

AI 1103 - Assignment 1

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Download all python codes from

https://github.com/rohanthota/Assignment_1/codes/Assignment_1.py

and latex codes from

https://github.com/rohanthota/Assignment_1/Assignment_1.tex

Question

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X ? Find the mean, variance and standard deviation of X .

Solution

There are a total of 15 students in the class, each equally likely to be selected. Hence, we could say that the probability of each student to be chosen is $\frac{1}{15}$.

Number of students of age **14** = 2

Number of students of age **15** = 1

Number of students of age **16** = 2

Number of students of age **17** = 3

Number of students of age **18** = 1

Number of students of age **19** = 2

Number of students of age **20** = 3

Number of students of age **21** = 1 We are assigning **$X=0$** for the case when a student of age **14** is picked, **$X=1$** for the case when a student of age **15** is picked, **$X=2$** for the case when a student of age **16** is picked, **$X=3$** for the case when a student of age **17** is picked, **$X=4$** for the case when a student of age **18** is picked, **$X=5$** for the case when a student of age **19** is picked, **$X=6$** for the case when a student of age **20** is picked, **$X=7$** for the case when a student of age **21** is picked,

Since we know that probability of an outcome to happen is $P(\text{outcome}) =$

$$\frac{\text{No. of favourable outcomes}}{\text{Total No. of outcomes}}$$

Therefore,

$$\Pr(X = 0) = \frac{2}{15} = 0.13333334$$

$$\Pr(X = 1) = \frac{1}{15} = 0.06666667$$

$$\Pr(X = 2) = \frac{2}{15} = 0.13333334$$

$$\Pr(X = 3) = \frac{3}{15} = 0.20000000$$

$$\Pr(X = 4) = \frac{1}{15} = 0.06666667$$

$$\Pr(X = 5) = \frac{2}{15} = 0.13333334$$

$$\Pr(X = 6) = \frac{3}{15} = 0.20000000$$

$$\Pr(X = 7) = \frac{1}{15} = 0.06666667$$

Therefore, the p

X	0	1	2	3	4	5	6	7
No. of students	2	1	2	3	1	2	3	1
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

1) The mean $E(X)$ of the distribution is given by

$$E(X) = \sum_{i=1}^n x_i \Pr(X = i)$$

$$= 14 \times \frac{2}{15} + 15 \times \frac{1}{15} + 16 \times \frac{2}{15} + 17 \times \frac{3}{15} + 18 \times \frac{1}{15} + 19 \times \frac{2}{15} + 20 \times \frac{3}{15} + 21 \times \frac{1}{15}$$

$$= \frac{28+15+32+51+18+38+60+21}{15}$$

$$= \frac{263}{15}$$

Therefore **$E(X) = 17.53$**

- 2) The variance of X is given by $\text{Var}(X) = E(X^2) - [E(X)]^2$.

Here, $E(X^2) = \sum_{i=1}^n x_i^2 \Pr(X = i)$

$$= 14^2 \times \frac{2}{15} + 15^2 \times \frac{1}{15} + 16^2 \times \frac{2}{15} + 17^2 \times \frac{3}{15} + 18^2 \times \frac{1}{15} + 19^2 \times \frac{2}{15} + 20^2 \times \frac{3}{15} + 21^2 \times \frac{1}{15}$$

$$= \frac{392+225+512+867+324+722+1200+441}{15}$$

$$= \frac{4683}{15}$$

$$\text{Hence, } \text{Var}(X) = \frac{4683}{15} - \left(\frac{263}{15}\right)^2$$

$$\text{Var}(X) = \frac{4683}{15} - \frac{69169}{225}$$

Therefore **Var (X) = 4.78**

$$\sigma_x = \sqrt{4.78}$$

Hence, **Standard Deviation, $\sigma_x = 2.18$**

Drawing the comparison graph with ages on x-axis, probabilities on y-axis, blue bar representing simulations and orange bar representing theoretical value, we get

