Quiz 1 Solution

Question 1. Prove that Gale Shapley algorithm for stable marriage is female pessimal.

Solution

Gale-Shapley (GS) algorithm

while (some man is free and hasn't proposed to every woman)

Choose such a man m,

w = 1st woman in m's list to whom m has not proposed yet

if w is free

w accepts m

else if w is engaged to m'

if m >w (m')

w accepts m. m' becomes free.

else w rejects m

Means once a woman is engaged, she never becomes free.

We will prove it in four ways one by one.

1. **GS algorithm will terminate in n^2 iterations** as In each iteration, a man proposes a woman, whom he has not proposed yet. So, there are only n^2 proposals possible.
2. **GS algorithm always outputs a perfect matching** as Assume we doesn't get a perfect matching after the completion of GS algorithm. $\exists$ m $\in$ M, m is free. Then $\exists$ w $\in$ W, w is free. By remark earlier, w was never proposed. But m proposes to every woman and so we get a contradiction.
3. **GS algorithm always output a stable matching** as Assume GS algorithm outputs a matching N, which is unstable. ie $\exists$ (m, w),(m' , w') $\in$ N, s.t.(m, w') is a blocking pair. But according to our algorithm, m must have proposed w' before w. If m had proposed w' , w' must have retained m as the partner (m >w' m'). This contradicts the fact that (m, w') is a blocking pair. Hence N will not contain any blocking pair and it will be a stable matching.
4. **GS algorithm outputs a stable matching which is men-optimal and women-pessimal** as Men will start proposing women in the decreasing order of their preferences. In this stable matching, every man will get the best partner in any stable matching. This comes at the expense of women and each woman will have the worst partner she can have in any stable matching.

   Step-4 can also be proved by giving an example or method of contradiction.

Rubric:

1] Algo - 2 marks

2] Stable Match and Perfect match explanation - 2 marks

3] Logical and Correct proof - 4 marks (If partially correct 2 marks)

Reference: http://www.imsc.res.in

Question 2. Give algorithm for detecting the articulation points in a connected, undirected graph using DFS. (5 Marks)

There are two possible algorithms for detecting articulation points in a connected, undirected graph using DFS. Any one of the following solutions is acceptable.

1. **Algorithm 1:**
    a. For every vertex v in the Graph G
        i. Remove v from G and its corresponding edges. OR create a graph G' excluding v and the edges associated with it. **(2 Marks)**
        ii. Use DFS to check if graph is connected / all vertices reachable and determine if v is articulation point or not. **(2 Marks)**
        iii. Add v and the associated edges back in the graph G (not needed if creating G' in step i) **(1 Mark)**

Time complexity = O(V*(V+E))

2. ALgorithm 2 (Tarjan's Algorithm)

Let vertex u be a parent of v in the DFS tree.
A vertex u is an articulation point if **(1 Mark)**

1. u is root of the DFS tree and has at least 2 children. **(2 Marks)**
2. u is not the root of the DFS tree and it has a child vertex v such that no vertex in subtree of rooted at v has a backedge to one of the ancestors of u. **(2Marks)**

Rubric : As mentioned against the steps of the algorithms
References : https://www.geeksforgeeks.org/articulation-points-or-cut-vertices-in-a-graph/

Q3) Use of Radix sort/count Sort or Bubble Sort

Count Sort is not comparison based algorithm. It has the complexity of $O(n+k)O(n+k)$, where kk is the maximum element of the input array.

So, if k is $O(n)$,Count Sort becomes linear sorting.

Algorithm:

For each digit ii where ii varies from the least significant digit to the most significant digit of a number
Sort input array using countsort algorithm according to ith digit.

We used count sort because it is a stable sort.

Example: Assume the input array is:
10,21,17,34,44,11,654,123
Based on the algorithm, we will sort the input array according to the one's digit (least significant digit).
0: 10
1: 21 11
2:
3: 123
4: 34 44 654
5:
6:

7: 17
8:
9:

So, the array becomes 10,21,11,123,24,44,654,17
Now, we'll sort according to the ten's digit:
0:
1: 10 11 17
2: 21 123
3: 34
4: 44
5: 654
6:
7:
8:
9:

Now, the array becomes : 10,11,17,21,123,34,44,654
Finally , we sort according to the hundred's digit (most significant digit):
0: 010 011 017 021 034 044
1: 123
2:
3:
4:
5:
6: 654
7:
8:
9:

The array becomes : 10,11,17,21,34,44,123,654 which is sorted. This is how our algorithm works.
Rubric : Answer is no (0 marks)
Answer is yes, with algorithm name (1 mark)
Proper explanation of algorithm (6 marks)

Q-4 What is inplace sorting.Explain with example.
Ans. It is a type of sorting in which no auxiliary space is required to sort the given elements.
Eg. bubble,insertion etc
Rubric :- 1 mark(definition)
1 mark(Example,but it should be in context of inplace sorting)

Q5) $T(n) = 3T(n/4) + O(n^2)$

Rubric :-
If drawn the tree to only 0th level (2 Marks)

If drawn the tree to one level (2 Marks)
If drawn the tree to two levels (4 Marks)
If complete tree is drawn with end condition (2 Marks)

References :- https://web.stanford.edu/class/archive/cs/cs161/cs161.1168/lecture3.pdf