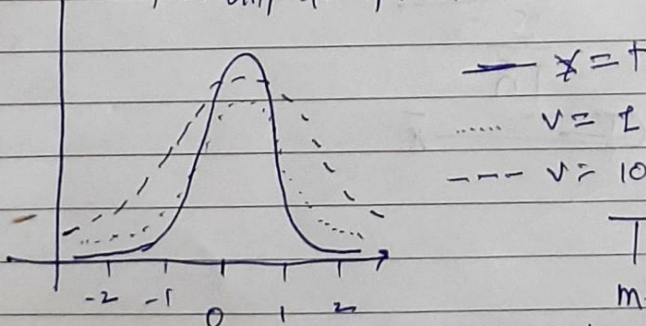
CI for Populatn standard deviatn unknown or Small Sample size

When we have large sample size we used sample std s as an estimate for σ (populatn std). However we donot get accurate results when sample size is small.

Student t distributn:-
If we take a sample of n observations from a normal distributn then t-distribution with $v = n-1$ degrees of freedom.
$(n-1)$ degrees of freedom because when we calculate sample std s, in numerator we have sum of n deviations. Sum of ~~deviations~~ is all n deviations is zero because sample was taken from a normal distributn & it is symetric abt mean. So sum of deviations is zero. So we just need $(n-1)$ deviations because once we know $(n-1)$ deviation terms we can calculate the last deviation as the sum of deviations is zero.
t distributn is symetric & bell shaped like normal.
However t distributn has heavier tails meaning it is more prone to producing values that fall far from mean.

Student t
Probablity density functn



for $v = \infty$ the t distributn is a normal distributn

T distributn has zero mean &

Exact shape depends on degrees of freedom
As degrees of freedom↑ graph of t distributn becomes like normal distributn

There is also a probability table to calculate t-values at various levels of confidence

As sample size increases student's t distributⁿ becomes more like a normal distributⁿ.
when sample size of 30 is reached normal distributⁿ is substituted for student's t bcoz they are so much alike.

student's t is a ratio of normal distributⁿ & chi squared distributⁿ. chi squared dist itself is a ratio of 2 variances the sample variance & the unknown populatⁿ variance.

$$t = \frac{z}{\sqrt{\frac{\chi^2}{\nu}}}$$

z = standard Normal Distributⁿ
χ^2 = chi-squared distributⁿ with ν degrees of freedom

$$t = \frac{\left(\frac{\bar{x}-\mu}{\sigma}\right)}{\sqrt{\frac{\frac{s^2}{(n-1)}}{\frac{\sigma^2}{(n-1)}}}}$$

$$\therefore t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

~~Now CI~~ $\bar{x}$

Given- sample of 10 observations of children ages in some area.
We want to ~~estimate~~ give ~~estimate of for~~ CI for populatⁿ avg age of children in that area with 95% confidence.

As $n=10$ we cannot use Normal distributⁿ so here we have to use student t distributⁿ bcoz it has heavier tails ~~& bcoz sample is whe very small~~ ~~the avg~~

We will see the student's t table to get t value
we will look at (9, 95%) or (9, 0.05)
9: degrees of freedom
$0.05 = \alpha$

Now we will calculate CF like before
$\bar{x}$ = sample mean
s = sample std
n = sample size = 10

$$\left(\bar{x} - t \times \frac{s}{\sqrt{n}} \; , \; \bar{x} + \frac{t \times s}{\sqrt{n}}\right)$$

The calculatⁿ is same, just to calculate the t value we have to use a ~~different~~ distributⁿ based on sample size.
If sample size > 30 use Normal Distributⁿ
else use student's t "