

# AI 1103 - Assignment 1

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Download all python codes from

[https://github.com/rohanthota/Assignment\\_1/codes/Assignment\\_1.py](https://github.com/rohanthota/Assignment_1/codes/Assignment_1.py)

and latex codes from

[https://github.com/rohanthota/Assignment\\_1/Assignment\\_1.tex](https://github.com/rohanthota/Assignment_1/Assignment_1.tex)

## Question

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age  $X$  of the selected student is recorded. What is the probability distribution of the random variable  $X$ ? Find the mean, variance and standard deviation of  $X$ .

## Solution

There are a total of 15 students in the class, each equally likely to be selected. Hence, we could say that the probability of each student to be chosen is  $\frac{1}{15}$ . Number of students of age **14** = 2

Number of students of age **15** = 1

Number of students of age **16** = 2

Number of students of age **17** = 3

Number of students of age **18** = 1

Number of students of age **19** = 2

Number of students of age **20** = 3

Number of students of age **21** = 1 We are assigning  **$X=0$**  for the case when a student of age **14** is picked,  **$X=1$**  for the case when a student of age **15** is picked,  **$X=2$**  for the case when a student of age **16** is picked,  **$X=3$**  for the case when a student of age **17** is picked,  **$X=4$**  for the case when a student of age **18** is picked,  **$X=5$**  for the case when a student of age **19** is picked,  **$X=6$**  for the case when a student of age **20** is picked,  **$X=7$**  for the case when a student of age **21** is picked, Since we know that probability of an outcome

to happen is  **$P(\text{outcome}) = \frac{\text{No. of favourable outcomes}}{\text{Total No. of outcomes}}$**   
Therefore,

$$P(X=0) = \frac{2}{15} = 0.13333334$$

$$P(X=1) = \frac{1}{15} = 0.06666667$$

$$P(X=2) = \frac{2}{15} = 0.13333334$$

$$P(X=3) = \frac{3}{15} = 0.20000000$$

$$P(X=4) = \frac{1}{15} = 0.06666667$$

$$P(X=5) = \frac{2}{15} = 0.13333334$$

$$P(X=6) = \frac{3}{15} = 0.20000000$$

$$P(X=7) = \frac{1}{15} = 0.06666667$$

Therefore, the p

<b>X</b>	0	1	2	3	4	5	6	7
<b>No. of students</b>	2	1	2	3	1	2	3	1
<b>P(X)</b>	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

1) The mean  $E(X)$  of the distribution is given by

$$E(X) = \sum_{i=1}^n x_i P_i$$

$$= 14 \times \frac{2}{15} + 15 \times \frac{1}{15} + 16 \times \frac{2}{15} + 17 \times \frac{3}{15} + 18 \times \frac{1}{15} + 19 \times \frac{2}{15} + 20 \times \frac{3}{15} + 21 \times \frac{1}{15}$$

$$= \frac{28+15+32+51+18+38+60+21}{15}$$

$$= \frac{263}{15}$$

Therefore  **$E(X) = 17.53$**

2) The variance of  $X$  is given by  **$\text{Var}(X) = E(X^2) - [E(X)]^2$** .

Here  $E(X^2)$  is given by

$$E(X^2) = \sum_{i=1}^n x_i^2 P_i$$

$$= 14^2 \times \frac{2}{15} + 15^2 \times \frac{1}{15} + 16^2 \times \frac{2}{15} + 17^2 \times \frac{3}{15} + 18^2 \times \frac{1}{15} + 19^2 \times \frac{2}{15} + 20^2 \times \frac{3}{15} + 21^2 \times \frac{1}{15}$$

$$= \frac{392+225+512+867+324+722+1200+441}{15}$$

$$= \frac{4683}{15}$$

$$\text{Hence, Var (X)} = \frac{4683}{15} - \left(\frac{263}{15}\right)^2$$

$$\text{Var (X)} = \frac{4683}{15} - \frac{69169}{225}$$

Therefore **Var (X) = 4.78**

3) Standard deviation is given by  $\sigma_x = \sqrt{\text{Var}(X)}$

$$\sigma_x = \sqrt{4.78}$$

Hence, **Standard Deviation,  $\sigma_x = 2.18$**

Drawing the comparison graph with ages on x-axis, probabilities on y-axis, blue bar representing simulations and orange bar representing theoretical value, we get

