AI 1103 - Assignment 1

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Question

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X ? Find the mean, variance and standard deviation of X.

Solution

There are a total of 15 students in the class, each equally likely to be selected.Hence, we could say that the probability of each student to be chosen is $\frac{1}{15}$.

Number of students of age 14 = *2*
Number of students of age 15 = *1*
Number of students of age 16 = *2*
Number of students of age 17 = *3*
Number of students of age 18 = *1*
Number of students of age 19 = *2*
Number of students of age 20 = *3*
Number of students of age 21 = *1*

We are assigning
X=0 for the case when a student of age 14 is picked,
X=1 for the case when a student of age 15 is picked,
X=2 for the case when a student of age 16 is picked,
X=3 for the case when a student of age 17 is picked,
X=4 for the case when a student of age 18 is picked,
X=5 for the case when a student of age 19 is picked,
X=6 for the case when a student of age 20 is picked,
X=7 for the case when a student of age 21 is picked,

Since we know that probability of an outcome to happen is P(outcome) = $\frac{\text{No. of favourable outcomes}}{\text{Total No. of outcomes}}$

The probabilities of choosing are :

$$\Pr(X = 0) = \frac{2}{15} = 0.13333334$$

$$\Pr(X = 1) = \frac{1}{15} = 0.06666667$$

$$\Pr(X = 2) = \frac{2}{15} = 0.13333334$$

$$\Pr(X = 3) = \frac{3}{15} = 0.20000000$$

$$\Pr(X = 4) = \frac{1}{15} = 0.06666667$$

$$\Pr(X = 5) = \frac{2}{15} = 0.13333334$$

$$\Pr(X = 6) = \frac{3}{15} = 0.20000000$$

$$\Pr(X = 7) = \frac{1}{15} = 0.06666667$$

X	0	1	2	3	4	5	6	7
No. of students	2	1	2	3	1	2	3	1
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

The mean E(X) of the distribution is given by

$$E(X) = \sum_{i=1}^{n} x_i \Pr(X = i)$$

$= 14 \times \frac{2}{15} + 15 \times \frac{1}{15} + 16 \times \frac{2}{15} + 17 \times \frac{3}{15} + 18 \times \frac{1}{15} + 19 \times \frac{2}{15} + 20 \times \frac{3}{15} + 21 \times \frac{1}{15} = \frac{28+15+32+51+18+38+60+21}{15} = \frac{263}{15} \quad \therefore \quad E(X) = 17.53$

The variance of X is given by Var(X) = $E(X^2) - [E(X)]^2$.

Here, $E(X^2) = \sum_{i=1}^{n} x_i^2 \Pr(X = i)$

$= 14^2 \times \frac{2}{15} + 15^2 \times \frac{1}{15} + 16^2 \times \frac{2}{15} + 17^2 \times \frac{3}{15} + 18^2 \times \frac{1}{15} + 19^2 \times \frac{2}{15} + 20^2 \times \frac{3}{15} + 21^2 \times \frac{1}{15} = \frac{392+225+512+867+324+722+1200+441}{15} = \frac{4683}{15}$

$$Hence, Var(X) = \frac{4683}{15} - \left(\frac{263}{15}\right)^2 = \frac{4683}{15} - \frac{69169}{225}$$

$$\therefore Var(X) = 4.78$$

Standard deviation is given by $\sigma_x = \sqrt{Var(X)}$

$\sigma_x = \sqrt{4.78}$

$\therefore Standard Deviation, \sigma_x = 2.18$

Drawing the comparison graph with ages on x-axis, probabilities on y-axis, blue bar representing simulations and orange bar representing theoretical value, we get

Comparison plot between simulated and theoritical estimations.