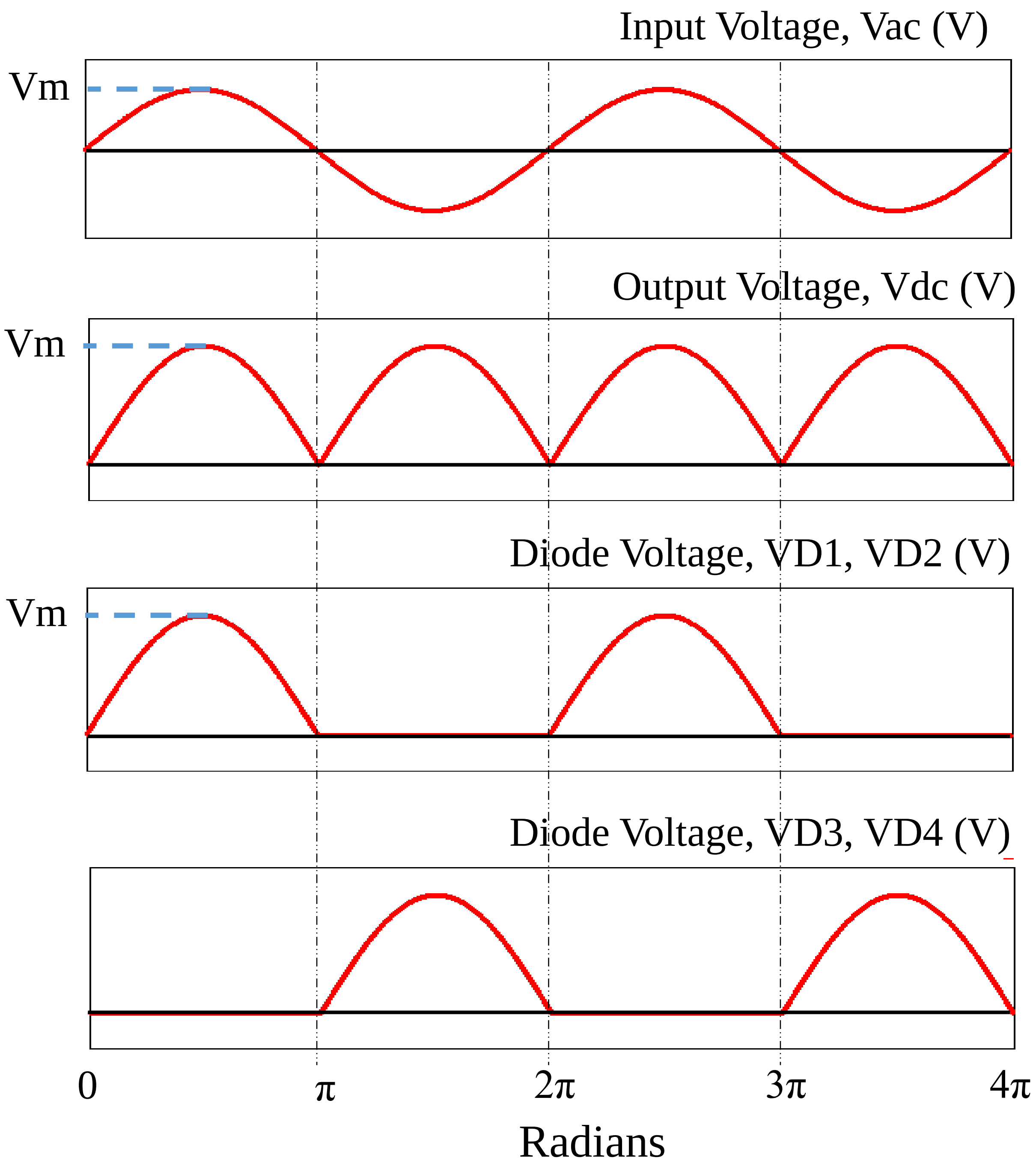
Single phase Full wave uncontrolled rectifier





- During positive cycle of the input voltage, Vac, diodes, D1, D2 conducts, and diodes D3, D4 is in blocking condition.

- During negative cycle of the input voltage, Vac, diodes, D3, D4 conducts, and diodes D1, D2 is in blocking condition.



Average DC output voltage, Vdc = $\frac{1}{2\pi}\int_0^{\pi}\left(V_m \sin\omega t\right)d\omega t$

$$= \frac{V_m}{2\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$\boxed{Vdc = \frac{V_m}{\pi}}$$

RMS DC output voltage, Vrms = $\sqrt{\left[\frac{1}{2\pi}\int_0^{\pi}\left(V_m \sin\omega t\right)^2 d\omega t\right]}$

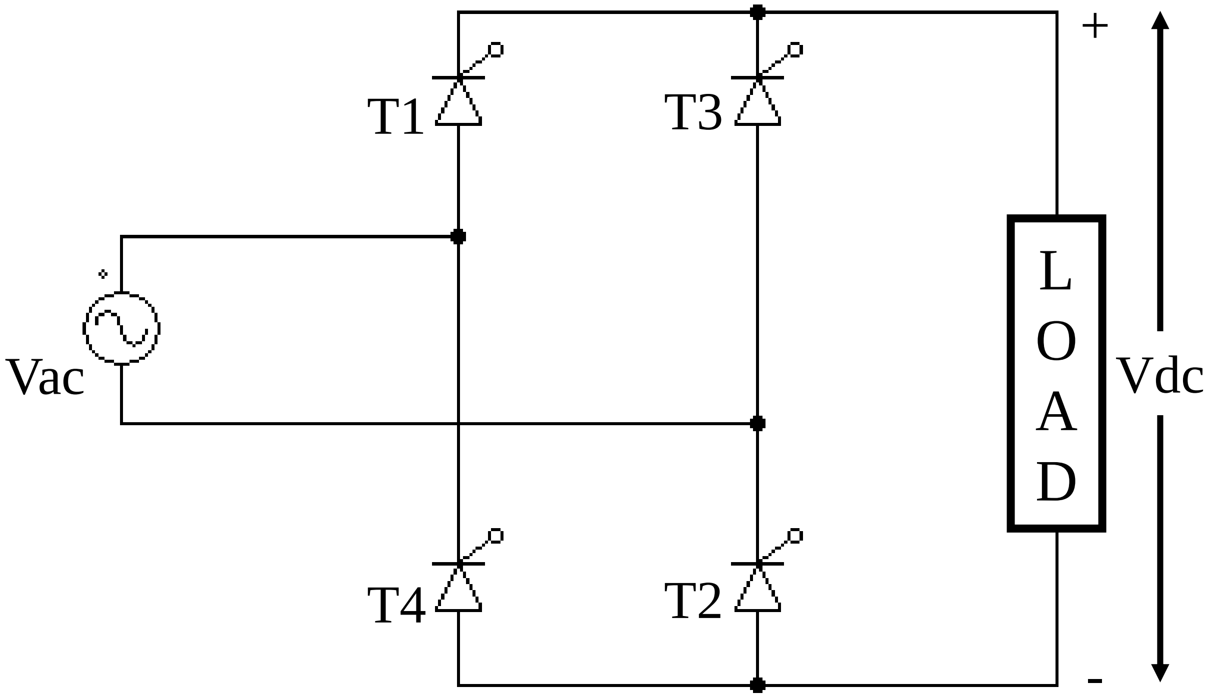
$$= \sqrt{\left[\frac{V_m^2}{2\pi}\int_0^{\pi}\sin^2\omega t\, d\omega t\right]}$$

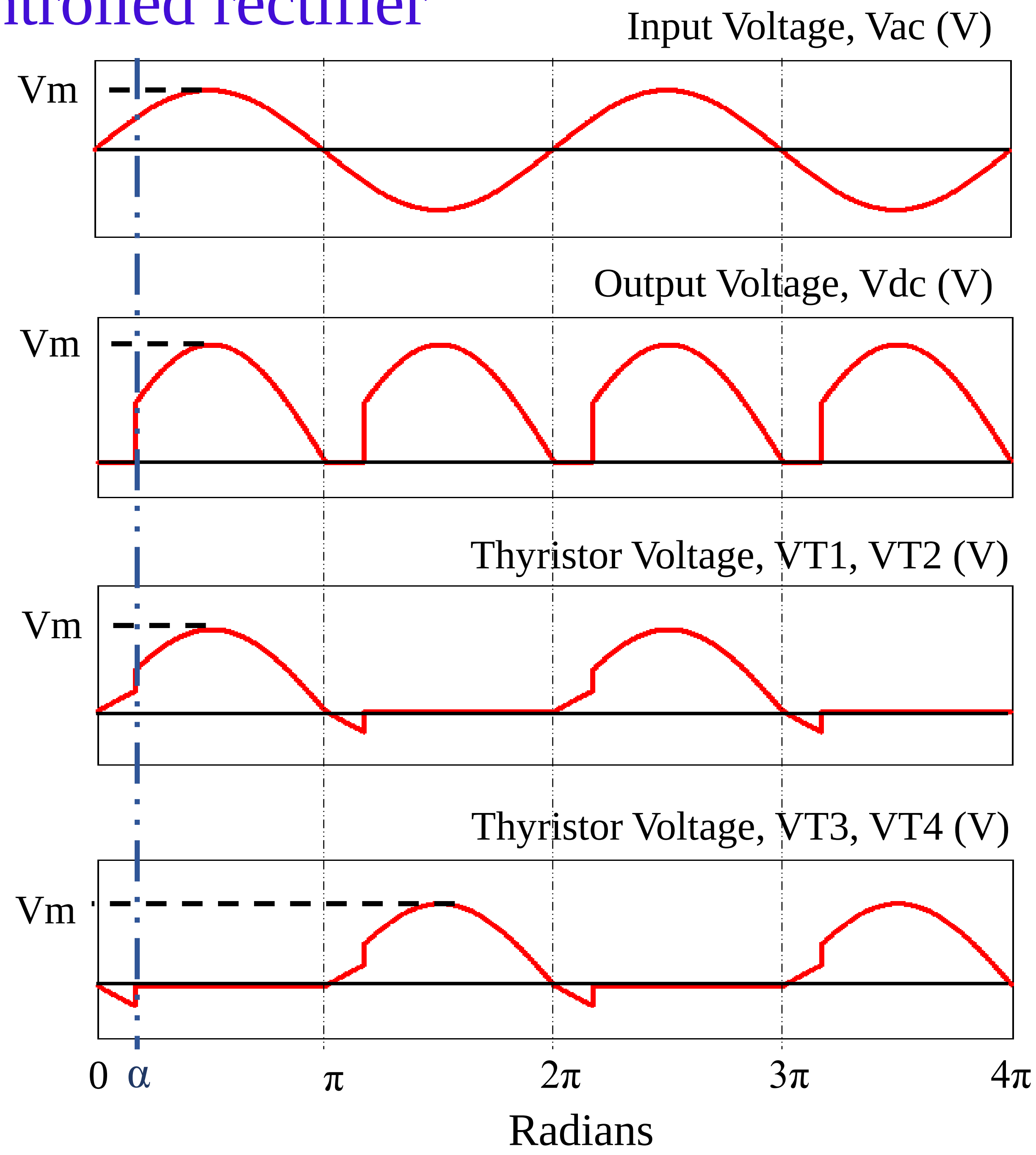
$$= \sqrt{\left[\frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t\right]}$$

$$= \sqrt{\left[\frac{V_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}\right]}$$

$$\boxed{Vrms = \sqrt{\left[\frac{V_m^2}{4\pi}[\pi]\right]} = \frac{V_m}{2}}$$

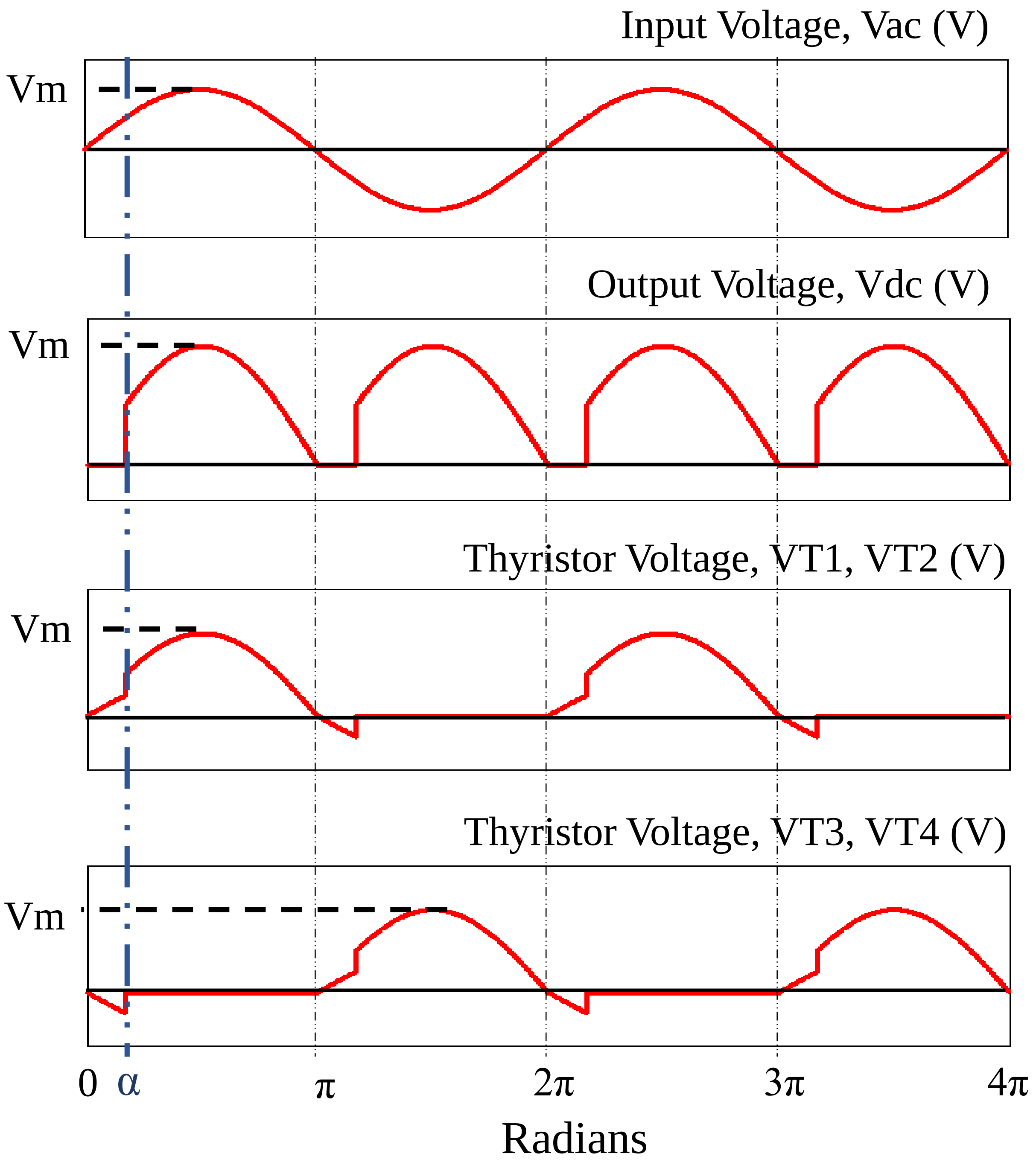
Single phase Full wave controlled rectifier





- Thyristors T1 and T2 are triggered at an angle α in the positive cycle of the input voltage, Vac. In this condition, Thyristors T3 and T4 remain in forward blocking state.

- While in negative cycle of the input voltage, Vac, Thyristors T3 and T4 are triggered at an angle 180 + α. In this condition, Thyristors T1 and T2 remain in forward blocking state.



Average DC output voltage, $Vdc = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}\left(V_m \sin\omega t\right)d\omega t$

$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi+\alpha}$$

$$\boxed{Vdc = \frac{2V_m}{\pi}\cos\alpha}$$

RMS DC output voltage, $Vrms = \sqrt{\left[\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}\left(V_m \sin\omega t\right)^2 d\omega t\right]}$

$$= \sqrt{\left[\frac{V_m^2}{\pi}\int_{\alpha}^{\pi+\alpha}\sin^2\omega t\, d\omega t\right]}$$

$$= \sqrt{\left[\frac{V_m^2}{\pi}\int_{\alpha}^{\pi+\alpha}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t\right]}$$

$$= \sqrt{\left[\frac{V_m^2}{2\pi}\left[\omega t-\frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi+\alpha}\right]}$$

$$= \sqrt{\left[\frac{V_m^2}{2\pi}[\pi]\right]}$$

$$\boxed{Vrms = \frac{V_m}{\sqrt{2}}}$$

Problem 2.1

The single phase full converter is connected to a 120V, 60 Hz supply. The load current Ia is continuous and its ripple content is negligible.

(a) Express the input current in a Fourier series. Determine the rms value and displacement angle of the nth harmonics and fundamental component. Calculate the rms value of the input current.

(b) If the delay angle is $\alpha = \pi/3$, calculate average output voltage (V_{dc}), RMS output voltage (V_{rms}), HF, DF, and PF.

Solution:

(a) The instantaneous sinusoidal input current can be expressed in Fourier series as,

$$i_s(t) = a_0 + \sum_{n=1,2,\ldots}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t\right) \qquad (1)$$

Where,

$$a_0 = \frac{1}{2\pi}\int_{\alpha}^{2\pi+\alpha} i_s(t)\ d(\omega t) = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} I_a\ d(\omega t) - \int_{\pi+\alpha}^{2\pi+\alpha} I_a\ d(\omega t)\right] = 0 \qquad (2)$$

- *Harmonic Factor (HF)* is a measure of the distortion of a waveform and is also known as *total harmonic distortion (THD)*. The HF of the input current is defined as

$$HF = \left(\frac{I_s^2 - I_{s1}^2}{I_{s1}^2}\right)^{1/2} = \left[\left(\frac{I_s^2}{I_{s1}^2}\right) - 1\right]^{1/2}$$

Where I_{s1} is the fundamental component of the input current I_s in RMS.

- If ϕ is the angle between the fundamental components of the input current and voltage, ϕ is called the *displacement angle*. The *displacement factor* is defined as

$$DF = \cos\phi$$

- The *input power factor (PF)* is defined as

$$PF = \frac{V_s I_{s1}}{V_s I_s}\cos\phi = \frac{I_{s1}}{I_s}\cos\phi$$

$$a_n = \frac{1}{\pi}\int_{\alpha}^{2\pi+\alpha} i_s(t)\cos n\omega t\, d(\omega t) = \frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} I_a \cos n\omega t\; d(\omega t) - \int_{\pi+\alpha}^{2\pi+\alpha} I_a\; \cos n\omega t\, d(\omega t)\right]$$

$$a_n = \frac{I_a}{\pi}\left[\left(\frac{\sin n\omega t}{n}\right)_{\alpha}^{\pi+\alpha} - \left(\frac{\sin n\omega t}{n}\right)_{\pi+\alpha}^{2\pi+\alpha}\right]$$

$$a_n = \frac{I_a}{n\pi}\left[\left(\sin(n\pi+n\alpha) - \sin n\alpha - \sin(2n\pi+n\alpha) + \sin(n\pi+n\alpha)\right)\right]$$

$$a_n = -\frac{4I_a}{n\pi}\sin n\alpha \qquad \text{for } n = 1,3,5..... \qquad (3)$$

$$a_n = 0 \qquad \text{for } n = 2,4,6.....$$

$$b_n = \frac{1}{\pi}\int_{\alpha}^{2\pi+\alpha} i_s(t)\sin n\omega t\, d(\omega t) = \frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} I_a \sin n\omega t\; d(\omega t) - \int_{\pi+\alpha}^{2\pi+\alpha} I_a\; \sin n\omega t\, d(\omega t)\right]$$

$$b_n = \frac{I_a}{\pi}\left[\left(\frac{-\cos n\omega t}{n}\right)_{\alpha}^{\pi+\alpha} - \left(\frac{-\cos n\omega t}{n}\right)_{\pi+\alpha}^{2\pi+\alpha}\right]$$

$$b_n = \frac{I_a}{n\pi}\left[\left(-\cos(n\pi+n\alpha) + \cos n\alpha + \cos(2n\pi+n\alpha) - \cos(n\pi+n\alpha)\right)\right]$$

$$b_n = \frac{4I_a}{n\pi}\cos n\alpha \qquad \text{for } n = 1,3,5..... \qquad (4)$$

$$b_n = 0 \qquad \text{for } n = 2,4,6.....$$

Substituting (2)-(4) in (1),

$$i_s(t) = \sum_{n=1,3,5,....}^{\infty}\left(\frac{4I_a}{n\pi}\right)\left(\cos n\alpha - \sin n\alpha\right) \qquad (5)$$

From (5), the rms value and displacement angle of the nth harmonics content can be expressed as,

$$I_{sn} = \frac{1}{\sqrt{2}}\left(a_n^2 + b_n^2\right)^{1/2} = \frac{4I_a}{\sqrt{2}n\pi} = \frac{2\sqrt{2}I_a}{n\pi} \qquad (6)$$

$$\phi_n = \tan^{-1}\frac{a_n}{b_n} = -n\alpha \qquad (7)$$

The rms value of the fundamental current and displacement angle (from (6)-(7)) is

$$I_{s1} = \frac{2\sqrt{2}I_a}{\pi} \qquad (n=1) \qquad (8)$$

$$\phi_1 = -\alpha \qquad (n=1) \qquad (9)$$

The input current rms can be calculated from,

$$I_s = \left[\frac{2}{2\pi}\int_{\alpha}^{\pi+\alpha} I_a^2\; d(\omega t)\right]^{1/2} = I_a \qquad (10)$$

(b) $\alpha = \pi/3$

$$V_{dc} = \frac{2V_m}{\pi}\cos\alpha = 54.02\,\text{V} \tag{11}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = V_s = 120\,\text{V} \tag{12}$$

$$HF = \left[\left(\frac{I_s}{I_{s1}}\right)^2 - 1\right]^{1/2} \tag{13}$$

Substituting (8) and (10) in (13),

$$HF = \left[\left(\frac{I_a}{\frac{2\sqrt{2}I_a}{\pi}}\right)^2 - 1\right]^{1/2} = 0.4834 \text{ or } 48.34\% \tag{15}$$

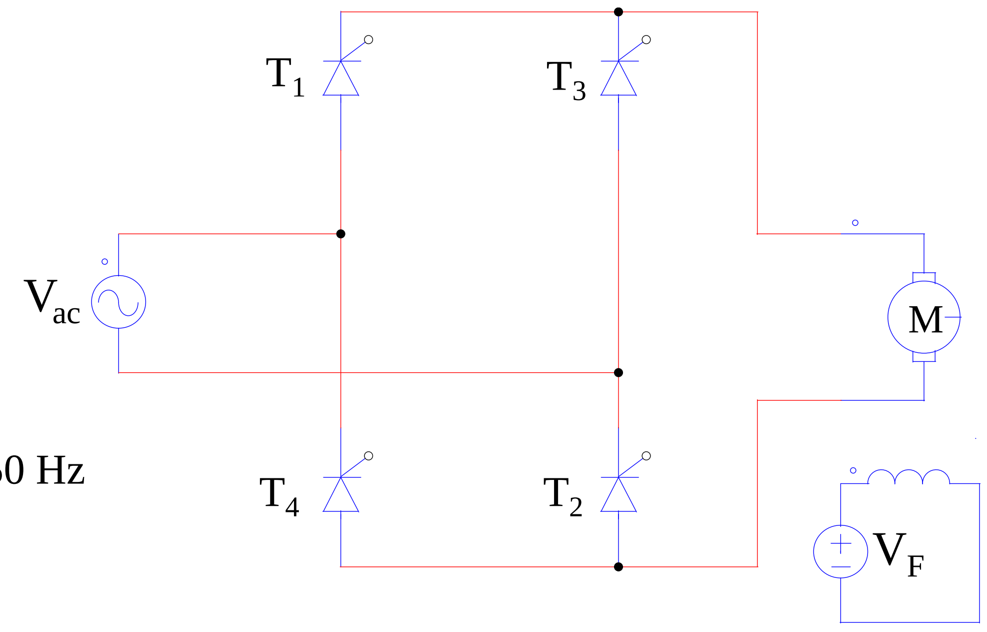
$$DF = \cos\phi_1 = \cos(-\alpha) = \cos\frac{-\pi}{3} = 0.5 \tag{16}$$

$$PF = \frac{I_{s1}}{I_s}\cos(-\alpha) = 0.45\,(\text{lagging}) \tag{17}$$

Problem 2.2

A 200 V, 875 rpm, 150 A separately excited DC motor has an armature resistance of 0.06 Ω. It is fed from a single phase fully-controlled rectifier with an AC source voltage of 220 V, 50 Hz. Assuming continuous conduction, calculate

(i) Firing angle for rated motor torque and 750 rpm
(ii) Firing angle for rated motor torque and (-500) rpm
(iii) Motor speed for $\alpha=160^o$ and rated torque.



Field circuit

Solution:

<u>Given:</u>

<u>DC Motor</u>: Rated voltage – 200V; Rated current – 150 A; Rated speed – 875 rpm; Armature resistance – 0.06 Ω

<u>Single phase fully controlled converter</u>: AC source voltage – 220 V, 50 Hz

We know that, And at rated operation,

$$E = V - (I_a R_a)$$

$$E = 200 - (150 \times 0.06) = 191V$$

(i) To find firing angle for rated motor torque and 750 rpm

Back emf at 750 rpm, $\frac{E_1}{E_2} = \frac{N_1}{N_2} \Rightarrow E_{750} = \frac{750}{875} \times 191 = 163.7\,V$

$$V_{750} = E_{750} + I_a R_a \Rightarrow V_{750} = 163.7 + (150 \times 0.06) = 172.7\,V$$

Now, for a single phase fully controlled converter the average output voltage can be expressed as

$$\frac{2V_m}{\pi}\cos\alpha = V_a$$

$$\frac{2 \times 220 \times \sqrt{2}}{\pi}\cos\alpha = 172.7$$

$$\therefore \cos\alpha = 0.872 \quad \text{or} \quad \alpha = 29.3^o$$

(ii) To find firing angle for rated motor torque and -500 rpm

Back emf at -550 rpm,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \Rightarrow E_{-500} = \frac{-500}{875} \times 191 = -109\,V$$

$$V_{-500} = E_{-500} + I_a R_a \Rightarrow V_{-500} = -109 + (150 \times 0.06) = -100\,V$$

Now, for a single phase fully controlled converter the average output voltage can be expressed as

$$\frac{2V_m}{\pi}\cos\alpha = V_a$$

$$\frac{2 \times 220 \times \sqrt{2}}{\pi}\cos\alpha = -100$$

$$\therefore \cos\alpha = -0.5 \quad \text{or} \quad \alpha = 120^o$$

(iii) To find the motor speed for a firing angle of 160^o and rated torque

$$\frac{2V_m}{\pi}\cos\alpha = V_a$$

$$\frac{2 \times 220 \times \sqrt{2}}{\pi}\cos\alpha = \frac{2 \times 220 \times \sqrt{2}}{\pi}\cos(160^o) = -186\,V$$

As, $V_a = E + I_a R_a$

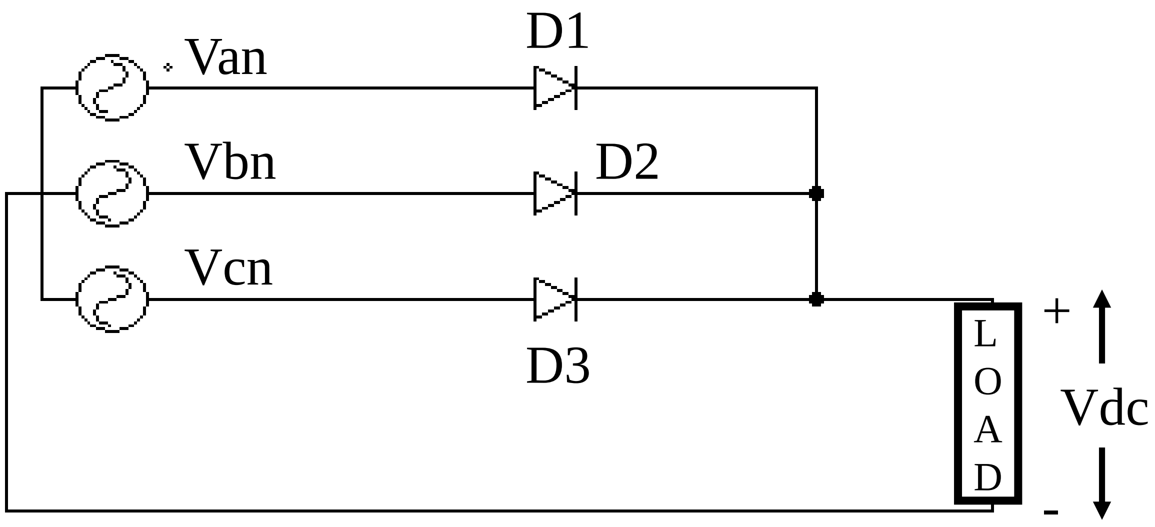
$$-186 = E + (150 \times 0.06) \Rightarrow E = -195\ V$$

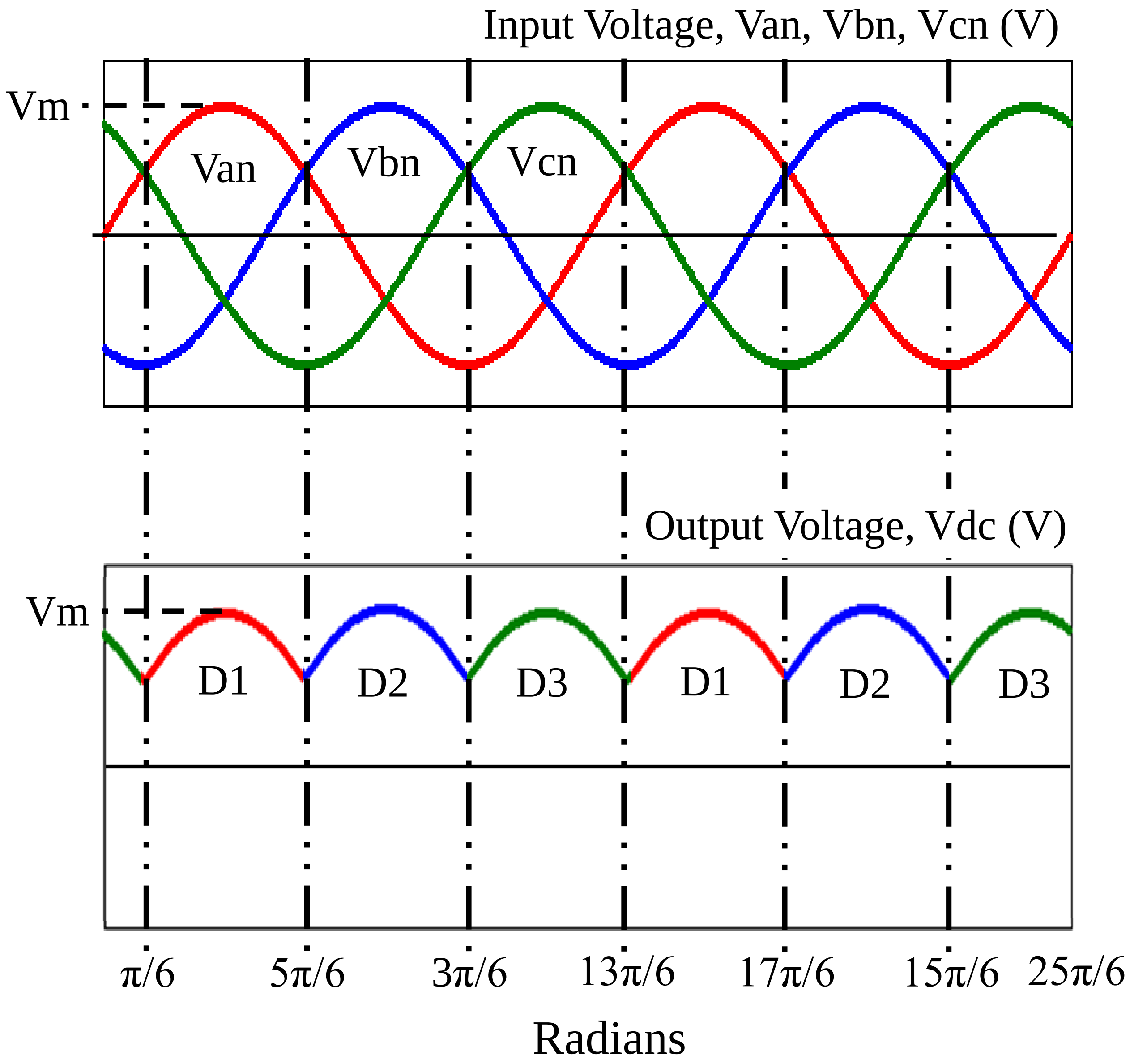
As,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$N_2 = \frac{-195}{191} \times 875 = -893.2\ \text{rpm}$$

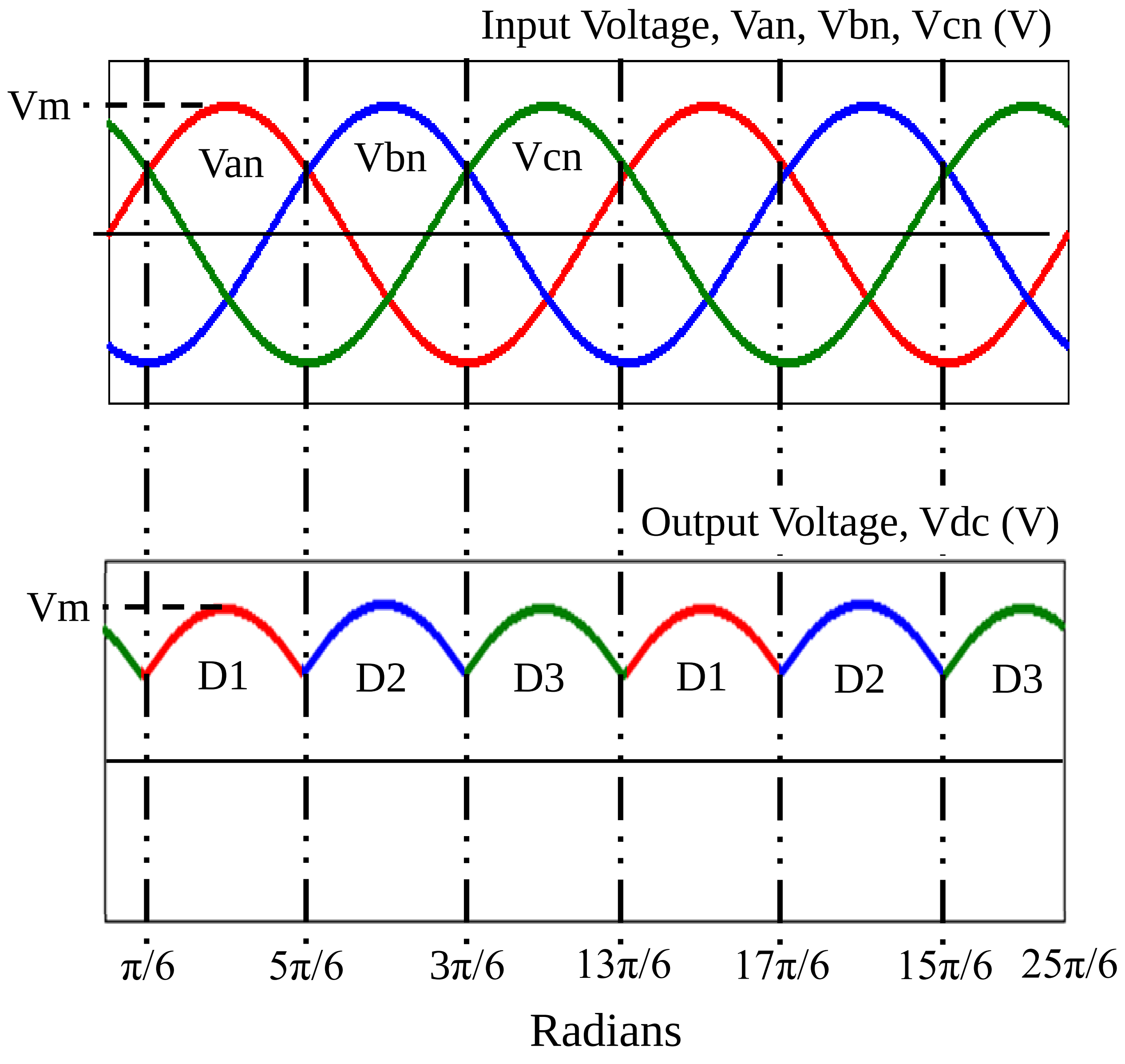
Three phase half wave uncontrolled rectifier





- The diodes D1, D2 and D3 are in conduction state during positive cycle of their respective input phase voltages Va, Vb and Vc.

- The output voltage depends on the maximum magnitude of the positive phase voltage among three phases at that instant.



Average DC output voltage, $\displaystyle V_{dc} = \frac{3}{2\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(V_m \sin\omega t \right) d\omega t$

$$= \frac{3V_m}{2\pi}\left[-\cos\omega t\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$\boxed{V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}}$$

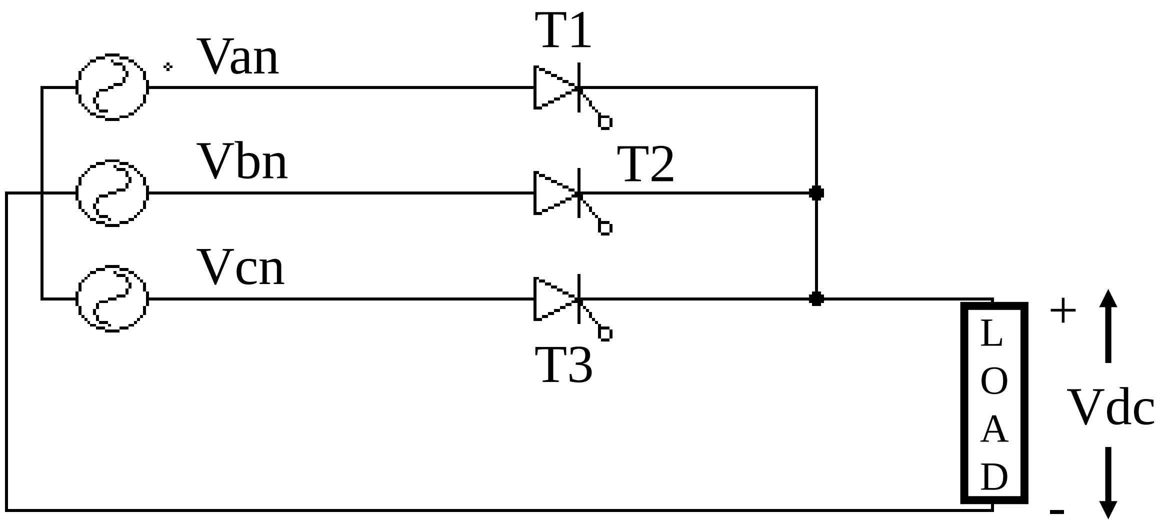
RMS DC output voltage, $\displaystyle V_{rms} = \sqrt{\left[\frac{3}{2\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(V_m \sin\omega t \right)^2 d\omega t\right]}$

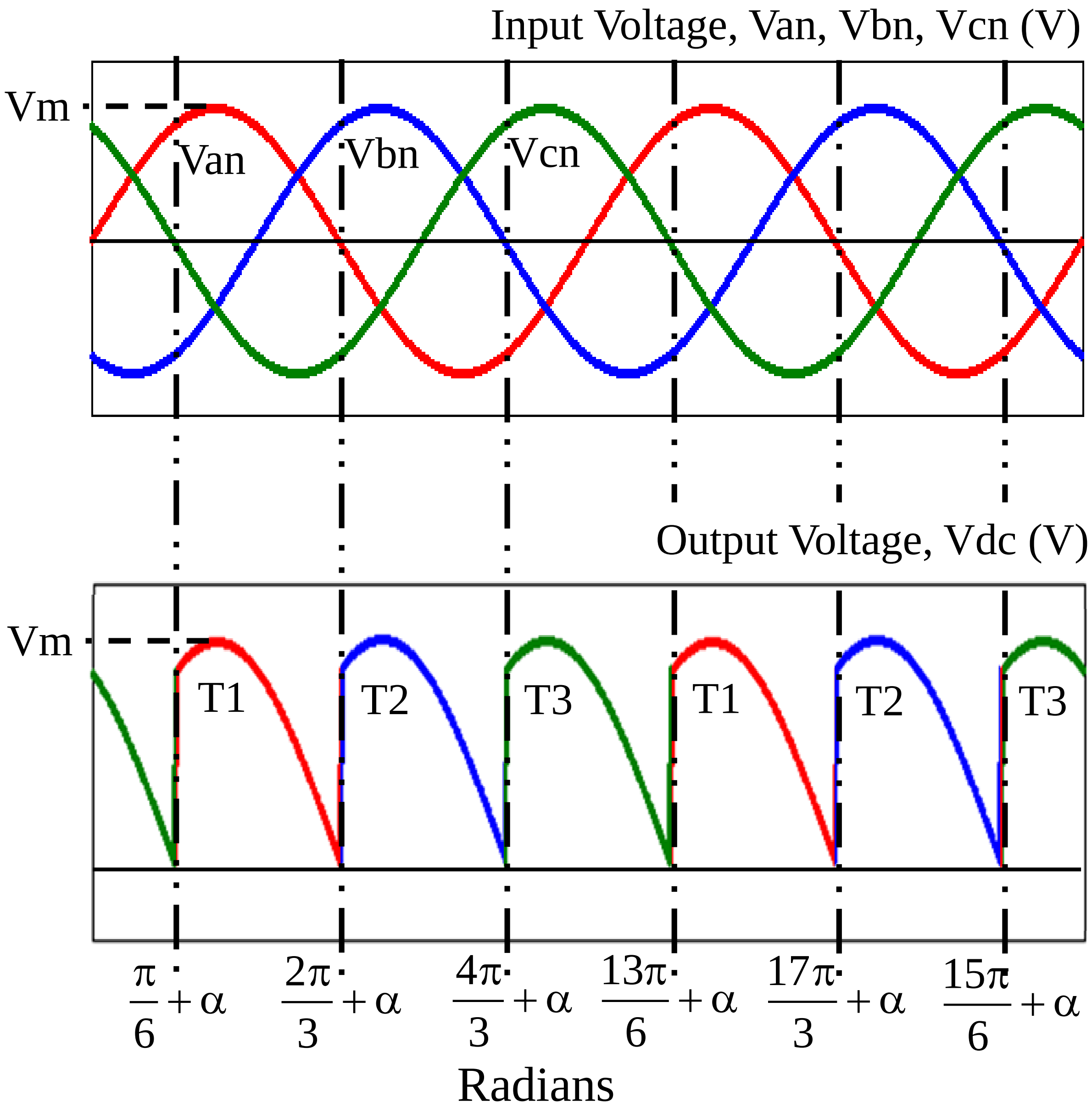
$$= \sqrt{\left[\frac{3V_m^2}{4\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left[1-\cos 2\omega t\right] d\omega t\right]}$$

$$= \sqrt{\left[\frac{3V_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\right]}$$

$$\boxed{V_{rms} = \sqrt{\left[\frac{V_m^2}{4\pi}\left[\frac{5\pi}{6} - \frac{\sin\frac{10\pi}{6}}{2} - \frac{\pi}{6} + \frac{\sin\frac{2\pi}{6}}{2}\right]\right]} = 0.84068 V_m}$$

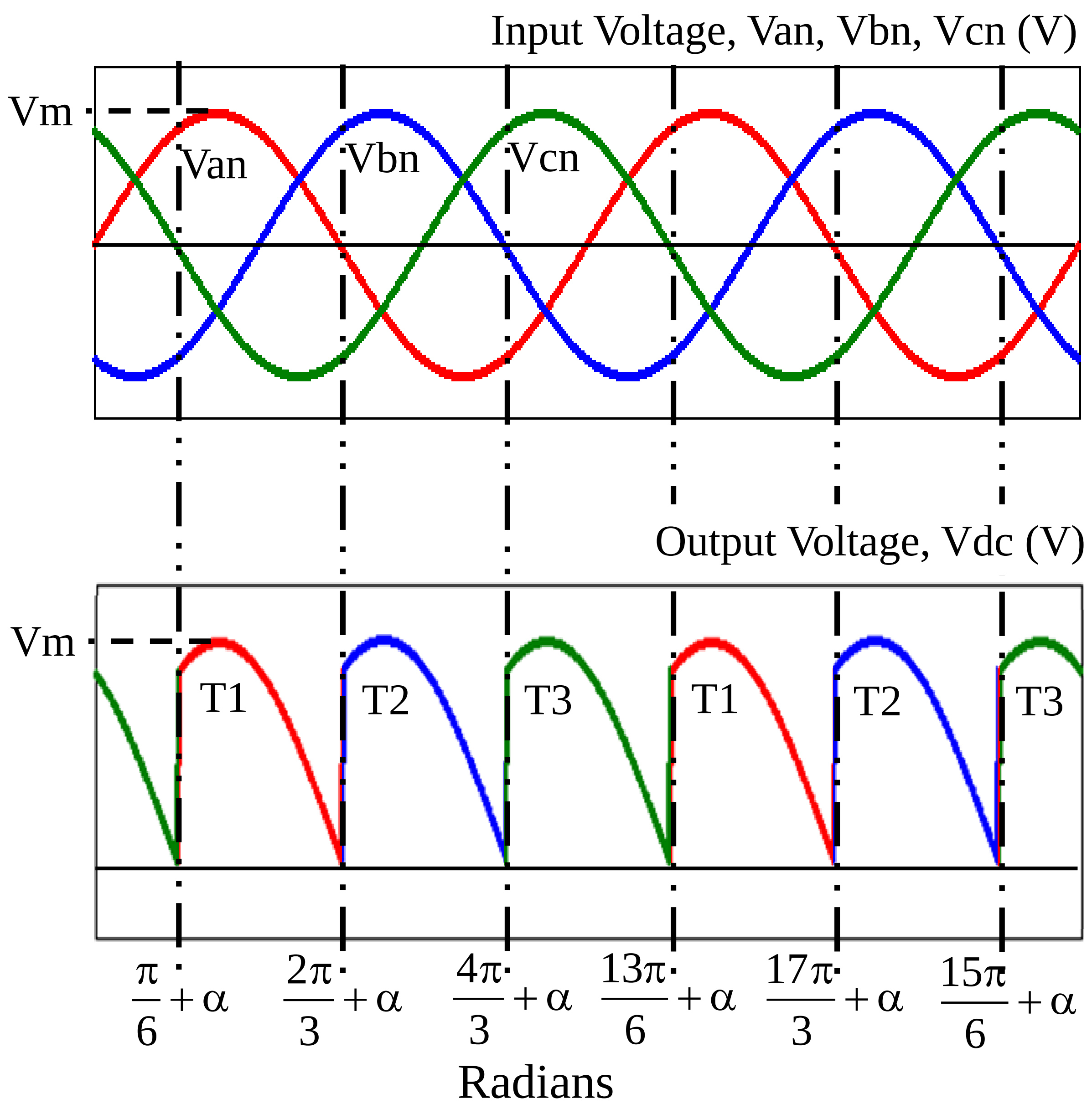
Three phase half wave controlled rectifier





- Based on the triggering angle α, thyristors T1, T2 and T3 conduct, during positive cycle of their respective input phase voltages Va, Vb and Vc.

- The previous thyristor continues to be in conduction state until the next thyristor is triggered or the input phase voltage magnitude becomes negative.



Average DC output voltage, $V_{dc} = \frac{3}{2\pi}\int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{5\pi}{6}} (V_m \sin\omega t)\, d\omega t$

$$= \frac{3V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{5\pi}{6}}$$

$$\boxed{V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha}$$

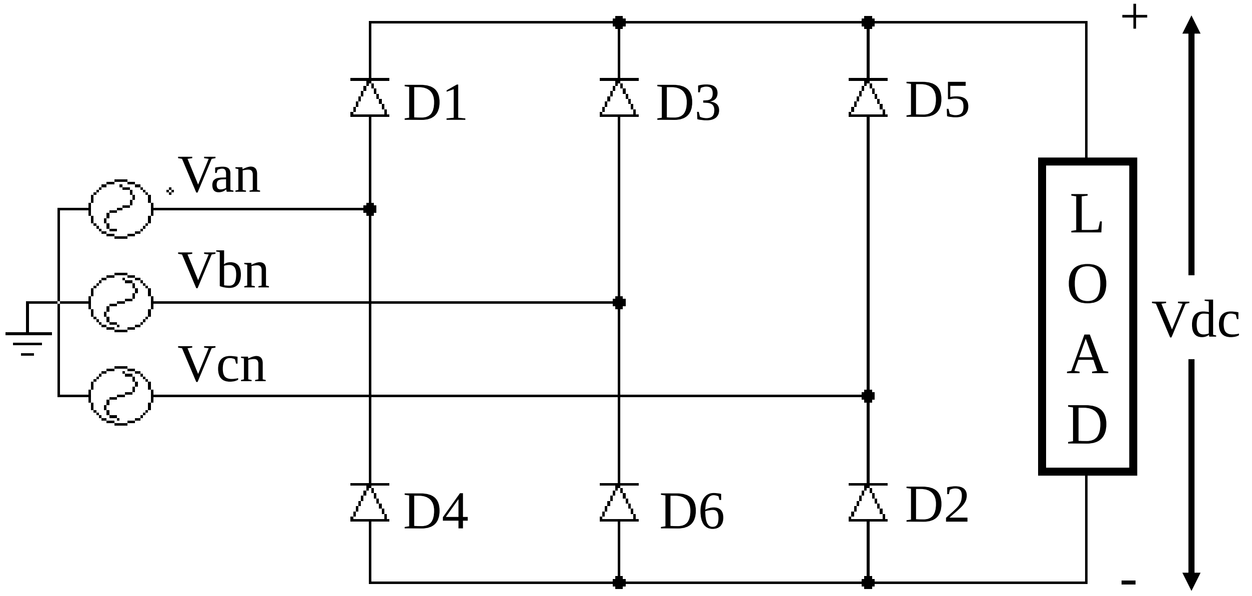
RMS DC output voltage, $V_{rms} = \sqrt{\left[\frac{3}{2\pi}\int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{5\pi}{6}} (V_m \sin\omega t)^2\, d\omega t\right]}$

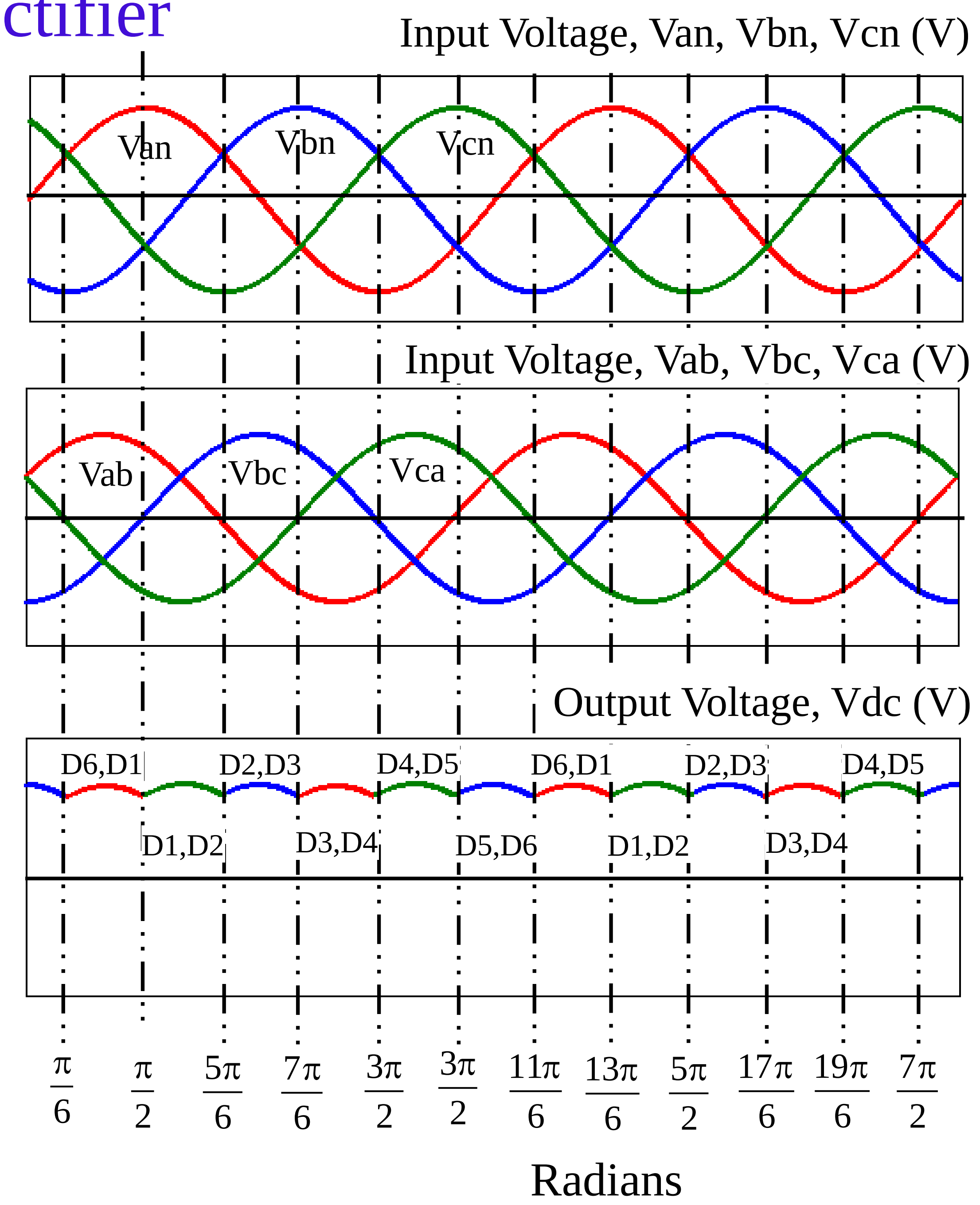
$$= \sqrt{\left[\frac{3V_m^2}{4\pi}\int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{5\pi}{6}} [1-\cos 2\omega t]\, d\omega t\right]}$$

$$= \sqrt{\left[\frac{3V_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{5\pi}{6}}\right]}$$

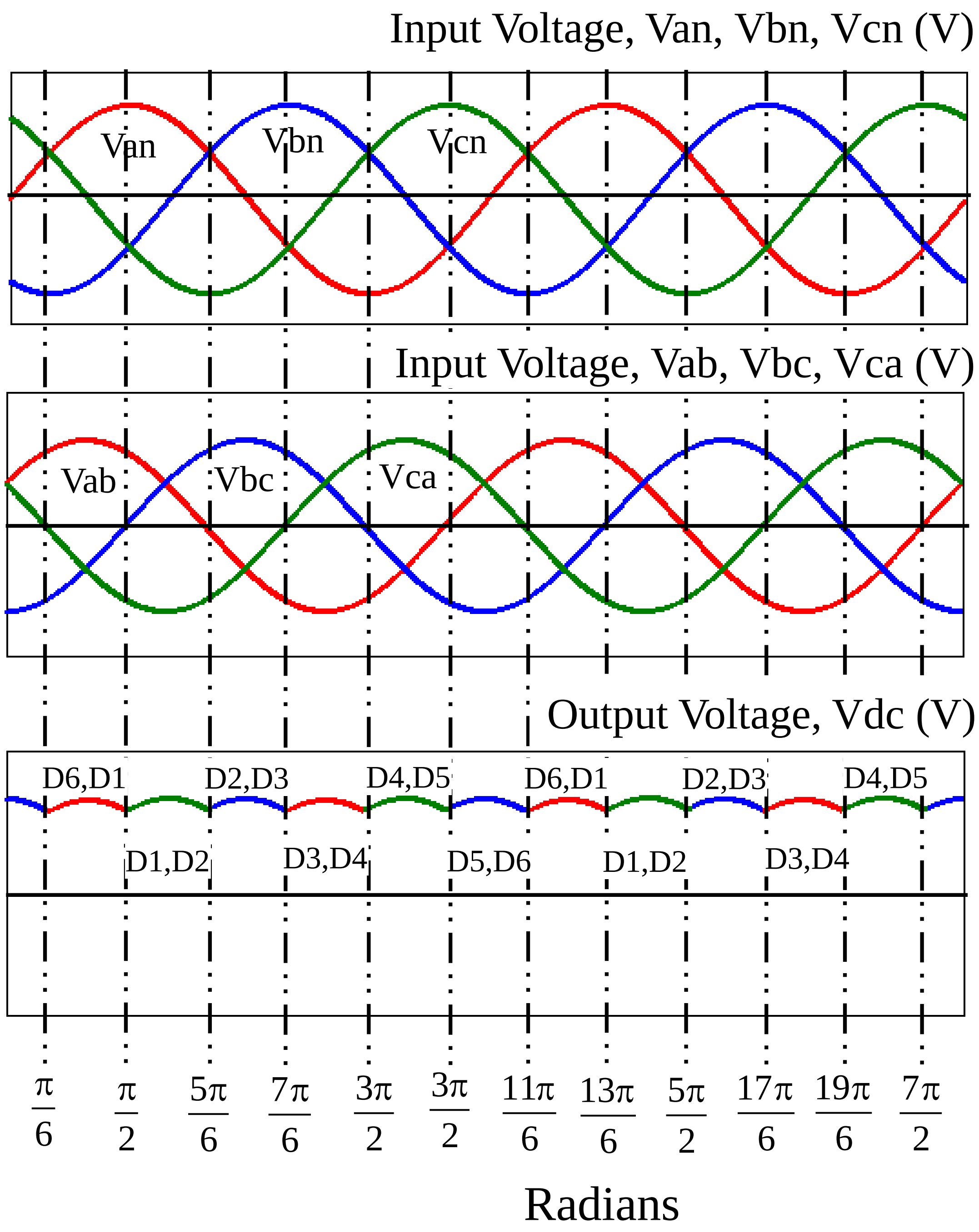
$$\boxed{V_{rms} = \sqrt{\frac{3V_m^2}{4\pi}\left[\frac{2\pi}{3}+\frac{\sqrt{3}}{2}\cos 2\alpha\right]}}$$

Three phase Full wave uncontrolled rectifier





- The pair of diodes which are connected between that pair of supply lines having the highest amount of instantaneous line-to-line voltage will conduct.
- The line-to-line voltage is $\sqrt{3}$ times the phase voltage of a three-phase Y-connected source.



Average DC output voltage, $\displaystyle Vdc = \frac{2}{2\pi/6}\int_0^{\frac{\pi}{6}}\left(\sqrt{3}V_m\cos\omega t\right)d\omega t$

$$= \frac{6\sqrt{3}V_m}{\pi}\left[\sin\omega t\right]_0^{\pi/6}$$

$$\boxed{Vdc = \frac{3\sqrt{3}V_m}{\pi}}$$

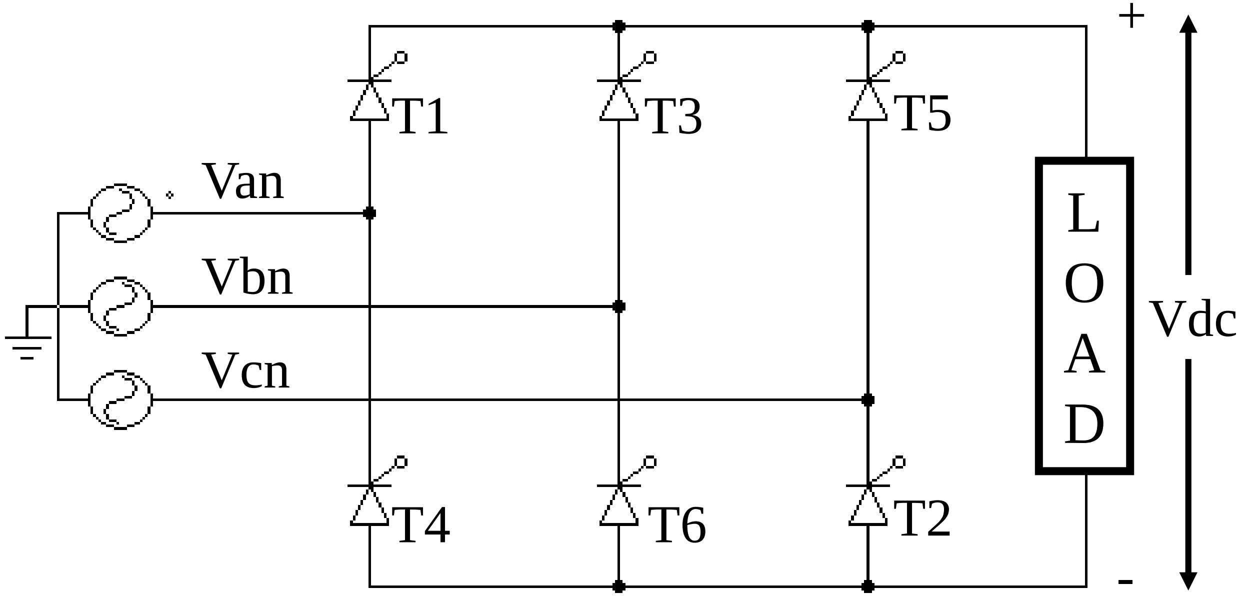
RMS DC output voltage, $\displaystyle Vrms = \sqrt{\left[\frac{2}{2\pi/6}\int_0^{\frac{\pi}{6}}\left(\sqrt{3}V_m\cos\omega t\right)^2 d\omega t\right]}$

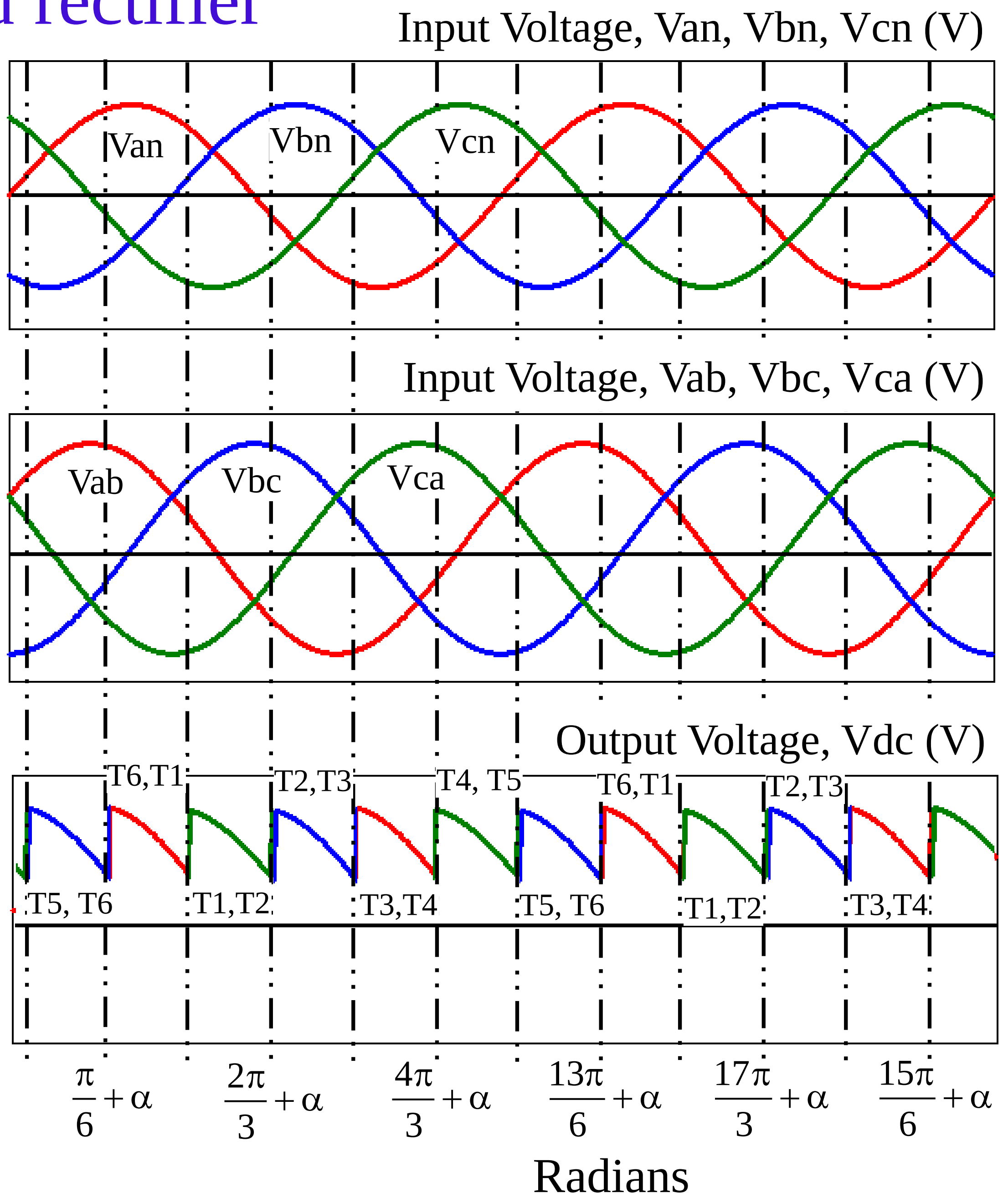
$$= \sqrt{\left[\frac{9V_m^2}{\pi}\int_0^{\frac{\pi}{6}}\left[1+\cos 2\omega t\right]d\omega t\right]}$$

$$= \sqrt{\left[\frac{9V_m^2}{\pi}\left[\omega t+\frac{\sin 2\omega t}{2}\right]_0^{\frac{\pi}{6}}\right]}$$

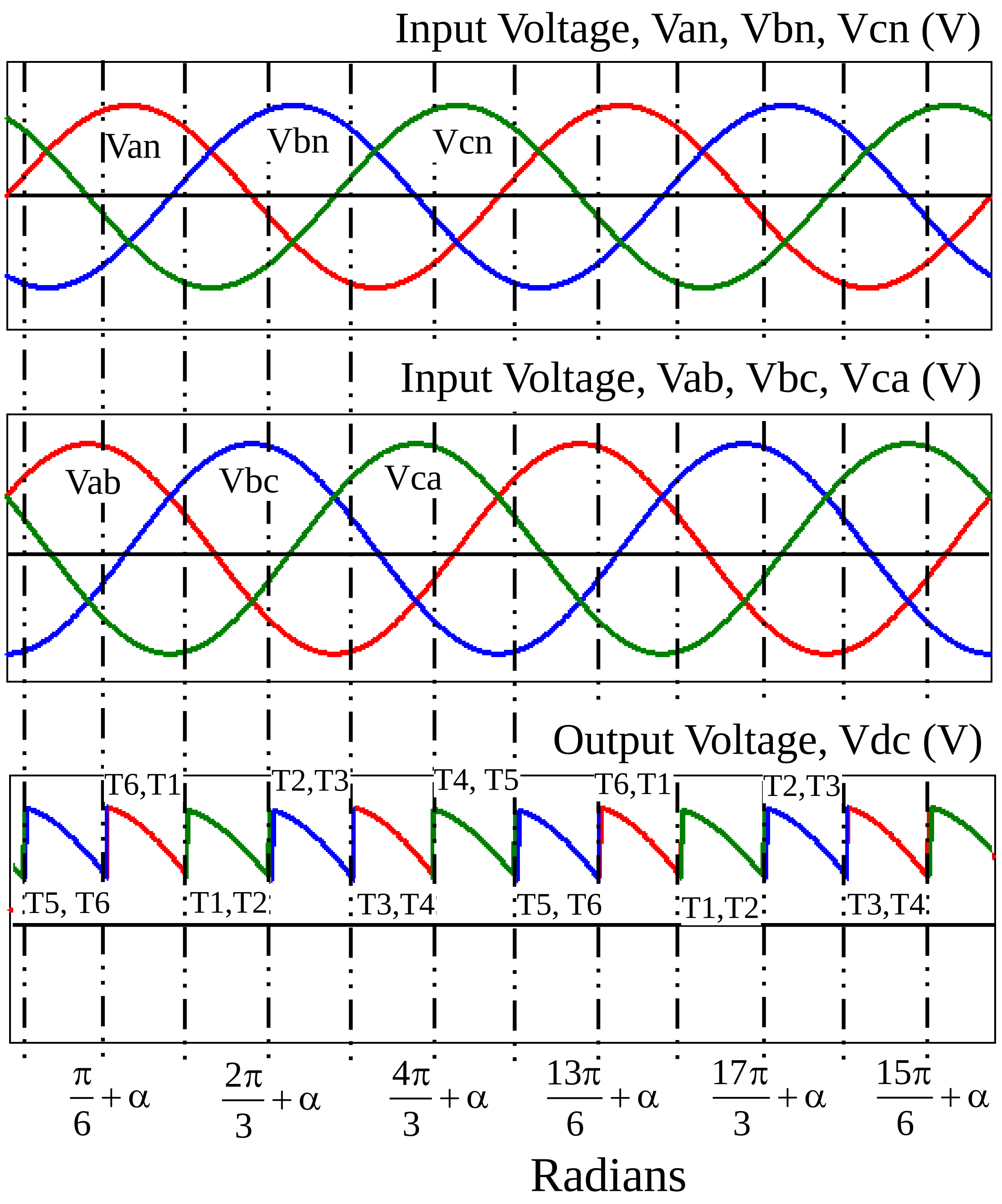
$$\boxed{Vrms = \sqrt{\left[\frac{9V_m^2}{\pi}\left[\frac{\pi}{6}+\frac{\sin\frac{2\pi}{6}}{2}-0-\frac{\sin 0}{2}\right]\right]} = 1.6554V_m}$$

Three phase Full wave controlled rectifier





- The thyristors are turned on at an interval of $\pi/3$.
- The frequency of output ripple voltage is 6 times the supply frequency and the filtering requirement is less than that of half-wave converters.



Average DC output voltage, $Vdc = \frac{3}{\pi}\int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{\pi}{2}}\left(\sqrt{3}V_m \sin\left(\omega t+\frac{\pi}{6}\right)\right)d\omega t$

$$\boxed{Vdc = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha}$$

RMS DC output voltage, $Vrms = \sqrt{\left[\frac{3}{\pi}\int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{\pi}{2}}\left(\sqrt{3}V_m \sin\left(\omega t+\frac{\pi}{6}\right)\right)^2 d\omega t\right]}$

$$\boxed{Vrms = \sqrt{3}V_m\sqrt{\left[\frac{1}{2}+\frac{3\sqrt{3}}{4\pi}\cos 2\alpha\right]}}$$

Problem 2.3

The three phase full converter is operated from a three-phase Y-connected 208-V, 60-Hz supply. The load current is continuous with a negligible ripple content.

(a) Express the input current in a Fourier series. Determine the rms value and displacement angle of the nth harmonics and fundamental component. Calculate the rms value of the input current.

(b) If the delay angle is $\alpha = \pi/3$, calculate average output voltage (V_{dc}), RMS output voltage (V_{rms}), HF, DF, and PF.

Solution:

(a) The instantaneous sinusoidal input current can be expressed in Fourier series as,

$$i_s(t) = a_0 + \sum_{n=1,2,....}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t\right) \qquad (1)$$

Where,

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} i_s(t)\ d(\omega t) = \frac{1}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_a\ d(\omega t) - \int_{\frac{7\pi}{6}+\alpha}^{\frac{11\pi}{6}+\alpha} I_a\ d(\omega t)\right] = 0 \qquad (2)$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} i_s(t)\cos n\omega t\, d(\omega t) = \frac{1}{\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_a \cos n\omega t\, d(\omega t) - \int_{\frac{7\pi}{6}+\alpha}^{\frac{11\pi}{6}+\alpha} I_a \cos n\omega t\, d(\omega t)\right]$$

$$a_n = -\frac{4I_a}{n\pi}\sin\frac{n\pi}{3}\sin n\alpha \qquad \text{for } n = 1,3,5..... \qquad (3)$$

$$a_n = 0 \qquad \text{for } n = 2,4,6.....$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} i_s(t)\sin n\omega t\, d(\omega t) = \frac{1}{\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_a \sin n\omega t\, d(\omega t) - \int_{\frac{7\pi}{6}+\alpha}^{\frac{11\pi}{6}+\alpha} I_a \sin n\omega t\, d(\omega t)\right]$$

$$b_n = \frac{4I_a}{n\pi}\cos\frac{n\pi}{3}\cos n\alpha \qquad \text{for } n = 1,3,5..... \qquad (4)$$

$$b_n = 0 \qquad \text{for } n = 2,4,6.....$$

Substituting (2)-(4) in (1),

$$i_s(t) = \sum_{n=1,3,5,....}^{\infty}\left(\frac{4I_a}{n\pi}\right)\left(\cos\frac{n\pi}{3}\cos n\alpha - \sin\frac{n\pi}{3}\sin n\alpha\right) \qquad (5)$$

From (5), the rms value and displacement angle of the nth harmonics content can be expressed as,

$$I_{sn} = \frac{1}{\sqrt{2}}\left(a_n^2 + b_n^2\right)^{1/2} = \frac{2\sqrt{2}I_a}{n\pi}\sin\frac{n\pi}{3} \qquad (6)$$

$$\phi_n = \tan^{-1}\frac{a_n}{b_n} = -n\alpha \qquad (7)$$

The rms value of the fundamental current and displacement angle (from (6)-(7)) is

$$I_{s1} = \frac{\sqrt{6}I_a}{\pi} = 0.7797 I_a \qquad (n = 1) \qquad (8)$$

$$\phi_1 = -\alpha \qquad (n = 1) \qquad (9)$$

The input current rms can be calculated from,

$$I_s = \left[\frac{2}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_a^2\, d(\omega t)\right]^{1/2} = I_a\sqrt{\frac{2}{3}} = 0.8165 I_a \qquad (10)$$

(b) $\alpha = \pi/3$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha =$$

$$V_{rms} = \sqrt{\frac{3V_m^2}{4\pi}\left[\frac{2\pi}{3} + \frac{\sqrt{3}}{2}\cos 2\alpha\right]}$$

$$HF = \left[\left(\frac{I_s}{I_{s1}}\right)^2 - 1\right]^{1/2} = \left[\left(\frac{\pi}{3}\right)^2 - 1\right]^{1/2} = 0.3108 \quad \text{or} \quad 31.08\%$$

$$DF = \cos\phi_1 = \cos(-\alpha) = \cos\frac{-\pi}{3} = 0.5$$

$$PF = \frac{I_{s1}}{I_s}\cos(-\alpha) = 0.478(\text{lagging})$$

Problem 2.4

A 220 V, 1500 rpm, 50 A separately excited motor with armature resistance of 0.5 Ω, is fed from a 3-phase fully-controlled rectifier. Available AC source has a line voltage of 440 V, 50 Hz. A star-delta connected transformer is used to feed the armature so that the motor terminal voltage equals rated voltage when converter firing angle is zero. Assuming continuous conduction Calculate,

(i) Transformer turns ratio;

(ii) Firing angle when, a) motor is running at 1200 rpm and rated torque

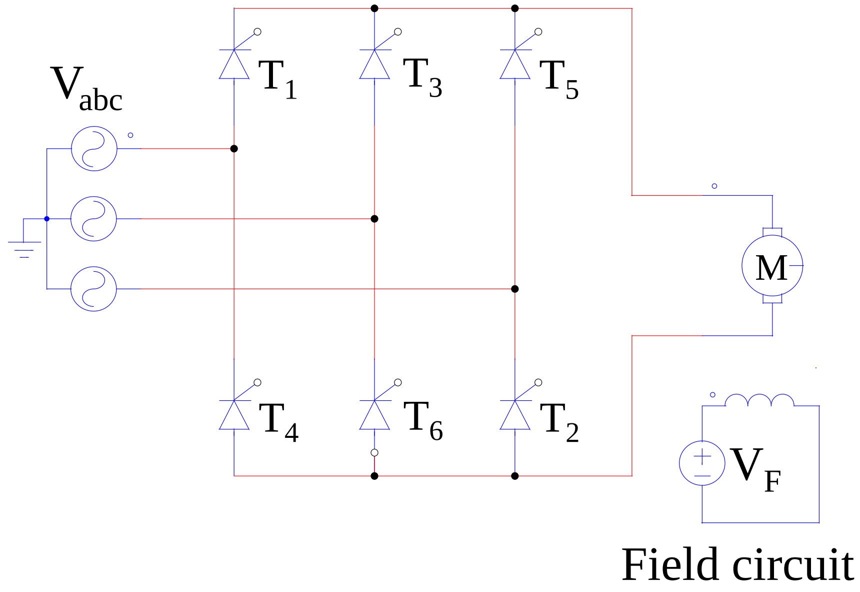
b) when motor is running at -800 rpm and twice the rated torque.

Solution:

Given: **DC Motor:** Rated voltage – 220V; Rated current – 50 A;

Rated speed – 1500 rpm; Armature resistance – 0.5

Single phase fully controlled converter: 3Φ AC source voltage – 440 V, 50Hz



(i) To find turns ratio for the star-delta transformer:

For a 3Φ fully controlled converter the maximum DC voltage is given by $V_m = \frac{\pi}{3}\frac{V_a}{\cos\alpha}$

For rated motor voltage α=0°, $V_m = \frac{\pi}{3}\frac{220}{\cos 0^o} = 230.4\ \text{V}$

rms converter input voltage between lines = $230.4/\sqrt{2} = 162.9\,\text{V}$

For star-delta transformer connection, ratio of turns between phase windings of primary and secondary = $\frac{440/\sqrt{3}}{162.9} = 1.559$

(ii) a) To find firing angle at 1200 rpm and rated torque

At rated 1500 rpm, $E = 220 - (0.5 \times 50) = 195V$

Now, at 1200 rpm $E = \frac{1200}{1500} \times 195 = 156V$

$V_a = E + I_a R_a \Rightarrow V_a = 156 + (50 \times 0.5) = 181V$

As, $V_a = 3\frac{V_m}{\pi}\cos\alpha$

$\cos\alpha = \frac{\pi}{3}\frac{V_a}{V_m} = \frac{\pi}{3} \times \frac{181}{230.4} = 0.8227$

$\alpha = 34.65^o$

b) To find firing angle at -800 rpm and rated torque

At -800 rpm

$E = \frac{-800}{1500} \times 195 = -104V$

$V_a = E + I_a R_a \Rightarrow V_a = -104 + (100 \times 0.5) = -54V$

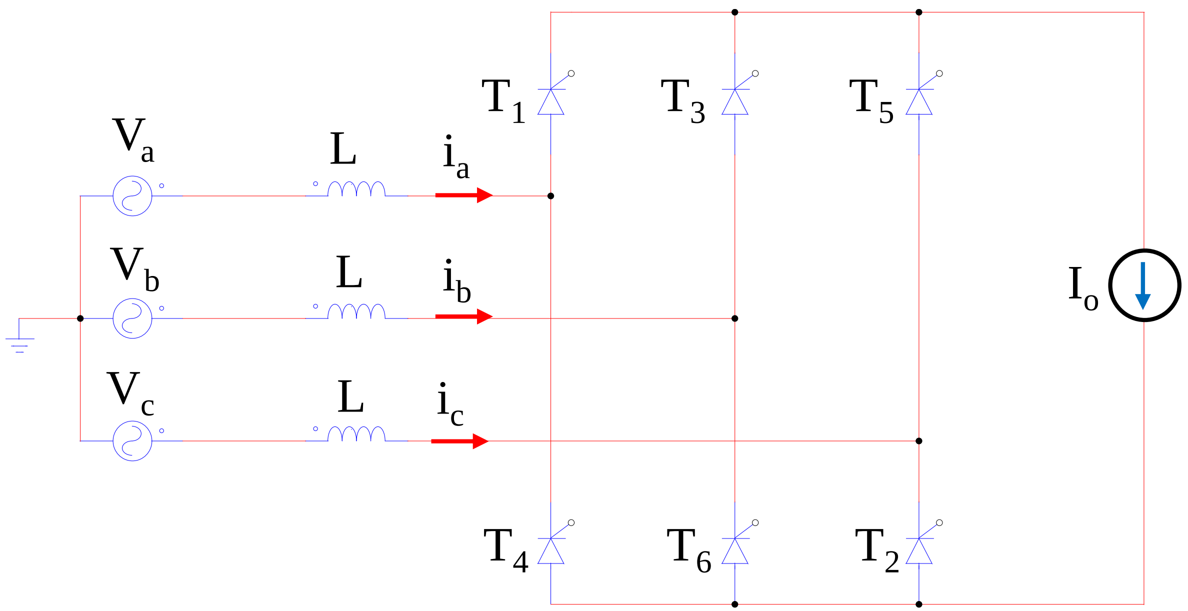
As, $V_a = 3\frac{V_m}{\pi}\cos\alpha$

$\cos\alpha = \frac{\pi}{3}\frac{V_a}{V_m} = \frac{\pi}{3} \times \frac{-54}{230.4} = -0.2454$

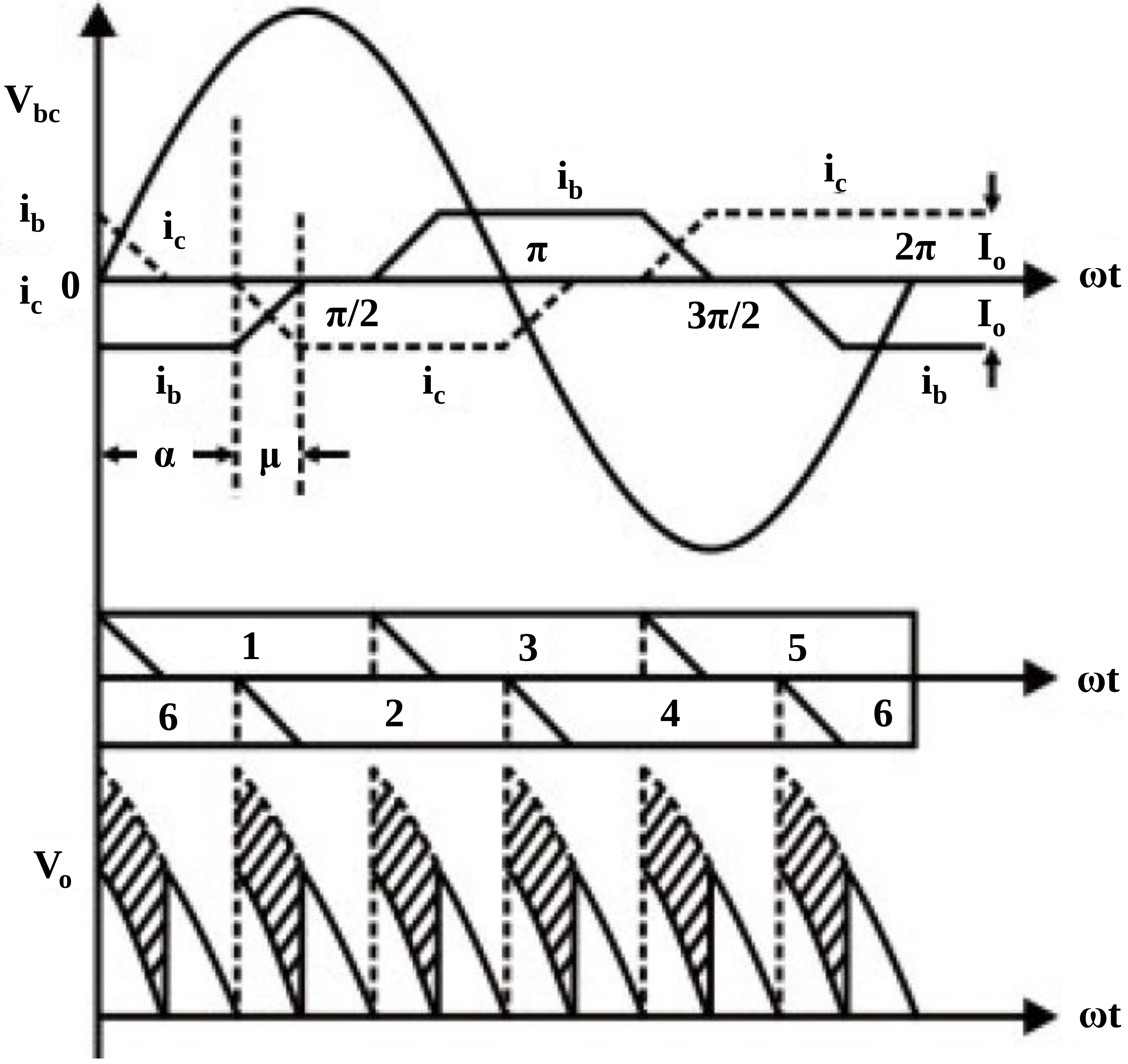
$\alpha = 104.20^o$

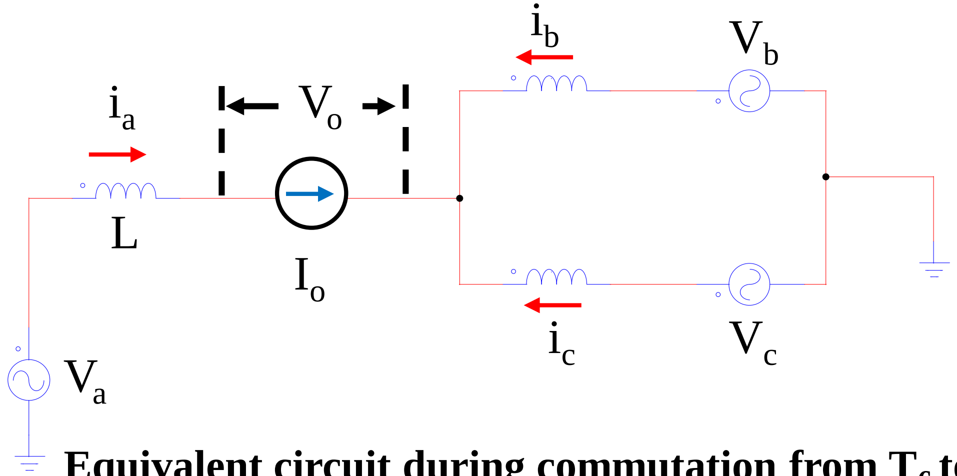
Effect of Source and Load inductances

- In the previous study of controlled and uncontrolled rectifier, the source was considered to be ideal and load to be resistive.
- Practically, the source has internal resistance and inductance, and load is not purely resistive.



- As in the case of a single phase converter the load is assumed to be highly inductive such that the load can be replaced by a current source.
- Commutations are not instantaneous due to the presence of source inductances. Instead, it takes place over an overlap period of "μ".
- During the overlap period three thyristors instead of two conducts.
- Current in the outgoing thyristor gradually decreases to zero while the incoming thyristor current increases and equals the total load current at the end of the overlap period.
- In the time interval $\alpha < \omega t \leq \alpha + \mu$, T_6 and T_2 from the bottom group and T_1 from the top group conducts.





Equivalent circuit during commutation from T_6 to T_2

In the interval $\alpha < \omega t \leq \alpha + \mu$

$$v_b = L\frac{di_b}{dt} - L\frac{di_c}{dt} + v_c \qquad (1)$$

or,
$$v_{bc} = L\frac{d}{dt}(i_b - i_c) \qquad (2)$$

but $\quad i_b + i_c + I_o = 0 \qquad \therefore \qquad \frac{di_b}{dt} = -\frac{di_c}{dt} \qquad (3)$

$$\therefore \qquad 2L\frac{d}{dt}i_b = v_{bc} = \sqrt{2}V_L \sin\omega t \qquad (4)$$

$$\therefore \qquad i_b = C - \frac{\sqrt{2}V_L}{2\omega L}\cos\omega t \qquad (5)$$

at $\omega t = \alpha$, $\qquad i_b = -I_0 \qquad \therefore \quad C = \frac{\sqrt{2}V_L}{2\omega L}\cos\alpha - I_0 \quad (6)$

$$\therefore \qquad i_b = \frac{\sqrt{2}V_L}{2\omega L}(\cos\alpha - \cos\omega t) - I_0 \qquad (7)$$

at $\omega t = \alpha + \mu, \quad i_b = 0$

$$\therefore \qquad \frac{\sqrt{2}V_L}{2\omega L}\left(\cos\alpha - \cos(\alpha+\mu)\right) = I_0 \qquad (8)$$

$$\text{Or,} \qquad \cos\alpha - \cos(\alpha+\mu) = \frac{\sqrt{2}\omega L}{V_L} I_0 \qquad (9)$$

Equation (9) holds for $\mu \leq 60^o$. It can be shown that for this condition to be satisfied

$$I_0 \leq \frac{V_L}{\sqrt{2}\omega L}\cos\left(\alpha - \frac{\pi}{3}\right) \qquad (10)$$

To calculate the dc voltage

For $\alpha \leq \omega t \leq \alpha + \mu$

$$v_0 = v_a - v_b + L\frac{di_b}{d_t} = \frac{3}{2}v_a \qquad (11)$$

for $\alpha + \mu \leq \omega t \leq \alpha + \frac{\pi}{3}$ $\qquad v_0 = v_{ac}$

$$\therefore \qquad V_0 = \frac{3}{\pi}\left[\int_{\alpha}^{\alpha+\mu}\frac{3}{2}v_a \, d\omega t + \int_{\alpha+\mu}^{\alpha+\frac{\pi}{3}} v_{ac} \, d\omega t\right]$$

$$= \frac{3}{\pi}\left[\int_{\alpha}^{\alpha+\mu}\left(v_{ac} + \frac{3}{2}v_a - v_{ac}\right) + \int_{\alpha+\mu}^{\alpha+\frac{\pi}{3}} v_{ac} \, d\omega t\right]$$

$$= \frac{3}{\pi}\left[\int_{\alpha}^{\alpha+\frac{\pi}{3}} v_{ac} \, d\omega t + \int_{\alpha}^{\alpha+\mu}\left(\frac{v_a}{2} + v_c\right) d\omega t\right]$$

$$= \frac{3\sqrt{2}}{\pi}V_L\cos\alpha - \frac{3}{2\pi}\int_{\alpha}^{\alpha+\mu} v_{bc} d\omega t \qquad (12)$$

$$\text{or} \qquad V_0 = \frac{3\sqrt{2}}{\pi}V_L\cos\alpha - \frac{3\sqrt{2}V_L}{2\pi}\int_{\alpha}^{\alpha+\mu}\sin\omega t \, d\omega t$$

$$= \frac{3\sqrt{2}}{\pi}V_L\cos\alpha - \frac{3\sqrt{2}V_L}{2\pi}\left[\cos\alpha - \cos(\alpha+\mu)\right] \qquad (13)$$

Substituting equation (9) into (13)

$$V_0 = \frac{3\sqrt{2}}{\pi}V_L\cos\alpha - \frac{3}{\pi}\omega L \, I_0 \qquad (14)$$

Problem 2.4

A 220V, 1450 RPM, 100A separately excited dc motor has an armature resistance to 0.1Ω. It is supplied from a 3 phase fully controlled converter connected to a 3 phase 50 Hz ac source. The ac source has an inductive reactance of 0.5Ω at 50 Hz. The line voltage is adjusted such that at α = 0; the motor operates at rated speed and torque. The motor is to be braked regeneratively in the reverse direction at rated speed using the converter. What is the maximum braking torque the motor will be able to produce under this condition without causing commutation failure?

Solution: Under rated operating condition, the motor terminal voltage is 220 V and it draws 100A current.

Therefore from equation (14),

$$V_o = \frac{3\sqrt{2}}{\pi} V_L \cos\alpha - \frac{3}{\pi}(\omega L) I_o$$

$$220 = \frac{3\sqrt{2}}{\pi} V_L \cos 0^o - \frac{3}{\pi} \times 0.5 \times 100 \Rightarrow V_L = 198V \qquad (1.1)$$

$$\text{Back EMF at rated speed} = E_b = V_a - I_a R_a$$

$$E_b = 220 - 100 \times 0.1 = 210V \qquad (1.2)$$

Under regenerative braking in the reverse direction at rated speed,

$$E_b = V_o - I_o R_a \qquad (1.3)$$

Therefore from equation (14),

$$E_b = \frac{3\sqrt{2}}{\pi} V_L \cos\alpha - \frac{3}{\pi}(\omega L) I_o - I_o R_a \qquad (1.4)$$

$$\frac{3\sqrt{2}}{\pi} \times 198 \times \cos\alpha - \frac{3}{\pi} \times 0.5 I_o + 0.1 I_o = -210V \qquad (1.5)$$

also from equation (9),

$$\cos\alpha - \cos(\alpha + \mu) = \frac{\sqrt{2}(\omega L)}{V_L} I_o \qquad (1.6)$$

$$\cos\alpha - \cos(\alpha + \mu) = \frac{\sqrt{2} \times 0.5}{198} I_o \qquad (1.7)$$

At the limiting condition of commutation failure,

$$\alpha + \mu \approx 180^o \qquad (1.8)$$

From equation (9),

$$\therefore \cos\alpha = \frac{I_o}{198 \times \sqrt{2}} - 1 \qquad (1.9)$$

$$\frac{3\sqrt{2}}{\pi} \times 198 \times \left(\frac{I_0}{198 \times \sqrt{2}} - 1 \right) - \frac{3}{\pi} \times 0.5 I_o + 0.1 I_o = -210V$$

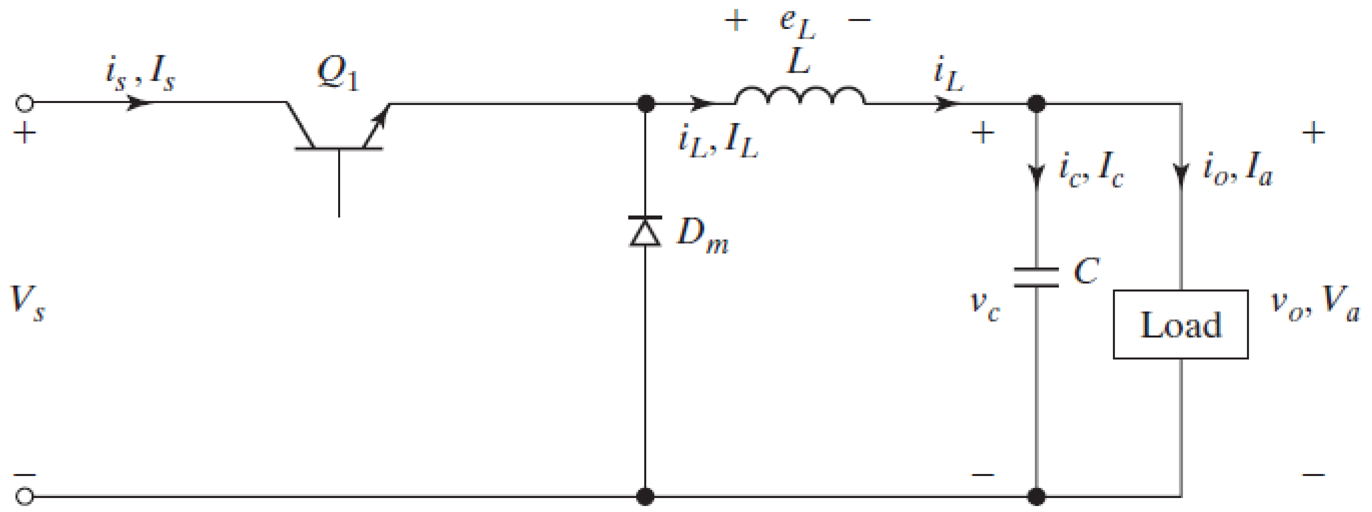
$$0.377 I_o = 57.4$$

$$\therefore I_o = 152.24\,A$$

Torque is directly proportional to current. Since current has increased to 150% of rated current, the maximum braking torque will be 150 % of the rated motor torque

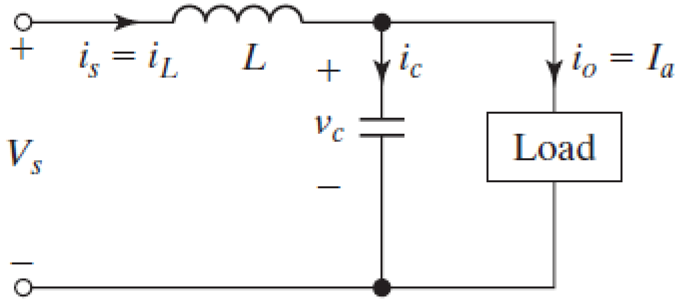
DC-DC Converters

Buck Converter

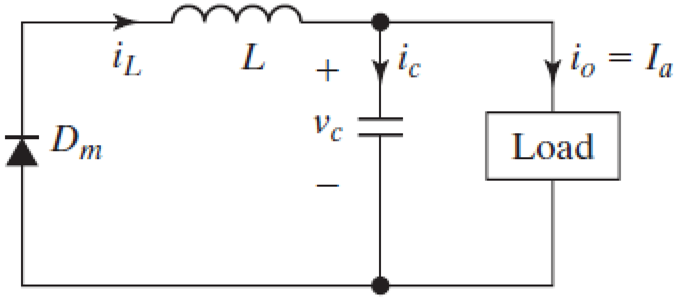


Circuit Diagram

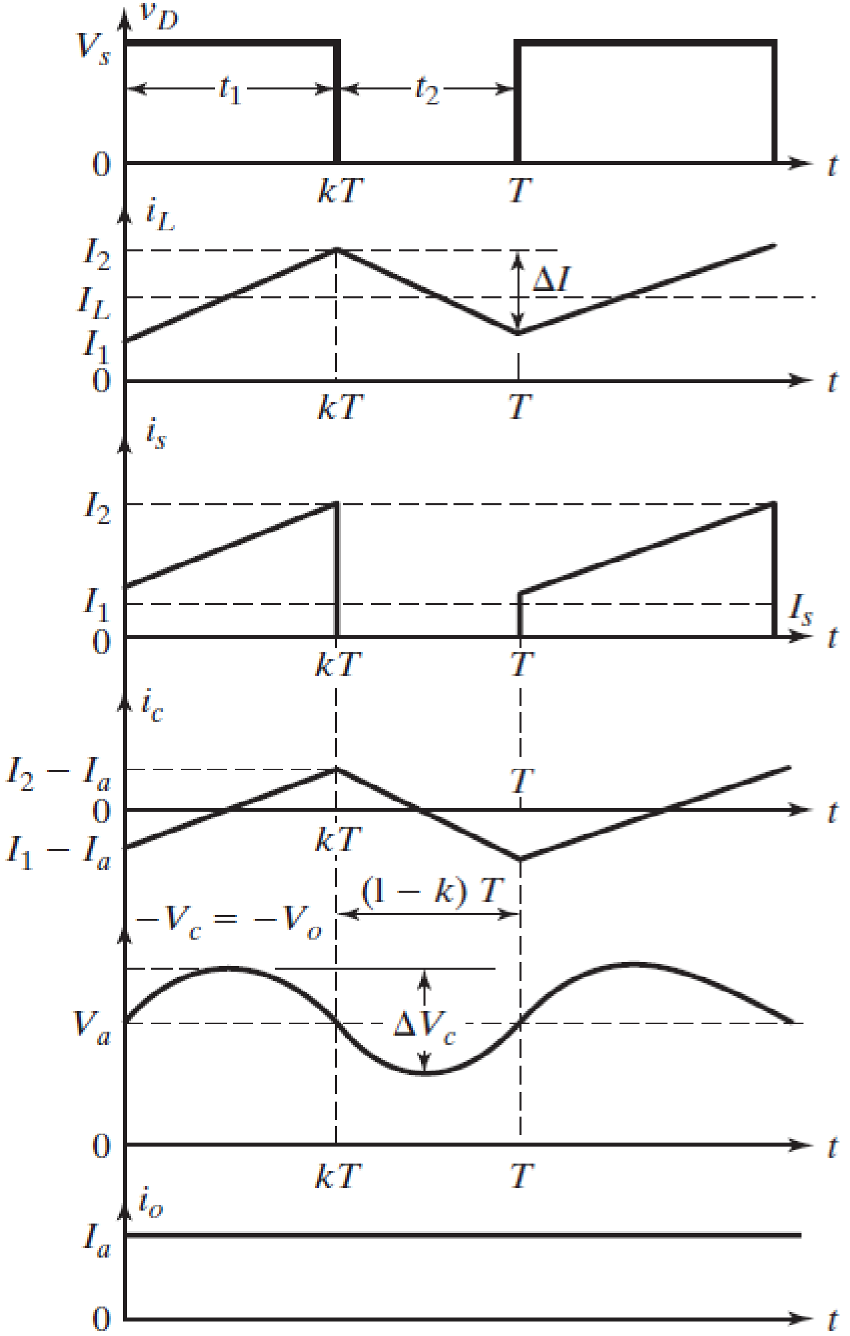
Operating modes

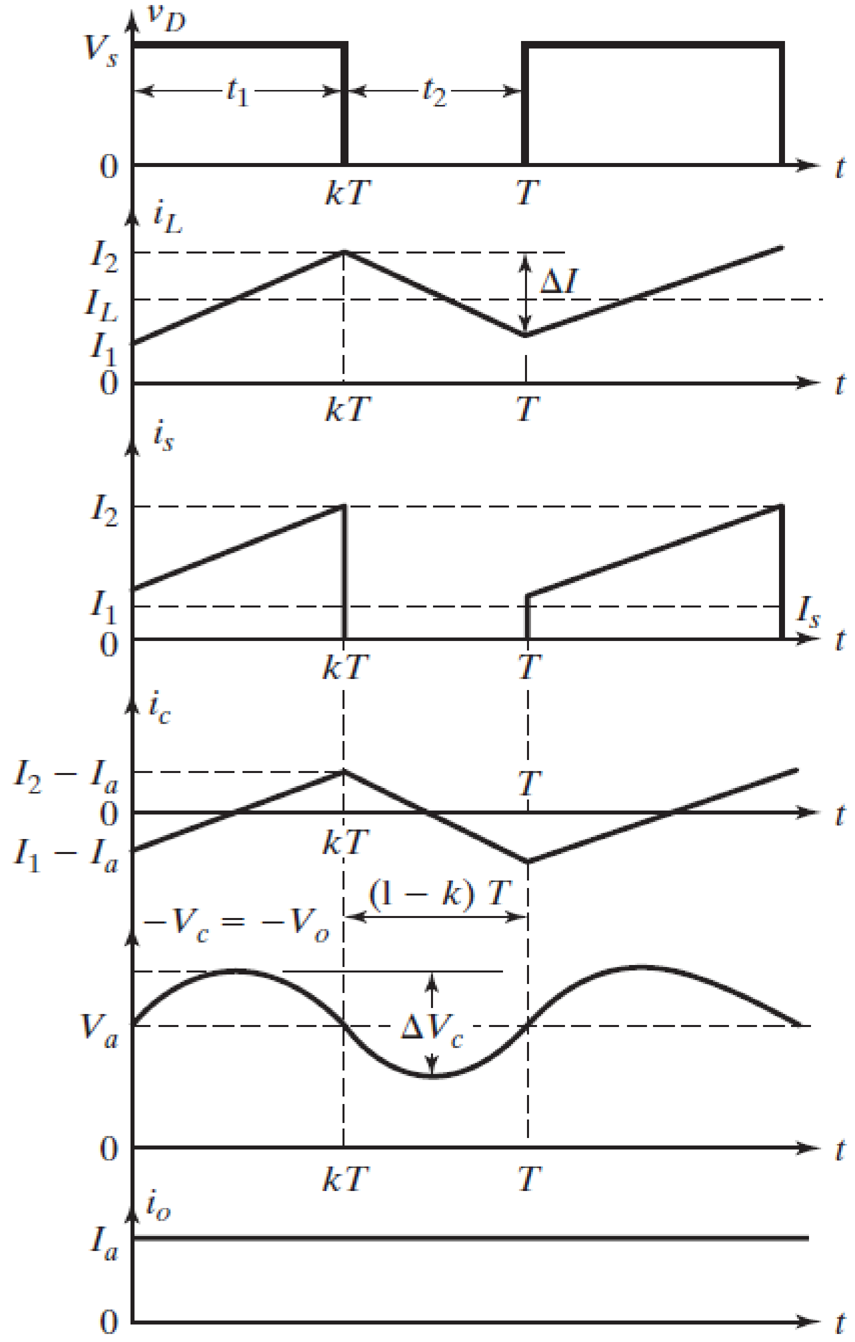


Mode 1



Mode 2





The voltage across the inductor L is, in general,

$$e_L = L\frac{di}{dt}$$

Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1,

$$V_s - V_a = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1} \tag{1}$$

$$t_1 = \frac{\Delta I\, L}{V_s - V_a} \tag{2}$$

And the inductor current falls linearly from I_2 to I_1 in time t_2,

$$-V_a = -L\frac{\Delta I}{t_2} \tag{3}$$

$$t_2 = \frac{\Delta I\, L}{V_a} \tag{4}$$

Where $\Delta I = I_2 - I_1$ is the peak to peak ripple current of the inductor L. Equating the value of ΔI in equations (1) and (3) gives

$$\Delta I = \frac{(V_s - V_a)t_1}{L} = \frac{V_a t_2}{L}$$

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$ yields the average output voltage as

$$V_a = V_s \frac{t_1}{T} = kV_s \qquad (5)$$

Assuming a lossless circuit, $V_s I_s = V_a I_a = kV_s I_a$ and the average input current

$$I_s = kI_a \qquad (6)$$

From equations (2) and (4), the switching period T can be expressed as,

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I\, L}{V_s - V_a} + \frac{\Delta I\, L}{V_a} = \frac{\Delta I\, L V_s}{V_a(V_s - V_a)} \qquad (7)$$

$$\Delta I = \frac{V_a(V_s - V_a)}{fLV_s} \quad \text{or} \quad \Delta I = \frac{V_s k(1-k)}{fL} \qquad (8)$$

$$L = \frac{V_s k(1-k)}{f\,\Delta I} \qquad (9)$$

Using Kirchoff's current law, inductor current i_L is

$$i_L = i_c + i_o$$

Assuming the load ripple Δi_o is very small and negligible,

$$\Delta i_L = \Delta i_c.$$

The average capacitor current, which flows into for $t_1/2 + t_2/2 = T/2$ *is*

$$I_c = \frac{\Delta I}{4}$$

The capacitor voltage is expressed as

$$v_c = \frac{1}{C}\int i_c\, dt + v_c(t=0)$$

And the peak to peak ripple voltage of the capacitor is

$$\Delta V_c = v_c - v_c(t=0) = \frac{1}{C}\int_0^{T/2} \frac{\Delta I}{4}\, dt = \frac{\Delta I\, T}{8C} = \frac{\Delta I}{8fC} \qquad (10)$$

$$C = \frac{\Delta I}{8f\,\Delta V_c} \qquad (11)$$

Substituting the value of ΔI from equations (8) yields, $\Delta V_c = \frac{V_s k(1-k)}{8LCf^2}$ or $\Delta V_c = \frac{V_a(V_s - V_a)}{8LCf^2 V_s}$ (12)

Condition for continuous inductor current and capacitor voltage. If I_L is the average inductor current, the inductor ripple current $\Delta I = 2I_L$.

Using equations (5) and (8), we get

$$\frac{V_S(1-k)k}{fL} = 2I_L = 2I_a = \frac{2kV_s}{R} \qquad (13)$$

which gives the critical value of the inductor L_c as

$$L_c = L = \frac{(1-k)R}{2f}$$

If V_c is the average capacitor voltage, the capacitor ripple voltage $\Delta V_c = 2V_a$.

Using equations (5) and (12), we get

$$\frac{V_s(1-k)k}{8LCf^2} = 2V_a = 2kV_s$$

which gives the critical value of the capacitor C_c as

$$C_c = C = \frac{1-k}{16Lf^2} \qquad (14)$$

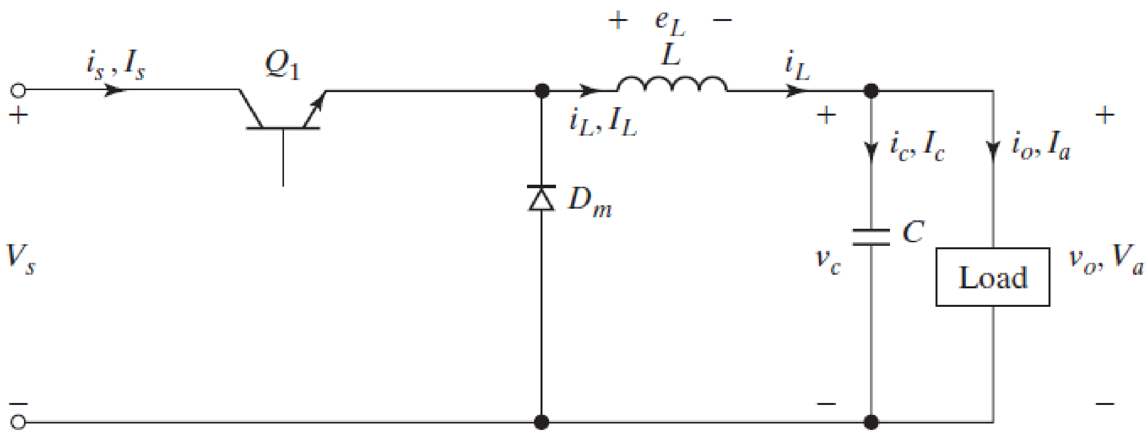
Problem 2.5

The buck converter has an input voltage of $V_s = 12$ V. The required output voltage is $V_a = 5$ V at R = 500 Ω and the peak-to-peak output voltage ripple voltage is 20 mV and the switching frequency is 25 kHz. If the peak-to-peak ripple current of the inductor is limited to 0.8 A, Determine (a) the duty cycle k; (b) the filter inductance L ; (c) the filter capacitance C ; and (d) the critical values of L and C .

Given: $V_s = 12$ V, $\Delta V_c = 20$ mV, $\Delta I = 0.8$ A, f = 25 kHz, and $V_a = 5$ V.

To Find:

(a) The duty cycle, k; (b) The filter inductance, L;
(c) The filter capacitance, C; (d) critical values of *L* and *C*



Formulae used:

(a) From Average Output Voltage, $V_a = V_s k$, we get $k = V_a / V_s$

(b) The filter inductance is $L = \dfrac{V_s k(1-k)}{f\,\Delta I}$

(c) The filter capacitance is $C = \dfrac{V_s k(1-k)}{8Lf^2 \Delta V}$

(d) (i) The critical value of inductance is $L_c = \dfrac{(1-k)R}{2f}$

(ii) The critical value of capacitance is $C_c = \dfrac{1-k}{16Lf^2}$

Solution:

(a) From Average Output Voltage $V_a = V_s k$,we get $k = V_a / V_s = 5/12 = 0.416$

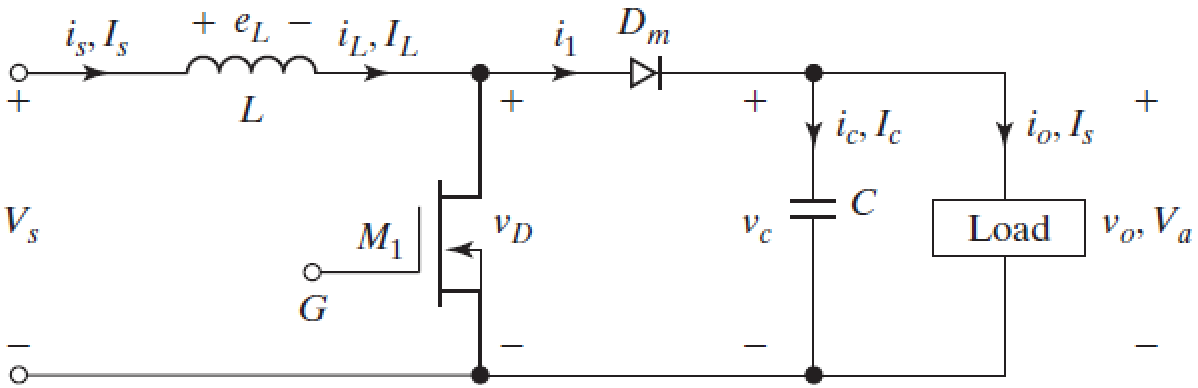
(b) The filter inductance is $L = \dfrac{V_s k(1-k)}{f \,\Delta I} = \dfrac{12\times 0.416\times(1-0.416)}{25\times 10^{3}\times 0.8} = 145.76\mu H$

(c) The filter capacitance is $C = \dfrac{V_s k(1-k)}{8Lf^{2}\Delta V} = \dfrac{12\times 0.416\times(1-0.416)}{8\times 145.76\times 10^{-6}\times(25\times 10^{3})^{2}\times 20\times 10^{-3}} = 200\mu F$

(d) (i) The critical value of inductance is $L_c = \dfrac{(1-k)R}{2f} = \dfrac{(1-0.416)\times 500}{2\times 25\times 10^{3}} = 5.83mH$

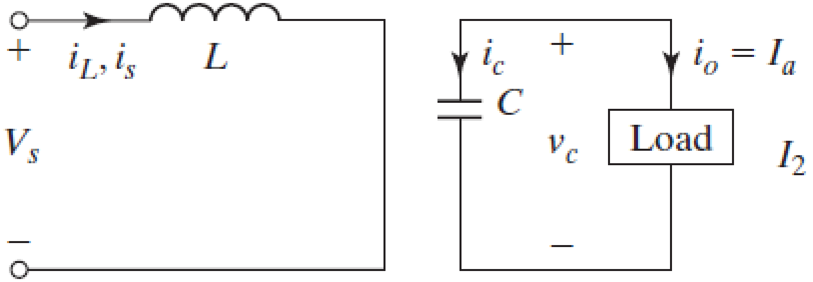
(ii) The critical value of capacitance is $C_c = \dfrac{1-k}{16Lf^{2}} = \dfrac{1-0.416}{16\times 145.76\times 10^{-6}\times(25\times 10^{3})^{2}} = 0.4\mu F$

Boost Converter

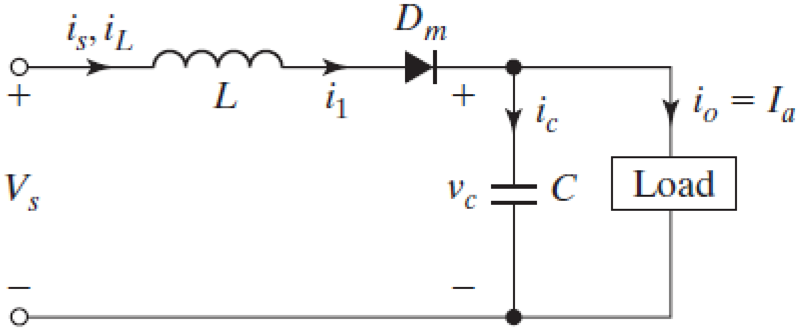


Circuit Diagram

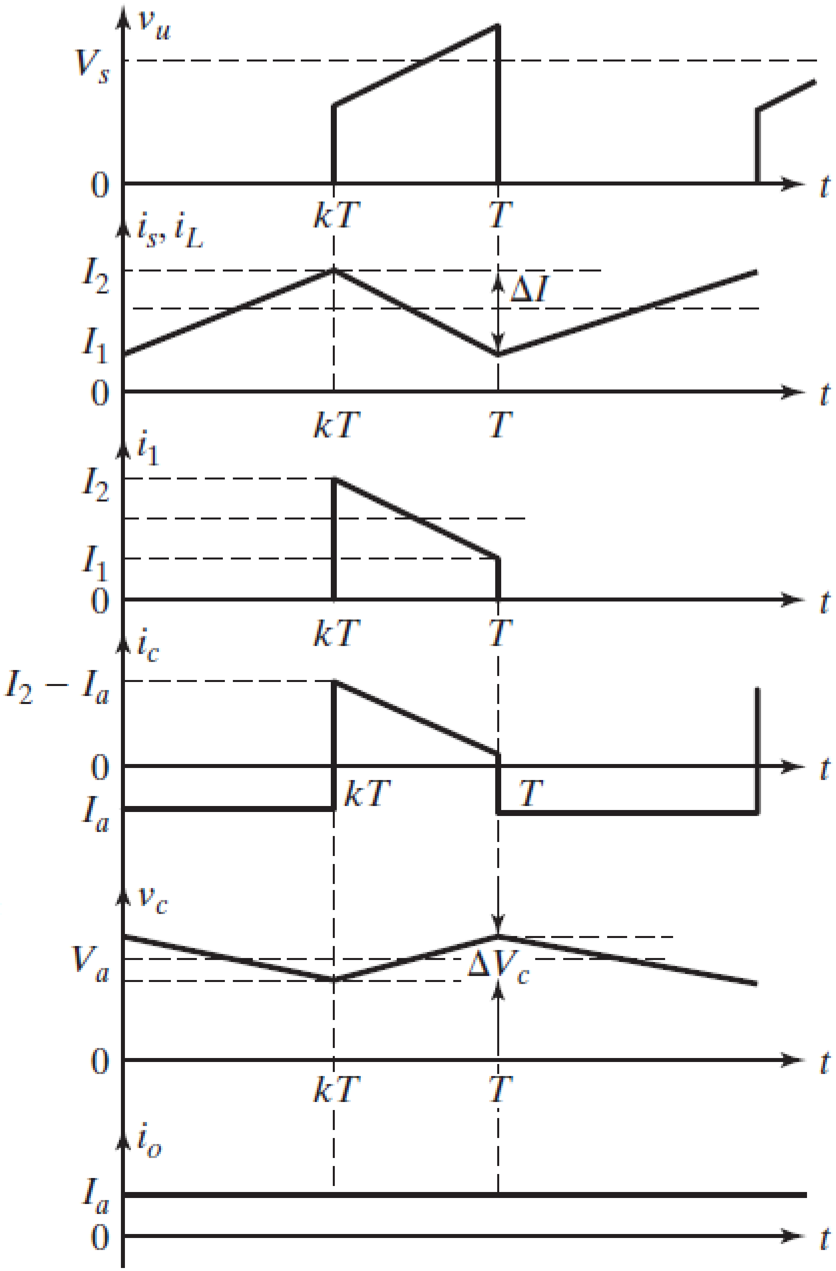
Operating modes

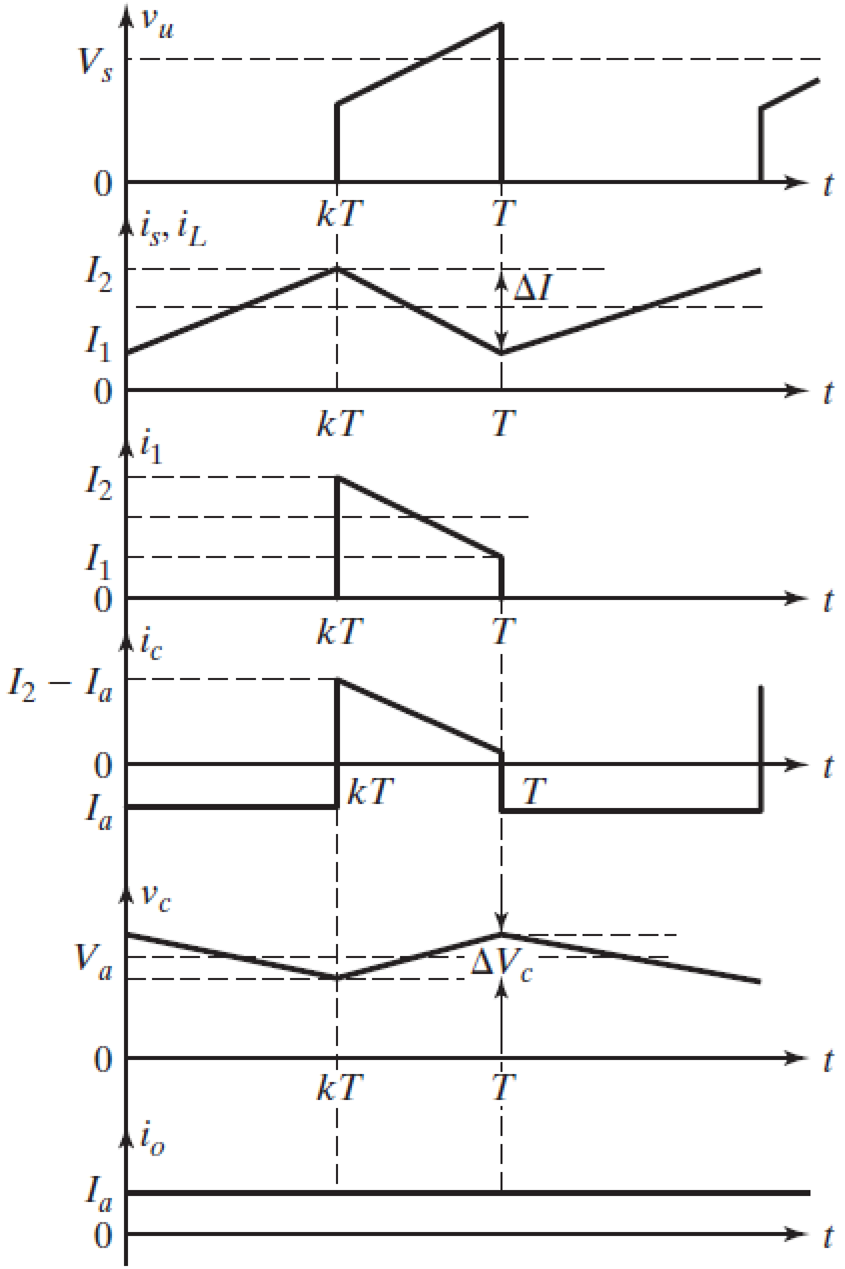


Mode 1



Mode 2





Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1,

$$V_s = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1} \tag{1}$$

$$t_1 = \frac{\Delta I L}{V_s} \tag{2}$$

and the inductor current falls linearly from I_2 to I_1 in time t_2,

$$V_s - V_a = -L\frac{\Delta I}{t_2} \tag{3}$$

$$t_2 = \frac{\Delta I L}{V_a - V_s} \tag{4}$$

where $\Delta I = I_2 - I_1$ is the peak-to-peak ripple current of inductor L.

Using equations (1) and (3)

$$\Delta I = \frac{V_s t_1}{L} = \frac{(V_a - V_s)t_2}{L}$$

Substituting $t_1 = kT$ and $t_2 = (1-k)\, T$ yields the average output voltage,

$$V_a = V_s \frac{T}{t_2} = \frac{V_s}{1-k} \qquad (5)$$

which gives

$$(1-k) = \frac{V_s}{V_a} \qquad (6)$$

Substituting $k = t_1/T = t_1 f$ into Equations (6) yields

$$t_1 = \frac{V_a - V_s}{V_a f} \qquad (7)$$

Assuming a lossless circuit, $V_s I_s = V_a I_a = V_s I_a/(1-k)$ and the average input current is

$$I_s = \frac{I_a}{1-k} \qquad (8)$$

Peak-to-peak inductor ripple current.

The switching period T can be found from

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I L}{V_s} + \frac{\Delta I L}{V_a - V_s} = \frac{\Delta I L V_a}{V_s (V_a - V_s)}$$

and this gives the peak-to-peak ripple current: (9)

$$\Delta I = \frac{V_s (V_a - V_s)}{f L V_a} \qquad (10)$$

$$\Delta I = \frac{V_s k}{f L} \qquad (11)$$

$$L = \frac{V_s k}{f \Delta I} \qquad (12)$$

Peak-to-peak capacitor ripple voltage.

When the transistor is on, the capacitor supplies the load current for $t = t_1$. the average capacitor current during time t1 is $Ic = Ia$

and the peak-to-peak ripple voltage of the capacitor is

$$\Delta V_c = v_c - v_c(t=0) = \frac{1}{C}\int_0^{t_1} I_c\,dt = \frac{1}{C}\int_0^{t_1} I_a\,dt = \frac{I_a t_1}{C} \quad (13)$$

Substituting $t_1 = (V_a - V_s)/(V_a f)$ from equations (7) gives

$$\Delta V_c = \frac{I_a(V_a - V_s)}{V_a f C} \quad (14)$$

$$\Delta V_c = \frac{I_a k}{fC} \quad (15)$$

$$C = \frac{I_a k}{f\Delta V_c} \quad (16)$$

Condition for continuous inductor current and capacitor voltage

If I_L is the average inductor current, at the critical condition for continuous conduction the inductor ripple current $\Delta I = 2I_L$

Using equations (5) and (11), we get

$$\frac{kV_s}{fL} = 2I_L = 2I_s = \frac{2V_s}{(1-k)^2}$$

which gives the critical value of the inductor L_c as

$$L_c = L = \frac{k(1-k)R}{2f} \quad (17)$$

If V_c is the average capacitor current, at the critical condition for continuous conduction the capacitor ripple voltage $\Delta V_c = 2V_a$

Using equation (15), we get

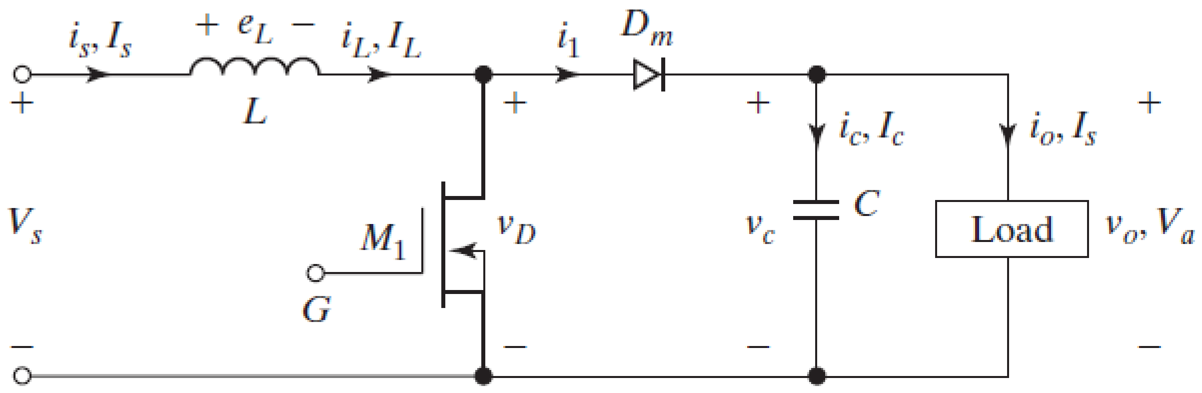
$$\frac{I_a k}{Cf} = 2V_a = 2I_a R$$

which gives the critical value of the capacitor C_c as

$$C_c = C = \frac{k}{2fR} \quad (18)$$

Problem 2.6

The boost converter shown has an input voltage of $V_s = 5$ V. The average output voltage $V_a = 15$ V and the average load current of $I_a = 0.5$ A. The switching frequency is 25 kHz. If L = 150 µH and filter capacitance C = 220 µF, Determine (a) the duty cycle k ; (b) the peak-to-peak ripple current of inductor, ΔI ; (c) the peak-to-peak output voltage ripple, ΔV_c ; and (d) the critical values of L and C .



Given:

$V_s = 5$ V, $V_a = 15$ V, $I_a = 0.5$ A, f = 25 kHz, L = 150 µH, C = 220 µF.

To Find:

(a) The duty cycle, k
(b) peak-to-peak ripple current of inductor, ΔI
(c) peak-to-peak output voltage ripple, ΔV_c
(d) critical values of L and C

Formulae used:

(a) Average Output Voltage, $\frac{V_a}{V_s} = \frac{1}{1-k}$ from which, $k = 1 - \frac{V_s}{V_a}$

(b) Peak-to-peak inductor current ripple is $\Delta I = \frac{V_s k}{f L}$

(c) Peak-to-peak output voltage ripple is $\Delta V_c = \frac{I_a k}{f C}$

(d) The load resistance is $R = \frac{V_a}{I_a}$

(i) The critical value of inductance is $L_c = \frac{k(1-k)R}{2f}$

(ii) The critical value of capacitance is $C_c = \frac{k}{2f\ R}$

Solution:

(a) Average Output Voltage is $V_a = V_s \dfrac{1}{1-k}$, from which the duty cycle is $k = 1 - \dfrac{V_s}{V_a} = 1 - (5/15) = 2/3 = 0.667$

(b) Peak-to-peak inductor ripple is $\Delta I = \dfrac{V_s k}{f\,L} = \dfrac{5 \times 0.667}{25{,}000 \times 150 \times 10^{-6}} = 0.89\,A$

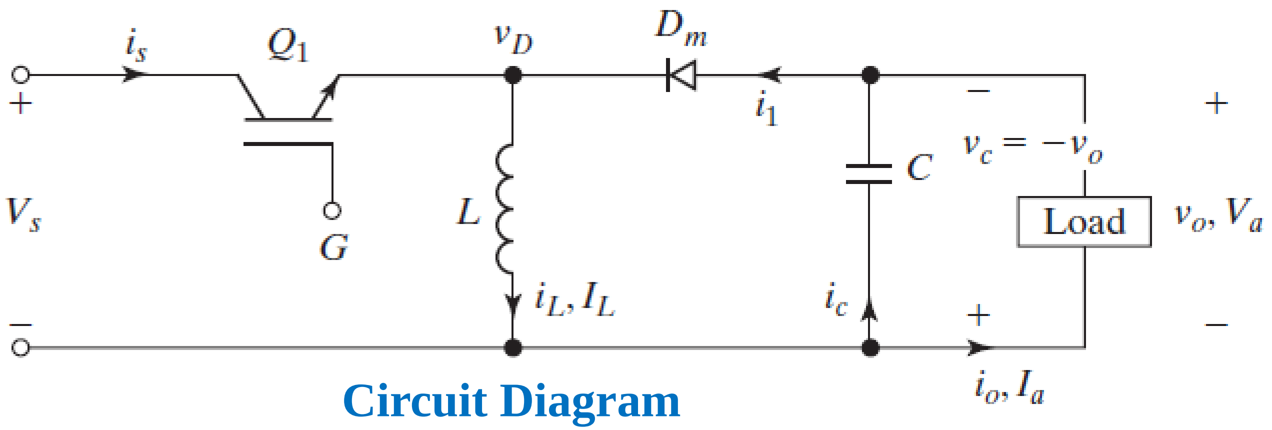
(c) Peak-to-peak output voltage ripple is $\Delta V_c = \dfrac{I_a k}{f\,C} = \dfrac{0.5 \times 0.667}{25{,}000 \times 220 \times 10^{-6}} = 60.61\text{mV}$

(d) The load resistance is $R = \dfrac{15}{0.5} = 30\Omega$

(i) The critical value of inductance is $L_c = \dfrac{k(1-k)R}{2f} = \dfrac{(1-0.667) \times 30 \times 0.667}{2 \times 25 \times 10^{3}} = 133\mu H$

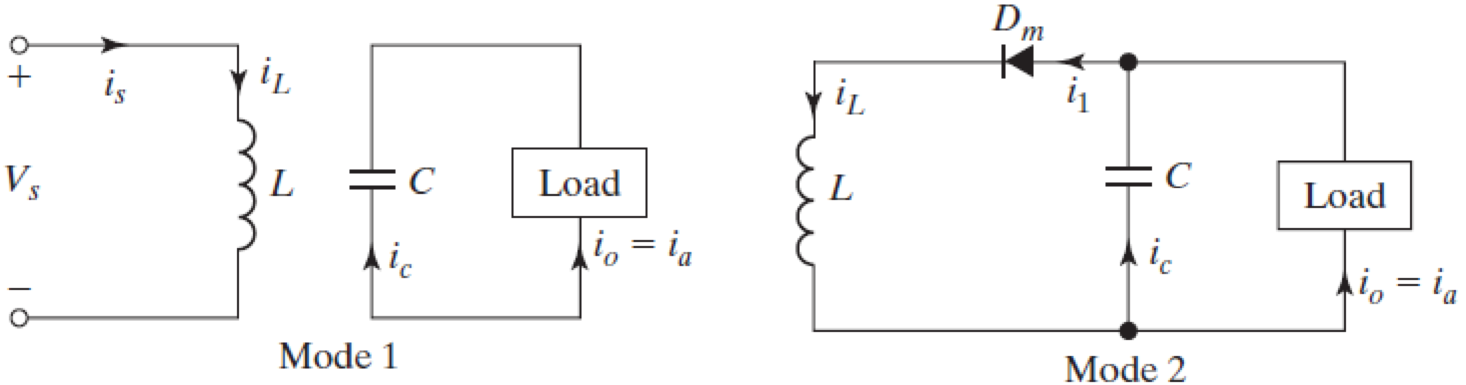
(ii) The critical value of capacitance is $C_c = \dfrac{k}{2f\,R} = \dfrac{0.667}{2 \times 25 \times 10^{3} \times 30} = 0.44\,\mu F$

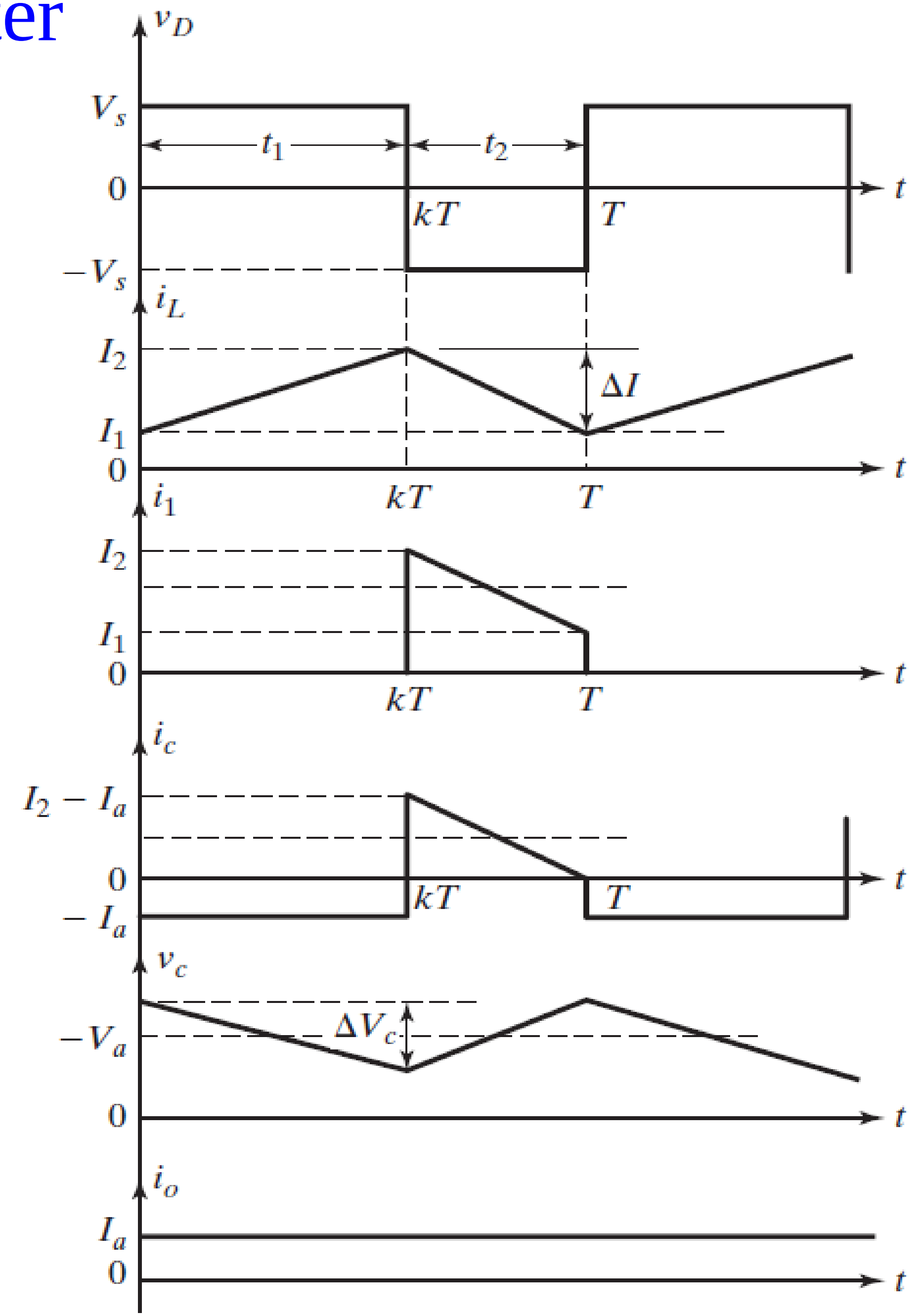
Buck-Boost Converter

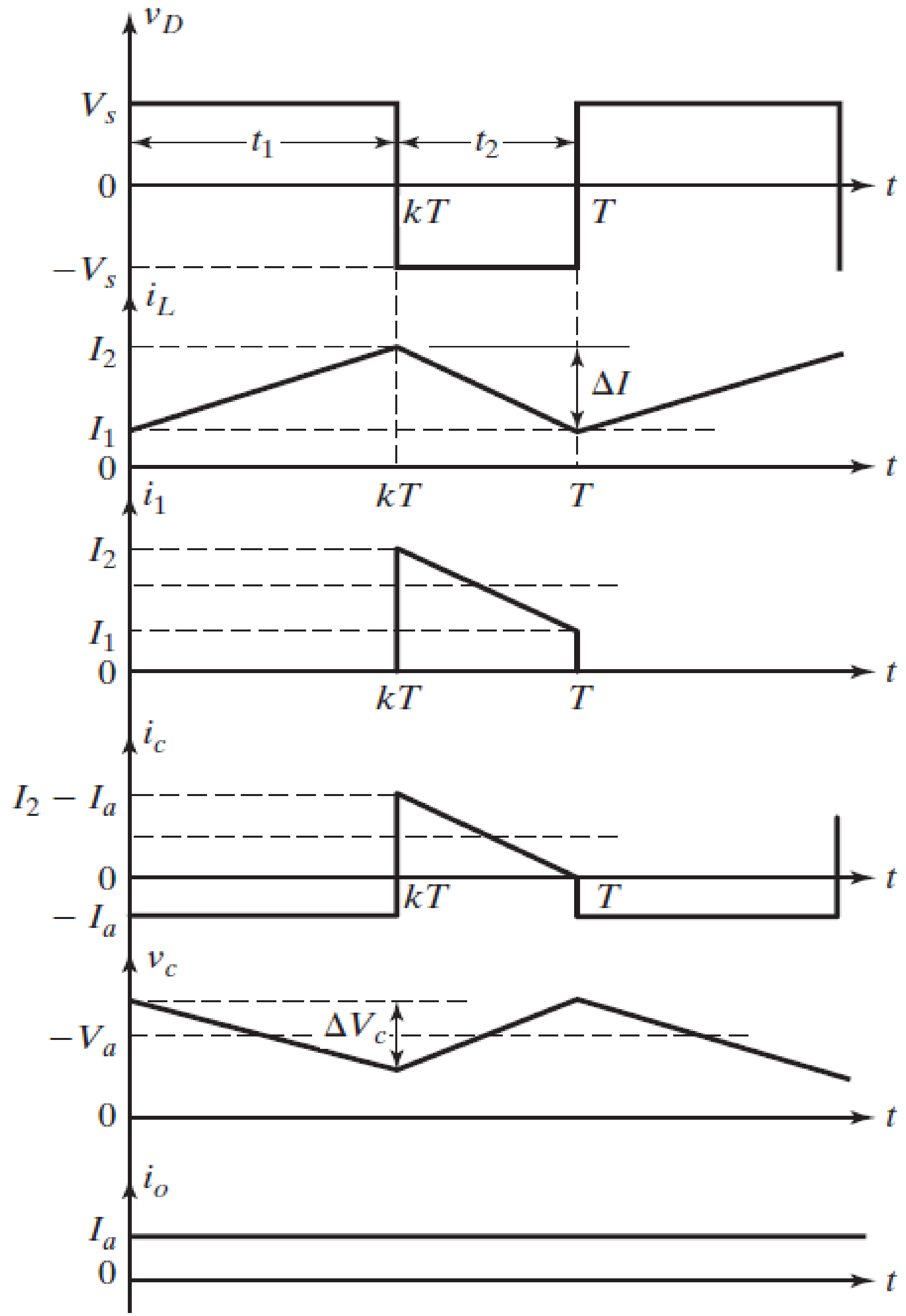


Circuit Diagram

Operating modes







Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1,

$$V_s = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1} \tag{1}$$

$$t_1 = \frac{\Delta I L}{V_s} \tag{2}$$

and the inductor current falls linearly from I_2 to I_1 in time t_2,

$$V_a = -L\frac{\Delta I}{t_2} \tag{3}$$

$$t_2 = \frac{-\Delta I L}{V_a} \tag{4}$$

where $\Delta I = I_2 - I_1$ is the peak-to-peak ripple current of inductor L.

From equations (1) and (3),

$$\Delta I = \frac{V_s t_1}{L} = \frac{-V_a t_2}{L}$$

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$, the average output voltage is

$$V_a = -\frac{V_s k}{1 - k} \tag{5}$$

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$ into Equations (5) yields

$$(1 - k) = \frac{-V_s}{V_a - V_s} \tag{6}$$

Substituting $t_2 = (1 - k)\,T$, and $(1 - k)$ from Equation (6) into (5) yields,

$$t_1 = \frac{V_a}{(V_a - V_s)f} \tag{7}$$

Assuming a lossless circuit, $V_s I_s = -V_a I_a = V_s I_a k/(1 - k)$

And the average input current I_s is related to the average output current I_a by

$$I_s = \frac{I_a k}{1 - k} \tag{8}$$

The switching period T can be found from

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta IL}{V_s} - \frac{\Delta IL}{V_a} = \frac{\Delta IL(V_a - V_s)}{V_s V_a} \tag{9}$$

and this gives the peak-to-peak ripple current,

$$\Delta I = \frac{V_s V_a}{fL(V_a - V_s)} \tag{10}$$

$$\Delta I = \frac{V_s k}{fL} \tag{11}$$

$$L = \frac{V_s k}{f\,\Delta I} \tag{12}$$

The average inductor current is given by

$$I_L = I_s + I_a = \frac{kI_a}{1 - k} + I_a = \frac{I_a}{1 - k} \tag{13}$$

When transistor Q1 is ON, the filter capacitor supplies the load current for t = t1. The average discharging current of the capacitor is Ic = -Ia and the peak to peak ripple voltage of the capacitor is

$$\Delta V_c = \frac{1}{C}\int_0^{t_1} -I_c\,dt = \frac{1}{C}\int_0^{t_1} I_a\,dt = \frac{I_a t_1}{C} \tag{14}$$

Substituting $t_1 = V_a/[(V_a - V_s)f]$ from equation (7) becomes

$$\Delta V_c = \frac{I_a V_a}{(V_a - V_s)fC} \quad \text{or} \quad \Delta V_c = \frac{I_a k}{fC} \tag{15}$$

$$C = \frac{I_a k}{f\,\Delta V_c} \tag{16}$$

Condition for continuous inductor current and capacitor voltage.

If I_L is the average inductor current, at the critical condition for continuous conduction the inductor ripple current $\Delta I = 2I_L$.
Using Equations (5) and (11), we get

$$\frac{kV_s}{fL} = 2I_L = 2I_a = \frac{2kV_s}{(1-k)R}$$

which gives the critical value of the inductor L_c as

$$L_c = L = \frac{(1-k)R}{2f} \tag{17}$$

If V_c is the average capacitor voltage, at the critical condition for continuous conduction the capacitor ripple voltage $\Delta V_c = -2V_a$.
Using Equations (15), we get

$$-\frac{I_a k}{Cf} = -2V_a = -2I_a R$$

which gives the critical value of the capacitor C_c as

$$C_c = C = \frac{k}{2fR} \tag{18}$$

Problem 2.7

The buck-boost converter has an input voltage of $V_s = 12$ V. The duty cycle k = 0.25 and the switching frequency is 25 kHz. The inductance L = 150 μH and filter capacitance C = 220 μF. The average load current $I_a = 1.25$ A. Determine (a) the average output voltage; (b) the peak-to-peak output voltage ripple, ΔV_c ; (c) the peak-to-peak ripple current of inductor, ΔI ; (d) the critical values of L and C.

Given:

$V_s = 12$ V, k = 0.25, $I_a = 1.25$ A, f = 25 kHz, L = 150 μH, C = 220 μF.

To Find:

(a) Average Output Voltage V_a
(b) peak-to-peak output voltage ripple, ΔV_c
(c) peak-to-peak ripple current of inductor, ΔI
(d) (e) critical values of L and C

Formulae used:

(a) Average Output Voltage $V_o = V_s \frac{(-k)}{1-k}$

(b) Peak-to-peak output voltage ripple is $\Delta V_c = \frac{I_a V_a}{(V_a - V_s) f\, C}$

(c) Peak-to-peak inductor current ripple is $\Delta I = \frac{V_s V_a}{(V_a - V_s) f\, L}$

(d) (i) The load resistance is $R = \frac{-V_a}{I_a}$

(ii) The critical value of inductance is $L_c = \frac{(1-k)R}{2f}$

(iii) The critical value of capacitance is $C_c = \frac{k}{2f\, R}$

Solution:

(a) Average Output Voltage $V_a = 12 \times \frac{(-0.25)}{1-0.25} = -4\,V$

(b) Peak-to-peak output voltage ripple is $\Delta V_c = \frac{I_a V_a}{(V_a - V_s) f\, C} = \frac{I_a k}{f\, C} = \frac{1.25 \times 0.25}{25{,}000 \times 220 \times 10^{-6}} = 56.8\,mV$

(c) Peak-to-peak inductor ripple is $\Delta I = \frac{V_s V_a}{(V_a - V_s) f\, L} = \frac{V_s k}{f\, L} = \frac{12 \times 0.25}{25{,}000 \times 150 \times 10^{-6}} = 0.8\,A$

(d) (i) The load resistance is $R = \frac{4}{1.25} = 3.2\,\Omega$

(ii) The critical value of inductance is $L_c = \frac{(1-k)R}{2f} = \frac{(1-0.25) \times 3.2}{2 \times 25 \times 10^3} = 450\,\mu H$

(iii) The critical value of capacitance is $C_c = \frac{k}{2f\;R} = \frac{0.25}{2 \times 25 \times 10^3 \times 3.2} = 1.56\,\mu F$

Problem 2.8

A 230 V, 960 rpm, 200 A separately excited DC motor with armature resistance of 0.02 Ω, is fed from a chopper which provides both motoring and braking operations. The source has a voltage of 230 V. Assuming continuous conduction, calculate

(i) duty ratio for the chopper in motoring mode at 350 rpm and rated torque
(ii) duty ratio for the chopper in braking mode at 350 rpm and rated torque
(iii) If the maximum duty ratio of the chopper is limited to 0.95 and the maximum permissible motor current is twice the rated, determine the maximum permissible motor speed obtainable without field weakening and power fed to the source
(iv) If the motor field is also controlled in (iii), calculate the field current as a fraction of its rated value for a speed of 1200 rpm

Solution:

Given: DC Motor: Rated voltage – 230V; Rated current – 200 A; Rated speed – 960 rpm; Armature resistance – 0.02 Ω
DC Chopper: DC voltage – 230 V;

(i) To find Duty ratio of the chopper:

At rated operation, $E = V_a - I_a R_a = 230 - (0.02 \times 200) = 226V$

Operating condition: motoring mode, 350 rpm and rated torque

$$E_{350} = \frac{350}{960} \times 226 = 82.4V$$

Motor terminal voltage, $V_a = E + I_a R_a \Rightarrow V_a = 82.4 + (200 \times 0.02) = 86.4V$

Duty ratio, $\delta = \frac{86.4}{230} = 0.376$

(ii) To find Duty ratio of the chopper:

Operating condition: braking mode, 350 rpm and rated torque

$$V_a = E - I_a R_a \Rightarrow V_a = 82.4 - (200 \times 0.02) = 78.4\text{V}$$

$$\delta = \frac{78.4}{230} = 0.34$$

(iii) To find the maximum speed without field weakening at δ=0.95 and at $I=2I_m$

Maximum available voltage,

$$V_a = \delta V_s \Rightarrow V_a = 0.95 \times 230 = 218.5\,\text{V}$$

$$E = V_a + I_a R_a = 218.5 + (200 \times 2 \times 0.02) = 226.5\,\text{V}$$

Maximum permissible motor speed = $\frac{226.5}{226} \times 960 = 962$ rpm

Assuming lossless chopper, power fed into the source

$$V_a I_a = 218.5 \times 400 = 87.4\ \text{kW}$$

(iv) To find the field current fraction

As in (iii), E=226.5 V for which at rated filed current speed= 960 rpm. Assuming linear magnetic circuit, E will be inversely proportional to field current. Field current as a ratio of its rated value = 960/1200 = 0.8

Problem 2.9

The motor in problem 2.8 is now operated in dynamic braking with chopper control with a braking resistance of 2 Ω.

(i) Calculate the duty ratio of the chopper for a motor speed of 600 rpm and braking torque of twice the rated value.

(ii) What will be the motor speed for a duty ratio of 0.6 and motor torque equal to twice it rated torque?

Solution:

$$E_{600} = \frac{600}{960} \times 226 = 141.25\text{V}$$

$$R_{BE} = (1-\delta)R_B = \frac{E}{I_a} - R_a$$

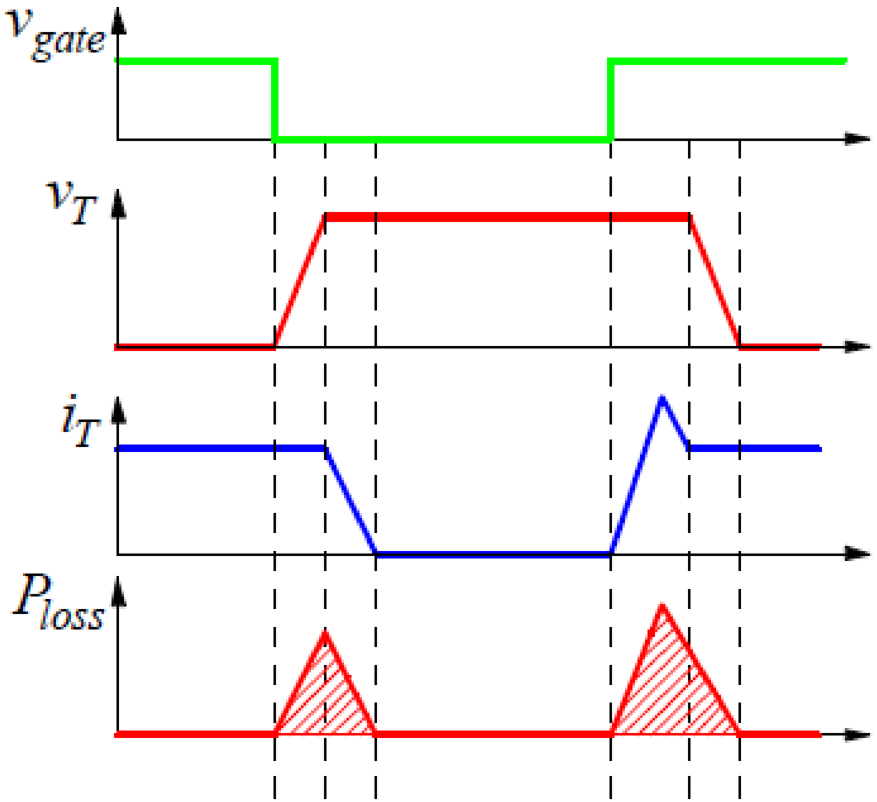
$$(1-\delta)2 = \frac{141.25}{400} - 0.02 \Rightarrow \delta = 0.83$$

$$E \; = I_a[(1-\delta)R_B + R_a] = 400[(1-0.60)\times 2 + 0.02] = 328\,\text{V}$$

$$\text{Speed} = \frac{328}{226} \times 960 = 1393.3\text{rpm}$$

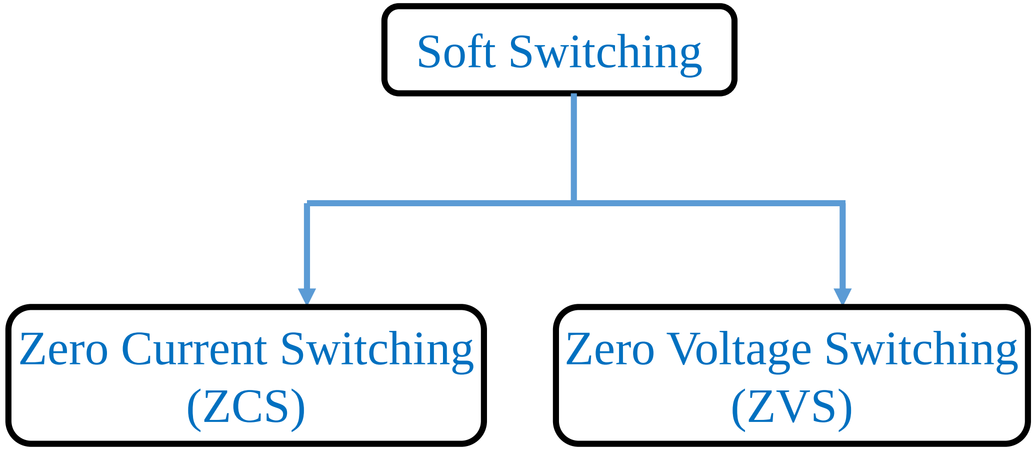
What is Soft Switching ?

Hard Switching Characteristics

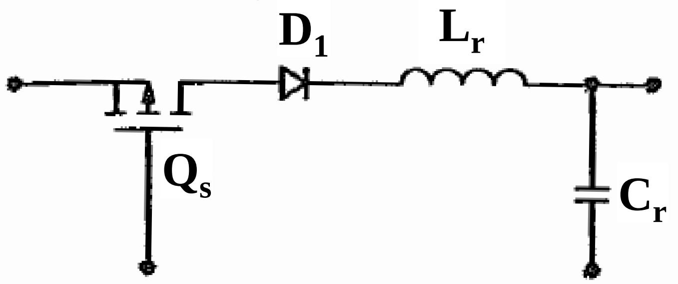


- Switching transitions under conditions where device voltage or current is zero
- Reduced switching losses, switch stress, possibly low EMI, easier thermal management

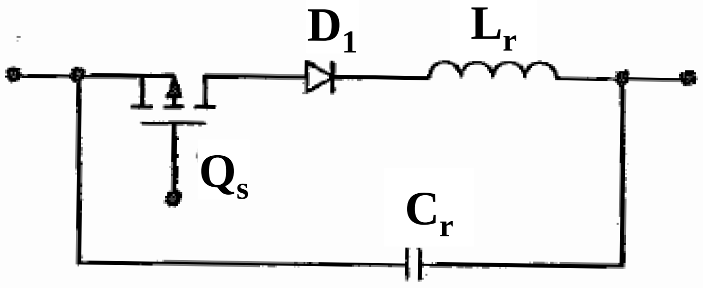
- At turn-on, the switching loss is mainly caused by the dissipation of energy stored in the capacitor of the power switch.
- At turn-off, the series inductance produces high *di/dt*, which results in a high voltage spike across the power switch.



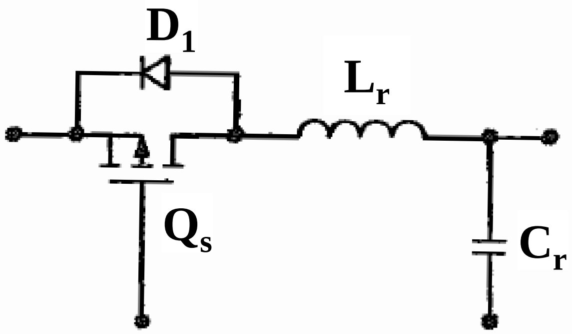
Zero current switching can be implemented using following resonant switch configurations.



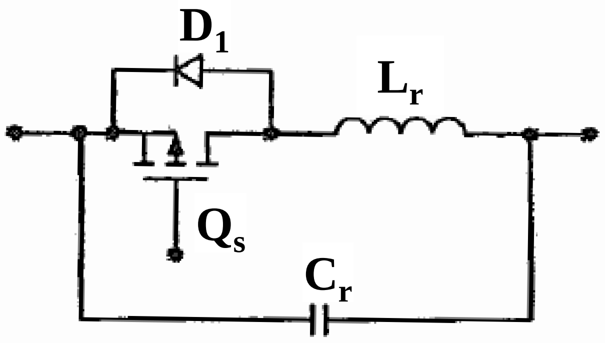
L-type, Half wave



M-type, Half wave



L-type, Full wave

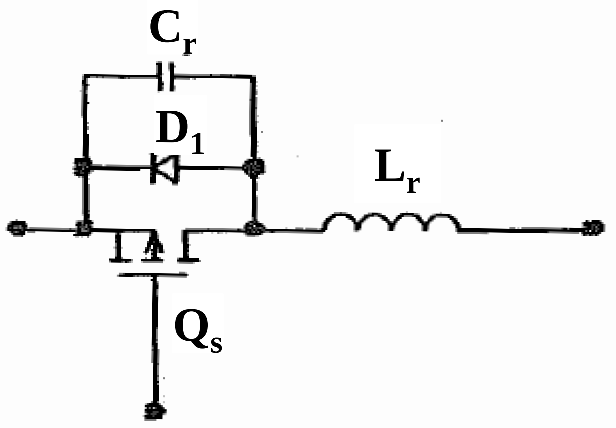


M-type, Full wave

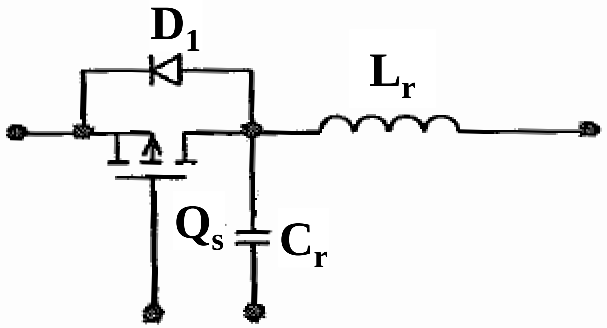
ZCS Power switches are turn ON and OFF at zero current condition

ZVS Power switches are turn ON and OFF at zero voltage condition

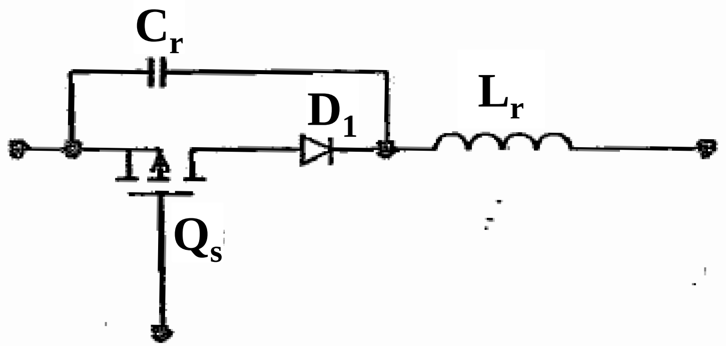
Zero voltage switching can be implemented using following resonant switch configurations.



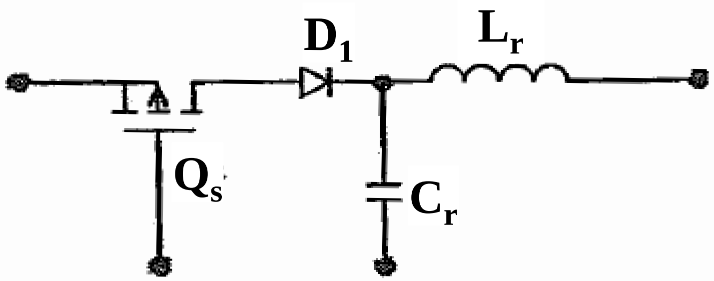
L-type, Half wave



M-type, Half wave



L-type, Full wave



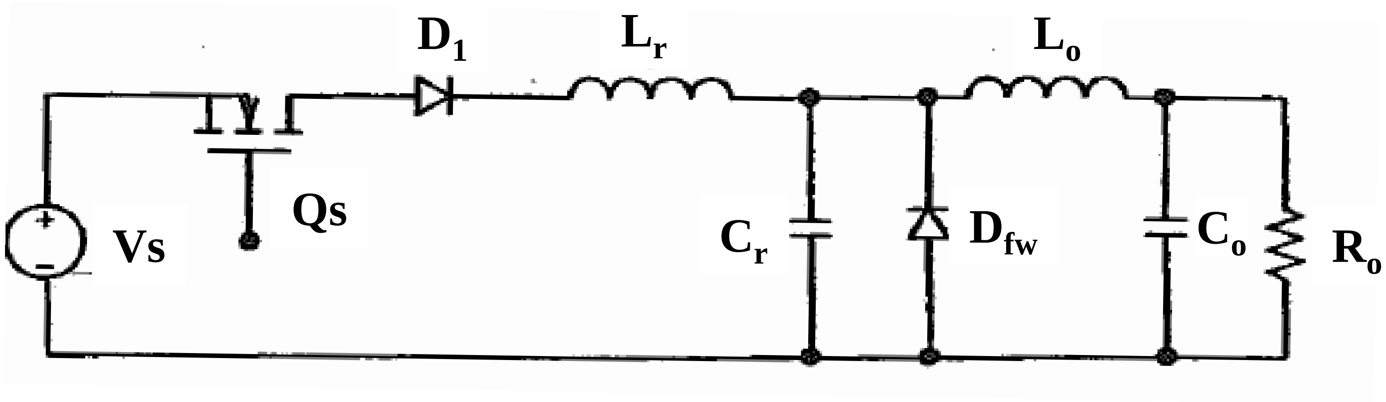
M-type, Full wave

In practical conditions, L-type configuration are preferred over M-type configuration.

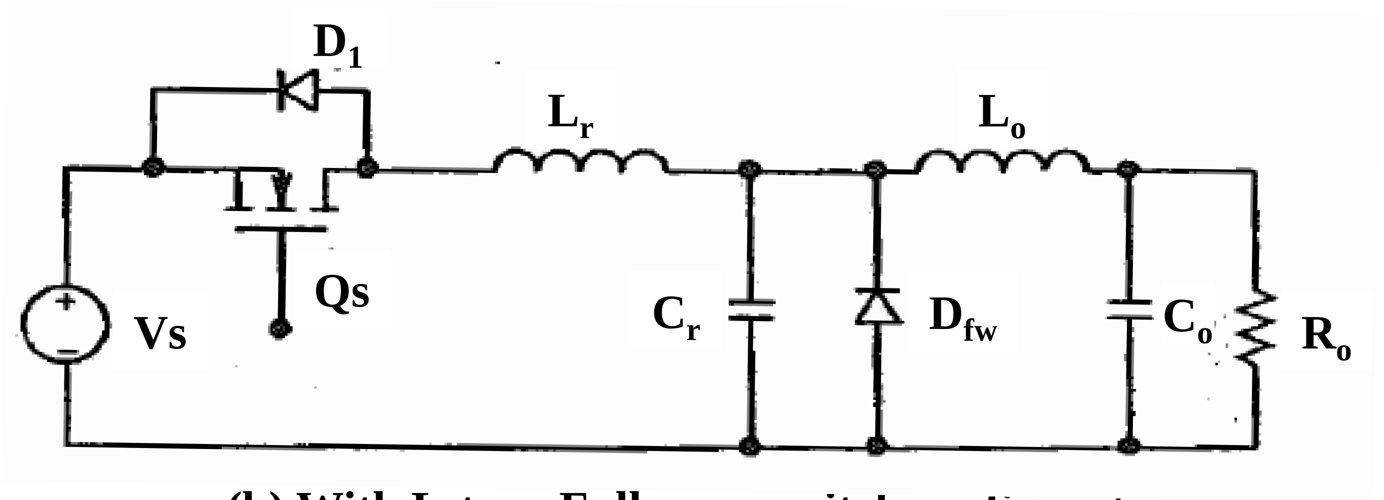
This is due to the fact that during turn OFF, the energy trapped in the inductor of the M-type configuration causes voltage transients across the switch. Thus, the switch may fail.

- In half wave type, the resonant current is not allowed to flow back to the source.
- In full wave type, an antiparallel diode, D1 is connected across the switching transistor and the resonant current can flow bi-directionally to both the load and source.

Zero Current Switching of Buck Converter



(a) With L-type Half wave switch configuration



(b) With L-type Full wave switch configuration

Assumptions

- The output current, I_o is constant.
- The output inductor (L_o) is much larger than resonant inductor (L_r).
- The switching devices are ideal.
- The resonant inductor (L_r) and capacitor (C_r) are ideal circuit elements with no lossy or parasitic elements.

Initial conditions

- The transistor Q_s is in OFF state.
- Freewheeling diode, D_{fw} carries the steady state output current of I_o, and resonant capacitor voltage, V_{Cr} at ($t \le 0$) is clamped at zero voltage by the freewheeling diode.
- No current is flowing through the resonant inductor, ie., $i_{Lr}(0) = 0$.

Mode 1 ($0 < t \le t_1$)

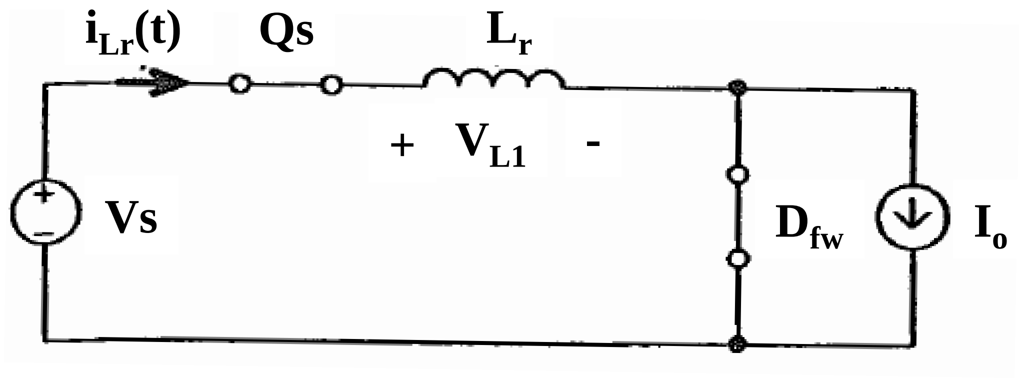
- Switching transistor Q_s is switched ON
- At the end of mode 1, the voltage across the resonant inductor, $V_{Lr}(t)$ is given by,

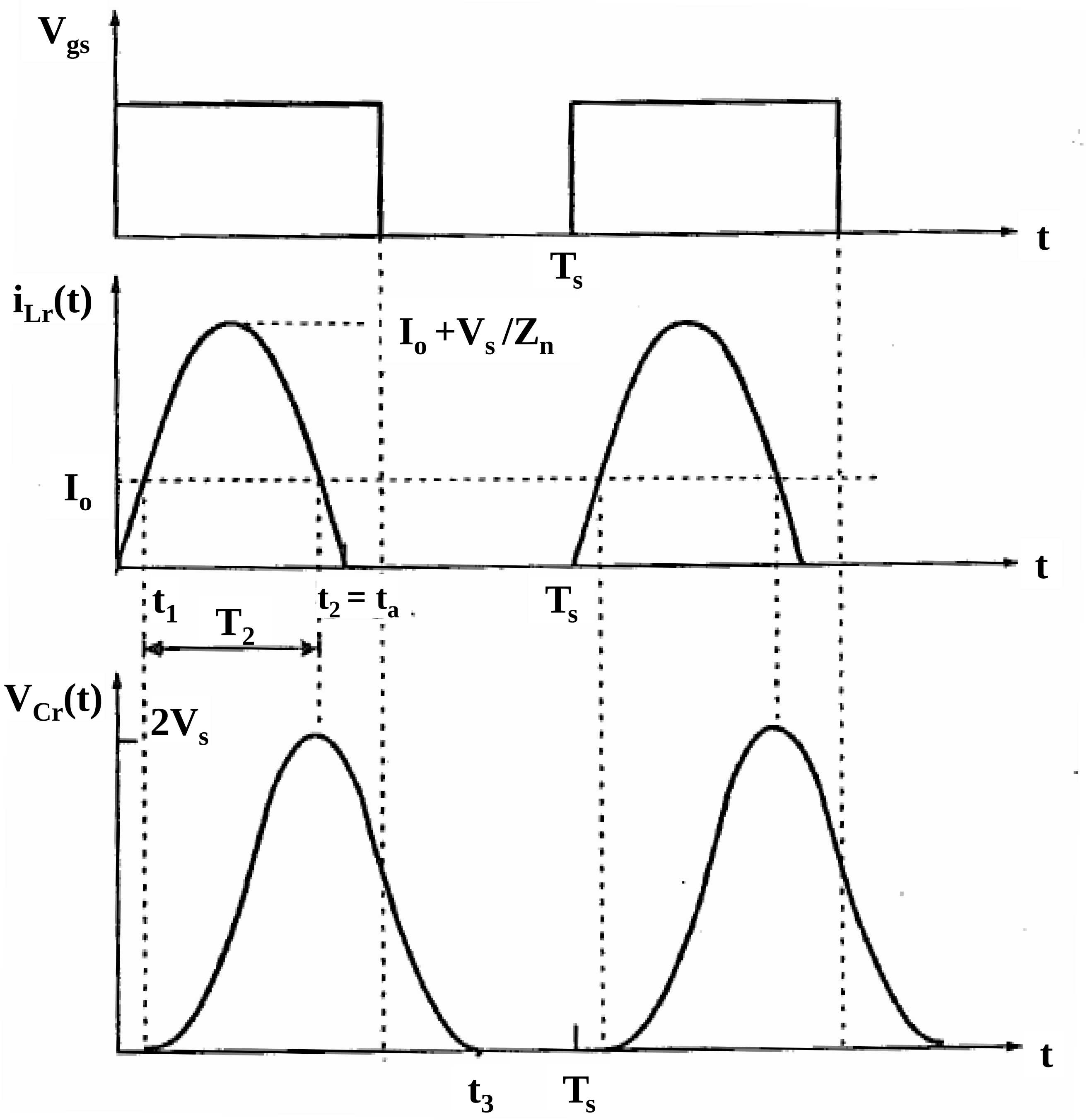
$$V_s = L_r \frac{I_o}{T_1} \qquad (1)$$

Thus, the duration of mode 1 is given by,

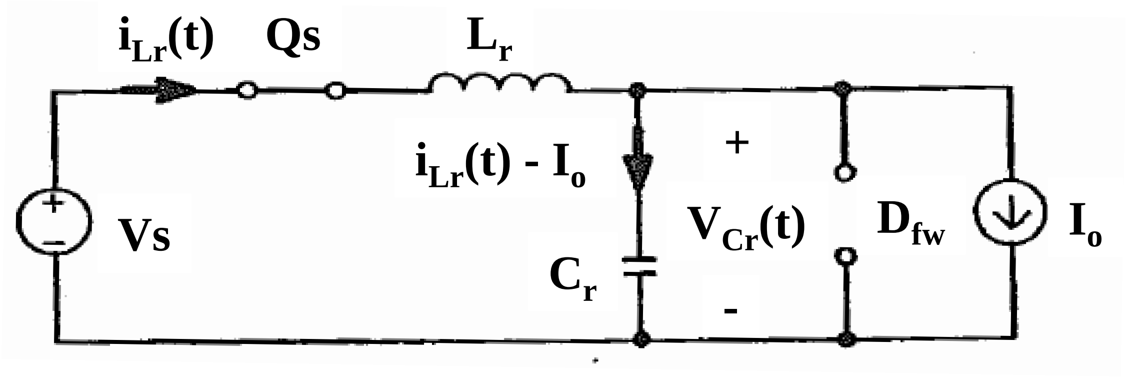
$$T_1 = L_r \frac{I_o}{V_s} \qquad (2)$$

- Mode 1 is characterized by inductor charging and storage of electrical energy, in magnetic form, in the resonant inductor.





Mode 2 ($t_1 < t \leq t_2$)



- Mode 2 begins when the current flow through the resonant inductor, $i_{Lr}(t)$ reaches the steady state output current, I_o
- The rate of increase of the resonant inductor current is given by,

$$\frac{di_{Lr}}{dt} = \frac{\left(V_s - V_{Cr}(t)\right)}{L_r} \qquad (3)$$

- The rate of increase of the resonant capacitor voltage is given by,

$$\frac{dV_{Cr}}{dt} = \frac{\left(i_{Lr}(t) - I_o\right)}{C_r} \qquad (4)$$

At time t_1, $V_{Cr}(t_1) = V_{Cr}(0) = 0$ (5)

$$i_{Lr}(t_1) = I_{Lr}(0) = I_o \qquad (6)$$

On solving (3) and (4) with initial conditions (5) and (6),

$$i_{Lr}(t) = I_o + \frac{V_s}{Z_n}\sin\omega_n t \qquad (7)$$

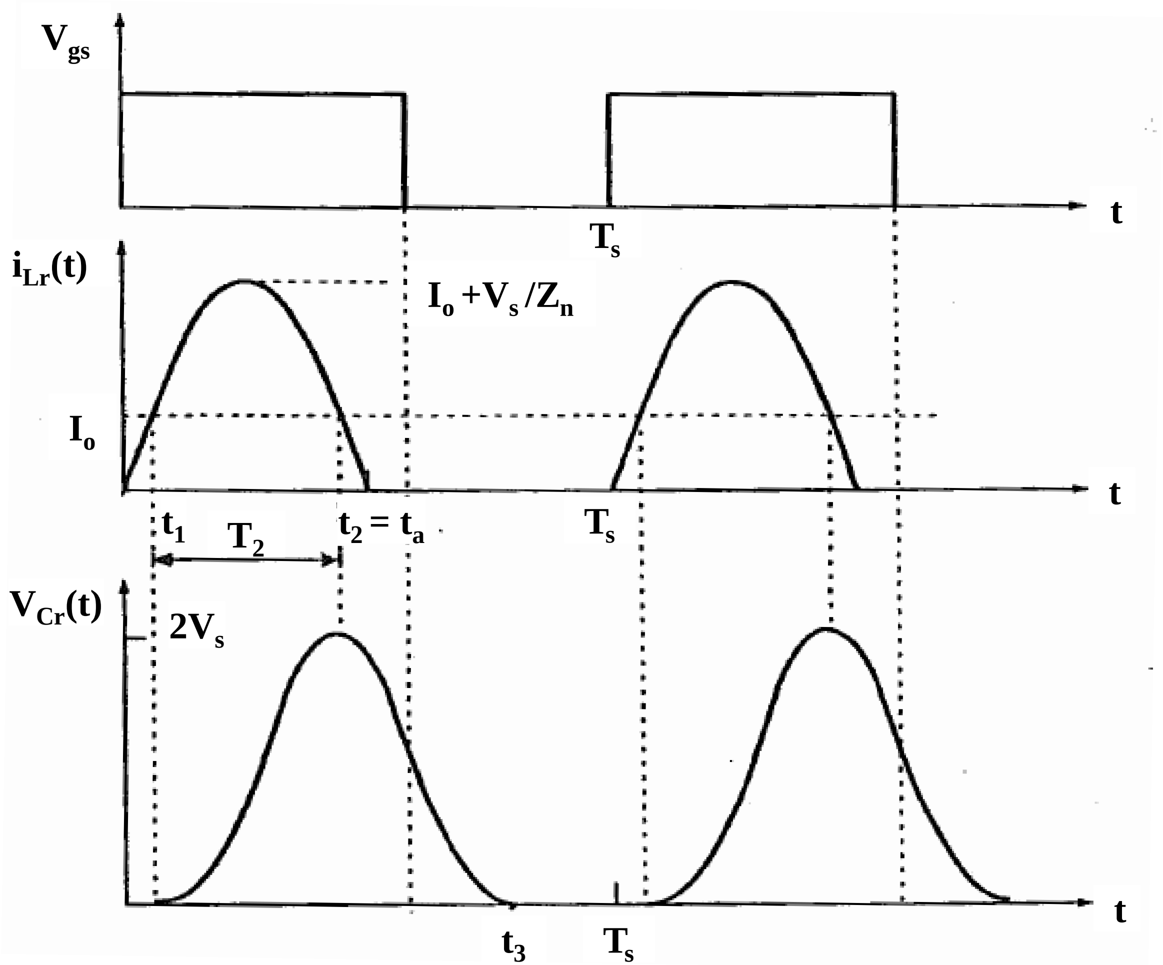
$$V_{Cr}(t) = V_s(1 - \cos\omega_n t) \qquad (8)$$

Where,

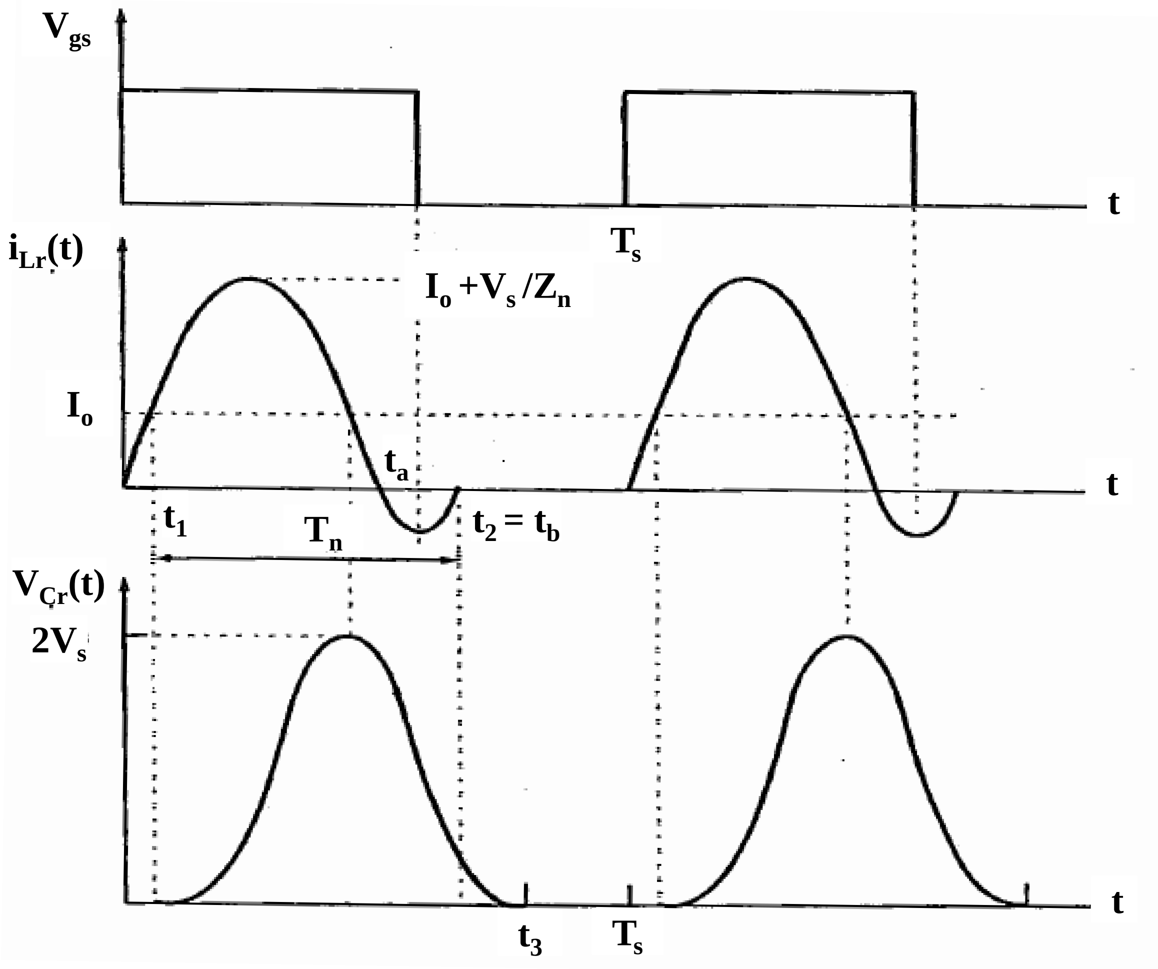
$$Z_n = \sqrt{L_r / C_r} \Rightarrow \text{Characteristic impedance}$$

$$\omega_n = 1/\sqrt{L_r C_r} \Rightarrow \text{Resonant tank frequency}$$

- Mode 2 is resonant stage.
- In half wave resonant buck converter, transistor Q_s will be naturally commutated at time $t_{2,}$ when the resonant inductor current, i_{Lr} (t) reduces to zero.
- In full wave resonant buck converter, the resonant inductor current will continue to oscillate and feed energy back to back source V_s through antiparallel diode D_1.



(a) ZCS waveforms in L-type Half wave switch configuration



(b) ZCS waveforms in L-type Full wave switch configuratio

- The duration of this resonant mode,

$$T_2 = t_2 - t_1$$

At T_2, $\quad i_{Lr}(T_2) = 0$

Therefore, from (7),

$$i_{Lr}(T_2) = 0 = I_o + \frac{V_s}{Z_n}\sin\omega T_2 \qquad (9)$$

$$T_2 = \frac{\sin^{-1}(-I_o Z_n / V_s)}{\omega_n} = \frac{\alpha}{\omega_n} \qquad (10)$$

$$V_{Cr}(t_2) = V_s\left(1-\cos\alpha\right) \qquad (11)$$

- At $I_o < V_s / Z_n$, transistor Q_s is switched OFF during zero current condition.

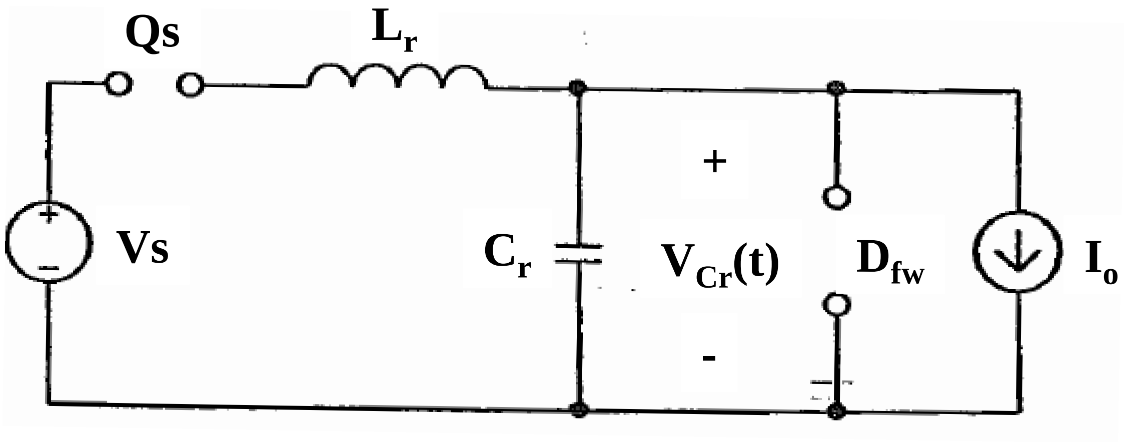
Mode 3 ($t_2 < t \leq t_3$)

- The resonant capacitor, C_r begin to discharge through the output loop and $V_{Cr}(t)$ decreases linearly to zero at time t_3,

$$C_r \frac{dV_{Cr}}{dt} = -I_o \qquad (12)$$

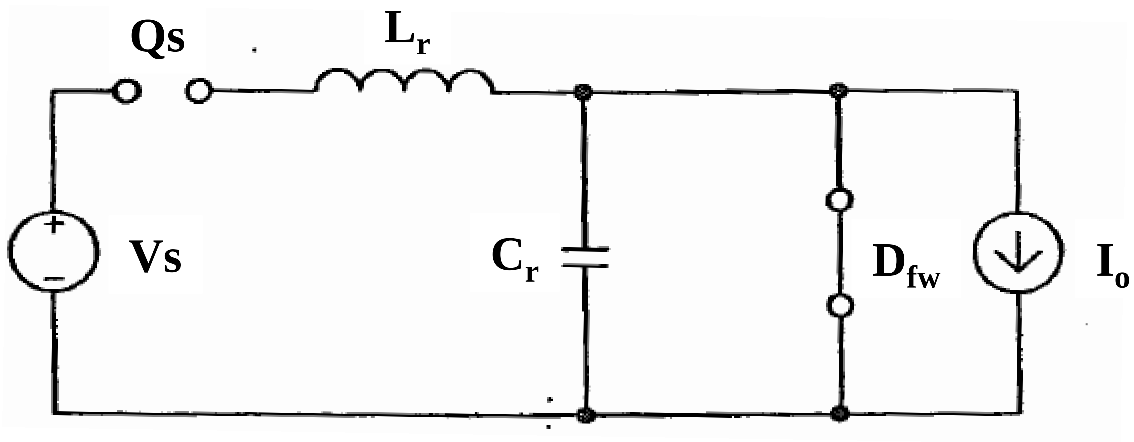
From (11) and (12),

$$T_3 = t_3 - t_2 = \frac{C_r V_{Cr}(t_3) - V_{Cr}(t_2)}{-I_o} = \frac{C_r V_s\left(1-\cos\alpha\right)}{I_o} \qquad (13)$$



Mode 4 ($t_3 < t \le t_4$)

- Mode 4 begins when the voltage across the V_{Cr} reduces to zero at time $t = t_3$.
- The freewheeling diode D_{fw} switches ON and the output current now flows through D_{fw}.



The duration of the mode 4,

$$T_4 = T_s - T_3 - T_2 - T_1 \qquad (14)$$

Where T_s is the switching period.

Calculation of voltage conversion ratio

- Using volt-sec relationship at output inductor, Lo,

$$(V_s - V_a)T_n - V_a(T_s - T_n) = 0 \qquad (15)$$

In resonant period   Non resonant period

On rearranging equation (15),

$$\frac{V_a}{V_s} = \frac{T_n}{T_s} = \frac{f_s}{f_n} \qquad (16)$$

- Output voltage is regulated by changing the switching frequency.
- In buck converter $f_s < f_n$ and insensitive to load variations.

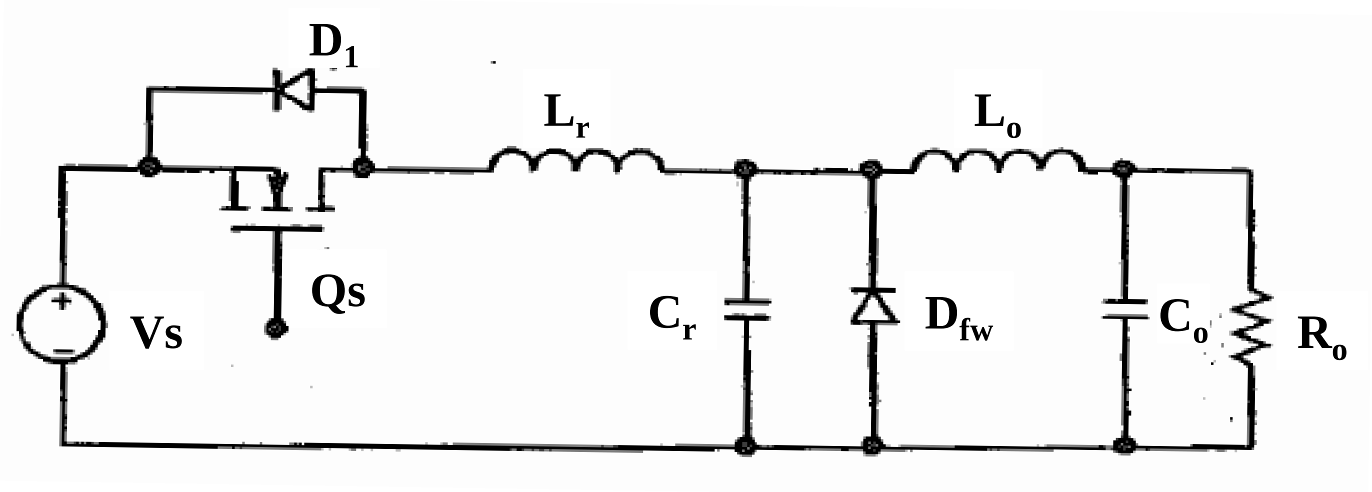
Problem 2.10

The zero-current-switching quasi-resonant BUCK converter shown in Figure has an input voltage of 12V. The values of the resonant inductor, L_r, and resonant capacitor, C_r, are 2 μH and 79 nF, respectively. The average output voltage is 9 V across a 9Ω resistor. The output inductor and output capacitor are 10 mH and 100 μF, respectively. Determine (a) the switching frequency, f_s , (b) the duration that the resonant inductor is charged, (c) the peak current in the resonant inductor, and (d) peak voltage across the resonant capacitor.

Solution:

(a) The resonant frequency is

$$f_n = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{2\times10^{-6}(79\times10^{-9})}} = 400\,\text{kHz}$$



and, the switching frequency, f_s is

$$f_s = \frac{V_a}{V_s} f_n = \frac{9}{12} 400\,\text{kHz} = 300\,\text{kHz}$$

(b) The average output current is

$$I_a = \frac{V_a}{R} = \frac{9}{9} = 1\text{A}$$

using which the resonant inductor is charged for

$$T_1 = \frac{L_s I_o}{V_s} = \frac{2\times10^{-6}(1)}{12} = 0.167\,\mu\text{s}$$

(c) The peak current in the resonant inductor is

$$I_{Lr,max} = I_o + \frac{V_s}{Z_n} = 1 + \frac{12}{\sqrt{\left(L_r / C_r\right)}} = 3.385\,A$$

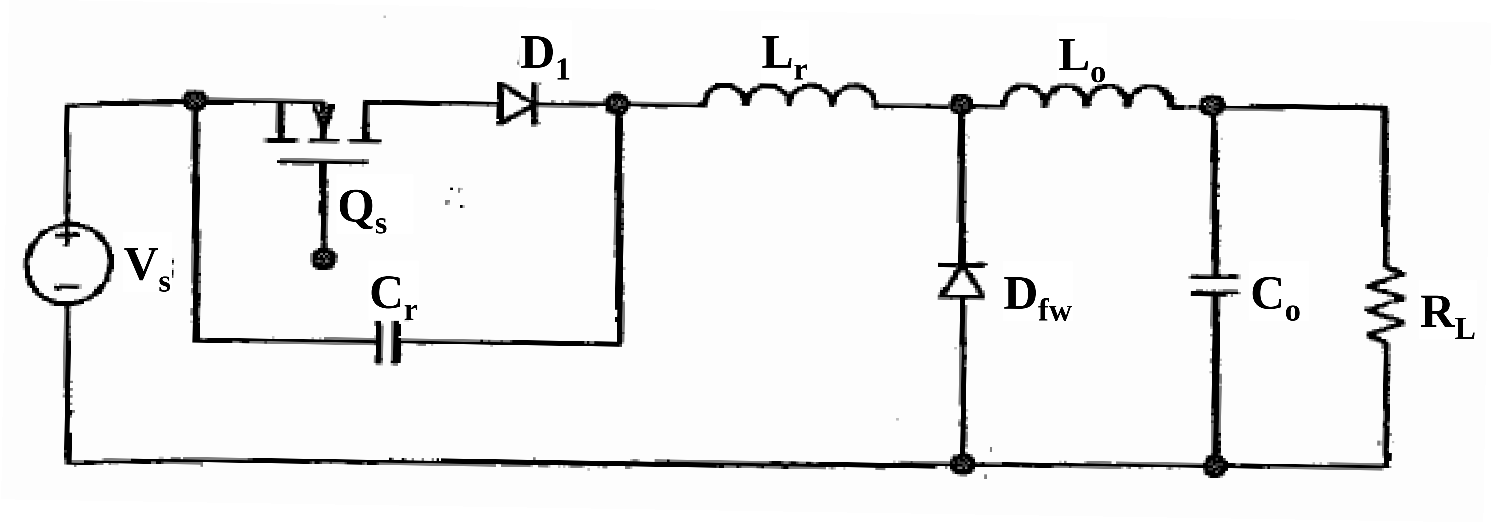
(d) The expression for the voltage across the resonant capacitor is

$$V_{Cr} = V_s\left[1 - \cos\frac{t}{\sqrt{L_r C_r}}\right] = 12\left[1 - \cos\left(2.516\times10^6\,t\right)\right]$$

The peak voltage across the resonant capacitor is

$$V_{Cr,max} = 12(1+1) = 24V$$

Zero Voltage Switching of Buck Converter



With L-type full wave switch configuration

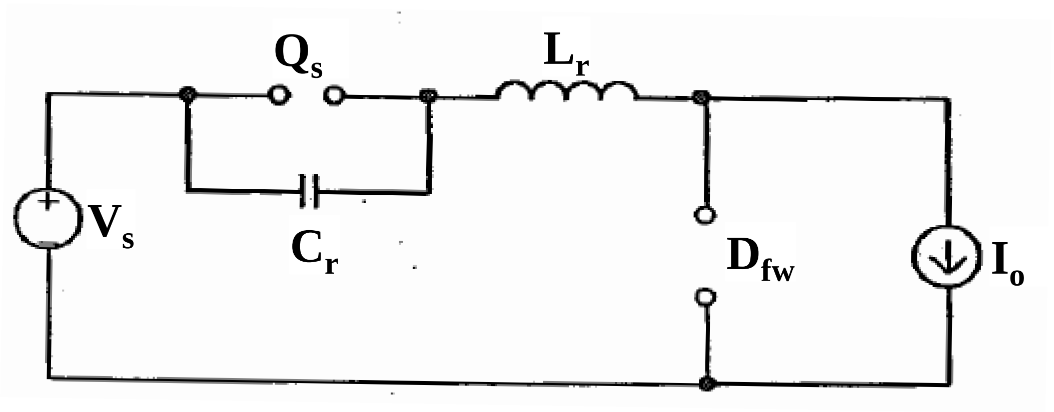
Assumptions

- The output current, I_o is constant.
- The output inductor (L_o) is much larger than resonant inductor (L_r).
- The switching devices are ideal.
- The resonant inductor (L_r) and capacitor (C_r) are ideal circuit elements with no lossy or parasitic elements.

Initial conditions

- The transistor Q_s is in ON state.
- Resonant inductor L_r carries the load current, I_o.
- The resonant capacitor C_r is clamped to zero voltage.
- Freewheeling diode, D_{fw} is switched OFF.

Mode 1 $(0 < t \leq t_1)$



- Switching transistor Q_s is switched OFF and the resonant capacitor C_r begins to charge.
- At the end of mode 1, the current flowing through the resonant capacitor is,

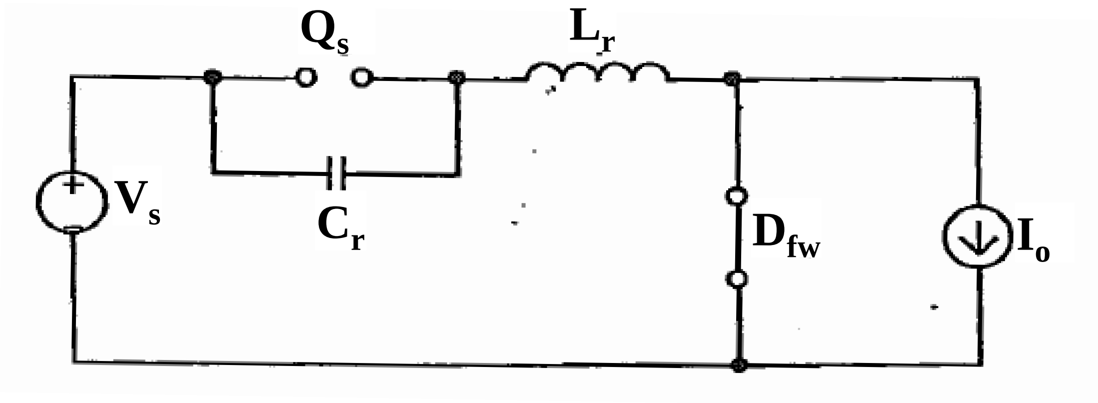
$$I_o = C_r \frac{V_s}{T_1} \qquad (1)$$

Thus, the duration of mode 1 is given by,

$$T_1 = C_r \frac{V_s}{I_o} \qquad (2)$$

- Mode 1 is characterized by capacitor charging and storage of electrical energy, in electrostatic form, in the resonant capacitor.

Mode 2 $(t_1 < t \leq t_2)$

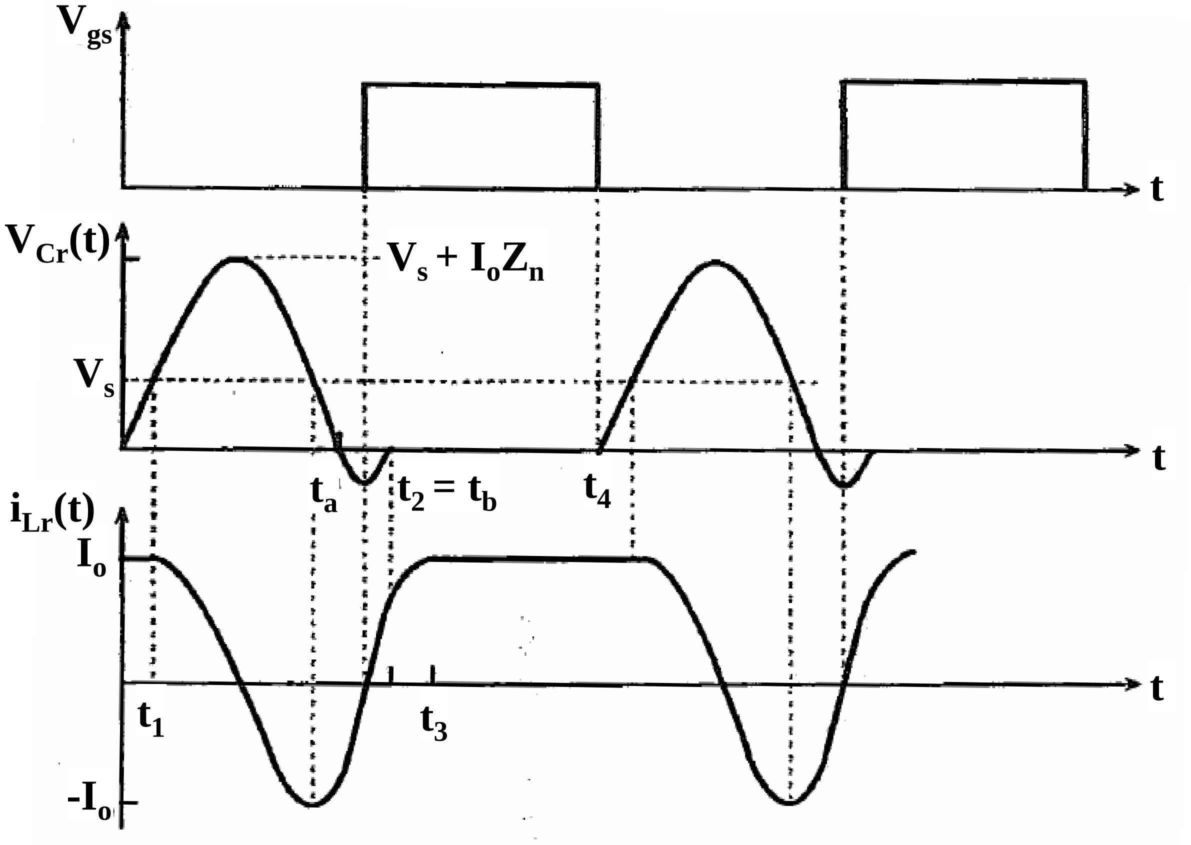


- Mode 2 begins when the voltage across the resonant capacitor, $V_{Cr}(t)$ reaches the input supply voltage V_a at time t_1
- The rate of increase of the resonant capacitor voltage is given by,

$$\frac{dV_{Cr}}{dt} = \frac{i_{Lr}(t)}{C_r} \qquad (3)$$

- The rate of decrease of the resonant inductor current is given by,

$$\frac{di_{Lr}}{dt} = \frac{V_s - V_{Cr}(t)}{C_r} \qquad (4)$$



ZVS waveforms in L-type Full wave switch configuration

At time t_1, $V_{Cr}(t_1) = V_{Cr}(0) = V_s$ (5)

$$i_{Lr}(t_1) = I_{Lr}(0) = I_o \qquad (6)$$

On solving (3) and (4) with initial conditions (5) and (6),

$$V_{Cr}(t) = V_s + Z_n I_o \sin \omega_n t \qquad (8)$$

$$i_{Lr}(t) = I_o \cos \omega_n t \qquad (7)$$

Where, $Z_n = \sqrt{L_r / C_r} \Rightarrow$ Characteristic impedance

$\omega_n = 1/\sqrt{L_r C_r} \Rightarrow$ Resonant tank frequency

- Mode 2 is resonant mode.
- At the end of mode 2, V_{Cr} oscillated to zero and current through the resonant inductor i_{Lr} increases towards I_o.
- The duration of this resonant mode,

$$T_2 = t_2 - t_1$$

At T_2, $V_{Cr}(T_2) = 0$

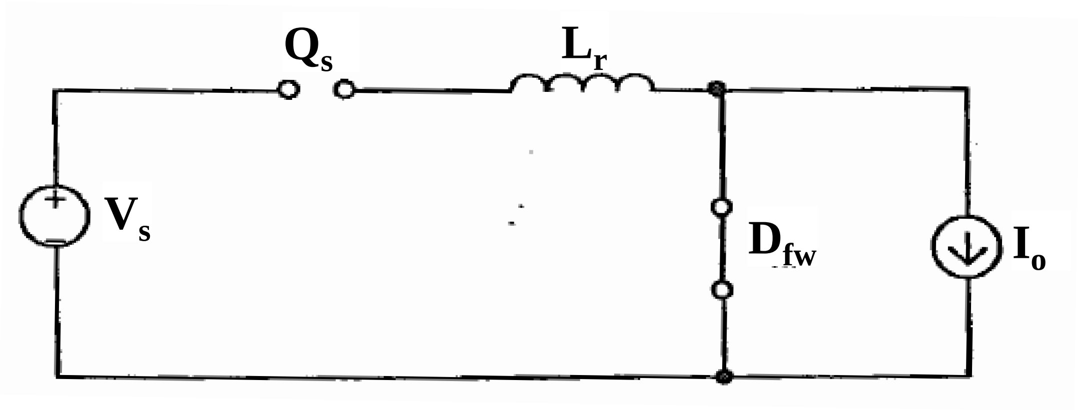
Therefore, from (8),

$$V_{Cr}(T_2) = 0 = V_s + Z_n I_o \sin \omega T_2 \qquad (9)$$

$$T_2 = \frac{\sin^{-1}(-V_s / Z_n I_o)}{\omega_n} = \frac{\alpha}{\omega_n} \qquad (10)$$

- At $I_o > V_s / Z_n$, transistor Q_s is switched ON during zero voltage condition.

Mode 3 ($t_2 < t \leq t_3$)

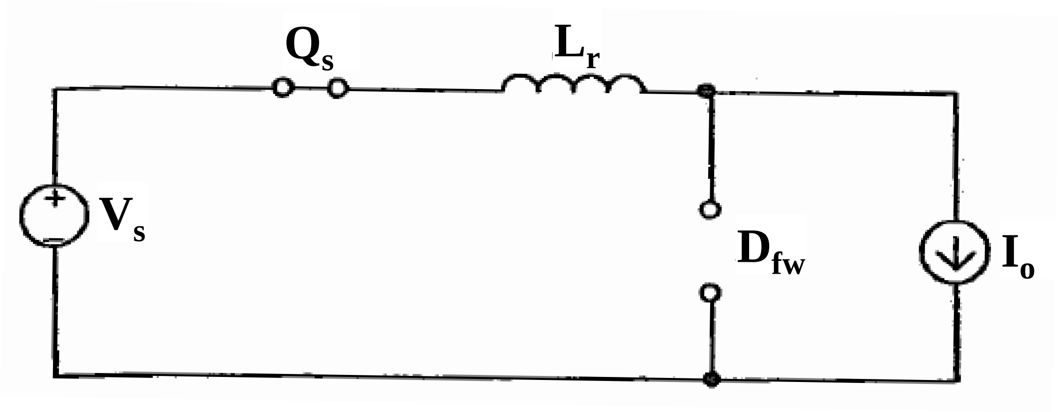


- The resonant inductor, L_r continues to increase towards the steady state output current I_o.
- The rate of increase of the resonant inductor current is

$$\frac{di_{Lr}}{dt} = \frac{V_s}{L_r} \qquad (11)$$

$$T_3 = t_3 - t_2 = \frac{i_{Lr}(t_3) - i_{Lr}(t_2)}{V_s / L_r} = \frac{L_r I_o \left(1 - \cos\alpha\right)}{V_s} \qquad (12)$$

Mode 4 ($t_3 < t \leq t_4$)



- Mode 4 begins when the current in the resonant inductor i_{Lr} reaches the steady state output current, I_o at time t_3.
- The freewheeling diode, D_{fw} is switched OFF at time t_3.

The duration of the mode 4,

$$T_4 = T_s - T_3 - T_2 - T_1 \qquad (13)$$

Where T_s is the switching period.

Calculation of voltage conversion ratio

- Using volt-sec relationship at output inductor, Lo,

$$- V_a T_n + (V_s - V_a)(T_s - T_n) = 0 \qquad (14)$$

In resonant period Non resonant period

On rearranging equation (15),

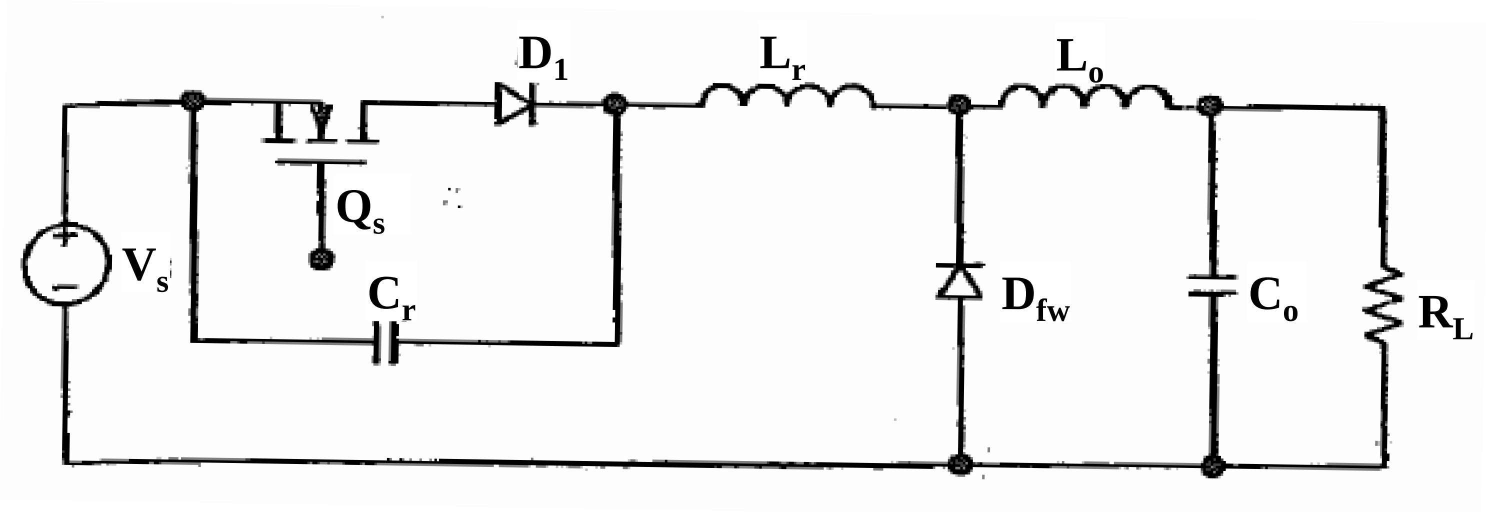
$$\frac{V_a}{V_s} = 1 - \frac{T_n}{T_s} = 1 - \frac{f_s}{f_n} \qquad (15)$$

- Output voltage is regulated by changing the switching frequency.
- In buck converter $f_s < f_n$ and insensitive to load variations.

Problem 2.11

The ZVS quasi-resonant buck converter shown in Figure has an input voltage of 12 V and a resistive load of 2Ω. The values of the resonant inductor, L_r, and resonant capacitor, C_r, are 2 μH and 79 nF, respectively. The switching frequency is 200 kHz. The output inductor and output capacitor are 10 mH and 100 μF, respectively. Determine (a) the average output voltage, V_a, (b) the duration that the resonant capacitor is charged, (c) the peak voltage across the resonant capacitor, and (d) the expression for the resonant inductor current.

Solution:



(a) The average output voltage is

$$V_a = \left(1 - \frac{f_s}{f_n}\right) V_s = \left(1 - \frac{200k}{400k}\right) 12 = 6V$$

(b) The average output current is

$$I_o = \frac{V_a}{R} = \frac{6}{2} = 3V$$

The duration that the resonant capacitor is charged is

$$T_1 = \frac{C_r V_s}{I_o} = \frac{79 \times 10^{-9}(12)}{3} = 0.316\mu s$$

(c) The peak voltage across the resonant capacitor is

$$V_{Cr,max} = V_s + Z_n I_o = V_s + \frac{V_a}{R}\sqrt{\frac{L_r}{C_r}} = 12 + \frac{6}{2}\sqrt{\frac{2\times10^{-6}}{79\times10^{-9}}} = 27.1\,V$$

(d) The expression for the voltage across the resonant capacitor is

$$i_{Lr} = I_o\cos(\omega_n t) = 3\cos\left(\frac{t}{\sqrt{L_r C_r}}\right) = 3\cos\left(2.516\times10^{6}\right)t\ A$$