$(-1)^3 \frac{d^3}{ds^3}(s^2+1)^{-1}$

$= (-1)\frac{d^2}{ds^2}(-1)\cdot(s^2+1)^{-2}\cdot(2s)$

$= 2\frac{d^2}{ds^2}\left[s\cdot(s^2+1)^{-2}\right]$

$= 2\frac{d}{ds}\left[(s^2+1)^{-2}\cdot 1 + s\cdot(-2)\cdot(s^2+1)^{-3}\cdot 2s\right]$

$= 2\frac{d}{ds}\left[(s^2+1)^{-2} - 4s^2(s^2+1)^{-3}\right]$

$= 2\left[(-2)(s^2+1)^{-3}\cdot(2s) - 4(s^2+1)^{-3}\cdot 2s + 4(s^2)\cdot(-3)\cdot(s^2+1)^{-4}\cdot(2s)\right]$

$= 2\left[-4s(s^2+1)^{-3} - \left[8s(s^2+1)^{-3} - 24s^3(s^2+1)^{-4}\right]\right]$

~~$= 2[-4s$~~

$= \left[-\frac{8s}{(s^2+1)^3} - \frac{16s}{(s^2+1)^3} + \frac{48s^3}{(s^2+1)^{+4}}\right]$

$= \frac{48s^3}{(s^2+1)^4} - \frac{16s}{(s^2+1)^3} - \frac{8s}{(s^2+1)^3}$

$= \frac{48s^3 - 16s^3 - 16s - 8s^3 - 8s}{(s^2+1)^4}$

$\frac{24-8}{16} = \frac{16}{16} = 1$

$= \frac{24s^3 - 8s}{(s^2+1)^4}$

Convolution Th

17/5/22

If $L^{-1}[F(s)] = f(t)$ and $L^{-1}[G(s)] = g(t)$ then

$$L^{-1}[F(s)G(s)] = \int_0^t f(u)g(t-u)\,du$$

$$= \int_0^t g(u)f(t-u)\,du$$

$$= f * g$$

Q-1) Using Convolution th, evaluate $L^{-1}\left[\frac{1}{(s-1)(s-3)}\right]$

Q-2) Using Convolution th, evaluate $L^{-1}\left[\frac{1}{(s+2)(s-6)}\right]$

Soln-2) Here, $L^{-1}\left[\frac{1}{(s+2)(s-6)}\right]$

Let, $F(s) = \frac{1}{s+2} \Rightarrow L^{-1}[F(s)] = L^{-1}\left[\frac{1}{s+2}\right]$

$= e^{-2t}$

$= f(t)$

and $G(s) = \frac{1}{s-6} \Rightarrow L^{-1}[G(s)] = L^{-1}\left[\frac{1}{s-6}\right]$

$= e^{6t}$

$= g(t)$

Using convolution th,

$L^{-1}\left[\frac{1}{(s+2)(s-6)}\right] = L^{-1}\left[\frac{1}{s+2}\cdot\frac{1}{s-6}\right]$

$= L^{-1}[F(s)\cdot G(s)]$

$= \int_0^t f(u)g(t-u)\,du$

$$= \int_0^t e^{-2u} e^{6(t-u)}\, du$$

$f(t) = e^{-2t}$

$f(u) = e^{-2u}$

$g(t) = e^{6t}$

$g(t-u) = e^{6(t-u)}$

$$= \int_0^t e^{-2u} \cdot e^{6t} \cdot e^{-6u}\, du$$

$$= e^{6t} \int_0^t e^{-8u}\, du$$

$$= e^{6t} \left[\frac{e^{-8u}}{-8}\right]_0^t$$

$$= e^{6t} \cdot \left(\frac{e^{-8t} - e^0}{-8}\right)$$

$$= e^{6t} \frac{(1 - e^{-8t})}{8}$$

$$= \frac{e^{6t} - e^{-2t}}{8} \quad \text{(Ans)}$$

Sol^n - 1)

Here, $L^{-1}\left[\frac{1}{(s-1)(s-3)}\right]$

Let, $F(s) = \frac{1}{s-1} \Rightarrow L^{-1}[F(s)] = L^{-1}\left[\frac{1}{s-1}\right] = e^t = f(t)$

Let, $G(s) = \frac{1}{s-3} \Rightarrow L^{-1}[G(s)] = L^{-1}\left[\frac{1}{s-3}\right] = e^{3t} = g(t)$

Using Convolution thm,

$$L^{-1}\left[\frac{1}{(s-1)(s-3)}\right] = L^{-1}\left[\frac{1}{s-1} \cdot \frac{1}{s-3}\right]$$

$f(t) = e^t$

$f(u) = e^u$

$g(t) = e^{3t}$

$g(t-u) = e^{3(t-u)}$

$$= L^{-1}[F(s) \cdot G(s)]$$

$$= \int_0^t f(u)\, g(t-u)\, du$$

$$= \int_0^t e^u \cdot e^{3(t-u)}\, du$$

$$= \int_0^t e^u \cdot e^{3t} \cdot e^{-3u}\, du$$

$$= e^{3t} \int_0^t e^{-2u}\, du$$

$$= e^{3t} \left[\frac{e^{-2u}}{-2}\right]_0^t$$

$$= e^{3t} \left[\frac{e^{-2t} - e^0}{-2}\right]$$

$$= e^{+3t} \left[\frac{1 - e^{-2t}}{2}\right]$$

$$= \frac{e^{3t} - e^t}{2} \quad \text{(Ans)}$$

(i) $L^{-1}\left[\frac{1}{s^2 - s + 6}\right]$ (ii) $L^{-1}\left[\frac{1}{s^2 + s - 3}\right]$

Evaluate (1) $L^{-1}\left[\frac{1}{(s-1)(s+2)}\right]$, (2) $L^{-1}\left[\frac{1}{(s-1)(s^2+8)}\right]$

(1) $L^{-1}\left[\frac{1}{(s-1)(s+2)}\right]$

$\underline{s+2-s+1}$

$$= L^{-1}\left[\frac{1}{3}\left(\frac{1}{s-1} - \frac{1}{s+2}\right)\right]$$

$$= \frac{1}{3}\left\{L^{-1}\left[\frac{1}{s-1}\right] - L^{-1}\left[\frac{1}{s+2}\right]\right\}$$

$$= \frac{1}{3}\left\{e^t - e^{-2t}\right\} \quad \text{(Ans)}$$

$\cancel{s+3}$

$s-4-s-3$

Q) $L^{-1}\left[\frac{1}{(s+3)(s-4)}\right]$

$= L^{-1}\left[-\frac{1}{7}\left(\frac{1}{s+3} - \frac{1}{s-4}\right)\right]$

$= -\frac{1}{7}\left\{L^{-1}\left[\frac{1}{s+3}\right] - L^{-1}\left[\frac{1}{s-4}\right]\right\}$

$= -\frac{1}{7}\left\{e^{-3t} - e^{4t}\right\}$ (Ans)

$s^2+8-(s^2-1)$

② $L^{-1}\left[\frac{1}{(s-1)(s^2+8)}\right]$

$= L^{-1}\left[\frac{1}{9}\left(\frac{1}{s-1} - \frac{s+1}{s^2+8}\right)\right]$

$= \frac{1}{9}\left\{L^{-1}\left[\frac{1}{s-1}\right] - L^{-1}\left[\frac{s}{s^2+8}\right] - L^{-1}\left[\frac{1}{s^2+8}\right]\right\}$

$= \frac{1}{9}\left\{e^{t} - \cos\sqrt{8}t - \frac{\sin\sqrt{8}t}{\sqrt{8}}\right\}$ (Ans)

H/W

$s^2+4-(s^2-9)$

$L^{-1}\left[\frac{1}{(s-3)(s^2+4)}\right]$

$= L^{-1}\left[\frac{1}{13}\left(\frac{1}{s-3} - \frac{s-3}{s^2+4}\right)\right]$

$= \frac{1}{13}\left\{L^{-1}\left[\frac{1}{s-3}\right] - L^{-1}\left[\frac{s}{s^2+4}\right] + L^{-1}\left[\frac{3}{s^2+4}\right]\right\}$

$= \frac{1}{13}\left\{e^{3t} - \cos 2t - \frac{3\sin 2t}{2}\right\}$

Vector Calculus

There are two type of f^ns — (i) scaler point function
(ii) vector point function

(i) $f(x,y,z) = x^2y + y^2z + z^2x$

$f(t) = t^2 - 3t - 4$

(ii) $\overrightarrow{f(x,y,z)} = (x^2y)i + (y^2z)j + (z^2x)k$

$f(\vec{t}) = t^2 i - 3t\hat{j} - 4k$

$\text{grad}(f) = \nabla f$

$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)f$

$= i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$

grad(f) = gradient of scaler point f^n f

Q) Find the grad $(x^3+y^3+z^3-xyz)$

Ans) grad $(x^3+y^3+z^3-xyz)$

$= \nabla(x^3+y^3+z^3-xyz)$

$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)(x^3+y^3+z^3-xyz)$

$= i\frac{\partial}{\partial x}(x^3+y^3+z^3-xyz) + j\frac{\partial}{\partial y}(x^3+y^3+z^3-xyz) + k\frac{\partial}{\partial z}(x^3+y^3+z^3-xyz)$

$= i(3x^2-yz) + j(3y^2-xz) + k(3z^2-xy)$ (Ans)

Divergence of a vector point funⁿ

The divergence of a vector point. f^n is defined as, $\vec{f}$

$$\nabla \cdot \vec{f} = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\cdot(\cancel{f_1 i + f_2}\ f_1 i + f_2 j + f_3 k)$$

$$= \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

Curl of a vector point f^n

A curl of vector point f^n $\vec{f}$ is defined as —

$$\vec{\nabla} \times \vec{f} = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\times(f_1 i + f_2 j + f_3 k)$$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

Q1> Find grad of the function, $f = x^3 + y^3 + z^3 - 3xyz$

Solⁿ Here, $f = x^3 + y^3 + z^3 - 3xyz$

$$\text{grad}(f) = \vec{\nabla} f$$

$$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) f$$

$$= i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$$

$$= i\frac{\partial}{\partial x}(x^3+y^3+z^3-3xyz) + j\frac{\partial}{\partial y}(x^3+y^3+z^3-3xyz) + k\frac{\partial}{\partial z}(x^3+y^3+z^3-3xyz)$$

Q2> Find the grad of the f^n, $f = x^3+y^3+z^3-3xyz$ at $(1,1,2)$

Ans $[\text{grad}(f)]_{(1,1,2)} = i(3-6) + j(3-6) + k(12-3)$

$= -3i - 3j + 9k$

Q3> Find ~~divergence~~ div of curl of the following vector point f^n.

$$\vec{f} = i(3x^2 - 3yz) + j(3y^2 - 3xz) + k(3z^2 - 3xy) = if_1 + jf_2 + kf_3$$

Solⁿ Here, $\vec{f} = i(3x^2-3yz) + j(3y^2-3xz) + k(3z^2-3xy)$.

Now, $\text{div}(\vec{f}) = \vec{\nabla}\cdot\vec{f}$

$$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)(f_1 i + f_2 j + f_3 k)$$

$$= \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$= \frac{\partial}{\partial x}(3x^2-3yz) + \frac{\partial}{\partial y}(3y^2-3xz) + \frac{\partial}{\partial z}(3z^2-3xy)$$

$$= 6x + 6y + 6z$$

Curl $(\vec{f}) = \vec{\nabla} \times \vec{f}$

$$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\times(f_1 i + f_2 j + f_3 k)$$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2-3yz & 3y^2-3xz & 3z^2-3xy \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(3z^2-3xy) - (3y^2-3xz)\frac{\partial}{\partial z}\right] - j\left[\frac{\partial}{\partial x}(3z^2-3xy) - \frac{\partial}{\partial z}(3x^2-3yz)\right] + k\left[\frac{\partial}{\partial x}(3y^2-3xz) - \frac{\partial}{\partial y}(3x^2-3yz)\right]$$

$$= i(-3x+3x) - j(-3y+3y) + k(-3z+3z) = 0$$

$= i(0) - j(0) + k(0)$

$= 0i + 0j + 0k$

$= \vec{0}$

23/05/22

Double Integration

Q1> Evaluate $\iint_R y\,dx\,dy$

where R is the region bounded by the curve

(i) $y^2 = x$ and $x^2 = y$

Q2> Evaluate $\iint_R y\,dx\,dy$

where R is the region bounded by the curves

$y^2 = x$ and $y = x$

Q3> Evaluate $\iint_R y\,dx\,dy$

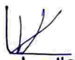
where R is the region bounded by the curves

$y = x$ and $x^2 = y$

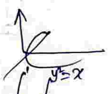
Q4> Evaluate $\iint_R y\,dx\,dy$

where R is the region bounded by the curves

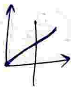
$y = 0,\ y = x$ and $x = 1$



$\int_{x=0}^{1} \int_{x^2=y}^{y=x}$

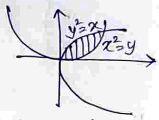


$\int \int_{y=x}^{y^2=x}$



$\int_{x=0}^{1} \int_{y=0}^{y=x}$

Sol^n-1



where R bounded by the curves $y^2 = x$ and $x^2 = y$

$\iint_R y\,dx\,dy$

$= \int_{x=0}^{1} \int_{x^2=y}^{y^2=x} y\,dx\,dy$

$= \int_{x=0}^{1} \left[\int_{x^2=y}^{y^2=x} y\,dy \right] dx$

$= \int_{x=0}^{1} \left[\frac{y^2}{2} \right]_{x^2=y}^{y^2=x} dx$

$= \frac{1}{2} \int_{x=0}^{1} (x - x^4)\,dx$

$= \frac{1}{2} \left(\left[\frac{x^2}{2} \right]_0^1 - \left[\frac{x^5}{5} \right]_0^1 \right)$

$= \frac{1}{2} \times \left(\frac{1}{2} - \frac{1}{5} \right)$

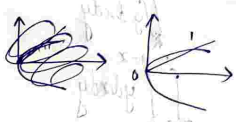
$= \frac{1}{2} \times \frac{5-2}{10} = \frac{3}{20}$

Sol^n-2 $\iint_R y\,dx\,dy$

$\int_{x=0}^{1} \int_{y=x}^{y^2=x} y\,dx\,dy$

$= \int_{x=0}^{1} \left[\frac{y^2}{2} \right]_{y=x}^{y^2=x} dx$

$= \frac{1}{2} \int_{x=0}^{1} (x - x^2)\,dx$

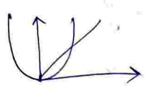


$= \frac{1}{2}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]$

$= \frac{1}{2}\left[\frac{1}{2} - \frac{1}{3}\right]$

$= \frac{1}{12}$

Sol^n-3



$\iint_R y \, dxdy$

$\int_{x=0} \int_{x^2=y}^{y=x} y \, dxdy$

$= \int_{x=0} \left[\frac{y^2}{2}\right]_{x^2=y}^{y=x}$

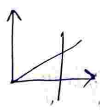
$= \frac{1}{2}\int_{x=0}^{1} (x^2 - x^4)\, dx$

$= \frac{1}{2}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_{x=0}^{1}$

$= \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right)$

$= \frac{1}{15}$

Sol^n-4



$\iint_R y \, dxdy$

$\int_{x=0}^{1} \int_{y=0}^{y=x} y \, dxdy$

$= \int_{x=0}^{1} \left[\frac{y^2}{2}\right]_{y=0}^{y=x} dx$

$= \int_{x=0}^{1} \frac{x^2}{2}\, dx$

$= \frac{1}{2}\left[\frac{x^3}{3}\right]_0^1$

$= \frac{1}{6}$

$\int_{x=1}^{2} \int_{y=2}^{3} y \, dxdy$

$= \int_{x=1}^{2} \left[\frac{y^2}{2}\right]_{y=2}^{3} dx$

$= \int_{x=1}^{2} \left[\frac{9}{2} - 2\right] dx$

$= \int_{x=1}^{2} \frac{5}{2}\, dx$

$= \frac{5}{2}[x]_1^2$

$= \frac{5}{2}$

$\int_{x=1}^{2} \int_{y=2}^{3} xy \, dxdy$

$= \frac{1}{2}\int_{x=1}^{2} (xy^2)_{y=2}^{3}\, dx$

$= \frac{1}{2}\int_{x=1}^{2} (9x - 4x)\, dx$

$= \frac{1}{2}\left[\frac{5x}{2}\right]_1^2$

$= \frac{5}{4}(4-1) = \frac{15}{4}$

Variation of parameter method

1) $(D^2+1)y = \sec x$

2) $(D^2+1)y = \sec^3 3x \tan x$

3) $(D^2+4)y = \sec 2x$

(i) D - separation method

(ii) V.P method

(iii) Ca·

Homogeneous higher order D.E

Q-1

1st part (C.P) - comple

24/5/22

D- Operator

$x^2, x+2, x^2-3x$
$e^x, e^{2x},$
$\cos x, \cos 2x, \sin 3x,$
$e^x \cos x, e^{2x} \sin x$

Solve, $(D^2+4)y = (e^{2x})$

C.F →

P I (Particular Integral) $= \frac{1}{D^2+4} e^{2x}$

e^{ax}
$D \to a$
$D \to 2$

$\frac{1}{(2)^2+4} = \frac{1}{8}$

$\Rightarrow \frac{1}{(2)^2+4} \cdot e^{2x}$

$= \frac{1}{8} e^{2x}$

∴ the general solⁿ is

$y = CF + PI$

$= C.F + \frac{1}{8} e^{2x}$

Q-2

$(D^2-9)y = e^{3x}$

e^{ax}
$D \to a$
$D \to 3$

$\frac{1}{(3)^2-9} = \frac{1}{0}$

C.F →

P.I → $\frac{1}{D^2-9} e^{3x}$

$= x\left[\frac{1}{2D}\right] e^{3x}$ (derivative of D^2-9 w.r.t D)

$= \frac{x}{2} \cdot \frac{1}{D}(e^{3x})$

$= \frac{x}{2}\left(\frac{e^{3x}}{3}\right) = \frac{xe^{3x}}{6}$

the general solⁿ is.

$y = CF + PI = CF + \frac{xe^{3x}}{6}$

Q4) Solve, $(D^2-4D+4)y = e^{2x}$

$D \to 2$

$\frac{1}{(2)^2-4\times2+4}$

$\frac{1}{4-8+4} = \frac{1}{0}$

$\frac{1}{2\times2-4} = \frac{1}{0}$

PI $= \frac{1}{D^2-4D+4} e^{2x}$

$= x\left[\frac{1}{2D-4}\right] e^{2x}$

$= x\left[x \frac{1}{2}\right] e^{2x}$

$= \frac{x^2}{2} e^{2x}$

Sin/cos x

cos ax/sin ax

$D^2 = -a^2$

$D^2 = -(1)^2$

$= -1$

$\frac{1}{-1+4} = \frac{1}{3}$

Q-1 $(D^2+4)y = \cos x$

CF →

P.I → $\frac{1}{D^2+4} \cos x$

$= \frac{1}{-1+4} \cos x$

$= \frac{1}{3} \cos x$

The general solⁿ is

$y = C.F + PI = C.F + \frac{1}{3}\cos x$

Q-2 Solve, $(D^2+9)y = \sin 3x$

$D^2 = -a^2 = -(3)^2 = -9$

$\frac{1}{-9+9} = \frac{1}{0}$

C.F →

P.I → $\frac{1}{D^2+9} \sin 3x$

$= x\left[\frac{1}{2D}\right] \sin 3x$

$= \frac{x}{2}\frac{1}{D}(\sin 3x) = \frac{x}{2}\left[-\frac{\cos 3x}{3}\right) = -\frac{x\cos 3x}{6}$

∴ The general solⁿ is C.F + P.I

Q> $(D^2+3D-4)y = \sin x$

C.F →

P.I → $\frac{1}{D^2+3D-4} \sin x$

$\cos ax / \sin ax$

$D^2 = -a^2$

$D^2 = -a^2 = -1$

$= \frac{1}{-1+3D-4} \sin x$

$= \frac{1}{3D-5} \sin x$ * *

$= \frac{3D+5}{(3D+5)(3D-5)} \sin x$

$= \frac{3D+5}{9D^2-25} \sin x$

$= \frac{3D+5}{9(-1)-25} \sin x$

$= \frac{3D+5}{-34} \sin x$

$= -\frac{1}{34} \{3D(\sin x) + 5(\sin x)\}$

$= -\frac{1}{34}(3\cos x + 5\sin x)$

The general solⁿ is

$y = C.F + P.I$

Q> $(D^2-4D+6)y = \cos 2x$

$\cos ax / \sin ax$

$D^2 = -a^2$

$= -(2)^2$

$= -4$

P.I → $\frac{1}{D^2-4D+6} \cos 2x$

$= \frac{1}{-4-4D+6} \cos 2x$

$= \frac{1}{-4D+2} \cos 2x$

$= \frac{2+4D}{(2-4D)(2+4D)} \cos 2x$

$= \frac{2+4D}{4-16D^2} \cos 2x$

$= \frac{2+4D}{4-16(-4)} \cos 2x$

$= \frac{2+4D}{68} \cos 2x$

$= \frac{1}{68} \{2(\cos 2x) + 4D(\cos 2x)\}$

$= \frac{1}{68} (2\cos 2x + 4\cdot(-\sin 2x).2\}$

$= \frac{1}{68} (2\cos 2x - 8\sin 2x)$

Q> $(D^2+3D-4)y = e^x \cos x$

e^{ax}

$D \to D+a$

$D+1$

$PI = \frac{1}{D^2+3D-4} e^x \cos x$

$= e^x \left[\frac{1}{(D+1)^2+3(D+1)-4} \cos x\right]$

$= e^x \left[\frac{1}{D^2+2D+1+3D+3-4} \cos x\right]$

$\cos ax$

$D^2 = -a^2$

$= e^x \left[\frac{1}{D^2+5D} \cos x\right]$

$= e^x \left[\frac{1}{-1+5D} \cos x\right]$

$= e^x \left[\frac{1}{5D-1} \cos x\right]$

$= e^x \left[\frac{5D+1}{(5D+1)(5D-1)} \cos x\right]$

$= e^x \left[\frac{5D+1}{25D^2-1} \cos x\right]$

$= e^x\left[\frac{5D+1}{-25-1}\cos x\right]$

$= \frac{e^x}{-26}\left[5D(\cos x)+\cos x\right]$

$= \frac{e^x}{-26}\left[-5\sin x+\cos x\right]$

Q) $(D^2-4D+6)y = e^{2x}\sin x$

$P.I = \frac{1}{D^2-4D+6}e^{2x}\sin x$

$= \frac{1}{(D+2)^2-4(D+2)+6}e^{2x}\sin x$

$= e^{2x}\left[\frac{1}{D^2+4D+4-4D-8+6}\sin x\right]$

$= e^{2x}\left[\frac{1}{D^2+2}\sin x\right]$

$= e^{2x}\left[\frac{1}{-1+2}\sin x\right]$

$= e^{2x}\sin x$

Q) $(D^2+3D+4)y = x^2-x$ 25/5/22

$(1+x)^{-1} = 1-x+x^2-x^3+\dots$

$(1-x)^{-1} = 1+x+x^2+x^3+\dots$

$P.I = \frac{1}{D^2+3D+4}x^2-x$

$= \frac{1}{4\left(1+\frac{D^2+3D}{4}\right)}x^2-x$

$= \frac{1}{4}\left[1+\frac{D^2+3D}{4}\right]^{-1}x^2-x$

$= \frac{1}{4}\left[1-\left(\frac{D^2+3D}{4}\right)+\left(\frac{D^2+3D}{4}\right)^2\dots\right]x^2-x$

$= \frac{1}{4}\left[1-\left(\frac{D^2+3D}{4}\right)+\left(\frac{D^4+6D^3+9D^2}{16}\right)\dots\right]x^2-x$

$= \frac{1}{4}\left[(x^2-x)-\left(\frac{2+3(2x-1)}{4}\right)+\left(\frac{0+6(0)+9(2)}{16}\right)\right]$

$= \frac{1}{4}\left[(x^2-x)-\left(\frac{6x-1}{4}\right)+\frac{18}{16}\right]$

$y = C.F + P.I$

Q) $(D^2-D-3)y = 2x+3$

$P.I = \frac{1}{D^2-D-3}2x+3$

$\cancel{\frac{1}{3\left(\frac{D^2-D}{3}-1\right)}2x+3} = \frac{1}{-3\left(1-\frac{D^2-D}{3}\right)}2x+3$

$= -\frac{1}{3}\left(1-\frac{D^2-D}{3}\right)^{-1}2x+3$

$= -\frac{1}{3}\left[1+\left(\frac{D^2-D}{3}\right)\dots\right]2x+3$

$= -\frac{1}{3}\left[(2x+3)+\left(\frac{0-2}{3}\right)\right]$

$= -\frac{1}{3}\left[2x+3-\frac{2}{3}\right]$

Q) $(D^2-D-3)y = e^x x^2$

e^{ax}

$D \to D+a$

$P.I = \frac{1}{D^2-D-3}e^x x^2$

$= e^x\left[\frac{1}{(D+1)^2-(D+1)-3}x^2\right]$

$= e^x\left[\frac{1}{D^2+2D+1-D-1-3}x^2\right]$

$= e^x\left[\frac{1}{D^2+D-3}x^2\right]$

$$= e^x\left[\frac{1}{-3\left[1-\frac{D^2+D}{3}\right]}x^2\right]$$

$$= e^x\left[-\frac{1}{3}\left(1-\frac{D^2+D}{3}\right)^{-1}x^2\right]$$

$$= e^x\left[-\frac{1}{3}\left(1+\left(\frac{D^2+D}{3}\right)+\left(\frac{D^2+D}{3}\right)^2\right)\right]x^2$$

$$= e^x\left[-\frac{1}{3}\left(1+\left(\frac{D^2+D}{3}\right)+\left(\frac{D^4+2D^3+D^2}{9}\right)\right]x^2$$

$$= e^x\left[-\frac{1}{3}\left(x^2+\left(\frac{2+2x}{3}\right)+\left(\frac{0+2(0)+2}{9}\right)\right]$$

$$= e^x\left[-\frac{1}{3}\left\{x^2+\left(\frac{2+2x}{3}\right)+\left(\frac{2}{9}\right)\right\}\right]$$

Q) $(D^2+3D+4)y = e^x x^2$

$$P.I = \frac{1}{D^2+3D+4}e^x x^2$$

$$= e^x\left[\frac{1}{(D+1)^2+3(D+1)+4}x^2\right]$$

$$= e^x\left[\frac{1}{D^2+2D+1+3D+3+4}x^2\right]$$

$$= e^x\left[\frac{1}{D^2+5D+8}x^2\right]$$

$$= e^x\left[\frac{1}{8\left(1+\frac{D^2+5D}{8}\right)}x^2\right]$$

$$= e^x\left[\frac{1}{8}\left(1+\frac{D^2+5D}{8}\right)^{-1}x^2\right]$$

$$= e^x\left[\frac{1}{8}\left(1-\frac{D^2+5D}{8}+\left(\frac{D^2+5D}{8}\right)^2\right)x^2\right]$$

$$= e^x\left[\frac{1}{8}\left(1-\frac{D^2+5D}{8}+\frac{D^4+10D^3+25D^2}{64}\right)x^2\right]$$

$$= e^x\left[\frac{1}{8}\left(x^2-\frac{2+5(2x)}{8}+\frac{0+10(0)+25(2)}{64}\right)\right]$$

$$= e^x\left[\frac{1}{8}\left(x^2-\frac{2+10x}{8}+\frac{50}{64}\right)\right]$$

Cauchy – Euler's Homogeneous Higher order differential eqn — 30/5/22

1) Q) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 4y = x^2\sin(\log x)$

$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = x^2e^x, \cos x.$

2) Q) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = \cos(\log x)$

Sol^n) Let. $z = \log x$ —— ①

$x\frac{dy}{dx} = \frac{dy}{dz}$ —— ②

$x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz}$ —— ③

$x = e^z$

$\Rightarrow x^2 = e^{2z}$

Substituting all in eq^n ①, we get,

$$\left[\frac{d^2y}{dz^2}-\frac{dy}{dz}\right] - 2\left[\frac{dy}{dz}\right] + 4y = e^{2z}.\sin(z)$$

$\Rightarrow \frac{d^2y}{dz^2} - 3\frac{dy}{dz} + 4y = e^{2z}\sin z$ —— ④ which is a higher order differential eq^n with constant coefficient.

Let, $y = e^{mz}$ be a trial solution of the reduced eq^n

$\frac{d^2y}{dz^2} - 3\frac{dy}{dz} + 4y = 0$

$\Rightarrow e^{mz}(m^2-3m+4) = 0$

$y = e^{mz}$

$\Rightarrow \frac{dy}{dz} = m.e^{mz}$

$\Rightarrow \frac{d^2y}{dz^2} = m^2e^{mz}$

$\Rightarrow m^2 - 3m + 4 = 0$

$$\therefore m = \frac{3 \pm \sqrt{9 - 4.1.4}}{2.1}$$

$$= \frac{3 \pm \sqrt{9-16}}{2}$$

$$= \frac{3 \pm \sqrt{7}i}{2}$$

$$\underline{C.F} = e^{\frac{3}{2}z}\left(C_1 \cos\frac{\sqrt{7}}{2}z + C_2 \sin\frac{\sqrt{7}}{2}z\right)$$

$$= e^{3/2 \log x}\left(C_1 \cos\left(\frac{\sqrt{7}}{2}\log x\right) + C_2 \sin\left(\frac{\sqrt{7}}{2}\log x\right)\right)$$

where C_1 & C_2 are constant.

$$\underline{P.I} = \frac{1}{D_1^2 - 3D_1 + 4} e^{2z} \sin z \qquad D_1 = \frac{d}{dz}$$

$$= e^{2z}\left[\frac{1}{(D_1+2)^2 - 3(D_1+2)+4}\right] \sin z$$

$$= e^{2z}\left[\frac{1}{D_1^2 + 4D_1 + 4 - 3D_1 - 6 + 4}\right] \sin z$$

$$= e^{2z}\left[\frac{1}{D_1^2 + D_1 + 2} \sin z\right]$$

$$= e^{2z}\left[\frac{1}{-1 + D_1 + 2}\right] \sin z$$

$$= e^{2z}\left[\frac{1}{D_1 + 1} \sin z\right]$$

$$= e^{2z}\left[\frac{D_1 - 1}{D_1^2 - 1} \sin z\right]$$

$$= e^{2z}\left[\frac{\cancel{D_1} \sin z . D_1 - \sin z}{-1-1}\right]$$

$$= \frac{e^{2z}}{-2}\left[\cos z - \sin z\right]$$

$$= \frac{e^{2 \log x}}{-2}\left[\cos(\log x) - \sin(\log x)\right]$$

The required general solⁿ is: $y = CF + PI$

$$= \left[e^{3/2 \log x}\left(C_1 \cos\left(\frac{\sqrt{7}}{2}\log x\right) + C_2 \sin\left(\frac{\sqrt{7}}{2}\log x\right)\right)\right] + \frac{e^{2 \log x}}{-2}\left[\cos(\log x) - \sin(\log x)\right]$$

Ans

Solⁿ-2

Let, $z = \log x$ — (i)

$$\Rightarrow x\frac{dy}{dx} = \frac{dy}{dz} \text{ — (ii)}$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz} \text{ — (iii)}$$

Substituting all in given ~~eqⁿ (i)~~ we get,

$$\cancel{\left[x^2\frac{d^2y}{dx^2}\right.}$$

$$\left[\frac{d^2y}{dz^2} - \frac{dy}{dz}\right] + \frac{dy}{dz} + 4y = \cos z$$

$\Rightarrow \frac{d^2y}{dz^2} + 4y = \cos z$ — (4) which is a higher order differential eqⁿ with constant coefficient.

Let, $y = e^{mz}$ be a trial solution of the reduced eqⁿ

$\frac{d^2y}{dz^2} + 4y = 0$	$y = e^{mz}$
$\Rightarrow m^2 e^{mz} + 4e^{mz} = 0$	$\Rightarrow \frac{dy}{dz} = m.e^{mz}$
$\Rightarrow e^{mz}(m^2+4) = 0$	$\Rightarrow \frac{d^2y}{dz^2} = m^2 e^{mz}$

$\Rightarrow m^2+4=0$

$m = \frac{-0 \pm \sqrt{0^2-4.1.4}}{2.1} = \frac{\pm 4i}{2} = \pm 2i$

C.F $= C_1 \cos 2z + C_2 \sin 2z$, where C_1 & C_2 are constants.

$= C_1 \cos 2(\log x) + C_2 \sin 2(\log x)$

P.I $= \frac{1}{D_1^2+4} \cos z$ where, $D_1 = \frac{d}{dz}$

$= \frac{1}{-1+4} \cos z$

$= \frac{1}{3} \cos z$

$= \frac{1}{3} \cos(\log x)$

$\therefore$ The required general solution is —

$y = C.F + P.I = C_1 \cos 2(\log x) + C_2 \sin 2(\log x) + \frac{1}{3}\cos(\log x)$

Ans

Q.4 $(D^2+3D+2)y = e^{e^x}$

Q-3

Solve, $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y = x \sin(\log x)$

Let, $z = \log x$ — (i)

$\Rightarrow x\frac{dy}{dx} = \frac{dy}{dz}$ — (ii)

$\Rightarrow x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz}$ — (iii)

Substituting all in given eq^n we get —

$\left[\frac{d^2y}{dz^2} - \frac{dy}{dz}\right] + 3\left[\frac{dy}{dz}\right] - 3y = e^z \sin[z]$

$\Rightarrow \frac{d^2y}{dz^2} + 2\frac{dy}{dz} - 3y = e^z \sin z$ — (iv) which is a higher order differential eq^n with constant coefficient.

Let, $y = e^{mz}$ be a trial solution of the reduced eq^n

$\frac{d^2y}{dz^2} + 2\frac{dy}{dz} - 3y = 0$

$y = e^{mz}$

$\Rightarrow \frac{dy}{dz} = m.e^{mz}$

$\Rightarrow \frac{d^2y}{dz^2} = m^2 e^{mz}$

$\Rightarrow m^2 e^{mz} + 2me^{mz} - 3e^{mz} = 0$

$\Rightarrow e^{mz}(m^2+2m-3) = 0$

$\therefore m^2+2m-3=0$

$\Rightarrow m^2+3m-m-3=0$

$\Rightarrow (m+3)(m-1)=0$

$\therefore m = -3, 1$

$\therefore$ C.F $= C_1 e^{-3z} + C_2 e^{z}$ where, C_1 & C_2 are constants.

$= C_1 e^{-3\log x} + C_2 e^{\log x}$

P.I $= \frac{1}{D_1^2+2D_1-3} e^z \sin z$

$= e^z \left[\frac{1}{(D_1+1)^2+2(D_1+1)-3} \sin z\right]$

$= e^z \left[\frac{1}{D_1^2+2D_1+1+2D_1+2-3} \sin z\right]$

$= e^z \left[\frac{1}{D_1^2+4D_1} \sin z\right]$

$= e^z \left[\frac{1}{-1+4D_1} \sin z\right]$

$= e^z \left[\frac{1}{4D_1-1} \sin z\right]$

$= e^z \left[\frac{4D_1+1}{16D_1^2-1} \sin z\right]$

$$= e^{z}\left[\frac{4D_1 + 1}{-16 - 1}\sin z\right]$$

$$= e^{z}\left[\frac{4\sin z\, D_1 + \sin z}{-17}\right]$$

$$= -\frac{e^{z}}{17}\left(+4\cos z + \sin z\right) = \frac{e^{\log x}}{-17}\left(\sin(\log x) + 4\cos(\log x)\right)$$

$$\cancel{\frac{e^{z}}{17}\left(2\cos z + \sin z\right) = \frac{e^{\log x}}{10}\left(2\cos(\log x) + \sin(\log x)\right)}$$

$\therefore$ The required general solution is —

$$y = C.F + P.I$$

$$= C_1 e^{-3\log x} + C_2 e^{\log x} + \cancel{\frac{e^{\log x}}{10}\left(2\cos(\log x) + \sin(\log x)\right)}$$

$$+ \frac{e^{\log x}}{-17}\left[\sin(\log x) + 4\cos(\log x)\right]$$

$$z = \log x \quad \text{—①}$$

$$\frac{dz}{dx} = \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dz} \times \frac{1}{x}$$

$$\Rightarrow \boxed{x\frac{dy}{dx} = \frac{dy}{dz}} \quad \text{—②}$$

$$\frac{d}{dx}\left[x\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{dy}{dz}\right] **$$

$$\Rightarrow x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{d}{dz}\left[\frac{dy}{dz}\right] \times \frac{dz}{dx} = \frac{d^2y}{dz^2} \times \frac{1}{x}$$

$$\cancel{\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \frac{d^2y}{dz^2}$$

$$\Rightarrow \boxed{x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz}}$$

$$P.I = \frac{1}{D^2+1}\, x\sin x \qquad \cos x + i\sin x = e^{ix}$$

Let, $X = \frac{1}{D^2+1}\, x\cos x$

$$Y = \frac{1}{D^2+1}\, x\sin x$$

$$X + iY = \frac{1}{D^2+1}\, x(\cos x + i\sin x)$$

$$= \frac{1}{D^2+1}\, x e^{ix}$$

$$= e^{ix}\frac{1}{(D+i)^2+1}\, x$$

$$= e^{ix}\frac{1}{D^2+2Di-1+1}\, x$$

$$= e^{ix}\frac{1}{D^2+2Di}\, x$$

$$= e^{ix}\frac{1}{2Di\left(1+\frac{D}{2i}\right)}\, x$$

$$= \frac{e^{ix}}{2iD}\left(1+\frac{D}{2i}\right)^{-1} x$$

$$= \frac{e^{ix}}{2iD}\left(1 - \frac{D}{2i} \cdots\cdots\right) x$$

$$= \frac{e^{ix}}{2iD}\left(x - \frac{1}{2i}\right)$$

$$= \frac{e^{ix}}{2i}\left(\frac{x^2}{2} - \frac{x}{2i}\right)$$

$$= -i\,\frac{e^{ix}}{2}\left(\frac{x^2}{2} + \frac{x}{2}i\right)$$

$$X + iY = \left(\frac{\cos x + i\sin x}{2}\right)\left(\frac{x}{2} - \frac{x^2}{2}i\right)$$

$$Y = \frac{x\sin x}{4} - \frac{x^2\cos x}{4} = P.I$$

31/5/22

1st order and higher degree differential equation :-

① solvable for p

② solvable for y : $y = f(x, p)$, $p = \frac{dy}{dx}$

③ solvable for x $\quad x = f(y, p)$

Solve ;

① $y = px + p^2$

② $y = px + \frac{1}{p}$

③ $y = px + \sqrt{p^2+1}$

④ $y = px + \sqrt{a^2p^2+b^2}$

$y^2 = 4ax$

parametric eq^n

$x = at^2$

$y = 2at$

t is a parameter

Sol^n

① $y = px + p^2$ —— ①

$$\Rightarrow \frac{dy}{dx} = \frac{dp}{dx}\cdot p + x\frac{dp}{dx} + 2p\frac{dp}{dx}$$

put $\frac{dy}{dx} = p$

$$\Rightarrow p = p + \frac{dp}{dx}(x+2p)$$

$$\Rightarrow \frac{dp}{dx}(x+2p) = 0$$

$$\therefore \frac{dp}{dx} = 0 \quad \text{or} \quad x + 2p = 0$$

$$\Rightarrow \int \frac{dp}{\cancel{dx}} = 0$$

$$\Rightarrow \boxed{p = C}$$

The general sol^n is $y = cx + c^2$

2nd part : $x + 2p = 0$

$\Rightarrow x = -2p$

The singular sol^n is $\left.\begin{array}{l} x = -2p \\ y = px + p^2 \end{array}\right\}$ Ans

where p is a parameter.

---

② $y = px + \frac{1}{p}$

$$\Rightarrow \frac{dy}{dx} = p + x\frac{dp}{dx} - \cancel{\frac{dp}{p}}\ \cancel{\log p \frac{dp}{dx}}\ \frac{1}{p^2}\frac{dp}{dx}$$

put $\frac{dy}{dx} = p$

$$\Rightarrow p = p + \frac{dp}{dx}\left(x - \frac{1}{p^2}\right)$$

$$\Rightarrow \frac{dp}{dx}\left(x - \frac{1}{p^2}\right) = 0$$

$$\therefore \frac{dp}{dx} = 0 \quad \text{or} \quad x - \frac{1}{p^2} = 0$$

$$\Rightarrow \int dp = c$$

$$\Rightarrow \boxed{p = c}$$

The general sol^n is $y = cx + \frac{1}{c}$

2nd part. $x - \frac{1}{p^2} = 0$

$\Rightarrow x = +\cancel{\log p}\ \frac{1}{p^2}$

The singular sol^n is $x = +\frac{1}{p^2}$; $y = px + \frac{1}{p}$ where p is a parameter. (Ans)

③ $y = px + \sqrt{p^2+1}$ ——— ①

$\Rightarrow \frac{dy}{dx} = p + x\frac{dp}{dx} + \frac{1}{\cancel{2}} \cdot \frac{1}{\sqrt{p^2+1}} \cdot \cancel{2}p \frac{dp}{dx}$

put $\frac{dy}{dx} = p$

$\Rightarrow p = p + x\frac{dp}{dx} + \frac{p}{\sqrt{p^2+1}}\frac{dp}{dx}$

$\Rightarrow \frac{dp}{dx}\left(x + \frac{p}{\sqrt{p^2+1}}\right) = 0$

$\therefore \frac{dp}{dx} = 0$ or $x + \frac{p}{\sqrt{p^2+1}} = 0$

$\Rightarrow \int dp = 0$

$\Rightarrow p = C$

The general solⁿ is $y = cx + \sqrt{c^2+1}$

2nd part: $x + \frac{p}{\sqrt{p^2+1}} = 0$

$\Rightarrow x = -\frac{p}{\sqrt{p^2+1}}$

The singular solution is : $x = -\frac{p}{\sqrt{p^2+1}}$
$y = px + \sqrt{p^2+1}$ } Ans

where p is a parameter.

④ $y = px + \sqrt{a^2p^2+b^2}$

$\Rightarrow \frac{dy}{dx} = p + x\frac{dp}{dx} + \frac{1}{2} \cdot \frac{1}{\sqrt{a^2p^2+b^2}} \cdot a^2 \cdot 2p\frac{dp}{dx}$

put $\frac{dy}{dx} = p$

1/6/22

Solvable for x :-

$x = f(y, p)$

1) Q) Solve, $x = py - p$

differentiating both side w.r.t y

$$\frac{dx}{dy} = p + y\frac{dp}{dy} - \frac{dp}{dy}$$

put $\frac{dy}{dx} = p$

$$\Rightarrow \frac{1}{p} = p + \frac{dp}{dy}(y-1)$$

$$\Rightarrow \frac{1}{p} - p = (y-1)\frac{dp}{dy}$$

$$\Rightarrow \frac{dy}{y-1} = \frac{dp}{\frac{1}{p} - p}$$

$$\Rightarrow \int \frac{dy}{y-1} = \int \frac{p\,dp}{1-p^2}$$

$1 - p^2 = z$

$\Rightarrow -2p\,dp = dz$

$\Rightarrow p\,dp = -\frac{dz}{2}$

$$\Rightarrow \log(y-1) = -\frac{1}{2}\int \frac{dz}{z}$$

$$\Rightarrow \log(y-1) = -\frac{1}{2}\log z + \log c$$

$$\Rightarrow y - 1 = \frac{c}{\sqrt{z}}$$

$$\Rightarrow y - 1 = \frac{c}{\sqrt{1-p^2}}$$

The general solution is $x = \cancel{px} = py - p$

$$y - 1 = \frac{c}{\sqrt{1-p^2}}$$

where p is a parameter, c is a constant

Ex: $x = 4p + 4p^3$

~~differentiati~~

1) $L[e^{-3t}\cos 2t]$

2) $L[t\cos 3t]$

3) $L[e^{-2t}t\sin t]$

4) $L[\int_0^t \cos 2t\,dt]$

5) $L[\frac{1}{s^2+s+6}]$

6) $L^{-1}[\frac{s}{s^2-s+6}]$

7) $L^{-1}[\frac{1}{(s+1)(s+4)}]$

8) $L^{-1}[\frac{1}{(s+3)(s^2+3)}]$

1) $L[e^{-3t}\cos 2t]$

Let, $f(t) = \cos 2t$

$\therefore L[f(t)] = L[\cos 2t]$

$$= \frac{s}{s^2+4} = F(s)$$

By using first shifting th,

$L[e^{-3t}\cos 2t] = L[e^{-3t}f(t)]$, $a = -3$

$$= F(s-a)$$

$$= F(s+3)$$

$$= \frac{s+3}{(s+3)^2+4} \quad \text{Ans}$$