

Tutorial-3
IIIT Delhi
Instructor: Debarka Sengupta

Problem 1.

There are n balls of identical weights except one, means $n-1$ balls of 1 kg and one ball of 2 kg, there is one scale to measure balls as shown in figure. But every time when you will compare the balls using scale than it will count 1 step.

1. Calculate the minimum number of steps required to find out that heavy ball.
2. Write algorithm for above mentioned problem using recursion and analyze complexity. Prove the correctness of your algorithm.

**Problem 2.**

Solve the following recurrences using the master method where possible:

- a. $T(n) = 9T(n/3) + n$
- b. $T(n) = 2T(n/2) + n \log(n)$
- c. $2. T(n) = 16T(n/4) + n!$

Problem 3.

Analyze the recurrence function and find its time complexity via master's theorem. (Assume constant time taken for comparing and returning the value = 1)

```
A(n)
{
    if (n <= 1)
T (n) = (0.5)T (n/2) + n^n
        return 1;
    else
```

```

        return A(\n);
    }
    ,

```

Problem 4.

You are given a sorted array A with n integers and an integer w and you want to determine whether there exists two distinct indices i, j in the array such that $A[i] + A[j] = w$. Design a recursive algorithm for this problem. Analyze the running time of your algorithm.

Problem 5.

Master's theorem is not applicable in the following cases. Explain why?

- $T(n) = (2^n)T(n/2) + n^n$
- $f(n)$ is smaller than $n^{\log_b a}$ but not polynomially smaller.
- $T(n) = 2T(n/2) + n/\log n$
- $T(n) = 64T(n/8) - 2n \log n$
- $T(n) = (0.5)T(n/2) + n^n$