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(1) Consider a random sample $(x_1, x_2, x_3 \cdots x_n)$ $\mu = \theta_1$ (mean) and $\sigma^2 = \theta_2$ (variance)

Likelihood function $L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \exp\left(-\frac{(x_i - \theta_i)^2}{2\theta_2}\right)$

To maximise take log on both sides

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[\frac{-1}{2} \ln(2\pi\theta_2) - \frac{(x_i - \theta_i)^2}{2\theta_2} \right]$$

(i) Differentiate w.r.t θ_1 (for θ_1)

$$\frac{d}{d\theta} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$X_i - n\theta_1 = 0$$

$$\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n} \quad \text{[mean]}$$

(ii) Differentiate w.r.t θ_2 (for θ_2)

$$\frac{d}{d\theta} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[\frac{-1}{2\theta_2} + \frac{(x_i - \theta_1)^2}{2\theta_2^2} \right] = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 \quad \text{[Variance]}$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

Binomial distribution $B(n, \theta)$ $p = \theta$, $q = 1-\theta$

pmf (probability mass function)

$$f(x; n, \theta) = {}^nC_x \, \theta^x (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^nC_{x_i} \, \theta^{x_i} (1-\theta)^{n-x_i}$$

Taking log on both sides

$$\ln L(\theta) = \sum_{i=1}^{n} \left[\ln {}^nC_{x_i} + x_i \ln\theta + (n-x_i) \ln(1-\theta) \right]$$

differentiate w.r.t θ

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right] = 0$$

Find θ

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right] = 0$$

$$\sum_{i=1}^{n} \left[\frac{(1-\theta)x_i - \theta(n-x_i)}{\theta(1-\theta)} \right] = 0$$

$$\sum_{i=1}^{n} (1-\theta)\, x_i - (n-x_i)\,\theta = 0$$

$$\theta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i \cdot m$$

$$\boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{n \cdot m}} \rightarrow \text{MLE}$$