Assignment 3- Solutions and rubric

Q1]

Part 1: Yes, it can be modelled using graph where circuit boards will form vertices and wires as edges.

Let $G = (V, E)$ be the directed graph in which V is the set of board , and there is a directed edge $(u, v) \in E$ for each wire oriented from board u to board v

Part 2: Either apply Kosaraju's Algorithm for SCC or BFS on G then BFS on G' i.e., you need to apply BFS twice

To determine whether the condition holds: ·

i)Pick an arbitrary node $v \in V$.

ii) Run BFS in G from v and check that all of V is reachable from v; if not, return "no".

iii) Construct the reverse graph $G' = (V, E')$, where $E' = \{(v, u) : (u, v) \in E\}$.

iv) Run BFS in G' from v and check that all of V is reachable from v; if not, return "no". Otherwise return "yes".

Part 3: Apply same algorithm in part 2 by removing the edge corresponding to the wire removed.

For each edge $e \in E$, run the algorithm of part (a) on $G \setminus \{e\}$, the graph obtained by removing e. Return "no" if any of these returns "no"; otherwise return "yes".

Proof of correctness.

Part 4: The minimum number of edges is $2n$. To see this, note that:

i) It is necessary that for every node $v \in V$, indegree(v) ≥ 2; that is, at least 2 edges pointing towards it. If not, then if we removed the sole edge (u, v) from G, then v cannot reach any other nodes. The same argument shows that we require outdegree(v) ≥ 2. If this is the case, then the number of edges is

m = 1 2 ∑ $v \in V$ deg(v) ≥ 1 2 ∑ $v \in V$ 4 ≥ 2n

where the 1/2 comes from the fact that each edge contributes to the degrees of two nodes.

ii) The condition above is also sufficient. To see this, number the vertices $v1, \cdots, vn$ and consider the "double-cycle" graph containing all edges (vi , vi+1) and (vi+1, vi). This graph remains strongly connected if any single edge is removed, and it has exactly 2n edges.

Rubric: (Binary Marking)

Part 1: explained both vertices and edges in graph - 5 Marks

If not explained about vertices and edges explicitly - 0 Marks

Part 2: Algorithm well explained with no. of times BFS was used

(in case you have used BFS) - 5 Marks

Part 3: Algorithm - 2.5 Marks

Correctness - 2.5 Marks

Part 4: Correct answer with explanation - 5 Marks (Binary Marking)

Q2]

Algo: Maximum Bipartite Matching Algorithm or Ford Fulkerson Algorithm (with min cut) satisfying the given conditions where the modification were simple.
Standart Algo links:
https://www.geeksforgeeks.org/ford-fulkerson-algorithm-for-maximum-flow-problem/
https://www.geeksforgeeks.org/maximum-bipartite-matching/

Correctness: Need to prove the correctness of the graph and satisfaction of all the conditions.
The below statement need to be proved:

There is a way to assign teachers to vacation days in a way that respects all constraints if and only if there is a feasible circulation in the flow network we have constructed.

Apart from the claim also need to prove the algorithm terminates correctly and all the contitions are covered.

Time Complexity: Need to derive it

Rubric:
Algorithm:10 marks (subjective)
Time Complexity: 2 Marks
Proof of Correctness: 8 marks (subjective)