Indraprastha Institute of Information Technology
Dept. of Computer Science and Engineering

CSE222– Solutions for Assignment #5
Analysis and Design of Algorithms

Instructor: Dr. Debarka Sengupta

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Each question carries **5 points**.
Plagiarism: All submitted codes/text are expected to be the result of your individual effort. You should never misrepresent someone else's work as your own. In case any plagiarism case is detected you will get one grade reduction in final examination.Cite the resource wherever using other's code/text.
Instructions:
1) No extensions on deadline. If you fail to submit within the time limit then your solution will not be evaluated.
2) No handwritten solution will be considered for evaluation. Use latex or other commonly used text editors to draft the solutions.
3) Submit the pdf file containing all the solutions and at top mentioning all the group members names and roll no.
4) While members of the a group can come up with solution strategies for the problems through discussion, they are not encouraged to submit identical copies. Blatant, mindless duplication will be considered as plagiarism.
5) If you have any doubt, use backpack discussion for clarification.

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1. Given a set $\{x_1, x_2, \ldots, x_n\}$ of points on the real line, determine the smallest set of unit-length closed intervals (e.g. the interval [x,x+1]) that contains all of the points. Give the most efficient algorithm you can to solve this problem, prove it is correct and analyze the time complexity.

   1. Start with the leftmost(The point with minimum value) point on the line.
   2. Consider a unit closed interval starting from this point.
   3. Remove all the points in the interval to obtain the set of remaining points.
   4. Repeat from one till all the points are covered.

2. Consider the problem of making change from n cents using the fewest coins when the available coins are quarters, dimes, nickels and pennies. Design a greedy algorithm for this problem and prove its correctness. Also analyze the running time of your algorithm.

   1. Find the highest denomination coin available. 2. Subtract the value of the coin from *current_remaining_value* to obtain the new *current_remaining_value*. 3. Repeat this till the value is zero.

3. The HAM-PATH problem is the following: Given an undirected graph G, is there a path in G that visits all vertices exactly once.
   The HAM-CYCLE problem is the following: Given an undirected graph G, is there a cycle in G that visits all vertices exactly once. Give a sketch of a proof that HAM-PATH is in NP. Now show that HAM-PATH is NP-complete by reducing HAM-CYCLE to HAM-PATH. (Assume the HAM-CYCLE is NP-HARD.)

   `http://www.cs.umd.edu/~meesh/351/mount/lectures/lect28-npcomplete-reductions.pdf`

4. Given an integer k, divide a set of n objects into k coherent clusters such that spacing, i.e., Min distance between any pair of points in different clusters, is maximized. Write an efficient algorithm for this problem.

   1.Run prims algo to get MST.
   2.Delete maximum weight k-1 edges in it.

5. Prove the cut property and the cycle property for the MST. `https://en.wikipedia.org/wiki/Minimum_spanning_tree#Cycle_property`

6. You have 2 random variables X_1 and X_2. You compute 500 Y values following the equation:

   $Y = aX_1 + bX_2 + c+$ small Gaussian noise where $a = b = c = 2$.

   Now consider that you do not know values of a, b and c. Consider a, b and c as unknowns. Given the 500 Y values, try to estimate those using gradient descent such that the plane fits the points best.

   Use linear regression to fit the best line. Optimize the cost function by gradient decent. (You can check the video lecture by Andrew Ng)