# AI 1103 - Assignment 1

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### *Question*

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X ? Find the

1) mean of X
2) variance of X
3) standard deviation of X.

### *Solution*

There are a total of 15 students in the class, each equally likely to be selected.

Hence, we could say that the probability of each student to be chosen is $\frac{1}{15}$.

Number of students of age **14** = *2*
Number of students of age **15** = *1*
Number of students of age **16** = *2*
Number of students of age **17** = *3*
Number of students of age **18** = *1*
Number of students of age **19** = *2*
Number of students of age **20** = *3*
Number of students of age **21** = *1*

We are assigning

**X=0** for the case when a student of age **14** is picked,
**X=1** for the case when a student of age **15** is picked,
**X=2** for the case when a student of age **16** is picked,
**X=3** for the case when a student of age **17** is picked,
**X=4** for the case when a student of age **18** is picked,
**X=5** for the case when a student of age **19** is picked,

**X=6** for the case when a student of age **20** is picked,
**X=7** for the case when a student of age **21** is picked,
Since we know that probability of an outcome to happen is **P(outcome) = $\frac{\text{No. of favourable outcomes}}{\text{Total No. of outcomes}}$**
Therefore,

$$P(\textbf{X=0}) = \frac{2}{15} = 0.13333334$$

$$P(\textbf{X=1}) = \frac{1}{15} = 0.06666667$$

$$P(\textbf{X=2}) = \frac{2}{15} = 0.13333334$$

$$P(\textbf{X=3}) = \frac{3}{15} = 0.20000000$$

$$P(\textbf{X=4}) = \frac{1}{15} = 0.06666667$$

$$P(\textbf{X=5}) = \frac{2}{15} = 0.13333334$$

$$P(\textbf{X=6}) = \frac{3}{15} = 0.20000000$$

$$P(\textbf{X=7}) = \frac{1}{15} = 0.06666667$$

Therefore, the p

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| P(X) | $\frac{2}{15}$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{3}{15}$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{3}{15}$ | $\frac{1}{15}$ |

1) The mean E(X) of the distribution is given by
   $$E(X) = \Sigma_{i=1}^{n} \; x_i \; P_i$$

$$= 14 * \tfrac{2}{15} + 15 * \tfrac{1}{15} + 16 * \tfrac{2}{15} + 17 * \tfrac{3}{15} + 18 * \tfrac{1}{15} + 19 * \tfrac{2}{15} + 20 * \tfrac{3}{15} + 21 * \tfrac{1}{15}$$

$$= \frac{28+15+32+51+18+38+60+21}{15}$$

$$= \frac{263}{15}$$

Therefore **E(X) = 17.53**

2) The variance of X is given by **Var(X) = E($X^2$) - $[E(X)]^2$.**

Here $E(X^2)$ is given by

$$E(X^2) = \Sigma_{i=1}^{n} \; x_i^2 \; P_i$$

$$= \; 14^2 * \tfrac{2}{15} + 15^2 * \tfrac{1}{15} + 16^2 * \tfrac{2}{15} + 17^2 * \tfrac{3}{15} + 18^2 * \tfrac{1}{15} + 19^2 * \tfrac{2}{15} + 20^2 * \tfrac{3}{15} + 21^2 * \tfrac{1}{15}$$

$$= \frac{392+225+512+867+324+722+1200+441}{15}$$

$$= \frac{4683}{15}$$

Hence, Var (X) $= \frac{4683}{15} - (\frac{263}{15})^2$

Var (X) $= \frac{4683}{15} - \frac{69169}{225}$

Therefore **Var (X) = 4.78**

3) Standard deviation is given by $\sigma_x = \sqrt{Var(X)}$

$\sigma_x = \sqrt{4.78}$

Hence, **Standard Deviation,** $\sigma_x$ **= 2.18**

Drawing the comparison graph with ages on x-axis, probabilities on y-axis, blue bar representing simulations and orange bar representing theoretical value, we get



Comparison plot between simulated and