Rohan Rajpal 2017084

# Assignment - 1

(1) $N(t+1) = a\,N(t)\,(1 - N(t))$ — ①

$F(N) = a\,N\,(1-N)$

Differentiating wrt N to get the stretching factor

$F'(N) = a(1-N) + aN(-1)$

$= \underbrace{a(1-2N)}_{\lambda \to \text{stretching factor}}$

At equilibrium $N(t+1) = N(t) = N^*$ (say)

$N^* = a N^* (1 - N^*)$

$N^* - aN^*(1-N^*) = 0$

$\cancel{N^*(1 - a(1-N^*)) = 0}$

$N^* - aN^* + a(N^*)^2$

$N^*(1 - a + aN^*) = 0$

$N^* = 0$ , $N^* = \frac{a-1}{a} = 1 - \frac{1}{a}$

~~For stable equilibrium point $0<\lambda<1$ ($\lambda \to$ stretching factor)~~

| $N^* = 0$ | $N^* = 1 - 1/a$ |
|---|---|
| $F'(0) = \underbrace{a}_{\lambda}$ | $F'(1 - 1/a) = \underbrace{2-a}_{\lambda}$ |
| a) $0 < a < 1$ | $0 < 2-a < 1$ |
| | $-2 < -a < -1$ |
| | $2 > a > 1$ |

As both possible ranges can give equilibrium we take their union.

$a \in (0,2) - \{1\}$ Ans

For stable oscillations $-1 < \lambda < 0$

| $N^* = 0$ | $N^* = 1 - 1/a$ |
|---|---|
| (b) $-1 < a < 0$ | $-1 < 2 - a < 0$ <br> $-3 < -a < -2$ <br> $3 > a > 2$ |

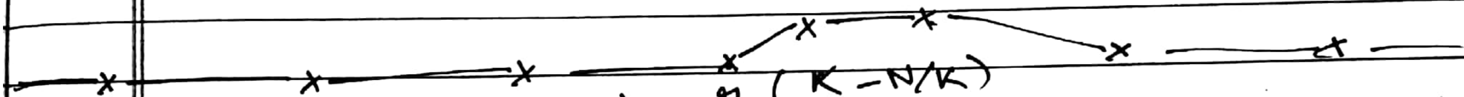
We take union as both ranges give stable oscillations.

$a \in (-1, 0) \cup (2, 3)$ ] Ans (b)

For unstable oscillations $\lambda < -1$

| | |
|---|---|
| (c) $a < -1$ | $2 - a < -1$ <br> $a > 3$ |

$a \in (-\infty, -1) \cup (3, \infty)$ ] Ans (c)



(3) $N(t+1) = N(t)\, e^{r(K - N/K)}$

At equilibrium : $N(t+1) = N(t) = N^*$ (say)

$$N^* = N^* e^{r(K - N^*/K)}$$

$$N^* \left(1 - e^{r(K - N^*/K)}\right) = 0$$

$$\boxed{N^* = 0} \quad \text{or} \quad r(K - N^*/K) = 0$$

$$\boxed{\Rightarrow N^* = K^2}$$

Differentiating to get the stretching factor ($\lambda$)

$$F(N) = N e^{r(K - N/K)}$$

$$F'(N) = e^{r(K - N/K)} + N e^{r(K - N/K)} \left(\frac{-r}{K}\right)$$

$$F'(N) = \underbrace{e^{r(K - N/K)} \left[1 - \frac{Nr}{K}\right]}_{\lambda}$$

For the model to exhibit oscillations the streching factor $\lambda$ has to be negative

~~$N^* = 0$~~

$$F'(0) = e^{rK}[1]$$

As $e^{rK} \not< 0$ We don't get any value for $r$ here which gives oscillations

~~$N^* = K^2$~~

$$F'(K^2) = e^{r(K - K^2/K)}\left[1 - \frac{K^2 r}{K}\right] < 0$$

$$= \left[1 - Kr\right] < 0$$

$$r > 1/K$$

For $r > 1/K$ the equation will exhibit oscillation.

Using the given eq$^n$ to calculate $e^{-rt}$

(4) $$1 + \left(\frac{K - N(0)}{N(0)}\right)e^{-rt} = \frac{K}{N(t)}$$

$$\left(\frac{K - N(0)}{N(0)}\right)e^{-rt} = \frac{K}{N(t)} - 1$$

$$e^{-rt} = \left(\frac{N(0)}{K - N(0)}\right)\left(\frac{K - N(t)}{N(t)}\right) \quad \text{— (1)}$$

Dividing $N(t+1)$ by $N(t)$

$$\frac{N(t+1)}{N(t)} = \frac{\dfrac{K}{1 + \left(\frac{K - N(0)}{N(0)}\right)e^{-r(t+1)}}}{\dfrac{K}{1 + \left(\frac{K - N(0)}{N(0)}\right)e^{-rt}}}$$

Scanned by CamScanner

$$\frac{N(t+1)}{N(t)} = \frac{1+\left(\frac{K-N(0)}{N(0)}\right)e^{-rt}}{1+\left(\frac{K-N(0)}{N(0)}\right)e^{-r(t+1)}}$$

Using (1)

$$\frac{N(t+1)}{N(t)} = \frac{1+\left(\cancel{\frac{K-N(0)}{N(0)}}\right)\left(\cancel{\frac{N(0)}{K-N(0)}}\right)\left(\frac{K-N(t)}{N(t)}\right)}{1+\left(\cancel{\frac{K-N(0)}{N(0)}}\right)e^{-r}\left(\cancel{\frac{N(0)}{K-N(0)}}\right)\left(\frac{K-N(t)}{N(t)}\right)}$$

$$= \frac{\frac{N(t)+K-N(t)}{\cancel{N(t)}}}{\frac{N(t)+e^{-r}(K-N(t))}{\cancel{N(t)}}}$$

$$N(t+1) = \frac{K e^{r}}{e^{r}N(t)+(K-N(t))} N(t)$$

$$N(t+1) = \frac{K e^{r}}{K+N(t)[e^{r}-1]} N(t) \Big\} \underline{Ans}$$

~~Here~~ Above we have $N(t+1)$ as a function of $N(t)$